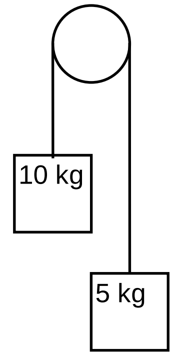
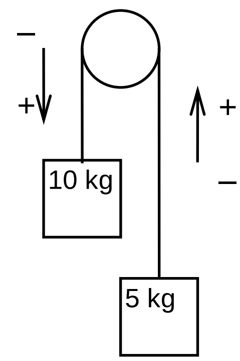
# Physics 20   <u>Lesson 18</u>  Pulleys and Systems

## I. Pulley and system problems

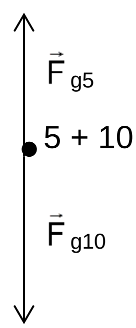



In this lesson we learn about dynamics problems that involve several masses that are connected and accelerating together. Using the pulley system illustrated to the right below as an example, the **basic method** for solving problems involving a system of masses is discussed. As in Lessons 15, 16 and 17, the basic method is to draw a free body diagram of the forces involved, write an expression for the net force, and then solve for the acceleration.




In a pulley system two masses are strung over a pulley. Note that downward acceleration for one of the masses results in upward acceleration for the other mass. Therefore, when we **assign directions** to the forces acting on the masses we have to keep in mind that downward motion on one side of the pulley is equivalent to upward motion on the other side of the pulley. Most people choose (+) to be the direction that the system will accelerate. In the example, therefore, down on the 10 kg side is (+) and down on the 5 kg side is (–).

To calculate the acceleration of the masses, we note that the masses will accelerate at the same rate together. If the masses accelerated at different rates, the string or rope between the masses would snap and there would no longer be a system of masses. Therefore, to calculate the **acceleration** we first draw a free body diagram for **the system** as a whole. From the free body diagram we can write an expression for the net force:



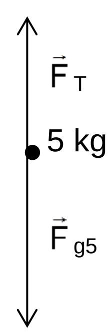

$$\vec{F}_{NET} = \vec{F}_{g10} + \vec{F}_{g5}$$

We then use Newton's 2nd law to calculate the acceleration of the system

$$\vec{a} = \frac{\vec{F}_{NET}}{m}$$

$$\vec{a} = \frac{\vec{F}_{g10} + \vec{F}_{g5}}{m_{10} + m_5}$$

If you are asked to find the **tension** in the rope or string, select **one** of the masses and draw the free body diagram for the forces acting on that mass alone. (In our example I selected the 5 kg mass.) Note that the forces acting **directly** on the mass are $F_T$ and $F_{g5}$. $F_{g10}$ only acts on the 5 kg mass via the string or rope. From the free body diagram write the expression for $F_{NET}$.




$$\vec{F}_{NET\,5} = \vec{F}_T + \vec{F}_{g5}$$

Since we have calculated the acceleration for the system we can calculate $F_{NET\,5}$

$$\vec{F}_{NET\,5} = m_5\vec{a}$$

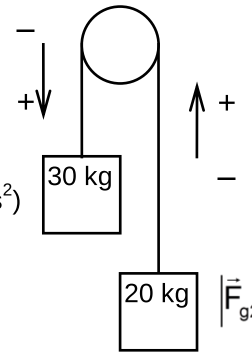
and then solve for the tension.

$$\vec{F}_{NET\,5} - \vec{F}_{g5} = \vec{F}_T$$

*Example 1*

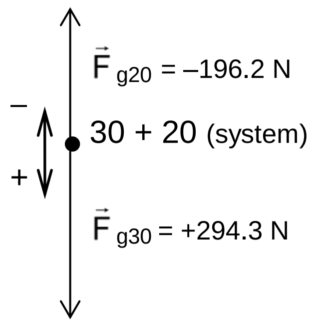
A 30 kg mass is connected over a pulley with a 20 kg mass. What is the resulting acceleration when the masses are released? What is the tension in the rope?




$|\vec{F}_{g30}| = 30\text{ kg }(9.81\text{ m/s}^2)$
$= 294.3\text{ N}$

$|\vec{F}_{g20}| = 20\text{ kg }(9.81\text{ m/s}^2)$
$= 196.2\text{ N}$

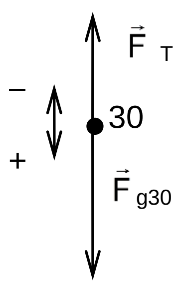
To calculate **acceleration** we draw the free body diagram for the **system**, write the net force relationship and then use Newton's 2nd law.




$$\vec{F}_{NET} = \vec{F}_{g30} + \vec{F}_{g20}$$
$$\vec{F}_{NET} = (+294.3\text{N}) + (-196.2\text{N})$$
$$\vec{F}_{NET} = +98.1\text{N}$$

$$\vec{a} = \frac{\vec{F}_{NET}}{m}$$
$$\vec{a} = \frac{+98.1\text{N}}{30\text{kg} + 20\text{kg}}$$
$$\boxed{\vec{a} = +1.96\,\text{m/s}^2}$$

To calculate the **tension** force we choose one of the masses, draw the free body diagram for the mass, write the net force relationship, calculate the net force, and then calculate the tension force. I chose the 30 kg mass.

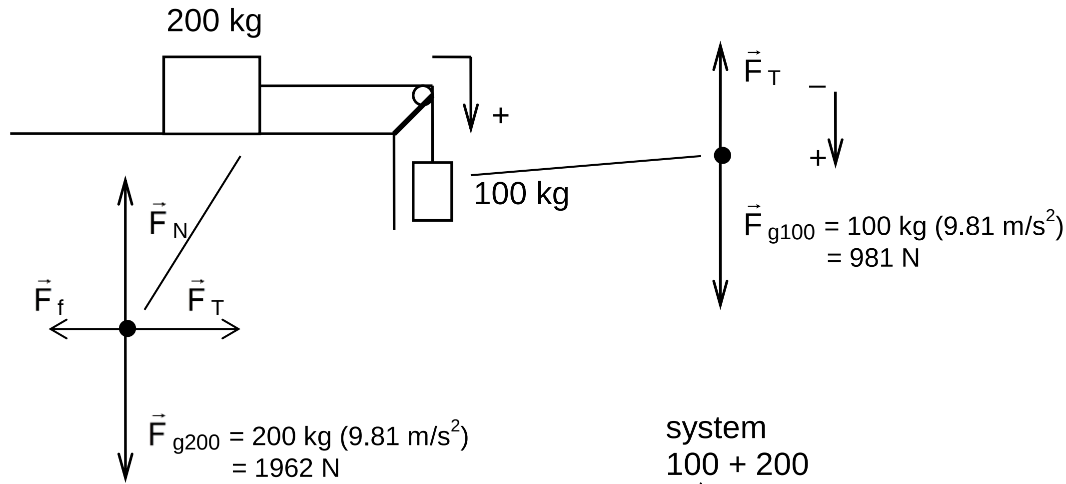


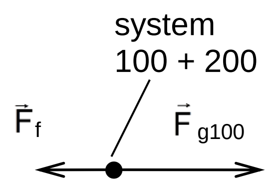

$$\vec{F}_{NET30} = \vec{F}_T + \vec{F}_{g30}$$
$$\vec{F}_T = \vec{F}_{NET30} - \vec{F}_{g30}$$
$$\vec{F}_T = (+58.9\text{N}) - (+294.3\text{N})$$
$$\vec{F}_T = -235\text{N}$$
$$\boxed{|\vec{F}_T| = 235\text{N}}$$

$$\vec{F}_{NET30} = m\vec{a}$$
$$\vec{F}_{NET30} = 30\text{kg}(+1.96\,\text{m/s}^2)$$
$$\vec{F}_{NET30} = +58.9\text{N}$$

*Example 2*

A 200 kg mass rests on a surface which has a coefficient of friction of 0.25. It is connected to a 100 kg mass over a pulley as shown in the diagram below. When the masses are released what is the resulting tension in the rope?




$$\left|\vec{F}_f\right| = \mu\left|\vec{F}_N\right| \longleftarrow \left|\vec{F}_N\right| = \left|\vec{F}_{g200}\right|$$

$$\left|\vec{F}_f\right| = 0.25(1962\text{N})$$

$$\left|\vec{F}_f\right| = 490.5\text{N}$$

From the free body diagram for the system note that the force acting against $F_{g100}$ is $F_f$ on the table

$$\vec{F}_{NET} = \vec{F}_{g100} + \vec{F}_f$$

$$\vec{F}_{NET} = (+981\text{N}) + (-490.5\text{N})$$

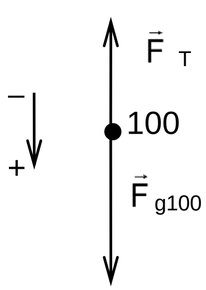
$$\vec{F}_{NET} = +490.5\text{N}$$

$$\vec{a} = \frac{\vec{F}_{NET}}{m}$$

$$\vec{a} = \frac{+490.5\text{N}}{100\text{kg} + 200\text{kg}}$$

$$\vec{a} = +1.635\,{}^{m}\!/_{s^2}$$

Calculate tension. (I chose the 100 kg mass).




$$\vec{F}_{NET100} = \vec{F}_T + \vec{F}_{g100}$$

$$\vec{F}_T = \vec{F}_{NET100} - \vec{F}_{g100}$$

$$\vec{F}_T = (+163.5\text{N}) - (+981\text{N})$$

$$\vec{F}_T = -817.5\text{N}$$

$$\boxed{\left|\vec{F}_T\right| = 818\text{N}}$$

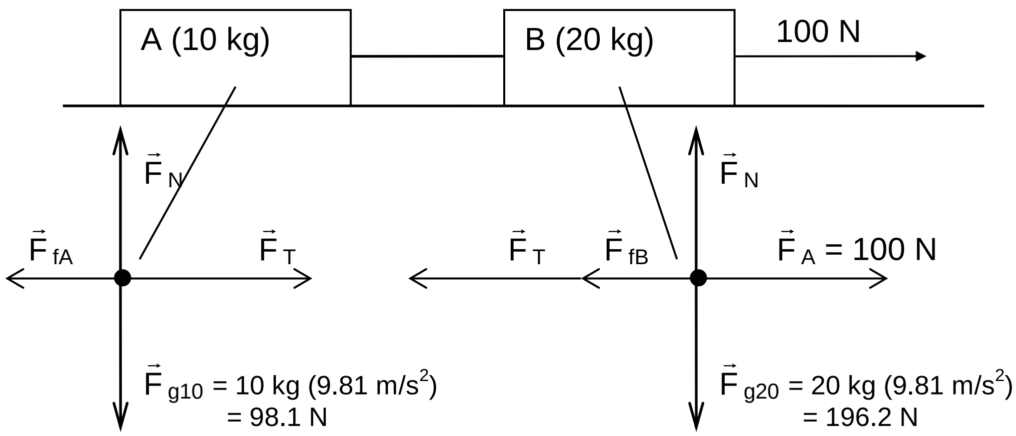
$$\vec{F}_{NET100} = m\vec{a}$$

$$\vec{F}_{NET100} = 100\text{kg}(+1.635\,{}^{m}\!/_{s^2})$$

$$\vec{F}_{NET100} = +163.5\text{N}$$

*Example 3*

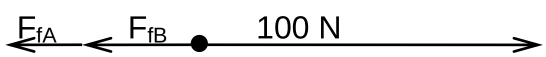
For the system below, find the acceleration of the masses if the coefficient if friction is 0.12.




$F_f = \mu F_N = \mu F_{g10}$
$F_f = 0.12(98.1\text{ N})$
$F_f = 11.8\text{ N}$

$F_f = \mu F_N = \mu F_{g20}$
$F_f = 0.12(196\text{ N})$
$F_f = 23.5\text{ N}$

the free body diagram for the system is



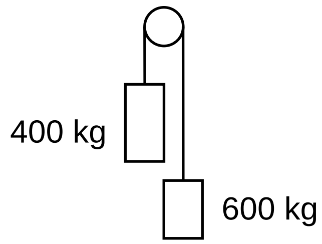

$\vec{F}_{NET} = \vec{F}_{fA} + \vec{F}_{fB} + \vec{F}_A$

$\vec{F}_{NET} = -11.8\text{ N} + -23.5\text{ N} + 100\text{ N} = +64.7\text{ N}$

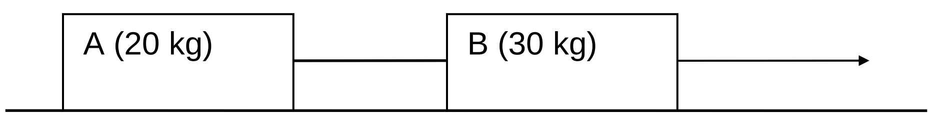
$\vec{a} = \frac{\vec{F}_{NET}}{m} = \frac{+64.7\text{ N}}{30\text{ kg}}$ = $\mathbf{+2.16\ m/s^2}$

## II. Practice problems

1. A 600 kg mass is connected over a pulley to a 400 kg mass. What is the resulting acceleration when the masses are released? What is the tension in the rope? (1.96 m/s$^2$, 4709 N)

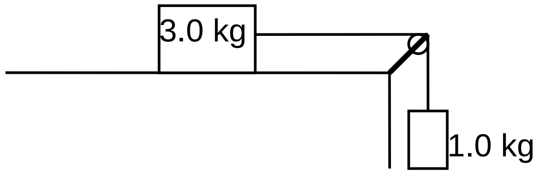



2. A 150 N force is applied to a system of masses. If mass A has a 50 N frictional force and mass B has a 70 N frictional force, find the <u>acceleration</u> of the system. (0.60 m/s$^2$)



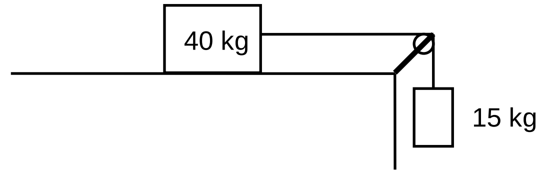

  

3. What is the acceleration and tension in the rope for the pulley system shown if the coefficient of friction is 0.23? (0.76 m/s$^2$, 9.05 N)

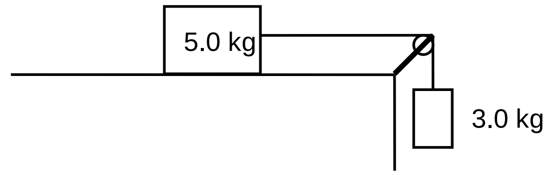


# III. Hand-in assignment

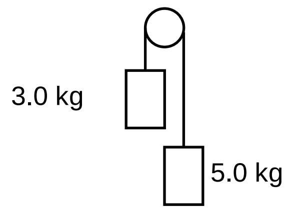
1. A 40 kg block on a level, frictionless table is connected to a 15 kg mass by a rope passing over a frictionless pulley. What will be the acceleration of the 15 kg mass when it is released? (2.7 $m/s^2$)



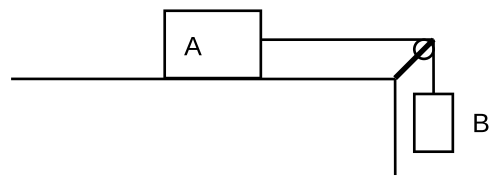

2. A 5.0 kg mass rests on a level frictionless table and is attached to a 3.0 kg mass by a light string that passes over a frictionless pulley. Calculate the tension in the string when the masses are released. (18 N)



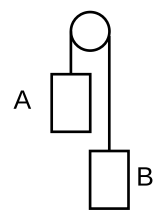

3. A 3.0 kg mass is attached to a 5.0 kg mass by a strong string that passes over a frictionless pulley. When the masses are released what will be (a) the acceleration of the masses, and (b) the magnitude of the tension in the string? (2.5 $m/s^2$, 37 N)




4. In the diagram, mass A is 100 kg and mass B is 25 kg and $\mu = 0.20$. What is the acceleration of mass A and what is the tension in the rope? (0.392 $m/s^2$, 235.4 N)

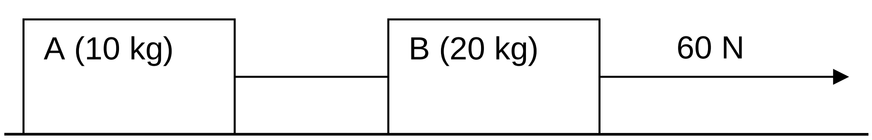



5. In the diagram, mass A is 5.0 kg and mass B is 7.0 kg. What is the acceleration of the 5.0 kg mass and what is the tension in the rope? (1.64 $m/s^2$, 57.2 N)



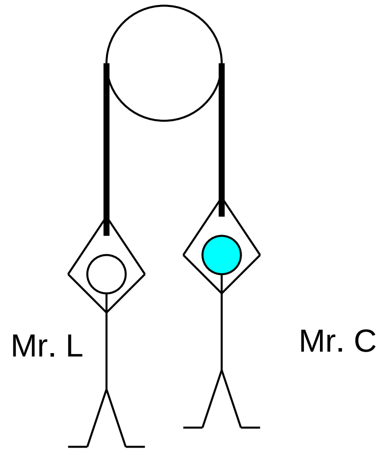

6. Two girls, one of mass 40 kg and the other of mass 60 kg, are standing side by side in the middle of a frozen pond. One pushes the other with a force of 360 N for 0.10 s. The ice is essentially frictionless.
   (a) What is each girl's acceleration? (9.0 $m/s^2$, -6.0 $m/s^2$)
   (b) What velocity will each girl acquire in the 0.10 s that the force is acting? (0.90 m/s, -0.60 m/s)
   (c) How far will each girl move during the same time period? (4.5 cm, -3.0 cm)

7. Find the acceleration of masses A and B if the coefficient of friction is 0.15. (0.53 $m/s^2$)

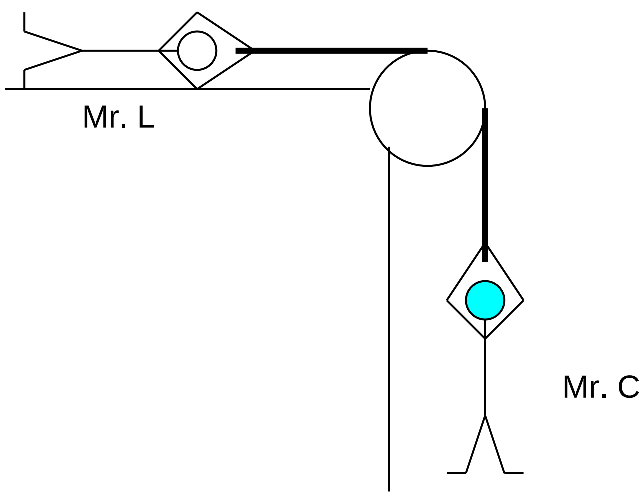


8. Mr. “Iron man” Licht, weight = 883 N, and Mr. “The man” Christison, weight = 932 N, enjoy doing physics experiments and often find themselves in odd situations. What is the resulting acceleration in each of the following situations?

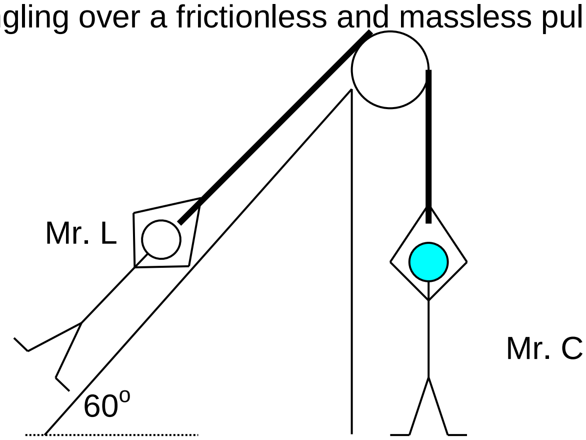
a. The first situation is where both Mr. L and Mr. C find themselves dangling over a frictionless and weightless pulley system.




b. The second situation is where Mr. L is on the ground ($\mu$ = .90) at the top of a cliff and Mr. C is dangling over a frictionless and massless pulley system over the cliff. (What should Mr. L do?)

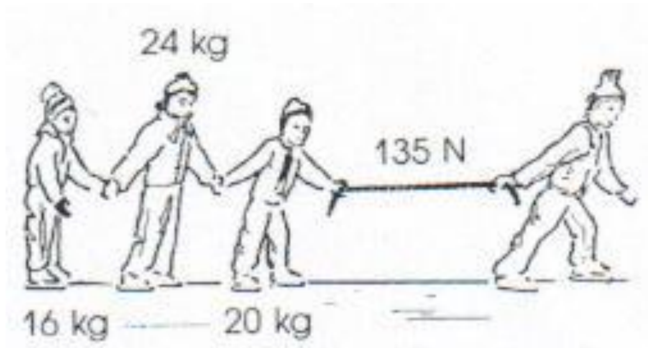



c. The final situation is where Mr. L is on a 60° incline ($\mu$ = .40) and Mr. C is dangling over a frictionless and massless pulley.



# IV. Bonus questions

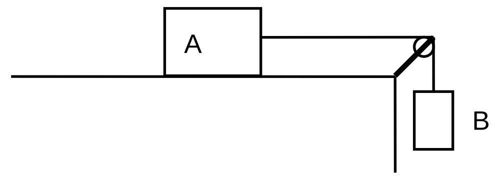
1. Three small children of mass 20.0 kg, 24.0 kg, and 16.0 kg, respectively, hold hands, as shown, and are pulled across a smooth frozen pond by a larger boy on skates, who pulls a horizontal rope being held by the first child. The skater pulls on the rope with a force of 135 N. Calculate each of the following.



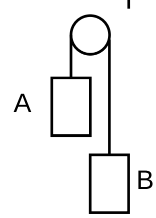

   (a) the acceleration of the skater (2.25 m/s$^2$)

   (b) the force with which each pair of children must hold hands, to ensure that the chain is not broken (90.0 N, 36.0 N)

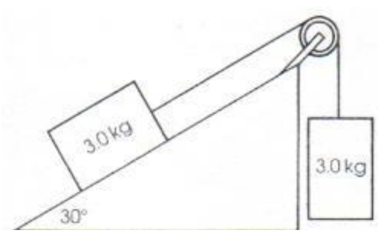
2. In the diagram, mass A is 170 kg. If the acceleration of the system is 2.5 m/s$^2$ towards B, what is the mass of B? (58.1 kg)



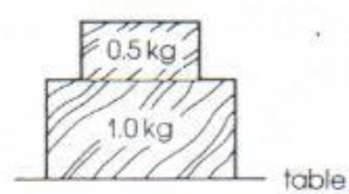

3. In the diagram, mass A is 5.0 kg. If the system accelerates toward B at 4.0 m/s$^2$, what is mass B? In addition, what is the tension in the rope? (11.9 kg, 69 N)




4. If the coefficient of friction is 0.20, what is the acceleration of the system? (1.6 m/s$^2$)




5. A 0.5 kg wooden block is placed on top of a 1.0 kg wooden block. The coefficient of static friction between the two blocks is 0.35. The coefficient of kinetic friction between the lower block and the level table is 0.20. What is the maximum horizontal force that can be applied to the lower block without the upper block slipping? (8.1 N)




6. A skier skiing downhill reaches the bottom of a hollow with a velocity of 20 m/s, and then coasts up a hill with a 10$^o$ slope. If the coefficient of kinetic friction is 0.10, how far up the slope will she travel before she stops? (75 m)