Physics 307 Laboratory

Experiment 5: Attenuation of γ-rays in Matter

Motivation:

In this experiment we will make a precision measurement of the attenuation coefficient for 662 keV γ-rays in aluminum. The error analysis is an <u>essential</u> part of this experiment. You should propagate the statistical uncertainty on your raw measurements through your calculations to a statistical uncertainty on the attenuation coefficient. Some discussion of possible systematic errors is also necessary.

References:

"Experiments in Modern Physics" by Melissinos 5.1, 5.2.1, 5.2.5, and 5.4.3.
"Foundations of Modern Physics" by Tipler 3.4, 3.5, and 4.2
"Data Reduction and Error Analysis..." by Bevington Ch. 4, 5.1, 5.2, and 6

Theory:

The purpose of shielding is to attenuate a beam of radiation. If the radiation consists of γ-rays, no gradual energy loss occurs, but there is a finite probability (proportional to the cross-section) for an interaction. Interactions (electromagnetic) of a gamma-ray beam with matter are the photoelectric effect, or Compton scattering, or pair production, depending on the energy of the y-rays. As explained in detail in Chapter 5 of Melissinos, through a series of such processes a fraction of the y-ray beam becomes completely absorbed in the material used for shielding. Since the interaction probability is proportional to the amount of material present, we have

$$\frac{dI}{dx} = KI$$

$$\frac{dI}{I} = -Kdx$$

hence $\quad I = I_0 e^{-Kx}$

where x is the length (of the shield), and $K = \dfrac{1}{L} = \dfrac{\sigma_t \rho N_0}{A}$, the attenuation coefficient.

L = the attenuation length,

N_0 = Avogadro's number,

ρ = density of the material,

σ_t = total cross section,

A = molecular or atomic weight,

And $\dfrac{\rho N_0}{A}$ = number of atoms/cm^3 .

    Actually, the expression for K is more complicated, see p. 169 in Melissinos. A more accurate calculation of the attenuation coefficient for 662 keV γ-rays in aluminum follows. The energy of the γ-ray, hf = 662 keV, is sufficiently low, such that only two of the three possible interactions occurs. Pair production is energetically forbidden. The photoelectric effect occurs primarily on the two K-shell electrons of each $\rho N_0/A$ nuclei/volume. The cross section is Equation 2.18 on page 166 of Melissinos:

$$\sigma_P = \sigma_T \frac{Z^5}{(137)^4} 2\sqrt{2}\left(\frac{hf}{m_e^2}\right)^{-7/2}$$

where σ_T is the Thomson cross section, Z is the nuclear charge, m_e is the electron mass, and c the speed of light. The Thomson cross section is:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)$$

where e is a unit charge in Gaussian units. Compton scattering occurs off of all the $Z\rho N_0/A$ electrons/volume in the solid. The differential cross section for Compton scattering is the Klein-Nishina formula given as Equation 3.11 on page 256 of Melissinos. The total or integral cross section for Compton scattering, σ_C is plotted Fig. 5.8 on page 167 of Melissinos. The calculation of K is outlined below.

$$K = (2\sigma_P + Z\sigma_C)\, \rho\, N_0/A, \quad \sigma_P = 1.21 \times 10^{-3}\, \sigma_T, \quad \sigma_C = 0.39\, \sigma_T,$$
$$\sigma_T = 6.57 \times 10^{-25}\ cm^2, \quad Z = 13, \quad \rho = 2.70\ gm/cm^3$$

$$N_0 = 6.023 \times 10^{23}, \qquad A = 27$$

Thus we find that the attenuation coefficient $K = 0.201 \text{ cm}^{-1}$ for 662 keV γ-rays in aluminum.

Experimental Procedure and Analysis:

This experiment can be performed using the SCA (single channel analyzer) and FC (frequency counter) or using the personal computer as a MCA (multichannel analyzer). Plot the Cs^{137} γ-ray spectrum again to be certain everything is working well. Use <u>only</u> the 662 keV photopeak in attenuation measurements. If you are using the SCA and FC, then the photopeak is isolated using the LLD and ULD settings. If you are using the personal computer as an MCA, then the photopeak is isolated using the region of interest option.

Record unattenuated counts Zi, attenuated counts Yi and background counts Bi, for each of seven or more aluminum attenuators of thickness li . The background Bi can be recorded using the lead attenuator or by removing the source. The thickness li should vary from 1 cm to 15 cm. The total sample time must be <u>exactly</u> the same when recording Zi, Yi, and Bi. The distance from the source to the detector must be exactly the same when recording Zi and Yi. Set up the experiment such that Zi is $\sim 10^4$. Statistical uncertainties will be $\sim 1\%$ for 10^4 counts.

Plot the transmittance T versus l on semilog paper where

$$T(l_i) = \frac{Y_i - B_i}{Z_i - B_i} \ .$$

A properly weighted least-square fit can be used to extract K, which is the slope of ln T versus l. The weighted least-square fit can also be used to determine a statistical uncertainty ΔK in K.

An alternate analysis involves using each set of measurements Yi, Zi, Bi, and li to provide one measurement Ki with uncertainty ΔKi Use the formula:

$$T(l_i) = \frac{Y_i - B_i}{Z_i - B_i} = \exp(-Kl_i) \ .$$

How is DKi determined from the uncertainties DYi, DZi, DBi, and Dli ?You should find that very small and very large values of li give a larger ΔKi. Combine all Ki measurements with uncertainties ΔKi into a final <u>weighted</u> mean K with final statistical uncertainty DK.

Questions

Is the density of the aluminum cylinders consistent with known density of pure, solid aluminum?

Students desiring extra credit, or taking the course for Honors Credit, should explain any systematic errors in their measurement of K and identify the physical effect which causes the error. They should also propose a better measurement technique to eliminate the problem.

Final Question:

Why do very small and very large values of 1, the thickness of the aluminum attenuator, produce larger uncertainties ΔK in the attenuation coefficient K?

Appendix

2.1 GENERAL REMARKS

As mentioned in the previous section the interaction of charged particles and photons with matter is electromagnetic and results either in a gradual reduction of energy of the incoming particle (with a change of its direction) or in the absorption of the photon. Particles coupled with the nuclear field, such as nuclei, protons, neutrons, and π-mesons, are subject to a nuclear interaction as well, which is, however, of much shorter range than the electromagnetic one. The nuclear interaction may become predominant only when the particles have enough energy to overcome Coulomb-barrier effects, and when the amount of matter traversed is large enough to be of the order of a nuclear mean free path—which is approximately 60 gm/cm^2.

Heavy charged particles lose energy through collisions with the atomic electrons of the material, while electrons lose energy both through collisions with atomic electrons and through radiation when their trajectory is altered by the field of a nucleus (*bremsstrahlung*—see Sec. 2.5). Photons lose energy through collisions with the atomic electrons of the material, either through the photoelectric or Compton effect; at higher energies photons interact by creating electron-positron pairs in the field of a nucleus.

Since by necessity the ensuing discussion will be brief, the reader is referred to the excellent description of these phenomena in Fermi's *Nuclear Physics*,[†] Chapter II, or to one of the other references given at the end of the chapter.

A brief review of definitions will be helpful.

Cross Section. We define the cross section, σ, for scattering from a single particle

$$\sigma = \frac{\text{scattered flux}}{\text{incident flux per unit area}} \qquad (2.1a)$$

Thus σ has dimensions of area (usually cm^2) and can be thought of as the area of the scattering center projected on the plane normal to the incoming beam. If the density of scatterers is n (particles/cm^3), there will be $n\,dx$ scatterers per unit area in a thickness dx of material, and the probability $dP = I_s/I_0$ of an interaction in the thickness dx is

$$dP = \frac{\sigma I_0/S}{I_0}(Sn\,dx) = \sigma n\,dx \qquad (2.1b)$$

where S is the area covered by the scattering material and I_0 is the total flux incident on the target; thus I_0/S is the flux per unit area as shown[‡]

[†] Enrico Fermi, *Nuclear Physics*. Notes compiled by Orear, Rosenfeld, and Shluter. Univ. of Chicago Press, 1950.

[‡] Occasionally confusion arises because the area of the incoming beam may be smaller than the area presented by the target as shown in Fig. 5.1b. Clearly the definition of Eq. 2.1a is valid in either case and always leads back to Eq. 2.1b.
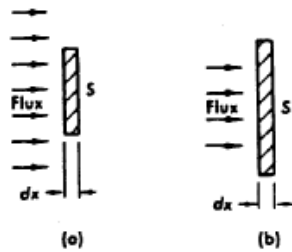
FIG. 5.1 Scattering of an incoming flux of particles by a target. (a) Area covered by flux larger than target area. (b) Area covered by flux smaller than target area.

in Fig. 5.1a. The result of Eq. 2.1b is not surprising since dP *must* be proportional to n and dx;

$$dP \propto n\,dx$$

σ is then the factor that transforms this proportionality into an equality. Atomic and nuclear cross sections are of the order of 10^{-24} cm^2 (one barn), which is not surprising, given the geometrical size (cross section) of the nucleus

$$\sigma_{geom} = \pi R^2 = 3.14 \times 10^{-26}\, A^{2/3}\, \text{cm}^2$$

Differential Cross Section. For a single scatterer we define

$$\frac{d\sigma(\theta, \phi)}{d\Omega} = \frac{\text{flux scattered into element } d\Omega \text{ at angles } \theta, \phi}{\text{incident flux per unit area}}$$

It follows that

$$\int_0^{2\pi} d\phi \int_0^{\pi} \frac{d\sigma}{d\Omega}\, d(\cos\theta) = \sigma$$

where the integration is over all angles. If after the scattering process a variety of energies is possible,

$$\frac{d^2\sigma(\theta, \phi, E)}{d\Omega\, dE} = \frac{\text{flux with energy } E, \text{ within } dE, \text{ scattered into } d\Omega \text{ at } \theta, \phi}{\text{incident flux per unit area}}$$

It follows that

$$\int_0^{\infty} \frac{d^2\sigma(\theta, \phi, E)}{d\Omega\, dE}\, dE = \frac{d\sigma(\theta, \phi)}{d\Omega}$$

where the integration is over all possible energies of the scattered flux.

Absorption Coefficient. To obtain the probability for scattering in a length x of some material, we consider an incident flux per unit area I_0; $I(x)$ represents the flux at a distance x into the material. According to Eq. 2.1b

$$-dI(x) = dP\, I(x) = I(x)\sigma n\, dx \qquad (2.1c)$$

thus

$$\frac{dI}{I} = -\sigma n \, dx \qquad I(x) = I_0 \, e^{-\sigma n x}$$

If we designate $P(x)$ the probability for scattering in a length x, we have

$$P(x) = 1 - (\text{probability for survival in a length } x) = 1 - e^{-\sigma n x} = 1 - e^{-\kappa x}$$

where $\kappa = \sigma n$ is the absorption coefficient. Similarly $\lambda = 1/\sigma n$, which has dimensions of length, is called the absorption length, or, mean free path.

The density of scattering centers n is given by

$$
\begin{aligned}
n &= \rho N_0/A & &\text{if we consider scattering by nuclei} \\
n_e &= \rho N_0 Z/A & &\text{if we consider scattering by electrons} \\
n_n &= \rho N_0 & &\text{if we consider scattering by nucleons}
\end{aligned}
\qquad (2.2)
$$

where N_0 is Avogadro's number 6.023×10^{23} and ρ is the density of the material in gm/cm^3; Z and A are the atomic and mass number respectively.

Often we wish to express the absorption in terms of the equivalent matter traversed, namely, $\xi = $ gm/cm^2. Then the thickness of the material can be expressed by $d\xi$, where

$$d\xi = \rho \, dx$$

The *mass* absorption coefficient is defined by

$$\mu = \frac{\kappa}{\rho} \qquad (2.3)$$

so that the fraction of a beam *not* absorbed is

$$\frac{I}{I_0} = e^{-\mu\xi} \qquad (2.4)$$

Similarly, if the region of interaction is very thin, the scattered flux is given directly by

$$I_s = I_0 \sigma n \, dx \qquad \text{for example, for nuclei,} \qquad I_s = I_0 \frac{N_0}{A} \sigma \, d\xi$$

2.5 Passage of Electromagnetic Radiation (Photons) through Matter

As mentioned in the introduction to this section photons lose energy or are absorbed in matter by one of the following three mechanisms:

(a) Photoelectric effect, which predominates at low energies.
(b) Compton effect, which predominates at medium energies (below a few MeV)
(c) Pair production of electrons and positrons, which predominates in the high-energy region

The relative importance of these processes and the energies at which they set in are best seen in Fig. 5.7, which gives the cross section for the interaction of a photon as a function of its energy (in units of the electron's rest mass). We will now briefly consider each process separately.

(a) *Photoelectric effect.* We speak of the photoelectric effect when the photon is completely absorbed and all its energy is transferred to an atomic electron. Consequently the photon must have enough energy to excite the bound electron from its quantum state to a higher state or into the continuum; the latter process (ionization of the atom) is much more probable.

† Calculated from Moliére theory; see U.C.R.L. Report 8030 by W. Barkas and A. H. Rosenfeld for tables of ε.
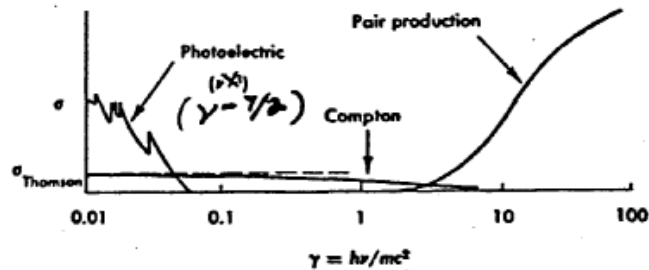
FIG. 5.7 The cross section for the interaction of photons with matter as a function of their energy (expressed in units of the electron's rest mass).

Since the binding energy of the inner electrons in atoms is of the order of keV, as the frequency of the photon is increased and it reaches the value of the binding energy of a particular shell,† a new "channel" opens, and we expect a sudden rise in the absorption cross section. Apart from the onset of new channels, the over-all variation of the photoeffect is a rapid decrease as the third power of the photon frequency (as $\gamma^{-7/2}$) thus resulting in the curve shown on the left in Fig. 5.7. The cross sections for photoeffect are derived in Heitler,‡ from where we give the nonrelativistic value for the ejection of *one* electron from the K shell, when the photon energy is not too close to the absorption edge:

$$K\,shell \rightarrow \quad \sigma_p = \sigma_T \frac{Z^5}{(137)^4} 2\sqrt{2} \left[\frac{h\nu}{mc^2} \right]^{-7/2} \quad (cm^2) \qquad (2.18)$$

where we note the dependence on the Z of the nucleus, indicating that L-shell and higher-shell ejection is less probable because of the screening of the nuclear charge. Here σ_T is the classical Thomson cross section, which is derived on the simplified assumption of a plane polarized electromagnetic wave scattering from a free electron (it is assumed that the displacement of the electron is much smaller than the wavelength); we obtain

$$\sigma_T = \frac{8\pi}{3} \left[\frac{e^2}{mc^2} \right]^2 = \frac{8\pi}{3} r_0^2 \quad (cm^2) \qquad (2.19)$$

where $r_0 = e^2/mc^2$ is the classical radius of the electron $= 2.8 \times 10^{-13}$ cm. Note that the Thomson cross section is independent of the frequency of the incoming photon. Near the absorption edge, Eq. 2.18 is to be modified

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† Note that $n = 1$ electrons are said to be in the K shell, $n = 2$ in the L shell, $n = 3$ in the M shell, etc.

‡ W. Heitler, *The Quantum Theory of Radiation*, 3rd ed., Oxford Univ. Press, 1954, pp. 207 and 208.

as follows:

$$\sigma_p = \frac{128\pi}{3}\frac{e^2}{mc}\frac{\nu_K^3}{\nu^4}\frac{\exp(-4\epsilon\cot^{-1}\epsilon)}{1-e^{-2\pi\epsilon}} \quad (\text{cm}^2) \qquad (2.20)$$

where ν_K is the frequency of the K-shell absorption edge and $\nu > \nu_K$ the frequency of the photon; $\epsilon = \sqrt{\nu_K/(\nu-\nu_K)}$.

(b) *Compton effect.* In the Compton effect, the photon scatters off an atomic electron and loses only part of its energy. This phenomenon, which is one of the most striking quantum effects, is described in detail in Chapter 6; the cross section for Compton scattering is given by the Klein–Nishina (K–N) formula, shown in an expanded scale in Fig. 5.8. The energy of the photon is given on the abscissa in units of the electron rest mass $\gamma = h\nu/mc^2$, and the ordinate gives the ratio of the Compton cross section σ_c to the classical Thomson cross section σ_r.

We give below the asymptotic approximations to the (K–N) Compton scattering cross section:

For low energies:

$$\sigma_c = \sigma_r(1 - 2\gamma + \tfrac{26}{5}\gamma^2 + \cdots) \quad \gamma \ll 1$$

For high energies:

$$\sigma_c = \frac{3}{8}\sigma_r\frac{1}{\gamma}\left(\ln 2\gamma + \frac{1}{2}\right) \qquad \gamma \gg 1 \qquad (2.21)$$

(c) *Pair production.* In pair production a photon of sufficiently high energy is annihilated and an electron-positron pair is created. For a free
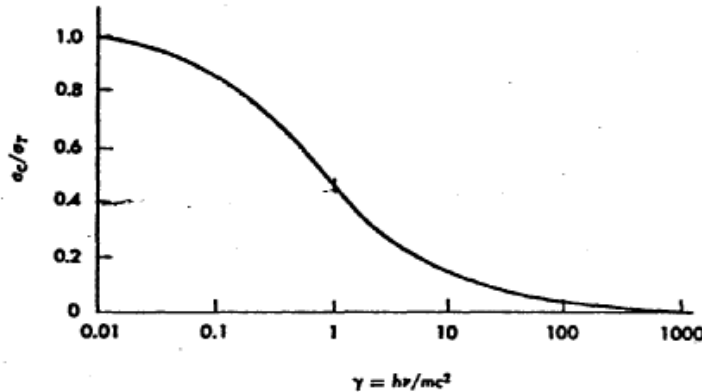

FIG. 5.8 The ratio of the Compton scattering cross section, σ_c, to the constant Thomson cross section, σ_T, as a function of photon energy expressed in units of the electron's rest mass.

photon conservation of energy and momentum would not be possible in this process, so pair production must take place in the field of a nucleus (or of another electron) which will take up the balance of momentum. Clearly the threshold for this process is $2mc^2$ (where m is the mass of the electron), hence 1022 keV. The cross section for pair production rises rapidly beyond the threshold, and reaches a limiting value for $h\nu/mc^2 \approx 1000$ given by the following equation:†

$$\sigma_{pair} = \frac{Z^2}{137} r_0^2 \left[\frac{28}{9} \ln \frac{183}{Z^{1/3}} - \frac{2}{27} \right] \quad (\text{cm}^2) \quad (2.22)$$

Since both the photoelectric and Compton effect cross sections decrease as the photon energy rises, pair production is the predominant interaction mechanism for very high-energy photons.

It is advantageous to introduce the mean free path (L_{pair}) for pair production; when a photon traverses a material with density of nuclei n,

$$L_{pair} = \frac{1}{n\sigma_{pair}} = \frac{1}{(28/9)(Z^2n/137)r_0^2 \ln (183/Z^{1/3})} \quad (\text{cm}) \quad (2.23)$$

where we have dropped the small term 2/27. Thus, the attenuation of a beam of I_0 photons will proceed as
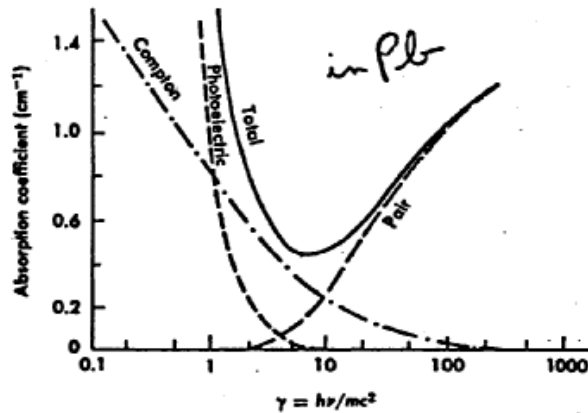
$$I(x) = I_0 e^{-x/L_{pair}} \quad (2.24)$$

FIG. 5.9 The relative contribution of the three effects responsible for the interaction of photons with matter. The absorption coefficient in lead is plotted against the logarith of photon energy (in units of the electron's rest mass).

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† See Heitler, loc. cit. p. 260.

In conclusion, Fig. 5.9 gives the total absorption coefficient for a photon traversing lead as a function of its energy (in units of the electron rest mass). Note that

$$\kappa_p = \sigma_p\, 2n \qquad \text{because there are 2 } K\text{-shell electrons per nucleus}$$
$$\kappa_c = \sigma_c\, n_e \qquad \text{electron density}$$
$$\kappa_{pair} = \sigma_{pair}\, n \qquad \text{density of nuclei}$$

$$n = \# \text{ of atoms}/cm^3$$

The dotted curves in Fig. 5.9 indicate the relative contributions of each of the three interaction mechanisms.