Physics 43 HW 2 Chapter 39
Problems given from 7th Edition

Problems: 4, 7, 8, 9, 14, 20, 22, 24, 29, 33, 35, 38, 40, 45,

4. How fast must a meter stick be moving if its length is measured to shrink to 0.500 m?

P39.4 $L = L_p\sqrt{1-\frac{v^2}{c^2}}$ $\qquad v = c\sqrt{1-\left(\frac{L}{L_p}\right)^2}$

Taking $L = \frac{L_p}{2}$ where $L_p = 1.00\text{ m}$ gives $v = c\sqrt{1-\left(\frac{L_p/2}{L_p}\right)^2} = c\sqrt{1-\frac{1}{4}} = \boxed{0.866c}$

7. An atomic clock moves at 1 000 km/h for 1.00 h as measured by an identical clock on the Earth. How many nanoseconds slow will the moving clock be compared with the Earth clock, at the end of the 1.00-h interval?

P39.7 $\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1-v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1-\frac{v^2}{c^2}}\right)\Delta t \cong \left(1-\frac{v^2}{2c^2}\right)\Delta t$

and $\Delta t - \Delta t_p = \left(\frac{v^2}{2c^2}\right)\Delta t$

If $v = 1\,000\text{ km/h} = \frac{1.00\times10^6\text{ m}}{3\,600\text{ s}} = 277.8\text{ m/s}$

then $\frac{v}{c} = 9.26\times10^{-7}$

and

$(\Delta t - \Delta t_p) = (4.28\times10^{-13})(3\,600\text{ s}) = 1.54\times10^{-9}\text{ s} = \boxed{1.54\text{ ns}}$

8. A muon formed high in the Earth's atmosphere travels at speed $v = 0.990c$ for a distance of 4.60 km before it decays into an electron, a neutrino, and an antineutrino $(\mu^- \to e^- + \nu + \bar{\nu})$. (a) How long does the muon live, as measured in its reference frame? (b) How far does the muon travel, as measured in its frame?

P39.8 For $\frac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is

$\Delta t = \frac{4.60\text{ km}}{0.990c}$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60\times10^3\text{ m}}{0.990(3.00\times10^8\text{ m/s})}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{L_p}{\gamma} = \frac{4.60\times10^3\text{ m}}{7.09} = \boxed{649\text{ m}}$

9. A spacecraft with a proper length of 300 m takes 0.750 μs to pass an Earth observer. Determine the speed of the spacecraft as measured by the Earth observer.

P39.9 The spaceship is measured by the Earth observer to be length-contracted to

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \quad \text{or} \quad L^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Also, the contracted length is related to the time required to pass overhead by:

$$L = vt \quad \text{or} \quad L^2 = v^2t^2 = \frac{v^2}{c^2}(ct)^2$$

Equating these two expressions gives $L_p^2 - L_p^2\frac{v^2}{c^2} = (ct)^2\frac{v^2}{c^2}$

or $\left[L_p^2 + (ct)^2\right]\frac{v^2}{c^2} = L_p^2$

Using the given values: $L_p = 300\text{ m}$ and $t = 7.50\times10^{-7}\text{ s}$

this becomes $\left(1.41\times10^5\text{ m}^2\right)\frac{v^2}{c^2} = 9.00\times10^4\text{ m}^2$

giving $v = \boxed{0.800c}$

14. The identical twins Speedo and Goslo join a migration from the Earth to Planet X. It is 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spacecraft. Speedo's craft travels steadily at 0.950*c*, and Goslo's at 0.750*c*. Calculate the age difference between the twins after Goslo's spacecraft lands on Planet X. Which twin is the older?

P39.14 In the Earth frame, Speedo's trip lasts for a time

$$\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 21.05\text{ yr}$$

Speedo's age advances only by the proper time interval

$$\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1-0.95^2} = 6.574\text{ yr during his trip}$$

Similarly for Goslo,

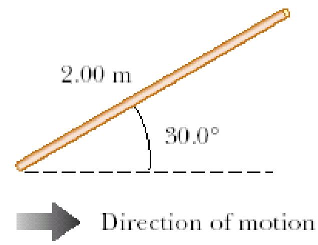
$$\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}} - \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 5.614\text{ yr}$$

Then $\boxed{\text{Goslo}}$ ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = \boxed{5.45\text{ yr}}$.

20. A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion, as shown in Figure P39.23. The rod has a speed of 0.995*c*. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?



P39.20 $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-0.995^2}} = 10.0$

We are also given: $L_1 = 2.00\text{ m}$, and $\theta = 30.0°$ (both measured in a reference frame moving relative to the rod).

Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00\text{ m})(0.867) = 1.73\text{ m}$

and $L_{1y} = L_1 \sin\theta_1 = (2.00\text{ m})(0.500) = 1.00\text{ m}$

L_{2x} is a proper length, related to L_{1x} by $L_{1x} = \dfrac{L_{2x}}{\gamma}$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3\text{ m}$

and $L_{2y} = L_{1y} = 1.00\text{ m}$ (Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4\text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

22 A red light flashes at position $x_R = 3.00$ m and time $t_R = 1.00 \times 10^{-9}$ s, and a blue light flashes at $x_B = 5.00$ m and $t_B = 9.00 \times 10^{-9}$ s, all measured in the S reference frame. Reference frame S′ has its origin at the same point as S at $t = t' = 0$; frame S′ moves uniformly to the right. Both flashes are observed to occur at the same place in S′. (a) Find the relative speed between S and S′. (b) Find the location of the two flashes in frame S′. (c) At what time does the red flash occur in the S′ frame?

P39.22 (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$\Delta x' = \gamma(\Delta x - v\Delta t)$ $\qquad 0 = \gamma\left[2.00\text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$

$v = \dfrac{2.00\text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^8\text{ m/s}}$ $\qquad \gamma = \dfrac{1}{\sqrt{1-\left(2.50\times10^8\text{ m/s}\right)^2/\left(3.00\times10^8\text{ m/s}\right)^2}} = 1.81$

(b) Again from the Lorentz transformation, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\text{ m} - \left(2.50\times10^8\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right]$$

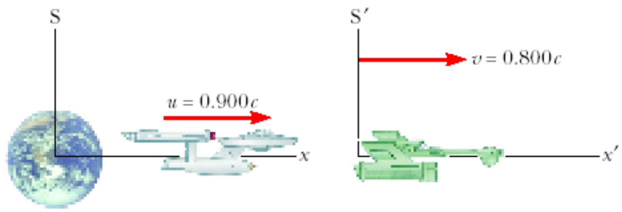
$$x' = \boxed{4.97\text{ m}}$$

(c) $t' = \gamma\left(t - \dfrac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^8\text{ m/s}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

24. A Klingon spacecraft moves away from the Earth at a speed of 0.800*c* (Fig. P39.26). The starship *Enterprise* pursues at a speed of 0.900*c* relative to the Earth. Observers on the Earth see the *Enterprise* overtaking the Klingon craft at a relative speed of 0.100*c*. With what speed is the *Enterprise* overtaking the Klingon craft as seen by the crew of the *Enterprise*?



P39.24 u_x = Enterprise velocity

v = Klingon velocity

From Equation 39.16

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$$

29. An unstable particle at rest breaks into two fragments of unequal mass. The mass of the first fragment is 2.50 × 10^{-28} kg, and that of the other is 1.67 × 10^{-27} kg. If the lighter fragment has a speed of 0.893*c* after the breakup, what is the speed of the heavier fragment?

P39.29 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$

or $$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50 \times 10^{-28}\text{ kg}}{\sqrt{1 - (0.893)^2}} \times (0.893c)$$

or $$\frac{\left(1.67 \times 10^{-27}\text{ kg}\right) u_2}{\sqrt{1 - (u_2/c)^2}} = \left(4.960 \times 10^{-28}\text{ kg}\right) c$$

Proceeding to solve, we find $$\left(\frac{1.67 \times 10^{-27}}{4.960 \times 10^{-28}} \frac{u_2}{c}\right)^2 = 1 - \frac{u_2^2}{c^2}$$

$$12.3 \frac{u_2^2}{c^2} = 1 \text{ and } u_2 = \boxed{0.285c}$$

---

33. Find the momentum of a proton in MeV/*c* units assuming its total energy is twice its rest energy.

P39.33 $$E = \gamma mc^2 = 2mc^2 \text{ or } \gamma = 2$$

Thus, $$\frac{u}{c} = \sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = \frac{\sqrt{3}}{2} \text{ or } u = \frac{c\sqrt{3}}{2}$$

The momentum is then $$p = \gamma mu = 2m\left(\frac{c\sqrt{3}}{2}\right) = \left(\frac{mc^2}{c}\right)\sqrt{3}$$

$$p = \left(\frac{938.3\text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63 \times 10^3 \frac{\text{MeV}}{c}}$$

35. A proton moves at 0.950*c*. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.

P39.35 (a) $E_R = mc^2 = \left(1.67\times10^{-27}\text{ kg}\right)\left(2.998\times10^{8}\text{ m/s}\right)^2 = 1.50\times10^{-10}\text{ J} = \boxed{938\text{ MeV}}$

(b) $E = \gamma mc^2 = \dfrac{1.50\times10^{-10}\text{ J}}{\left[1-(0.950c/c)^2\right]^{1/2}} = 4.81\times10^{-10}\text{ J} = \boxed{3.00\times10^{3}\text{ MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\text{ J} - 1.50\times10^{-10}\text{ J} = 3.31\times10^{-10}\text{ J} = \boxed{2.07\times10^{3}\text{ MeV}}$

38. In a typical color television picture tube, the electrons are accelerated through a potential difference of 25 000 V. (a) What speed do the electrons have when they strike the screen? (b) What is their kinetic energy in joules?

P39.38 (a) $q(\Delta V) = K = (\gamma - 1)m_e c^2$

Thus, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 1 + \dfrac{q(\Delta V)}{m_e c^2} = 1 + \dfrac{25\,000\text{ eV}}{511\,000\text{ eV}} = 1.0489$

so $1 - (u/c)^2 = 0.9089$ and $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1)m_e c^2 = q(\Delta V) = \left(1.60\times10^{-19}\text{ C}\right)\left(2.50\times10^{4}\text{ J/C}\right) = \boxed{4.00\times10^{-15}\text{ J}}$

40. Consider electrons accelerated to an energy of 20.0 GeV in the 3.00-km-long Stanford Linear Accelerator. (a) What is the γ factor for the electrons? (b) What is their speed? (c) How long does the accelerator appear to them?

P39.40 (a) $E = \gamma mc^2 = 20.0\text{ GeV}$ with $mc^2 = 0.511\text{ MeV}$ for electrons.

Thus, $\gamma = \dfrac{20.0\times10^{9}\text{ eV}}{0.511\times10^{6}\text{ eV}} = \boxed{3.91\times10^{4}}$.

(b) $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 3.91\times10^{4}$ from which $\boxed{u = 0.999\,999\,999\,7c}$

(c) $L = L_p\sqrt{1-\left(\dfrac{u}{c}\right)^2} = \dfrac{L_p}{\gamma} = \dfrac{3.00\times10^{3}\text{ m}}{3.91\times10^{4}} = 7.67\times10^{-2}\text{ m} = \boxed{7.67\text{ cm}}$

45. The power output of the Sun is 3.77 × 10^{26} W. How much mass is converted to energy in the Sun each second?

P39.45 $\mathcal{P} = \dfrac{dE}{dt} = \dfrac{d\left(mc^2\right)}{dt} = c^2\dfrac{dm}{dt} = 3.85\times10^{26}\text{ W}$

Thus, $\dfrac{dm}{dt} = \dfrac{3.85\times10^{26}\text{ J/s}}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = \boxed{4.28\times10^{9}\text{ kg/s}}$