Physics Learning Achievement Study: Projectile, using Mathematica program of Faculty of Science and Technology Phetchabun Rajabhat University Students

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Abstract
The propose of this study is to study Physics Learning Achievement, projectile motion, using the Mathematica program of Faculty of Science and Technology Phetchabun Rajabhat University students, comparing with Faculty of Science and Technology Phetchabun Rajabhat University students who study the projectile motion experiment set. The samples are Faculty of Science and Technology Phetchabun Rajabhat University Technology students, studying in the first semester of academic year 2011, consisting of 50 samples who study by using the Mathematica program and 50 controlled samples, studying by using the projectile motion experiment set. The instrument used for collecting data is the 30 items achievement test of the projectile motion. Analyze to compare the achievement between the samples and the controlled samples by t-test (Lord, F.M. 1967; Airasian, P.W. and Madaus, G.F., 1972) independent sample. The result finds that the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students who study projectile motion, using Mathematica is higher than the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students who study projectile motion, using projectile motion experiment set at 0.01 level
Keywords: projectile motion, Mathematica program , pre-test and post-test, t-test, Learning-Classroom

Introduction

Nowadays, science and technology play the important role in human living and develop the nationwide extremely in teaching management. Physics is the subject for every student who wants to pass only, there are very few students who intuitively want to know and understand physics, especially the Mathematica (Bauman, 1996) program, Faculty of Science and Technology Phetchabun Rajabhat University students don't pay intention to experiment. So the researcher gets interested in using Mathematica program (Bauman, 1996; Bauman, 2005) which is Mathematica program (Wolfram, 1994; Maeder, 1991) as the aid for projectile motion (Arya, 1990; Ginsberg, 1995; Fishbane et al. 2005; Bueche and Hecht, 2006; Marion and Thornton, 1995) experiment to assist Faculty of Science and Technology Phetchabun Rajabhat University students to experiment the projectile understandingly, to be able to learn and revise it all the time, to learn effectively and to solve the students unequal knowledge basic problem.

In order to study Physics learning achievement, the projectile by using the Mathematica program at Faculty of Science and Technology Phetchabun Rajabhat University students.

The achievement of the Technology students, studying projectile, using the Mathematica program is higher than the achievement of the Technology students, studying projectile, using projectile motion experiment set.

Projectile Motion

Projectile motion is two - dimensional motion, moving on horizon and vertical simultaneously. It is speedy motion because of the steady gravity, being near the surface of the earth, whereas there is no speed on the horizon motion owing to lacking of force on horizon, so the speed of the horizon movement is steady. The moving route of projectile motion will be parabola curve. The projectile motion as the following will be considered without braking power and other results such as, rotation, the change of material figures. It is considered only the particle motion, which is speedy on only vertical direction. When the motion is specified on one plain, two - dimension motion is specified as *x*-*y* coordinate system. The origin is zero as *x*-axis, being on *y* plain. A *y*-axis is vertical, the gravity acceleration is on -y, or the acceleration component is

$$\left.\begin{aligned} a_x &= 0 \\ a_y &= -y \end{aligned}\right\} \quad (1)$$

According to the motion equation, we obtain the velocity on *x*-axis and *y*-axis is

$$\left.\begin{aligned} v_x(t) &= v_{x0} \\ v_y(t) &= v_{yo} - gt \end{aligned}\right\} \quad (2)$$

Using equation (2) into

$$\left.\begin{aligned} x(t) &= x_0 + \int_{t_0}^{t} v_x(t)\,dt \\ y(t) &= y_0 + \int_{t_0}^{t} v_y(t)\,dt \end{aligned}\right\} \quad (3)$$

And use the origin to be start point or $x_0 = y_0 = 0$, we get the particle position for projectile is

$$\left.\begin{aligned} x(t) &= v_{x0}t \\ y(t) &= v_{y0}t - \frac{1}{2}gt^2 \end{aligned}\right\} \quad (4)$$

Restricting *t* of both equators in (4), the relation between coordinate *x* and *y* is (Arya, A, 1990; Ginsberg, E.S. 1995)

$$y = -\frac{g}{2v_{x0}^2}x^2 + \frac{v_{y0}}{v_{x0}}x \quad (5)$$

The relation between x and y of equation (5) is the motion a parabolic trajectory, crossing at *x* axis on

$$\left.\begin{aligned} x_0 &= 0 \\ R &= \frac{2v_{x0}v_{y0}}{g} \end{aligned}\right\} \quad (6)$$

R Interval, called the range of projectile is the interval that particle move furthest on the plain. When the derivative of y in equation (5) comparing with x, the value of derivative is zero to obtain the x interval to be $\frac{R}{2}$. The interval of y is highest; the value is equal (Fishbane et al. 2005)

$$H = \frac{v_{y0}^2}{2g} \qquad (7)$$

The figures of parabola mentioned in equation (5) to equation (7) shown as the figure (1) R is replaced as the interval on x- axis in the first equation (4), it is the time that the particle being in the air.

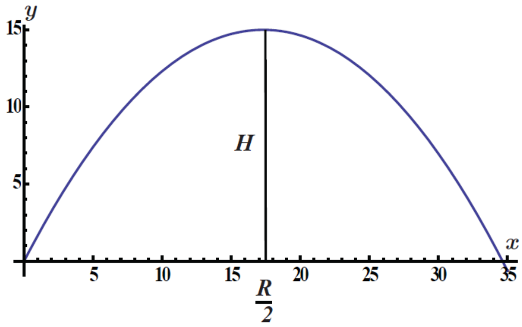

Fig. 1. *A projectile moving under the force of gravity is at its maximum height when* $v_y = 0$.

The motion on the plain is equal

$$t_R = \frac{2v_{y0}}{g}. \qquad (8)$$

$\frac{R}{2}$ is replaced as the interval on x- axis in the first equation (4), it is the time that the particle being in the air. The motion on the plain is equal

$$t_H = \frac{v_{y0}}{g}. \qquad (9)$$

Projectile motion is generally specified the initial velocity at $t = 0$ is v_0, being an angle with x-axis, the angle θ_0 so the components of the velocity on x-axis and y-axis is

$$\left.\begin{aligned} v_{x0} &= v_0 \cos\theta_0 \\ v_{y0} &= v_0 \sin\theta_0 \end{aligned}\right\} \qquad (10)$$

Different quantities related with projectile motion which are in term of v_{x0} and v_{y0} from equation (2) to equation (9) can be written in form of v_0 and θ_0 for instance, the range of R in equation (6) in form of v_0 and θ_0 is (Bueche and Hecht, 2006; Marion and Thornton, 1995) in Fig. (2), Fig. (3), Fig. (4), Fig. (5).

$$\left.\begin{aligned} R &= \frac{2v_0^2\cos(\theta_0)\sin(\theta_0)}{g} \\ R &= \frac{v_0^2\sin(2\theta_0)}{g} \end{aligned}\right\}. \qquad (11)$$

It indicates that v_0 is steady value, θ_0 angle that the highest range value is $\pi/4\ rad.$ at the position projectile motion, with the velocity v, being angle with x-axis and y-axis is

$$\left.\begin{aligned} v_x &= v\cos(\theta) = v_{x0} \\ v_y &= v\sin(\theta) \end{aligned}\right\} \qquad (12)$$

The value of θ angle is specified to be plus, being measured from $+x$-axis parallel to v on anticlockwise direction and to be minus on the opposite direction. The mentioned specification conforming to the direction of v_y is minus. The angle value and the velocity component of projectile motion at different positions are indicated as the figure (2).

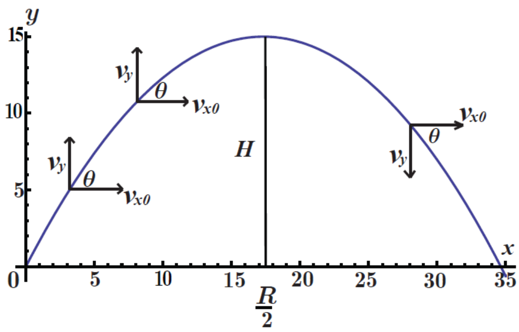

***Fig. 2** The velocity component v_{x0} and v_y of projectile motion at different positions of projectile motion.*

The remark of figure (2), the velocity of x-axis is unchangeable. The size of velocity depends on the highest velocity of y-axis. The least of velocity size value is equal the x-axis size of velocity. Besides, the curve type, motion, the size of velocity and the size of angle θ angle are symmetrically around the highest interval of motion.

For example the first, numerical evaluation of the projectile motion (Bauman, 1996; Bauman, 2005).

- In[1]:= eq1= {m x$^{\mathbb{ZZ}}$[t] == 0, m y$^{\mathbb{ZZ}}$[t] == -m g}; t == t1;
- In[2]:= initial1 ={x[0] == 0, x[t1] == x1, y[0] == 0, y[t1] == y1};
- In[3]:= dsol1 = DSolve[Join[eq1, initial1], {x[t], y[t]}, t] // Flatten

- Out[3]= {x[t] ‖ $\frac{t\,x1}{t}$, y[t] ‖ $\frac{-g t^2\, t1 + g\ t\ t1^2 + 2 t\, y1}{2 t1}$}
- In[4]:= Map[Collect[#, t] &, dsol1, {2}]
- Out[4]= {x[t] ‖ $\frac{t\,x1}{t}$, y[t] ‖ $-\frac{g\ t^2}{2} + \frac{t\left(g\ t1^2 + 2\ y1\right)}{2\ t1}$}
- In[5]:= Map[Collect[#, t] &, dsol1, {2}]
- Out[5]= {collect[x[t], t] ‖ collect[$\frac{t\,x1}{t}$, t], collect[y[t], t] ‖ collect[$\frac{-g t^2\, t1 + g\ t\ t1^2 + 2 t\, y1}{2\ t1}$, t]}
- In[6]:= values1 = {x1 ‖ 350, y1 ‖ 0, g ‖ 9.8};
- In[7]:= coord1[t_, t1_] ={x[t], y[t]} /. Dsol1 /. values1
- Out[7]= {$\frac{350\ t}{t}$, $\frac{-9.8\ t^2\, t1 + 9.8\ t\ t1^2}{2\ t1}$}
- In[8]:= points[t1_]:= Table[coord1[t, t1], {t, 0, t1, 0.5}]
- In[9]:= plot1[t1_]:= ListPlot[points[t1] // Evaluate, PlotStyle ‖ {Hue[0.9], PointSize[0.02]}, GridLines ‖ Automatic, DisplayFunction ‖ Identity]
- In[10]:= Show[{plot1[5], plot1[10], plot1[15], plot1[20]}, Epilog ‖ {Text["t1 = 5", coord1[5/2, 6.5], TextStyle ‖ {FontWeight ‖ "Bold"}], Text["t1 = 10", coord1[10/2, 11.5], TextStyle ‖ {FontWeight ‖ "Bold"}], Text["t1 = 15", coord1[15/2, 16.0], TextStyle ‖ {FontWeight ‖ "Bold"}], Text["t1 = 20", coord1[20/2, 19.5], TextStyle ‖ {FontWeight ‖ "Bold"}] }, DisplayFunction ‖ $ DisplayFunction, PlotRange ‖ All]
- Out[10]= graph see Fig. (3), Fig. (4) and Fig. (5)

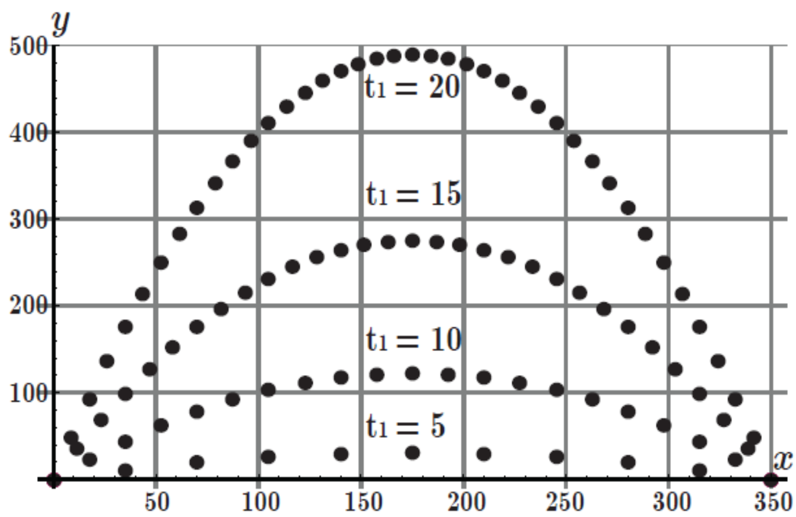

Fig. 3 A projectile(a ball) moving under the force of gravity is at its maximum height when $t1 = 20$ and $v_y = 0$ and minimum height when $t1 = 5$. At that moment, the ball is traveling horizontally.

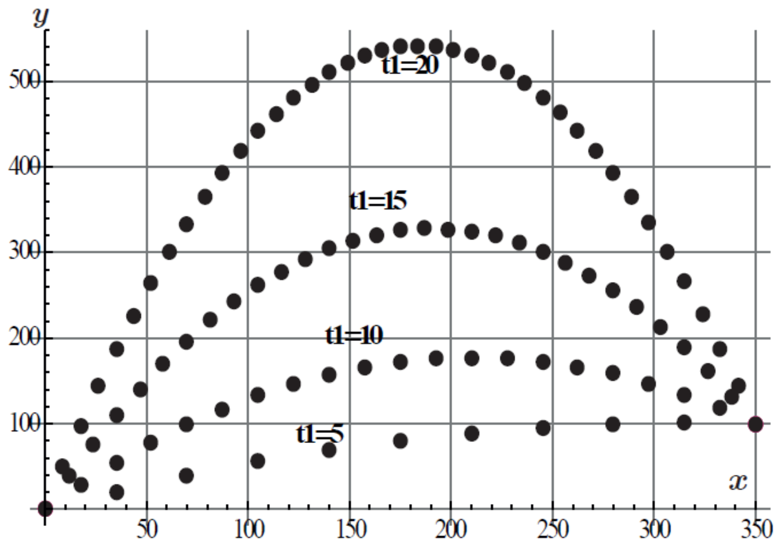

Fig. 4 *A projectile(a ball) moving under the force of gravity is at its maximum height when* $t1 = 20$ *s. and* $v_y = 0$ *m/s and minimum height when* $t1 = 5$ *s. . By Setting x1=350 m., y1=100 m., g=9.8m/s*^{2}.

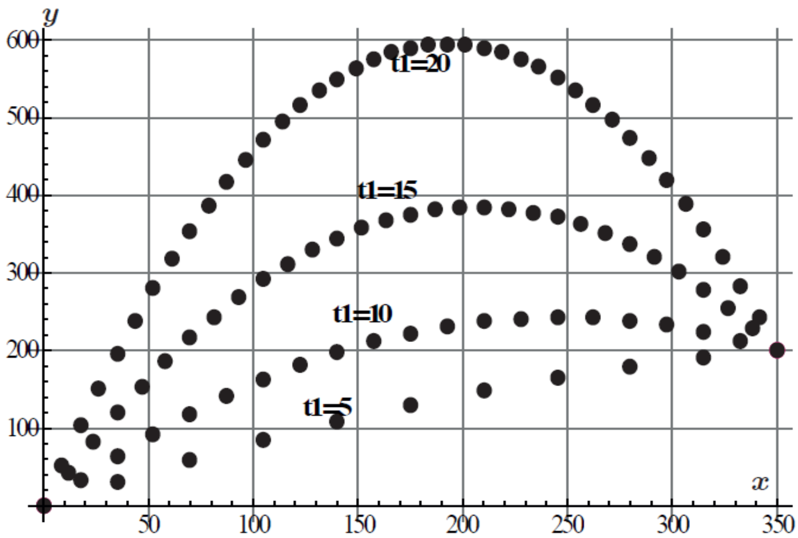

Fig. 5. *A projectile(a ball) moving under the force of gravity is at its maximum height when* $t1 = 20$ *s. and* $v_y = 0$ *m/s and minimum height when* $t1 = 5$ *s.. By Setting x1=350m, y1=200m, g=9.8m/s*^{2}.

For example the second, numerical evaluation of the projectile motion (Wolfram, S. 1994; Maeder, R. 1991).

- In[11]:= initial2 = {x[0] == 0, $x^{\mathbb{Z}}$[0] == v0 Cos[θ], y[0] == 0, $y^{\mathbb{Z}}$[0] == v0 Sin[θ]};
- In[12]:= dsol2 = DSolve[Join[eq1, initial2] , {x,y} , t] // Flatten
- Out[12]= {x ‖ Function[{t} , t v0 Cos[θ]] , y ‖ Function[{t} , $\frac{1}{2}$(-g t^2 + 2 t v0 Sin[θ])]}
- In [13]:= eq2 = $y^{\mathbb{Z}}$[t] == 0/. Dsol2

- Out[13]= $\frac{1}{2}$ (- 2 g t + 2 v0 Sin[θ]) == 0
- In[14]:= $y^{\mathbb{Z}}$[t] == 0/. Dsol1
- Out[14]= $y^{\mathbb{Z}}$[t] == 0
- In[15]:= tsol = Solve[eq2, t] // Flatten
- Out[15]= { $\mathbf{t} \rightarrow \frac{\mathbf{v0\,Sin[\theta]}}{g}$ }
- In[16]:= {x[t] , y[t]} /. Dsol2 /. tsol // Simplify
- Out[16]= { $\frac{\mathbf{v0^2 Cos[\theta] Sin[\theta]}}{g}$, $\frac{\mathbf{v0^2 (Sin[\theta])^2}}{2\,g}$ }
- In[17]:= values2 = {v0 ‖ 100, g ‖ 9.8};
- In[18]:= coord2[t_, θ_] ={x[t], y[t]} /. Dsol2 /. Values2
- Out[18]= {100 t Cos[θ], $\frac{1}{2}$ (-9.8 t^2 + 200 t Sin[θ])}
- In[19]:= plot2[θ_]:= ListPlot[Table[coord2[t, θ] , {t , 0 , 20 , ½}] // Evaluate, PlotStyle ‖ {Hue[0.9], PointSize[0.02]}, GridLines ‖ Automatic, DisplayFunction ‖ Identity]
- In[20]:= Show[{plot2[$\frac{\pi}{3}$], plot2[$\frac{\pi}{6}$], plot2[$\frac{\pi}{4}$], plot2[$\frac{5\pi}{12}$]}, Epilog ‖ {Text[“ θ = $\frac{\pi}{3}$ ”, coord2[4.5, $\frac{\pi}{3}$], TextStyle ‖ {FontWeight ‖ “Bold”}], Text[“ θ = $\frac{\pi}{6}$ ”, coord2[5, $\frac{\pi}{6}$], TextStyle ‖ {FontWeight ‖ “Bold”}], Text[“ θ = $\frac{\pi}{4}$ ”, coord2[5, $\frac{\pi}{4}$], TextStyle ‖ {FontWeight ‖ “Bold”}], Text[“ θ = $\frac{5\pi}{12}$ ”, coord2[5, $\frac{5\pi}{12}$], TextStyle ‖ {FontWeight ‖ “Bold”}] }, DisplayFunction ‖ $ DisplayFunction, PlotRange ‖ {{0 , 1100} , {0 , 500}} , FrameLabel ‖ {“x[t]” , “y[t]”}]
- Out[20]= graph see figure (4) (Ginsberg, E.S. 1995)

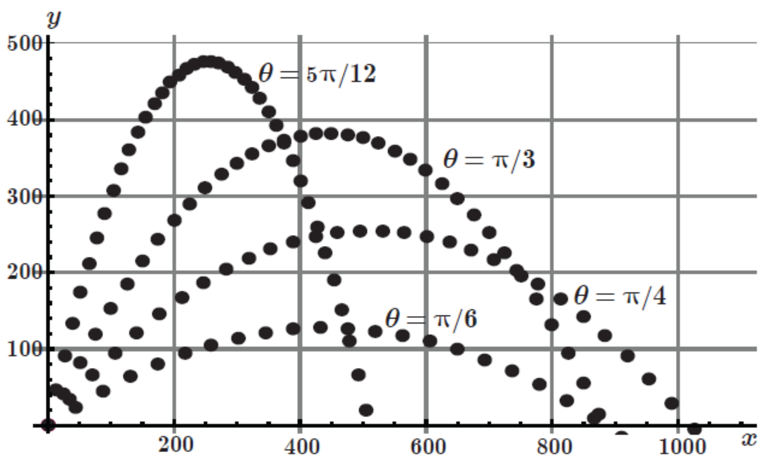

Fig. 6. *For a fixed initial velocity (ball) and if air resistance is ignored, a projectile’s trajectory will have a maximum range for an elevation angle of* $\pi/4$ *rad*. *(Ginsberg, E.S. 1995). The range is the horizontal distance the projectile travels to reach the same height from which it started.*

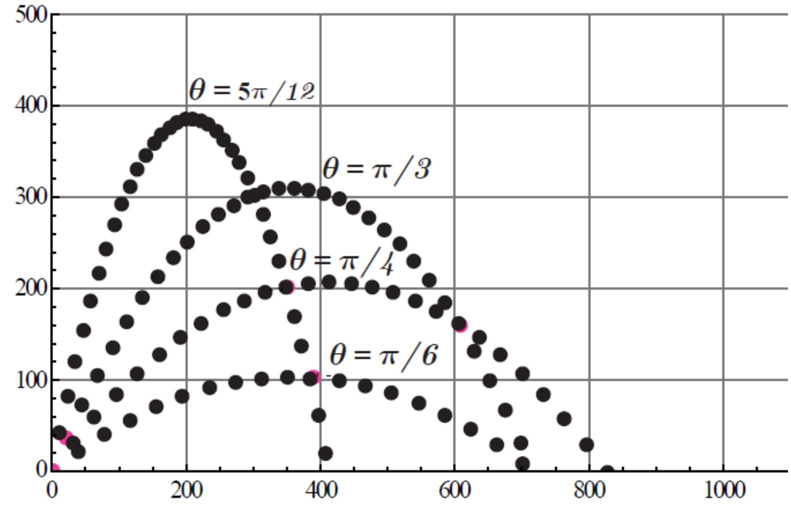

Fig. 7. *For a fixed initial velocity (ball) and if air resistance is ignored, a projectile's trajectory will have a maximum range for an elevation angle of* $\pi/4$ *rad. (Ginsberg, E.S. 1995). From this is Fig. 7, if the value initial velocity (V0) has increase, the range is the horizontal distance the projectile travels to has lessen. By setting V0=90 m/s.*

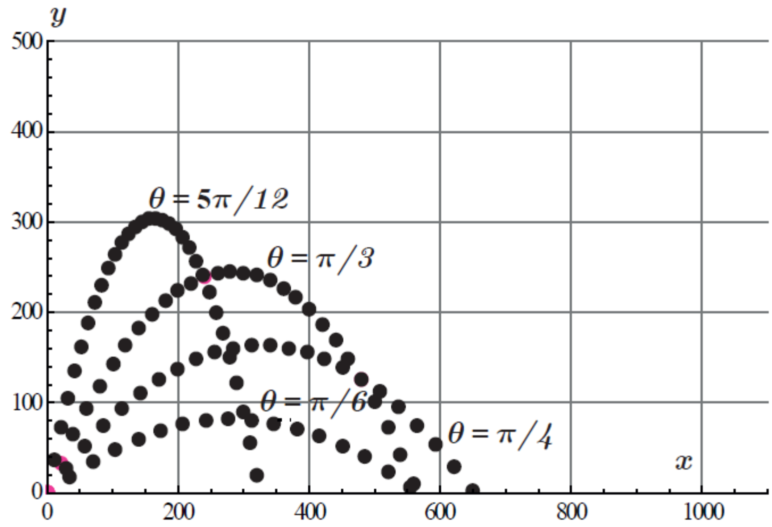

Fig. 8. *For a fixed initial velocity (ball) and if air resistance is ignored, a projectile's trajectory will have a maximum range for an elevation angle of* $\pi/4$ *rad. (Ginsberg, E.S. 1995). From this is Fig. 8, if the value initial velocity (V0) has increase, the range is the horizontal distance the projectile travels to has lessen. By setting V0=80 m/s*

For example the third, numerical evaluation of the projectile motion (Wolfram, 1994; Maeder, 1991).

```
In[1]:= Manipulate[ParametricPlot[{{V Cos[θ] t, V Sin[θ] t - 9.8 t² / 2}},
    {t, 0, tf}, PlotStyle → {{Thick, Black}}, AxesOrigin → {0, 0},
    FrameLabel → {Style[x, Bold, Black, Large], Style[y, Bold, Black, Large]},
    LabelStyle → Directive[Black, Bold], LabelStyle → Directive[Black, Bold],
    Frame → True, FrameStyle → Directive[Thick, Black, 16], Axes → True,
    PlotRange → {{0, 400}, {0, 300}}, ImageSize → 500, Epilog →
     {Black, PointSize[.02], Point[{{V Cos[θ] tf, V Sin[θ] tf - 9.8 tf² / 2}}]}],
  {{V, 48.8, "initial velocity"}, 1, 100, Appearance → "Labeled"},
  {{θ, 1.09, "angle"}, .1, π / 2, Appearance → "Labeled"},
  {{vt, 100, "terminal velocity"}, .01, 300, Appearance → "Labeled"},
  {{tf, 8.75, "time"}, .1, 17, Appearance → "Labeled"}]
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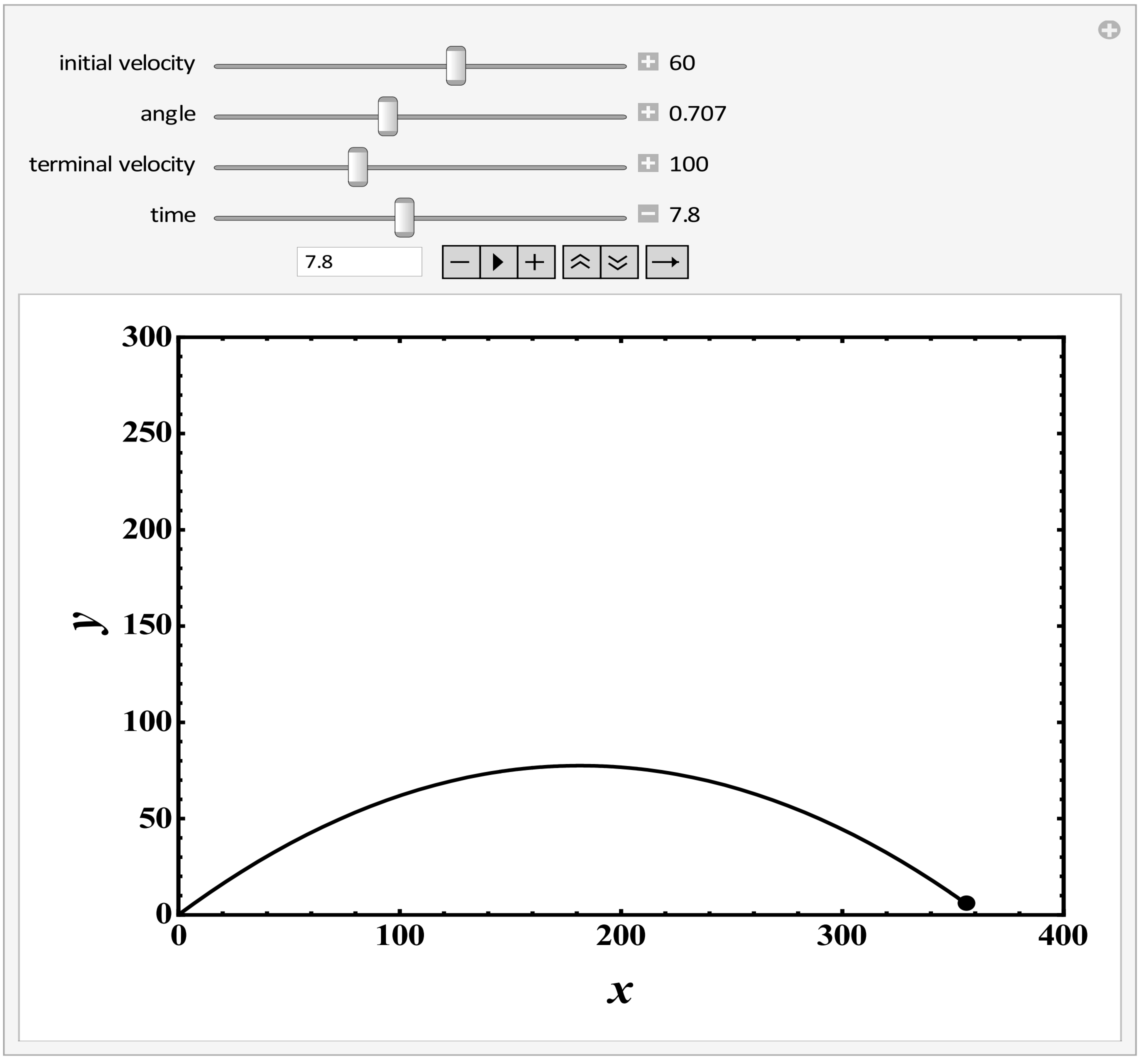

Fig. 9*: A projectile fire from the origin with an initial speed of 60 m/s at various time of 9 second.*

Method

The researcher divided the samples into 2 groups, experiment group and controlled group by random. Fifty students of experiment group study projectile motion by using Mathematica program and fifty students of controlled group study projectile motion by using projectile motion experiment.

Get both experiment group and controlled group to take pre-test by using 30 items of achievement test.

Let experiment group study projectile motion, using Mathematica program and controlled group study projectile motion, using projectile motion experiment set.

Get both experiment group and controlled group to take 60 minutes post-test by using 30 items of achievement test.

Instruments

The instruments used for projectile motion experiment are Mathematica program and projectile motion experiment set.

The instrument used for collecting data is the 30 items of achievement test.

Data analysis

Analyze the physics achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using Mathematica program and Faculty of Science and Technology Phetchabun Rajabhat University students, studying projectile motion by using experiment set of projectile motion, finding the average $\left(\bar{X}\right)$ (The simplest number used to characterize a sample is the *mean*, which for N values x_i, $i = 1, 2, \ldots, N$.) is defined by (Riley, K.F. et al., 2006)

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad (13)$$

and the sample standard deviation (SD) is the positive square root of the sample variance, i.e.

$$SD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2}. \qquad (14)$$

We may therefore write the sample variance SD^2 as

$$SD^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i^2\right)^2, \qquad (15)$$

from which the sample standard deviation is found by taking the positive square root. Thur, by evaluating the quantities $\sum_{i=1}^{N} x_i$ and $\sum_{i=1}^{N} x_i^2$ for our sample, we can calculate the sample mean and sample standard deviation at the same time.

Compare pre-test and post-test (Lord, F.M. 1967; Airasian, P.W. and Madaus, G.F., 1972) of physics achievement of Faculty of Science and Technology Phetchabun Rajabhat University students, studying projectile motion experiment by using Mathematica program and Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using and experiment set of projectile by t-test independent.

The Results

The results of analyzing Pre-test and post-test physics achievement; projectile of Faculty of Science and Technology Phetchabun Rajabhat University students are as the following. From table (1) finding an average of the pre-test of physics achievement, projectile motion of experiment group, studying projectile motion experiment by using Mathematica program is 12.04 and an average of the pre-test of physics of controlled group studying projectile motion by using projectile motion experiment set is 13.30. Using t-test independent Sample to compare achievement finds that the experiment group and controlled group's achievement is different insignificantly. From table (2), Finding the average of physics achievement, projectile motion by using Mathematica program is 24.38 and the average of controlled group studying projectile motion by using projectile motion experiment set is 20.82 Using t-test independent sample to compare achievement finds that controlled group studying projectile motion by using Mathematica program is significantly higher at level of 0.01. It means the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using Mathematica program is higher than the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using projectile motion.

Table 1. *The result of physics achievement study, projectile pre-test*

Statistics *Sample*	*N*	$\bar{X}$	*SD*	*t-test*
Experiment group	50	12.04	2.821	3.743
Controlled group	50	13.30	1.982	

Table 2. *The result of physics achievement study, projectile post-test*

Statistics *Sample*	*N*	$\bar{X}$	*SD*	*t-test*
Experiment group	50	24.38	1.772	12.04
Controlled group	50	20.82	1.289	

Conclusion and Discussion

The achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using Mathematica is higher than the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using projectile motion experiment set at level of 0.01.

The achievement of Faculty of Science and Technology Phetchabun Rajabhat University Technology students who use Mathematica program is higher than the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using projectile motion experiment set. Examining hypothesis finds that the achievement of Faculty of Science and Technology Phetchabun Rajabhat University Technology students using Mathematica program is higher than the achievement of Faculty of Science and Technology Phetchabun Rajabhat University students studying projectile motion by using projectile motion experiment set at level of 0.01 which relates to the specified hypothesis of study since Mathematica program is an easy program. It can be experimented at home or at university. It inspires students to study because it's strange and new for studying which related to the act of education 1999 B.E. part 4 section 22-23 to encourage students to study naturally and efficiently, let students study autonomously, encourage students to enjoy learning, to understand lessons well and make students creative as the purposes of study.

Acknowledgements

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