# Physics: Mathematical Basis and Intuition

**Lu Kannan**
**Physics, S.H. Ho College**

## 1. Introduction

After reading *Science and Method* written by Henri Poincaré, a physics student would probably wonder that if "mind seems to borrow least from the exterior world" when human beings do mathematics (167), how can physicists dare to involve so much mathematics when their job is to understand the exterior world? This essay aims to unveil the role of mathematics in physics so as to answer the preceding question and subsequently to ponder something required in physics which is more than mathematics.

## 2. Role of Mathematics in Physics

### *2.1 View on Mathematics*

First, mathematics needs to be re-understood so as to better understand its role in physics. Mathematics is composed by both postulates and rigorous reasoning. This is quite evident in Euclid's masterpiece *Elements* where the great mathematician placed five postulates and then worked out the subsequent numerous derivations, namely different theorems. The result is that Euclid has built up an axiomatic system which is a network of geometric theorems, an aspect of mathematics, just basing on a few postulates.

The essence of the whole network of Euclidean geometry is that though the reasoning done in *Elements* has little to do with the exterior world, the internal logic through reasoning guarantees the consistence of the whole system and also makes predictions of other possible conclusions which may be extremely vague through superficial observation. For instance, one may not be prone to discover that the sum of the three internal angles of the triangle is equal to two right-angles just through observation. Here, one point needs to be clarified. Actually, in mathematics, such network is not unique. In other words, one may find several different mathematical systems. For example, Euclidean geometry is a widely-accepted one, while Riemannian geometry is another system based on different postulates from Euclid's ("Riemannian Geometry"). More importantly, for mathematics, the logic itself is true and thus the mathematical network itself is closely-bonded and vigorous whatever mathematical system it is.

### *2.2 Motivation of Physics*

Now we come to a step to explain why physics ought to be based on mathematics and this may attribute to the primary motivation of physics. Physics is always intended to study the world with only one general law system. The systematisation of four fundamental forces has been in progress for years ("Fundamental Interaction"). As a physicist, Newton had the insight to regard the falling of an apple from a tree and the circulation of the moon around the earth as the motion governed by the same law—the law of universal gravitation.

### *2.3 The reason of the Combination*

Apparently, according to the elaboration above, mathematics can serve as a huge and vigorous network for physics to rely on because mathematics, in terms of every system itself, is already a consistent system and more cheerfully, the reasoning is internally true. Now it seems reasonable that physical laws are built on mathematics because it is the systematical networks of mathematics that lay a foundation for physics to explain the world within one general system which just fulfills the motivation of physics.

### *2.4 The Benefits*

Moreover, since mathematics plays such a role in physics, the natural facts in physics are automatically connected. And many physicists just have done the same work as Euclid has done in his masterpiece, derivation. For instance, Newton managed to derive the elliptical trajectory of planets of the solar system through mathematical calculation in two different ways (Cohen 52). This is quite acceptable because the mathematics does resemble a network, therefore one can just move from one node to another in different ways. Another example is able to verify the vigor of the mathematical network. Einstein applied mathematical derivations to obtain the sensational equation known as the mass-energy equivalence, $E = mc^2$, from his special relativity equations ("Albert Einstein"). The astonishing fact is that the first real experimental verification was carried out later by John Cockcroft and Ernest Walton ("Mass-energy Equivalence"). This can be seen as a complete triumph of mathematics over simple observation or sense. Mathematics, just as being stated above, is like a huge, closely-related and vigorous network and the shape of it is determined by the postulates. Hence, different mathematical systems based on different postulates can just be seen as networks with different shapes. One point should be emphasised here is that the contents in each mathematical system are consistent despite different postulates. And which mathematical system a physical system should be based on depends on the convenience to do so. For instance, Einstein's special relativity is most conveniently formulated on Minkowski space, one of the mathematical space settings ("Minkowski Space"). Now if the Epicureans (Dunham 271) had seen this result, he would never criticise what Euclid did. If there had not been so many mathematical derivations on physics, human beings would keep stumbling in the path of understanding the world and would never think of some deep relations of the world such as the equivalence of mass and energy because our senses have limitations and can be deceiving. Recall the contents in the Allegory of the Cave in Plato's *Republic*. The essence of it is that what one sees and hears may not be utterly real. Up to now, if one is asked about what the Scientific Revolution is, that it is a revolution of exploration of the world from mere observation to mathematical reasoning, experiment and observation can be a satisfactory answer. Taking the advantage of the mathematical network thus can be regarded as an intellectual exploration for physicists.

## 3. Beside Mathematics, What Is Physics About?

### 3.1. Finding Out the Truth

Now there may be little confusion to refer to physics as a combination of physical postulates, also known as initial assumptions of the world, and mathematical reasoning. And since the mathematical reasoning is guaranteed, for physics, the question goes back to testify the postulates. We are now supposed to step back to think about the Allegory of the Cave. A more difficult question may be that how the prisoner is able to know that the outside of the cave is not another cave, namely the reality because he can only be sure that the cave where he escaped is not the highest level of reality. Similarly, how can we be sure that the contemporary physics is most real rather than just another cave because we can only be sure that some old physical systems are not of the highest level of truth? This kind of questioning is depressing but crucial.

### *3.2 Experiments Have Limitations*

So far, on behalf of modern science, especially physics, much has been focused on putting physical meanings onto the mathematical network, thus the system itself seems to be flawless but this actually attributes to the internal consistence of mathematics. Most of the time, physicists do experiments to testify some conclusions and once these conclusions are verified, it is plausible to verify the postulates which produce the preceding conclusions because of the consistence of the mathematical system but is it enough? The answer is apparently not enough because the experiments always just cover a certain range and always contain errors. For instance, the real number is continuous, but for experiments, the data are always limited and discrete, so the conclusions usually known as theorems may not be testified completely. A more vivid example is just about the special relativity raised by Einstein. As mentioned above, no experiments had shown such phenomenon before the theory came into being. The reason is that the phenomenon of special relativity is evident only when an object moves at a speed which is extremely close to the speed of light, whose magnitude is extremely big and at that time no facility could meet the requirements. Moreover, most physicists at that time just had not thought about it because Newtonian mechanics seemed to be perfect after numerous experiments.

### *3.3 Intuition Matters*

So the most important duty for physicists is to have an appropriate sense of reality, of the truth. For the time being, Einstein's view is the relatively highest level of reality but is it the absolutely highest level of reality? After reading Kandel's *In Search of Memory* on the textbook *In Dialogue with Nature*, there appear some more challenges on the postulates of physics. Can we find building blocks for subjectivity, namely the "elements of subjectivity"

(Kandel 187)? This is a basic idea or approach in physics which conveys a kind of philosophy or physical assumption that all the stuffs can be reduced, namely reductionism. So can we trust in such postulates in physics? If we can, how are we going to deal with consciousness and mind? These are all beyond the capacity of mathematical networks. They are all about physical postulates or in other words, the outlook of the world and for a physicist, his or her intuition matters. Then where does the intuition come from? For a physicist, having an understanding of different philosophies is crucial and thus he or she can make some judgments and modifications so as to form his or her own understanding which may serve as the intuition. Moreover, reflecting on the existing physical systems is also important. Once some fallacies are found out in a certain system, at least a relatively low level of reality is detected, which is helpful to direct us to a better understanding of the world. All the preceding points can be summarised into one statement that the experience contributes to the intuition. And this procedure is time-consuming and twisted but worthwhile. If there really is something prompting this process, it may be a kind of criterion, a criterion for one to make judgments and gradually form the intuition. And the essence of this criterion is the sense of beauty described by Poincaré in *Science and Method*. When one reflects on old works, he may modify his underlying understanding according to the sense of beauty and it is more favourable to believe that the high level of reality is beautiful and elegant.

## 4. Conclusion

To conclude, though mathematics has little to do with the exterior world, it can serve as a consistent network for physics to rely on, which just accounts for the starting question. As a result, mathematicians should always try to enlarge and enhance the network of mathematics thus maybe one day, hopefully, difficult problems of this time can be solved through derivations while physicists should always pursuit more appropriate outlook of the world so as to ensure that the physical system with internally consistency would not be in vain. One thing is for sure modern science is never just about observation anymore.

## Works Cited

"Albert Einstein." *Wikipedia: The Free Encyclopedia*. 7 May 2013.

Cohen, I. Bernard. *The Birth of a New Physics.* 1960. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 49 – 62.

Dunham, William. *The Mathematical Universe*. 1994. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 259–273.

"Fundamental Interaction." *Wikipedia: The Free Encyclopedia*. 7 May 2013.

"Isaac Newton." *Wikipedia: The Free Encyclopedia*. 7 May 2013.

Kandel, Eric R. *In Search of Memory.* 2006. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 179–194.

"Mass-energy Equivalence." *Wikipedia: The Free Encyclopedia*. 8 Aug 2013.

"Minkowski Space." *Wikipedia: The Free Encyclopedia*. 26 Jul 2013.

Poincaré, Henri. *Science and Method.* 2001. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 161–178.

"Riemannian Geometry." *Wikipedia: The Free Encyclopedia*. 7 May 2013.

## Reference

Euclid. *Elements*. 1956. Trans. Thomas L. Heath. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 275–290.

Plato. *Republic*. 2004. Trans. C.D.C. Reeve. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese Univesrity of Hong Kong, 2012. 5–9.

Sivin, Nathan. "Why the Scientific Revolution Did Not Take Place in China—or Didn't It?" 1982. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme*. Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 219–243.

Watson, James D. *DNA: The Secret of Life*. 2003. Rpt. in *In Dialogue with Nature: Textbook for General Education Foundation Programme.* Ed. Chi-wang Chan, Wai-man Szeto, and Wing-hung Wong. 2nd ed. Hong

Kong: Office of University General Education, The Chinese University of Hong Kong, 2012. 97–141.

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## Teacher's comment:

"How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?" Albert Einstein wondered. Inspired by the course "In Dialogue with Nature" and his major subject, physics, Kannan is intrigued by the same question and offers a brilliant attempt to tackle this challenging question. He points out that physicists are building axiomatic systems—each system starts with its own set of postulates with physical meaning and develops into a whole via mathematical reasoning. However, how do we know that the postulates, which are the pillars of a system, are infallible? Kannan argues that experiments do not guarantee absolute reliability and the intuition of physicists also plays an important role in determining the reliability of the postulates. (Szeto Wai Man)