Master‘s Program in Medical Physics

Physics of Imaging Systems

Basic Principles of Magnetic Resonance Imaging II

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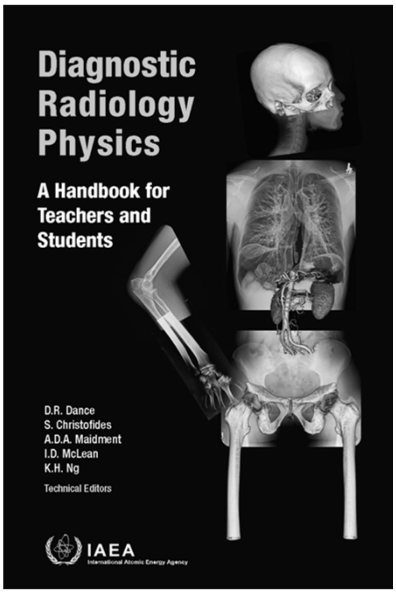
Literature I



Dance et al.:
“Diagnostic Radiology Physics”
Publisher: International Atomic Energy Agency

https://www-pub.iaea.org/MTCD/Publications/PDF/Pub1564webNew-74666420.pdf

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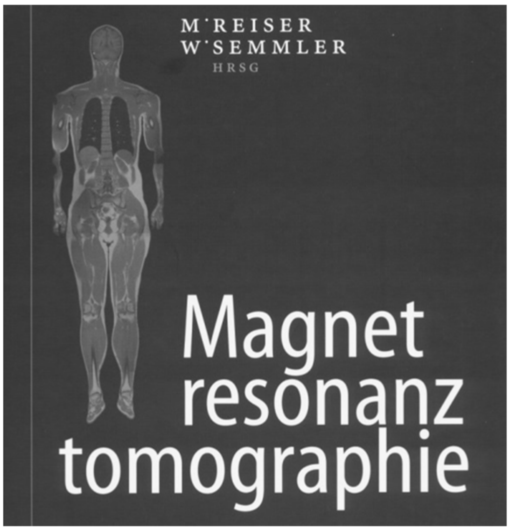
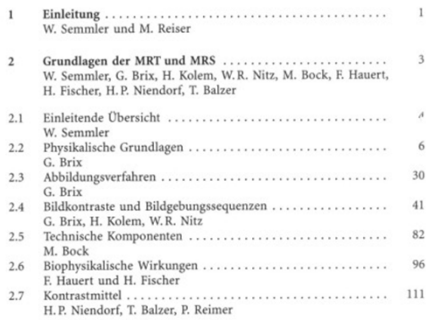
Literature II





Reiser and Semmler: “Magnetresonanztomographie” Chapter 2, 2002

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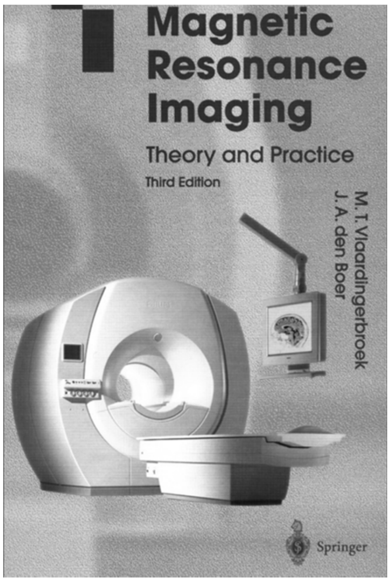
Literature III

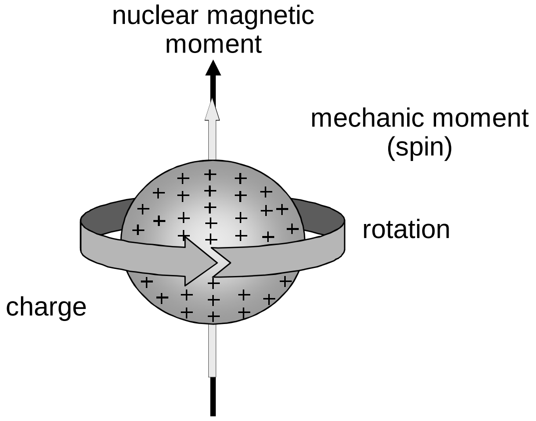


Vlaardingerbroek and den Boer:
“Magnetic Resonance Imaging
Theory and Practice”, 2003

Physics: Nuclei

Physics: Nuclei

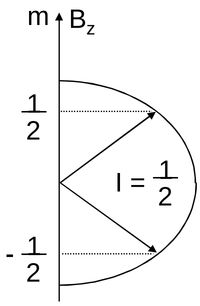
Proton

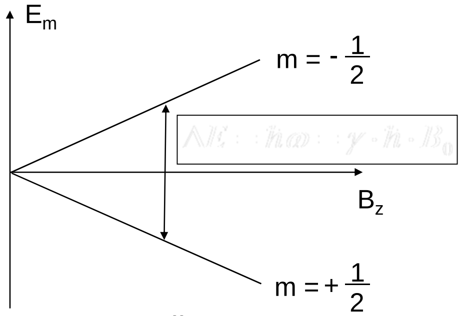


Spin Quantum Mechanics I

$|\vec{I}| = \hbar \sqrt{I(I+1)}$ norm of nuclear spin *I*
with $\hbar = 1.05 \cdot 10^{-34}$ Js Planck's constant

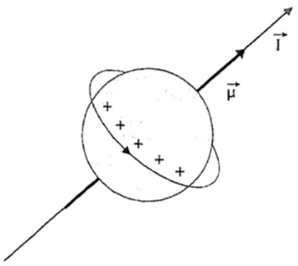
with *m* the magnetic quantum number and
discrete energy levels -*I*, -*I*+1,..., *I*-1, *I*
in total 2*I*+1 possibilities





Zeeman effect

Spin Quantum Mechanics II

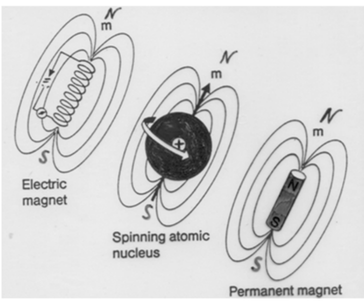


magnetic moment μ
is defined by nuclear spin I:

$$\vec{\mu} = \gamma \vec{I}$$

γ gyromagnetic ratio
proton: $\gamma/2\pi = 42.6$ MHz/T

only nuclei with I ≠ 0
are visible by MRI !!



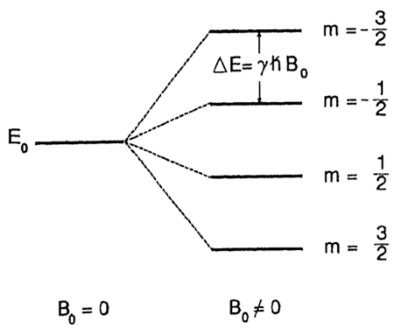
analogy of nuclear magnetism

NMR Nuclei

nucleus	spin *I*	gyromagnetic ratio γ [10^8 rad s^{-1} T^{-1}]	natural abundance of isotope in %	sensitivity $\tilde{S}$ for B_0 = const. in % (rel. to ^1H)		
^{1}H	1/2	2,675	99,98	100,00	MRI:	110 mol
^{19}F	1/2	2,518	100,00	83,40	(MRS:	< 10^{-3} mmol)
^{23}Na	3/2	0,708	100,00	9,27	MRI:	50 mmol
^{31}P	1/2	1,084	100,00	6,65	MRS:	40 mmol
^{2}H	1	0,410	0,01	$9{,}60 \times 10^{-1}$		
^{12}C	0	-	98,89	-		
^{13}C	1/2	0,673	1,11	$1{,}75 \times 10^{-2}$		
^{14}N	1	0,193	99,63	$1{,}00 \times 10^{-1}$		
^{16}O	0	-	99,76	-		
^{17}O	5/2	-0,363	0,04	$1{,}11 \times 10^{-3}$		
^{35}Cl	3/2	0,262	75,77	$3{,}58 \times 10^{-1}$		
^{39}K	3/2	0,125	93,26	$4{,}76 \times 10^{-2}$		
^{25}Mg	5/2	-0,164	10,00	$2{,}68 \times 10^{-2}$		
^{43}Ca	7/2	-0,180	0,14	$8{,}68 \times 10^{-4}$		
^{33}S	3/2	0,205	0,75	$1{,}70 \times 10^{-3}$		

Nuclei in an External Magnetic Field

Zeeman energy levels of nuclei with *I* = 3/2



$$\omega_0 = \gamma B_0$$

ω_0 : Larmor frequency
= 64 MHz for protons at 1.5 T

γ : gyromagnetic ratio
= 42.6 MHz/T for protons

- potential energy in external B_0:

$$E = -\mu_Z B_0$$

$$\mu_Z = \gamma \hbar m$$

$$E_m = -\gamma \hbar m B_0$$

- external RF can induce transition between energy levels if $\Delta m = \pm 1$:

$$E_{RF} = \hbar \omega_{RF}$$

$$\Delta E = \hbar \omega_0 = \gamma \hbar B_0$$

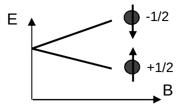
NMR History: Discovery

Germany - Columbia

1938

Isidor Rabi

- rebuilt a molecular beam apparatus (Otto Stern)
- detected nuclear resonance in a stream of Lithium Chloride molecules



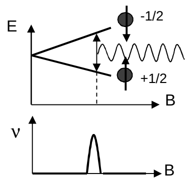
- Nobel prize for physics in 1944

Harvard

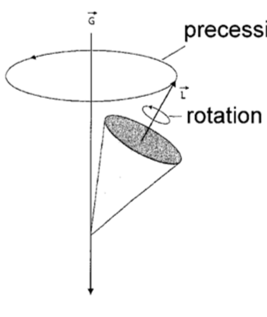
1946

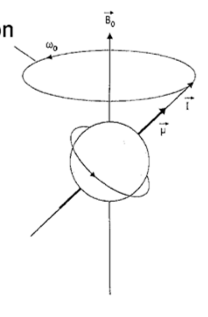
Edward Purcell, Torrey and Pound

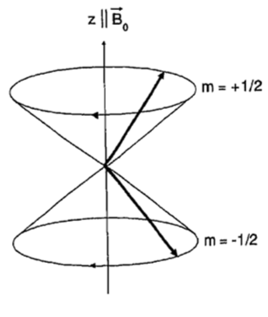
- applied radar technology in investigating magnetic resonance
- achieved the first resonance in a practical sample, a block of paraffin



Semi Classic Description







mechanic spinner

atomic spinner

double precession cone for proton with *I* =1/2

problem of semi classic description:
- mechanic precession = reaction of spinner to external force G
- atomic precession = intrinsic properties of proton resulting from QM

problem solved by macroscopic quantity: M_0 magnetization

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Movie: Summary

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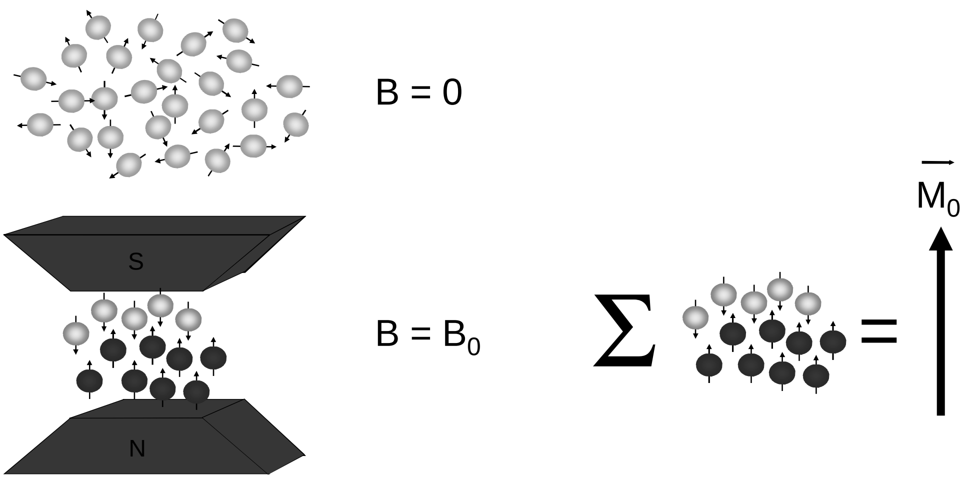
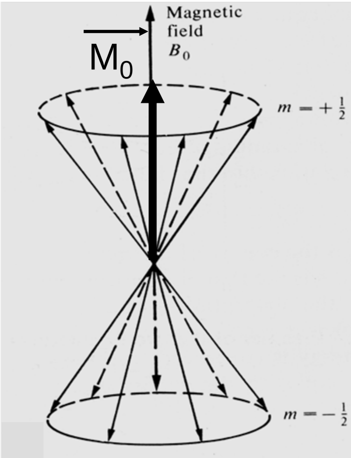
Magnetization M_0 for Spin 1/2



Boltzmann Statistic M_0



Boltzmann statistic protons:

$N_{+1/2} / N_{-1/2} = \exp(\Delta E / kT) = \exp(\gamma\hbar B_0 / kT)$

k : Boltzmann constant = 1.4 10^{-23} J/K
$N_{+1/2}$: number of spins parallel to B_0
lower energy level
$N_{-1/2}$: number of spins anti-parallel to B_0
higher energy level

since $\gamma\hbar B_0 << kT \rightarrow$ Taylor series:

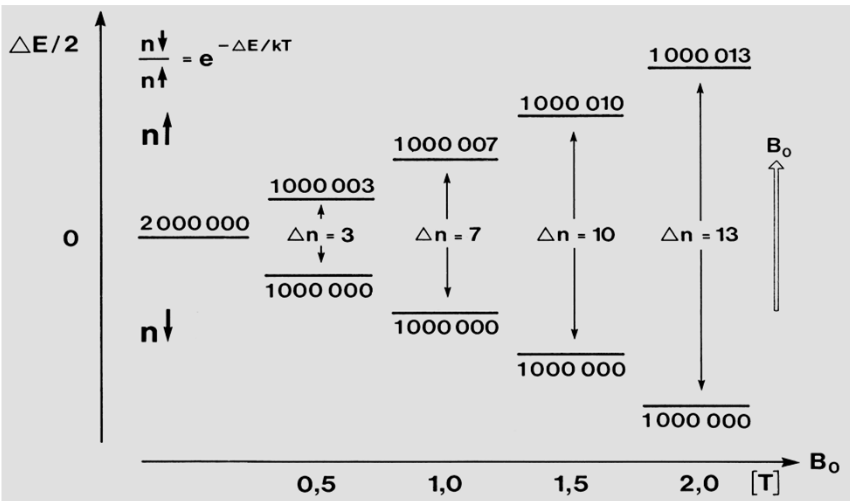
$N_{+1/2} / N_{-1/2} = 1 + \gamma\hbar B_0 / kT = 1.0000066 = 6.6$ ppm

at $B_0 = 1.0$ T and T = 37°C = 310 K.

$M_0 = (N_{+1/2} - N_{-1/2}) <\mu_z> /$ volume

$= (N/V)(\gamma^2\hbar^2/4kT)B_0$

Energy Level Ratio



source: Lissner and Seiderer. "Klinische Kernspintomographie" 1987

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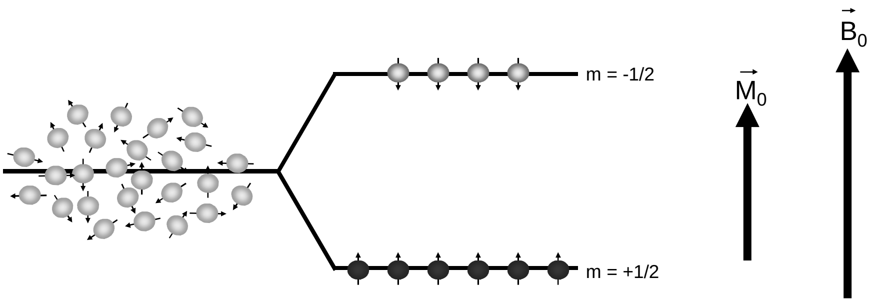
Zeeman Effect

Curie's law: $M_0 = \frac{\rho \cdot I(I+1) \cdot \gamma^2 \cdot \hbar^2 \cdot B_0}{3kT}$



B = 0

$B = B_0$

splitting of energy levels
(Zeeman effect)

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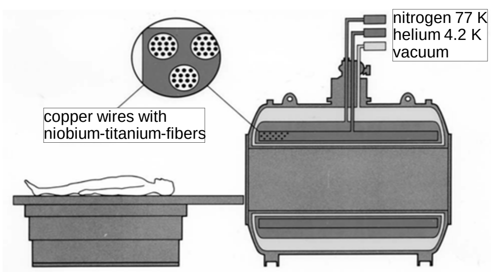
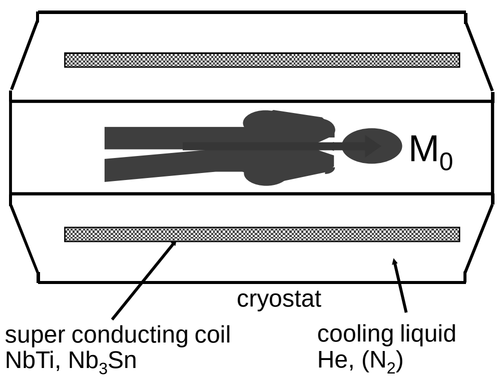
Magnetic Field B_0

static magnetic field B_0

field strength	1.5 – 3.0 Tesla
homogeneity	< 1.0 ppm



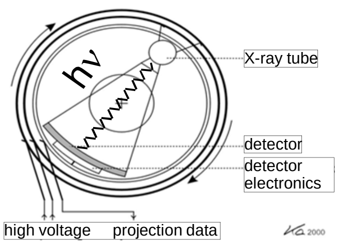


Magnetic Field B_0: Construction

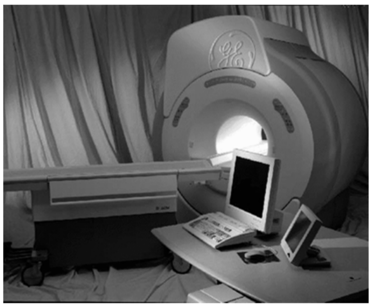
courtesy: Overweg, Philips

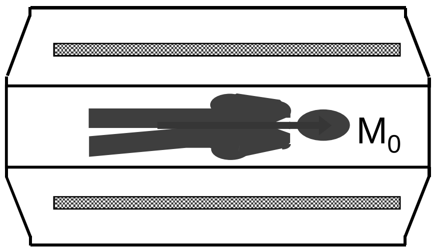
Comparison: CT - MRI

CT = transmission tomography



MRI = “direct” tomography





Correspondence Principle

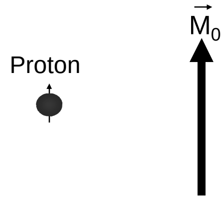
in 1 mm^3 water about $6x10^{19}$ protons ($6x10^{23}$ protons / mol Avogadro number)

10 ppm (10^{-5}) energy level ratio at 1.5 T

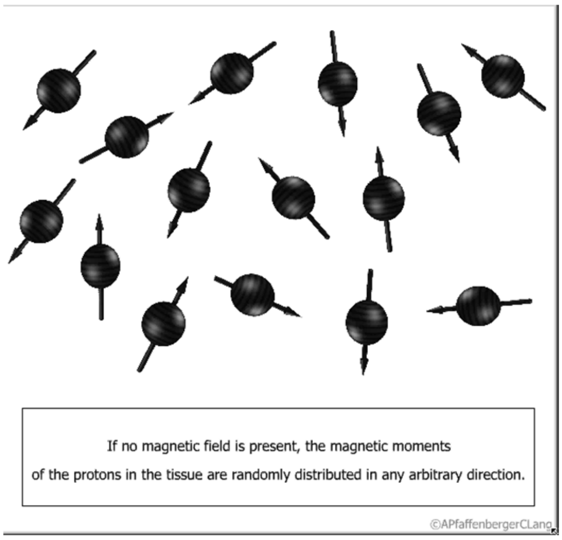
→ $6x10^{14}$ parallel spins in M_0

Bohr's correspondence principle

$\lim_{n \to \infty}$ QM → classical physics



Summary: Proton Bulk



source: Schlegel and Mahr. "3D Conformal Radiation Therapy: A Multimedia Introduction to Methods and Techniques" 2007

Quantum Mechanic ↔ Classical Mechanic

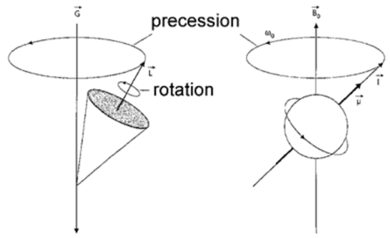
quantum mechanic

Schrödinger equation:

$$\gamma \boldsymbol{B}\hat{\boldsymbol{I}}\psi(t) \;=\; i\hbar\frac{d\psi(t)}{dt}$$

$$\psi(t) \;=\; a(t)e^{-i\omega_0 t/2}\psi_\uparrow + b(t)e^{i\omega_0 t/2}\psi_\downarrow$$

classical mechanic



$$\frac{d\vec{M}(t)}{dt} = \gamma \cdot \vec{M} \times \vec{B}$$

Bloch equation

notice: M is a macroscopic quantity, all nutation angles are allowed → CM
μ is a microscopic quantity, only +1/2 and -1/2 are allowed → QM

Slichter. “Principles of Magnetic Resonance” 1978

NMR History: Theory

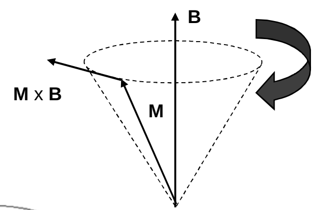
Leipzig - Stanford

1946

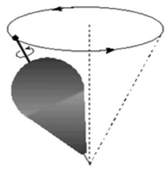
Felix Bloch

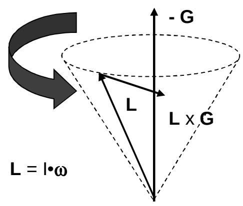
- achieved the same in a sample of water
- provided the mathematical characterization of the nuclear magnetic resonance phenomenon
- Nobel Prize for physics (Bloch & Purcell) in 1952

the Bloch equations



$$\frac{d\mathbf{M}}{dt} = \gamma \cdot (\mathbf{M} \times \mathbf{B})$$





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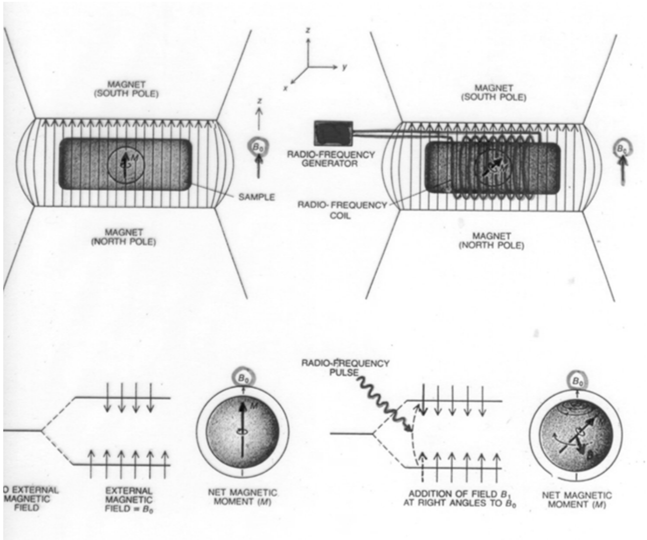
Radiofrequency: Resonance



N Nuclear

M Magnetic

R Resonance

calculation of RF wave length:
$c_{H2O} = c/\sqrt{\varepsilon_{H2O}} \sim c/7$

$\lambda = c/\nu$

$\lambda \sim$ 67 cm at 1.5 T (64 MHz)
$\lambda \sim$ 14 cm at 7.0 T (298 MHz)

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Electromagnetic Spectrum

frequency [Hz]	wave length [m]	photon energy [eV]	radiation	molecular impact
10^{26}	10^{-18}	10^{12}		
10^{24}	10^{-16}	10^{10}		
10^{22}	10^{-14}	10^{8}	x- and γ-ray	DNA break
10^{20}	10^{-12}	10^{6}		
10^{18}	10^{-10}	10^{4}		
10^{16}	10^{-8}	10^{2}	UV-radiation	e^--excitation (orbital)
10^{14}	10^{-6}	10^{0}	visible light	oscillation
10^{12}	10^{-4}	10^{-2}	IR-radiation	rotation
10^{10}	10^{-2}	10^{-4}		
[redacted]				
10^{4}	10^{4}	10^{-10}	MW	
10^{2}	10^{6}	10^{-12}	LW	
10^{0}		10^{-14}		

source: Lissner and Seiderer. “Klinische Kernspintomographie” 1987

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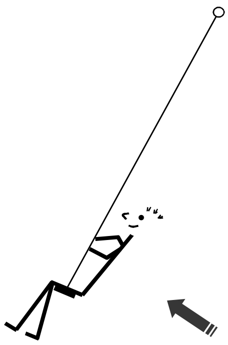
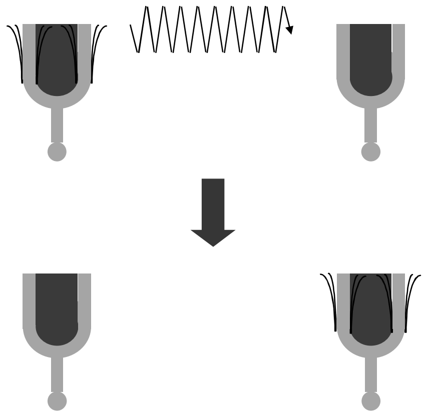
Resonance: Basic Principle



swing



tuning fork

- energy transfer between A and B is only possible if both systems are resonant
- RF system has to work at 64 MHz at 1.5 T

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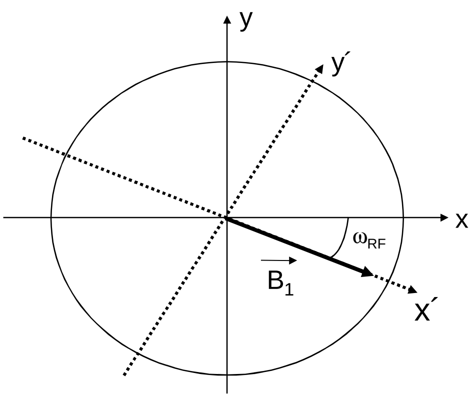
Radiofrequency: Rotating Frame



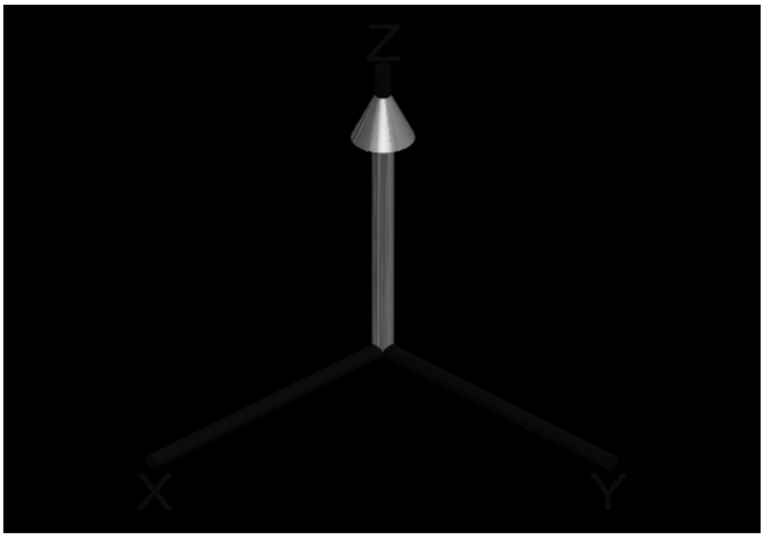
radiofrequency RF:
$\omega_{RF} = \gamma B_0$

in a rotating coordinate system the x' – y' – plane is rotating synchronous with a circular polarized RF-field

→ B_1-vector is not moving in this system !

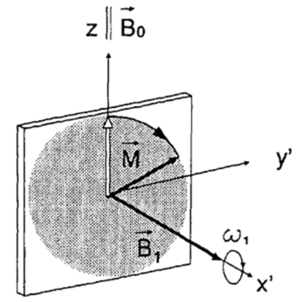
→ rotating M_0-vector only "sees" B_1 !

Movie: Rotating Frame



Magnetization Dynamic

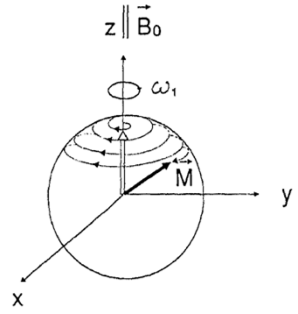
rotating system



in the rotating frame the M_0-vector starts to precede with $\omega_1 = \gamma B_1$ around the direction of B_1

flip angle: $\alpha = \omega_1 t_p = \gamma B_1 t_p$

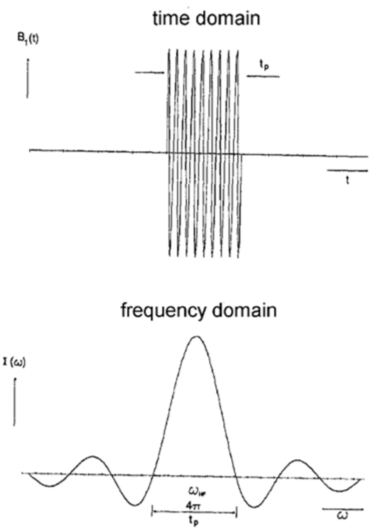
laboratory system



in the laboratory frame the M_0-vector is moving spirally in the direction of the x,y – plane

$B_0 >> B_1 \rightarrow \omega_0 >> \omega_1$

RF-Pulse Characteristics

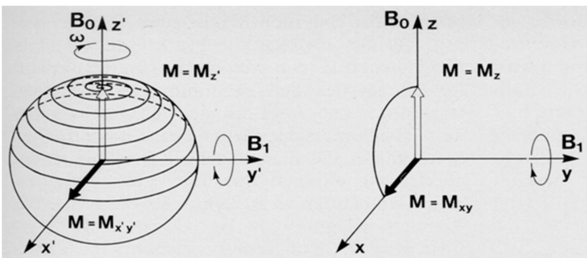


since t_p is a finite quantity the frequency distribution of the excited spins after Fourier transformation does have a frequency shape and bandwidth called

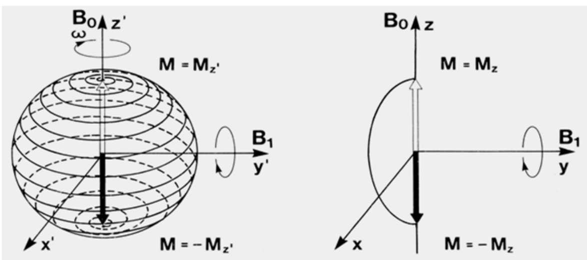
→ sinc-pulse

for $t_p \rightarrow \infty$: frequency spectrum gets monochromatically

90°- and 180°- Pulses



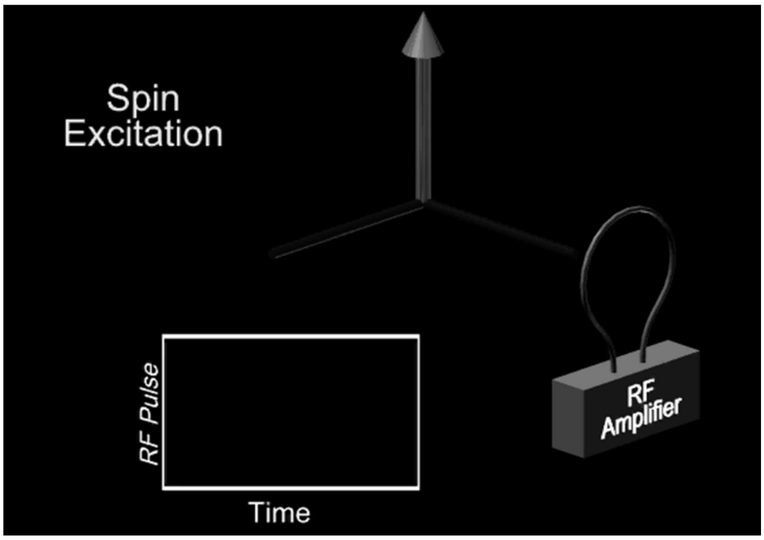
90°-pulse (π/2-pulse) in the laboratory and rotating coordinate system
$N_{-1/2} = N_{+1/2}$
↑ to ↓ : $3x10^{14}$ spins per 1 mm^3 at 1.5 T



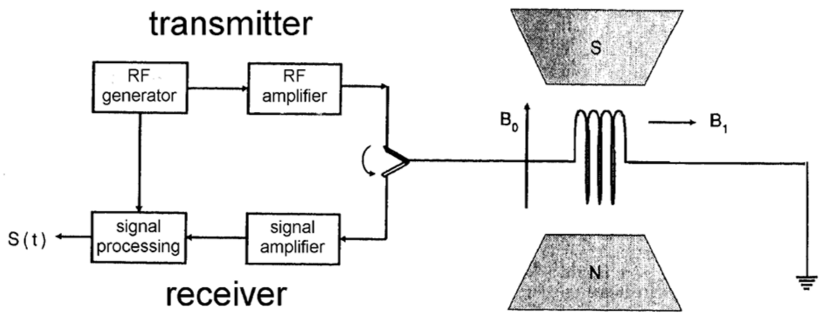
180°-pulse (π-pulse) in the laboratory and rotating coordinate system
$N_{-1/2} > N_{+1/2} = -M_0$
↑ to ↓ : $6x10^{14}$ spins per 1 mm^3 at 1.5 T

source: Lissner and Seiderer. “Klinische Kernspintomographie” 1987

Movie: Spin Excitation

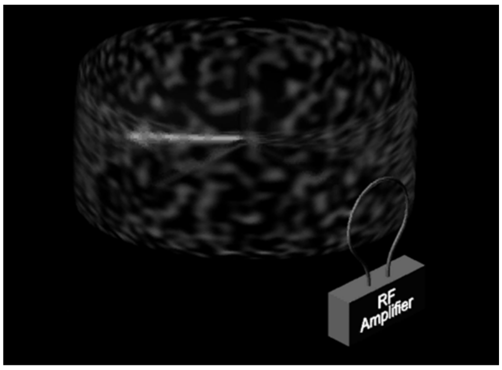


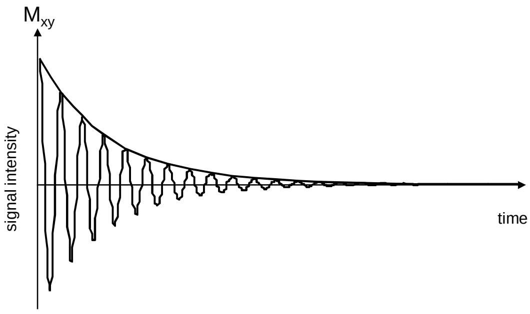
NMR Experiment: Signal Detection



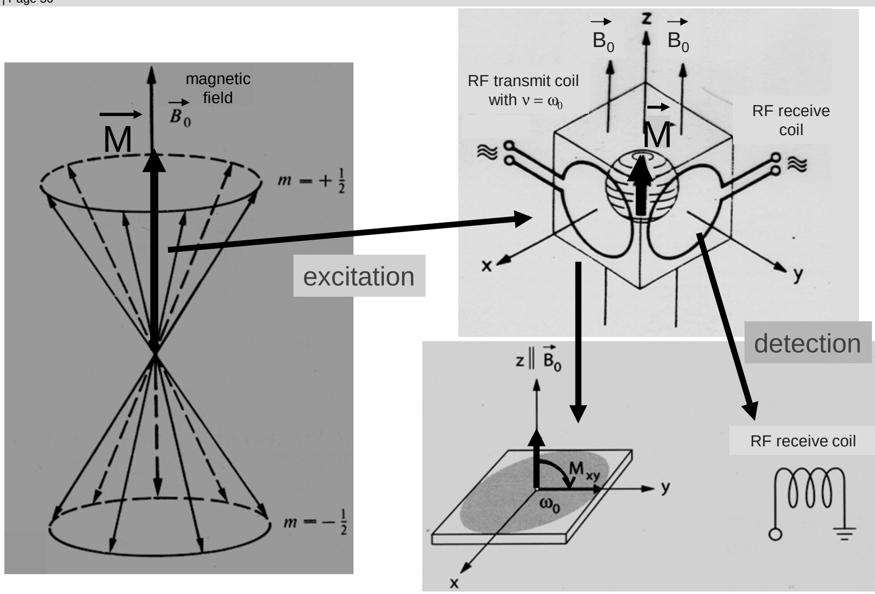
- object is located in a homogeneous static magnetic field B_0
- RF-coil creates a magnetic field B_1 perpendicular to B_0 → transmitter
- after excitation the received signal of the object is transferred by the receiving electronic to the computer → receiver

Movie: Free Induction Decay FID

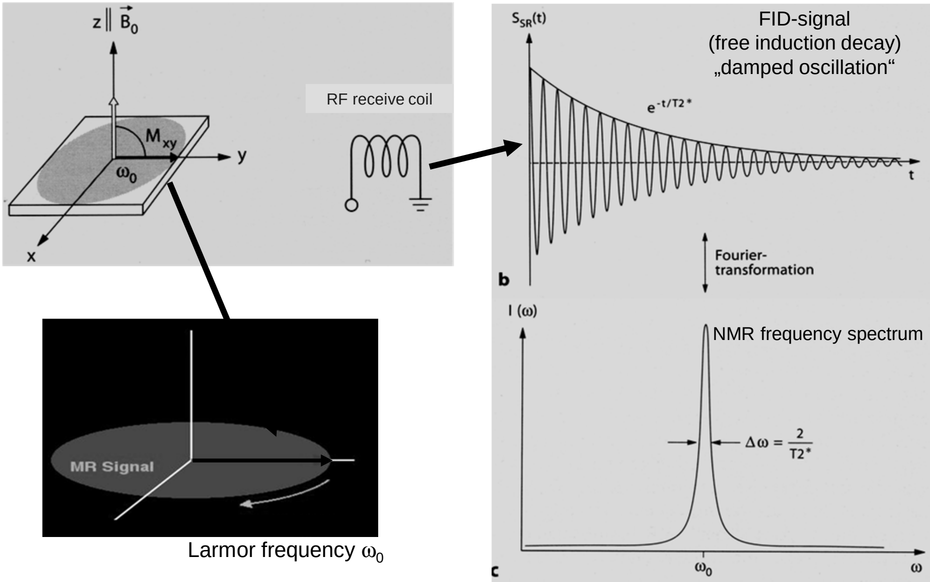




NMR Excitation and Signal Detection



FID Signal and Frequency Spectrum

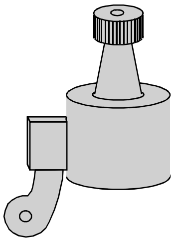


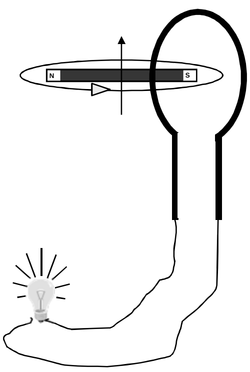
Faraday Induction

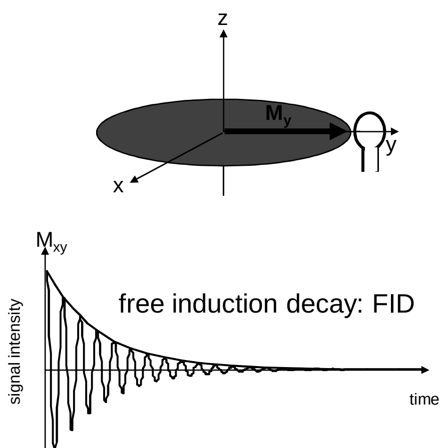
bicycle
dynamo

loop with
rotating magnet

rotating magnetic
moments







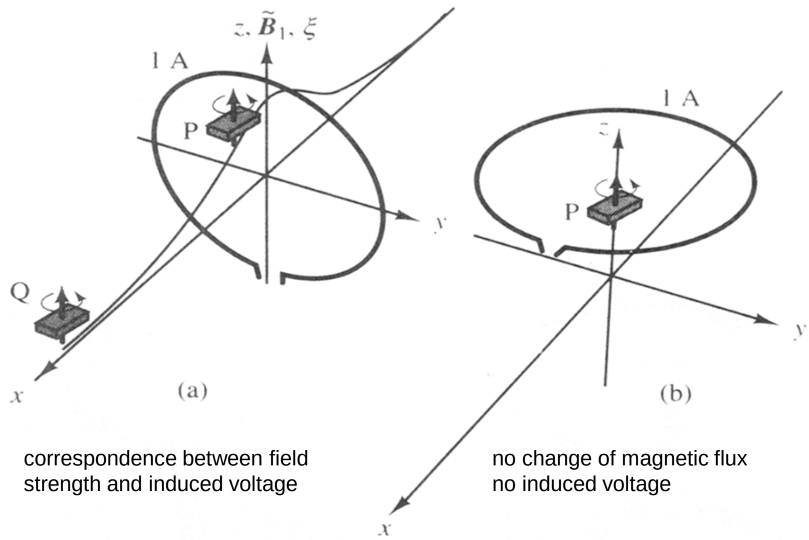
Signal Detection

Based on:
- Faraday law of electromagnetic induction
and
- principal of reciprocity

Electromagnetic Induction: a temporally variable magnetic flux in a loop (receiver coil) induces a charge in this loop which is proportional to the rate of change of the magnetic flux in the loop

Principal of Reciprocity: the sensitivity for detecting a rotating magnetic moment in space is directly proportional to a corresponding electric current in the coil which is necessary for generating the same magnetic field at this point in space

Principal of Reciprocity



correspondence between field strength and induced voltage

no change of magnetic flux
no induced voltage

Hoult. Encyclopedia of Nuclear Magnetic Resonance 1996

Coil Signal

magnetic flux through the coil:

$$\Phi(t) = \int_{Objekt} \vec{B}_r(\vec{r}) \cdot \vec{M}(\vec{r},t) \cdot dr$$

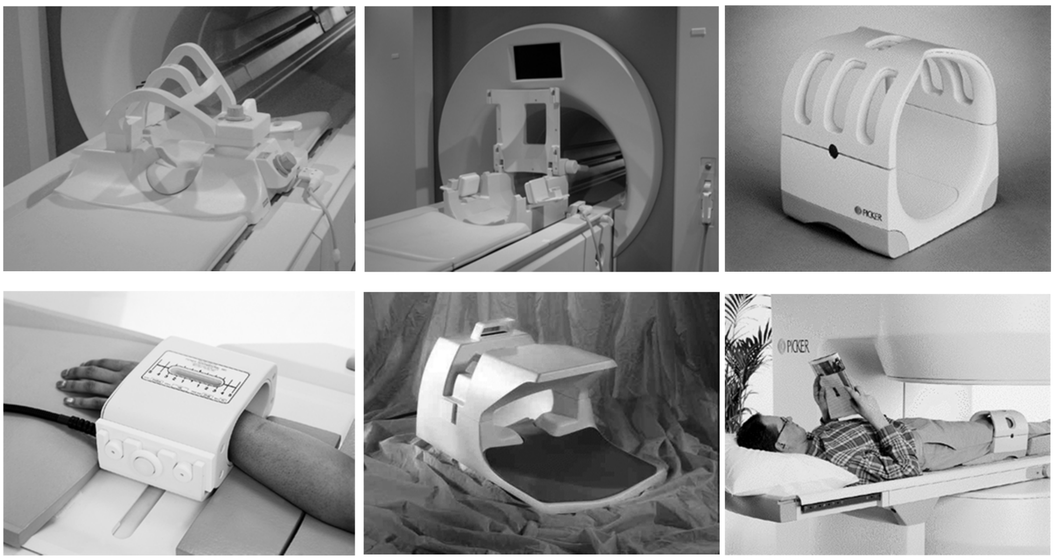
$\vec{B}_r(\vec{r})$

sensitivity of receiving coil

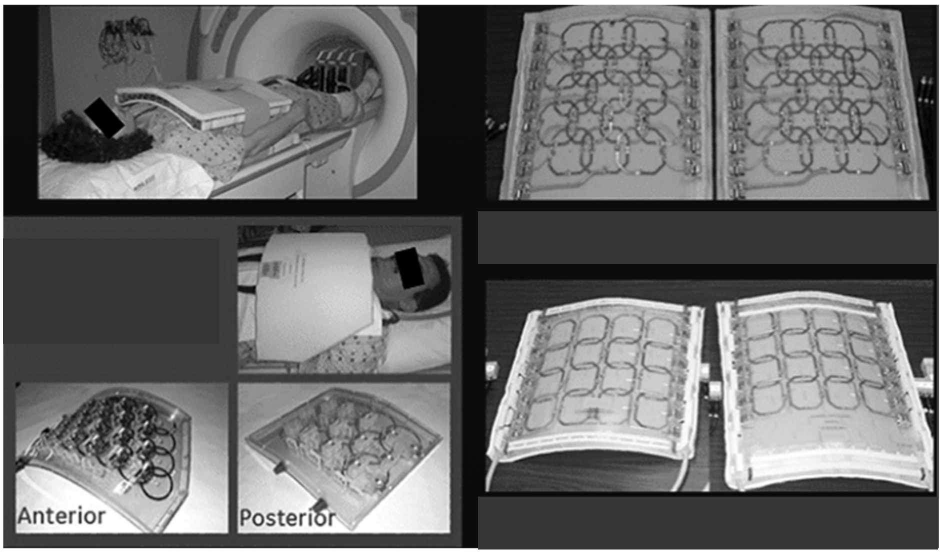
Faraday induction:

$$U_{ind}(t) = -\frac{\partial \Phi(t)}{\partial t} = -\frac{\partial}{\partial t} \int_{Objekt} \vec{B}_r(\vec{r}) \cdot \vec{M}(\vec{r},t) \cdot dr$$

Radio Frequency Coils: Volume Resonators



Radio Frequency Coils: Coil Arrays I

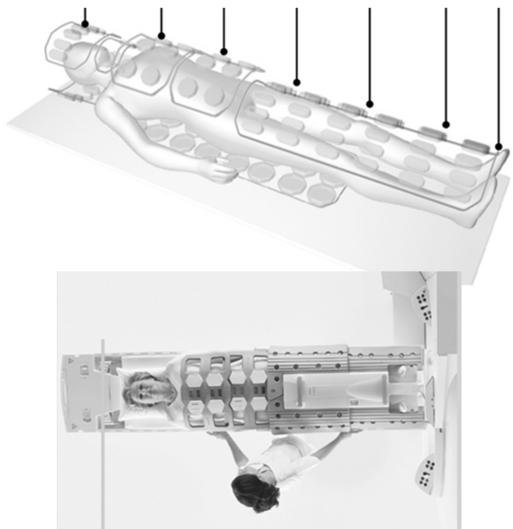


Hardy et al. MRM 2006

Zhu et al. MRM 2004

Radio Frequency Coils: Coil Arrays II

102 seamlessly integrated coil elements
at 32 receiving channels



matrix coils:

head

neck

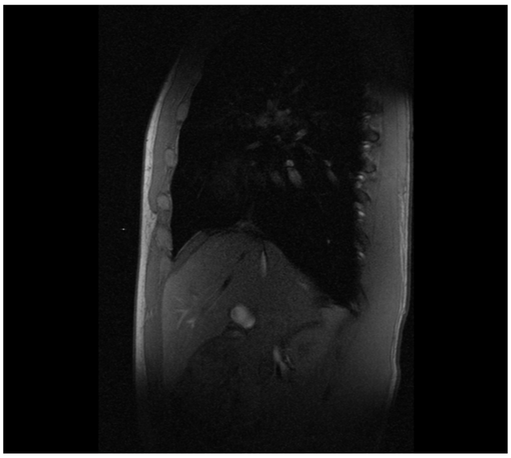
stem

leg

courtesy: Siemens AG, Erlangen

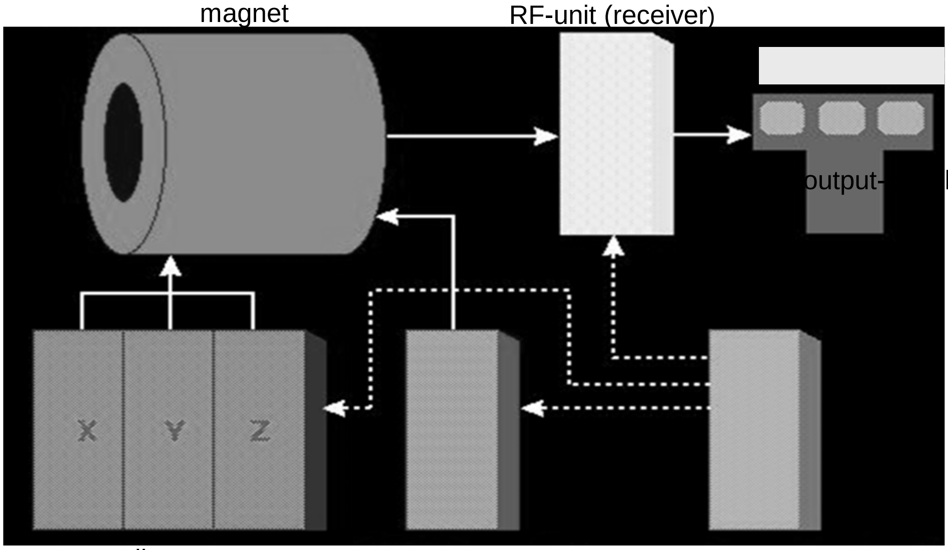
Radio Frequency Coils: Coil Sensitivity

- surface coils
- inhomogeneity correction
- phased array coils
- image combination
- parallel imaging: SMASH / SENSE

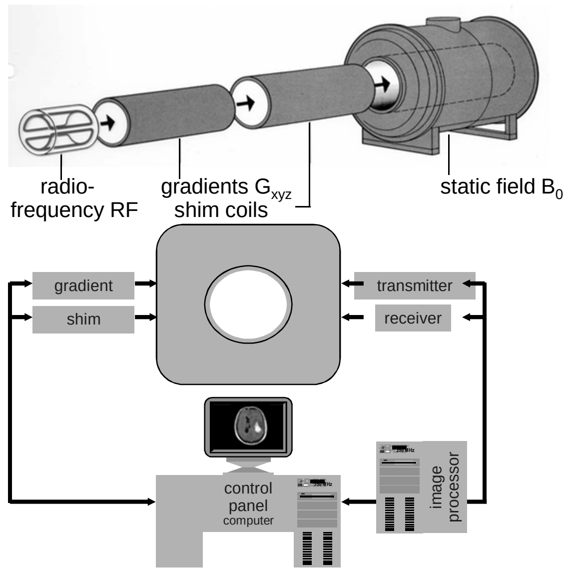


array combination
body coil

MRI Components: Schema



MRI Components: Physical Parameters



technical component	→	physical parameter
static field B_0	→	M_0
radiofreq. RF	→	signal
gradients G_{xyz}	→	image