

Physics of the tractor pull.  
How to use the tractor pull as a  
context rich setting to cover  
Minnesota Physical Science  
Standards.

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Abstract: Whether the source of power is a diesel tractor or a 4-year old girl, the “sled” in a pull competition represents a context rich physics/engineering problem that students at many levels can explore theoretically and verify experimentally. Its simple but elegant design involves a variable mass cart that starts over the wheels and moves forward as the cart is pulled forward until it comes to rest over a skid plate, allowing an increasing resistance force.

In addition to deriving the general equation for the apparatus and verifying it with the apparatus, ideas for how this context can be used to meet Minnesota Academic Standards in Science (as well as national science standards and engineering standards) for all grade levels will be presented.

Science requirements in MN are changing, maybe context should too?

Physics students, 2011. 25% of graduates take physics.

Physics students, 2015. All Minnesota students have to take chemistry OR physics to graduate. How do we get these people who have traditionally avoided physics interested in a “nerdy” subject like physics?

Context rich problems in a real world setting might be one way.

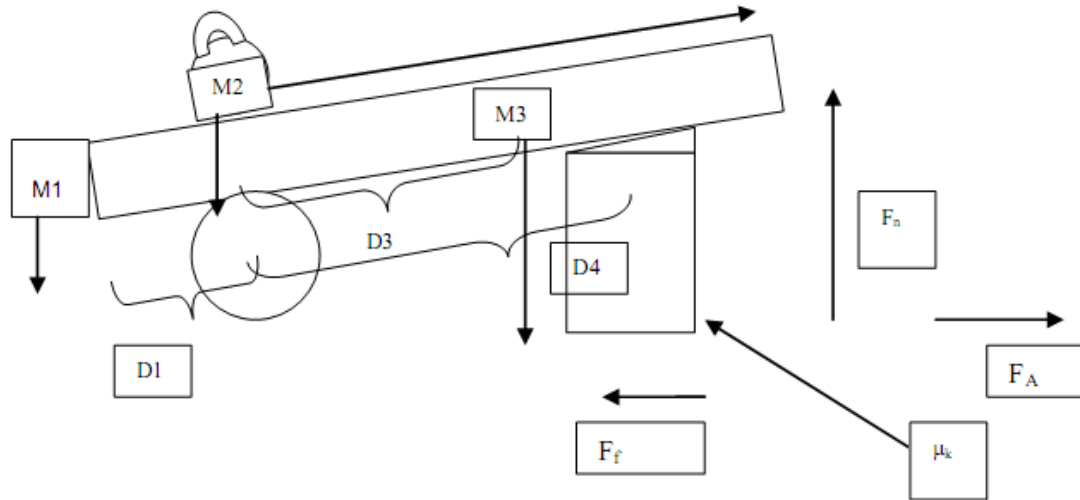
# Real school science

- Teachers are under more pressure than ever to teach to the standards because of laws like No Child Left behind. What follows shows Minnesota Academic Standards in Science that could be addressed by looking at a pedal pull.
- They include physical science content standards and the relatively new nature of engineering standards.

When you do the pull, it goes easy ...  
until it is like hitting a brick wall.

- The nerd in me immediately started thinking, “What would a graph of force vs. time look like for a pull? It is obviously increasing, but is it linear? This would make a great physics problem!”
- The “guy” in me started thinking, “I wonder how much horsepower the puller is generating? And I bet other dads would think this information would be pretty cool to know.”
- Finally, the teacher in me asked, “Could this experience be used as a teaching tool? For what levels?”

# Physics of the sled



Assumptions:  
No weight on tongue.  
No friction on wheels

Also: ramp is inclined, so  
torques are not at 90°,  
but they are =

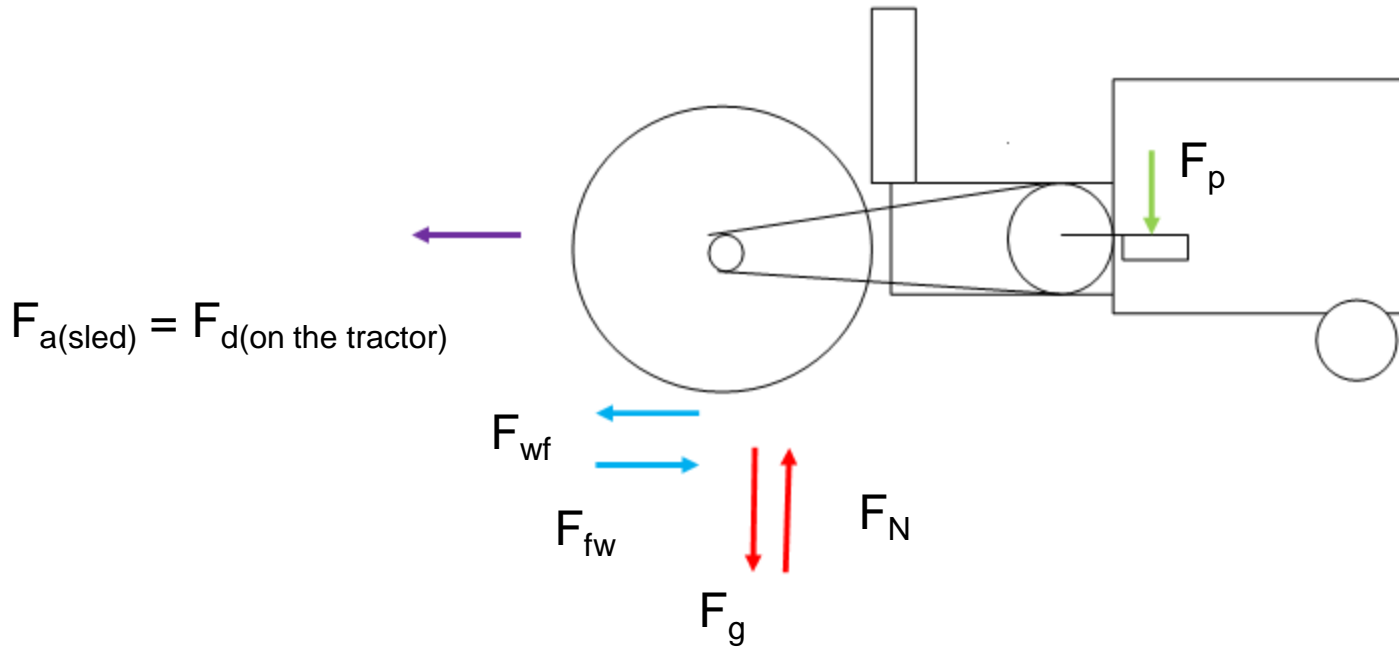
- Assume the sled is free to rotate around the axel, but does not because it remains in rotational equilibrium.
- M1 represents a variable mass off the back of the sled, more mass makes pedaling easier since it lifts the front of the sled, decreasing  $F_N$ . D1 is its distance from the axel.
- M2 represents the mass of a cart + additional mass that moves along the sled from a position over the wheels to a position forward of the skid plate. D2 (not shown) would represent its position from the axel (and it could even be negative, initially, in some designs).
- M3 is the mass of the cart itself, which would act at a distance D3 from the axel.
- The torque from  $F_N$  (normal force) from the skid plate resting on the ground is what keeps the sled in rotational equilibrium. However, as M2 moves forward,  $F_N$  must increase.
- The force of friction ( $F_f$ ) between the skid plate and the ground depends on the surfaces (skid plate and cement) and the  $F_N$ . As  $F_N$  increases,  $F_f$  should increase. To keep the cart in motion,  $F_A$  would have to increase as well.

# Physics of the tractor

- Mass of tractor + puller is distributed over the 4 wheels with more **weight/normal force ( $F_N$ )** in the back.
- **Force applied to pedal ( $F_p$ )**.
- Ratio of pedal radius to front gear radius.
- Ratio of rear gear radius to rear wheel diameter.
- Coefficient of friction between floor and the wheel.
- **Force of floor on wheel ( $F_{fw}$ ), and of wheel on floor ( $F_{wf}$ ).**  
= and opposite, Newton's 3<sup>rd</sup> Law.
- ( $F_d$ ) **Drag of the sled (constantly increasing)**

Assumptions:

- No frictional forces slowing the tractor down.
- Friction between floor and rear wheels is greater than drag of the sled.



# What is the force, horsepower, etc.?

- Find the general equation for the force that the puller must apply to the pedals based on the variables available. What is the maximum force needed at the end of a “Full Pull” of 32 feet (~10 meters)?
- What is the general equation for the work that the puller does over the course of a Full Pull?
- What is the general equation for the average power generated over the course of a pull?



# The tractor side:

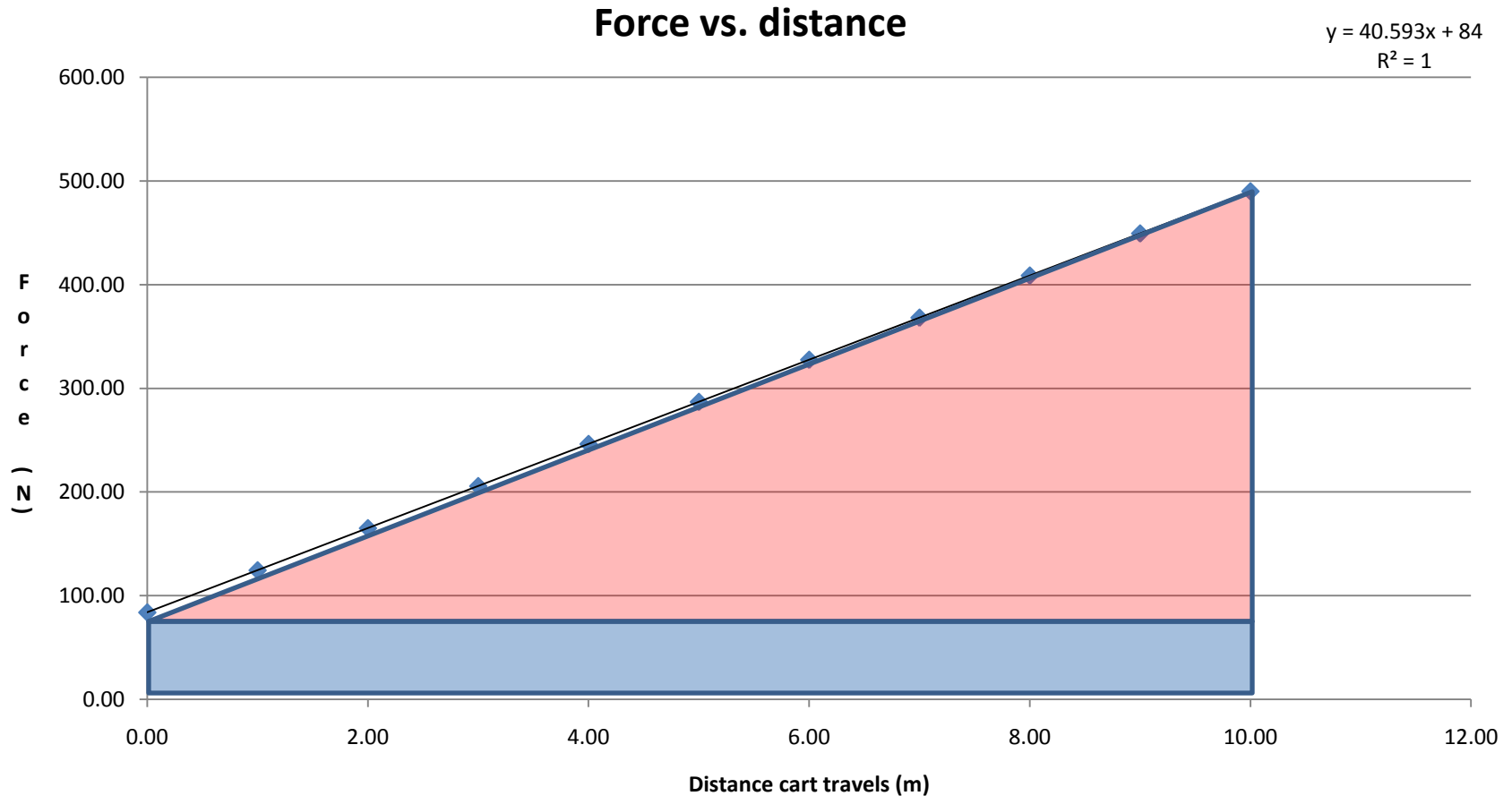
- The friction ( $F_{f_w}$ ) of the rear wheel against the floor would need to equal the drag of the sled to keep the tractor in uniform forward motion (Newton's First Law). When  $\text{drag} > F_{f_w}$  the tractor slows to a stop.
- Using the concept of 2 wheel and axels (one at the pedal/front gear, the other at the rear wheel/rear gear), you get 2 torque equations. Solving these, the force that the puller needs to apply to the pedal will be the force applied to the sled multiplied by the ratio of both wheel/axel combos:

$$F_p = F_a (r_{r.w.}/r_{r.g.})(r_{f.g.}/r_{f.p.})$$

# From the sled side

- $F_T = F_f$ ,  $F_f = F_N \cdot \mu_k$
- $m_1 \cdot g \cdot d_1 + F_n \cdot d_4 = m_2 \cdot g \cdot d_2 + m_3 \cdot g \cdot d_3$
- Rearranging and combining gives:
- $F_T = ((m_2 \cdot d_2 + m_3 \cdot d_3 - m_1 \cdot d_1)g / d_4) \cdot \mu_k$
- Or
- $F_T = m_2 \cdot g \cdot \mu_k / d_4 \cdot d_2 + ((m_3 \cdot d_3 - m_1 \cdot d_1)g / d_4) \cdot \mu_k$
- $F_T = m \cdot d_3 + b$  where  $m$  is the slope and  $b$  is the intercept and both can be calculated from the apparatus itself by measuring masses and distances.
- Thus it appears the force needed to be applied by the tractor should increase linearly.

# Force vs. Distance, find work



Work = Force x distance but also is the area under the curve. Area of rectangle + area of triangle.  $\sim 100\text{N} \times 10\text{m} + 1/2 \times \sim 500\text{N} \times 10\text{m} = 3500\text{N}$

# Engineering design questions

- How do you design a device that can be used to rate a tractor's ability to pull?
- Design constraints:
  - Easily adjustable (small weight added = big force increase) – don't want the workers to get hurt.
  - Finite distance – winners decided within 10m. If the resistance does not change, the puller could pull for miles...
  - Limit the number of runs – If you have to change the weights between each run, it would take forever – ever watched a high jump event?

# Physics ideas that influence design

- Newton's First Law
- Torque, particularly in the gear ratios
- Friction – kinetic, static

# Elementary nature of engineering standards

- 2.1.2.2.1 Identify a need or problem and construct an object that helps to meet the need or solve the problem.
  - *How does one design a device that sorts pullers by strength within a relatively short distance?*
- 2.1.2.2.2 Describe why some materials are better than others for making a particular object and how materials that are better in some ways may be worse in other ways.
  - *What to make the wheels out of so they grip. Does the surface matter?*
- 6.1.2.2.1 Apply and document an engineering design process that includes identifying criteria and constraints, making representations, testing and evaluation, and refining the design as needed to construct a product or system that solves a problem.
  - *Have students design and test models of other designs of a variable resistance sled ... test with toy truck.*
- 6.1.3.1.1 Describe a system in terms of its subsystems and parts, as well as its inputs, processes and outputs.
  - *Breaking down the forces in the sled and tractor. This could be done qualitatively with 6<sup>th</sup> graders as opposed to the quantitative treatment earlier.*

# Nature & Process of Science

- 3.1.1.2.3 Maintain a record of observations, procedures and explanations, being careful to distinguish between actual observations and ideas about what was observed.
- 3.1.1.2.4 Construct reasonable explanations based on evidence collected from observations or experiments.
- 6.1.3.4.1 Determine and use appropriate safe procedures, tools, measurements, graphs and mathematical analyses to describe and investigate natural and designed systems in a physical science context.

Students could make measurements of the force with a large scale. How does it change if weight is added to the back of the sled? To the cart that moves up the sled? To the front of the sled? Hypothesize and test.

How does the force change over distance? Hook the scale to the sled and pull, measuring how force changes as distance changes.

How does the force change depending on surface? Try the sled on different surfaces (asphalt vs. concrete vs. dirt vs. indoors).

# Nature and process of science, cont.

- 7.1.1.2.2 Plan and conduct a controlled experiment to test a hypothesis about a relationship between two variables, ensuring that one variable is systematically manipulated, the other is measured and recorded, and any other variables are kept the same (controlled).
- 8.1.3.3.1 Explain how constraints like scientific laws and engineering principles ... must be taken into account in designing engineering solutions.
- 9.1.1.2.1 Formulate a testable hypothesis, design and conduct an experiment to test the hypothesis, analyze the data, consider alternative explanations and draw conclusions supported by evidence from the investigation.



# Elementary standards

- 2.2.2.2.1 Describe how push and pull forces can make objects move.

*For example: Push and pull objects on smooth and rough surfaces.*

# Elementary school questions

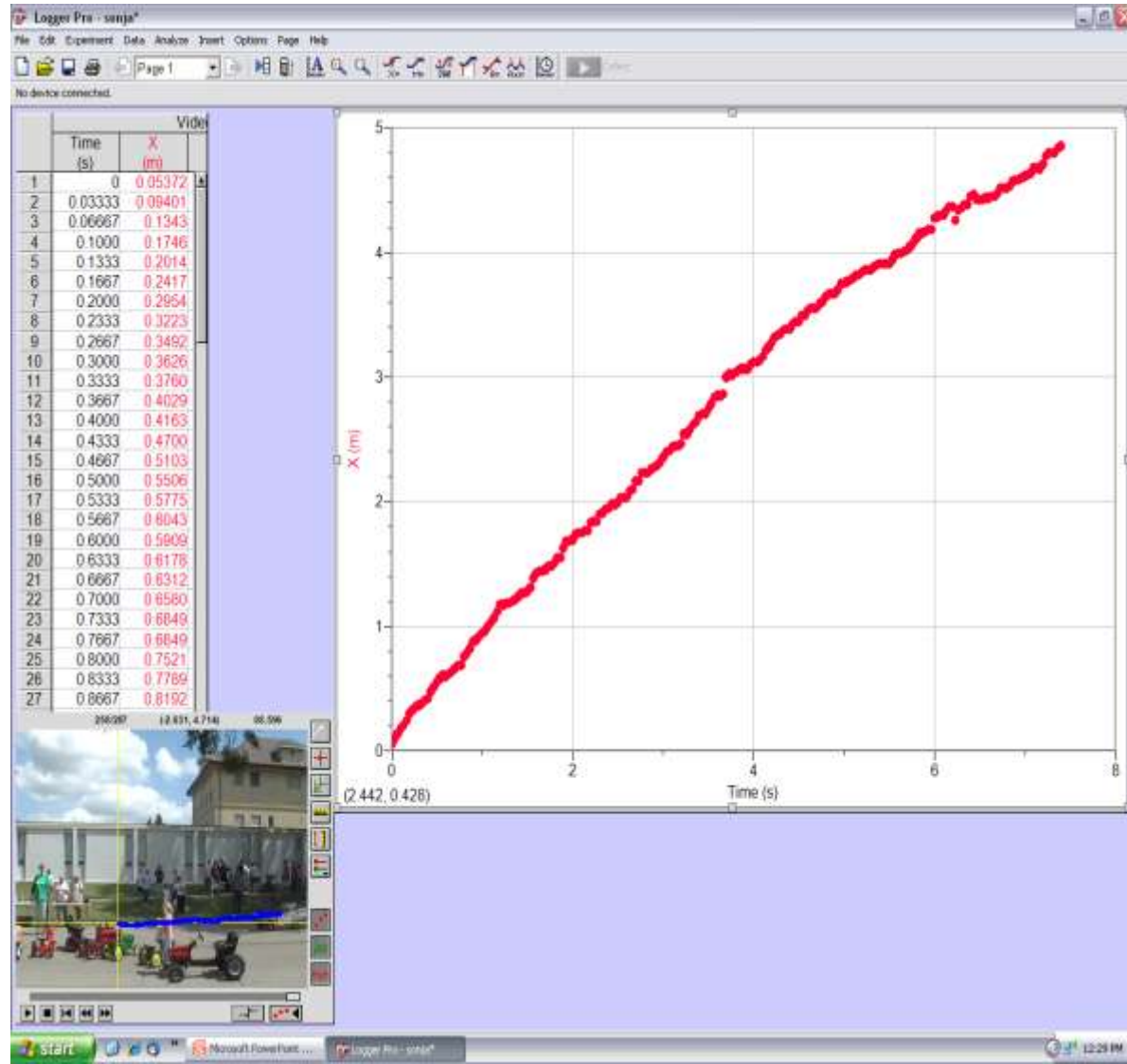
- Does it matter what kind of a surface that the sled is on?
- Does pushing on the pedal always make the sled go? Why or why not?
- Does adding weight to the sled make pulling it harder or easier?

# Select middle school standards

- 5.2.2.1.1 Give examples of simple machines and demonstrate how they change the input and output of forces and motion (*2 sets wheel/axel on tractor*).
- 5.2.2.1.2 Identify the force that starts something moving or changes its speed or direction of motion. (*force applied to the pedals to speed up/friction slows down*)
- 6.2.2.1.1 Measure and calculate the speed of an object that is traveling in a straight line.
- 6.2.2.1.2 Graph an object's position as a function of time and an object's speed as a function of time for an object traveling in a straight line and use the graphs to describe the object's motion. (*See Logger-Pro*).
- **6.2.2.2.1 Recognize that when the forces acting on an object are balanced, the object remains at rest or continues to move at a constant speed in a straight line, and that unbalanced forces cause a change in the speed or direction of the motion of an object.**
- **6.2.2.2.2 Identify the forces acting on an object and describe how the sum of the forces affects the motion of the object.**

# Logger Pro graph of the motion of a tractor and sled.

Notice slope (speed) decreases as the tractor and sled move forward.



# Middle school questions

- Does adding weight to the sled make pulling it harder or easier? Does it matter where the weight is added? Why? Design an experiment to test this relationship.
- Does the tractor make it easier or harder to pull the sled? Why?
- What should the graph of the sled's position/time, velocity/time look like for a pull to failure? Why?
- If the tractor comes to a stop, why should it be virtually impossible to restart it? ( $\Sigma F$  answer – the drag being greater than pull is what stopped it, so unless pull somehow increases to be bigger than drag...).
- Why does the weight need to shift forward during the pull?

# Select high school standards

- 9.2.2.2.2 Explain and calculate the acceleration of an object subjected to a set of forces in one dimension ( $F=ma$ ).
- 9.2.2.2.3 Demonstrate that whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted by the second object back on the first object.
- 9.2.3.2.2 Calculate and explain the energy, work and power involved in energy transfers in a mechanical system.
- 9P.2.2.1.1 Use vectors and free-body diagrams to describe force, position, velocity and acceleration of objects in two-dimensional space.
- 9P.2.2.1.2 Apply Newton's three laws of motion to calculate and analyze the effect of forces and momentum on motion.
- 9P.2.2.2.1 Explain and calculate the work, power, potential energy and kinetic energy involved in objects moving under the influence of gravity and other mechanical forces.

# High school and up questions

- Identify & measure all of the forces present in the system.
- Solve for the work done during a pull.
  - Algebraically ( $W=Fd$ ) or by finding the area under the curve (geometry or using an integral), depending on the math ability of the students in the class.
- Solve for the average power generated in a pull.
  - $P = \text{Work}/\text{time of the pull}$
- If the tractor comes to a stop, why should it be virtually impossible to restart it?
  - $\Sigma F$  answer. In order to decelerate it in the first place,  $F_{\text{friction}} > F_{\text{wt (ground against the wheel)}}$ . But, to accelerate it forward again from a stop the  $F_{\text{wt}}$  would need to be greater than  $F_f$ , which it is not.
  - $\mu_s > \mu_k$ . The coefficient of static friction is generally larger than the coefficient of kinetic friction, meaning it is easier to keep a moving object moving than to get one started from rest (static).

# Some measurements to help you with problems

- $D1 = 0.25\text{m}$
- $D2 = (1.5/9.7) \times \text{Distance the sled has moved}$
- $D3 = 0.6\text{m}$
- $D4 = 1.25\text{m}$
- $m1 = \text{varies } \sim 10\text{Kg per weight added}$
- $m2 = \text{varies } \sim 10\text{kg per big weight, smaller weights are sometimes used.}$
- $m3 = 40\text{kg}$
- $\mu_k = .45$  (steel on asphalt)



# Extensions, etc.

- It is interesting to note that the mass of the person and the mass of the tractor themselves are largely irrelevant, since the friction on the tractor is near zero (compared to the sled). So while the tractor itself is heavy, it does not appear in the equation  $a = \Sigma F/m$ , only  $\Sigma F$  can cause  $a$  to be zero, not  $m$ . If  $F_{\text{pull}} > F_{\text{drag}}$ , it *will* accelerate.

# Too light to pull, friction too small

- One interesting situation is if a puller is too light. If the puller is too light, they do not provide enough normal force with the floor (since  $F_N = \text{mass} \times 9.8\text{m/s/s}$ ) and since the force of friction with the floor (which propels the tractor forward) depends on  $F_N$  ( $F_f = F_N \times \mu_s$ ), the tractor's forward progress might be stopped because the rear wheels might start to spin instead of because the legs fail to provide adequate force. Likewise, if  $\mu_s$  is too small (wet, smooth ground) the wheels may spin.

# Thank you.

- I would like to thank Terry and Tammy Sperr and all the others at T&T Pedal Pulls for their help on this project!
- Check out their website at <http://tandtpedalpulls.com/>