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Matching to Produce Information

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Matching to Produce Information: A Model of Self-Organized Research Teams*

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Abstract

In recent decades, research organizations have brought the "market inside the firm" by allowing workers to sort themselves into teams. How do research teams form absent a central authority? We introduce a model of team formation in which workers first match and then non-cooperatively produce correlated signals about an unknown state. We uncover a novel form of moral hazard: an efficient team of workers producing complementary signals may be disrupted if one of its members can form an inefficient team in which she exerts less effort. This inefficiency rationalizes targeted management interventions which designate specific workers as "project leaders" with more assumed responsibilities.

Keywords: Matching, Teams, Information Acquisition, Correlation.
JEL Classification: C78, L23, D83.

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1 Introduction

Self-organized teams are playing an increasingly important role in economic activity. From 1987 to 1996, the fraction of Fortune 1000 firms with workers in self-managed work teams rose from 27 percent to 78 percent (Lawler, Mohrman and Benson (2001) and Lazear and Shaw (2007)). More recently, a 2016 survey of more than 7,000 executives in over 130 countries indicates that organizations are increasingly operating as a network of teams in which workers engage in self-directed research (Deloitte, 2016). These human resources trends are particularly important in organizations such as Universities (Wuchty, Jones and Uzzi (2007)) and large technology companies, like Google and Amazon, that rely on flexible internal labor markets in order to take advantage of informational complementarities among workers with diverse backgrounds. Yet while the free-ridership problem within teams has garnered considerable theoretical attention (see, for instance, Hölmstrom (1982), Legros and Matthews (1993), and Winter (2004)), less has been devoted to the study of how moral hazard within teams affects sorting. Furthermore, to our knowledge, no existing work studies this interaction in the context of the production of information.

To fix ideas, consider the case of the Danish hearing-aid manufacturer Oticon. In 1987, Oticon lost almost half of its equity when its competitors began selling cosmetically superior devices. In an attempt to regain its competitive advantage, Oticon restructured its research department, replacing vertical, hierarchical production with horizontal, project-based team production. At first, these changes were profitable. Eliminating hierarchies and allowing workers to lead their own teams enabled the firm to take advantage of the existing information dispersed among its workers (Kao, 1996).[1] However, new problems arose. First, some teams were far better than others "in terms of how well the team members worked together and what the outcome of team effort was" (Larsen, 2002). Second, competition meant that "anybody [at a project] could leave at will, if noticing a superior opportunity in the internal job market" (Foss, 2003). These problems eventually led Oticon to selectively intervene in the assignment of workers to teams, designating particular workers as "project leaders".

We show that the types of inefficiencies observed at Oticon, and in other organizations which decentralize information production, arise naturally in a model in which workers

[1] Oticon's CEO commented that decentralization "improved markedly [Oticon's] ability to invent new ideas, concepts, and make use of what [Oticon] actually [had]" (Kao, 1996). In particular, the firm was able to revive old projects that later turned out to be profitable.

cooperatively form teams and non-cooperatively produce information. In the setting we study, workers form teams (match) in order to forecast the value of a Gaussian state. Each worker then acquires any number of costly Gaussian signals about it. The moral hazard problem *within* teams affects the efficiency of sorting *across* teams in two ways. First, productive teams composed of workers producing complementary information may form at the expense of excluded workers who must form relatively unproductive teams composed of workers producing substitutable information. Second, productive teams composed of workers producing complementary information may not form even when efficient; a worker in such a team may prefer to join a less productive team if, in this deviating team, she can exert sufficiently less effort. The latter inefficiency rationalizes the selective management intervention in teams observed at Oticon; by designating specific workers as project leaders, management could eliminate opportunistic deviations by workers in its internal labor market.

To derive these results, we proceed as follows. First, we characterize the (Pareto-Efficient Nash) equilibrium correspondence of the signal-acquisition game played within teams. Our characterization consists of cutoff values on the (state-conditional) pairwise correlation between workers' signals. Intuitively, more positively correlated signals contain more redundant information. Thus, the marginal value of producing a signal when one's teammate has already produced one is decreasing in correlation. It follows that, if the cost of producing a signal is small enough, there is a cutoff above which there is a unique asymmetric equilibrium, and another cutoff below which there is a unique symmetric equilibrium. More subtly, when signals are not too revealing, there is a third, intermediate cutoff above which all equilibria are asymmetric and below which there is at least one symmetric equilibrium (Proposition 1).

Given this characterization, we turn to sorting. Defining, and proving the existence of, a notion of equilibrium in our environment is non-trivial: workers face a one-sided matching problem in which an equilibrium correspondence determines their matching payoffs. Nonetheless, while a stable matching may not exist, as in the Roommate Problem of Gale and Shapley (1962)), we show that by *fixing* non-cooperative equilibria played within each feasible team, we can always find a self-enforcing matching (Proposition 2). We call a collection of such equilibria and a self-enforcing matching a *Coalitional Subgame Perfect Equilibrium (CSPE)*.

We then study the welfare efficiency of equilibrium sorting. For a fixed strategy profile in which each worker produces at least one signal, minimizing pairwise correlation maximizes team productivity. Hence, one might guess that forming teams composed of workers with the lowest feasible pairwise correlations is efficient. But this need not be the case; matching such workers might cause excluded workers to form highly unproductive teams composed of workers with high pairwise correlations. We call this phenomena *Stratification Inefficiency*.

Sometimes, however, a team composed of workers with a low pairwise correlation need *not* form even when it is efficient. A worker in such a team may prefer to match with another worker with whom she has a *higher* pairwise correlation if in that team she can produce relatively fewer signals than her partner in equilibrium. Moral hazard thus generates an additional sorting inefficiency, which we call *Asymmetric Effort Inefficiency*. Hence, while Stratification Inefficient CSPE feature too much inequality in productivity *across* teams, Asymmetric Effort Inefficient CSPE feature too much inequality of effort *within* teams.

We conclude by showing that each inefficiency occurs in an open set of correlation parameters (Proposition 3). Our formal definitions and proofs reveal two important insights relevant to our motivating applications. First, whenever a CSPE is Stratification Inefficient, there is no other efficient CSPE (Observation 1). Hence, Stratification Inefficiency is a robust phenomenon that can only be eliminated by actively assigning workers to teams, in which case self-enforced teams are *not* an optimal organizational structure. Second, in many cases, when there is an Asymmetric Effort Inefficient CSPE, there is multiplicity and an efficient CSPE exists as well. That an efficient CSPE exists suggests a simple resolution to incentive problems: make particular workers more responsible for team output (Observation 2). Then, opportunities to free ride can be eliminated and so the efficient outcome can be obtained as an equilibrium.

Literature

Matching with Nontransferable Utility. Legros and Newman (2007) consider general two-sided matching environments in which, for each matched pair, there is an exogenously specified utility possibility frontier.[2] As matching is two-sided, a stable matching–the core of an assignment game–exists, as established by Kaneko (1982). As we consider a

[2] A well-known application of this framework is to risk-sharing within households. Legros and Newman (2007) and Chiappori and Reny (2016) show that if couples share risk efficiently, then all stable matchings are negative assortative. Gierlinger and Laczó (2018) show that if the assumption of perfect risk-sharing is relaxed, then positive assortative matching can occur. Schulhofer-Wohl (2006) finds necessary and sufficient conditions for preferences under which risk-sharing problems admit a transferable utility representation.

one-sided matching problem, however, the core may be empty; in the absence of restrictions on the expected utility possibilities frontier within each team, cycles can arise (see Online Appendix B for an example). Hence, we define a new, weaker solution concept, Coalitional Subgame Perfect Equilibrium (CSPE). In a CSPE, the non-cooperative equilibria played within teams–even those not formed in equilibrium– are *fixed*. Our existence proof thus demonstrates how after-match equilibrium selection can be used to prevent off-path deviations that undermine stability.[3]

Sorting and Bilateral Moral Hazard. Our paper joins a small literature that considers matching settings in which the utility possibility frontier of each matched pair is affected by the presence of bilateral moral hazard.[4] Kaya and Vereshchagina (2015) study one-sided matching between partners who, after matching, play a repeated game with imperfect monitoring (due to moral hazard) and transfers. While moral hazard limits the achievable joint surplus attainable by a matched pair, transfers ensure that the Pareto-frontier is linear, i.e. payoffs are transferable. Hence, stable matchings exist and (constrained) efficiency is ensured by standard arguments, in contrast to our setting.[5]

Vereshchagina (2019) studies two-sided matching between financially-constrained entrepreneurs in the presence of bilateral moral hazard and incomplete contracts; entrepreneurs can only sign contracts under which the realized revenue is split between the partners according to an equity-sharing rule.[6] Non-transferability of output gives rise to inefficient positive sorting through the following channel: wealthy entrepreneurs, whom contribute more resources to joint production, are willing to form partnerships with poor entrepreneurs only if they receive a high equity share. But, joint surplus maximizing equity shares may be constant across all partnerships. Hence, wealthy entrepreneurs prefer to match even if the overall benefit of re-matching with poor entrepreneurs is large. The logic behind inefficiency thus resembles that of Stratification Inefficiency.[7]

[3]In Section 2.3, we compare our definition and that of the core in detail. It is worth noting that our constructive proof bears resemblance to that of Farrell and Scotchmer (1988), who prove that the core is non-empty in a market for partners whom divide output equally.

[4] Wright (2004), Serfes (2005), Serfes (2007), and Sperisen and Wiseman (2016) study the assortativity of stable matchings in the presence of one-sided moral hazard, i.e. principals matching agents.

[5]Kaya and Vereshchagina (2014) study a special case of their model in which workers form partnerships that may involve "money burning" to provide incentives. They then ask whether workers would prefer to work for an entrepreneur, i.e. hire a budget-breaker, as in Franco, Mitchell and Vereshchagina (2011) to avoid this problem. Chakraborty and Citanna (2005) consider a model similar to that of Kaya and Vereshchagina (2015) in which partners play asymmetric roles.

[6]Two-sidedness again ensures that a stable matching exists, in the sense of Legros and Newman (2007), unlike in our setting.

[7]We note, however, that there is no analog to Asymmetric Effort Inefficiency in her model. A related, earlier contribution is that of Sherstyuk (1998), who shows that equal-sharing equity rules may preclude

Finally, Kräkel (2017) considers a very different channel through which moral hazard leads to inefficient endogenous sorting. He studies an environment in which a firm posts an initial contract that determines both wages and a sorting protocol (workers either endogenously sort into teams or are randomly assigned to teams). The firm then receives interim information about the efficiency of the matches formed and can re-negotiate the initial contract. Under endogenous sorting, workers may form inefficient teams in order to force the firm to re-negotiate the initial contract.

Team Theory. The seminal work of Marschak and Radner (1972) investigates the behavior of a team of agents whom share a common prior and objective function, but possess different information when taking actions. As in this work, we assume that workers in a team have no conflict of interest: they all want to choose an action closest to the realized state. However, in our setting, effort is costly and these costs have implications for the composition of teams that form in equilibrium.

Like us, Chade and Eeckhout (2018) study teams in a matching setting. They study the optimal assignment of workers to teams in a canonical Gaussian environment with two important features: (i) each worker produces exactly one signal within a team and (ii) utility is transferable. In our environment, in contrast to (i), workers can acquire any number of signals and, in contrast to (ii), utility is non-transferable. We are thus able to study the impact of moral hazard on sorting, a "relevant open problem with several economic applications" (Chade and Eeckhout, 2018). Our analysis, consequently, focuses on the *efficiency* of equilibrium teams as opposed to their assortativity, as is the focus of Chade and Eeckhout (2018).[8]

An additional difference between our setup and that of Chade and Eeckhout (2018) is that they assume that signals between workers possess a common correlation parameter, but differ in variance, whereas we assume the opposite. We make this assumption to capture research settings in which workers are identical in their level of "expertise", but may come from different backgrounds. Our work, therefore, contributes to the literature on diversity in teams, i.e. Prat (2002), Hong and Page (2001), and Hong and Page (2004).[9] In particular, Asymmetric Effort Inefficient CSPE are characterized by excessive

efficient heterogeneous partnerships.

[8] As the latter question is of independent interest, however, in Online Appendix A we discuss how endogenous effort might affect the equilibrium assortativity of teams. Fixing the signal structure of Chade and Eeckhout (2018), we show that, once effort choice is endogenous, optimal matching must simultaneously diversify, while incentivizing effort.

[9] Prat (2002) finds conditions under which a team should be comprised of homogenous information structures when these information structures are priced according to market forces. Hong and Page (2004) and Hong and Page (2001) consider the performance of heterogeneous non-Bayesian problem solvers. In

homogeneity, i.e. high correlation, within teams. Our results thus illustrate a new channel through which moral hazard can cause homogenous teams to form even when they are suboptimal.

Correlation and Information Acquisition. More broadly, our analysis of the information acquisition game played within teams is related to recent work defining notions of complementary and substitutable information. In the environment we consider, lower correlation implies higher complementarity in terms of the value of information. Börgers, Hernando-Veciana and Krähmer (2013) define signals as complements or substitutes in terms of their value across *all* decision problems, therefore requiring stronger conditions. Liang and Mu (2020) adapt the definition of Börgers, Hernando-Veciana and Krähmer (2013) to a multivariate Gaussian environment and use it characterize the learning outcomes of a sequence of myopic players.

2 Model

2.1 Environment

Four workers, indexed by the set $\mathcal{N} := \{1,2,3,4\}$, are uncertain about a state θ and share a common Gaussian prior with mean μ_θ and variance σ_θ^2.[10] Each worker can obtain unbiased, conditionally independent Gaussian signals with variance σ^2. Within a team, however, signals are correlated; $\rho_{ij} \in [-1,1]$ is the state-conditional correlation coefficient between worker i's and worker j's signal when they work together.

Prior to production, workers form teams of at most two workers; forming a team of two incurs a cost of $K > 0$ on each member. The final assignment of workers to teams is therefore described by a matching function $\mu : \mathcal{N} \to \mathcal{N}$ such that the teammate of worker i's teammate, j, is i–that is, if $j = \mu(i)$, then $\mu(j) = i$.[11] Let $\mathcal{M}$ denote the set of all such functions. After teams have been formed, each worker i simultaneously and independently chooses a number of signals to produce, $n_i \in \mathbb{N} \cup \{0\}$, at cost $c(n_i)$, where $c : \mathbb{N} \cup \{0\} \to \mathbb{R}$ is an increasing function satisfying increasing marginal costs, i.e. $c(n) - c(n-1) \geq c(n-1) - c(n-2)$ for any $n \geq 2$, and $c(0) = 0$.

The correlation structure in the signal-acquisition stage captures the economics of a situation in which joint and simultaneous effort is affected by complementarities, while

contrast, we consider the *endogenous* formation of teams by Bayesian workers within a firm with a fixed information structure.

[10]The analysis extends easily to the case of N workers.

[11]We interpret (i,i) as a single-worker team.

unilateral effort is not. In particular, we interpret n_i as a decision by worker i to produce a single signal in each of n_i consecutive "periods", starting from period 1; if $n_i \geq n_j > 0$, then workers i and j produce signals jointly in periods $t \in \{1, ..., n_j\}$. Hence, signals drawn in these periods are conditionally correlated according to ρ_{ij}. If $n_i > n_j$, however, then worker i produces a signal alone in periods $t \in \{n_j + 1, ..., n_i\}$. Consequently, in these periods, workers cannot exploit correlation between signals to learn about the state. Figure 1 depicts the case in which $n_i = 3$ and $n_j = 2$.

Finally, after observing the signal realizations of every team member, each team takes an action $a^* \in \mathbb{R}$ to minimize the expected value of a quadratic loss function. Formally,

$$a^* \in \arg\min_{a \in \mathbb{R}} E_\theta \left[(a - \theta)^2 \mid x^S \right],$$

where x^S denotes the concatenation of signals observed in the team.

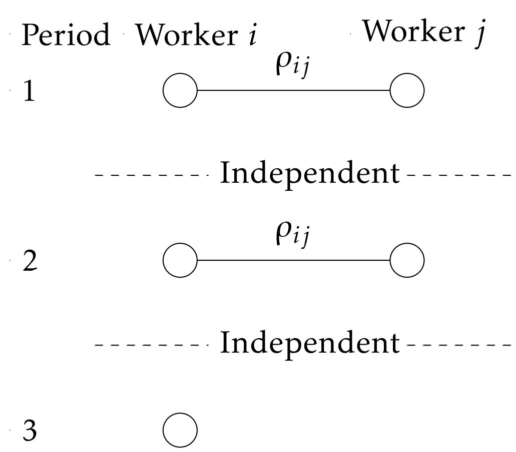


Figure 1: Signal structure when $n_i = 3$ and $n_j = 2$.

2.2 Solution Concepts

A signal-acquisition strategy for worker i is a function mapping teammate identity to a non-negative integer, $n_i : \mathcal{N} \to \mathbb{N} \cup \{0\}$.[12] Given a strategy for each player, we denote the profile of signals chosen in team (i, j) by $n(i, j) := (n_i(j), n_j(i))$. The payoff to worker i in team (i, j) given the strategy profile $n(i, j)$ is

$$v_i(n(i,j); \rho_{ij}) := -E_x \left[\min_{a \in \mathbb{R}} E_\theta \left[(a - \theta)^2 \mid x^S \right] \right] - c(n_i(j)) - K \mathbb{1}_{i \neq j}. \tag{1}$$

[12] We consider pure strategies for ease of interpretation and tractability.

To ease notation, we denote $n_i(j)$ and $n_j(i)$ by n_i and n_j and drop the dependence of v_i on ρ_{ij} whenever there is no confusion that j is i's teammate.

The strategy spaces for each player, $\mathbb{N} \cup \{0\}$, and the payoff functions defined in Equation 1 constitute a normal-form game–call it the **Production Subgame**.[13] To account for pre-play communication, in each team (i,j), we require that the strategy profile $n^*(i,j)$ is a **Pareto-Efficient Nash Equilibrium (PEN)** of the Production Subgame. For the two-stage game, we introduce a new solution concept called *Coalitional Subgame Perfect Equilibrium (CSPE)*.

Definition 1. *A matching $\mu \in \mathcal{M}$ and a collection of PEN, $N^* = \{n^*(i,j)\}_{i,j \in \mathcal{N}}$, is a **Coalitional Subgame Perfect Equilibrium (CSPE)** if there does not exist a matching, $\mu' \in M$, and a worker i for which i and $j = \mu'(i)$ are strictly better off under μ' given the PEN:*

$$v_i(n^*(i,j)) > v_i(n^*(i,\mu(i))), \text{ and}$$

$$v_j(n^*(i,j)) > v_j(n^*(j,\mu(j))).$$

A matching $\mu \in \mathcal{M}$ and a collection of PEN, one for every feasible team, is a CSPE if no worker(s) can form a deviating team in which, given the prescribed PEN in that team, each worker obtains a strictly higher payoff.

2.3 Relationship to the core

The standard solution concept in the literature on matching with imperfectly transferable utility is the core. A matching function and a point in the utility possibility frontier for each matched pair is in the **core** if (i) no matched worker is better off alone and (ii) no pair can match and pick a point in their utility possibility frontier that makes both strictly better off.[14] While the core is certainly well-defined in our environment– the set of PEN payoffs within a team is its utility possibility frontier–condition (ii) is problematic; if workers are free to play *any* PEN in a deviating team, then cycles of re-negotiation may arise and cause the core to be empty. To circumvent this problem, we define a new solution concept, CSPE, in which each off-path team plays a *fixed* PEN. This limits the set of payoffs achievable by a deviating pair of workers and enables us to prove existence (Proposition 2). In addition to the advantage of existence, we find CSPE both intuitive

[13]If a worker i decides to work alone, then the Production Subgame is to be interpreted as a decision problem.

[14]See Legros and Newman (2007) for a general definition in two-sided environments and Kaya and Vereshchagina (2015) for a definition in a one-sided environment.

and plausible; every core allocation is also a CSPE and those which are not are sustained by credible "off-path" behavior, i.e. Pareto-Efficient Nash Equilibria.

3 Production Subgame Analysis

3.1 Preliminaries

Because each worker's payoff function is quadratic, her optimal action given any signal realization is the posterior mean. Hence, her expected payoff when signals are costless is the negative posterior variance. Lemma 1 states these observations and provides a closed-form solution for the posterior variance.

Lemma 1. *Suppose workers i and j form a team and acquire (n_i, n_j) signals with $n_i \le n_j$. Each worker's optimal action is $a = E(\theta \mid x)$ and the expected payoff of worker i is*

$$v_i(n_i, n_j) = Var(\theta \mid (n_i, n_j)) - c(n_i) - K\mathbb{1}_{i \neq \mu(i)},$$

where x is the concatenation of realized signals and

$$Var(\theta \mid (n_i, n_j)) := \begin{cases} 0 & \text{if } i \neq j, n_i > 0, n_j > 0 \text{ and } \rho_{ij} = -1 \\ \left(\left(\frac{2n_i}{1+\rho_{ij}} + (n_j - n_i)\right)\sigma^{-2} + \sigma_\theta^{-2}\right)^{-1} & \text{otherwise.} \end{cases}$$

The pairwise correlation coefficient $\rho := \rho_{ij}$, for $i \neq j$, measures the complementarity between workers: as ρ increases, the value of working together decreases. For intuition, consider the extreme cases. When $\rho = -1$, by producing $(1, 1)$ signals, a team can match the state by choosing an action equal to the sample average. On the other hand, when $\rho = 1$, working together to produce $(1, 1)$ signals is equivalent to having only one worker produce a signal. So, to rule out uninteresting cases, we assume that the cost of a single signal satisfies the following two properties: (i) in a two-worker team in which $\rho = -1$, both workers have an incentive to produce a single signal (and so perfectly learn the state), and (ii) in any team, at least one worker has an incentive to produce at least one signal.

Assumption 1. $c(1) < \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2} \min\{\sigma_\theta^2, \sigma^2\}$.

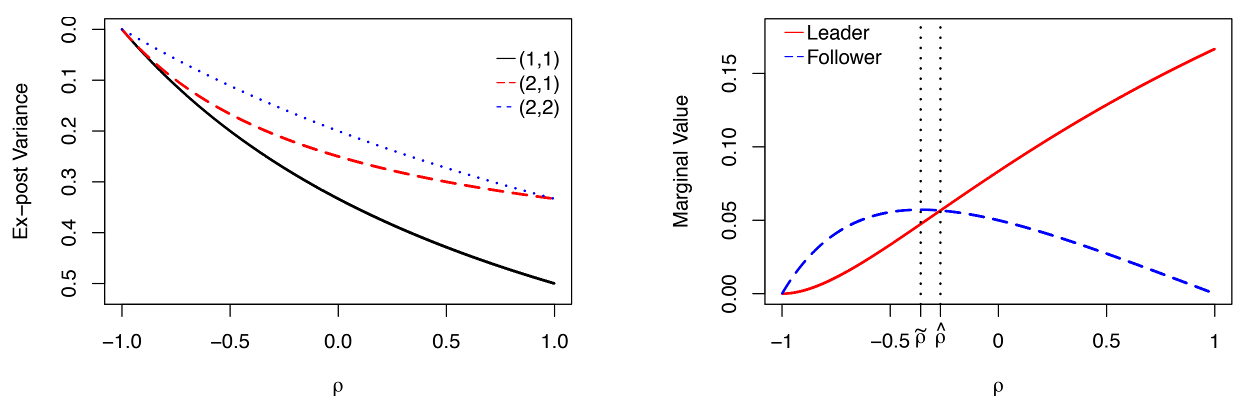
3.2 The Marginal Value of Information

To characterize PEN, we define and analyze the **marginal value of information** to worker i of producing a signal in the n_i-th period given that worker j produces a signal in the first n_j periods. This marginal benefit corresponds to the reduction of the ex-post variance generated by the last signal:

$$MV(n_i; n_j, \rho) \equiv Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)).$$

If $n_i \geq n_j$, we call worker i a **leader**. If the inequality is strict, we call worker j a **follower**.

Figure 2a illustrates the posterior variance $Var(\theta \mid (n_i, n_j))$ for different correlations, ρ, and strategy profiles, (n_i, n_j), in the case in which $\sigma = \sigma_\theta = 1$. In Figure 2a, the difference between the dashed red line and the solid black line is the marginal value of information to a leader of producing a signal in period two, while the difference between the dotted blue line and the dashed red line is the marginal value of information to a follower of producing a signal in period two, given that the leader is already producing one in the first two periods. The former difference is represented by the solid, red line in Figure 2b, while the latter is represented by the dashed, blue line in Figure 2b.



(a) Ex-post Variance.

(b) Marginal Value of Second Signal.

Figure 2: Ex-post Variance and Marginal Values.

We make three observations about the figures, which generalize beyond the parameterization we consider, and which we exploit in proving our main characterization result. First, the marginal value of information to the leader is strictly increasing in ρ. This happens because the value of the information obtained from working together with the follower in previous periods *decreases*. By concavity of the information production function, the marginal value of information left to learn increases.

Second, the marginal value of information to a follower is non-monotonic in ρ. Indeed, we see the *difference* between the blue line and red line in Figure 2a is non-monotonic, and so the blue line in Figure 2b is hump-shaped. The marginal value of the follower is increasing in an initial region for the same reason the leader's marginal value is increasing; when ρ increases, the value of work done together in past periods decreases and so the marginal value of information left to learn increases. However, there is another effect to consider. When ρ increases, the value of working together with the leader in a future period *decreases*– the leader and follower's information is less complementary. After an interior cutoff value $\tilde{\rho}$, the second effect dominates and the marginal value of information to the follower decreases.

Third, the marginal value of a leader is higher than the marginal value to a follower above a negative cutoff value, $\hat{\rho}$. It turns out that the relationship between $\hat{\rho}$ and $\tilde{\rho}$ is the key to ordering the equilibrium correspondence in terms of symmetry. We discuss this in detail after stating our main characterization result.

3.3 PEN Characterization

Proposition 1. *Let ρ denote the pairwise correlation between workers in a two-worker team. For each $\rho \in [-1, 1]$, there exists a PEN of the Production Subgame. If Assumption 1 is satisfied, there exist interior cutoff values $\rho^* \leq \rho^{**}$ for which the following properties hold:*

1. *For $\rho \leq \rho^*$, there is a unique PEN. It is symmetric and each worker produces a strictly positive number of signals.*

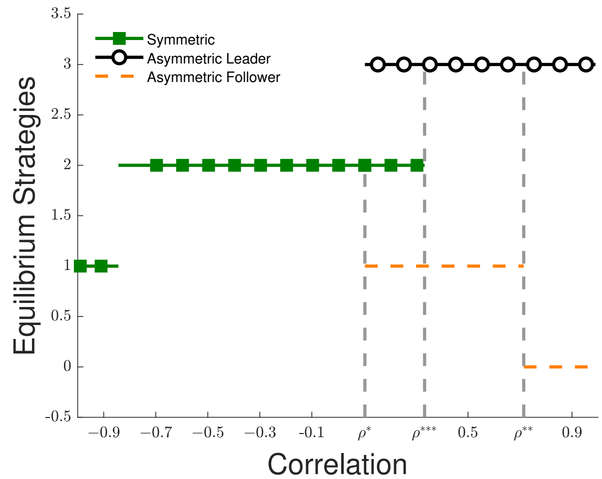
2. *For $\rho > \rho^{**}$, generically, there is a unique PEN up to the identity of each worker. In it, one worker produces a strictly positive number of signals and the other produces none.*

If, in addition, $\sigma^2 \geq \sigma_\theta^2$, there exists another cutoff value ρ^{***} for which $\rho^* \leq \rho^{***} \leq \rho^{**}$ and the following properties hold:

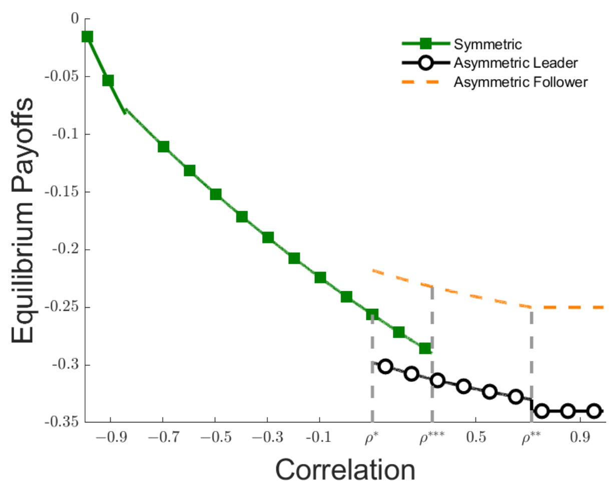
3. *For $\rho \in (\rho^*, \rho^{***}]$, there is at least one symmetric and one asymmetric PEN.*

4. *For $\rho > \rho^{***}$, all PEN are asymmetric.*

Figure 3 illustrates the Proposition in a case in which $\sigma^2 \geq \sigma_\theta^2$, so that all four properties apply: below $\rho^* \approx .10$, the unique PEN is symmetric; above $\rho^{**} \approx .71$, there is a unique PEN, up to identity, in which only one worker exerts effort; for $\rho \in (\rho^*, \rho^{***}]$, where



(a) Equilibrium Correspondence.



(b) Payoff Correspondence.

Figure 3: PEN Correspondence with $\sigma^2 = \sigma_\theta^2 = 1$ and $c(n) = 0.01n^2$.

$\rho^{***} \approx 0.33$, there is both a symmetric and an asymmetric PEN; and, for $\rho > \rho^{***}$ all PEN are asymmetric.

The intuition for the first two properties is simple. Under Assumption 1, (i) $(1,1)$ is the unique equilibrium when $\rho = -1$ and (ii) for any pairwise correlation, at least one worker produces at least one signal. Since posterior variance is continuous in pairwise correlation, (i) implies that there exists a cutoff, strictly above negative one, below which the unique equilibrium is (1,1), establishing the first property of the proposition. Property (ii) implies that when $\rho = 1$ there is, generically, a unique equilibrium, up to identity, in which only one worker produces a strictly positive number of signals.[15] As the marginal

[15]The genericity qualifier rules out the case in which a worker is indifferent between two positive integers

value of a follower's signal is close to zero for correlations near one, continuity of the posterior variance in correlation again implies that there is a cutoff strictly below one above which only one worker acquires signals.

The intuition for the last two properties is subtle. It turns out that if signals are sufficiently noisy, $\sigma^2 \geq \sigma_\theta^2$, then the marginal value of information for a follower is maximized at a correlation $\tilde{\rho}$ strictly below the value at which the marginal value of information for a leader exceeds that of the follower, $\hat{\rho}$, as in Figure 2b. Hence, increasing ρ past $\hat{\rho}$ increases the marginal value to the leader while decreasing the marginal value to the follower. Behavior then coheres with intuition; higher correlations drive equilibria to be asymmetric because leaders have an increasing incentive to acquire more information, while followers have a decreasing incentive to match the signals produced by leaders. We thus obtain a strong result: there is an intermediate cutoff above which all PEN are asymmetric and below which there is at least one symmetric and one asymmetric PEN.[16]

We conclude our analysis by pointing out two properties satisfied by the equilibrium correspondence in Figure 3, but which are *not* ensured by the condition $\sigma^2 \geq \sigma_\theta^2$ alone: (i) $\rho^* < \rho^{***}$ and (ii) there is a non-trivial asymmetric equilibrium, i.e. an equilibrium in which each worker produces a strictly positive number of signals, for correlations above ρ^*. The latter property is satisfied whenever a worker produces at least three signals by herself, i.e. whenever the marginal value of a third signal exceeds the marginal cost. Assumption 2 thus ensures that both (i) and (ii) are satisfied; we will require it to prove the robustness of Asymmetric Effort Inefficiency.

Assumption 2. $\rho^* < \rho^{***}$ *and* $c(3) - c(2) < MV(3;0,0)$.[17]

3.4 Existence of CSPE

Exploiting symmetry of the equilibrium correspondence of the Production Subgame, we provide an algorithm that identifies a Pareto-Efficient CSPE.

when $\rho = 1$ and her teammate takes zero draws.

[16] If $\hat{\rho} < \tilde{\rho}$, a counterintuitive phenomena emerges. In this case, there is a region in which increasing ρ past $\hat{\rho}$ increases the marginal value for both the leader *and* the follower. Hence, if an asymmetric equilibrium is played at some correlation ρ above $\hat{\rho}$, but below $\tilde{\rho}$, it may be the case that for a higher correlation a symmetric equilibrium may be played. Why? The increase in the value of information left to learn for the follower might induce her to match the leader's signal. If this happens, the leader's incentive to produce another signal may decline enough so that she does not produce another one herself. For such an example, we direct the reader to Online Appendix C.

[17] While we have not stated the condition $\rho^* < \rho^{***}$ in terms of primitives, it holds whenever the cost function is not "too convex". For instance, if costs are linear, it is always satisfied. As our running example demonstrates, however, the condition is satisfied beyond the case of linear costs.

Proposition 2. *A Pareto-Efficient CSPE exists.*

Two comments are in order. First, Proposition 2 generalizes to environments beyond the one we consider; as long as the after-match game is symmetric, a CSPE exists. Second, as previously discussed, while a CSPE exists the core may be empty. See Online Appendix B for a formal definition of the core and an example of its emptiness.

4 Inefficient Sorting

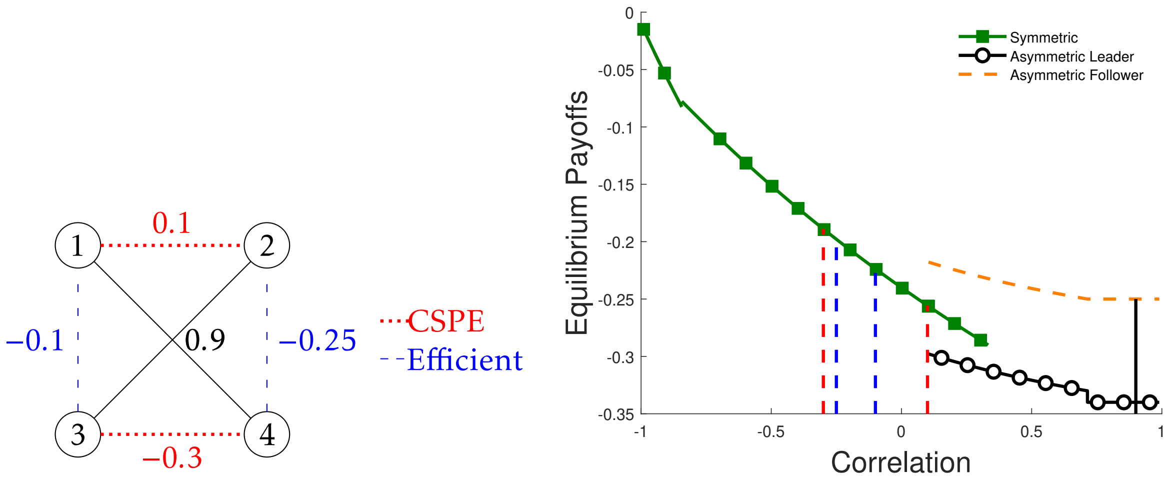
Our analysis of the Production Subgame yields two important insights. First, fixing a strategy profile within teams, reducing correlation increases the value of information the team generates. Second, increasing correlation decreases the symmetry of equilibria; as signals become more substitutable, the marginal value of matching a leader's signal decreases. We now show how these two within-team properties influence the efficiency of sorting across teams.

4.1 Stratification Inefficiency

We first exposit an inefficiency, Stratification Inefficiency, that arises because two highly productive workers, i.e. workers with low pairwise correlation, match at the expense of the two excluded workers, whom must form a less productive team with a relatively high pairwise correlation. After illustrating it with a numerical example, we observe that management intervention within teams cannot restore efficient sorting. The presence of Stratification Inefficiency thus suggests that self-organizing teams may not be an optimal organizational structure.

Suppose, for simplicity, that the parameters are as in Figure 3, so that all properties of Proposition 1 apply. Suppose further that the network in Figure 4a describes the correlation matrix; numbers next to adjacent edges depict pairwise correlation. Then, the unique PEN in teams composed of workers connected by dotted or dashed lines is the symmetric profile $(2,2)$ and the unique PEN (up to identity) in teams composed of workers connected by solid lines is the asymmetric profile $(0,3)$.[18] Corresponding payoffs are depicted in Figure 4b. We argue that, if team membership costs are small enough, the unique CSPE matching pairs worker 1 (worker 3) and worker 2 (worker 4), while the efficient matching pairs worker 1 (worker 2) and worker 3 (worker 4).

[18]This can be seen by referring back to Figure 3.



(a) Correlation Matrix. (b) PEN Payoffs.

Figure 4: Stratification Inefficiency.

First, observe that teams $(1,4)$ and $(2,3)$ can never form in a CSPE; fixing any PEN, the worker producing a positive number of signals would be better off alone. Second, observe that worker 3 and worker 4 must match in any CSPE. Inspecting the green line in Figure 4b, we see that worker 3 and worker 4 each obtains a higher payoff in a team together than in any PEN in any other team; indeed, their pairwise correlation, -0.3, is the smallest among all feasible teams and all teams other than $(1,4)$ and $(2,3)$ play the same equilibrium. But if worker 3 and worker 4 match, worker 1 and worker 2 are left with only two options: they can either work alone or form a team together. For a small enough team membership cost $K > 0$, the utility each obtains from working together exceeds that of working alone. Hence, worker 1 and worker 2 must match in any CSPE.

While we have argued that in the unique CSPE, worker 1 (worker 3) and worker 2 (worker 4) match, it remains to argue that matching worker 1 (worker 2) and worker 3 (worker 4) is welfare improving. Why might this be the case? Though the most productive team, $(3,4)$, forms in the CSPE matching, this comes at the cost of preventing workers 1 and 2 from joining teams with significantly lower pairwise correlations. In particular, while the team $(1,2)$ produces positively correlated signals, the teams $(1,3)$ and $(2,4)$ do not. It turns out that the gain in productivity obtained from re-matching worker 1 with worker 3, and worker 2 with worker 4, outweighs the cost of disrupting the most productive team $(3,4)$.

We now formalize the logic just described and define our first notion of inefficiency.

Definition 2 (Stratification Inefficiency)**.** *A CSPE (μ, N^*) is* ***Stratification Inefficient*** *if*

1. *there exist two workers $i, j \in \mathcal{N}$, $i \neq j$, for which which $\mu(i) = j$ and $v_\ell(n^*(i,j))$ is the highest payoff worker $\ell \in \{i,j\}$ can obtain in any PEN in any team; and,*

2. *there exists a matching $\mu' \neq \mu \in \mathcal{M}$ and a collection of PEN $\hat{N} = \{\hat{n}(i,j)\}_{i,j\in\mathcal{N}}$ such that,*

$$\sum_{\ell\in\mathcal{N}} v_\ell(n^*(\ell,\mu(\ell))) < \sum_{\ell\in\mathcal{N}} v_\ell(\hat{n}(\ell,\mu'(\ell))).$$

The first condition requires that, in any Stratification Inefficient CSPE matching, a pair of teammates are each as well off as in *any* other feasible team playing *any* other PEN, i.e. worker 3 and worker 4 in our example. The second condition requires that there exists another matching, i.e. $\hat{\mu}$ such that $\hat{\mu}(3) = 1$ and $\hat{\mu}(4) = 2$ in our example, and a collection of PEN in each team that increases utilitarian welfare. Stratification Inefficiency therefore arises because two (possibly highly productive) workers each obtains a higher payoff together than in any other team, but do not internalize the "externality" they generate on the productivity of other matches. An efficiency-minded manager, in contrast, prefers them not to match so that she can better exploit the entire correlation matrix.

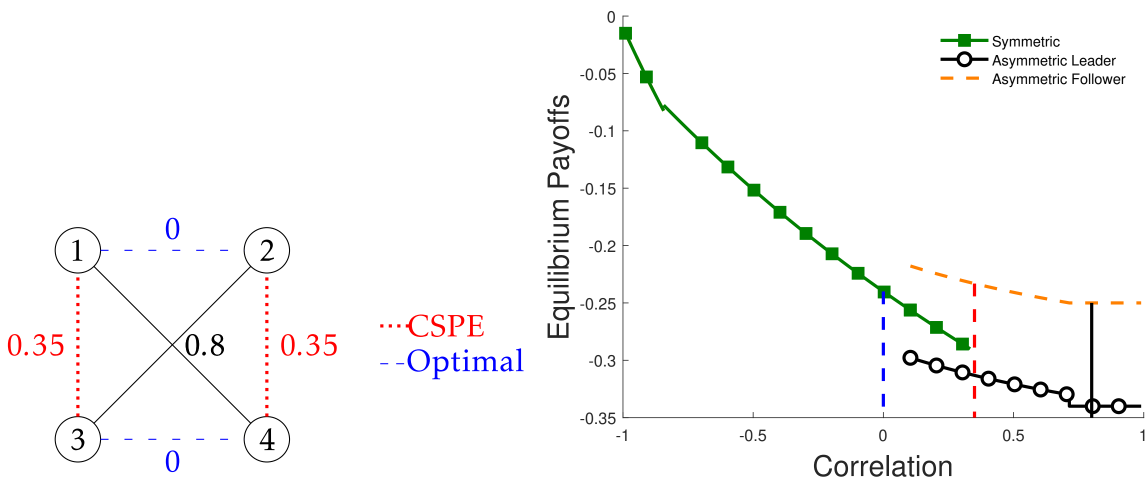
An immediate implication of our definition is that Stratification Inefficiency is a phenomenon that can only be eliminated by a manager that actively intervenes in the assignment of workers to teams. In particular, if two workers obtain a higher payoff together than in any other team playing any PEN, there is no way to select PEN *within* teams to induce either to form a more efficient team. Put differently, whenever a Stratification Inefficient CSPE exists, then *no* efficient CSPE exists.

Observation 1. *If a Stratification Inefficient CSPE exists, then no efficient CSPE exists. Hence, no management intervention within teams can restore efficiency.*

4.2 Asymmetric Effort Inefficiency

Stratification Inefficiency is *not* driven by free riding. Indeed, in the inefficient matching we illustrated, each worker works as hard as she would in the efficient matching. We now focus on the implications of free riding within teams for sorting across teams. In contrast to Stratification Inefficiency, we show that the type of inefficiency that arises due to free riding *can* be prevented by active management of workers within teams.

For illustration, suppose again that the parameters are as in Figure 3. But now, suppose the network in Figure 5a describes the correlation matrix. The unique PEN in teams composed of workers connected by blue dashed lines is the symmetric profile $(2,2)$, the unique PEN (up to identity) in teams composed of workers connected by dotted red lines is the asymmetric profile $(1,3)$, and the unique PEN (up to identity) in teams composed of workers connected by solid lines is the asymmetric profile $(0,3)$. Corresponding payoffs are depicted in Figure 5b. We argue that, if team membership costs are small enough, there is a CSPE matching that pairs worker 1 (worker 2) and worker 3 (worker 4), even though the efficient matching pairs worker 1 (worker 3) and worker 2 (worker 4).



(a) Correlation Matrix (b) PEN Payoffs.

Figure 5: Asymmetric-Effort Inefficiency.

To see why such a CSPE exists, consider the incentives of worker 1. She has two relevant options: form a team with worker 2, with whom she produces uncorrelated signals, or form a team with worker 3, with whom she produces positively correlated signals.[19] In the team with worker 2, worker 1 produces two signals in any PEN. On the other hand, in a team with worker 3, worker 1 either produces three signals (so that worker 3 produces one signal) or *one* signal (so that worker 3 produces three signals). In the case in which worker 1 produces three signals when matched with worker 3, it is clear that she would rather form a team with worker 2; not only is the value of information produced lower in the team with worker 3, she is exerting more effort. But, if worker 1 produces one signal when matched with worker 3, so that she is the follower in that team, then she would rather form a team with worker 3; though the value of information produced is lower, she is exerting *less* effort (see Figure 5b for the payoff comparison).

[19] Again, teams $(1,4)$ and $(2,3)$ never form in any CSPE.

So, for argument's sake, *fix* the PEN within team $(1,3)$ to be the strategy profile $(1,3)$ so that worker 1 would rather match with worker 3 than worker 2. Worker 3 would only deem such a team acceptable if she could not persuade worker 4 to form a team with her. But, fixing the PEN within team $(2,4)$ to be the asymmetric profile $(3,1)$, worker 4 prefers to work with worker 2 (worker 4 has the "same" options as worker 1). Hence, there is a CSPE in which worker 1 matches worker 3 (and worker 2 matches worker 4) because worker 3 (worker 2) has no better option. The CSPE outcome is inefficient, however, because not only does the total value of information produced in the firm increase by forming teams $(1,2)$ and $(3,4)$, but total effort costs decrease weakly.[20]

We again formalize the logic just described and define our second notion of inefficiency.

Definition 3 (Asymmetric Effort Inefficiency)**.** *A CSPE (μ, N^*) is **Asymmetric Effort Inefficient** if*

1. *there exist two workers $i, j \in \mathcal{N}$, $i \neq j$, for which $\mu(i) = j$, and a PEN $\hat{n}(i,i')$, $i' \neq i$, satisfying*
$$n_i^*(j)\hat{n}_{i'}(i) < \hat{n}_i(i')n_j^*(i);$$
and,

2. *there exists a matching $\hat{\mu} \in \mathcal{M}$ satisfying $\hat{\mu}(i) = i'$ and a collection of PEN $\hat{N} = \{\hat{n}(i,j)\}_{i,j\in\mathcal{N}}$, including $\hat{n}(i,i')$, such that*
$$\sum_{\ell\in\mathcal{N}} v_\ell(n^*(\ell,\mu(\ell))) < \sum_{\ell\in\mathcal{N}} v_\ell(\hat{n}(\ell,\mu'(\ell))).$$

To understand the definition, consider again the example. Let $(i,j) = (1,3)$ and $(i',j') = (2,4)$. The manager prefers to match worker 1 with worker 2 because there is a symmetric PEN inside the team, $\hat{n}(1,2) = (2,2)$, in which worker 1 exerts *relatively* more effort than her partner when compared to the "on-path" PEN, $n^*(1,3) = (1,3)$. In particular, $\frac{n_1^*(3)}{n_3^*(1)} = \frac{1}{3} < 1 = \frac{\hat{n}_1(2)}{\hat{n}_2(1)}$ so that, by cross-multiplying, we see that the first inequality of the definition is satisfied. The second part of the definition ensures that, upon re-matching worker 1 and worker 2 and fixing $\hat{n}(1,2)$, the manager can select a PEN and matching of the other two workers so that utilitarian welfare increases.

[20] Recall, the cost of effort is increasing for an individual worker and the total number of signals produced in each team in the efficient and inefficient matching is the same.

Our discussion suggests a possible resolution to incentive problems in the case of Asymmetric Effort Inefficiency. If a manager can assign *roles* to individual workers, then it may be possible to enforce an efficient CSPE. For example, returning to Figure 5, if a manager designates worker 1 as a leader, quite literally in the terminology of our analysis, then she can enforce an efficient CSPE. In particular, if we choose a PEN in team $(1,3)$ so that worker 1 is the team leader, rather than worker 3, then worker 1 would rather form a team with worker 2. As worker 2 prefers this arrangement to the case in which she matches with worker 4 and is the leader, worker 1 and worker 2 match, leaving worker 3 and worker 4 to match.

Observation 2. *If an Asymmetric Effort Inefficient CSPE exists, then an efficient CSPE may also exist. In this case, a manager can select PEN within teams to restore efficiency.*

4.3 Robustness

We finally show that the inefficiencies we identify are robust in a formal sense.

Proposition 3 (Robustness)**.** *Suppose $K > 0$ is small.*

1. *If Assumption 1 holds, then there is an open set of correlation parameters for which there is a Stratification Inefficient CSPE.*

2. *If Assumption 1 and 2 hold, and $\sigma^2 \geq \sigma_\theta^2$, then there is an open set of correlation parameters for which there is an Asymmetric Effort Inefficient CSPE.*

The proof of Proposition 3 makes full use of our characterization of within-team equilibria to construct correlations leading to inefficiency. In particular, to construct a Stratification Inefficient CSPE, we choose four pairwise correlations strictly below ρ^*, the cutoff below which there is a unique and symmetric PEN, and all others above it. As long as there is no free-riding opportunity for the workers with the lowest correlation, they must match in any CSPE (for small $K > 0$). However, if the other two workers have a sufficiently high correlation, then re-matching workers can improve welfare.

To construct an Asymmetric Inefficient CSPE, we observe that, under Assumption 2, there is an open set of correlations above ρ^* for which there is a non-trivial asymmetric PEN of the Production Subgame. We then pick a worker, say worker 1, and two pairwise correlations– ρ_{13}, in this open set, and ρ_{12} below ρ^*–so that worker 1 prefers to free ride in the team with worker 3 than to match worker 2. If ρ_{12} is small enough, however, the sum of utilities in the team $(1,2)$ exceeds that in $(1,3)$, as in our illustrative example.

Moreover, in the open set of parameters we identify, a manager can always improve welfare by forcing worker 1 to be the leader when matched with worker 3, i.e. selecting a PEN in which she exerts relatively more effort than worker 3. We remark that such powers seem plausible in many organizational contexts; while it may be costly to assign all workers to teams, it may not be so costly to manage the behavior of particular workers.[21]

5 Discussion

Our paper is a first step towards understanding how research teams form absent a central authority. We shed light on how workers' incentives for effort within teams are affected by their skill complementaries and therefore impact equilibrium sorting. Our analysis uncovers two plausible forces leading to inefficient sorting. First, workers producing complementary information may match and force excluded workers to form highly unproductive teams composed of workers producing substitutable information. Hence, there is too much inequality in productivity *across* teams. Second, even when it is efficient for a team composed of workers producing complementary information to form, such a team may not arise in equilibrium if one of its members has an opportunity to form a less productive team in which she exerts relatively less effort. Hence, there is too much inequality in effort *within* teams. While the former inefficiency suggests conditions under which self-organized teams are not optimal, the latter provides foundation for targeted management interventions that designate specific workers as project leaders, as in the case of Oticon.

We conclude by commenting on the structure of our model leading to our results and on the extensions we have considered.

Gaussian Environment. We model information acquisition using a canonical quadratic-Gaussian set-up; workers obtain normally distributed signals to minimize a quadratic loss function and have normally distributed prior beliefs.[22] In this environment, the expected

[21] A careful reader should note that while we have demonstrated that Stratification Inefficiency and Asymmetric Inefficient CSPE occur in a range of non-trivial scenarios, we have *not* argued that they are the only sources of inefficiency in our model. This claim is *false* precisely because welfare inefficiency may exist *within* a team. In particular, a PEN may be selected within a team that is welfare dominated by another PEN. As the focus of our analysis is on inefficient *sorting*, however, we do not attempt to characterize such inefficiencies. We conjecture, but have not proven that, taking care of within-team inefficiency, the inefficiencies we have identified are exhaustive.

[22] Our results generalize to the case in which the joint distribution of signals and states for any number of draws is elliptical with finite second moments. In this case, the conditional expectation is still linear in signals and our characterization results will possess the same qualitative features.

value of the posterior distribution simplifies to the negative posterior variance (Lemma 1). Hence, we can derive comparative statics using a closed-form utility function. Our analysis uncovers how correlation between teammates affects the symmetry of the PEN correspondence–that we can order equilibria by symmetry in terms of correlation is the crucial property for our main inefficiency results. In Online Appendix D, we consider a binary signal, binary state environment, as is common in applied theoretical work. While the intuition that perfect negative correlation leads to perfect learning does not hold (because such correlation is statistically infeasible), it is still the case that low correlation in state-conditional signals is desirable. Consequently, the marginal value of a draw, and hence equilibrium predictions, satisfy the same properties as in the Gaussian case.

Draw Procedure. The procedure through which workers acquire and share information possesses two features which deserve comment. First, workers choose numbers of signals simultaneously. Methodologically, we abstract from dynamic considerations in order to isolate the key property relevant for team formation–namely, the relationship between correlation and the symmetry of equilibrium strategies. Nonetheless, it is worthwhile to explore the extent to which the intuitions we have provided hold in a more complex dynamic game. Towards an answer, in Online Appendix E, we study a finite extensive form game with sequential decisions. Our main conclusion is that for many, but not all, correlations there is a Subgame Perfect Equilibrium of the sequential game that coincides with the most symmetric equilibrium of the simultaneous game. Nonetheless, it may be the case that an equilibrium of the simultaneous game is more asymmetric than the most symmetric equilibrium of the extensive form game. Hence, sequential decisions do *not* eliminate asymmetric equilibria, the driving force behind Asymmetric Effort Inefficiency.

Second, the correlation between signals differs across "periods"; if (n_i, n_j) draws are taken in team (i, j) with correlation ρ, and $n_j > n_i > 0$, then the first n_i signals drawn by each worker are correlated according to ρ and the last $n_j - n_i$ signals are conditionally independent. We assume that pairwise correlation affects the value of effort within, but not across, periods in order capture the economics of a situation in which joint and simultaneous effort is affected by complementarities, while unilateral effort is not. In particular, it would *not* be equivalent to analyze a continuous choice model in which workers first choose precisions and then share a single signal. In this set-up, within and across period effects cannot be disentangled.[23]

[23]Nonetheless, a kind of "continuous draw" set-up may be imagined as follows. Suppose, relative to a single signal of fixed precision, a worker can draw many signals with lower precisions, but with the cost of information held constant. In Online Appendix F, we analyze the limit model obtained when such preci-

Size-Two Teams. We follow the matching literature in assuming that workers form teams with at most two workers. This restriction allows us to obtain a clean characterization of within-team equilibria; a single pairwise correlation coefficient captures intuitively the effects of skill complementarity. Extending the analysis to teams of more than two workers is not without its challenges. In subsequent work, Segura-Rodriguez (2019) shows that a team of three workers can perfectly learn the state even if each worker produces a single signal and all three signals are *highly* correlated. Characterizing within-team equilibria with many workers and exploring the implications of these equilibria for team formation is an interesting open problem we leave for future research.

Optimality of Decentralized Sorting. Our current framework illustrates the ways in which decentralized sorting within firms may be inefficient. We have assumed throughout, however, that workers are compensated equally for team output and that management does not play an active role in the assignment of workers to teams. In subsequent work, Kambhampati and Segura-Rodriguez (2020) study the problem of optimally assigning workers to teams and designing incentive contracts in the presence of both moral hazard and adverse selection. They characterize when creating incentives in a centralized organization becomes so costly that a profit-maximizing manager prefers to allow workers to sort themselves into teams and compensate them equally on the basis of team output alone. Nonetheless, the environment in Kambhampati and Segura-Rodriguez (2020) is simpler than the one considered in this paper and so their results do not directly apply. A complete analysis of the tradeoff between centralization and decentralization in informational settings thus awaits future research.

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sions become arbitrarily small and costs are adjusted. Workers can therefore be interpreted as choosing a real number of signals. In this limit model, the equilibrium correspondence can be ordered by symmetry using pairwise correlations as in our main characterization result, and hence our main inefficiency Propositions generalize. We do not use such a model in the main text, however, as we have not proven that the equilibrium correspondence of the sequence of discrete-draw models converges to that of the continuous-draw model.

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A Proofs

A.1 Proof of Lemma 1

For any measurable function $g : X \to \mathbb{R}$, where X is the set of possible realizations of signals,

$$-\mathbb{E}_{x,\theta}\left[(g(x)-\theta)^2\right] \leq -\mathbb{E}_x\left[(\mathbb{E}(\theta \mid x)-\theta)^2\right] = -\mathbb{E}_x\left[\mathbb{E}_\theta\left[(\mathbb{E}(\theta \mid x)-\theta)^2 \mid x\right]\right] = -Var(\theta \mid x).$$

The inequality follows because $\mathbb{E}\left[(b-\theta)^2|x\right]$ is minimized by setting $b = \mathbb{E}[\theta|x]$. The first equality follows from the Law of Iterated Expectations. The second equality follows from the definition of conditional variance.

Let Σ be the correlation matrix of joint signals x, and 1_N be a N-column vector of 1s. The likelihood function of the signals is, $p(x|\theta) = \det(2\pi\sigma^{-2}\Sigma)^{-\frac{1}{2}} \exp\Big(-\frac{1}{2}\Big[(\theta \cdot 1_N - x)'\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\Big)$ and the prior density is, $p(\theta) = (2\pi\sigma^{-2})^{-\frac{1}{2}} \exp\Big(-\frac{1}{2}\Big[(\theta - \mu_\theta)^2\sigma_\theta^{-2}\Big]\Big)$.

By Bayes rule, the posterior distribution of $\theta|x$ is proportional to,

$$
\begin{aligned}
p(x|\theta)p(\theta) &\propto \exp\Big(-\frac{1}{2}\Big[(\theta - \mu_\theta)^2\sigma_\theta^{-2} + (\theta \cdot 1_N - x)'\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\Big) \\
&\propto \exp\Big(-\frac{1}{2}\Big[\theta^2(\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}1_N) - \theta(2\mu_\theta\sigma_\theta^{-2} + \sigma^{-2}(x'\Sigma^{-1}1_N + 1_N'\Sigma^{-1}x)\Big]\Big) \\
&\propto \exp\Big(-\frac{1}{2}\Big[\theta - A\Big]' B\Big[\theta - A\Big]\Big),
\end{aligned}
$$

where $B = (\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}1_N)$, $A = B^{-1}(\mu_\theta\sigma_\theta^{-2} + \sigma^{-2}1_N'\Sigma^{-1}x)$, and the proportionality operator eliminates positive constants. Since the derived expression is the kernel of a normal distribution, $Var(\theta \mid x) = B^{-1}$.

We construct B^{-1} when workers take $n_j \geq n_i$ draws. The prior covariance matrix, Σ^{-1}, is block diagonal with n_i blocks of the form,

$$
\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},
$$

and $n_j - n_i$ scalar blocks each equal to 1. The inverse of a block diagonal matrix is equal to the block diagonal matrix formed by inverting each block. Then, $1_N'\Sigma^{-1}1_N$ is equal to $n_i 1_2'\Sigma_0^{-1}1_2 + (n_j - n_i)$. Since,

$$
\Sigma_0^{-1} = \frac{1}{1-\rho^2}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix},
$$

we have, $1_2'\Sigma_0^{-1}1_2 = \frac{2}{1+\rho}$. Hence,

$$
Var(\theta \mid n_i, n_j) = B^{-1} = \left(\sigma^{-2}1_N'\Sigma^{-1}1_N + \sigma_\theta^{-2}\right)^{-1} = \left(\sigma^{-2}\left(\frac{2n_i}{1+\rho} + (n_j - n_i)\right) + \sigma_\theta^{-2}\right)^{-1}.
$$

Finally, if $\rho = -1$, the average of two signals equals the realized state θ, and so the posterior variance is zero.

A.2 Proof of Proposition 1

A.2.1 Existence of Nash Equilibrium

Since information production exhibits diminishing marginal returns, eventually, the marginal value of producing a signal must be less than the marginal cost *regardless* of the behavior of one's partner. Therefore, it is without loss to bound the action space.

Lemma 2. *There is a positive integer $\bar{N}$ such that for each positive integer $n \geq \bar{N}$, n is a not best response by worker i to any strategy by worker j.*

Proof. For $n_i \leq n_j$,

$$Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)) = \frac{\left(\frac{1-\rho}{1+\rho}\right)\sigma^{-2}}{\left(\left(n_i \frac{1-\rho}{1+\rho} + n_j + 1 - \frac{2}{1+\rho}\right)\sigma^{-2} + \sigma_\theta^{-2}\right)\left(\left(n_i \frac{1-\rho}{1+\rho} + n_j\right)\sigma^{-2} + \sigma_\theta^{-2}\right)}$$

is strictly decreasing in n_j and in n_i because $\frac{1-\rho}{1+\rho} > 0$. For $n_i \geq n_j + 1$,

$$Var(\theta \mid (n_i - 1, n_j)) - Var(\theta \mid (n_i, n_j)) = \frac{\sigma^{-2}}{\left(\left(n_j \frac{1-\rho}{1+\rho} + n_i - 1\right)\sigma^{-2} + \sigma_\theta^{-2}\right)\left(\left(n_j \frac{1-\rho}{1+\rho} + n_i\right)\sigma^{-2} + \sigma_\theta^{-2}\right)}$$

is strictly decreasing in n_i and in n_j, again because $\frac{1-\rho}{1+\rho} > 0$.

Therefore, the marginal value of worker i is strictly decreasing in n_j, so that worker i's best response is decreasing in n_j. We only need to prove that worker i's best response to 0 draws by worker j is finite. It suffices to show that there is an $n_i \in \mathbb{Z}_+$ such that $Var(\theta \mid (n_i - 1, 0)) - Var(\theta \mid (n_i, 0))$ is smaller than $c(1)$. We have

$$Var(\theta \mid (n_i - 1, 0)) - Var(\theta \mid (n_i, 0)) \;\; = \frac{1}{(n_i - 1)\sigma^{-2} + \sigma_\theta^{-2}} - \frac{1}{n_i \sigma^{-2} + \sigma_\theta^{-2}} < \frac{\sigma^2}{n_i(n_i - 1)}.$$

Then, it is sufficient to have $n_i > \frac{\sigma^2}{c(1) n_i} + 1$. When $n_i > \frac{\sigma^2}{c(1)} + 1$ we obtain the desired inequality. Define $\bar{N} \in \mathbb{N}$ as the smallest value that satisfies the inequality. $\square$

Since we can bound the action space, we may redefine the game as a finite exact potential game to show that there exists a pure strategy Nash equilibrium.

Lemma 3. *There exists a pure strategy Nash equilibrium of the Production Subgame.*

Proof. Given that no worker optimally produces a number of signals larger than $\bar{N}$, we can redefine the Production Subgame as the finite normal form game $(\{0, 1, \ldots, \bar{N}\}^2, \{v_i, v_j\})$. Define the potential function,

$$\Phi(n_i, n_j, \rho_{ij}) = -Var(\theta \mid (n_i, n_j)) - c(n_i) - c(n_j),$$

where ρ_{ij} is the correlation for team (i,j). It is a potential function since

$$\begin{aligned}
v_i(n_i,n_j)-v_i(n_i',n_j) &= -Var(\theta\mid(n_i,n_j))-c(n_i)+Var(\theta\mid(n_i',n_j))+c(n_i') \\
&= \Phi(n_i,n_j,\rho_{ij})-\Phi(n_i',n_j,\rho_{ij}) \\
v_j(n_i,n_j)-v_j(n_i,n_j') &= -Var(\theta\mid(n_i,n_j))-c(n_j)+Var(\theta\mid(n_i,n_j'))+c(n_j') \\
&= \Phi(n_i,n_j,\rho_{ij})-\Phi(n_i,n_j',\rho_{ij}).
\end{aligned}$$

Hence, the redefined game is a finite exact potential game and is guaranteed to have a pure strategy Nash equilibrium by Corollary 2.2 of Monderer and Shapley (1996). □

A.2.2 Existence of Pareto-Efficient Nash Equilibrium

By Lemma 2, we can conclude that the set of Nash Equilibria is finite. Consider the subset of equilibria that maximizes worker i's payoff. Choose any equilibrium that (weakly) maximizes worker j's payoff within this subset. The chosen equilibrium must be Pareto-Efficient. Hence, a Pareto-Efficient Nash Equilibrium exists.

A.2.3 Comparative Statistics Preliminary Lemmas

Lemma 4 states that the marginal value of a signal by a leader is increasing in ρ.[24]

Lemma 4 (Leader Comparative Statics in ρ)**.** *For $n_i > n_j$, $MV(n_i;n_j,\rho)$ is increasing in ρ.*

Proof. For $n_i > n_j$,

$$\frac{\partial MV(n_i;n_j,\rho)}{\partial\rho} \propto \left(\left(n_j\frac{1-\rho}{1+\rho}+n_i\right)\sigma^{-2}+\sigma_\theta^{-2}\right)+\left(\left(n_j\frac{1-\rho}{1+\rho}+n_i-1\right)\sigma^{-2}+\sigma_\theta^{-2}\right)>0.$$

□

The same property does not hold for a follower.[25] We prove the follower's marginal benefit is strictly concave in the pairwise correlation ρ and has a unique maximizer. For the following lemmas it is useful to define the signal-to-prior variance ratio $\gamma := \frac{\sigma^2}{\sigma_\theta^2}$.

Lemma 5 (Follower Comparative Statics in ρ)**.** *For $n_i < n_j$ with $n_j \geq 1$, $MV(n_i+1;n_j,\rho)$ is strictly concave in ρ with unique maximizer,*

$$\tilde{\rho}(n_i+1,n_j,\gamma)=\frac{\left(n_j+\gamma-\sqrt{n_i(n_i+1)}\right)^2}{-(n_j+\gamma)^2+n_i(n_i+1)}.$$

[24] Recall, a *leader* is a teammate taking weakly more draws than her partner.

[25] Recall, a *follower* is a teammate taking strictly fewer draws than her partner.

Proof. If worker j produces n_j signals, the marginal benefit of the $n+1$-th signal for worker i, with $n < n_j$, is equal to,

$$MV(n+1;n_j,\rho) = \frac{\frac{1-\rho}{1+\rho}}{\left(n\frac{2}{1+\rho}+n_j-n+\gamma\right)\left((n+1)\frac{2}{1+\rho}+n_j-n-1+\gamma\right)}.$$

Differentiating with respect to ρ,

$$\frac{\partial MV(n+1;n_j,\rho)}{\partial\rho} = \frac{2\sigma^2\left(-(n_j+\gamma)^2(1+\rho)^2+n(n-1)(1-\rho)^2\right)}{\left(2n+(n_j-n+\gamma)(1+\rho)\right)^2\left(2(n+1)+(n_j-n-1+\gamma)(1+\rho)\right)^2}.$$

Differentiating again with respect to ρ,

$$\begin{aligned}\frac{\partial^2 MV(n+1;n_j,\rho)}{\partial\rho^2} &\propto 4n(n+1)\left(-(n_j+\gamma)^2(1+\rho)-n(n-1)(1-\rho)\right)\\ &+n(n+1)\left[2n(n_j-n-1+\gamma)+2(n+1)(n_j-n+\gamma)+(n_j-n-1+\gamma)(n_j-n+\gamma)\right](2\rho-2)<0.\end{aligned}$$

Hence, the marginal value $MV(n+1;n_j,\rho)$ is strictly concave in ρ. The unique maximizer $\tilde{\rho}$ must satisfy,

$$(n_j+\gamma)^2(1+\tilde{\rho})^2 = n(n+1)(1-\tilde{\rho})^2,$$

a quadratic equation in ρ with roots,

$$\rho = \frac{\left(n_j+\gamma\pm\sqrt{n(n+1)}\right)^2}{-(n_j+\gamma)^2+n(n+1)}.$$

Both solutions are negative because the denominator is negative. However, the smaller root (corresponding to the "plus" in the numerator) is less than -1 and therefore infeasible. Since $n+1 \le n_j$, the other root (corresponding to the "minus" in the numerator) is greater than -1. Set $\tilde{\rho}(n+1,n_j,\gamma) = \frac{\left(n_j+\gamma-\sqrt{n(n+1)}\right)^2}{-(n_j+\gamma)^2+n(n+1)}$. □

We now make stepwise comparisons between the marginal value of a signal by a leader and the marginal value of a signal by a follower. Workers initially produce $n-1$ signals. The leader's marginal value is the payoff of producing an n-th signal. The follower's marginal value is the payoff of producing an n-th signal, given that the leader already produced n signals. Lemma 6 states that for any number $n \ge 1$ and signal-to-prior variance ratio $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$, there is a unique correlation, $\hat{\rho}(n,\gamma)$, below which the marginal value of the leader is less than the marginal value of the follower, and above which the opposite holds.

Lemma 6 (Leader-Follower MV Comparison 1)**.** *Fix $n_j > n_i$ with $n_j \geq 1$ and γ. Then,*

$$\underbrace{MV(n_j;n_i,\rho)}_{\textit{Marginal Value Leader}} < \underbrace{MV(n_i+1;n_j,\rho)}_{\textit{Marginal Value Follower}}$$

if and only if,

$$\rho < \hat{\rho}(n_i+1,n_j,\gamma) = \frac{-(\gamma+n_i+n_j)+\sqrt{(\gamma+n_i+n_j)^2-4(\gamma+n_j-n_i-1)}}{2(\gamma+n_j-n_i-1)} < 0.$$

Proof.

$$\begin{aligned}
&MV(n_i+1;n_j,\rho) \geq MV(n_j;n_i,\rho)\\
\Leftrightarrow &\frac{\frac{1-\rho}{1+\rho}}{\left(n_i\frac{2}{1+\rho}+n_j-n_i+\gamma\right)\left((n_i+1)\frac{2}{1+\rho}+n_j-n_i-1+\gamma\right)} \geq \frac{1}{\left(n_i\frac{2}{1+\rho}+n_j-n_i-1+\gamma\right)\left(n_i\frac{2}{1+\rho}+n_j-n_i+\gamma\right)}\\
\Leftrightarrow &\frac{1-\rho}{1+\rho}\left(n_i\frac{2}{1+\rho}+n_j-n_i-1+\gamma\right) \geq \left((n_i+1)\frac{2}{1+\rho}+n_j-n_i-1+\gamma\right)\\
\Leftrightarrow &\ 0 \geq (\gamma+n_j-n_i-1)\rho^2+(\gamma+n_i+n_j)\rho+1.
\end{aligned}$$

The last inequality involves a quadratic concave function in ρ. The roots are:

$$\rho^+(n_i,n_j) = \frac{-(\gamma+n_i+n_j)+\sqrt{(\gamma+n_i+n_j)^2-4(\gamma+n_j-n_i-1)}}{2(\gamma+n_j-n_i-1)}$$

$$\rho^-(n_i,n_j) = \frac{-(\gamma+n_i+n_j)-\sqrt{(\gamma+n_i+n_j)^2-4(\gamma+n_j-n_i-1)}}{2(\gamma+n_j-n_i-1)}.$$

When $n_i \geq 1$ the expression inside the root is greater than $(\gamma+n_j-n_i)^2$, so that $\rho^-(n_i,n_j) < -1$ for all $n_i \geq 1$. If $n_i = 0$, the expression inside the root is equal to $(\gamma+n_j-2)^2$. If $\gamma+n_j < 2$ then $\rho^-(0,n_j) < -1$, and if $\gamma+n_j \geq 2$, then $\rho^-(0,n_j) = -1$. Therefore, ρ^- is an infeasible solution.

It is clear that $\rho^+(n_i,n_j) < 0$ for all n_i and n_j, and

$$\rho^+(0,n_j) = \frac{-(\gamma+1)+\sqrt{(\gamma-1)^2}}{2\gamma} = \begin{cases} \frac{-2}{2(\gamma+n_j-1)} > -1 & \text{if } \gamma+n_j \geq 2\\ \frac{2-2(\gamma+n_j)}{2(\gamma+n_j-1)} = -1 & \text{if } \gamma+n_j < 2. \end{cases}$$

Further, when $n_i \geq 1$ the expression inside the root is larger than $(\gamma+n_j-2)^2$. Therefore, $\rho^+(n_i,n_j) > -1$ for all $n_j > n_i \geq 0$. Then ρ^+ is a feasible solution and we set $\hat{\rho}(n_i+1,n_j) =$

ρ^+. $\square$

Lemma 7 states that if γ is sufficiently large, the pairwise correlation at which the marginal value of a follower is maximized, $\tilde{\rho}(n_i, n_i, \gamma)$, is less than $\hat{\rho}(n_i, \gamma)$. We use this property in the next section to order equilibria in terms of their symmetry.

Lemma 7 (Leader-Follower MV Comparison 2)**.** *Fix $n_j > n_i \geq 1$. Then for $\gamma \geq 1$*

$$\tilde{\rho}(n_i+1, n_j, \gamma) \leq \hat{\rho}(n_i+1, n_j, \gamma).^{26}$$

Proof. Define the function $g(n_i+1, n_j, \gamma) := \tilde{\rho}(n_i+1, n_j, \gamma) - \hat{\rho}(n_i+1, n_j, \gamma)$. We want to show that $g(n_i+1, n_j, \gamma) \leq 0$ for any $n_j > n_i \geq 1$ and any $\gamma \geq 1$. It suffices to show that $g(n_i+1, n_j, 1) \leq 0$ for any $n_j > n_i \geq 1$ and then show that $\frac{\partial g(n_i+1, n_j, \gamma)}{\partial \gamma} < 0$ for any $n_j > n_i \geq 1$.

We first show that $g(n_i+1, n_j, 1) \leq 0$. Notice,

$$g(n_i+1, n_j, 1) = \frac{\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2}{-(n_j+1)^2 + n_i(n_i+1)} - \frac{-(1+n_i+n_j) + \sqrt{(1+n_i+n_j)^2 - 4(n_j - n_i)}}{2(n_j - n_i)} \leq 0,$$

if and only if,

$$\frac{2(n_j - n_i)\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2 + \left((1+n_i+n_j) - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)\left(-(n_j+1)^2 + n_i(n_i+1)\right)}{2(n_j - n_i)\left(-(n_j+1)^2 + n_i(n_i+1)\right)} \leq 0.$$

For any $n_j > n_i \geq 1$, the denominator is negative. Hence, the expression holds if and only if,

$$2(n_j - n_i)\left(n_j + 1 - \sqrt{n_i(n_i+1)}\right)^2 + \left((1+n_i+n_j) - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)\left(-(n_j+1)^2 + n_i(n_i+1)\right) \geq 0.$$

Dividing by $n_j + 1 - \sqrt{n_i(n_i+1)} > 0$, we see that the inequality holds if and only if

$$\begin{aligned}
&2(n_j - n_i)(n_j + 1 - \sqrt{n_i(n_i+1)}) - \left(1 + n_i + n_j - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right)(n_j + 1 + \sqrt{n_i(n_i+1)}) \geq 0 \\
\Leftrightarrow\ &(n_j+1)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) \geq \sqrt{n_i(n_i+1)}\left(3n_j - n_i + 1 - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right).
\end{aligned}$$

Since $n_j \geq n_i + 1$, we have that $n_j + 1 \geq 2n_i - \sqrt{n_i(n_i+1)} + 2$. So, it is sufficient to show that

$$\begin{aligned}
&(2n_i - \sqrt{n_i(n_i+1)} + 2)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) \\
&\geq \sqrt{n_i(n_i+1)}\left(3n_j - n_i + 1 - \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) \\
\Leftrightarrow &(n_i+1)\left(n_j - 3n_i - 1 + \sqrt{(1+n_i+n_j)^2 - 4(n_j-n_i)}\right) - 2\sqrt{n_i(n_i+1)}(n_j - n_i) \geq 0.
\end{aligned}$$

[26] For $n_i = 0$, $\tilde{\rho}(n_i, n_j, \gamma) = -1$, so the inequality is satisfied for any γ.

To complete the argument, we show that (i) the left-hand side of the last expression is positive when $n_j = n_i + 1$ and (ii) increasing in n_j. To show (i), notice that when $n_j = n_i + 1$ the expression on the left-hand side is positive if and only if

$$\begin{aligned}
& -n_i + \sqrt{n_i^2 + 2n_i} \geq \sqrt{\frac{n_i}{n_i + 1}} \\
\Leftrightarrow\ & 2(n_i + 1) - \frac{1}{n_i + 1} \geq 2\sqrt{n_i^2 + 2n_i} \\
\Leftrightarrow\ & \frac{1}{(n_i + 1)^2} \geq 0,
\end{aligned}$$

where the second and third lines are the result of taking squares on both sides and simplifying. Clearly, the last inequality always holds. To show (ii), notice that the derivative of the left-hand side with respect to n_j is positive if and only if

$$(n_i + 1)\left(1 + \frac{n_i + n_j - 1}{\sqrt{(n_i + n_j + 1)^2 - 4(n_j - n_i)}}\right) \geq 2\sqrt{n_i(n_i + 1)}.$$

Since $n_j > n_i$, the left-hand side of this expression is greater than $\frac{2(n_i+1)(n_i+n_j)}{n_i+n_j+1}$. Hence, it is sufficient to show that,

$$\begin{aligned}
& \frac{n_i + n_j}{n_i + n_j + 1} \geq \sqrt{\frac{n_i}{n_i + 1}} \\
\Leftrightarrow\ & n_j^2 \geq n_i(1 + n_i),
\end{aligned}$$

where the second line is the result of taking squares on both sides and simplifying. But, this holds is as long as $n_j \geq n_i + 1$. We have thus completed the proof that $g(n_i+1, n_j, 1) \leq 0$.

To show that $\frac{\partial g(n_i+1,n_j,\gamma)}{\partial \gamma} < 0$, we first observe that for all $\gamma \geq 1$,

$$\begin{aligned}
\frac{\partial \hat{\rho}(n_i+1,n_j,\gamma)}{\partial \gamma} \propto & \left(-1 + \left((\gamma + n_j + n_j i)^2 - 4(\gamma + n_j - n_i - 1)\right)^{-0.5} 2(\gamma + n_i + n_j - 2)\right) 2(\gamma + n_j - n_i - 1) \\
& -2\left(-(\gamma + n_j + n_i) + \sqrt{(\gamma + n_j + n_i)^2 - 4(\gamma + n_j - n_i - 1)}\right) \\
= & \, 2(2n_j + 1)\sqrt{(\gamma + n_j + n_i)^2 - 4(\gamma + n_j - n_i - 1)} + 2(n_i + n_j + \gamma)^2 - 8(n_j + 1) > 0.
\end{aligned}$$

The inequality follows because the first term is positive and the second term minus the third term is non-negative: $2(n_i + \gamma)^2 - 8$ if $n_i = 0$, and greater than $2(2n_i + 1)^2 - 8(n_i + 1)$ if $n_i > 0$. Second, we observe that for all $\gamma \geq 1$,

$$\frac{\partial \tilde{\rho}(n_i+1,n_j,\gamma)}{\partial \gamma} \propto n_i(n_i + 1) - (n_j + \gamma)\sqrt{n_i(n_i + 1)} < 0,$$

since $\sqrt{n_i(n_i+1)} > n_i$, and $n_j \geq n_i + 1$. □

A.2.4 Proof of 1. and 2.

By Assumption 1, for $\rho = -1$ and correlations close to it, both workers produce at least one signal. Since $MV(2;1,\rho)$ is close to 0, for correlations close to -1, no worker has an incentive to produce a second signal when both are producing a single signal. Hence, there is a threshold $\rho^* > -1$ below which the unique equilibrium is symmetric.

As ρ approaches 1, the marginal benefit of matching the first signal of one's teammate, $MV(1;1,\rho)$, approaches zero. By continuity and monotonicity of $MV(1;1,\rho)$ in ρ, there exists a unique $\rho^{**} < 1$ such that $MV(1;1,\rho^{**}) = c(1)$. Since, by the proof of Lemma 2, $MV(1;n,\rho) = c(1)$ is decreasing in ρ when $n \geq 1$, the follower in the team has no incentive to match the leader's first signal. Hence, for $\rho > \rho^{**}$ one worker produces zero signals and the other produces a strictly positive number of signals. This PEN is unique up to identity, except in the case in which the leader is indifferent between two numbers of signals.

A.2.5 Proof of 3. and 4.

For this proof, we use the *Sequential Response Algorithm*:

1. Set $(n_i^0, n_j^0) = (0,0)$ and $t = 1$.

2. If $MV(t; n_j^{t-1}, \rho) > c(t) - c(t-1)$, set $n_i^t = n_i^{t-1} + 1$ and move to step 3, replacing t with $t+1$. If not, set $n_i^t = n_i^{t-1}$ and move to step 4, replacing t with $t+1$.

3. Set $n_j^t = \arg\max_{n \leq n_i^t} Var(\theta \mid (t,n)) - c(n)$ and go back to step 2, replacing t with $t+1$.

4. (Complement Effect) Set $(n_i^t, n_j^t) = (n_i^{t-1}+1, n_j^{t-1}+1)$ if (i) both workers are made weakly better off and (ii) the resulting profile is a Nash equilibrium. If either (i) or (ii) is not satisfied, set $(n_i^t, n_j^t) = (n_i^{t-1}, n_j^{t-1})$ and move to step 5, replacing t with $t+1$. Else, repeat step 4, replacing t with $t+1$.

5. (Substitution Effect) Consider the profile $(n_1^{t-1}+1, n_2^{t-1}-n)$, where $n_2^{t-1}-n$ is a best-response by worker j given $n_1^{t-1}+1$ subject to the constraint that $0 \leq n \leq n_2^{t-1}$. Set $(n_1^t, n_2^t) = (n_1^{t-1}+1, n_2^{t-1}-n)$ if (i) both workers are made weakly better off and (ii) the resulting profile is a Nash equilibrium. Then, repeat step 5, replacing t with $t+1$. If either (i) or (ii) are not satisfied, exit the algorithm and return (n_1^{t-1}, n_2^{t-1}).

The algorithm terminates in finite time for the following two reasons. First, the algorithm eventually exits the loop between step 2 and step 3 because the marginal value of a signal approaches zero and costs are increasing. Second, the algorithm eventually exits step 4 and step 5 because, by Lemma 2, there is a positive integer above which worker 1 no longer wants to produce a signal, no matter the number of signals produced by worker 2.

Lemma 8. *The Sequential Response Algorithm finds a PEN that minimizes* $\left|n_i - n_j\right|$.

Proof. We first claim that if step 4 is reached in iteration $t+1$, then (n_i^t, n_j^t) is a Nash equilibrium. To see this, note that, after step 1, the algorithm cycles between step 2 and step 3. We make two observations about this cycle. First, worker j either exits the loop having never produced a signal or she matches worker i's signal the first time step 3 is reached.[27] Second, if worker j does not match worker i's signal in step 3, then she never increases the number of signals she acquires in any future iteration in which step 3 is reached (her marginal value decreases each time step 3 is reached).

When step 4 is reached, worker j does not have a profitable deviation downwards by construction. Checking that worker i has no profitable downward deviation is more involved. First, suppose step 3 was never reached. Then, worker i exits having produced zero signals and cannot reduce the number of signals she produces further. Second, suppose step 3 was reached at least once. If during last time step 3 was reached worker j matched worker i's signal, then symmetry ensures that worker i has no profitable deviation. If during the last time step 3 was reached, worker j best responded by weakly decreasing the number of signals she produced, then the marginal value of information for worker i is larger after j's decision than before. Once again, worker i has no profitable downward deviation.

We now check for profitable upward deviations. When step 4 is reached, worker i does not have a profitable deviation upwards by construction. Moreover, $n_j^t \leq n_i^t$. If $n_j^t = n_i^t$, then j's incentives are the same as i's and so she has no profitable deviation upwards. If $n_j^t < n_i^t$, however, she has no incentive to produce $n_j^t + 1$ signals by construction. Moreover, since the marginal value of information is decreasing in the number of signals she produces, she does not want to produce any larger number of signals either.

[27] To understand why, consider the first time step 2 is reached. Worker i either (i) produces zero signals or (ii) produces one signal. In case (i), the algorithm proceeds to step 4 with both workers having produced zero signals. In case (ii), the algorithm proceeds to step 3. If worker j then matches worker i's first signal, we are done. Otherwise, worker j best responds by producing zero signals. But if she produces zero signals, she must exit the loop having produced zero signals; each future iteration at which step 3 is reached, the marginal value of producing a strictly positive number of signals decreases.

then our second observation about the algorithm suggests that $n_j^t + 1$ was not profitable. But, then, any larger number of signals is also not profitable. Hence, worker j has no profitable deviation upwards. We have thus established that the profile entering step 4 is a Nash equilibrium.

Step 4 and step 5 ensure that the algorithm finds a PEN. That this PEN is the most symmetric follows from the incremental construction in the cycle between step 2 and step 3.

□

The proof consists of two steps. First, we argue that if (n_1, n_2) with $n_1 > n_2$ is a PEN for correlation ρ, then for correlation $\rho' > \rho$ there exists an equilibrium (n_1', n_2') with $n_1' \geq n_1$ and $n_2' \leq n_2$. Second, we argue that if there is no symmetric PEN at ρ, then for $\rho' > \rho$ there is no symmetric PEN as well. These two properties together imply the result.

First, suppose that for correlation ρ there is a PEN (n_1, n_2) with $n_1 > n_2$. Then, it has to be that

$$MV(n_1; n_2, \rho) \geq MV(n_2 + 1; n_1, \rho).$$

Lemma 6 and Lemma 7 imply that $\rho > \hat{\rho}(n_2 + 1, n_1, \gamma) > \tilde{\rho}(n_2 + 1, n_1, \gamma)$. Hence, by Lemma 5, for any correlation $\rho' > \rho$, $MV(n_2+1; n_1, \rho') < MV(n_2+1; n_1, \rho)$. Pick $\rho' > \rho$ and start the *Sequential Response Algorithm* at step 3. Since the marginal value of draw n_2+1 is smaller at ρ' than at ρ, the optimal response of player 2 to n_1 draws is smaller than or equal to n_2 when the correlation is ρ' instead of ρ. Continuing with the algorithm, we find a PEN (n_1', n_2'). Since the number of signals produced by player 1 can only increase throughout the algorithm and the marginal value of a signal by player 2 decreases in the number of signals produced by player 1, we conclude that $n_1' \geq n_1$ and $n_2' \leq n_2$.

Second, suppose that for correlation ρ there is no symmetric PEN. Then, using the *Sequential Response Algorithm*, there must be an iteration t at which $n_2^t < t$ and $n_1^t = t$. Since player 1 has taken draw t, it means that

$$MV(t; t-1, \rho) \geq MV(t; t, \rho).$$

Then, Lemma 6 and Lemma 7 imply that $\rho > \hat{\rho}(t, t, \gamma) > \tilde{\rho}(t, t, \gamma)$. Hence, by Lemma 5, for any correlation $\rho' > \rho$, $MV(t; t, \rho') < MV(t; t, \rho)$. Therefore, at iteration t of the *Sequential Response Algorithm* when the pairwise correlation is ρ', it must be that $n_2^t < t$ as well. Furthermore, it must be that $n_1^t = t$; by Lemma 4, the marginal value of draw t for player 1 is larger at correlation ρ' than at correlation ρ. Since (n_1^t, n_2^t) is asymmetric, and any asymmetric profile at any iteration of the algorithm stays asymmetric, there is no symmetric

PEN.

A.3 Proof of Proposition 2

The following algorithm finds a Pareto-Efficient CSPE:

1. There are $\binom{4}{2} = 6$ possible two-worker teams. Within each of these teams, select a PEN in which the leader obtains the highest possible payoff. Among the set of all of these teams, identify the team, say $(1,2)$, in which the leader, say worker 1, obtains a higher payoff than any leader in any other team. If the PEN played in this team, $n^*(1,2)$, yields the leader a lower payoff than if she works alone, then in the unique CSPE each worker is alone and chooses an optimal number of signals. If this is not the case, fix the PEN $n^*(1,2)$ and set $\mu(1) = 2$. Proceed to the next step.

2. Fix $n^*(1,3)$, $n^*(1,4)$, $n^*(2,3)$, and $n^*(2,4)$ so that 1 and 2 are leaders in every PEN in every team to which they are not assigned. Fixing these PEN, worker 1 has no incentive to deviate from her current team since, by construction, she obtains a higher payoff than any leader in any team playing any PEN. And as worker 2 obtains a weakly higher payoff than worker 1 (she is acquiring weakly fewer signals), she has no incentive to deviate as well. Therefore, neither worker 3 nor worker 4 can persuade worker 1 or worker 2 to form a deviating team. If there is a PEN within the team $(3,4)$ that gives to both workers a higher utility than working alone, let $\mu(3) = 4$ and choose this PEN. Otherwise, let $\mu(3) = 3$ and $\mu(4) = 4$ and let each choose an optimal number of signals. In either case, we obtain a CSPE $(\mu, \{n^*(i,j)\}_{i \neq j \in \mathcal{N}})$. If the CSPE is Pareto-Efficient, we are done.

3. If the CSPE found in Step 2 is not Pareto-Efficient, there is another matching $\hat{\mu} \in \mathcal{M}$ and a collection of "on-path" PEN, $\hat{n}(i, \hat{\mu}(i))_{i \in \mathcal{N}}$, such that, for each worker,

$$v_i(\hat{n}(i, \hat{\mu}(i))) \geq v_i(n^*(i, \mu(i))),$$

and the inequality is strict for at least one worker. Consider the profile $(\hat{\mu}, \{\tilde{n}(i,j)\}_{i \neq j \in \mathcal{N}})$ where $(\tilde{n}_i, \tilde{n}_j) = (\hat{n}_i, \hat{n}_j)$ if $j = \hat{\mu}(i)$ and $(\tilde{n}_i, \tilde{n}_j) = (n_i^*, n_j^*)$ otherwise. This profile is a CSPE; on-path, each worker obtains a higher payoff than in the original CSPE and each worker has the same deviations as before. If it is not Pareto-Efficient, then repeat this step until a Pareto-Efficient CSPE is found. As there is a finite number of CSPE, the algorithm must end in a finite number of iterations.

A.4 Proof of Proposition 3

We make use of the following Lemma.

Lemma 9. *For any correlations, there exists a $K > 0$ such that any worker in a size-two team in which her partner produces a strictly positive number of signals strictly prefers that team to working alone.*

Proof. Suppose worker i and worker j play an arbitrary PEN (a, b), with $a, b > 0$. Suppose worker i optimally produces c signals alone. Clearly, absent membership costs, worker i is strictly better off in a team with worker j in which (c, b) is played. But, since (a, b) is a PEN, she is weakly better off under (a, b) than (c, b), and hence strictly better off under (a, b) than working alone. Hence, if the team membership cost is below some number $K_{ij} > 0$, the cost of forming the team is strictly smaller than the benefit for both i and j. Choosing $K = \min_{i,j \neq i} K_{ij} > 0$ completes the proof. □

1. We select correlations ρ_{12}, ρ_{13}, ρ_{14}, ρ_{23}, ρ_{24}, and ρ_{34} so that there is a CSPE in which teams $(1, 2)$ and $(3, 4)$ form, but for which there is a matching forming teams $(1, 3)$ and $(2, 4)$ and a collection of PEN that strictly increases welfare. By Assumption 1, the first two results of Proposition 1 apply. Hence, there exists a correlation $\rho^* > -1$ below which there is a unique and symmetric PEN in which each worker produces a strictly positive number of signals and a correlation $1 > \rho^{**} \geq \rho^*$ above which there is a unique and completely asymmetric PEN in which one worker produces zero signals. We utilize these correlations in the proof.

   First, choose $\rho_{12} < \rho^*$ so that the marginal value of a signal for each worker in the unique and symmetric PEN (n, n), $n > 0$, is strictly smaller than the marginal cost $c(n+1) - c(n)$. Second, choose ρ_{34} such that $\rho_{12} < \rho_{34} < \rho^*$ and the unique and symmetric PEN is (n, n) as well; by continuity of the marginal value of information, such a correlation is guaranteed to exist. Third, choose $\rho_{13}, \rho_{24} \in [\rho_{12}, \rho_{34}]$ close to ρ_{12} so that (n, n) is the unique PEN in teams $(1, 3)$ and $(2, 4)$. Fourth, choose ρ_{14} and ρ_{23} greater than ρ^{**} and select an arbitrary PEN in these teams. This ensures that teams $(1, 4)$ and $(2, 3)$ can never form in any CSPE; due to membership costs, the worker producing a strictly positive number of signals would prefer to work alone. Finally, utilizing Lemma 9, choose $K > 0$ small enough that any worker in a size-two team in which her partner produces a strictly positive number of signals strictly prefers that team to working alone.

By construction, the unique PEN in teams $(1,2)$, $(1,3)$, $(2,4)$, and $(3,4)$ is (n,n). Fixing this strategy profile, we observe that payoffs for any given worker are strictly decreasing in pairwise correlation,

$$\frac{\partial v(n,n;\rho)}{\partial \rho} = \frac{-2n\sigma^{-2}}{\left(2n\sigma^{-2} + (1+\rho)\sigma_\theta^{-2}\right)} < 0.$$

Hence, worker 1 and worker 2 each obtain a higher payoff together than in a team with either worker 3 or worker 4, and, by our restriction on $K > 0$, a strictly higher payoff than when alone. Team $(1,2)$ must therefore form in any CSPE. This leaves worker 3 and worker 4 with two options: form a team or work alone. But, again, the two workers prefer to form a team by our restriction on $K > 0$.

Notice, however, that if ρ_{13} and ρ_{24} are close enough to ρ_{12}, the gain from matching worker 1 (2) and worker 3 (4) (and selecting the unique PEN in these teams) strictly increases the total sum of utilities. Hence, we shown that our original CSPE is welfare dominated. Further, as incentives are strict everywhere, for $\epsilon > 0$ small, there is an ϵ-ball around our chosen correlations for which the same properties are satisfied.

2. We construct an Asymmetric Effort Inefficient CSPE in which teams $(1,3)$ and $(2,4)$ form. We again select correlations ρ_{12}, ρ_{13}, ρ_{14}, ρ_{23}, ρ_{24}, and ρ_{34}. By Assumption 1, the first two properties of Proposition 1 hold. Hence, we can choose correlations ρ_{14} and ρ_{23} above ρ^{**} so that teams $(1,4)$ and $(2,3)$ never form in any CSPE. Since $\sigma^2 \geq \sigma_\theta^2$, the third and fourth properties of Proposition 1 hold. Moreover, by Assumption 2, we can choose correlations $\rho_{13} > \rho^*$ and $\rho_{24} > \rho^*$ close to ρ^* in which there is a non-trivial asymmetric PEN in which each worker does strictly better as a follower than in the unique PEN at ρ^*. Fix these non-trivial PEN, $n^*(1,3)$ and $n^*(2,4)$, so that worker 1 and worker 4 are the followers in each team. Let the payoff to worker 1 in this team be denoted by u_1^* and the payoff to worker 4 be denoted by u_4^*.

   It remains to choose ρ_{12} and ρ_{34}. We claim that for any small $\epsilon > 0$, we can find a correlation $\rho_{12} < \rho^*$ such that the payoff to each worker in the unique symmetric PEN in team $(1,2)$ is $u_{12} \in (u_1^*, u_1^* - \epsilon)$. Why does such a correlation exist? First, observe that, at $\rho = -1$, where the unique PEN is $(1,1)$, each worker obtains a higher payoff than u_1^*. Next, notice that at ρ^*, the payoff to each worker, $\underline{u}$, is less than u_1^*. Any payoff between $\underline{u}$ and u_1^* is attainable for some $\rho \in [-1, \rho^*)$ since (i) ρ decreases below ρ^* payoffs increase, fixing a strategy profile and (ii) the number signals each worker produces decreases as ρ. (ii) implies that any discontinuities in the payoff correspondence must cause both workers to suffer a *decrease* in payoffs, as in Figure 3, thereby eliminating the possibility that the interval $(u_1^*, u_1^* - \epsilon)$ is "jumped over". A similar argument ensures we can find a correlation $\rho_{34} < \rho^*$ such that the payoff to each worker in the unique symmetric PEN in team $(3,4)$ is $u_{34} \in (u_4^*, u_1^* - \epsilon)$.

We argue that the matching μ satisfying $\mu(1) = 3$ and $\mu(2) = 4$, together with any collection of PEN that includes $n^*(1,3)$ and $n^*(2,4)$, is a CSPE. By construction, worker 1 (worker 4) is strictly better off playing $n^*(1,3)$ ($n^*(2,4)$) than if they formed a team with worker 2 (worker 3) and played the unique PEN in this team. Further, worker 2 (worker 3) prefers to match with worker 1 (worker 4) than to remain alone.

However, for $\epsilon > 0$ small enough, it is clear that the total sum of utilities is higher when matching worker 1 with worker 2 (worker 3 with worker 4) and playing the unique PEN in that team. Hence, the CSPE is welfare inefficient. Moreover, in these welfare-improving teams, worker 1 and worker 4 each exert relatively more effort than their partners in the original CSPE. Hence, the CSPE we constructed is Asymmetric Effort Inefficient. Again, as incentives are strict everywhere, for $\epsilon > 0$ small, there is an ϵ-ball around our chosen correlations for which the same properties are satisfied.

Online Appendix: "Matching to Produce Information: A Model of Self-Organized Research Teams"

Ashwin Kambhampati*, Carlos Segura-Rodriguez† and Peng Shao‡

A Assortative Matching

Chade and Eeckhout (2018) study optimal matching in an information environment related to ours. In theirs, the correlation between signals is constant, but precisions may be heterogeneous. They show that if utilities are transferable and each worker produces only one signal, the reduced form utility obtained from forecasting the state is submodular for a wide range of correlations. Therefore, if teams are composed of two workers, optimal matching is negative assortative: the best worker matches the worst worker, the second best matches the second worst, and so on.

In our environment, workers strategically choose the number of signals they produce and transfers are not possible. Moreover, correlation varies, but precisions are held constant. To isolate the effects of the first two features of our model, we assume in this section that precisions vary, but correlation is held to zero. Our main conclusion is that, perhaps unsurprisingly, it need not be true that the negative assortative matching maximizes welfare, nor that it emerges endogenously.[1]

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[1] That utilities are nontransferable is not necessary to revert their result, but we keep it to preserve the structure of the game we study. Following our approach, the equilibrium of the production game is inherently inefficient due to its public goods nature, while in a fully transferable world this inefficiency disappears. We focus on whether negative assortative matching is optimal given the equilibrium played inside each team.

Suppose each worker produces conditionally independent signals with precisions $\tau_1 < \tau_2 < \ldots < \tau_N$. As in the main text, suppose each agent receives the quadratic loss of her team's optimal forecast and that a team has at most two workers. Then, if workers i and j are in a team together, and produce n_i and n_j signals, the utility loss associated with their forecast is

$$-\frac{1}{\tau_\theta + n_i\tau_i + n_j\tau_j}.$$

An application of Proposition 2 of Chade and Eeckhout (2018) implies that the posterior variance is submodular in $n_i\tau_i$. Consequently, negative assortative matching with respect to $n_i\tau_i$ is optimal when workers are forced to choose one signal.

We consider what happens when i and j are free to choose the number of signals they produce. For simplicity, suppose worker i can produce signals with unit variance, the prior variance is equal to unity, and the cost of drawing n signals is $c(n) = 0.001n^2$. Figure 6 presents the resulting PEN correspondence and shows that, as worker j's signal variance increases, equilibria become asymmetric. Why? Since each of worker j's signals produce less information, fixing n_i and n_j, the marginal value of worker j's last signal decreases. On the other hand, the marginal value of a signal for worker i increases. Both forces lead to asymmetry.

The implications of this behavior for team formation are stark. Suppose that there are four workers with variances 0.25, 0.5, 1 and 1.25. If we match the best worker (the one with variance 0.25) with the worst worker (the one with variance 1.25), the unique PEN played within the team is $(2,0)$; the worst worker does not contribute at all. In contrast, when the worst worker is paired with the worker with variance 1, the unique PEN is $(2,1)$. Consequently, for small team membership costs, the optimal matching is $\{(0.25,1),(0.5,1.25)\}$, instead of the negative assortative matching, $\{(0.25,1.25),(0.5,1)\}$. Moreover, it turns out that the optimal matching can be decentralized as a CSPE, while the negative assortative matching cannot.

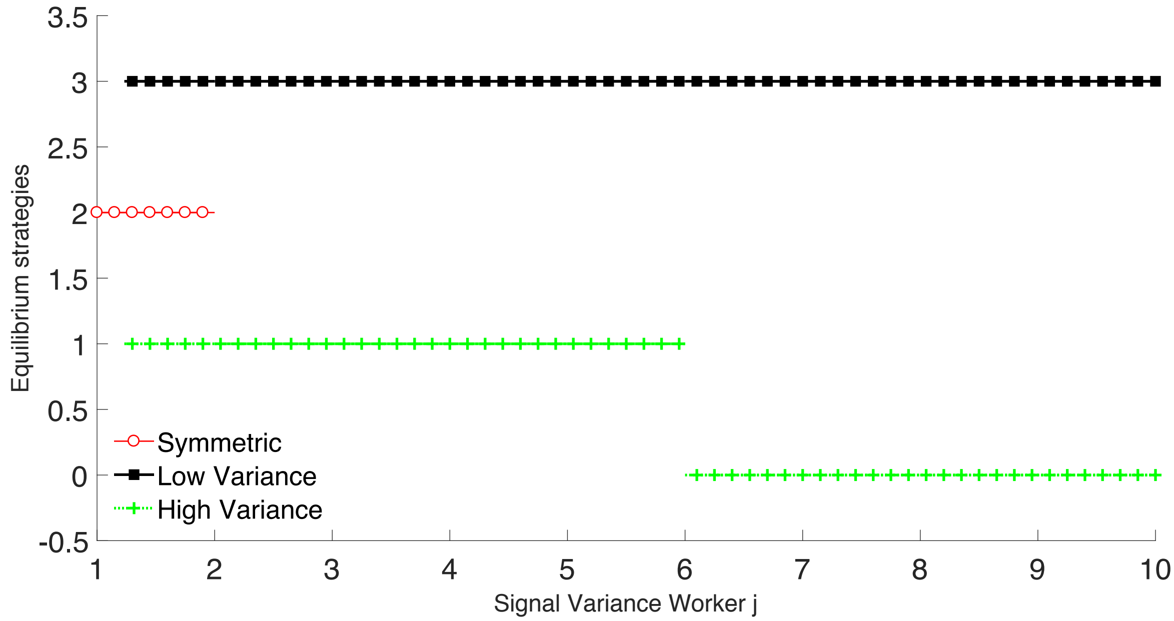


Figure 6: PEN Correspondence when $\rho_{ij} = 0$ and $\tau_i = 1$.

B Analysis of the core

We first formally define the core.

Definition 1 *A matching $\mu \in \mathcal{M}$ and a collection of PEN, $N^* = \{(n_i^*, n_j^*)\}_{i \in \mathcal{N}, j=\mu(i)}$, is in the* ***core*** *if there does not exist a matching, $\hat{\mu} \in M$, a worker k with match $\ell = \hat{\mu}(k)$, and a PEN $(\hat{n}_k, \hat{n}_\ell)$ for which:*

$$v_k(\hat{n}_k, \hat{n}_\ell) > v_k(n_k^*, n_{\mu(k)}^*), \text{ and}$$

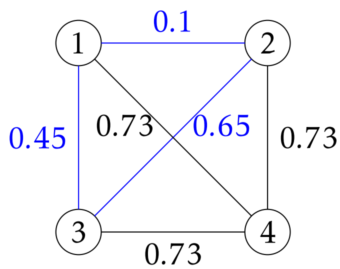
$$v_\ell(\hat{n}_k, \hat{n}_\ell) > v_\ell(n_\ell^*, n_{\mu(\ell)}^*).$$

Notice, the core coincides with the definition of a CSPE if there is a unique PEN within every feasible team. Furthermore, every core partition is a CSPE partition. However, in contrast to a CSPE, there may not exist any partition in the core.

We now present an example of an empty core. Suppose the four workers' technologies are correlated according to the network depicted in Figure 7 and the parameters of the model are those in Table 6. With positive membership costs, no worker forms a team with worker 4 in the core. We exhibit a preference cycle among the other three workers

Table 6: Parameters for example with empty core.

Parameter	Interpretation	Value
σ^2	Signal Variance	1
σ_θ^2	Prior Variance	1
$c(n)$	Cost of m Signals	$0.01n^2$
K	Cost of Teammate	0.01



Correlation	PEN	Payoffs
0.1	$(2,2)$	$(-0.256,-0.256)$
0.45	$(1,3)$	$(-0.238,-0.318)$
0.65	$(1,3)$	$(-0.247,-0.327)$
0.73	$(0,3)$	$(-0.25,-0.34)$

Figure 7: Empty core.

to show that the core is empty. Suppose that team $(1,2)$ forms and workers 3 and 4 work alone. Then, worker 3 and worker 1 can form a mutually beneficial deviating team in which worker 3 is the leader. Suppose that team $(1,3)$ is formed. Then, worker 2 can make an offer to the leader of team $(1,3)$ and form a mutually beneficial deviating team in which worker 2 is the leader. Suppose that team $(3,2)$ is formed. Then, worker 1 can form a mutually beneficial deviating team with its leader in which worker 1 is the leader. Finally, if all workers remain alone, worker 1 and worker 2 can form a team and be made better off. Hence, no matching is in the core.

C PEN Characterization Conditions are Necessary

Consider the equilibrium correspondence presented in Figure 8, where $\sigma^2 = \frac{1}{4} < 1 = \sigma_\theta^2$ violates the sufficient condition for the third and fourth properties in Proposition 1. In Figure 8, while for $\rho = -0.29$ there is a unique and asymmetric PEN, for a slightly higher correlation there is a unique and symmetric PEN.

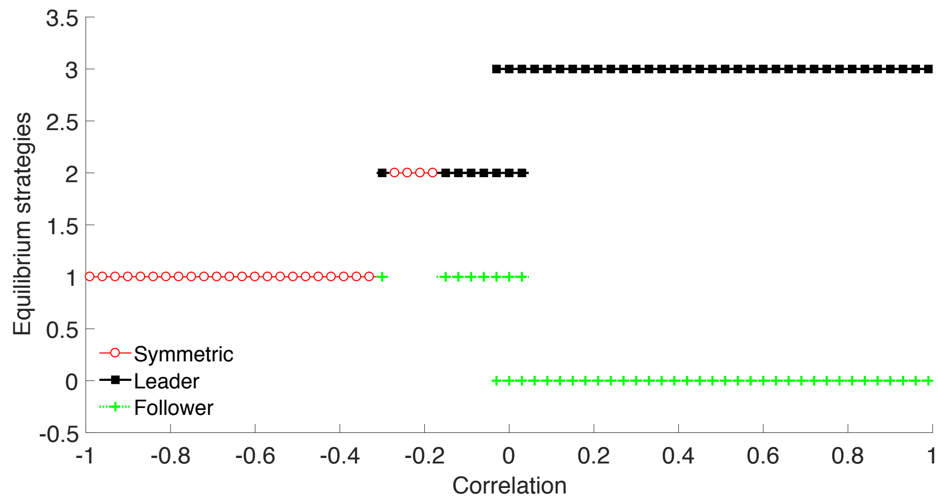


Figure 8: Equilibrium correspondence when $c(m) = 0.019m$, $\sigma^2 = \frac{1}{4}$ and $\sigma_\theta^2 = 1$.

Why does this happen? When $n = 1$, for $\rho = -0.29 \in (\hat{\rho}, \tilde{\rho})$ the marginal value of a signal for a leader is greater than the marginal value of a signal for a follower. We may then fix the marginal cost of a second signal so that the leader wants to produce it. But then, if ρ increases, the marginal value of the follower *increases* and may exceed the chosen marginal cost, so that she wants to produce a second signal as well. If the follower produces a second signal, however, the leader has no incentive to produce a third signal because the information left to learn decreases sufficiently. Hence, a symmetric equilibrium $(2, 2)$ is played.

D Binary States, Binary Signals

Suppose that the state θ is either High (H) or Low (L). For simplicity, suppose further that $Pr(\theta = H) = \frac{1}{2}$. Each worker can produce an informative signal, with realization H

		Worker i	
		H	L
Worker j	H	$p^2+\rho_{ij}p(1-p)$	$p(1-p)(1-\rho_{ij})$
	L	$p(1-p)(1-\rho_{ij})$	$(1-p)^2+\rho_{ij}p(1-p)$

Figure 9: Joint distribution when state is High (H).

or L realization, and it equals to the true state with probability $p > \frac{1}{2}$. Figure 9 presents the joint distribution over signal realizations when the state is H. If the state is L, the elements of the main diagonal are switched.

Notice that in this environment the feasible set of correlations is bounded below. In particular, statistical feasiblitiy requires that $\rho_{ij} \geq -\frac{1-p}{p}$. Hence, when a couple compares signals and has the most feasible negative correlation they need not learn the state; the state is revealed if HH (or LL) is observed, but not given any other realization. Further, for any correlation, there is a positive probability that HL or LH is observed.

Table 7: Expected Posterior Variance in the two-state model for some strategies.

# signals i	# signals j	Expected Posterior Variance
0	0	$\frac{1}{4}$
1	0	$p(1-p)$
1	1	$p(1-p)\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)} + \frac{1}{2}(1-\rho_{ij})\right)$
2	0	$p(1-p)\left(\frac{p(1-p)}{p^2+(1-p)^2} + \frac{1}{2}\right)$
2	1	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^3+(1-p)^3+\rho_{ij}p(1-p)} + 2(1-\rho_{ij}) + \frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{(1+\rho_{ij})p(1-p)}\right)$
2	2	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))^2(1-p+\rho_{ij}p)^2}{(p^2+\rho_{ij}p(1-p))^2+((1-p)^2+\rho_{ij}p(1-p))^2} + \frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{2p(1-p)} + (1-\rho_{ij})^2 + \frac{4(1-\rho_{ij})(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}\right)$
3	0	$p^2(1-p)^2\left(\frac{p(1-p)}{p^3+(1-p)^3} + 3\right)$
3	1	$p^2(1-p)^2\left(\frac{p(p-1)(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2(p^2+\rho_{ij}p(1-p))+(1-p)^2((1-p)^2+\rho_{ij}p(1-p))} + (1-\rho_{ij}) + \frac{2p(1-p)(1-\rho_{ij})}{p^2+(1-p)^2} + \frac{2(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)} + \frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p(1-p+\rho_{ij}p)+(1-p)(p+\rho_{ij}(1-p))}\right)$

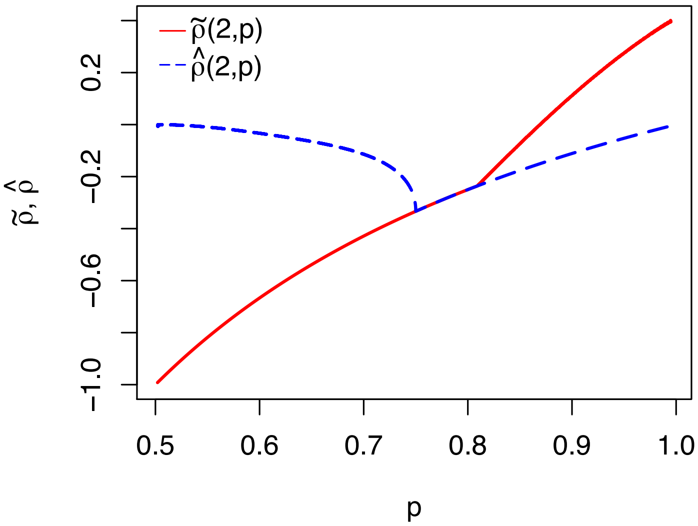


Figure 10: Values $\tilde{\rho}(2,p)$ and $\hat{\rho}(2,p)$ for different signal precisions p.

There is no simple expression for the expected posterior variance for an arbitrary profile of signals. Nonetheless, Table 7 computes it for a number of cases; these values are enough to find the PEN of the Production Subgame when each worker's best response is bounded by three. Defining $\tilde{\rho}(t,p)$ and $\hat{\rho}(t,p)$ as in the main text, Figure 10 displays their values when $t = 2$. The figure shows that it is still true that we have $\tilde{\rho}(2,p) > \hat{\rho}(2,p)$ if and only if the precision of the signal is high enough. We suspect a similar result is true for larger t.

E Sequential versus Simultaneous Decision

In this section, we present a finite sequential version of the game played within each team. We assume that the total number of periods $T \geq 2\bar{M}$, where $\bar{M}$ is the upper bound on best responses described in Lemma 2. In each period, each worker chooses whether or not to produce a signal $a_t^i \in \{0,1\}$. Signals across periods are conditionally independent and signals in the same period are correlated according to the pairwise correlation of teammates, ρ. In period t, all workers observe all actions a_{t-1} and signals x_{t-1} in periods $1,\ldots t-1$; the public history at period t is given by $h^{t-1} = (a_r, x_r)_{r=1}^{t-1}$ where $a_r = (a_r^1, a_r^2)$.

Let H^{t-1} denote the set of feasible histories up to period t. Then, a strategy for worker

i is a function $s_i : \bigcup_{t=1}^{T} H^{t-1} \to \{0,1\}$. The expected payoff of worker i given the history $(a_r, x_r)_{r=1}^{T}$ is:

$$v_i^{(i,j)}(((a_r)_{r=1}^{T})) = -\frac{1}{\left(\frac{2}{1+\rho_{ij}}\sum_{r=1}^{T} a_r^1 a_r^2 + \sum_{r=1}^{T}\left(a_r^1 + a_r^2 - 2a_s^1 a_r^2\right)\right)\sigma^{-2} + \sigma_\theta^{-2}} - c\left(\sum_{r=1}^{T} a_r^i\right).$$

We refer to the equilibrium outcome number of signals as (n_1, n_2), where $n_i = \sum_{r=1}^{T} a_r^i$.

We consider Subgame Perfect Equilibria that are not Pareto Dominated by any other Subgame Perfect Equilibrium– call such an equilibrium a Pareto-Efficient Subgame Perfect Equilibrium (PESP). The next proposition states that, if there is a PEN in the simultaneous game in which strategies differ by at most 1, there is an identical PESP outcome of the sequential game.

Proposition 4 *Let (m_1, m_2) be the most symmetric PEN in the simultaneous game. If $|m_1 - m_2| < 2$, there is a PESP of the sequential game with outcome (n_1, n_2), where $n_1 = m_1$ and $n_2 = m_2$.*

Proof After every history h^{t-1} each worker knows the posterior variance of θ, which we denote by $\sigma^t(h^{t-1})$. We define three automaton states: W_N, W_{D_1}, W_{D_2}. W_N is the state at which no worker deviates, W_{D_1} is the state at which worker 1 is the last deviator, and W_{D_2} is the state at which worker 2 is the last deviator. Consider the strategy profile

$$s_i(h^{t-1}) = \begin{cases} 1 \text{ if } n_i(\sigma^t(h^{t-1})) \geq T - t \\ 0 \text{ otherwise} \end{cases},$$

where $n_i(\sigma^t(h^{t-1}))$ is the number of the most symmetric equilibrium given the prior variance $\sigma^t(h^{t-1})$ and without loss $n_1(\sigma^t(h^{t-1})) \geq n_2(\sigma^t(h^{t-1}))$. Off the path of play choose any Nash equilibrium of the Subgame. If a worker deviates from the prescribed strategy profile then he takes the largest number of signals implied by this Nash equilibrium in the subgame that follows after.

To see why no worker has an incentive to deviate, notice if worker 1 does not produce a signal when she is prescribed to do so, then she can never produce as many signals as she was initially prescribed. But as $|n_1 - n_2| < 2$, worker 2 cannot compensate for worker 1's deviation. As worker 1 prefers to produce n_1 instead of $n_1 - 1$ signals in the simultaneous game, she has no incentive to deviate. A similar argument applies for worker 2. ■

The following example shows why we cannot extend the proposition to all correlations. Suppose $\sigma = \sigma_\theta = 1$ and $c(m) = 0.05m$. If $\rho = 0.15$, the only equilibrium in the simultaneous game is $(3,0)$. However, in the sequential game this cannot be a Subgame Perfect Equilibrium. Suppose worker 1 deviates and decides to produce only one signal in each of the last two periods. Then, the best response of worker 2 is to produce a signal in period $T-1$ or period T. This outcome gives worker 1 a payoff of -0.367 instead of -0.4.[2]

However, for large correlations, the same deviation is not profitable for worker 1 since worker 2 will never want to produce a signal in period T or period $T-1$. If both workers produce a signal during the same period, they would be highly correlated. Hence, worker 2 would not have incentive to produce a signal, since the extra information that is produced by her signal is almost zero. This observation illustrates that, for intermediate correlations, inefficiency due to asymmetric equilibria may be smaller in the extensive game than in the simultaneous game.

Although our intuition suggests that all equilibria of the simultaneous game are more asymmetric than all equilibria of the sequential game, this may not be true. In the following example, there is an asymmetric equilibrium of the sequential game that is more asymmetric than the most symmetric equilibrium of the simultaneous game. Furthermore, it is not an equilibrium of the simultaneous game. Consider the example in Figure 11 in which we graph the equilibrium correspondence of the simultaneous game. For correlation $\rho = 0.1$, the profile $(3,2)$ is the most symmetric equilibrium in the simultaneous game and $(4,1)$ is not an equilibrium. However, in the sequential game, the on-path sequence $(a_r)_{r=1}^{T}$, with $a_T^2 = 1$, $a_r^1 = 1$ for $r = T-4, T-3, T-2, T-1$ and $a_r^i = 0$ in any other period, is consistent with a PESP. Notice, all signals are taken in different periods and $(4,1)$ is the outcome number of signals. A deviation by worker 1 at period $T-4$ is not necessarily followed by an increase in the number of signals by worker 2, since an extra signal by her implies acquiring correlated information. It can be shown that a Nash

[2]In the unique Subgame Perfect Equilibrium, up to identity, worker 1 produces 2 signals and worker 2 produces 1 signal, with no signals taken in the same period.

equilibrium of the Subgame following such a deviation is $(3,1)$. As $(4,1)$ is preferred by worker 1 to $(3,1)$, worker 1 does not have the incentive to deviate at $T-4$. A similar argument applies for deviations in other periods.

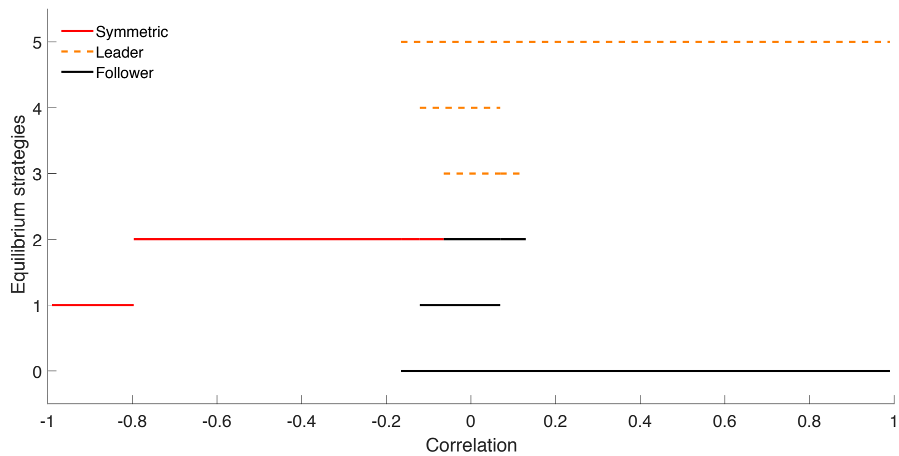


Figure 11: Equilibrium strategies when $c = 0.01n$, $\sigma = \frac{1}{2}$, and $\sigma_\theta = 1$.

F Continuous Action Space

In our model, the informativeness of a signal is scaled by its precision. In this section, we modify the production game by making signals more imprecise and scaling the cost so that there is no "free lunch" effect. This allows us to find a limit game where the action space is continuous.

Let us consider a sequence of games in which each signal becomes less informative. In the kth game, k signals are equivalent to a single signal of the original game. That is, the variance in the kth game, σ_k^2, is equal to $k\sigma^2$, where σ^2 is the variance of each signal in the original game. For simplicity, we assume that the cost of taking a signal is linear. No free lunch implies that in the kth game the cost of a signal is $\frac{c}{k}$, where c is the cost of a signal in the original game. Suppose workers i and j are in a team together and the correlation between their signals is ρ. Then in the kth game, if they choose n_i^k and n_j^k signals, worker i's payoff is given by

$$v_i^{(i,j)}(n_i^k, n_j^k) = \left(\left(\min\left\{ \frac{n_i^k}{k}, \frac{n_j^k}{k} \right\} \frac{2}{1+\rho} + \left| \frac{n_i^k}{k} - \frac{n_j^k}{k} \right| \right) \sigma^{-2} + \sigma_\theta^{-2} \right)^{-1} - c\frac{n_i^k}{k}.$$

Notice that for any real number z and fixed $\epsilon > 0$, there exist rational numbers k and n such $\left|\frac{n}{k} - z\right| < \epsilon$. Therefore, the sequence of games converges to the game where player i chooses $r_i \in \mathbb{R}_+$ and, if workers choose r_i and r_j signals, worker i's payoff is given by

$$v_i^{(i,j)}(r_i, r_j) = \frac{-\sigma^2}{(\underline{r}_{ij}(\phi_{ij} - 1) + \bar{r}_{ij}) + \gamma} - cr_i,$$

where $\underline{r}_{ij} = \min\{r_i, r_j\}$, $\bar{r}_{ij} = \max\{r_i, r_j\}$, $\phi_{ij} = \frac{2}{1+\rho_{ij}}$ and $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$.

As in the discrete game, workers i and j's payoff when in a team together depend on a factor $\phi_{ij} \in [1, \infty)$ that specifies the team's productivity. The equilibrium correspondence is similar to the one described in the main text and characterized in the following proposition.

Proposition 5

- *If $\phi_{ij} < 2$, the unique Nash equilibrium, up to the identity of the workers, is $\left(0, \sqrt{\frac{\sigma^2}{c}} - \gamma\right)$.*
- *If $\phi_{ij} = 2$, any strategy profile such that $r_i + r_j = \sqrt{\frac{\sigma^2}{c}} - \gamma$ is a PEN.*
- *If $\phi_{ij} \geq 2$, the only PEN is*

$$r_i = r_j = \frac{\sqrt{\frac{\sigma^2(\phi_{ij}-1)}{c}} - \gamma}{\phi_{ij}}.$$

Proof Suppose $r_i > r_j$. Then, the marginal value of r_i for worker i is,

$$\frac{\sigma^2}{\left(r_j(\phi_{ij} - 1) + r_i + \gamma\right)^2}$$

and the marginal value of r_j for worker j is,

$$\frac{(\phi_{ij} - 1)\sigma^2}{\left(r_j(\phi_{ij} - 1) + r_i + \gamma\right)^2}.$$

If $\phi_{ij} < 2$ the marginal value for worker j is always smaller than worker i's marginal value, so there is a corner solution in which $r_j = 0$. Given r_j, i's best-response is $r_i =$

$\sqrt{\frac{\sigma^2}{c}} - \gamma$.

If $\phi_{ij} = 2$, the marginal value of a signal is the same for both workers. Optimally, each chooses r so that the marginal value equals the marginal cost. Since any investment division between the workers does not affect the marginal output, any profile (r_i, r_j) such that $r_i + r_j = \sqrt{\frac{\sigma^2}{c}} - \gamma$ is an equilibrium.

If $\phi_{ij} > 2$, it cannot be the case that $r_i > r_j$ since the marginal benefit for worker j is strictly larger and both workers face the same marginal cost. Hence, all equilibria are symmetric. For (r, r) to be an equilibrium, it must be the case that:

$$\left.\frac{\sigma^2}{\left(r_j(\phi_{ij} - 1) + r_i + \gamma\right)^2}\right|_{r_i = r_j} \leq c,$$

and,

$$\left.\frac{(\phi_{ij} - 1)\sigma^2}{\left(r_j(\phi_{ij} - 1) + r_i + \gamma\right)^2}\right|_{r_i = r_j} \geq c.$$

The only PEN is the profile in which $r = r_i = r_j$ is maximized and satisfies the previous constraints. Hence, the second inequality binds. Re-arranging yields the equation stated in the proposition. ■

The proposition implies that for negative correlations the only equilibrium is symmetric, for conditionally independent signals there is multiplicity, and for positive correlations the only equilibrium is fully asymmetric.

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