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Pitch Control of Flight System using Dynamic Inversion and PID Controller

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***Abstract*—The paper presents the pitch control of aircraft in cruising stage using Dynamic Inversion concept and Proportional-Integral-Derivative controllers. Flight system has been designed using the linearized longitudinal dynamics of aircraft. Simulation result for the control of pitch angle is presented in time domain. The performance are investigated and analyzed based on common criteria of step response in order to identify whether the control strategy delivers better performance with respect to the desired pitch angle.**

Index Terms* — *Aircraft, Longitudinal dynamics, Dynamic inversion, PID controller.

I. INTRODUCTION

The rapid advancement of many technologies has contributed to the development of aircraft design from the very limited capabilities of the Wright brothers, first successful airplane to today's high performance military, commercial and general aviation aircraft. Development has made in its aerodynamics, structures, materials used, propulsion system and flight control systems. Modern aircraft have a variety of automatic control system [1], [2] that helps the flight crew in navigation, augmenting the stability characteristic and flight management of the airplane.

In the early days of aviation, in order to fly safely aircraft requires the continuous attention of a pilot. As aircraft range increases allowing many hours of flight, the constant attention of pilot may lead to serious fatigue. After many years of advancing technology aircraft soon adapt the concept of autopilot and it is designed to perform some tasks of the pilot. The first aircraft autopilot was developed by Sperry Corporation in 1912. The autopilot is also called as a pilot assistant, it assist the pilot during long journey flight. It permits the aircraft to fly straight and on a level course without a pilot's attention, thereby it greatly reduces the pilot's workload.

The paper focuses on the design of an autopilot that controls the pitch of aircraft, which can be used by the flight crew to lessen their workload during cruising stage. In the longitudinal plane, elevator controls the pitch of aircraft. Elevators are also called longitudinal controls. By moving the elevator control backwards, the rear of elevator deflects upwards and tail is given a negative camber resulting in a downward force which causes pitch down of the tail and pitch up of the airplane. Similarly, by moving the elevator control forwards, the rear of elevator deflects downwards and tail is given a positive camber resulting in an upward force which causes pitch up of the tail and pitch down of the airplane.

The paper presents the design of an autopilot that controls the pitch of the aircraft using PID with dynamic inversion concept, during cruising stage. The performance is analyzed based on common criteria of step response for a desired pitch angle. Performance of both PID and PID with Dynamic Inversion control strategy with respect to the pitch angle and pitch rate is examined.

II. MATHEMATICAL MODEL OF A PITCH CONTROL SYSTEM

The forces and moments acting on the aircraft are shown in Fig.1. The applied forces and moments on the aircraft and the resulting response of the aircraft are described by a set of equations known as equations of motion [1], [7]. The forces acting on the airplane includes gravitational force, thrust forces and aerodynamic forces. The gravitational force acts through CG and therefore does not contribute any moment about CG. The thrust force and aerodynamic force also can be assumed to act at CG and hence the moment contribution is taken as zero. In order to reduce the complexity of analysis, the six equations of motion can be decoupled into two sets of three equations each, namely the longitudinal and lateral equations.

For controlling the pitch of an aircraft, it is necessary to use only the longitudinal equations of motion. Longitudinal motion consists of those movements where the aircraft would only move within the x-z plane that is, translation in x and z directions and rotation about y axis. The three longitudinal equations of motion consist of X-force, Z-force and M-moment equations respectively.

$$X - mg\ sin\Theta = m(\dot{u} - rv + qw \qquad (1)$$

$$M = I_y\dot{q} + I_{xz}(p^2 - r^2) + rp(I_x - I_z) \qquad (2)$$

$$Z + mg\ cos\Theta\ cos\phi = m(\dot{w} + pv - qu) \qquad (3)$$

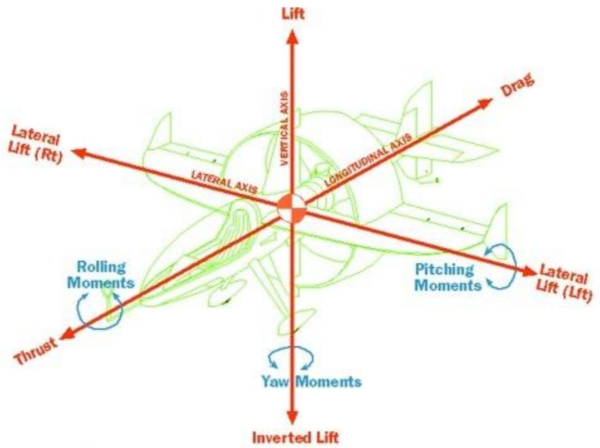

Figure 1. Forces and moments on aircraft

X and Z are the total aerodynamic and propulsive forces acting in the x and z directions. u, v, w-Linear velocities along x, y and z axes and p, q, r- components of angular velocity along x, y and z axes respectively i.e. roll rate, pitch rate and yaw rate respectively, $w = pi + qj + rk$. The pitch control system is shown in Fig.2, where X_b, Y_b and Z_b represent the aerodynamics force components. Θ, ϕ and δ_e represent the orientation of aircraft (pitch angle), roll angle in the earth-axis system and elevator deflection angle.

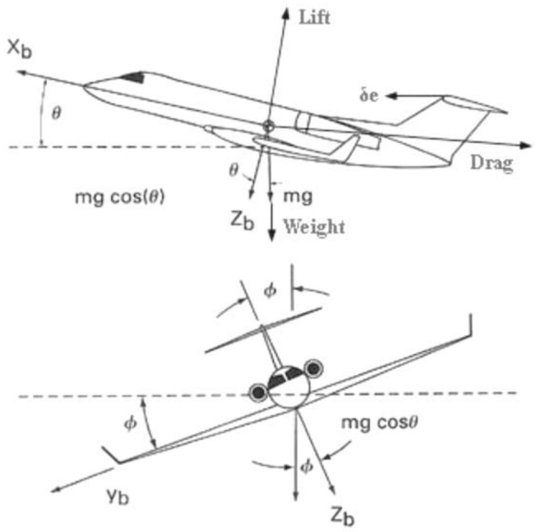

Figure 2. Pitch control system of aircraft

The equations of motion in (1),(2) and (3) can be linearized using small disturbance theory, assuming the motion of the airplane consists of deviations about a steady flight condition. All the variables in the equations of motion are replaced by a reference value plus a disturbance or perturbation, as shown below.

$u = u_o + \Delta u \qquad v = v_o + \Delta v \qquad w = w_o + \Delta w$
$p = p_o + \Delta p \qquad q = q_o + \Delta q \qquad r = r_o + \Delta r$
$X = X_o + \Delta X \qquad M = M_o + \Delta M \qquad Z = Z_o + \Delta Z$
$\delta_e = \delta_{e_o} + \Delta\delta_e$

The equations for perturbations can be obtained by substituting the values of perturbed variables in the governing equations. Assuming that the reference flight condition is to be symmetric and the propulsive forces remain constant. This assumption implies that, $v_o = p_o = r_o = \phi_o = \psi_o = w_o = 0$. After linearization the equations (4), (5) and (6) are obtained.

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta_w + (g\cos\Theta_o)\Delta\Theta = X_{\delta_e}\Delta\delta_e \tag{4}$$

$$-Z_u\Delta u + \left[(1 - Z_w)\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_o + Z_q)\frac{d}{dt} - g\sin\Theta_o\right]\Delta\Theta = Z_{\delta_e}\Delta\delta_e \tag{5}$$

$$-M_u\Delta u - \left(M_w\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\Theta = M_{\delta_e}\Delta\delta_e \tag{6}$$

From the above equations (4), (5) and (6) transfer function for the change in the pitch rate to the change in elevator deflection angle and the change in pitch angle to the change in elevator deflection are obtained as equation (7) and (8).

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{-\left(M_{\delta_e} + \frac{M_{\dot\alpha}Z_{\delta_e}}{u_o}\right)s - \left(\frac{M_\alpha Z_{\delta_e}}{u_o} - \frac{M_{\delta_e}Z_\alpha}{u_o}\right)}{s^2 - \left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{u_o}\right)s + \left(\frac{Z_\alpha M_q}{u_o} - M_\alpha\right)} \tag{7}$$

$$\frac{\Delta\Theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}\cdot\frac{-\left(M_{\delta_e} + \frac{M_{\dot\alpha}Z_{\delta_e}}{u_o}\right)s - \left(\frac{M_\alpha Z_{\delta_e}}{u_o} - \frac{M_{\delta_e}Z_\alpha}{u_o}\right)}{s^2 - \left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{u_o}\right)s + \left(\frac{Z_\alpha M_q}{u_o} - M_\alpha\right)} \tag{8}$$

In this study the data from General Aviation Airplane is used in system analysis and modeling. The parameter include in dimensional derivatives are;

$Q = 36.8\,lb/ft^2 \qquad QS = 6771lb \qquad QS\bar{c} = 38596ft.lb$
$(\bar{c}/2u_o) = 0.016s \qquad X_u = -0.045 \qquad Z_u = -0.369$
$M_u = 0 \qquad X_w = 0.036 \qquad Z_w = -2.02 \qquad M_w = -0.05$
$X_{\dot w} = Z_{\dot w} = 0 \qquad M_{\dot w} = -0.051 \qquad X_{\dot\alpha} = X_\alpha = Z_{\dot\alpha} = 0$
$Z_\alpha = -355.42 \qquad M_\alpha = -8.8 \qquad M_{\dot\alpha} = -0.8976 \qquad X_q = 0$
$Z_q = 0 \qquad M_q = -2.05 \qquad X_{\delta_e} = 0 \qquad Z_{\delta_e} = -28.15$
$M_{\delta_e} = -11.874$

Two assumptions made in this paper are: First, the aircraft is steady state cruise at constant altitude and velocity. Second, the change in pitch angle does not change the speed of an aircraft under any circumstance. Substituting the values of the stability derivatives in the equations (4), (5), (6), (7) and (8) transfer function and the state space form $\dot{x} = Ax + Bu$ is obtained as equation (9), (10) and (11).

$$\frac{\Delta\Theta(s)}{\Delta\delta_e(s)} = \frac{11.7304s + 22.578}{s^3 + 4.967s^2 + 12.941s} \tag{9}$$

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\Theta}\end{bmatrix} = \begin{bmatrix}-2.02 & 1 & 0\\ -6.9868 & -2.9476 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\Theta\end{bmatrix} + \begin{bmatrix}0.16\\ 11.7304\\ 0\end{bmatrix}[\Delta\delta_e] \quad (10)$$

$$y = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\Theta\end{bmatrix} + [0] \quad (11)$$

III. METHODOLOGY

In this section two control schemes are proposed in detail which is the conventional PID controller and PID controller with Dynamic Inversion (DI) concept. In this work the design considerations are rising time less than 3 second, settling time less than 5 second, percentage of overshot less than 12% and steady state error less than 2% for controlling the pitch angle of 0.2 radian (11.5 degree).

A. PID CONTROLLER

A PID controller is an extreme form of phase lag-lead compensator. It is a three-term controller, whose transfer function is generally written in the parallel form as given in equation (12)

$$G(s) = K_P + K_I\frac{1}{s} + K_D s = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \quad (12)$$

where, K_P is the proportional gain, K_I the integral gain, K_D the derivative gain, T_I the integral time constant and, T_D the derivative time constant. According to the size of error a proportional controller controls the output. Integral action reduces the steady state error through low frequency compensation and derivative term helps to improve transient response through high frequency compensation by a differentiator. A PID controller is used to control the pitch of aircraft [5] and the block diagram is shown in Fig: 3.

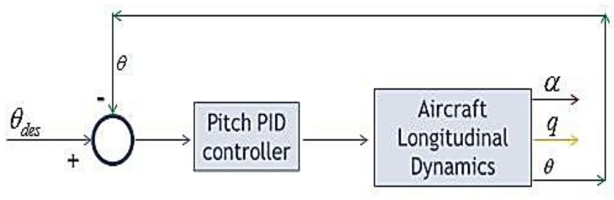

Figure.3 Block diagram of pitch control system using PID

There are different methods of tuning a PID controller; in this paper Automatic PID tuning using MATLAB/SIMULINK is used. The values of gain obtained using automated tuning of PID are, $K_P = 28.75$ $K_I = 17.81$ $K_D = 10.3$.

B. *PITCH CONTROL OF AIRCRAFT USING DYNAMIC INVERSION*

Dynamic inversion is a nonlinear control technique used to control aircraft; it is used when aircraft dynamics are linear and nonlinear. Dynamic inversion is a design technique used to synthesize flight controllers whereby the set of existing or undesirable dynamics are cancelled out and replaced by a designer selected set of desired dynamics. In this work, a linearized longitudinal dynamics of aircraft is used and the use of DI concept increases the generality character of pitch control system [3], [6]. Base on control effectiveness, there is slow dynamics and fast dynamics. Pitch rate control generally respond very fast to the changes in control input than pitch angle. So, pitch rate control forms the inner loop of dynamic inversion and outer loop is pitch angle, controlled by a PID controller. Using dynamic inversion, the command vector $\delta_e$is obtained from the state space equation $\dot{x} = Ax + Bu$ given as;

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\Theta}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\Theta\end{bmatrix} + \begin{bmatrix}b_{11}\\ b_{21}\\ b_{31}\end{bmatrix}[\Delta\delta_e] \quad (13)$$

Using the second differential equation, we get the command vector as;

$$\Delta\delta_e = b_{21}^{-1}[\Delta\dot{q}_c - a_{21}\Delta\alpha - a_{23}\Delta q] \quad (14)$$

where $\Delta\dot{q}_c$ is the calculated value of $\Delta\dot{q}$ and the equation for $\Delta\dot{q}_c$ is given as;

$$\Delta\dot{q}_c = \ddot{\Theta}_{des} + K_P^{\Theta}(\Theta_{des} - \Theta) + K_I^{\Theta}\int(\Theta_{des} - \Theta)\,dt + K_D^{\dot{\Theta}}\frac{d}{dt}(\Theta_{des} - \Theta) \quad (15)$$

The block diagram of the pitch control system using PID with Dynamic Inversion concept is shown in Fig: 4.

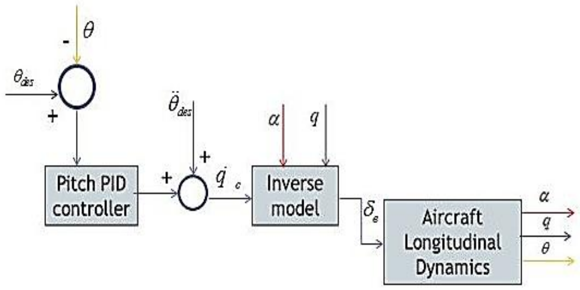

Figure.4 Block diagram of pitch control system using PID with DI

IV. SIMULATION RESULTS

The proposed control schemes are implemented and the corresponding results are presented for an aviation aircraft based on common criteria of step response. The pitch control system using PID and PID with DI has been simulated using MATLAB/SIMULINK software and the response of the system for a given pitch angle is obtained.

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The response of the system with PID controller is shown in Fig:5. The response of system with PID and Dynamic Inversion is shown in Fig: 6. A comparative assessment based on time response specification performance between PID and PID with DI for the pitch control of an aircraft system is presented in Table I. The results clearly shows that PID with Dynamic Inversion gives a good performance as compared to PID controller in terms of delay time that is 0.036s, rise time that is 0.08s, settling time that is 2.56s, percentage overshoot that is 11.5% and steady state error that is zero. For a PID controller delay time is obtained as 0.3s, rise time is 0.89s, settling time is 6.5s, percentage overshoot is 10.6%, and steady state error is 0.0001.

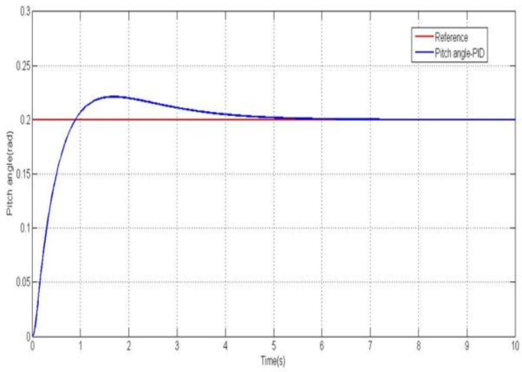

Figure.5 Response of system with PID controller

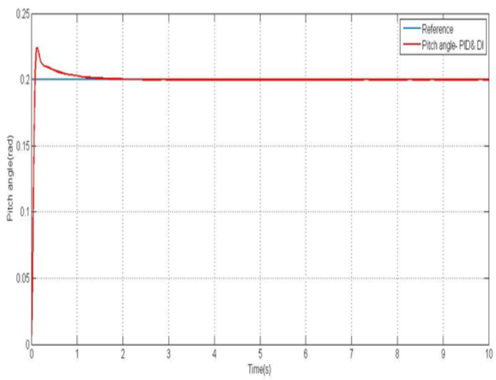

Figure.6 Response of system with PID and DI

From the result obtained in Fig:7, it can be concluded that the PID with DI controller provide higher ability in controlling the pitch angle as compared to the PID controller.

TABLE I.
PERFORMANCE CHARACTERISTICS FOR PITCH ANGLE

Response Characteristics	Pitch Angle	
	PID	PID & DI
Delay time (T_d)	0.3s	0.036s
Rise time (T_r)	0.89s	0.08s
Settling time (T_s)	6.5s	2.56s
Percentage Overshoot (OS%)	10.6%	11.5%
Steady state error (ess)	0.0001	0

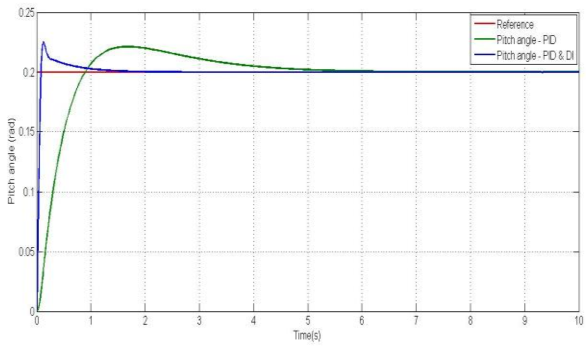

Figure.7 Response of system with PID & PID with DI

V. CONCLUSION

Pitch control system is one that requires a pitch controller to maintain the pitch angle at a desired value. Two controllers, PID and PID with Dynamic Inversion are successfully designed and presented. Simulation results show that, PID with Dynamic Inversion controller relatively gives the better performance compared to PID controller in controlling the pitch angle of an aircraft system. Based on the analysis it is found that the percentage overshoot for PID with Dynamic Inversion is greater so, for further research, effort can be devoted through adding another element that make up the control system to give better performance, followed by the development of more advanced and robust control techniques that can control the pitch of aircraft. Besides, the proposed control algorithm can be implements to real plant for validating the theoretical results.

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