6-6 Placing Figures in the Coordinate Plane

ALGEBRA

What You'll Learn

- To name coordinates of special figures by using their properties

. . . And Why

To examine a T-shirt design, as in Example 1

Check Skills You'll Need

GO for Help Lesson 6-1

Draw a quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral. 1–4. See back of book.

1. $H(-5, 0), E(-3, 2), A(3, 2), T(5, 0)$
2. $S(0, 0), A(4, 0), N(3, 2), D(-1, 2)$
3. $R(0, 0), A(5, 5), I(8, 4), N(7, 1)$
4. $W(-3, 0), I(0, 3), N(3, 0), D(0, -3)$

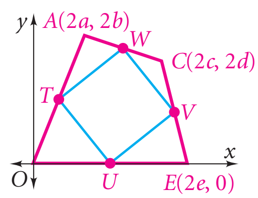
1 Naming Coordinates

When working with a figure in the coordinate plane, it generally is good practice to place a vertex at the origin and one side on an axis. You can also use multiples of 2 to avoid fractions when finding midpoints.

1 EXAMPLE Real-World Connection

T-Shirt Design An art class creates T-shirt designs by drawing quadrilaterals and connecting their midpoints to form other quadrilaterals. Tiana claims that everyone's inner quadrilateral will be a parallelogram. Is she correct? Explain.

Draw quadrilateral $OACE$ with one vertex at the origin and one side on the x-axis. Since you are finding midpoints, use coordinates that are multiples of 2. Find the coordinates of the midpoints T, W, V, and U.



$$T = \text{midpoint of } \overline{OA} = \left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) = (a, b)$$

$$W = \text{midpoint of } \overline{AC} = \left(\frac{2a+2c}{2}, \frac{2b+2d}{2}\right) = (a+c, b+d)$$

$$V = \text{midpoint of } \overline{CE} = \left(\frac{2c+2e}{2}, \frac{2d+0}{2}\right) = (c+e, d)$$

$$U = \text{midpoint of } \overline{OE} = \left(\frac{0+2e}{2}, \frac{0+0}{2}\right) = (e, 0)$$

Find the slopes of the sides of $TWVU$.

$$\text{slope of } \overline{TW} = \frac{b-(b+d)}{a-(a+c)} = \frac{d}{c} \qquad \text{slope of } \overline{VU} = \frac{d-0}{(c+e)-e} = \frac{d}{c}$$

The slopes are equal, so $\overline{TW} \parallel \overline{VU}$. Similarly, $\overline{WV} \parallel \overline{TU}$.

Since both pairs of opposite sides of $TWVU$ are parallel, $TWVU$ is a parallelogram, and Tiana is correct.

Quick Check 1 Find the slopes of $\overline{WV}$ and $\overline{TU}$. Verify that $\overline{WV}$ and $\overline{TU}$ are parallel.
Both $= \frac{b}{a-e}$, so $\overline{WV} \parallel \overline{TU}$.

6-6

1. Plan

Objectives

1 To name coordinates of special figures by using their properties

Examples

1 Real-World Connection
2 Naming Coordinates

Professional Development

Math Background

Coordinate geometry was introduced by Descartes and Fermat in the 17th century. It can sometimes be used to prove theorems that otherwise seem intractable.

More Math Background: p. 304D

Lesson Planning and Resources

See p. 304E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Classifying Special Quadrilaterals
Lesson 6-1: Example 2
Extra Skills, Word Problems, Proof Practice, Ch. 6

Differentiated Instruction Solutions for All Learners

Special Needs L1	Below Level L2
For Example 2, have students make a cutout of a 10 square unit rectangle and trace it in different positions on the coordinate grid. Have students place each vertex so that the coordinates will be integers. **learning style: tactile**	Work through Examples 1 and 2 using numerical coordinates before introducing variables. Generalize with variables only after students understand the arithmetic. **learning style: verbal**

2. Teach

Guided Instruction

1 EXAMPLE Teaching Tip

If necessary, review the slope and midpoint formulas.

2 EXAMPLE Visual Learners

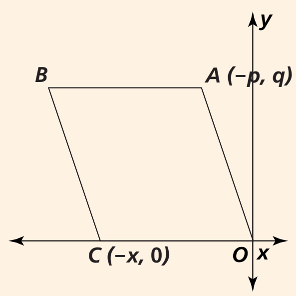
Suggest that students highlight the axes on a copied diagram to help them more readily find the coordinates of the vertices.

PowerPoint

Additional Examples

1 Show that *TWVU* in Example 1 is a parallelogram by proving both pairs of opposite sides congruent. **Use the Distance Formula to show *TW* = *VU* and *WV* = *TU*.**

2 Use the properties of parallelogram *OCBA* to find the missing coordinates. Do not use any new variables.



O(0, 0), B(−p − x, q)

Resources

- Daily Notetaking Guide 6-6 L3
- Daily Notetaking Guide 6-6—Adapted Instruction L1

Closure

A square with side length *a* is drawn with one vertex at the origin in a coordinate plane. Find possible coordinates for its other vertices. **Sample: (*a*, 0), (*a*, *a*), (0, *a*)**

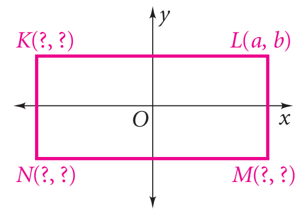
Centering figures at the origin can also be helpful when placing figures in the coordinate plane.

Problem Solving Hint

A rectangle centered at the origin has opposite sides equidistant from the origin.

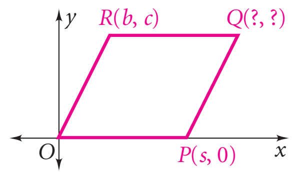
2 EXAMPLE Naming Coordinates

Algebra In the diagram, rectangle $KLMN$ is centered at the origin with sides parallel to the axes. Find the missing coordinates.



L has coordinates (a, b), so the coordinates of the other vertices are $K(-a, b)$, $M(a, -b)$, and $N(-a, -b)$.

Quick Check **2** Use the properties of parallelogram $OPQR$ to find the missing coordinates. Do not use any new variables. **$Q(s + b, c)$**



EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 343)

GO for Help

1. Claim: The midpoint of the hypotenuse of a right triangle is equidistant from the vertices of the triangle. Follow Steps 1 and 2 to place a right triangle in the coordinate plane. Complete Steps 3–5 to verify the claim. **See margin.**

Given: Right $\triangle ABC$ with M the midpoint of hypotenuse $\overline{AB}$

Prove: $MA = MB = MC$

Step 1: Draw right $\triangle ABC$ on a coordinate plane. Locate the right angle, $\angle C$, at the origin and leg $\overline{CA}$ on the positive x-axis.

Step 2: You seek a midpoint, so label coordinates using multiples of 2. The coordinates of point A are **a.** _?_. The coordinates of point B are **b.** _?_.

Step 3: By the Midpoint Formula, the coordinates of midpoint M are **c.** _?_.

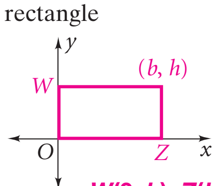
Step 4: By the Distance Formula, MA = **d.** _?_, MB = **e.** _?_, and MC = **f.** _?_.

Step 5: Conclusion: **g.** _?_

Example 2 (page 344)

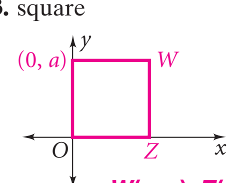
Algebra **Give coordinates for points *W* and *Z* without using any new variables.**

2. rectangle



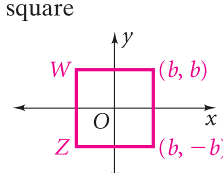
W(0, h); Z(b, 0)

3. square



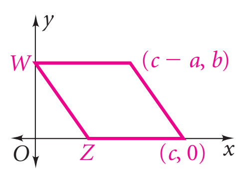
W(a, a); Z(a, 0)

4. square



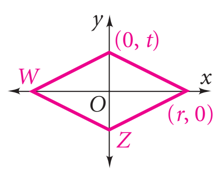
W(−b, b); Z(−b, −b)

5. parallelogram



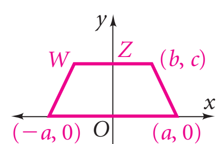
W(0, b); Z(a, 0)

6. rhombus



W(−r, 0); Z(0, −t)

7. isosceles trapezoid



W(−b, c); Z(0, c)

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Have students rework Example 1 without using coordinate geometry.

learning style: verbal

English Language Learners ELL
Review key terms used in the lesson such as *midpoint, slope, vertical,* and *horizontal.* Ask students for the formulas of midpoint and slope and to identify the slopes of vertical and horizontal lines.

learning style: verbal

B Apply Your Skills

8. **Writing** Choose values for r and t in Exercise 6. Find the slope and length of each side. State why the figure satisfies the definition of a rhombus. **See back of book.**

9. What property of a rhombus makes it convenient to place its diagonals on the x- and y-axes? (See Exercise 6.) **See back of book.**

Here are coordinates for eight points in the coordinate plane ($q > p > 0$).

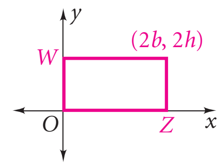
$A(0, 0)$, $B(p, 0)$, $C(q, 0)$, $D(p + q, 0)$, $E(0, q)$, $F(p, q)$, $G(q, q)$, $H(p + q, q)$

Which four points, if any, are the vertices for each type of figure?

10–15. Answers may vary. Samples are given.

10. parallelogram **A, C, H, F**
11. rhombus **B, D, H, F**
12. rectangle **A, B, F, E**
13. square **A, C, G, E**
14. trapezoid **A, C, F, E**
15. isosceles trapezoid **A, D, G, F**

Refer to the diagrams in Exercises 2–7. Use the coordinates given below in place of the ones shown. Then give the coordinates for points W and Z without using any new variables. The new diagram for Exercise 16 is shown here.



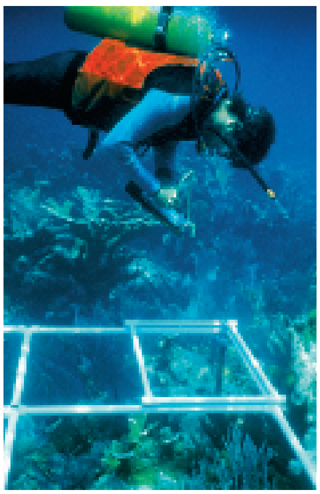
16. Ex. 2, $(2b, 2h)$
17. Ex. 3, $(0, 2a)$
18. Ex. 4, $(2b, 2b)$, $(2b, -2b)$
19. Ex. 5, $(2c, 0)$, $(2c - 2a, b)$
20. Ex. 6, $(2r, 0)$, $(0, 2t)$
21. Ex. 7, $(-2a, 0)$, $(2a, 0)$, $(2b, 2c)$

16–21. See back of book.

GO Online Homework Help
Visit: PHSchool.com
Web Code: aue-0605

22. **Multiple Choice** The coordinates of three vertices of a rectangle are $(-2a, 0)$, $(2a, 0)$, and $(2a, 2b)$. What are the coordinates of the midpoint of the diagonal joining one of these points with the fourth vertex? **A**

Ⓐ $(0, b)$ Ⓑ $(0, 2b)$ Ⓒ $(2a, b)$ Ⓓ $(-2a, 2b)$

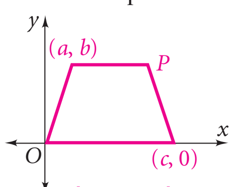


Real-World Connection

Careers Underwater archaeologists also spend time on land studying historical data to locate submerged sites or ships.

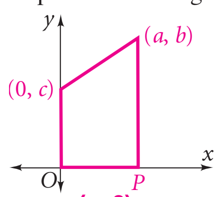
Give the coordinates for point P without using any new variables.

23. isosceles trapezoid



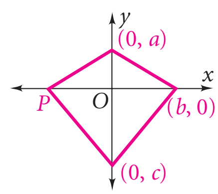
$(c - a, b)$

24. trapezoid with a right ∠



$(a, 0)$

25. kite



$(-b, 0)$

26. a. Draw a square whose diagonals of length $2b$ lie on the x- and y-axes.
b. Give the coordinates of the vertices of the square. **a–b, e. See back of book.**
c. Compute the length of a side of the square. $b\sqrt{2}$
d. Find the slopes of two adjacent sides of the square. **1, −1**
e. Do the slopes show that the sides are perpendicular? Explain.

27. Make two drawings of an isosceles triangle with base length $2b$ and height $2c$.
a. In one drawing, place the base on the x-axis with a vertex at the origin.
b. In the second, place the base on the x-axis with its midpoint at the origin.
c. Find the lengths of the legs of the triangle as placed in part (a).
d. Find the lengths of the legs of the triangle as placed in part (b).
e. How do the results of parts (c) and (d) compare? **a–e. See margin.**

C Challenge

28. **Marine Archaeology** Marine archaeologists sometimes use a coordinate system on the ocean floor. They record the coordinates of points where artifacts are found. Assume that each diver searches a square area and can go no farther than b units from the starting point. Draw a model for the region one diver can search. Assign coordinates to the vertices without using any new variables. **See margin.**

3. Practice

Assignment Guide

1 A B 1-27
C Challenge 28-29

Test Prep 30-33
Mixed Review 34-37

Homework Quick Check
To check students' understanding of key skills and concepts, go over Exercises 1, 5, 8, 20, 26.

Exercises 2–7 If you do these as class exercises, ask students to explain their choices of coordinates.

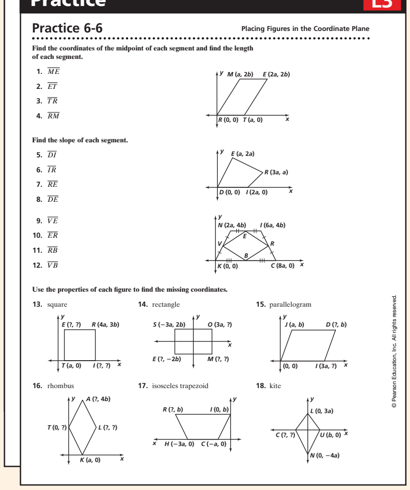
Exercise 22 Have students make a quick sketch of the rectangle. Before students identify the fourth vertex ask: *Is the midpoint the same for both diagonals?* **yes**

Error Prevention!

Exercise 23 Students may have trouble finding that the first coordinate of P is $c - a$. Have them refer to the parallelogram in Exercise 5

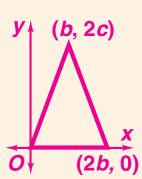
Differentiated Instruction Resources

GPS Guided Problem Solving L3
Enrichment L4
Reteaching L2
Adapted Practice L1
Practice L3

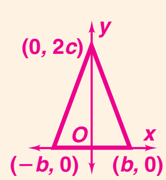


1. a. $(2a, 0)$
b. $(0, 2b)$
c. (a, b)
d. $\sqrt{b^2 + a^2}$
e. $\sqrt{b^2 + a^2}$
f. $\sqrt{b^2 + a^2}$
g. $MA = MB = MC$

27. a.



b.



c. $\sqrt{b^2 + 4c^2}$
d. $\sqrt{b^2 + 4c^2}$
e. The lengths are =.

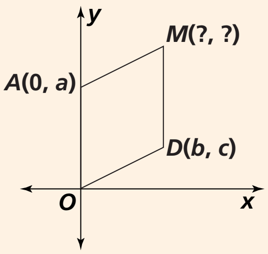
4. Assess & Reteach

PowerPoint

Lesson Quiz

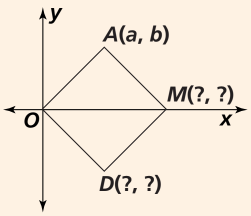
Find the missing coordinates of each figure.

1. parallelogram



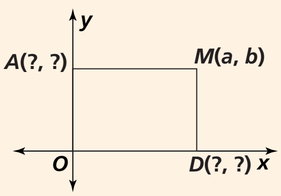
M(*b*, *c* + *a*)

2. rhombus



M(2*a*, 0), *D*(*a*, −*b*)

3. rectangle



A(0, *b*), *D*(*a*, 0)

Find the coordinates of the midpoint and the slope.

4. $\overline{OM}$ in Exercise 1 **midpoint: $\left(\frac{b}{2}, \frac{c+a}{2}\right)$; slope: $\frac{c+a}{b}$**

5. $\overline{AD}$ in Exercise 2 **midpoint: (*a*, 0); slope: undefined**

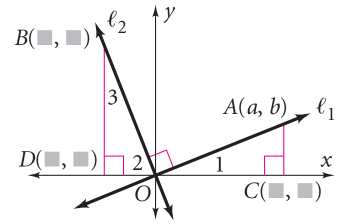
6. $\overline{AD}$ in Exercise 3 **midpoint: $\left(\frac{a}{2}, \frac{b}{2}\right)$; slope: $-\frac{b}{a}$**

Alternative Assessment

Have students draw two of these figures in a coordinate plane, label the vertices, and explain how they chose the coordinates: *parallelogram, square, isosceles trapezoid, kite.*

29. Coordinate Proof Follow the steps below to prove:

If two nonvertical lines are perpendicular, the product of their slopes is −1.



Step 1: Two nonvertical lines, ℓ_1 and ℓ_2, intersect. Which coordinate point might be the easiest to work with as the point of intersection? **(0, 0)**

Step 2: To work with the slope of a line, you need two points on the line. Choose one point $A(a, b)$ on ℓ_1. What are the coordinates of C? **(*a*, 0)**

Step 3: Notice that ∠1 and ∠3 are both complements of ∠2. Why? **See margin.**

Step 4: This means that the two triangles pictured have congruent angles. Thus, if any pair of sides are congruent, the two triangles are congruent. Congruent triangles are desirable, so what would be a good choice for the coordinates of point D? **(−*b*, 0)**

Step 5: If you made a choice for D so that $\triangle ACO \cong \triangle ODB$ what must be the coordinates of point B? **(−*b*, *a*)**

Step 6: Now, complete the proof that the product of slopes is −1. **See margin.**

Test Prep

Multiple Choice

30. The vertices of a rhombus are located at (*a*, 0), (0, *b*), (−*a*, 0), and (0, −*b*), where $a, b > 0$. What is the midpoint of the side that is in Quadrant II? **B**
A. $\left(\frac{a}{2}, \frac{b}{2}\right)$ B. $\left(-\frac{a}{2}, \frac{b}{2}\right)$ C. $\left(-\frac{a}{2}, -\frac{b}{2}\right)$ D. $\left(\frac{a}{2}, -\frac{b}{2}\right)$

31. The vertices of a kite are located at (0, *a*), (*b*, 0), (0, −*c*), and (−*b*, 0), where $a, b, c, d > 0$. What is the slope of the side in Quadrant IV? **F**
F. $\frac{c}{b}$ G. $\frac{b}{c}$ H. $-\frac{b}{c}$ J. $-\frac{c}{b}$

32. The vertices of a square are located at (*a*, 0), (*a*, *a*), (0, *a*), and (0, 0). What is the length of a diagonal? **C**
A. a B. $2a$ C. $a\sqrt{2}$ D. $2\sqrt{a}$

Short Response

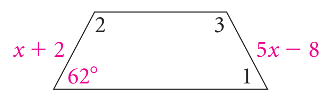
33. The vertices of a rectangle are (2*b*, 0), (2*b*, 2*a*), (0, 2*a*), and (0, 0). What are the coordinates of the midpoint of each diagonal? What can you conclude from your answers? **See margin.**

Mixed Review

GO for Help

Lesson 6-5

34. **Algebra** Find the measure of each angle and the value of x in the isosceles trapezoid. **62, 118, 118; 2.5**



Lesson 5-3

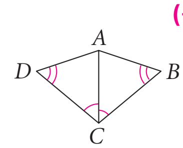
Find the center of the circle that circumscribes △ABC.

35. $A(1, 1), B(5, 3), C(5, 1)$ **(3, 2)**

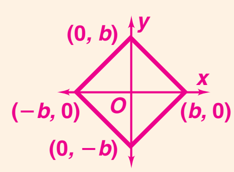
36. $A(-5, 0), B(-1, -8), C(-1, 0)$ **(−3, −4)**

Lesson 4-3

37. **Given:** $\angle ACD \cong \angle ACB$ and $\angle D \cong \angle B$
Prove: $\triangle ADC \cong \triangle ABC$
See back of book.



28.



29. **Step 3: Since $m\angle 1 + m\angle 2 + 90 = 180$, ∠1 and ∠2 must be compl. ∠3 and ∠2 are the acute ∠s of a rt. △.**

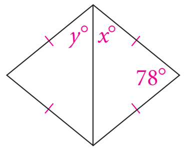
Step 6: Using the formula for slope, the slope for $\ell_1 = \frac{b}{a}$ and the slope for $\ell_2 = -\frac{a}{b}$. Mult. the slopes, $\frac{b}{a} \cdot -\frac{a}{b} = -1$.

Checkpoint Quiz 2 — Lessons 6-4 through 6-6

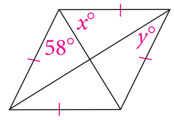
1. $x = 51, y = 51$
2. $x = 58, y = 32$
3. $x = 2, y = 4$

Algebra **Find the value(s) of the variable(s).**

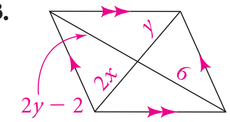
1.



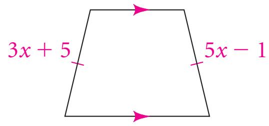
2.



3.

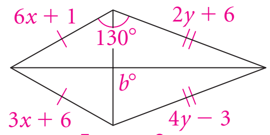


4.



$x = 3$

5.



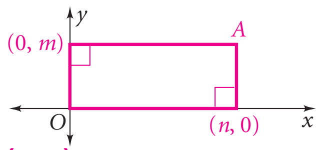
$x = \frac{5}{3}, y = \frac{9}{2}, b = 90$

In Exercises 6–8, decide whether the statement is *true* or *false*. If true, explain why. If false, show a counterexample. 6–8. See margin.

6. A quadrilateral with congruent diagonals is an isosceles trapezoid or rectangle.
7. A quadrilateral with congruent and perpendicular diagonals is a square.
8. Each diagonal of a kite bisects two angles of the kite.

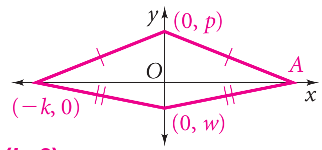
Give the coordinates for point *A* without using any new variables.

9.



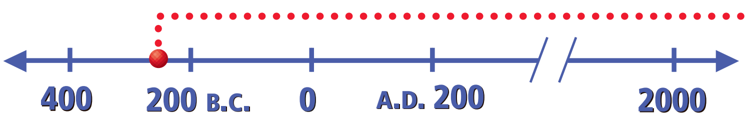
(n, m)

10.



(k, 0)

A Point in Time



Many walls in ancient Egypt were decorated with reliefs. The relief in the photo was created in the year 255 B.C. First, the artist sketched the scene on papyrus overlaid with a grid. Next, the wall was marked with a grid the size of the intended sculpture. To draw each line, a tightly stretched string that had been dipped in red ochre was plucked, like a guitar string.

Using the grid squares as guides, the artist transferred the drawing to the wall. Then, a sculptor cut the background away, leaving the scene slightly raised. Finally, an artist painted the scene.

Go Online PHSchool.com
For: Information about Egyptian reliefs
Web Code: aue-2032

33. [2] (b, a); the diagonals of a rectangle bisect each other.
[1] no conclusion given

Test Prep

Resources
For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 361
- Test-Taking Strategies, p. 356
- Test-Taking Strategies with Transparencies

Checkpoint Quiz

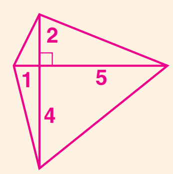
Use this Checkpoint Quiz to check students' understanding of the skills and concepts of Lessons 6-4 through 6-6.

Resources
Grab & Go
- Checkpoint Quiz 2

Checkpoint Quiz 2

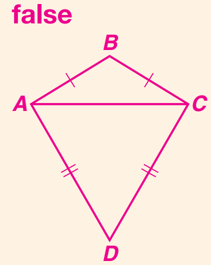
6–8. Counterexamples may vary.

6. false;



7. False; a kite can have ≅ ⊥ diags.

8. false



$\angle BAC \ncong \angle CAD$