PLANAR KINETICS OF A RIGID BODY: WORK AND ENERGY (Sections 18.1-18.4)

Today's Objectives:

Students will be able to:

a) Define the various ways a force and couple do work.

b) Apply the principle of work and energy to a rigid body.

In-Class Activities:

- Check homework, if any
- Reading quiz
- Applications
- Kinetic energy
- Work of a force or couple
- Principle of work and energy
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. Kinetic energy due to rotation is defined as

   A) $(1/2)\ m\ (v_G)^2$

   B) $(1/2)\ m\ (v_G)^2 + (1/2)\ I_G\ \omega^2$

   C) $(1/2)\ I_G\ \omega^2$

   D) $I_G\ \omega^2$

2. When calculating work done by forces, the work of an internal force does not have to be considered because

   A) internal forces do not exist.

   B) the forces act in equal but opposite collinear pairs.

   C) the body is at rest initially.

   D) the body can deform.

APPLICATIONS

The work of the torque (or moment) developed by the driving gears on the two motors on the concrete mixer is transformed into the rotational kinetic energy of the mixing drum.

If the motor gear characteristics are known, could the velocity of the mixing drum be found?

APPLICATIONS (continued)

The work done by the soil compactor's engine is transformed into the translational kinetic energy of the frame and the translational and rotational kinetic energy of its roller and wheels (excluding the additional kinetic energy developed by the moving parts of the engine and drive train).

Are the kinetic energies of the frame and the roller related to each other? How?

KINETIC ENERGY

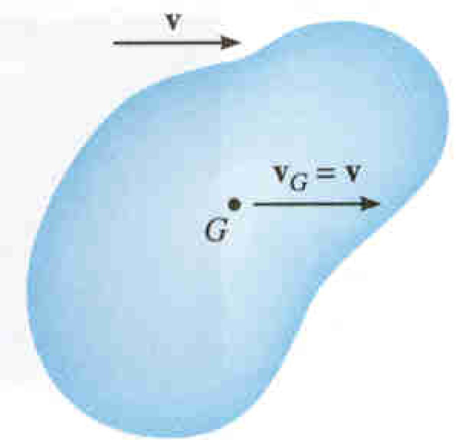
The kinetic energy of a rigid body can be expressed as the sum of its translational and rotational kinetic energies. In equation form, a body in general plane motion has kinetic energy given by

$$T = 1/2\ m\ (v_G)^2 + 1/2\ I_G\ \omega^2$$

Several simplifications can occur.

1. Pure Translation: When a rigid body is subjected to only curvilinear or rectilinear translation, the rotational kinetic energy is zero ($\omega = 0$). Therefore,

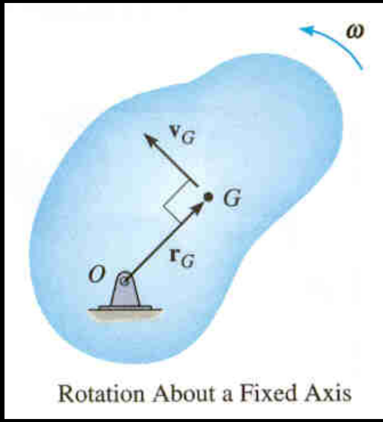
$$T = 0.5\ m\ (v_G)^2$$



Translation

KINETIC ENERGY (continued)

2. Pure Rotation: When a rigid body is rotating about a fixed axis passing through point O, the body has both translational and rotational kinetic energy. Thus,



Rotation About a Fixed Axis

$$T = 0.5m(v_G)^2 + 0.5I_G\omega^2$$

Since $v_G = r_G\omega$, we can express the kinetic energy of the body as

$$T = 0.5(I_G + m(r_G)^2)\omega^2 = 0.5I_O\omega^2$$

If the rotation occurs about the mass center, G, then what is the value of v_G?

In this case, the velocity of the mass center is equal to zero. So the kinetic energy equation reduces to

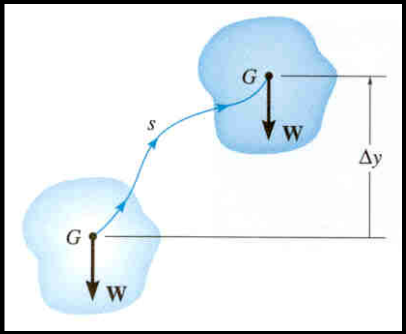
$$T = 0.5\, I_G\, \omega^2$$

WORK OF A FORCE

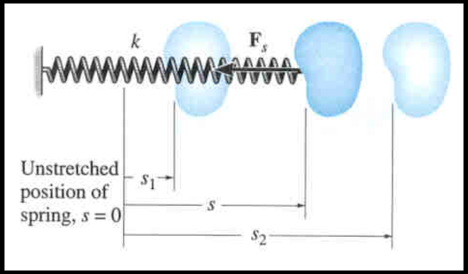
Recall that the work done by a force can be written as

$$U_F = \int \mathbf{F} \cdot d\mathbf{r} = \int_s (F \cos \theta)\, ds.$$

When the force is constant, this equation reduces to $U_{Fc} = (F_c \cos\theta)s$ where $F_c\cos\theta$ represents the component of the force acting in the direction of displacement s.



Work of a weight: As before, the work can be expressed as $U_W = -W\Delta y$. Remember, if the force and movement are in the same direction, the work is positive.

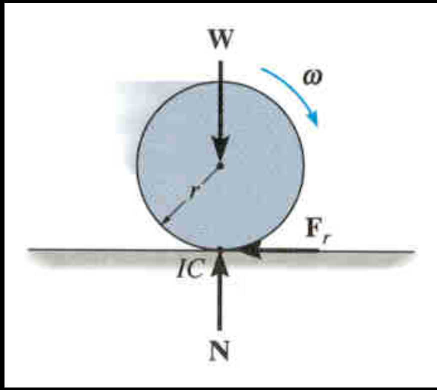


Work of a spring force: For a linear spring, the work is

$$U_s = -0.5k[(s_2)^2 - (s_1)^2]$$

FORCES THAT DO NO WORK

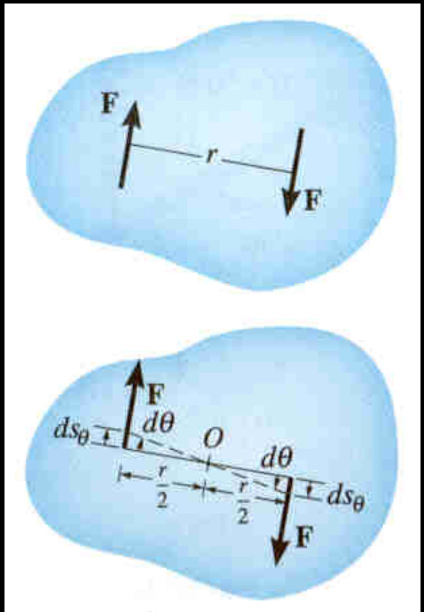
There are some external forces that do no work. For instance, reactions at fixed supports do no work because the displacement at their point of application is zero.



Normal forces and friction forces acting on bodies as they roll without slipping over a rough surface also do no work since there is no instantaneous displacement of the point in contact with ground (it is an instant center, IC).

Internal forces do no work because they always act in equal and opposite pairs. Thus, the sum of their work is zero.

THE WORK OF A COUPLE



Rotation

When a body subjected to a couple experiences general plane motion, the two couple forces do work only when the body undergoes rotation.

If the body rotates through an angular displacement $d\theta$, the work of the couple moment, M, is

$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

If the couple moment, M, is constant, then

$$U_M = M\,(\theta_2 - \theta_1)$$

Here the work is positive, provided M and $(\theta_2 - \theta_1)$ are in the same direction.

PRINCIPLE OF WORK AND ENERGY

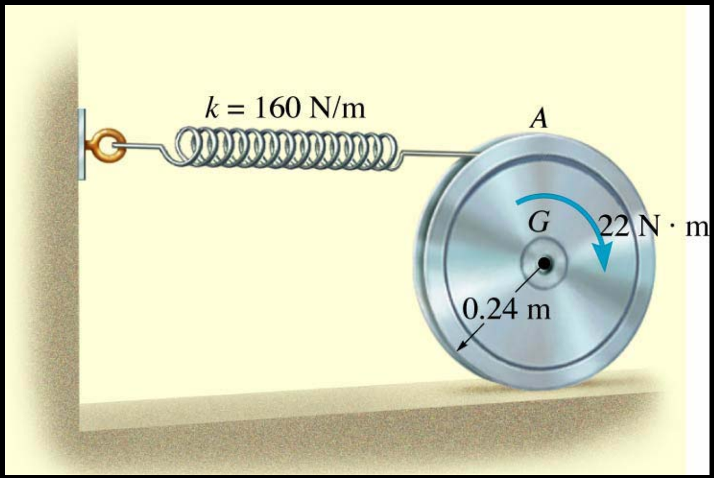
Recall the statement of the principle of work and energy used earlier:

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

In the case of general plane motion, this equation states that the sum of the initial kinetic energy (both translational and rotational) and the work done by all external forces and couple moments equals the body’s final kinetic energy (translational and rotational).

This equation is a scalar equation. It can be applied to a system of rigid bodies by summing contributions from all bodies.

EXAMPLE



Given: The disk has a mass of 40 kg and a radius of gyration (k_G) of 0.6 m. A 15 N·m moment is applied and the spring has a spring constant of 10 N/m.

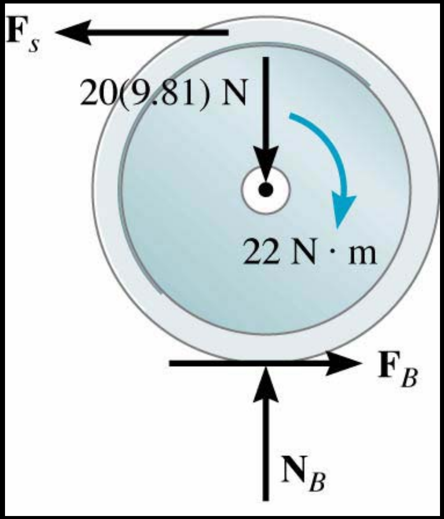
Find: The angular velocity of the wheel when point G moves 0.5 m. The wheel starts from rest and rolls without slipping. The spring is initially unstretched.

Plan: Use the principle of work and energy since distance is the primary parameter. Draw a free body diagram of the disk and calculate the work of the external forces.

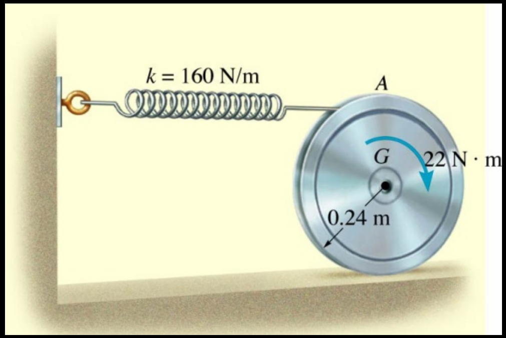
EXAMPLE (continued)

Solution:

Free body diagram of the disk:

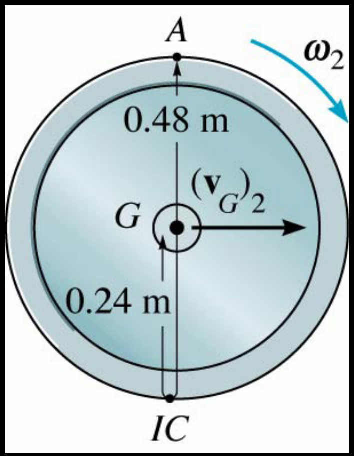


Since the body rolls without slipping on a horizontal surface, only the spring force and couple moment M do work. Why don't forces F_B and N_B do work?



Since the spring is attached to the top of the wheel, it will stretch twice the amount of displacement of G, or 1 m.

EXAMPLE (continued)



Work: $U_{1\text{-}2} = -0.5k[(s_2)^2 - (s_1)^2] + M(\theta_2 - \theta_1)$

$U_{1\text{-}2} = -0.5(10)(1^2 - 0) + 15(0.5/0.24) = 26.25\ \text{N}\cdot\text{m}$

Kinematic relation: $v_G = r\,\omega = 0.24\omega$

Kinetic energy: $T_1 = 0$

$T_2 = 0.5m\,(v_G)^2 + 0.5\,I_G\,\omega^2$

$T_2 = 0.5(40)(0.24\omega)^2 + 0.5(40)(0.6)^2\omega^2$

$T_2 = 8.352\,\omega^2$

Work and energy: $T_1 + U_{1\text{-}2} = T_2$

$0 + 26.25 = 8.352\,\omega^2$

$\omega = 3.14$ rad/s

CONCEPT QUIZ

1. If a rigid body rotates about its center of gravity, its translational kinetic energy is __________ at all times.

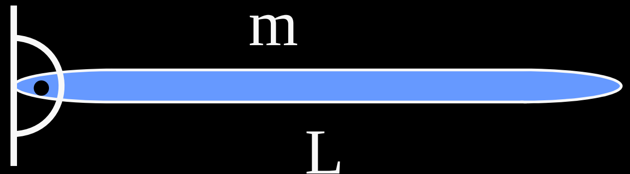
   A) constant

   B) zero

   C) equal to its rotational kinetic energy

   D) Cannot be determined.

2. A rigid bar of mass m and length L is released from rest in the horizontal position. What is the rod's angular velocity when it has rotated through 90°?

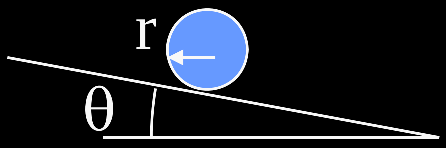


   A) $\sqrt{g/3L}$ B) $\sqrt{3g/L}$

   C) $\sqrt{12g/L}$ D) $\sqrt{g/L}$

ATTENTION QUIZ

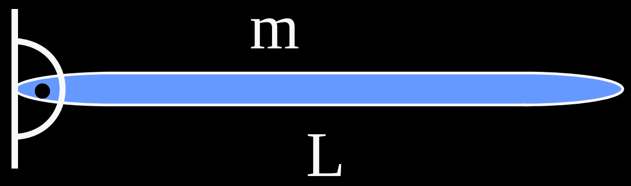
1. A disk and a sphere, each of mass m and radius r, are released from rest. After 2 full turns, which body has a larger angular velocity? Assume roll without slip.



A) Sphere B) Disk

C) The two are equal. D) Cannot be determined.

2. A slender bar of mass m and length L is released from rest in a horizontal position. The work done by its weight when it has rotated through 90° is



A) m g (π/2) B) m g L

C) m g (L/2) D) -m g (L/2)

End of the Lecture

Let Learning Continue

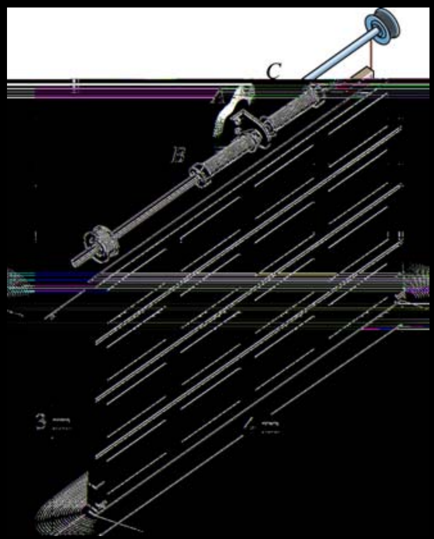
PLANAR KINETICS OF A RIGID BODY: CONSERVATION OF ENERGY (Section 18.5)

Today's Objectives:

Students will be able to:

a) Determine the potential energy of conservative forces.

b) Apply the principle of conservation of energy.



In-Class Activities:

- Check homework, if any
- Reading quiz
- Applications
- Potential energy
- Conservation of energy
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

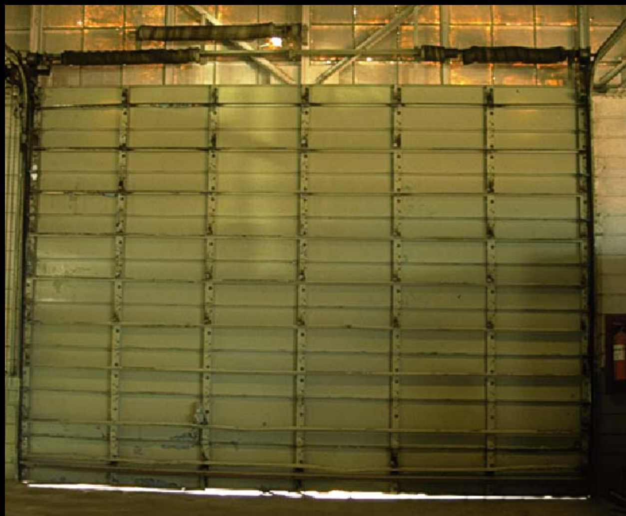
1. Elastic potential energy is defined as

   A) $+ (1/2)\, k\, (s)^2$.
   B) $- (1/2)\, k\, (s)^2$.
   C) $+ (1/2)\, k\, (v)^2$.
   D) None of the above.

2. The kinetic energy of a rigid body consists of the kinetic energy due to ___________.

   A) translational motion and rotational motion
   B) only rotational motion
   C) only translational motion
   D) the deformation of the body

APPLICATIONS



The torsional spring located at the top of the garage door winds up as the door is lowered. When the door is raised, the potential energy stored in the spring is transferred into the gravitational potential energy of the door's weight, thereby making it easy to open.

Are parameters such as the torsional spring stiffness and initial rotation angle of the spring important when you install a new door?

CONSERVATION OF ENERGY

The conservation of energy theorem is a "simpler" energy method (recall that the principle of work and energy is also an energy method) for solving problems. Once again, the problem parameter of distance is a key indicator of when conservation of energy is a good method for solving the problem.

If it is appropriate, conservation of energy is easier to use than the principle of work and energy. This is because the calculation of the work of a conservative force is simpler.   But, what makes a force conservative?

CONSERVATIVE FORCES

A force F is conservative if the work done by the force is independent of the path. In this case, the work depends only on the initial and final positions of the object with the path between positions of no consequence.

Typical conservative forces encountered in dynamics are gravitational forces (i.e., weight) and elastic forces (i.e., springs).

What is a common force that is not conservative?

CONSERVATION OF ENERGY

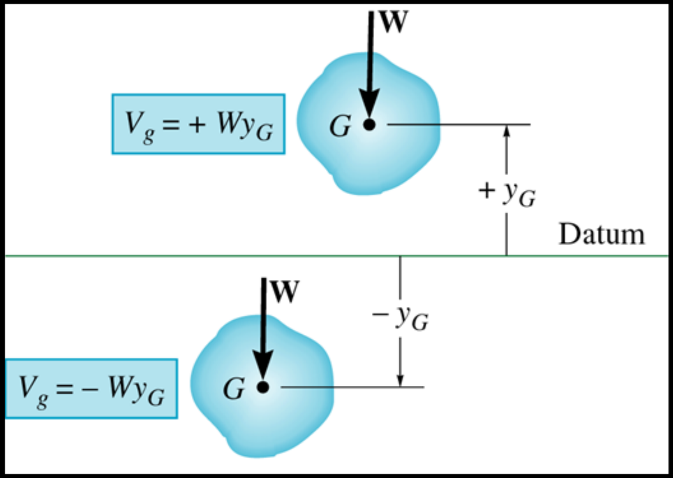
When a rigid body is acted upon by a system of conservative forces, the work done by these forces is conserved. Thus, the sum of kinetic energy and potential energy remains constant. This principle is called conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 = \text{Constant}$$

In other words, as a rigid body moves from one position to another when acted upon by only conservative forces, kinetic energy is converted to potential energy and vice versa.

GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is a function of the height of the body's center of gravity above or below a datum.



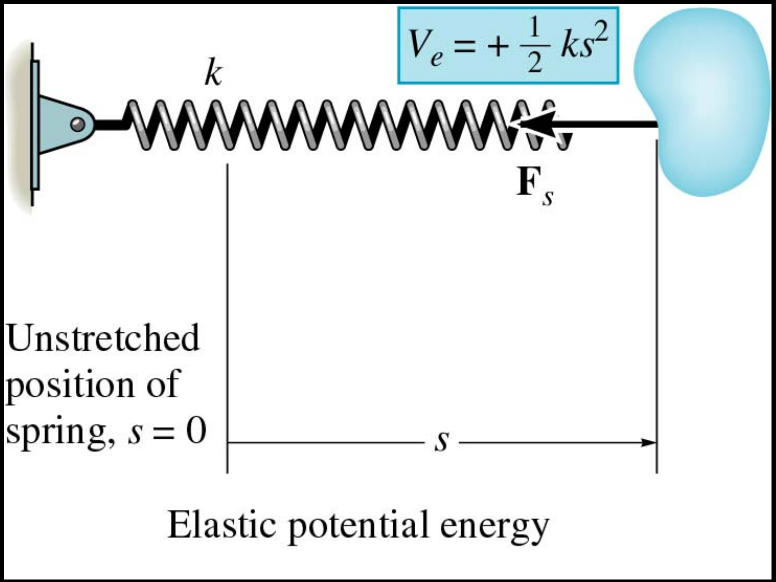
The gravitational potential energy of a body is found by the equation

$$V_g = W\, y_G$$

Gravitational potential energy is positive when y_G is positive, since the weight has the ability to do positive work when the body is moved back to the datum.

ELASTIC POTENTIAL ENERGY

Spring forces are also conservative forces.



The potential energy of a spring force (F = ks) is found by the equation

$$V_e = \frac{1}{2} ks^2$$

Notice that the elastic potential energy is always positive.

PROCEDURE FOR ANALYSIS

Problems involving velocity, displacement and conservative force systems can be solved using the conservation of energy equation.

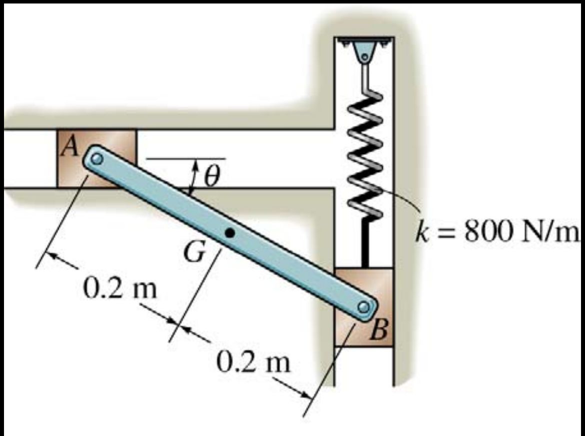
- Potential energy: Draw two diagrams: one with the body located at its initial position and one at the final position. Compute the potential energy at each position using

$$V = V_g + V_e, \text{ where } V_g = W\, y_G \text{ and } V_e = 1/2\, k\, s^2.$$

- Kinetic energy: Compute the kinetic energy of the rigid body at each location. Kinetic energy has two components: translational kinetic energy ($½\, m(v_G)^2$) and rotational kinetic energy ($½\, I_G\, \omega^2$).

- Apply the conservation of energy equation.

EXAMPLE 1



Given: The rod AB has a mass of 10 kg. Piston B is attached to a spring of constant k = 800 N/m. The spring is un-stretched when θ = 0°. Neglect the mass of the pistons.

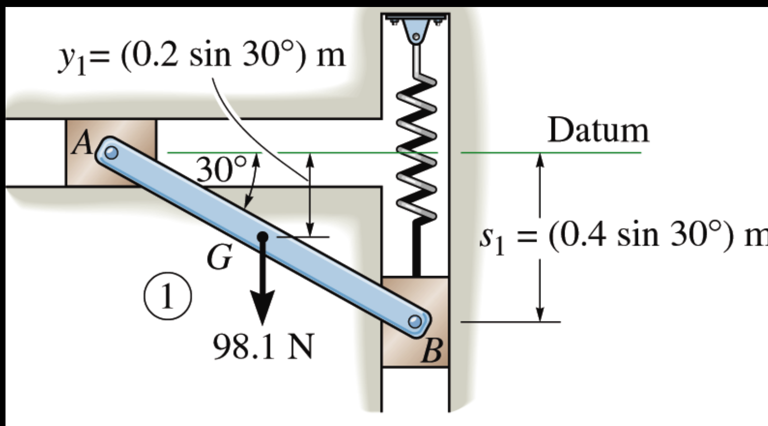
Find: The angular velocity of rod AB at θ = 0° if the rod is released from rest when θ = 30°.

Plan: Use the energy conservation equation since all forces are conservative and distance is a parameter (represented here by θ). The potential energy and kinetic energy of the rod at states 1 and 2 will have to be determined.

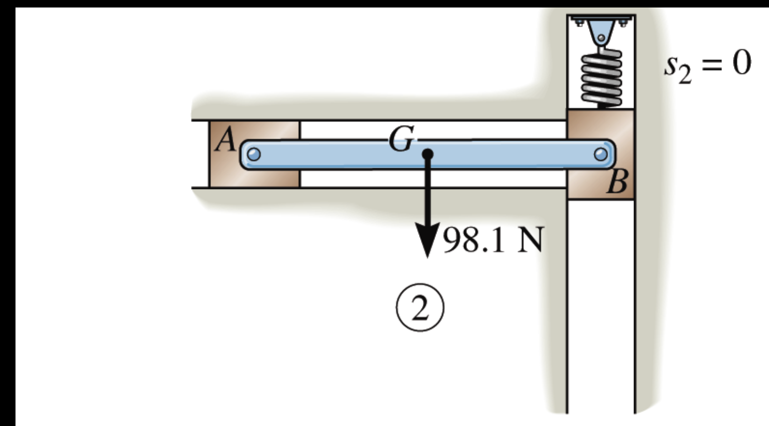
EXAMPLE 1 (continued)

Solution:

Initial Position



Final Position



Potential Energy:

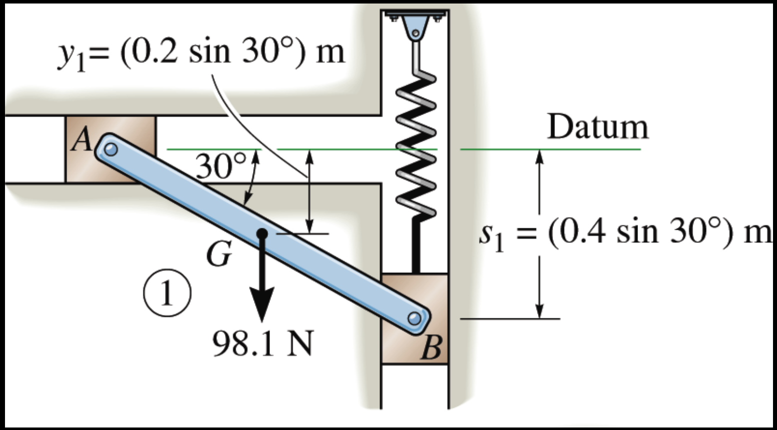
Let's put the datum in line with the rod when θ = 0°.
Then, the gravitational potential energy and the elastic potential energy will be zero at position 2. => $V_2 = 0$

Gravitational potential energy at 1: - (10)(9.81) ½ (0.4 sin 30)
Elastic potential energy at 1: ½ (800) $(0.4 \sin 30)^2$

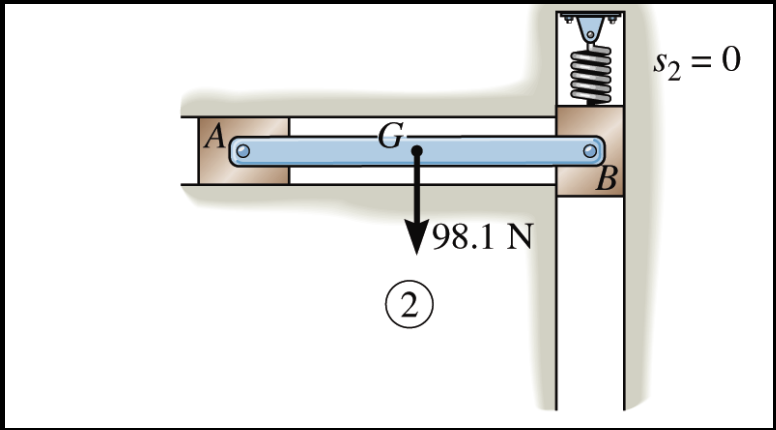
So $V_1 = -9.81\text{ J} + 16.0\text{ J} = 6.19\text{ J}$

EXAMPLE 1 (continued)

Initial Position



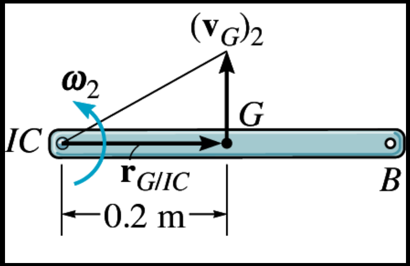
Final Position



Kinetic Energy:

The rod is released from rest from position 1 (so $v_{G1} = 0$, $\omega_1 = 0$). Therefore, $T_1 = 0$.

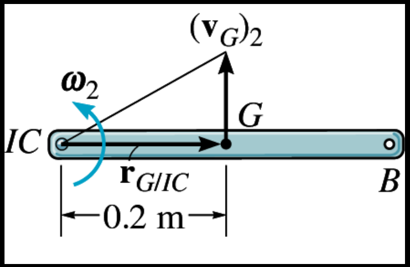
At position 2, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2}.



EXAMPLE 1 (continued)

Therefore,

$$T_2 = ½\,(10)(v_{G2})^2 + ½\,(1/12)(10)(0.4^2)(\omega_2)^2$$



At position 2, point A is the instantaneous center of rotation. Hence, $v_{G2} = r\,\omega = 0.2\,\omega_2$.

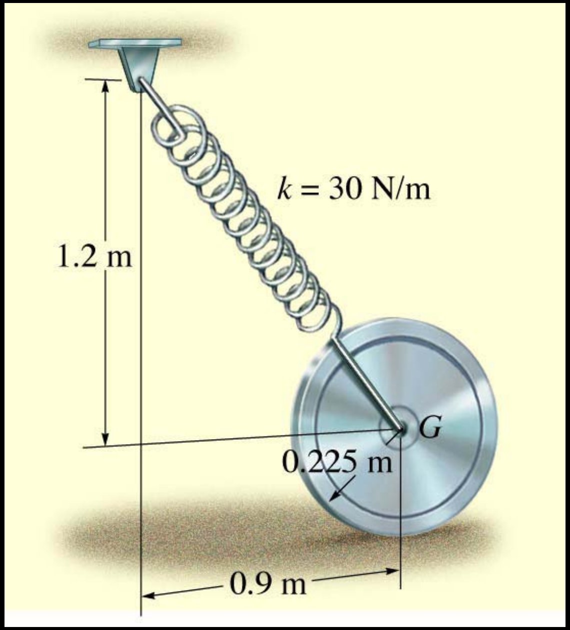
Then, $T_2 = 0.2\,\omega_2^2 + 0.067\,\omega_2^2 = 0.267\,\omega_2^2$

Now apply the conservation of energy equation and solve for the unknown angular velocity, ω_2.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6.19 = 0.267\omega_2^2 + 0 \quad \Rightarrow \quad \omega_2 = 4.82 \text{ rad/s}$$

EXAMPLE 2



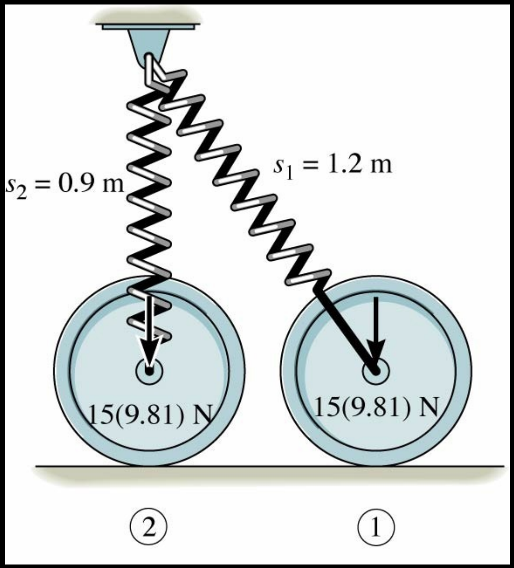
Given: The weight of the disk is 150 N and its k_G equals 0.18 m. The spring has a stiffness of 30 N/m and an unstretched length of 0.3 m.

Find: The velocity at the instant G moves 3 m to the left. The disk is released from rest in the position shown and rolls without slipping.

Plan: Since distance is a parameter and all forces doing work are conservative, use conservation of energy. Determine the potential energy and kinetic energy of the system at both positions and apply the conservation of energy equation.

EXAMPLE 2 (continued)

Solution:



(1) Initial Position
(2) Final Position

Potential Energy:

There are no changes in the gravitational potential energy since the disk is moving horizontally.

The elastic potential energy at position 1 is: $V_1 = 0.5\, k\, (s_1)^2$ where $s_1 = 1.2$ m.

Thus, $V_1 = 0.5\,(30)\,(1.2)^2 =$ 21.6 J

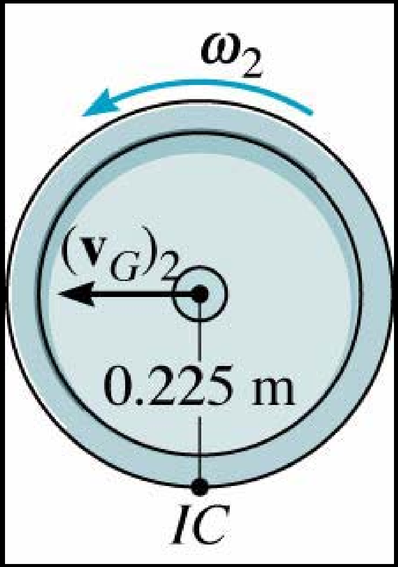
Similarly, the elastic potential energy at position 2 is

$V_2 = 0.5\,(30)\,(0.9)^2 =$ 12.15 J

EXAMPLE 2 (continued)

Kinetic Energy:

The disk is released from rest at position 1, so $v_{G1} = 0$ and $\omega_1 = 0$. Thus, the kinetic energy at position 1 is $T_1 = 0$.



At position 2, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

$$T_2 = 0.5m\,(v_{G2})^2 + 0.5\,I_G\,(\omega_2)^2$$

$$= 0.5(150/10)\,(v_{G2})^2 + 0.5\,(150/10)\,0.18^2\,(\omega_2)^2$$

The disk is rolling without slipping, so $v_{G2} = (0.225\,\omega_2)$.

$$T_2 = 0.5(150/10)(0.225\,\omega_2)^2 + \tfrac{1}{2}(150/10)\,0.18^2\,(\omega_2)^2 = 0.6227\,(\omega_2)^2$$

EXAMPLE 2 (continued)

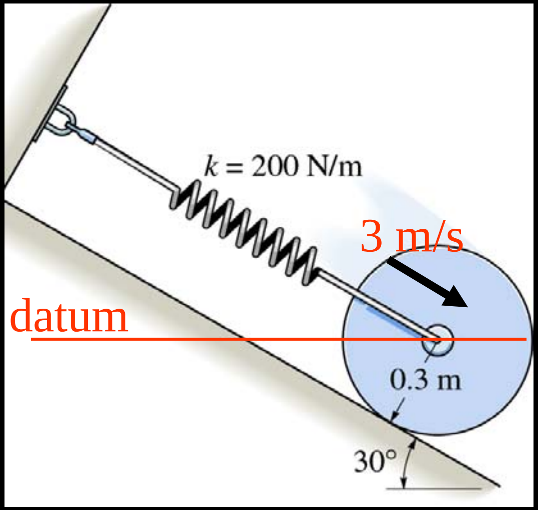
Now all terms in the conservation of energy equation have been formulated. First, writing the general equation and then substituting into it yields:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 21.6 \text{ J} = 0.6227\, \omega_2^2 + 12.15 \text{ J}$$

Solving , $\omega_2 = 3.90$ rad/s

CONCEPT QUIZ



1. At the instant shown, the spring is undeformed. Determine the change in potential energy if the 20 kg disk (k_G = 0.5 m) rolls 2 revolutions without slipping.

   A) $\frac{1}{2}(200)(1.2\pi)^2 + (20)9.81(1.2\pi \sin 30)$

   B) $-\frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30)$

   C) $\frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30)$

   D) $\frac{1}{2}(200)(1.2\pi)^2$

2. Determine the kinetic energy of the disk at this instant.

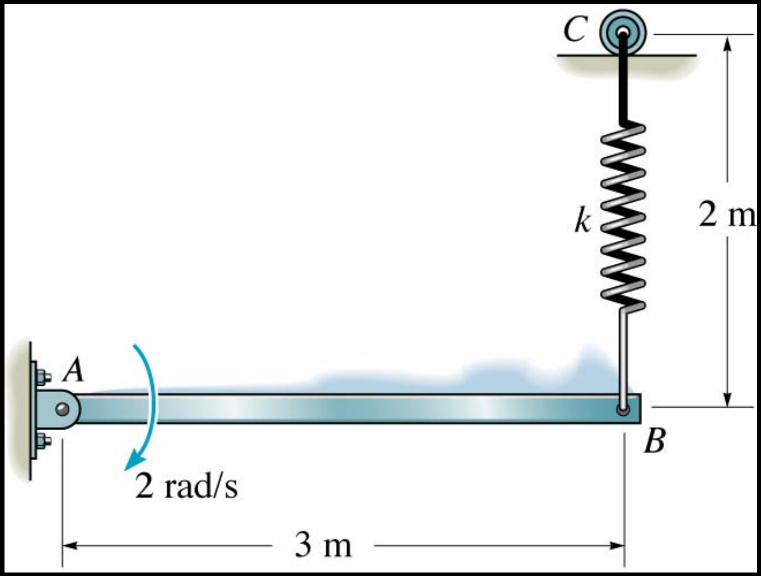
   A) $(\frac{1}{2})(20)(3)^2$

   B) $\frac{1}{2}(20)(0.5^2)(10)^2$

   C) Answer A + Answer B

   D) None of the above.

GROUP PROBLEM SOLVING



Given: A 50 N bar is rotating downward at 2 rad/s. The spring has an unstretched length of 2 m and a spring constant of 12 N/m.

Find: The angle (measured down from the horizontal) to which the bar rotates before it stops its initial downward movement.

Plan: Conservative forces and distance (θ) leads to the use of conservation of energy. First, determine the potential energy and kinetic energy for both positions. Then apply the conservation of energy equation.

GROUP PROBLEM SOLVING (continued)

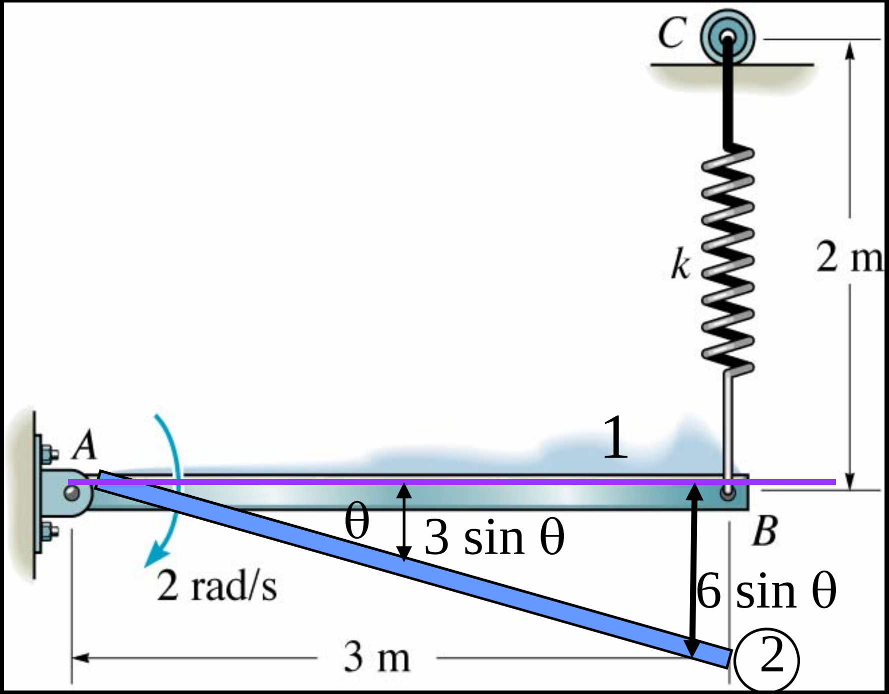
Solution:

Potential Energy:

Let's put the datum in line with the rod when θ = 0.

Then, at position 1, the gravitational potential energy is zero and the elastic potential energy will be

$$V_1 = 0.5\, k\, (s_1)^2$$
$$= 0.5\, (12)\, (4 - 2)^2$$

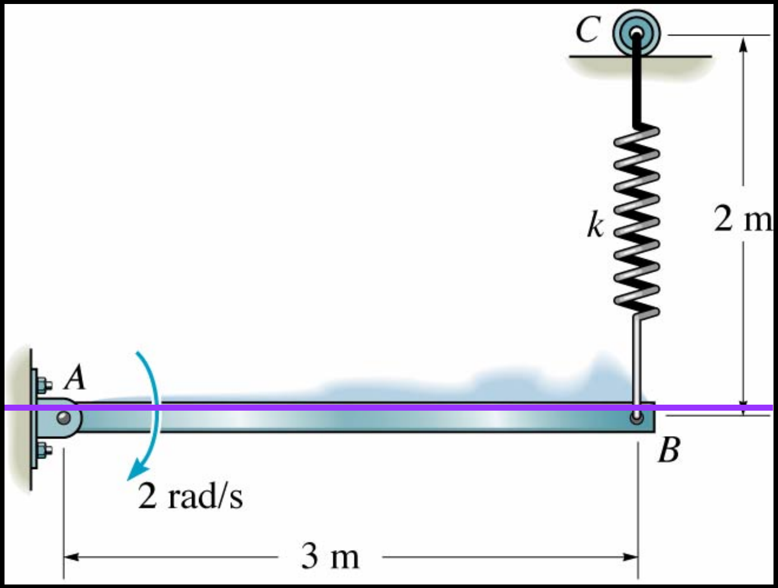


Gravitational potential energy at position 2: - (50) (3 sin θ)

Elastic potential energy at position 2: 0.5 (12) {4 + (6 sin θ) - 2}^{2}

So, $V_2 = -\,(50)\,(3\sin\theta) \;+\; 0.5\,(12)\,\{4 + (6\sin\theta) - 2\}^2$

GROUP PROBLEM SOLVING (continued)



Kinetic Energy:

At position 1 (when $\theta = 0$), the rod has a rotational motion about point A.

$$T_1 = 0.5\, I_A\, (\omega^2)$$

$$= 0.5\{1/3\, (50/9.81)\, 6^2\}\, (2^2)$$

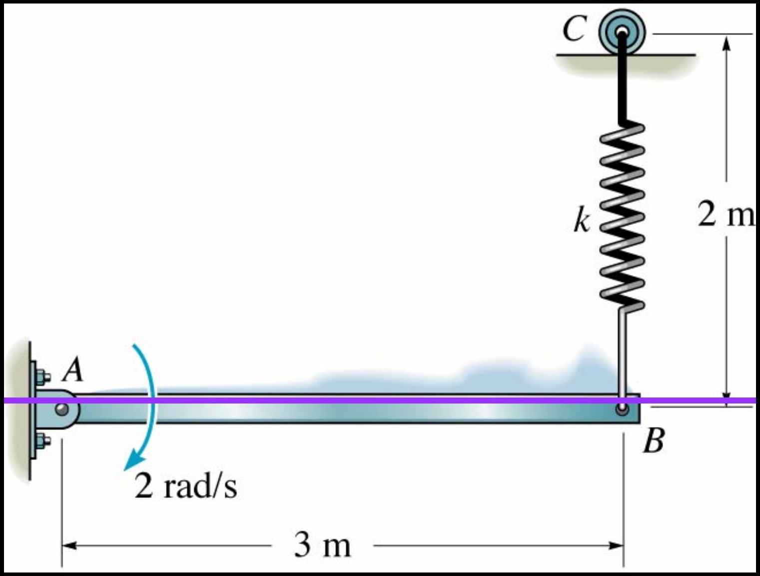
At position 2, the rod momentarily has no translation or rotation since the rod comes to rest.

Therefore, $T_2 = 0$.

GROUP PROBLEM SOLVING (continued)

Now, substitute into the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$



$$0.5\{1/3\ (50/9.81)\ 6^2\}(\ 2^2\ ) + 0.5\ (12)\ (4 - 2)^2$$

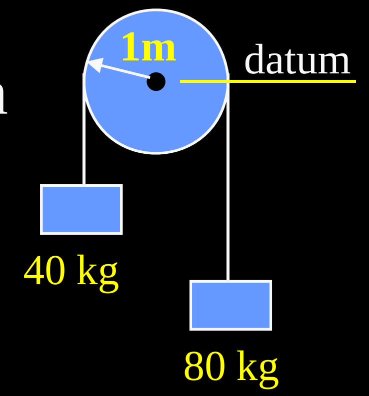
$$= 0.0 - (50)(3 \sin\theta) + 0.5(12)\{4 + (6 \sin\theta) - 2\}^2$$

Solving for sin θ yields

$\sin\theta = 0.7666.$ Thus, $\theta = 50.0$ deg.

ATTENTION QUIZ

1. Blocks A and B are released from rest and the disk turns 2 revolutions. The V_2 of the system includes a term for



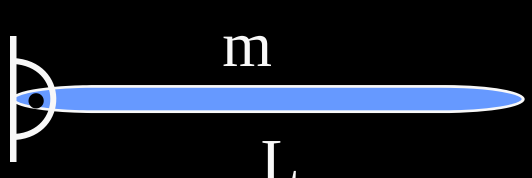
   A) only the 40 kg block.

   B) only the 80 kg block.

   C) the disk and both blocks.

   D) only the two blocks.

2. A slender bar is released from rest while in the horizontal position. The kinetic energy (T_2) of the bar when it has rotated through 90° is



   A) $\frac{1}{2}\, m\, (v_{G2})^2$

   B) $\frac{1}{2}\, I_G\, (\omega_2)^2$

   C) $\frac{1}{2}\, k\, (s_1)^2 - W\, (L/2)$

   D) $\frac{1}{2}\, m\, (v_{G2})^2 + \frac{1}{2}\, I_G\, (\omega_2)^2$

End of the Lecture

Let Learning Continue