Plane Motion of Rigid Bodies:

Forces and Accelerations

Reference:

Beer, Ferdinand P. et al, *Vector Mechanics for Engineers : Dynamics*, 8^{th} Edition, Mc GrawHill

Hibbeler R.C., *Engineering Mechanics: Dynamics, 11^{th} Edition*, Prentice Hall (Chapter 17)

Introduction

- In this chapter and in Chapters 17 and 18, we will be concerned with the *kinetics* of rigid bodies, i.e., relations between the forces acting on a rigid body, the shape and mass of the body, and the motion produced.

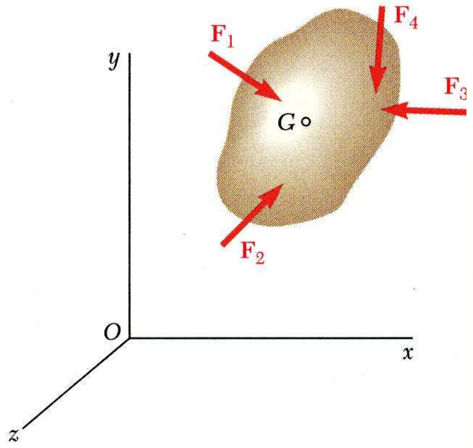
- Results of this chapter will be restricted to:
  - plane motion of rigid bodies, and
  - rigid bodies consisting of plane slabs or bodies which are symmetrical with respect to the reference plane.

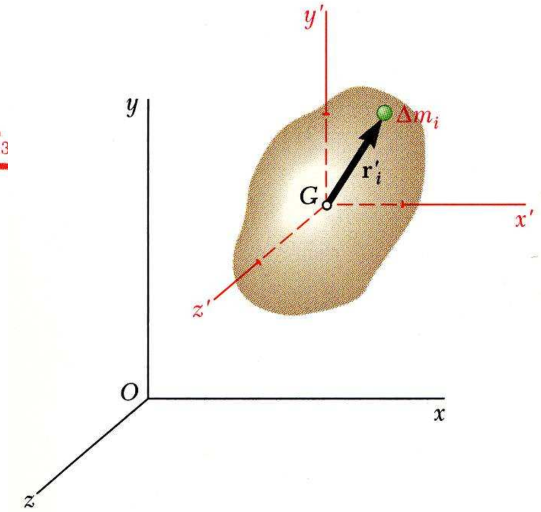
- Our approach will be to consider rigid bodies as made of large numbers of particles and to use the results of Chapter 14 for the motion of systems of particles. Specifically,

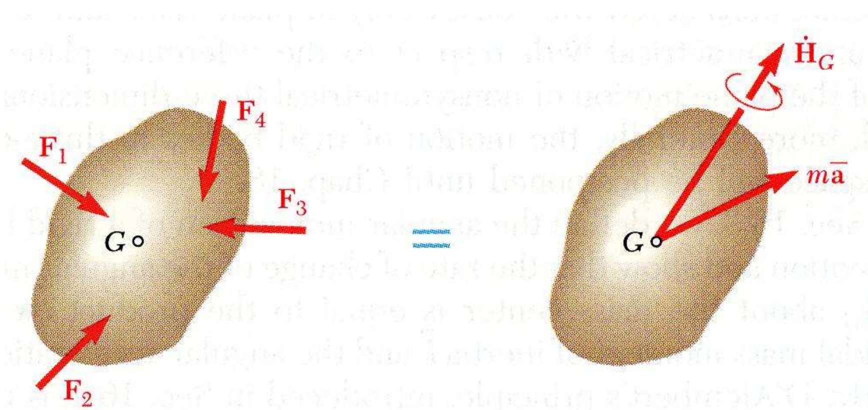
$$\sum \vec{F} = m\vec{\bar{a}} \qquad \text{and} \qquad \sum \vec{M}_G = \dot{\vec{H}}_G$$

- D'Alembert's principle is applied to prove that the external forces acting on a rigid body are equivalent a vector $m\vec{\bar{a}}$ attached to the mass center and a couple of moment $\bar{I}\alpha$.

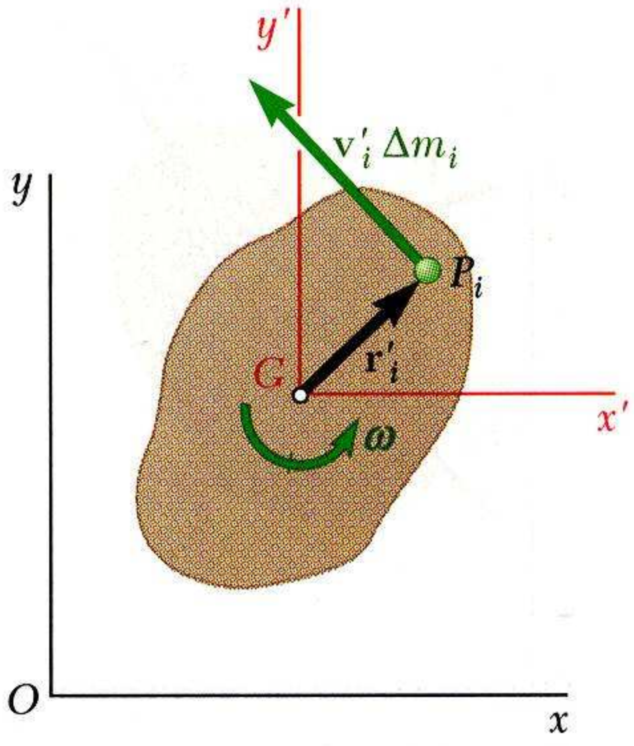
Equations of Motion for a Rigid Body



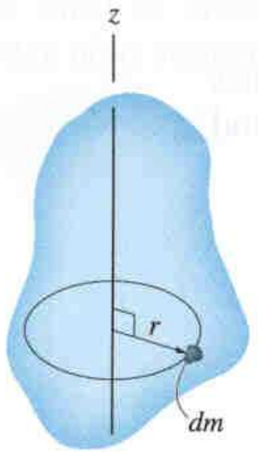




Angular Momentum of a Rigid Body in Plane Motion

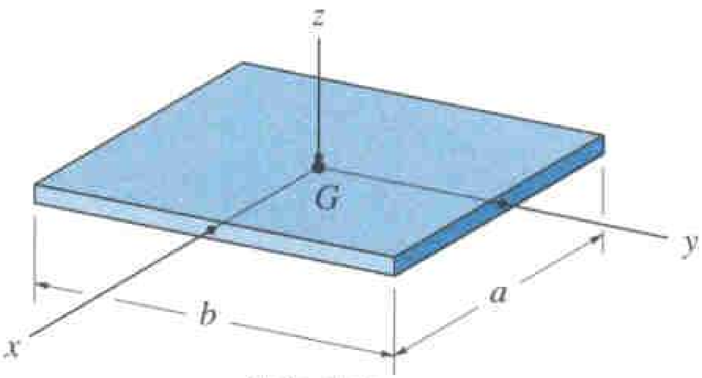


MOMENT OF INERTIA



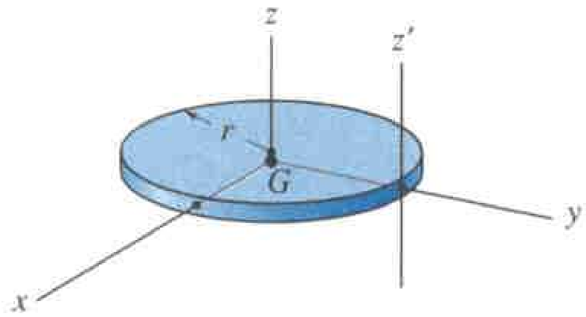
The mass moment of inertia is a measure of an object's resistance to rotation.

$I = \int r^2\, dm = \int r^2 \rho\, dV$



Thin plate

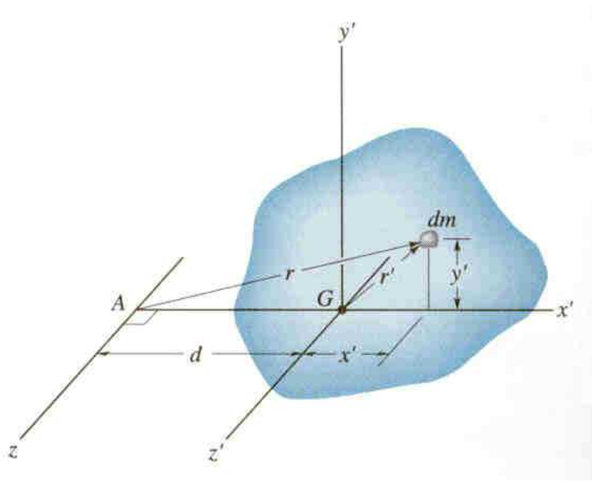
$I_{xx} = \frac{1}{12} m b^2 \quad I_{yy} = \frac{1}{12} m a^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



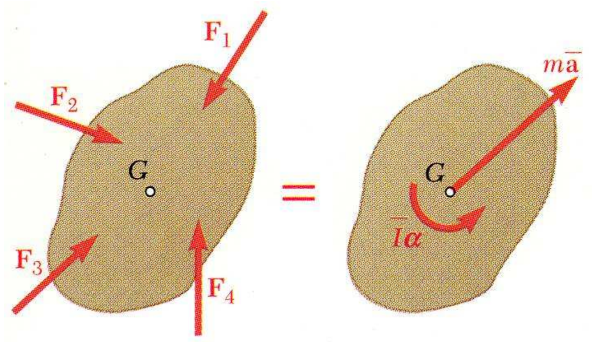
Thin circular disk

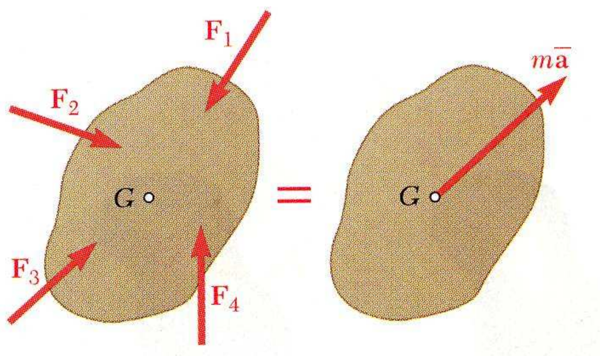
$I_{xx} = I_{yy} = \frac{1}{4} m r^2 \quad I_{zz} = \frac{1}{2} m r^2 \quad I_{z'z'} = \frac{3}{2} m r^2$

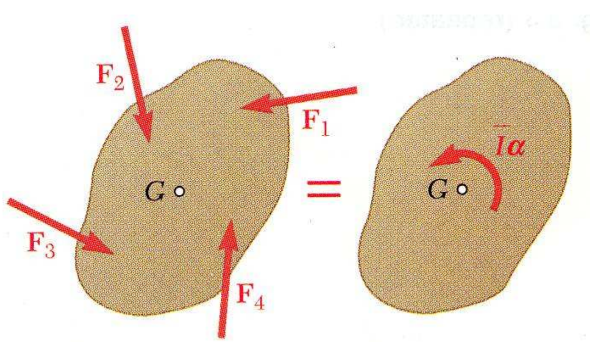
PARALLEL-AXIS THEOREM



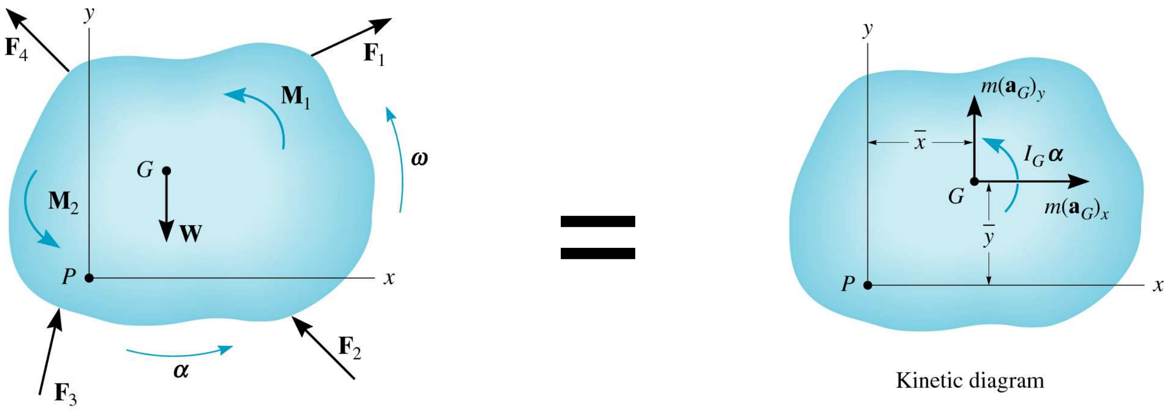
Plane Motion of a Rigid Body





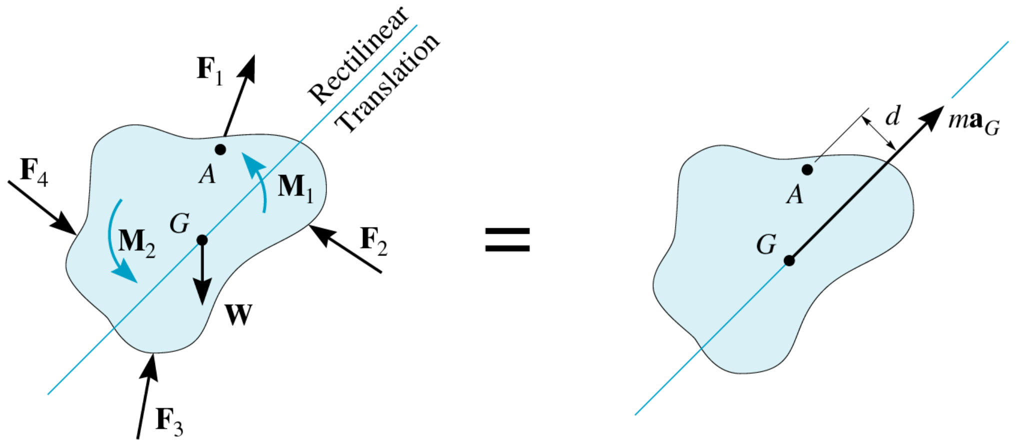


EQUATIONS OF TRANSLATIONAL MOTION

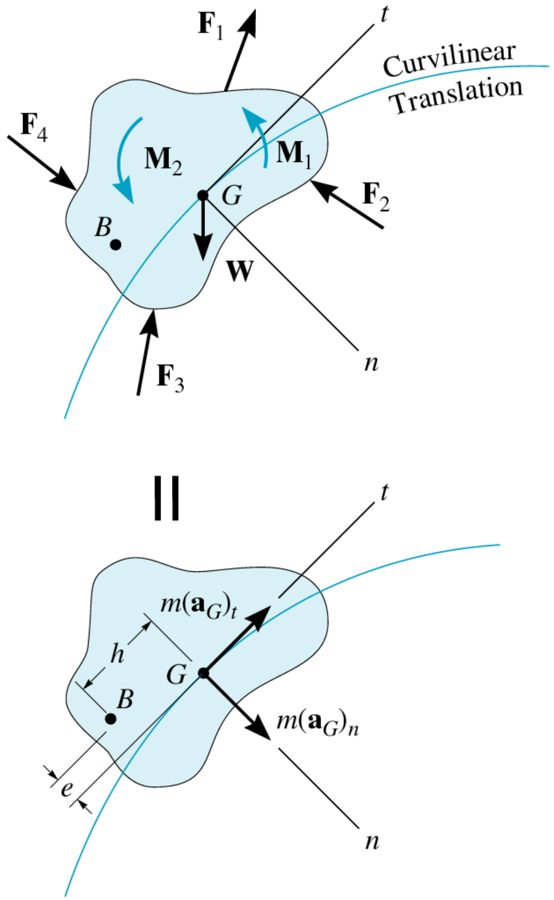


Kinetic diagram

EQUATIONS OF MOTION: TRANSLATION ONLY



EQUATIONS OF MOTION: TRANSLATION ONLY



PROCEDURE FOR ANALYSIS

Problems involving kinetics of a rigid body in only translation should be solved using the following procedure.

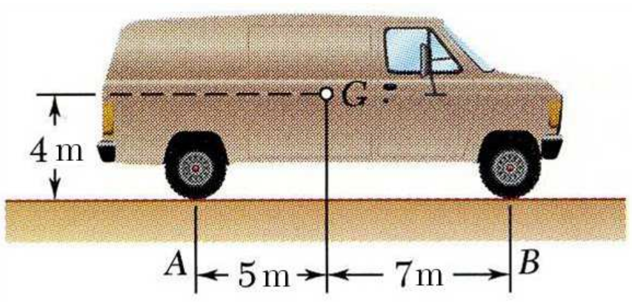
1. Establish an (x-y) or (n-t) inertial coordinate system and specify the sense and direction of acceleration of the mass center, $\boldsymbol{a}_G$.
2. Draw a FBD and kinetic diagram showing all external forces, couples and the inertia forces and couples.
3. Identify the unknowns.
4. Apply the three equations of motion:

$$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y \qquad \Big| \qquad \Sigma F_n = m(a_G)_n \quad \Sigma F_t = m(a_G)_t$$

$$\Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P \qquad \Big| \qquad \Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P$$

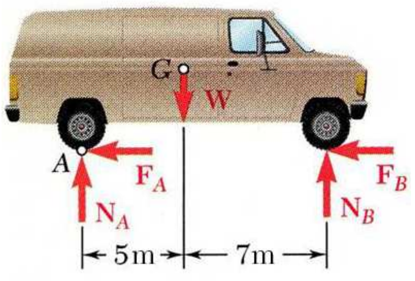
5. Remember, friction forces always act on the body opposing the motion of the body.

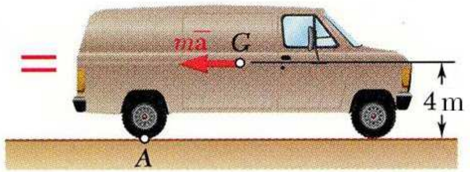
Sample Problem 16.1



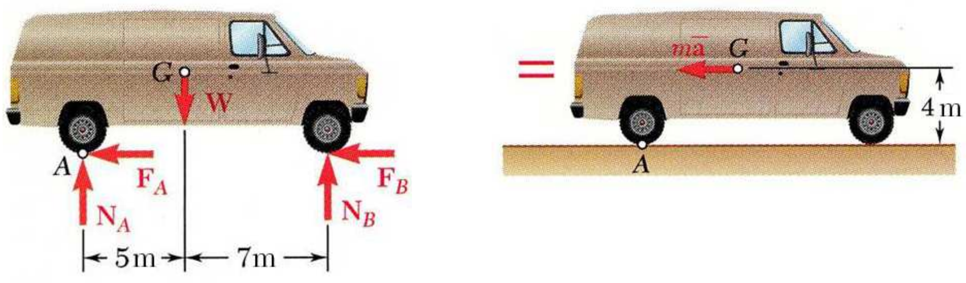
At a forward speed of 30 m/s, the truck brakes were applied, causing the wheels to stop rotating. It was observed that the truck to skidded to a stop in 200 m.

Determine the magnitude of the normal reaction and the friction force at each wheel as the truck skidded to a stop.





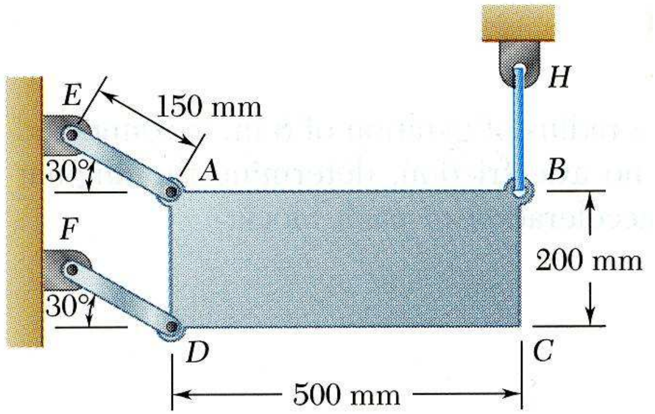
Sample Problem 16.1

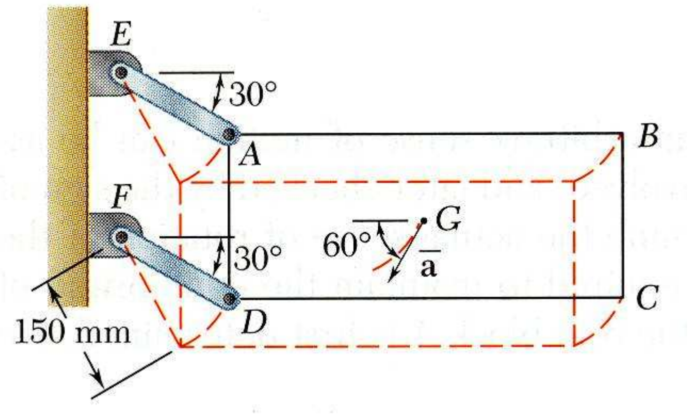


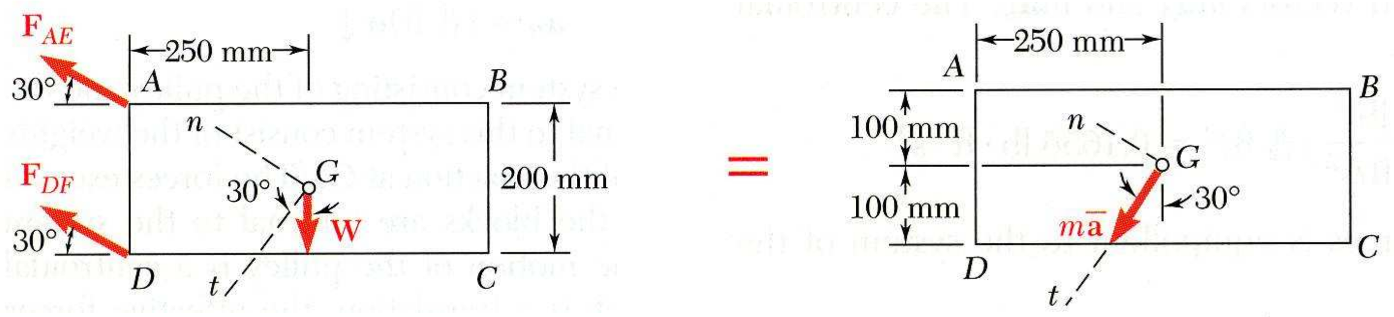
Sample Problem 16.2

The thin plate of mass 8 kg is held in place as shown.

Neglecting the mass of the links, determine immediately after the wire has been cut (*a*) the acceleration of the plate, and (*b*) the force in each link.

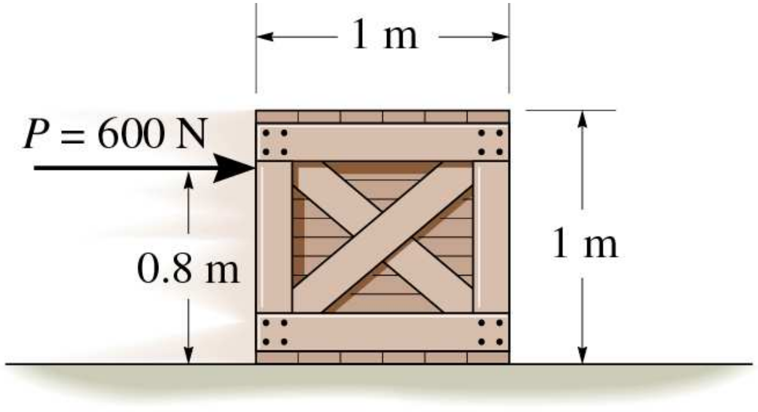






$\mathbf{F}_{AE}$
250 mm
30°
A
B
n
$\mathbf{F}_{DF}$
30°
G
200 mm
W
30°
D
t
C
=
250 mm
A
B
100 mm
n
G
100 mm
30°
$m\bar{\mathbf{a}}$
C
D
t

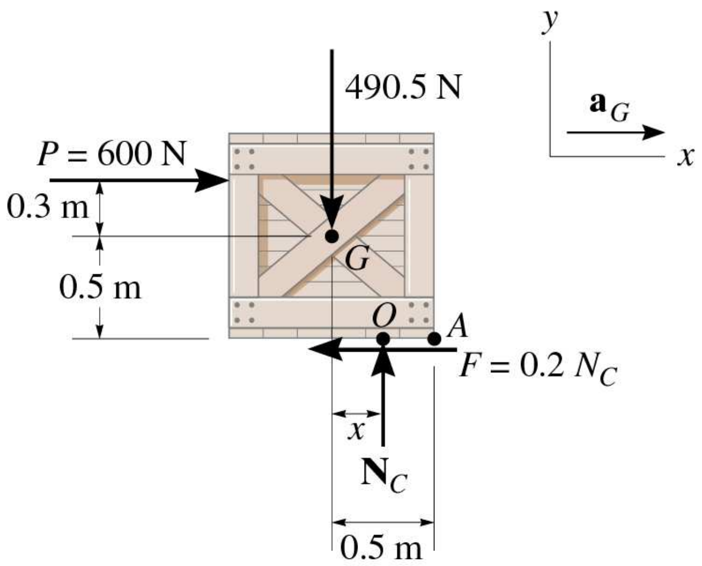
EXAMPLE



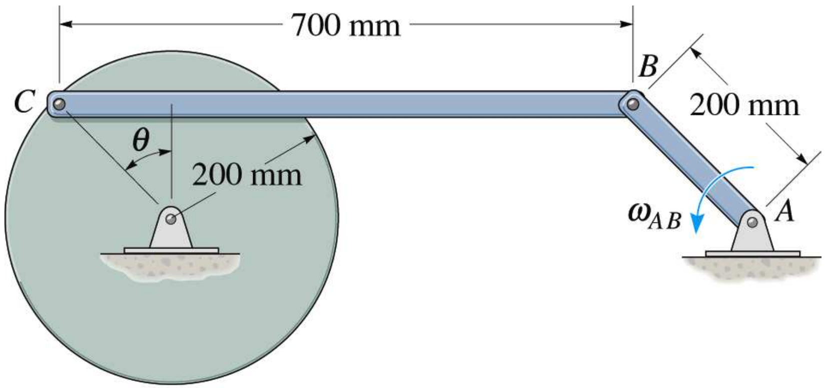
Given: A 50 kg crate rests on a horizontal surface for which the kinetic friction coefficient $\mu_k = 0.2$.

Find: The acceleration of the crate if P = 600 N.

EXAMPLE (continued)



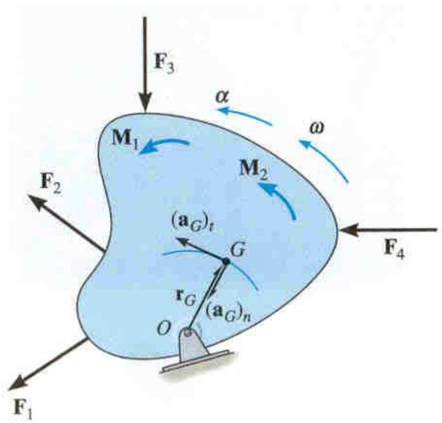
GROUP PROBLEM SOLVING



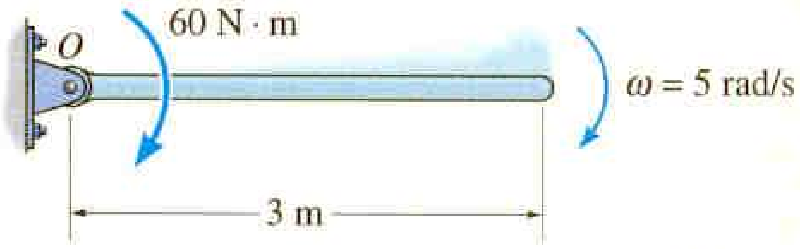
Given: A uniform connecting rod BC has a mass of 3 kg. The crank is rotating at a constant angular velocity of ω_{AB} = 5 rad/s.

Find: The vertical forces on rod BC at points B and C when θ = 0 and 90 degrees.

EQUATIONS OF MOTION FOR PURE ROTATION



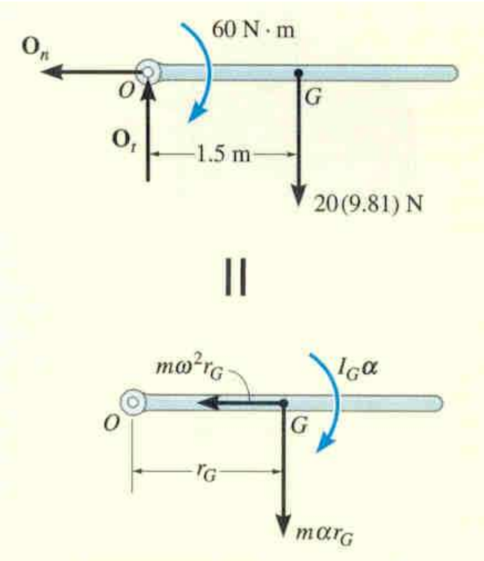
EXAMPLE

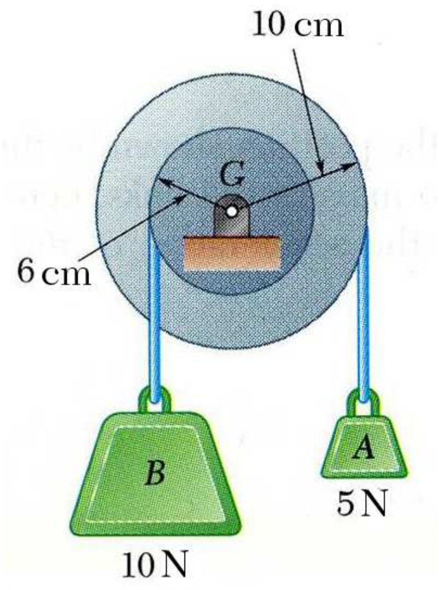


Given: A rod with mass of 20 kg is rotating at 5 rad/s at the instant shown. A moment of 60 N·m is applied to the rod.

Find: The angular acceleration α and the reaction at pin O when the rod is in the horizontal position.

EXAMPLE

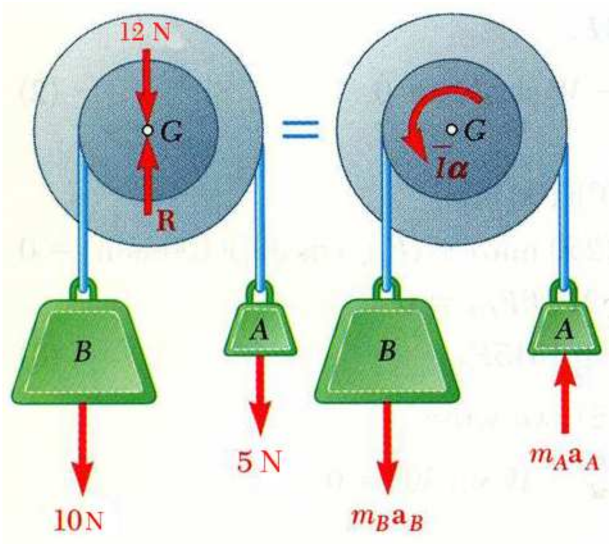




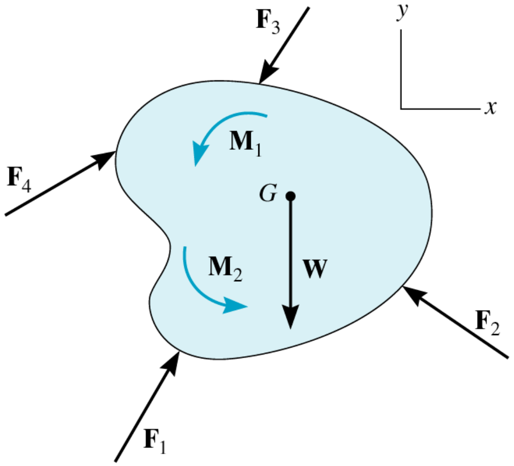
Sample Problem 16.3

A pulley weighing 12 N and having a radius of gyration of 8 cm is connected to two blocks as shown.

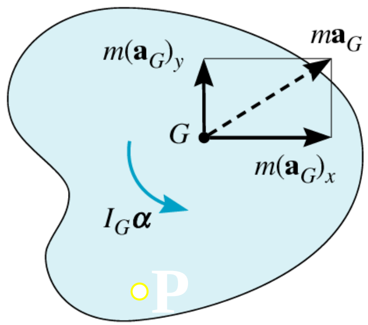
Assuming no axle friction, determine the angular acceleration of the pulley and the acceleration of each block.


12 N
G
R
=
G
$\bar{I}\alpha$
B
A
B
A
10N
5 N
$m_B a_B$
$m_A a_A$

EQUATIONS OF MOTION: GENERAL PLANE MOTION



When a rigid body is subjected to external forces and couple-moments, it can undergo both translational motion as well as rotational motion. This combination is called general plane motion.



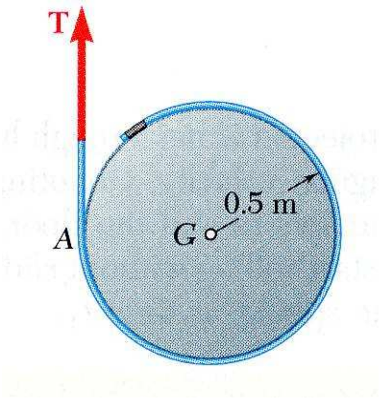
Using an x-y inertial coordinate system, the equations of motions about the center of mass, G, may be written as

$$\sum F_x = m\,(a_G)_x$$

$$\sum F_y = m\,(a_G)_y$$

$$\sum M_G = I_G\,\alpha$$

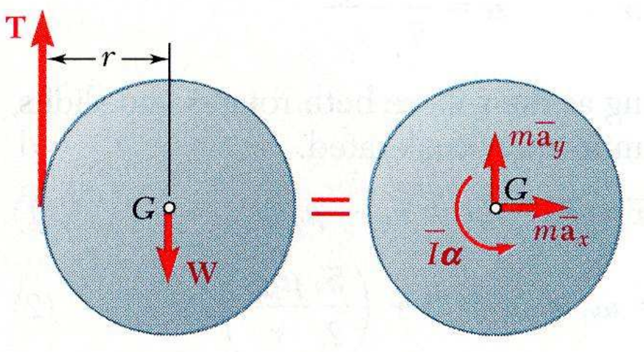
Sample Problem 16.4



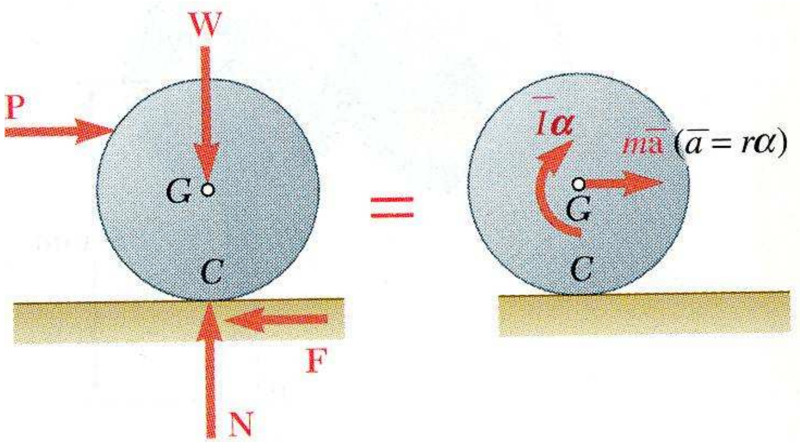
A cord is wrapped around a homogeneous disk of mass 15 kg. The cord is pulled upwards with a force T = 180 N.

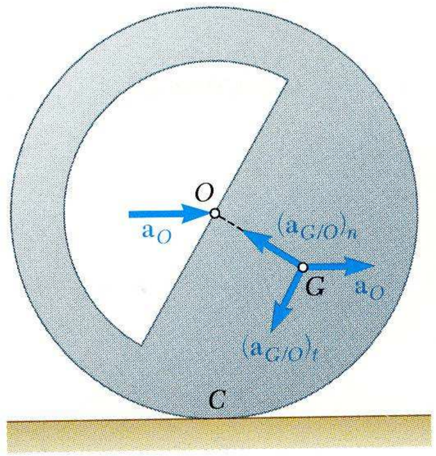
Determine: (*a*) the acceleration of the center of the disk, (*b*) the angular acceleration of the disk, and (*c*) the acceleration of the cord.

Sample Problem 16.4



Constrained Plane Motion: Rolling Motion





- For a balanced disk constrained to roll without sliding,

  $\bar{x} = r\theta \quad \rightarrow \quad \bar{a} = r\alpha$

- Rolling, no sliding:

  $F \le \mu_s N \qquad \bar{a} = r\alpha$

  Rolling, sliding impending:

  $F = \mu_s N \qquad \bar{a} = r\alpha$

  Rotating and sliding:

  $F = \mu_k N \qquad \bar{a}, r\alpha$ independent

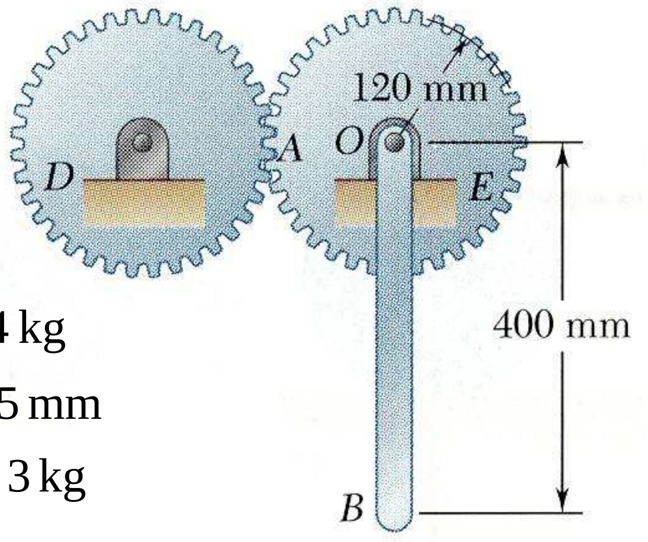
- For the geometric center of an unbalanced disk,

  $a_O = r\alpha$

  The acceleration of the mass center,

  $$\begin{aligned}\vec{a}_G &= \vec{a}_O + \vec{a}_{G/O} \\ &= \vec{a}_O + \left(\vec{a}_{G/O}\right)_t + \left(\vec{a}_{G/O}\right)_n\end{aligned}$$

Sample Problem 16.6



$$m_E = 4\,\text{kg}$$

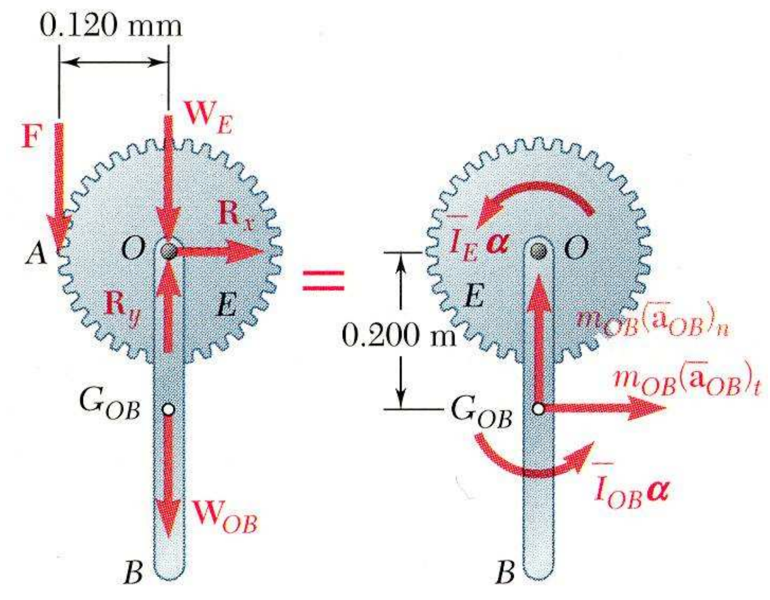
$$\bar{k}_E = 85\,\text{mm}$$

$$m_{OB} = 3\,\text{kg}$$

The portion *AOB* of the mechanism is actuated by gear *D* and at the instant shown has a clockwise angular velocity of 8 rad/s and a counterclockwise angular acceleration of 40 rad/s^2.

Determine: a) tangential force exerted by gear *D*, and b) components of the reaction at shaft *O*.

Sample Problem 16.6

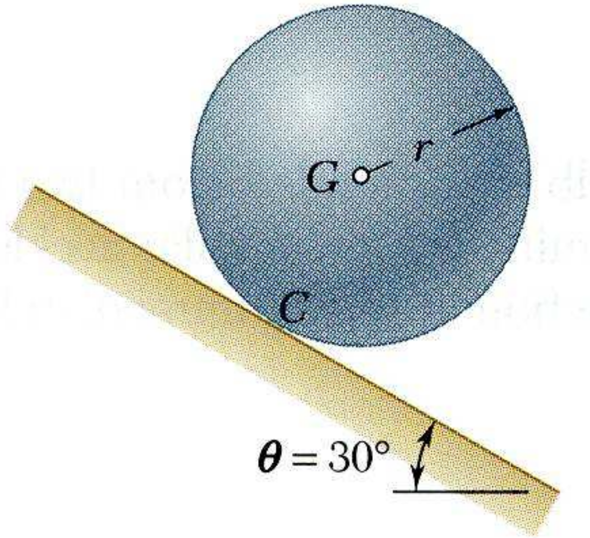


$$m_E = 4\,\mathrm{kg} \qquad \alpha = 40\,\mathrm{rad/s^2}$$

$$\bar{k}_E = 85\,\mathrm{mm} \qquad \omega = 8\,\mathrm{rad/s}$$

$$m_{OB} = 3\,\mathrm{kg}$$

Sample Problem 16.8



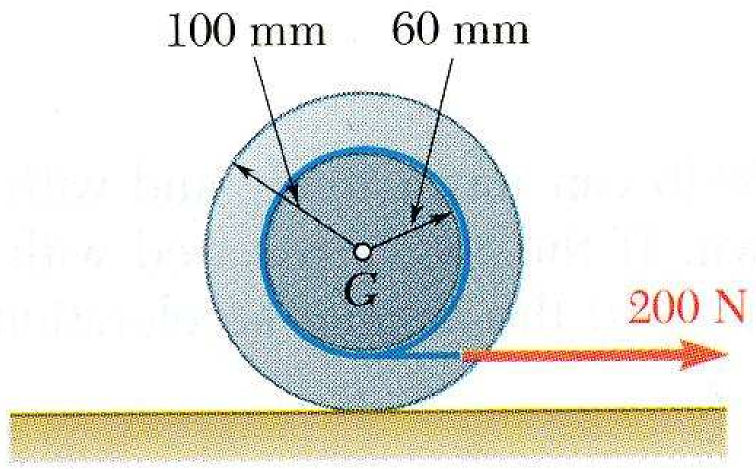
A sphere of weight W is released with no initial velocity and rolls without slipping on the incline.

Determine: *a*) the minimum value of the coefficient of friction, *b*) the velocity of G after the sphere has rolled 10 m and *c*) the velocity of G if the sphere were to move 10 m down a frictionless incline.

SOLUTION:

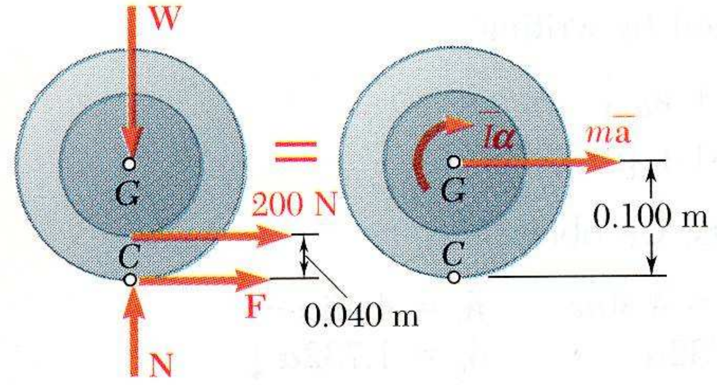
- Draw the free-body-equation for the sphere, expressing the equivalence of the external and effective forces.
- With the linear and angular accelerations related, solve the three scalar equations derived from the free-body-equation for the angular acceleration and the normal and tangential reactions at C.
- Calculate the friction coefficient required for the indicated tangential reaction at C.
- Calculate the velocity after 10 m of uniformly accelerated motion.
- Assuming no friction, calculate the linear acceleration down the incline and the corresponding velocity after 10 m.

Sample Problem 16.9

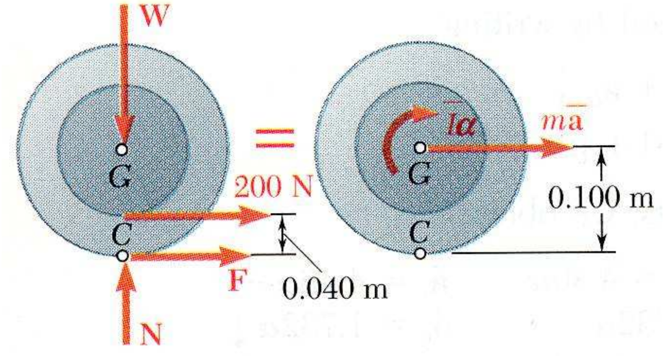


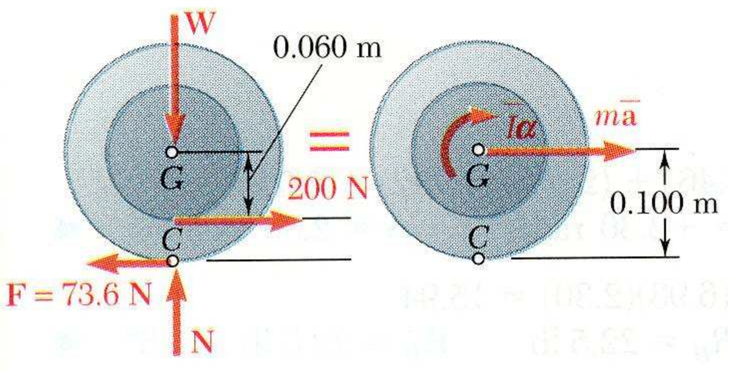
A cord is wrapped around the inner hub of a wheel and pulled horizontally with a force of 200 N. The wheel has a mass of 50 kg and a radius of gyration of 70 mm. Knowing $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the acceleration of G and the angular acceleration of the wheel.

Sample Problem 16.9

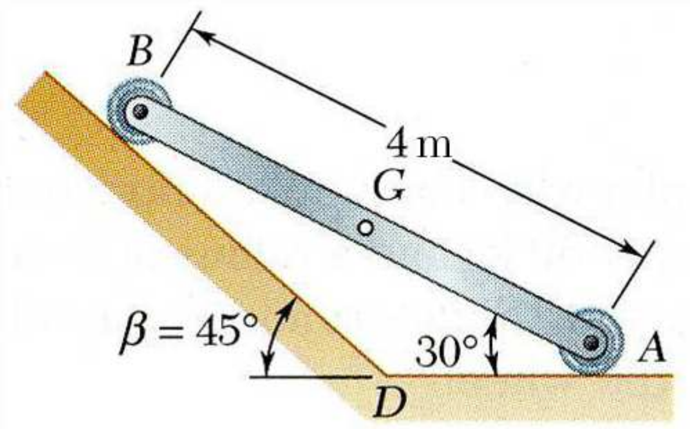


Sample Problem 16.9





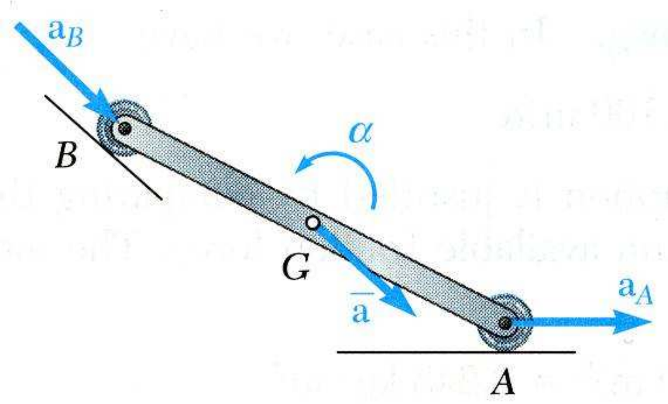
Sample Problem 16.10

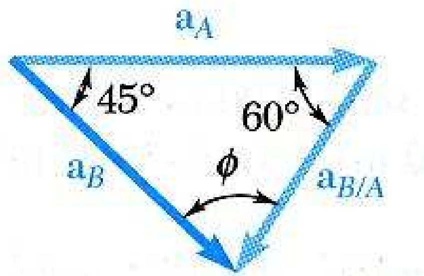


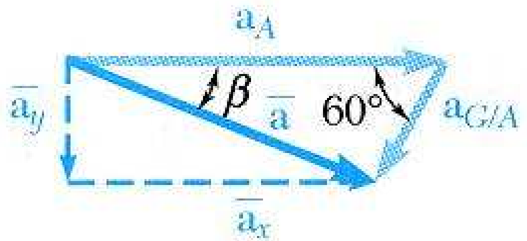
The extremities of a 4-m rod weighing 50 N can move freely and with no friction along two straight tracks. The rod is released with no velocity from the position shown.

Determine: *a*) the angular acceleration of the rod, and *b*) the reactions at *A* and *B*.

Sample Problem 16.10







Sample Problem 16.10