

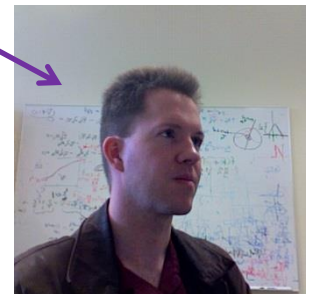
# PLASMA IN CONTACT WITH METAL: RF ANTENNA NEAR FIELD BEHAVIORS IN TOKAMAKS (AND INDUSTRIAL PLASMAS)

Presenter: David N. Smithe

Co-author: Tom Jenkins

**Tech-X Corporation**

**Plasma Physics Colloquium**  
*Applied Math and Physics Dept.*  
*Columbia University, NY*  
*September 5, 2014*





# More Acknowledgments

- Lodestar Research Corporation
  - Dan D’Ippolito, Jim Myra
- RF SciDAC (CSWPI) including
  - MIT (Paul Bonoli, John Wright)
  - PPPL (Cynthia Phillips, Nicola Bertelli)
  - ORNL (David Green)
  - CompX (Bob Harvey)
- TechX’ers (Scott Kruger, Jake King)
- Oak Ridge Leadership Computing Facility
  - Titan



# Funding Acknowledgements

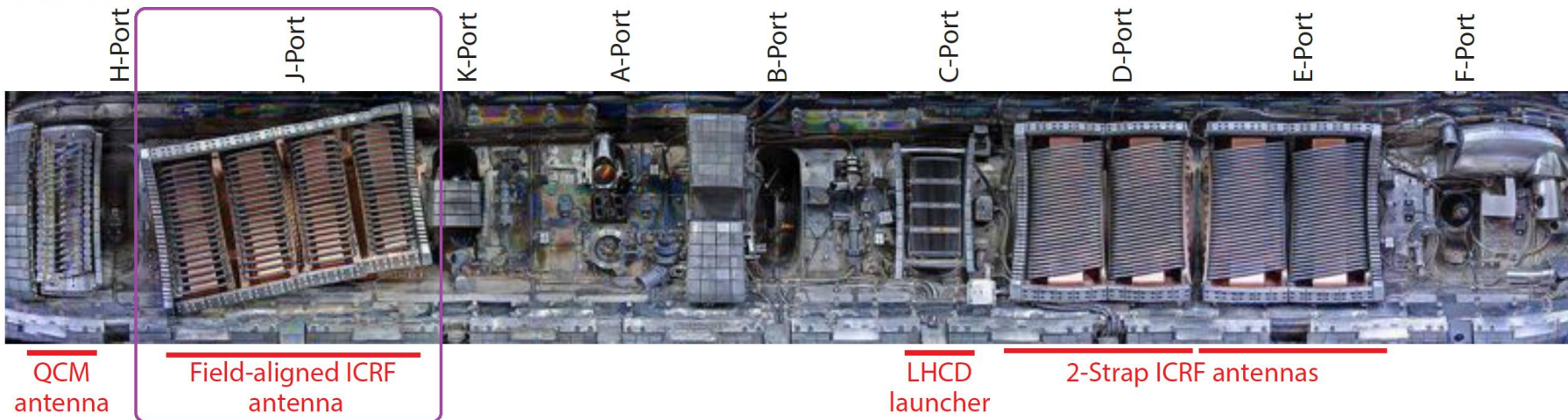
- DOE (SciDAC, Theory, Phase II SBIR)
  - DE-FC02-08ER54953
  - DE-FG02-09ER55006,
  - DE-FC02-05ER54823
- Tech X IR&D

# If You Don't Already Know Me

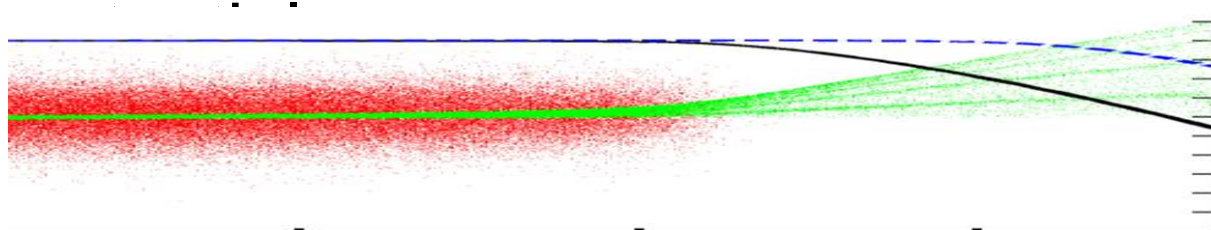
- '87 – Ph.D. (Univ. Mich.) PostDoc (PPPL)
  - ICRF Core Absorption, 1D Parallel “All Orders” (METS)
  - Early FEM 2.5D Core Solvers (SHOOT, SPRUCE)
- '89 Hired by Mission Research Corp.
  - Defense contractor, FDTD PIC codes and radar (MAGIC)
  - '95, PPPL (Phillips) 1D perp “All Orders” for HHFW (METS95)
- '05 – MRC bought by ATK, I went to Tech X
  - Back to Fusion (and some accelerator work) (Vorpal)
  - Trying to see what 3D FDTD can do in Fusion ... Antenna
- Unusual Talent
  - Good with 3D geometry

# The Original Problem

- We want to model this (~1 meter) antenna:



- Which has this (~10 microns) sheath





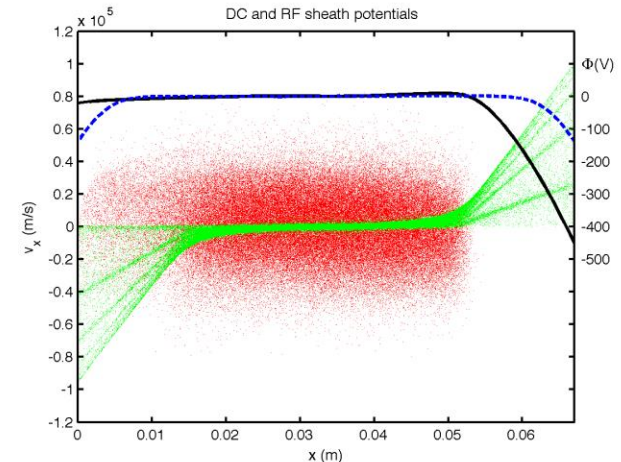
# Because ...

- RF antennas create impurities
  - which reduce the  $Z_{\text{effective}}$  of the plasma
  - Which reduces the fusion yield
  - Which is **bad**, especially in ITER
- Impurities are thought to arise from sputtering, when ions fall through the rectified RF sheath, hitting wall with
  - ~ 100 volts, a bit higher than  $T_{e,\text{edge}}$ .
- Work by D'Ippolito and Myra at Lodestar
  - Local electric fields ~  $10^4$  to  $10^5$  V/m
  - RF Rectified Sheaths get you ~ 100 Volts

# What is an RF Rectified Sheath?

- Additional plasma DC bias in the sheath, due to RF, so plasma potential never goes wrong way.

- (Show Movie 1)
- 1D Parallel Plate
- Start with Debye sheath (blue)
- Oscillating potential (black)



- Still very small (10 microns – 100 microns)
  - Antenna is a meter, grid size is 1 mm





# How to Defeat the Scale Problem

- Analogy: in EM-FDTD, we don't resolve the metallic skin depth,  $\delta = (\mu\sigma\omega)^{-1/2}$ , we just set  $\mathbf{E}_{\parallel} = 0$ , as a boundary condition.
- If we need ohmic losses in the skin depth, we use the local  $\mathbf{H}$  field to compute current and power, as in  $R_s |\mathbf{H}|^2$ .
  - E.g., the metallic skin depth is a “sub-grid model”
  - We model its effect,  $\mathbf{E}_{\parallel} = 0$ , not its full physics
- We “want” to do the same with the sheath.
  - Can we!? Not as simple as  $\mathbf{E}_{\parallel} = 0$ .
  - But lots of sheath work to draw from ...



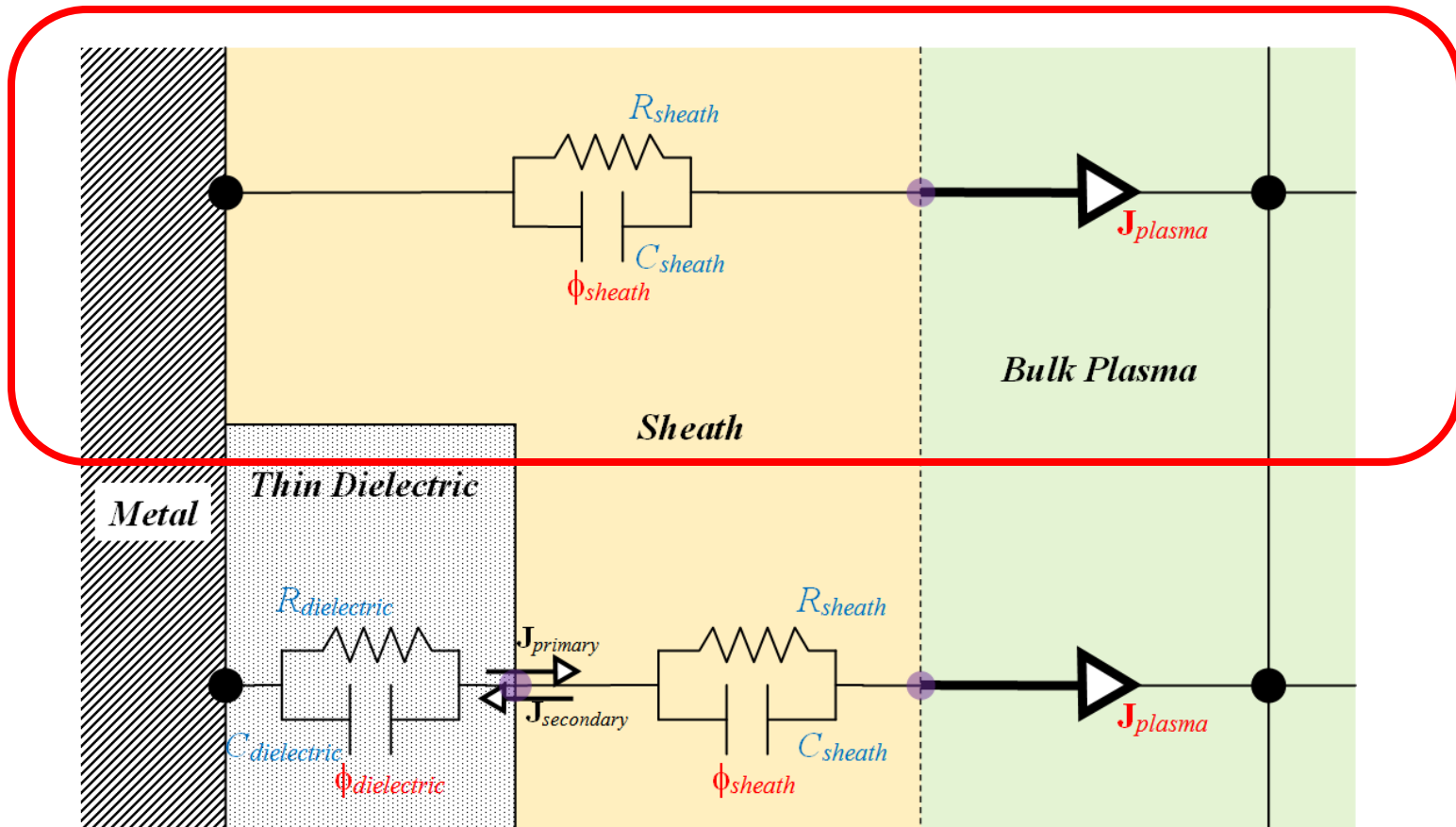
# Best Answer So Far

- D'Ippolito and Myra Boundary Condition
  - D. A. D'Ippolito and J. R. Myra, *Phys. Plasmas* 13, 102508 (2006)
- Model sheath as capacitive gap.
- Simple parameter,  $\Delta$ , characterizes capacitor.
- Follow the action:
  - Capacitor has oscillating  $\phi_{\text{sheath}}$  (RF sheath Voltage)
  - Nonlinear Capacitor spacing,  $\Delta(|\phi_{\text{sheath}}|)$ , Child-Langmuir
  - Rectified potential,  $\phi_{\text{DC,rectified}} \sim |\phi_{\text{sheath}}|$  is computed after the fact, usable for ion sputtering analysis

$$C_s = \frac{\epsilon_0 A}{\lambda_{De} \left[ 1 + \left( \frac{0.6 q_i \Delta V_s}{T_e} \right)^2 \right]^{3/8}}$$

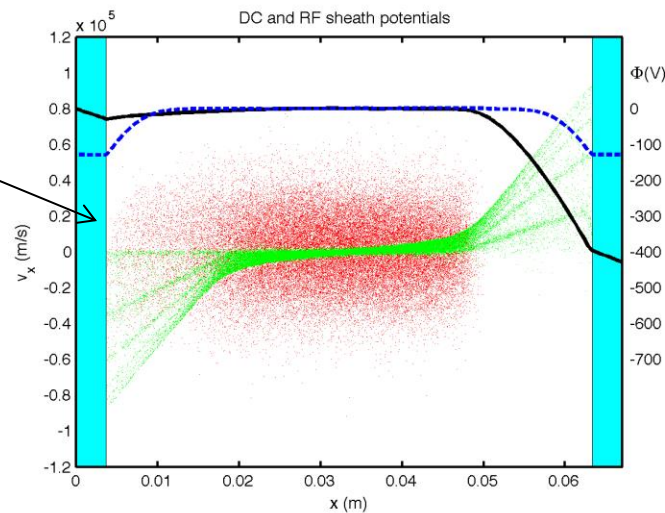
# Any EE's in the Audience ?

- Looks like this.



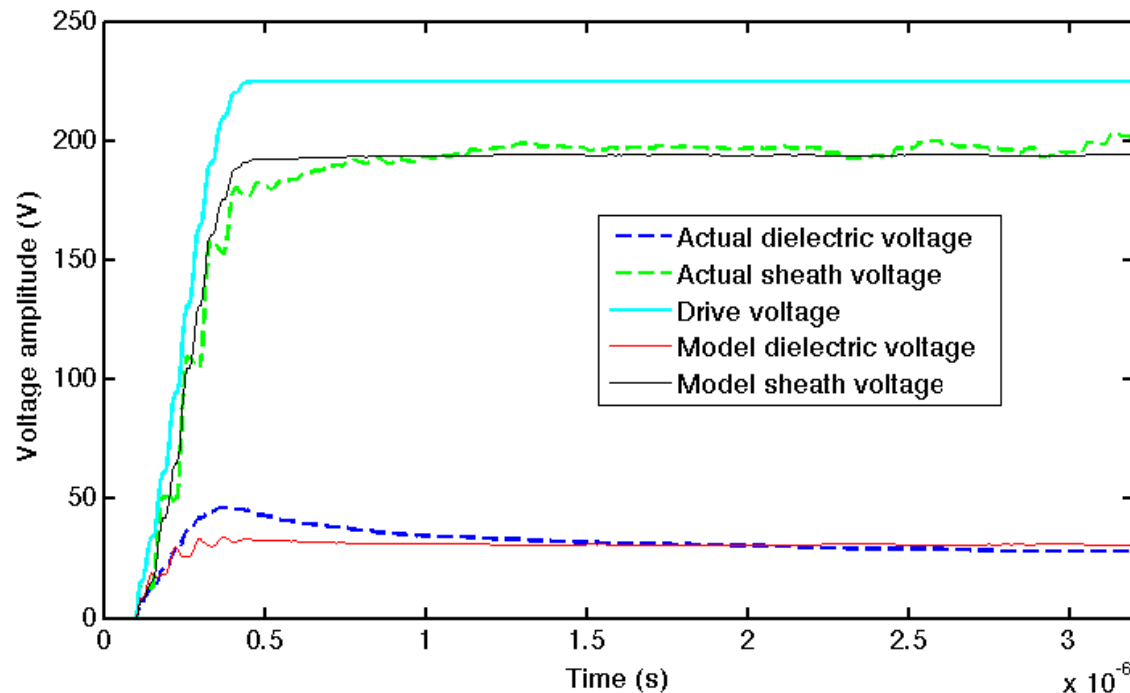
# Do We Believe This?

- We've done 1D PIC (particle-in-cell) benchmarking. From first principles.
- For industrial plasmas (and maybe fusion), we add a thin layer of dielectric to the metal wall.
- The Dielectric layer DOES behave like a capacitor, with well known  $C/\text{Area}=(d\epsilon)^{-1}$ .
- (Show Movie 2)
- Dielectric (blue)



# Yes, We Believe

- Voltage is split, from capacitances in series, we know one, we suspect we know the other. Sheath capacitance based on  $\Delta(|\phi_{\text{sheath}}|)$  gives correct split.



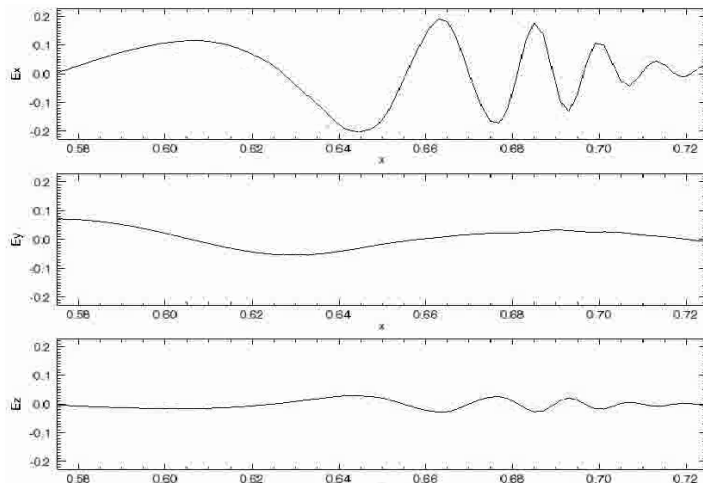


# Also Need a Plasma

- Sheath potentials will depend on local electric field.
- That is only correct if the plasma in front of the antenna is correct
  - $\epsilon_{plasma} \sim 1000 \dots \pm!$
- Time domain cold plasma:  
*“Finite-difference time-domain simulation of fusion plasmas at radiofrequency time scales”, Smithe, Physics of Plasmas 14, 1 2007*
- (This is actually where the story begins.)

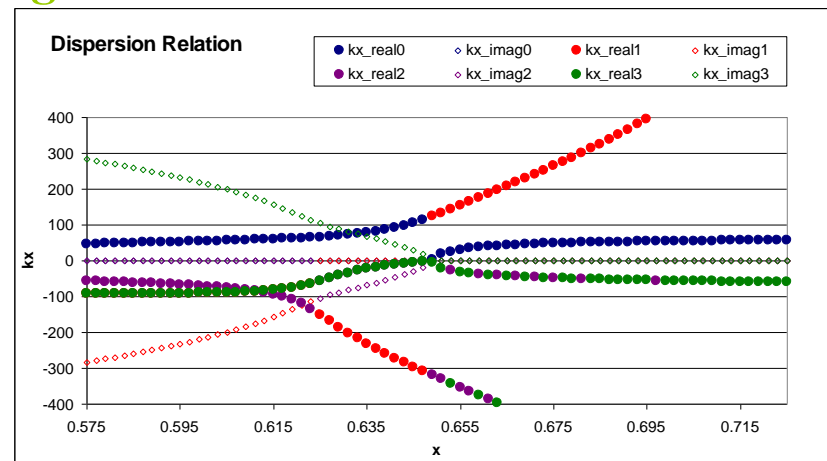
# 2007 PoP Paper

- Critical Realization: Cold Plasma Dielectric is zero-D – can make it point-wise implicit in time domain, to get all cold-plasma waves, **stable**, regardless of cutoff / resonances.
- Benchmarking: E.g., ICW mode conversion



*High Field*

*Low Field*



# Time Domain Cold Plasma Equations

- Convert frequency-domain linear plasma dielectric tensor to “auxiliary differential equation” (ADE) time domain method.

$$\epsilon = 1 - \sum_{\text{species}} \begin{bmatrix} \frac{\omega_p^2}{\omega^2 - \Omega^2} & \left(\frac{\Omega}{-i\omega}\right) \frac{\omega_p^2}{\omega^2 - \Omega^2} & 0 \\ \left(\frac{\Omega}{i\omega}\right) \frac{\omega_p^2}{\omega^2 - \Omega^2} & \frac{\omega_p^2}{\omega^2 - \Omega^2} & 0 \\ 0 & 0 & \frac{\omega_p^2}{\omega^2} \end{bmatrix}$$

- “Auxiliary Fields” are the linear plasma currents, one per species.

$$\{\partial_t + \nu_s\} \mathbf{J}_s = \epsilon_0 \omega_{ps}^2 \mathbf{E} - \boldsymbol{\Omega}_s \times \mathbf{J}_s$$



# “Super-Boris” Time Domain Cold Plasma Implicit Algorithm

- $3 \times (N_{\text{species}} + 1)$  matrix is analytically invertible.
- But messy!

$$\lambda_s \equiv \frac{1 - \frac{1}{2} \nu_s \delta t}{1 + \frac{1}{2} \nu_s \delta t}, \quad \omega_s \equiv \frac{\Omega_s \delta t}{1 + \frac{1}{2} \nu_s \delta t}, \quad \omega_{p0}^2 \equiv \sum_s \frac{\omega_{ps}^2 \delta t^2}{1 + \frac{1}{2} \nu_s \delta t}$$

$$\gamma^2 \equiv \sum_s \frac{\omega_{ps}^2 \Omega_s^2 \delta t^4}{(1 + \frac{1}{2} \nu_s \delta t)^3}, \quad \delta \equiv \sum_s \frac{\omega_{ps}^2 \Omega_s \delta t^3}{(1 + \frac{1}{2} \nu_s \delta t)^2}, \quad \Delta^2 \equiv \frac{\delta^2}{1 + \frac{1}{4} \omega_{p0}^2}$$

$$\mathbf{K} = \mathbf{L} - \frac{\delta t}{2} \sum_s (1 + \lambda_s) \left[ \frac{1}{1 + \frac{1}{4} \omega_s^2} \mathbf{J}_s^n + \frac{1}{4} \omega_s^2 \mathbf{b} \mathbf{b} \cdot \mathbf{J}_s^n - \frac{1}{2} \omega_s \mathbf{b} \times \mathbf{J}_s^n \right]$$

$$\mathbf{F}^{n+1} = \frac{1 - \frac{1}{4} \omega_{p0}^2 - \frac{1}{64} \Delta^2}{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Delta^2} \mathbf{F}^n + \frac{1}{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Delta^2} \mathbf{K}$$

$$+ \frac{\frac{1}{64} \Delta^2 - \frac{1}{16} \gamma^2}{\left(1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Delta^2\right) \left(1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{16} \gamma^2\right)} \mathbf{b} \mathbf{b} \cdot [2\mathbf{F}^n + \mathbf{K}]$$

$$+ \frac{\frac{1}{8} \delta}{\left(1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Delta^2\right) \left(1 + \frac{1}{4} \omega_{p0}^2\right)} \mathbf{b} \times [2\mathbf{F}^n + \mathbf{K}]$$

$$\mathbf{J}_s^{n+1} = \frac{\lambda_s - \frac{1}{4} \omega_s^2}{1 + \frac{1}{4} \omega_s^2} \mathbf{J}_s^n + \left( \frac{1}{\delta t} \right) \frac{\omega_p^2 \delta t^2}{1 + \frac{1}{2} \nu_s \delta t} \frac{1}{2} (\mathbf{F}^{n+1} + \mathbf{F}^n) + \frac{1}{4} \omega_s^2 \mathbf{b} \mathbf{b} \cdot [(\lambda_s + 1) \mathbf{J}_s^n + \frac{1}{2} (\mathbf{F}^{n+1} + \mathbf{F}^n)] - \frac{1}{2} \omega_s \mathbf{b} \times [(\lambda_s + 1) \mathbf{J}_s^n + \frac{1}{2} (\mathbf{F}^{n+1} + \mathbf{F}^n)]$$

- Steps over  $\omega_{pe}$ ,  $\Omega_{ce}$  times.

# In the Boundary Cells ...

- Key Realization is that Plasma Currents from Time Domain Plasma is Capacitor Current.
  - E.g., the sheath IS the boundary condition for  $\mathbf{J}_{\text{plasma}}$
- One More Equation, for sheath potential.
  - Ugh, redo all that math, make sure energy conserving

The following equations shows the energy balance.

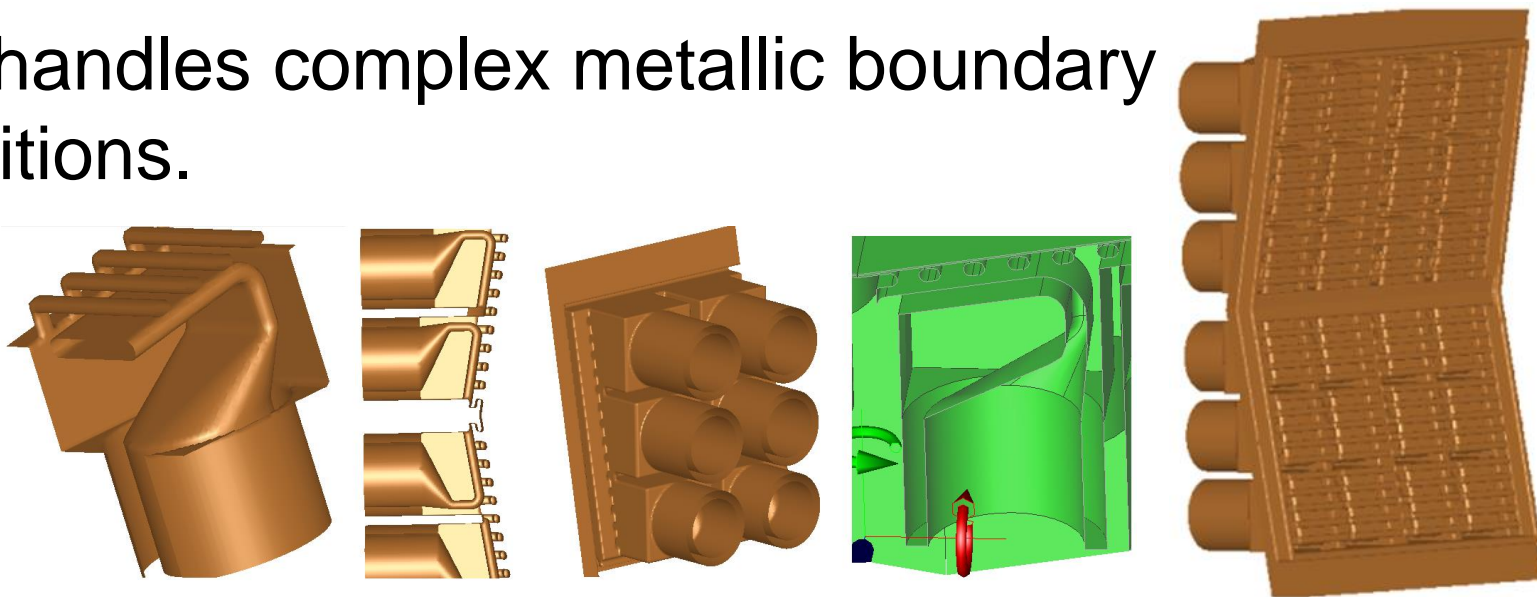
$$\begin{array}{l}
 (1/\delta x) \phi_{sheath} \quad \{ \quad (\epsilon_0/\Delta) \partial_t \phi_{sheath} = \mathbf{n} \cdot \mathbf{J} - (\epsilon_0 \omega v / \Delta) \phi_{sheath} \quad \} \\
 \mathbf{H} \quad \cdot \quad \{ \quad \mu_0 \partial_t \mathbf{H} = -\nabla \times \mathbf{E} \quad \} \\
 \mathbf{E} \quad \cdot \quad \{ \quad \epsilon_0 \partial_t \mathbf{E} = \nabla \times \mathbf{H} - \mathbf{J} \quad \} \\
 \mathbf{J} \quad \cdot \quad \{ \quad (1/\epsilon_0 \omega_p^2) \partial_t \mathbf{J} = [\mathbf{E} + \mathbf{n}(1/\delta x) \phi_{sheath}] \quad \}
 \end{array}$$

The sum of all of these gives

$$\partial_t \{ \frac{1}{2} \epsilon_0 (1 - \Delta/\delta x) |\mathbf{E}|^2 + \frac{1}{2} \epsilon_0 (\Delta/\delta x) |\phi/\Delta|^2 + \frac{1}{2} \mu_0 |\mathbf{H}|^2 + \frac{1}{2} (1/\epsilon_0 \omega_p^2) |\mathbf{J}|^2 \} = -v \omega \epsilon_0 (\Delta/\delta x) |\phi/\Delta|^2$$

# The Advantages of Time Domain

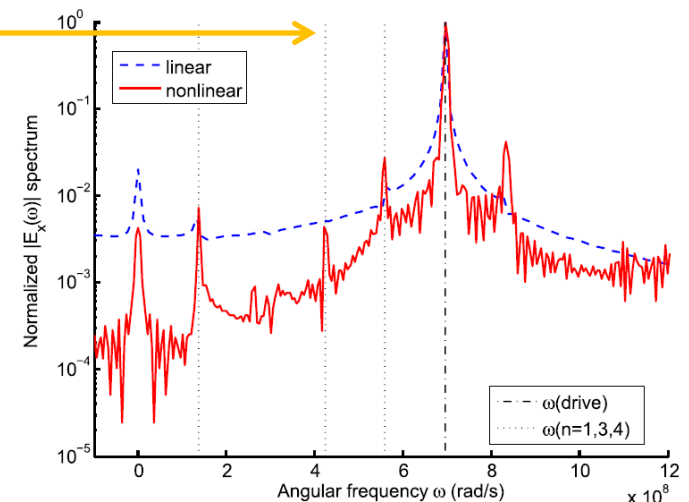
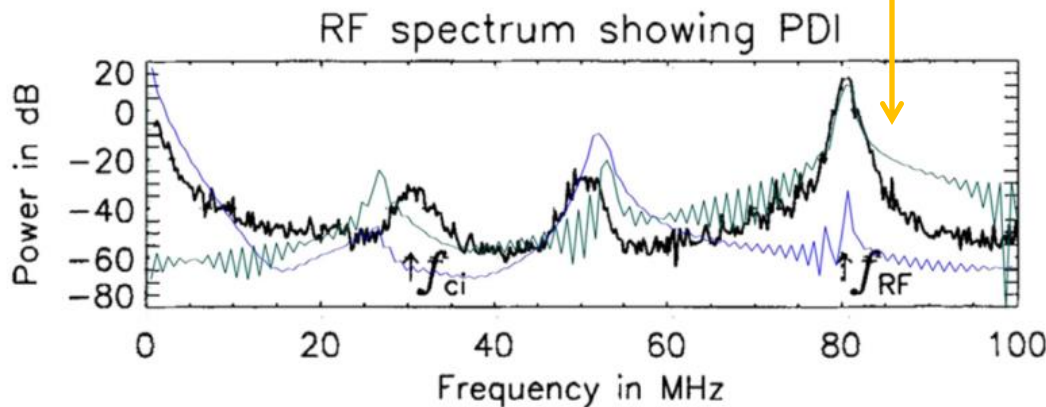
- Explicit FDTD EM code is very well established and fast. *(e.g., we already had the code!)*
- And handles complex metallic boundary conditions.



- Of course, multiple simultaneous frequencies.
- Which allows for inclusion of nonlinearity, without a priori guess at modes and frequencies

# An Aside: Example of Nonlinearity: Parametric Decay Instability

- Use linear plasma electrons, and kinetic ions, (needed for IBW wave in PDI)
  - Sometimes as expected (*comparison to J. Rost Thesis, MIT*)
  - Sometimes not



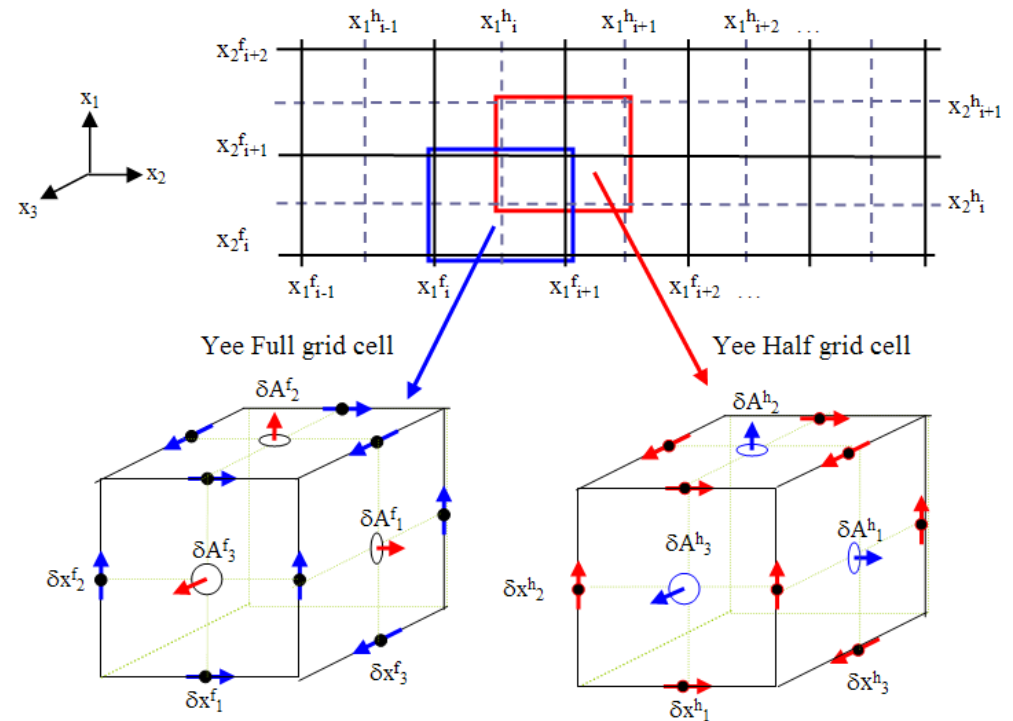
“Time-domain simulation of nonlinear radiofrequency phenomena,” Thomas G. Jenkins, Travis M. Austin, David N. Smithe, John Loverich, and Ammar H. Hakim, *Phys. Plasmas* 20, 012116 (2013)

# The Disadvantage of Time Domain

- **Yee Cell !** ( $B_x$  is at different location of  $B_y, B_z$ )
- Complicates the  $\mathbf{J} \times \mathbf{B}$  in plasma force equation.

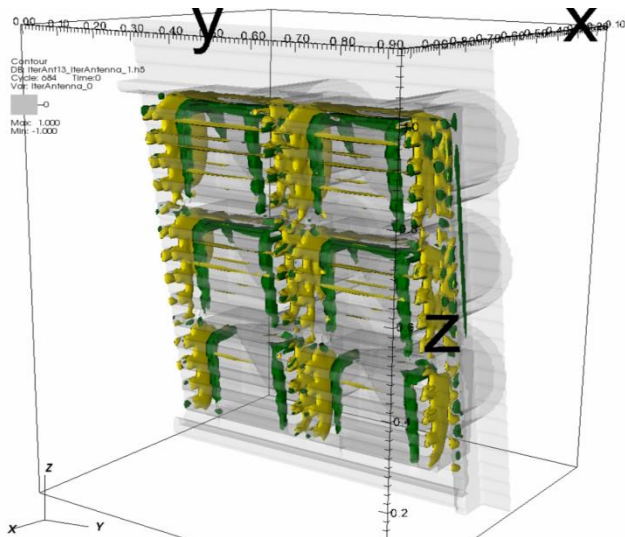
Also, no warm plasma effects, since no longer point physics,  $\nabla$ -operations.

So no IBW.



# ITER Antenna Simulations

- (Show Movie 3 and 4)
- Simulations of  $\frac{1}{4}$  ITER antenna, with surrounding box shows
  - slow-wave enhanced sheaths on the surrounding antenna box,
  - an interesting circulation pattern around the box that vanishes when field aligned





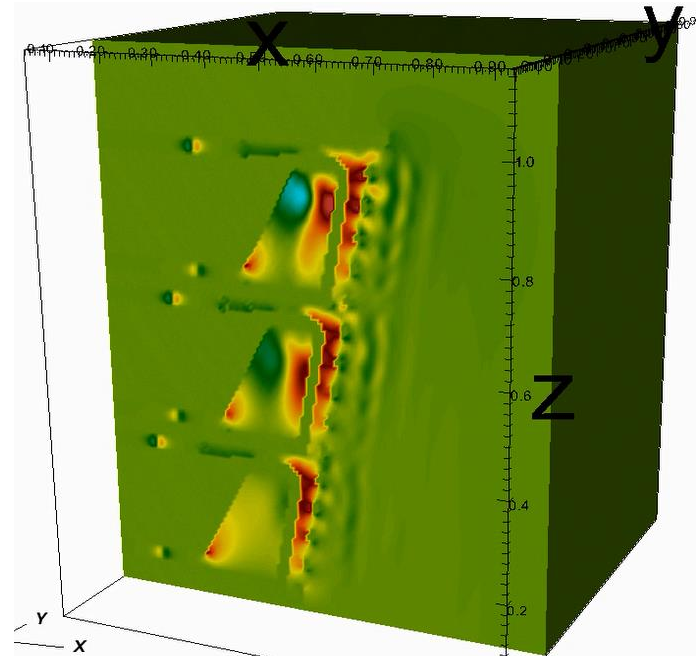
## New Work on ITER Antenna

- Simulations pre-date energy conserving form
  - So sheath voltages are not necessarily correct
- Simulations pre-date Titan usage
  - Can do all 24 straps now
- (Hopefully) more material for APS ITER session talk later this fall.
- And then there is the question of the slow wave...

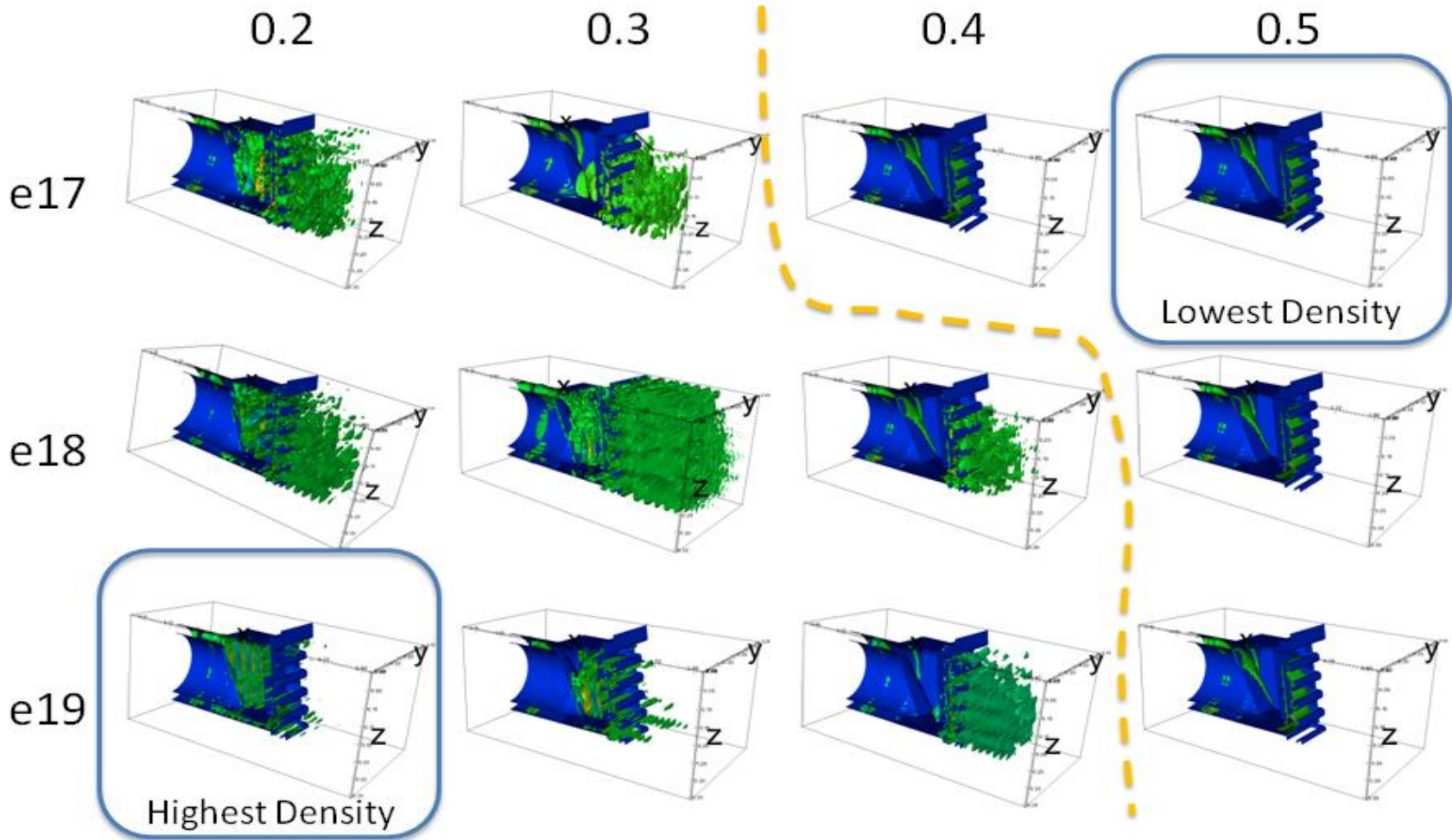


# The Slow Wave

- I was given a  $10^3$  range in edge densities to look at for ITER. (!)
- Some of those densities show a short wavelength mode in front of the antenna
- (Show Movie 5).



# “Layer” moves with Density



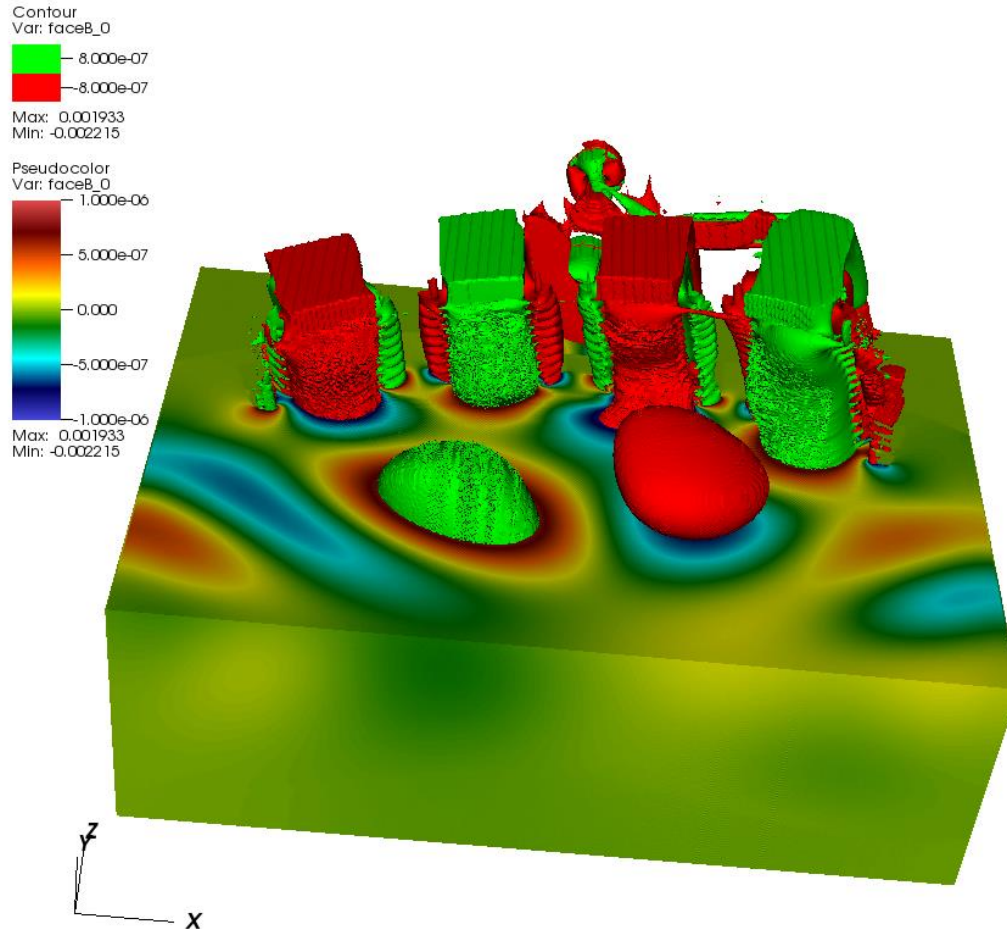


# Does This Have Anything to do with NSTX Anomalous Losses?

- This question is presently under study.
- Concern is that it might be there for ITER as well.
- I've been trying to get a better instinctive idea of what is going on with these slow waves.
- Cold Plasma, they have to essentially be lower hybrid resonances.
- Resonance in the edge is  $\omega \sim \omega_{LH} \sim \omega_{p,ion}$
- So there is a critical density, but  $\omega_{pi}$ , not  $\omega_{pe}$ .



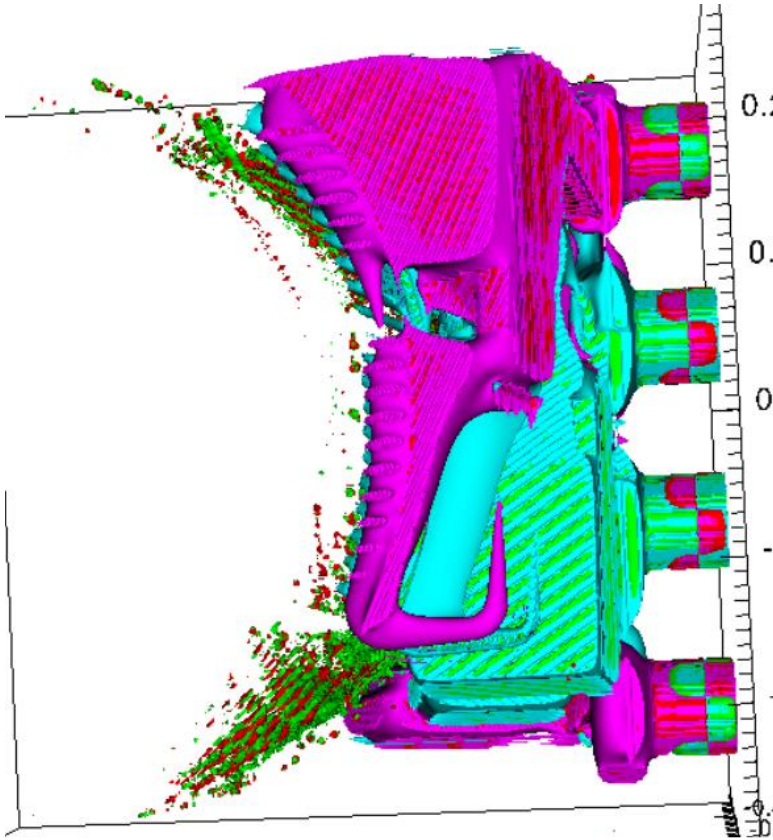
# We are looking at the CMod Field Aligned Antenna as well



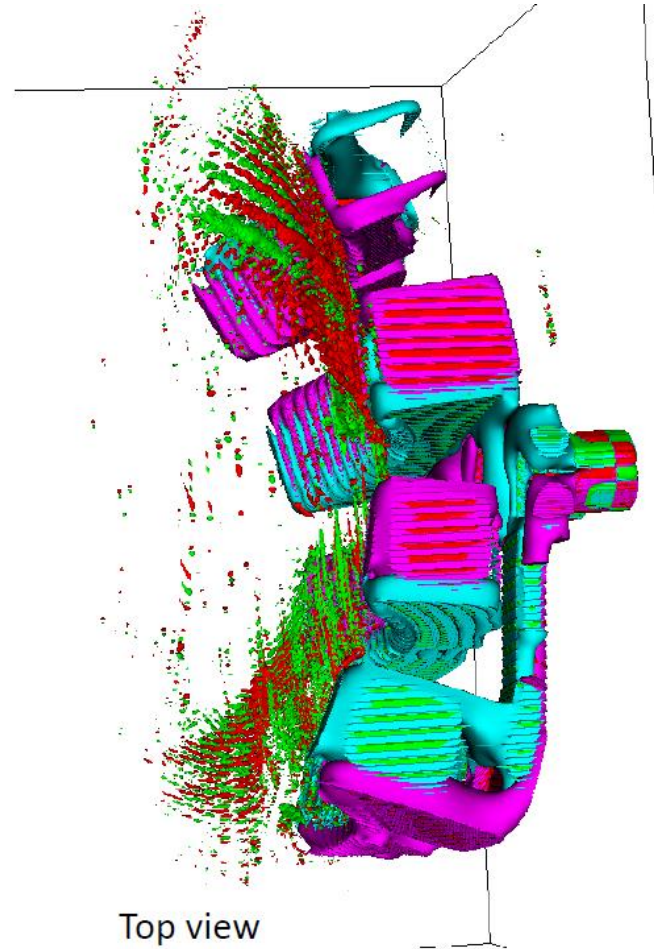




We can see slow waves there too,  
sometimes at high magnitude



Poloidal view



Top view

# Slow Wave Dispersion

- The slow waves are NOT a thin layer trapped between two cutoffs, they are a thin layer trapped between a cutoff and a resonance.
- The resonance is what allows for energy to be in the layer, otherwise would be too thin to excite, since need a  $k_{\perp}$ .

## $\omega_{LH}$ in the Edge

- From Stix:  $\omega_{LH}^{-2} = (\omega_{pi}^2 + \Omega_{ci}^2)^{-1} + (\Omega_{ci}\Omega_{ce})^{-1}$
- At edge density, last term neglects away.
- At edge,  $(\omega_{RF}/\Omega_{ci})^2 \sim (n_{Harm} R_{outer}/R_0)^2 \gg 1$
- So:  $\omega_{LH}^2/\omega_{RF}^2 \approx \omega_{pi}^2/\omega_{RF}^2 + \text{SmallNumber}$   
(0.2 - 0.02)





# Critical Density

- Since  $\omega_{LH} \sim \omega_{pi}$ , insensitive to B field, only density
  - Assume D plasma.
- For 30 MHz:  $1.7 \times 10^{17} / m^3$
- For 80 MHz:  $2.7 \times 10^{17} / m^3$

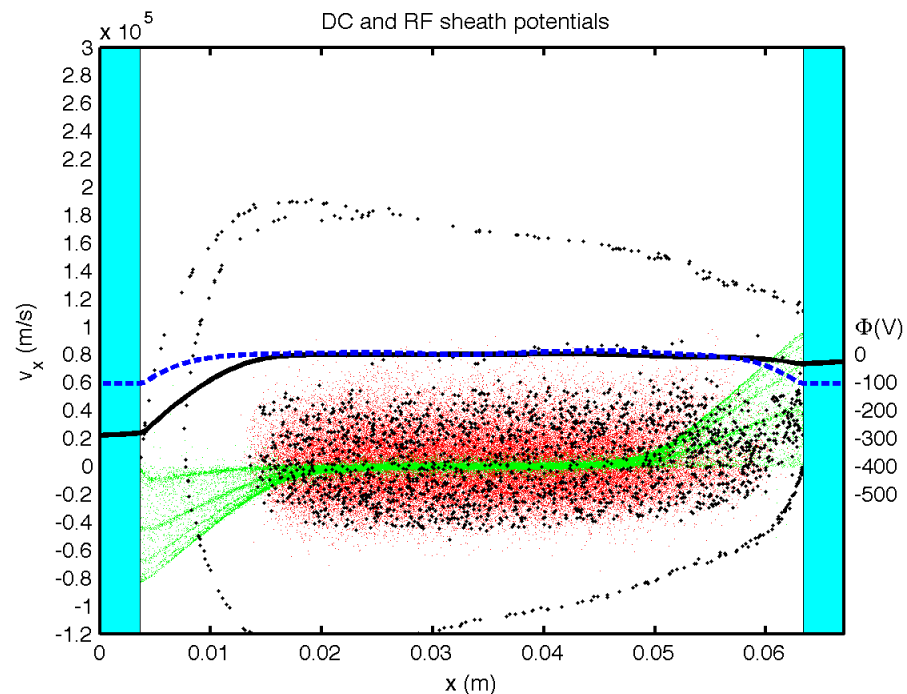


# What are the Reported Densities?

- NSTX: Expect  $5-10 \times 10^{17} m^3$ , at least 3x over critical density.
  - Is there some other explanation why density might be lower than reported, in front of antenna. Ponderomotive pressure?
  - The game is on ...
- ITER: Density is unknown. Hopefully high enough! The frequency *is* higher ... so happens at higher density.

# An Aside: Industrial Applications

- Many industrial plasmas are maintained by similar DC + RF fields, with energetic particles generated in the sheath as the ionization source.
- (Show Movie 6.)



# Phase II SBIR Work

- Still have the scale problem, so still want to use sub-grid sheath.
- Want EED, use “test” particles.
- Have created special particle boundaries that see the sheath “kick” before impact, or after secondary emission, in an attempt to reproduce the “beam” electrons without resolving the sheath.
- Much interest in estimating steady-state of such plasmas a priori.

# Summary

- Ultimate goal is estimation of impurity generating sputtering in ITER, and other tokamaks.
- Resolved scale problem with RF sheath sub-grid model, e.g., single  $\phi_{\text{sheath}}$  value represents sheath.
- Coupled with pre-existing time-domain cold plasma algorithm, and pre-existing 3D CAD-importing EM software.
- Slow wave is a very interesting tangent.
- SBIR work will also include particles and industrial concerns.



# THANK YOU!

- Questions, Comments?