1

Play and problem solving

This chapter covers:

- **how some theories of play have influenced early years mathematics education**
- **how play is an appropriate context for problem solving**
- **creativity and mathematical development**
- **including all children and families in playful mathematical experiences**
- **the transition from Foundation Stage (at age 5 years) to KS 1 (5–7 years)**
- **implications for an appropriate early years mathematics pedagogy.**

Why play?

Play is what young children are about, it is what preoccupies them and it can be considered both a mode of behaviour and a state of mind. We understand that play is a natural pursuit for young children, but what we seem less able to understand is how to harness appropriate play pedagogical processes and use them to support mathematical development.

In recent years, it has seemed that a playful approach to learning has been at odds with UK government directives. Teachers of mixed-age early years classes and those teaching Year 1 children (6-year-olds) are aware that the transition from the Early Years Foundation Stage, a distinct phase of play-based education and care from birth to the end of Reception (5-year-olds), to a more subject-specific Year 1 can be disjointed, bewildering and potentially detrimental (Ofsted, 2004). The *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools* (DCSF, 2008), led by Sir Peter Williams, however, described play as a feature of effective early years pedagogy and stressed the importance of connection making, creative recording and mark making, along with appropriate transition from the Foundation Stage to Year 1. Two further reviews, the *Independent Review of the Primary Curriculum: Final Report* (Rose, 2009) and *Towards a New Primary Curriculum: A Report from the Cambridge Primary Review, Part 2: The Future* (Alexander, 2009), led by Professor Robin Alexander, also regard play as a significant feature of good early years practice. They argue that, far from being trivial, play is an effective and age-appropriate pedagogy and one which concerns itself with *how* children learn, and not merely with *what* they learn. Although play is complex to define, some theories of play have had a particular impact on the teaching of mathematics.

Piaget

Piaget's constructivist theory states that active learning, first-hand experience and motivation are the catalysts for cognitive development. Learning develops through clearly defined ages and stages – a continuum from functional play, through symbolic play to play with rules. Piaget (1958) has not only influenced early years practitioners in their practice of allowing children self-choice, but his work has also been very influential in commercial maths schemes which assume a hierarchical view of mathematical development. This had the effect of advocating 'pre-number' activities, such as matching and sorting before a child could progress to counting and manipulating numbers.

Vygotsky

Vygotsky (1978), unlike Piaget, emphasizes the significance of social interaction, in particular the use of language, which assists learning and development. His social constructivist theory regards social interaction with peers and adults, through which children can make sense of their world and create meaning from shared experiences, as crucial. Learning occurs in the 'zone of proximal development', which represents the difference between what the child actually knows and what the child can learn with the assistance of a 'more knowledgeable other'. Play with others can provide these 'zones' because of the meaningful and motivating social context in which they occur. His influence on mathematics has been to encourage mathematics teaching to be related to the child's own experiences and to encourage talk about mathematics.

Bruner

Like Vygotsky, Bruner (1991) shares social constructivist theories which highlight the significance of interaction with others. Play serves as a vehicle for socialization and its contexts enable children to learn about rules, roles and friendships. The practitioner is proactive in creating interesting and challenging environments and in providing quality interactions, which act as a 'scaffold' for children's learning. He advocates a 'spiral curriculum' where children revisit play materials and activities over time, using them differently at each encounter as their increased development dictates. The structure of the National Numeracy Strategy (DfEE, 1999), with its repeated visits to specific learning objectives each half term, reflected the need for children to revisit ideas and consolidate their learning before moving on to the next stage.

Smilansky and Shefatya

Smilansky and Shefatya (1990) define socio-dramatic play as requiring interaction, communication and cooperation, which allow children to test out ideas and concepts, unlike dramatic play, where the child may play alone. Smilansky and Shefatya suggest that enriched learning comes from the adult working alongside children in their play, or 'play tutoring'. Through play, children can assimilate information and prepare for new situations. By selecting different role-play areas, practitioners can give access to different and appropriate areas of learning. This is a common approach in many early years settings, where the practitioner might establish a cafe or shop in the role-play area to provide a context for developing an understanding of specific mathematical concepts, often those involving the use of money.

Bruce (1991) argues that 'free-flow play' is the purest form of play where play is freely chosen by the child and without the confines of external expectation. During this 'pure' play, children will:

- initiate the activity in a meaningful context
- have control and ownership of the activity by imagining, making decisions and predictions
- experiment with strategies and take risks in this 'safe' context
- show curiosity
- repeat, rehearse and refine observed social behaviours and skills
- seek pleasure from the essence of the activity.

All of these processes, integral to play, are also essential for mathematical thinking and problem solving. Play and mathematics, therefore, seem natural partners, and their combination will allow the child to:

- gain an understanding of the cultural role of mathematics
- have a heightened awareness that mathematics can be useful in the real world
- recognize that mathematical activity can be both sociable and cooperative
- perceive mathematical activity to be enjoyable and purposeful.

In order to fully support mathematical development, playful activity requires adult involvement at some level. Early years curricula in the UK (DfE, 2012; SE, 2007; WAG, 2008) advocate that play which best supports learning is that in which there is a mix of child-initiated and adult-supported play. Indeed, a balance of practitioner-led, practitioner-initiated and child-initiated activity is desirable (Fisher, 2010; Pound, 2008). While practitioner-led activity can ensure the systematic teaching of skills, child-initiated learning, without adult control and dominance, can enable children to become self-regulated learners.

Creativity

There is growing evidence that it is *how* and not *what* a child learns that has greatest impact on their school achievement (Bronson, 2000). The revised EYFS emphasizes *how* children learn by the inclusion of the 'characteristics of effective learning', although it is Dame Tickell's *Independent Report on the Early Years Foundation Stage to Her Majesty's Government* (Tickell, 2011) that provides a more robust discussion about their inclusion. The inclusion of these characteristics – play and exploring, active learning and creating and critical thinking – promote key learning dispositions in the young child such as engagement, motivation and thinking, all of which are necessary for self-regulated learning and also vital for real mathematical enquiry. Arguably, creativity is at the heart of all young children's learning and although difficult to define, can be regarded as comprising of four main aspects: imagination, purpose, originality and value (NACCCE, 1999). When young children are being creative, they:

- are captivated and curious
- will be driven by this curiosity to achieve their goal
- make links in their learning in order to make sense of their activity, refine their thinking and thus give rise to new thinking

- evaluate the process they are engaged in so that they might adapt or refine their task in order to be satisfied with their activity.

Creativity can be encouraged in mathematical play by:

- using open-ended questions and ensuring sustained shared thinking (see 'Prompts and questions for problem solving' below)
- providing open-ended resources (see Chapter 2)
- ensuring time and space for children to explore and extend their enquiry
- allowing children to leave out resources such as wooden blocks, model-making materials, etc., so they can revisit them the following day and develop their thinking further
- ensuring that children have the opportunity to review their work and talk about their thinking using appropriate vocabulary (see 'Review time' below)
- encouraging children to make links between maths and other experiences.

Creating links or connections between areas of learning is widely recognized and advocated as a means of embedding learning and fostering creative thinking (DCSF, 2008; DfE, 2012; DfES, 2006; Rose, 2009). Not only do children need to make sense of the connections between mathematics and other learning, but they should be encouraged to make connections between the aspects of mathematics that bind the mathematical activity itself. Haylock and Cockburn (2013) argue that in order to understand mathematical concepts of number and number operations, children must build up a network of connections between four types of mathematical experience: manipulating concrete objects, symbols, language and pictures. This connection making between images, words and symbols at an early stage will often help children avoid misconceptions at a later stage (Gersten et al., 2005). Play situations can provide the very context in which mathematical vocabulary and symbols can be successfully introduced and practised by the children, enabling them to understand this more easily later on.

Principles into practice

Context: Bakery role play

Prompt: There are five buns on the shelf today. Can you make a record for the baker to show how many have been sold and how many are left?

The children will have opportunities to:

- 'sell' the buns (manipulate concrete objects)
- talk about the experience with peers and adults using the language of number and subtraction (use mathematical language)
- move the buns (create an image or picture)
- record responses using their own graphics or standard notation (use symbols and/or pictures).

Problem solving

Contextualizing mathematical learning and providing for real and purposeful mathematical enquiry lies at the heart of this book. Children are born problem-solvers and innately want to find out 'why?' and 'how?' Problem solving for young children is best provided for in the early years by real situations. Practitioners should be reassured that problem solving does not mean reaching for commercial mathematical investigation packs and worksheets, with their abstract symbols and de-contextualized subject matter. Far better is the frequent facilitation of real problem-solving opportunities. When young children begin formal schooling, they often lose interest and confidence in their mathematical abilities. Their experience of mathematics goes from the meaningful to the abstract very quickly and they can find it difficult to bridge the gap between 'real' and 'school' mathematics (Carruthers and Worthington, 2011; Pound, 2008). The early years practitioner, therefore, needs to provide experiences through which children can make connections and see a purpose in solving problems. Such opportunities will present themselves in everyday routines and experiences such as:

- counting the lunch preferences for the day, finding the most and least popular preference and the difference between the two
- finding how many more (or fewer) children are at school today than yesterday
- making sure there are enough pieces of fruit for everyone at snack time
- calculating the number of resources that have arrived in an order.

These experiences give children opportunities to count both in ones from numbers other than 0 and 1, and in given steps, skills crucial in problem solving (see Chapter 4). Practitioners should also take advantage of these real situations to:

- model mathematical language
- show the process of problem solving itself by allowing children to see them struggling with their own thinking
- provide time, space and open-ended resources
- support children's thinking with the use of sensitive prompts and open questions (see below)
- show that there is not always a 'right' answer and encourage 'draft' thinking
- support children to take risks.

Key to supporting children's ability to solve problems is practitioners having strong subject knowledge (DCSF, 2008), along with a stimulating learning environment (see Chapter 2).

Prompts and questions for problem solving

I wonder what would happen if…?

I wonder why…

Tell me more…

Can you explain your thinking?

How do you know this?

Why do you think that?

How did you know what to do next?

What could you try next?

Do you think that this is the only way…?

How does this make sense?

Do you think this will work?

Why/how do you think this will work?

What might you plan to do next?

How have you been reminded about other learning?

Can you talk about the links you have made in your learning?

Has this given you other ideas?

Is there anything we have talked about that has changed how you will do this next time?

What could you have done to improve your learning?

Real situations are ideal but practitioners can create scenarios where children are 'helping' a fictional character by solving problems and thinking mathematically (see Chapters 4–7).

Inclusion

Inclusive practice promotes the teaching of young children in physical and psychological environments where their abilities are encouraged and where there are high expectations of them as learners. Effective and positive relationships with staff, along with appropriate systems for the sharing of relevant information with families, will enable families to feel welcome and parents will be more likely to engage (see Chapter 9). Children will feel included if they can access all that is offered to them, including space, time and the attention of practitioners. Observation is key as it will:

- inform staff about the current interests of the children, thus enabling provision to be made for them
- monitor which children are accessing which learning contexts
- provide vital information to consider when auditing the learning environment
- provide the opportunity to reflect on whether the resources provided truly reflect the home cultures of the children in attendance.

Regular review of the learning environment is important to ensure that it is truly inclusive for all children and families.

Useful questions when considering inclusion

Do the activities and resources reflect all languages and cultures of the current group of children?

Are the mathematical displays/resources/books accessible and appropriate for the languages and cultures of current children and their families?

Are there areas or activities that are dominated by a particular group? If so, how can this be addressed?

Are there opportunities for children to share their mathematical thinking in different ways such as talking in a group, talking one-to-one, using chosen signing or visual prompts?

Do learning contexts such as play trays or the block area need repositioning to enable greater access by wheelchairs or walking aids?

Are there areas in the environment that are quiet?

Are there areas in the environment that are neutral in colour?

Can all of the children view recorded material on the same colour background?

Do the quiet children have opportunities to share their mathematical thinking in pairs or individually?

Do the very able mathematicians feel that their thinking is challenged and encouraged?

Is there a mix of large- and small-handled tools in the malleable material and painting areas, allowing appropriate access for all children?

Transition

Increased pressure on Year 1 teachers to deliver content and focus on *what* is learnt has undoubtedly contributed to the difficulties often encountered by young children moving from Reception (aged 5) to Y1 (5–6 years) (Ofsted, 2004). Transition should be regarded as a process, to be handled sensitively and not to be regarded as something which is 'done' to the child. Both the *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools* (DCSF, 2008) and the *Independent Review of the Primary Curriculum*: *Final Report* (Rose, 2009) suggest that Year 1 teachers should consult the Early Years Foundation Stage Profile passed on to them by the Foundation teachers to inform them about the whole child as well as mathematical attainment. Comments made about their learning dispositions with regard to the characteristics of effective learning will also help teachers to understand and support the child as a self-regulated learner (Stewart, 2011).

Principles into practice

- Continue to prepare a stimulating and inviting environment, both indoors and outdoors. If there is no immediate access outside, ensure there are sessions outside where children play creatively.
- Be mindful of a balance between practitioner-led, practitioner-initiated and child-initiated activity. Try to ensure that children have opportunities for all three over the course of a week.
- Avoid leaving play for Friday afternoons; ensure that there are opportunities during the week and that it becomes embedded in weekly practice.
- Give a high profile to play by ensuring that independent learning time is reviewed and evaluated by the children.
- Provide open-ended resources that lend themselves to exploration and natural problem solving.
- Ensure time and space for high quality play.
- Avoid abrupt changes in routines and pedagogy to enable children to feel secure.
- Allow children to get to know their new environment through regular visits prior to them beginning in Y1.

Implications for an effective early years mathematics pedagogy

As children have a natural propensity to play, it follows that effective provision should capitalize on play as a preferred learning style. Good practice would suggest that practitioners should both harness play as an appropriate learning style and ensure that the children they teach make agreed progress in their mathematical understanding. In order for children to make progress in mathematics, they need to be involved in systematic, focused teaching in small groups where adults are actively teaching (Siraj-Blatchford et al., 2002). This ensures frequent, repeated and appropriate experiences, made all the more effective if the teachers have good subject knowledge and can plan for progression (DCSF, 2008; Gifford, 2008).

Principles into practice

- Ensure a sensitive balance between focused teaching and practitioner-initiated play, practitioner-initiated–child-continued play and child-initiated play (see Glossary).
- Provide opportunities for children to evaluate their activity and discuss their thinking, allowing them to develop their self-efficacy and act as models for self-regulated learning.
- Provide group-focused teaching.
- Plan motivating mathematical activity within playful and meaningful contexts where children can rehearse and refine a range of skills.
- Support children to make links in their learning.
- Ensure that practitioners have strong mathematics subject knowledge and that they can plan for progression.

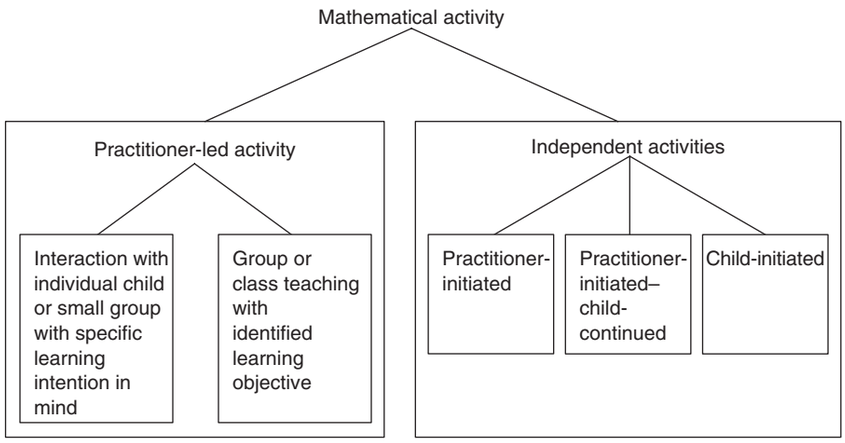

Figure 1.1 Mathematical activity

While the youngest children will be taught maths in integrated sessions, older children in Key Stages 1 and 2 (7–11-year-olds) are generally taught maths in a maths 'lesson'. By incorporating practitioner-led activity (direct teaching) in playful and meaningful contexts, with practitioner-initiated play and child-initiated play into a 'mathematics session', the session can provide a continuum *between* 'work' and 'play' in the following ways:

- Ensure mathematics occurs in a meaningful and stimulating context, such as linking it to the current theme or interests of the children. If the children are enjoying *Handa's Surprise* (see Resources) in their literacy curriculum, then they can 'help' Handa to solve mathematical problems involving fruit and animals (see Chapter 4).
- Link all learning contexts – play trays, role play, small worlds, etc. – by a common theme through which different areas of the curriculum can be accessed which will encourage connection making between experiences (see Chapter 2).
- Include a review time for children to evaluate both their directed and self-initiated activity.

Figure 1.1 summarizes playful mathematical activity, while Figure 1.2 suggests a structure for a playful teaching session for single or mixed-age classes from Reception (4–5-year-olds) to Year 3 (7–8-year-olds).

Review time gives opportunities for the *practitioner* to:

- raise the profile of independent activities
- assess levels of learning and engagement with the characteristics of effective learning
- model language for learning and problem solving
- assist the child to show that there is more than one way to tackle a problem
- give precise feedback about the child's learning processes and thinking.

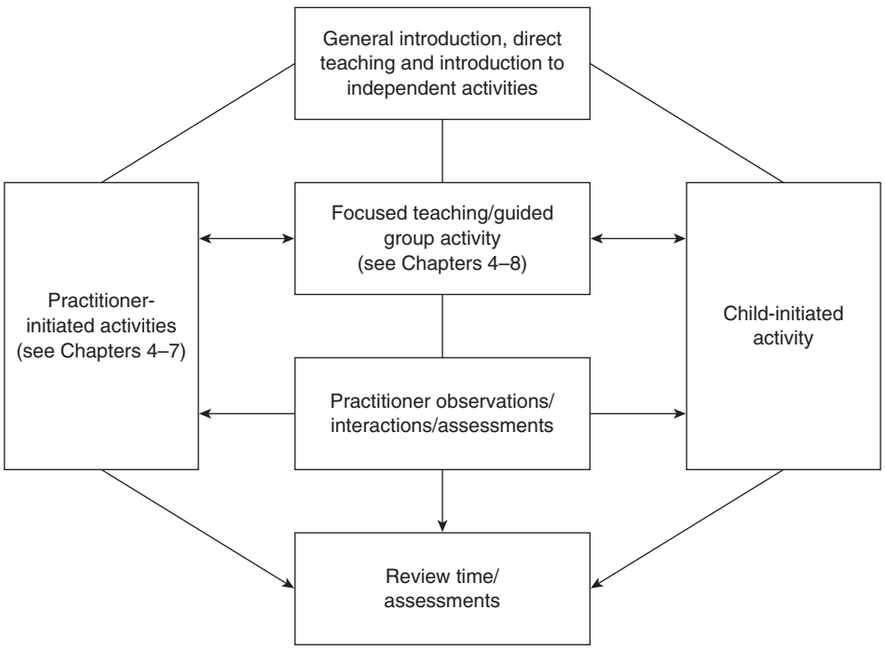

Figure 1.2 Model of an early years mathematics teaching session

Review time gives opportunities for the *child* to:

- practise using language for learning
- present their critical thinking to others
- make connections between their mathematical learning and other experiences and vocalize these to others
- evaluate their thinking and processes of working
- rethink their processes in the light of practitioners' feedback
- perceive themselves as an independent learner and mathematician
- develop their self-efficacy.

Children's language for learning

- I made a mistake, but I corrected my thinking…
- I managed my distractions so I could…
- I made links in my learning…
- I worked well on my own/with others…
- I solved the problem by…
- The questions I asked were…
- I predicted that … would happen and I found that…

Play, then, is endorsed as an appropriate context for mathematics teaching beyond Foundation Stage; the practice of teachers throughout the primary age range might be greatly informed by a study of early years mathematics pedagogy (Alexander, 2009; DCSF, 2008; Rose, 2009). The ensuing chapters provide examples of teaching mathematics by harnessing children's propensity for play and problem solving through independent play and playful contexts that are characteristic of effective early years pedagogy.

Glossary

Child-initiated: children select their own resources, plan their own activity and pursue their own learning agenda.

EYFS: Early Years Foundation Stage.

KS: Key Stage.

Practitioner-initiated: children follow a task or idea suggested by the practitioner who has a specific learning outcome in mind.

Practitioner-initiated–child-continued: children extend or develop a practitioner-initiated activity to pursue their own learning.

Further reading

Department of Children, Schools and Families (DCSF) (2008) *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools*. London: DCSF. Useful to read first-hand to identify how play is endorsed.

Fisher, J. (2010) *Moving on to Key Stage 1: Improving Transition from the Early Years Foundation Stage.* Maidenhead: OUP. Clear and accessible, this book describes strategies to support transition from FS to KS1.

Gifford, S. (2008) '"How do you teach nursery children mathematics?" In search of a mathematics pedagogy for the early years', in I. Thompson (ed.) *Teaching and Learning Early Number*, 2nd edition. Maidenhead: OUP. This chapter describes features of an early years mathematics pedagogy for nursery children.

Pound, L. (2008) *Thinking and Learning about Mathematics in the Early Years*. London: Routledge. This book provides the theory underpinning the benefits of a playful and creative approach to early years mathematics

Stewart, N. (2011) *How Children Learn: The Characteristics of Effective Early Learning*. London: The British Association for Early Childhood Education. This very readable book outlines the theory and practice supporting the characteristics of effective learning.