PLL EXERCISE

## PLL EXERCISE

## No. 1 FM demodulator Analysis


A) Draw a block diagram of the overall FM demodulator and derive its TF $\Delta V_{o}(S) / \Delta \omega_{\text {in }}(s)$
B) Determine $K_{d}, K_{v}, F_{n 1}, \zeta_{1}$ and $F_{n 2}$ of the PLL demodulator and then sketch its gain response - label with relevant parameters. Explain why the demodulator has a Butterworth response.
C) If $\mathrm{F}_{\mathrm{car}}=30 \mathrm{kHz}, \Delta \mathrm{F}=2 \mathrm{kHz}$ and $\mathrm{F}_{\text {mod }}=500 \mathrm{~Hz}$, sinewave modulation, what is the minimum, maximum and average phase error and the demodulated voltage? Use pages 10 or 11 of theory notes to read peak phase error.
D) Repeat for $F_{\text {mod }}=5 \mathrm{kHz}$.
E) Derive the general TF for $\Delta \Phi_{e}(S) / \Delta \omega_{\text {in }}(S)$ and then calculate the exact values of the phase error for steps C and D.
F) For what modulation frequency is the phase error maximum?
G) Explain the function of the last stage.
H) What is amplitude of the demodulated O/P signal, the HF ripple at the final output and the \% ripple if the FM input has $\mathrm{F}_{\text {mod }}=500 \mathrm{~Hz}, \Delta \mathrm{~F}=2 \mathrm{kHz}$ at $\mathrm{F}_{\text {car }}=30 \mathrm{kHz}$.
I) Simulate the demodulator gain response and the phase error response to an FM input with sinewave modulation.
J) Simulate for determination of lock range and capture range.
K) Simulate for determination of output HF ripple.

## No. 2 FM demodulator Design



Design a 3rd order FM demodulator that has a low-pass Bessel response, given the following input signal.
FM input: $\quad \mathrm{F}_{\mathrm{car}}=100 \mathrm{kHz} \Delta \mathrm{F}=0$ to $10 \mathrm{kHz} \quad$ Modulation: sinewave, $\mathrm{F}_{\mathrm{mod}}=0$ to 3 kHz Final output: 0 to $5 \mathrm{~V}_{\mathrm{p}}$

Select $F_{n}$ of PLL for a reasonable phase error and large enough to pass the demod signal.

## N0.3 FSK demodulator Design



Design a 4th order FSK demodulator that has a Bessel low-pass response with a bandwidth of 15 kHz .
The two input frequencies are 100 kHz and 110 kHz .
What is the maximum bit rate that can be demodulated?

## No. 4 PLL Frequency Synthesiser Analysis


A) Determine the two possible sets of output frequencies.
B) How do we ensure operation of the synthesiser on one set of frequencies only?
C) Why is there spurious FM sidetones in the spectrum of the output signal? Is this good or bad? How can they be minimized?
D) Are $\zeta$ and $\omega_{n}$ constant in this system? Explain.

## No. 5 RF PLL Synthesizer Design and Analysis

Re-do the example on pages 33 to 40 with the following data:
Using the same frequencies, re-design for a phase margin of $55^{\circ}$, an $\omega_{p}$ value of $2 \pi^{*} 10 \mathrm{kHz}$ and additional spurious sideband attenuation of 20 dB from the $\mathrm{R}_{3} \mathrm{C}_{3}$ section added to the loop filter.

Modify the SystemView file given for the theory example to simulate your new design.

## SOLUTIONS

No.1A)


$$
\begin{aligned}
& \frac{V_{o}(S)}{\Phi_{i n}(S)}=\frac{T_{1} N_{1}}{1-L(S)}=\frac{K_{d} \times \frac{\frac{1}{S C_{2}}}{R_{1}+R_{2}+\frac{1}{S C_{2}}} \times\left(\frac{R_{F}}{R_{E}} \times \frac{1}{1+S C_{F} R_{F}}\right)}{1-\left[-K_{d} \times \frac{R_{2}+\frac{1}{S C_{2}}}{R_{1}+R_{2}+\frac{1}{S C_{2}}} \times \frac{K_{V}}{S}\right]} \\
& \frac{V_{o}(S)}{\Phi_{\text {in }}(s)}=\frac{T_{1} N_{1}}{1-L_{(S)}}=\frac{K_{d} \times \frac{1}{1+S C_{2}\left(R_{1}+R_{2}\right)} \times\left(\frac{R_{F}}{R_{E}} \times \frac{1}{1+\left[K_{d} \times K_{V} \times \frac{1+S C_{F} R_{F}}{1+S C_{2}\left(R_{1}+R_{2}\right)}\right)} \times \frac{1}{S}\right]}{\frac{V_{o}(s)}{C_{2}\left(R_{1}+R_{2}\right)}}=\frac{K_{d}}{S} \times\left(\frac{R_{F}}{R_{E}} \times \frac{1}{1+S C_{F} R_{F}}\right) \\
& \left(S^{2}+S\left(\frac{1+K_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}\right) \\
& \frac{\Delta V_{o}(s)}{\Delta \omega_{i n}(S)}=\frac{K_{d}}{\left(S_{2}^{2}+S\left(\frac{1+R_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}\right)} \times\left(\frac{R_{F}}{R_{E}} \times \frac{1}{1+S C_{F} R_{F}}\right)
\end{aligned}
$$

B) $\quad \mathrm{K}_{\mathrm{d}}=0.68 \mathrm{~V} / \mathrm{r}$ if $\mathrm{V}_{\text {in }}>100 \mathrm{mV} \mathrm{V}_{\mathrm{pp}}$, that is if LM 565 phase detector $\mathrm{O} / \mathrm{P}$ is saturated - see data sheets.
$\mathrm{K}_{\mathrm{v}}=49.4 \mathrm{~F}_{\text {cen }} / \Delta \mathrm{V}_{\mathrm{cc}}=49.4 * 30 \mathrm{~K} / 18=82333 \mathrm{r} / \mathrm{s}$ or $13.1 \mathrm{kHz} / \mathrm{V}$.

$$
\begin{aligned}
& F_{n 1}=\frac{1}{2 \pi} \sqrt{\frac{K_{v} K_{d}}{C_{2}\left(R_{1}+R_{2}\right)}}=\frac{1}{2 \pi} \sqrt{\frac{82333 * 0.68}{12 n *(3.6 k+1.2 k)}}=4962 \mathrm{~Hz} \quad \zeta_{1}=\frac{1+K_{d} K C_{2} R_{2}}{2 \omega_{n 1} C_{2}\left(R_{1}+R_{2}\right)}=0.5029 \\
& F_{n 2}=\left(2 \pi R C_{F}\right)^{-1}=4977 \mathrm{~Hz}
\end{aligned}
$$

$\mathrm{F}_{\mathrm{n} 1}$ and $\mathrm{F}_{\mathrm{n} 2}$ are almost equal and $\zeta_{1} \approx 0.5$ which corresponds to a third order Butterworth response whose normalised poles are $\bar{S}_{1}=\underline{1 / 180^{\circ}}$ and $\bar{S}_{2,3}=\underline{1 / \pm 120^{\circ}} \cdot \frac{\Delta V_{o}(s)}{\Delta \omega_{i n}(S)}=\frac{1}{K_{V}} \times \frac{R_{F}}{R_{E}}=231.6 \mu$

C) $\Delta \mathrm{F}=2 \mathrm{kHz}, \mathrm{F}_{\text {mod }}=500 \mathrm{~Hz}, \mathrm{~F}_{\text {car }}=30 \mathrm{kHz}$
$\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=2 \pi \times 4962(12 n \times(3.6 k+1.2 k))=1.8 \quad$ and $\quad \frac{F_{\mathrm{mod}}}{F_{n 1}}=\frac{500}{4962}=0.10076$
For all phase error graphs, the LF frequency value of $\frac{\Delta \Phi_{e}}{\left(\Delta F_{\text {in }} / F_{n 1}\right)} \approx \frac{1}{\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)}=\frac{1}{1.8}$, therefore we have $\Delta \Phi_{e}($ peak $) \approx \frac{\left(\Delta F_{\text {in }} / F_{n 1}\right)}{1.8}=\frac{2000 / 4962}{1.8}=0.224 r_{p}$ Average phase error is $\pi / 2$ if $\mathrm{F}_{\text {cen }}=\mathrm{F}_{\text {car }}$ for LM565

D) $\quad \Delta \mathrm{F}=2 \mathrm{kHz}, \mathrm{F}_{\text {mod }}=5 \mathrm{kHz}, \mathrm{F}_{\text {car }}=30 \mathrm{kHz}$
$\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=2 \pi \times 4962(12 n \times(3.6 k+1.2 k))=1.8 \quad$ and $\quad \frac{F_{\mathrm{mod}}}{F_{n 1}}=\frac{5000}{4962}=1.0077$
At $\frac{F_{\text {mod }}}{F_{n 1}}=1$, we read on the phase error graphs (pp 10-11)
$\frac{\Delta \Phi_{e}(\text { peak })}{\left(\Delta F_{\text {in }} / F_{n 1}\right)} \approx 1.5 \quad$ for $\quad \omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=1$
$\frac{\Delta \Phi_{e}(\text { peak })}{\left(\Delta F_{\text {in }} / F_{n 1}\right)} \approx 1.0 \quad$ for $\quad \omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=5$
Using linear interpolation, we have
$\Delta \Phi_{e}($ peak $) \approx \frac{\left(\Delta F_{\text {in }} / F_{n 1}\right)}{1.4}=\frac{2000 / 4962}{1.4}=0.288 r_{p}$
Average phase error is $\pi / 2$ if $F_{\text {cen }}=F_{\text {car }}$ for LM565


E)


$$
\begin{aligned}
& \frac{\Phi_{e}(S)}{\Phi_{i n}(S)}=\frac{T_{1} N_{1}}{1-L(S)}=\frac{1 \times 1}{1-\left[-K_{d} \times \frac{R_{2}+\frac{1}{S C_{2}}}{R_{1}+R_{2}+\frac{1}{S C_{2}}} \times \frac{K_{V}}{S}\right]}=\frac{1}{1+\left[K_{d} \times K_{V} \times \frac{1+S C_{2} R_{2}}{1+S C_{2}\left(R_{1}+R_{2}\right)} \times \frac{1}{S}\right]} \\
& \frac{\Phi_{e}(S)}{\frac{\omega_{i n}(S)}{S}}=\frac{\frac{S\left(1+S C_{2}\left(R_{1}+R_{2}\right)\right)}{C_{2}\left(R_{1}+R_{2}\right)}}{\left(S^{2}+S\left(\frac{1+K_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}\right)}
\end{aligned} \frac{\Delta \Phi_{e}(S)}{\Delta \omega_{i n}(S)}=\frac{\left(S+\frac{1}{C_{2}\left(R_{1}+R_{2}\right)}\right)}{\left(S^{2}+S\left(\frac{1+K_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}\right)}
$$

$$
\Delta \Phi_{e}(S)=\Delta \omega_{i n}(S) \times \frac{\left(S+\frac{1}{C_{2}\left(R_{1}+R_{2}\right)}\right)}{\left(S^{2}+S\left(\frac{1+K_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}\right)}
$$

$$
\left.\Delta \Phi_{e}\left(j \omega_{m}\right)=\Delta \omega_{i n}\left(j \omega_{m}\right) \times \frac{\left(j \omega_{m}+\frac{1}{C_{2}\left(R_{1}+R_{2}\right)}\right)}{\left(\left(j \omega_{m}\right)^{2}+\frac{K_{d} K_{V}}{C_{2}\left(R_{1}+R_{2}\right)}+j \omega_{m}\left(\frac{1+K_{d} K_{V} R_{2} C_{2}}{C_{2}\left(R_{1}+R_{2}\right)}\right)\right.}\right)
$$

$$
\Delta \Phi_{e}\left(j \omega_{m}\right)=\Delta \omega_{i n}\left(j \omega_{m}\right) \times \frac{\left(j \omega_{m}+17361\right)}{\left(\left(j \omega_{m}\right)^{2}+972 \times 10^{6}+j \omega_{m}(31357.8)\right)}
$$

$$
\Delta \Phi_{e}(\text { peak })=\Delta \omega_{i n}(\text { peak }) \times \frac{\left(j \omega_{m}+17361\right)}{\left(\left(972 \times 10^{6}-\omega_{m}^{2}\right)+j \omega_{m} 31357.8\right)}
$$

For $\mathrm{F}_{\mathrm{m}}=500 \mathrm{~Hz}$ we have

$$
\Delta \Phi_{e}(\text { peak })=2 \pi \times 2000 \times \frac{(j 1000 \pi+17361)}{\left(\left(972 \times 10^{6}-(1000 \pi)^{2}\right)+j 1000 \pi \times 31357.8\right)}=0,2258 r \underline{/ 0.179 r}
$$

Compared to $0.224 r_{p}$ found in step C. The phase angle of the phase error signal is the phase shift between the modulation signal and the phase error signal.

For $\mathrm{F}_{\mathrm{m}}=5 \mathrm{kHz}$ we have
$\Delta \Phi_{e}($ peak $)=2 \pi \times 2000 \times \frac{(j 10000 \pi+17361)}{\left(\left(972 \times 10^{6}-(10000 \pi)^{2}\right)+j 10000 \pi \times 31357.8\right)}=0,2302 r \underline{1.066 r}$
Compared to $0.288 r_{p}$ found in step $D-I$ guess the graph was not that accurate.
F) For $\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)$ values of 5 and over the maximum phase error occurs at $\mathrm{F}_{\mathrm{m}}=\mathrm{F}_{\mathrm{n} 1}=4962 \mathrm{~Hz}$.

But for $\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=1$ the peak has shifted to about $0,8 \mathrm{~F}_{\mathrm{n} 1}$, so my estimate is that for $\omega_{n 1}\left(\tau_{1}+\tau_{2}\right)=1.8$ the maximum phase error will occur at approximately $0.9 \mathrm{~F}_{\mathrm{n} 1}=4466 \mathrm{~Hz}$
G) All yours to answer.
H) Ripple calculation (see chart page 19 of theory section)

From phase detector $\mathrm{O} / \mathrm{P}$ to final $\mathrm{O} / \mathrm{P}$, we have

$\frac{V_{o}(S)}{V_{d}(S)}=\frac{\frac{1}{S C_{2}}}{R_{1}+R_{2}+\frac{1}{S C_{2}}} \times \frac{R_{F} \| \frac{1}{S C_{F}}}{R_{E}} \approx \frac{1}{S C_{2}\left(R_{1}+R_{2}\right)} \times \frac{1}{S C_{F} R_{E}}=\frac{A_{1}}{S} \times \frac{A_{2}}{S}$
$\Delta V_{o}(p p)=\Delta V_{o}(p p) \times \frac{A_{1} A_{2}}{4 \times 8 \times F_{r i p}^{2}}=2.16 \times \frac{1}{\left(4300 \times 390 \times 10^{-12}\right) \times\left(12 \times 10^{-9} \times 4800\right) \times 4 \times 8 \times 60 K^{2}}=194.1 \mathrm{~m} V_{P P}$
O/P signal level $\quad V_{o}($ peak $)=\frac{\Delta \omega_{i n}}{K_{V}} \times \frac{R_{F}}{R_{E}}=\frac{2 \pi \times 2000}{82333} \times \frac{82 k}{4.3 k}=2.91 V_{p}$
$\%$ ripple $=194.1 \mathrm{~m} /(2.91 * 2) * 100=3.33 \%$



From the previous page, we obtained the following theoretical values:
$\mathrm{V}_{\mathrm{o}}=5.82 \mathrm{~V}_{\mathrm{pp}}$ and ripple of $191.4 \mathrm{mV}_{\mathrm{pp}}$
The ripple component is off because the duty cycle was assumed to be $50 \%$ at an average frequency of $\mathrm{F}_{\text {rip }}=2 \mathrm{~F}_{\text {car }}$ at the phase detector O/P. At $F_{\text {min }}$ and $F_{\text {max }}$ of the FM signal, the duty cycle is above and below $50 \%$ and $\mathrm{F}_{\text {rip }}=2 \mathrm{~F}_{\text {min }}$ and $2 \mathrm{~F}_{\text {max }}$ respectively.

Read ripple at top and bottom of demod O/P and average the two values which should come close to the predicted value for $50 \%$ duty cycle.


From the S/V simulation, the lock range is:
$F_{\text {max }}=\frac{274.3 \mathrm{k}}{2 \pi}=43.656 \mathrm{kHz}$
$F_{c e n}=30 \mathrm{kHz}$
$\Delta F_{L}=F_{\text {max }}-F_{\text {cen }}=F_{\text {cen }}-F_{\text {min }}$
$\Delta F_{L}=43.656 k-30 k=13.66 \mathrm{kHz}$
Theoretically we have:
$\Delta F_{\text {LOCK }} \approx \pm \frac{8 \times F_{c e n}}{\Delta V_{C C}}$
$\Delta F_{\text {LOCK }}= \pm \frac{8 \times 30 \mathrm{k}}{18}=13.33 \mathrm{kHz}$
NOTE: The simulation shown beside is not to scale - run sim file for a better look at results.

The above simulation was done starting at $\mathrm{F}_{\text {in }}=30 \mathrm{kHz}$ and then ramping up $\mathrm{F}_{\text {in }}$ at a rate of $100 \mathrm{kHz} / \mathrm{sec}$. If the ramp rate is too fast then the lock rage will be less - lock range is defined for a slow variation of $F_{i n}$. You can find $F_{\text {min }}$ by starting at 30 kHz again but now ramping down $\mathrm{F}_{\text {in }}$ at $-100 \mathrm{kHz} / \mathrm{sec}$ (Try it!)


Nos.2, 3 and 5 you are on your own - see theory examples for help.
No. 4 A) $\quad \mathrm{F}_{\mathrm{o}}=86 \mathrm{MHz}$ to 108 MHz or $\mathrm{F}_{\mathrm{o}}=152 \mathrm{MHz}$ to 174 MHz with steps of $\Delta \mathrm{F}_{\mathrm{o}}=200 \mathrm{kHz}$ $B, C$ and $D$ are yours.

Rev. 10/18/2002 Phase-Locked Loops Page CC-10

