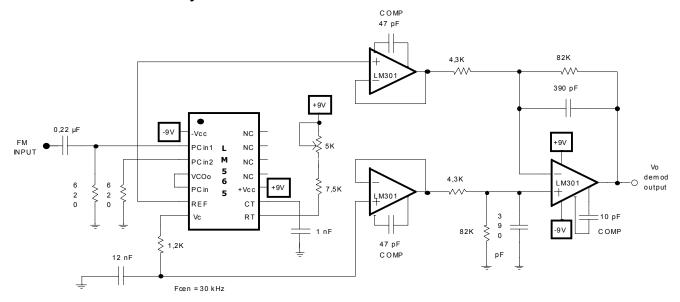
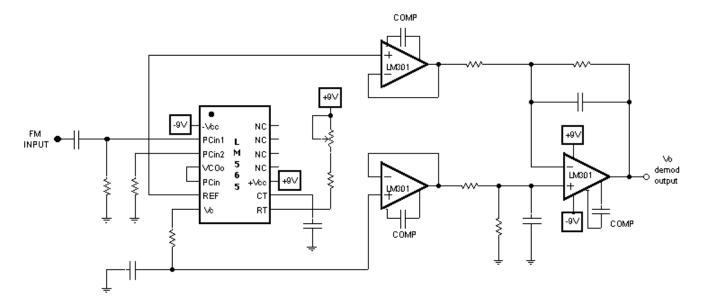
# PLL EXERCISE

#### No.1 FM demodulator Analysis



- A) Draw a block diagram of the overall FM demodulator and derive its TF  $\Delta V_{o}(s)/\Delta \omega_{in}(s)$
- B) Determine  $K_d$ ,  $K_v$ ,  $F_{n1}$ ,  $\zeta_1$  and  $F_{n2}$  of the PLL demodulator and then sketch its gain response label with relevant parameters. Explain why the demodulator has a Butterworth response.
- C) If  $F_{car} = 30$  kHz,  $\Delta F = 2$  kHz and  $F_{mod} = 500$  Hz, sinewave modulation, what is the minimum, maximum and average phase error and the demodulated voltage? Use pages 10 or 11 of theory notes to read peak phase error.
- D) Repeat for  $F_{mod} = 5 \text{ kHz}$ .
- E) Derive the general TF for  $\Delta\Phi_e(s)/\Delta\omega_{in}(s)$  and then calculate the exact values of the phase error for steps C and D.
- F) For what modulation frequency is the phase error maximum?
- G) Explain the function of the last stage.
- H) What is amplitude of the demodulated O/P signal, the HF ripple at the final output and the % ripple if the FM input has  $F_{mod}$  = 500 Hz,  $\Delta F$  = 2 kHz at  $F_{car}$  = 30 kHz.
- I) Simulate the demodulator gain response and the phase error response to an FM input with sinewave modulation.
- J) Simulate for determination of lock range and capture range.
- K) Simulate for determination of output HF ripple.

### No.2 FM demodulator Design



Design a 3rd order FM demodulator that has a low-pass Bessel response, given the following input signal.

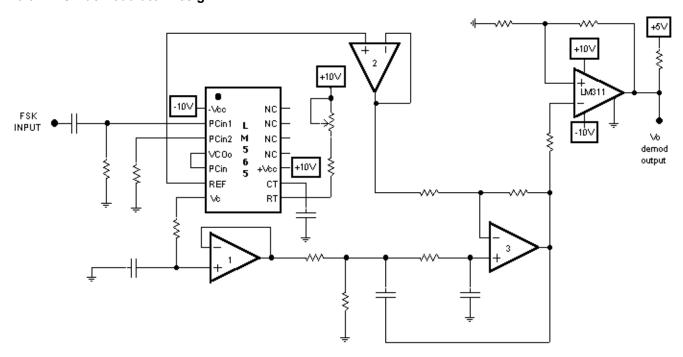
FM input:  $F_{car} = 100 \text{ kHz } \Delta F = 0 \text{ to } 10 \text{ kHz}$ 

Modulation: sinewave,  $F_{mod} = 0$  to 3 kHz

Final output: 0 to 5V<sub>p</sub>

Select F<sub>n</sub> of PLL for a reasonable phase error and large enough to pass the demod signal.

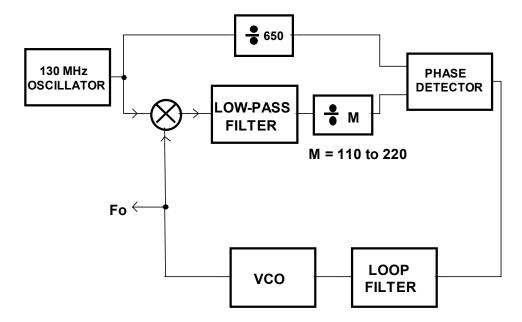
### N0.3 FSK demodulator Design



Design a 4th order FSK demodulator that has a Bessel low-pass response with a bandwidth of 15 kHz. The two input frequencies are 100 kHz and 110 kHz.

What is the maximum bit rate that can be demodulated?

### No.4 PLL Frequency Synthesiser Analysis



- A) Determine the two possible sets of output frequencies.
- B) How do we ensure operation of the synthesiser on one set of frequencies only?
- C) Why is there spurious FM sidetones in the spectrum of the output signal? Is this good or bad? How can they be minimized?
- D) Are  $\zeta$  and  $\omega_0$  constant in this system? Explain.

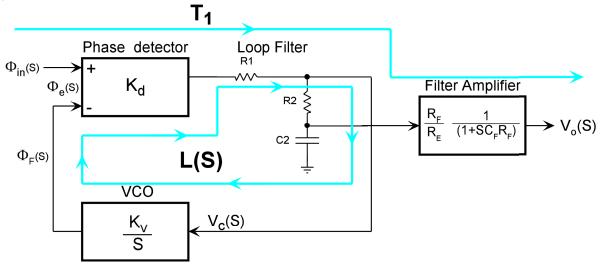
#### No.5 RF PLL Synthesizer Design and Analysis

Re-do the example on pages 33 to 40 with the following data:

Using the same frequencies, re-design for a phase margin of  $55^{\circ}$ , an  $\omega_p$  value of  $2\pi^*10$  kHz and additional spurious sideband attenuation of 20 dB from the  $R_3C_3$  section added to the loop filter.

Modify the SystemView file given for the theory example to simulate your new design.

No.1A)



$$\frac{V_o(s)}{\Phi_{in}(s)} = \frac{T_1 N_1}{1 - L(s)} = \frac{K_d \times \frac{1}{SC_2}}{1 - \left[ -K_d \times \frac{R_F}{R_1 + R_2} + \frac{1}{SC_2} \times \left( \frac{R_F}{R_E} \times \frac{1}{1 + SC_F R_F} \right) \right]}{1 - \left[ -K_d \times \frac{R_2 + \frac{1}{SC_2}}{R_1 + R_2 + \frac{1}{SC_2}} \times \frac{K_V}{S} \right]}$$

$$\frac{V_o(s)}{\Phi_{in}(s)} = \frac{T_1 N_1}{1 - L(s)} = \frac{K_d \times \frac{1}{1 + SC_2(R_1 + R_2)} \times \left( \frac{R_F}{R_E} \times \frac{1}{1 + SC_F R_F} \right)}{1 + \left[ K_d \times K_V \times \frac{1 + SC_2 R_2}{1 + SC_2(R_1 + R_2)} \times \frac{1}{S} \right]}$$

$$\frac{V_o(s)}{\frac{\Theta_{om}(s)}{S}} = \frac{\frac{S K_d}{C_2(R_1 + R_2)}}{\left[ S^2 + S \left( \frac{1 + K_d K_V R_2 C_2}{C_2(R_1 + R_2)} \right) + \frac{K_d K_V}{C_2(R_1 + R_2)} \right]}{\left( \frac{\Delta V_o(s)}{C_2(R_1 + R_2)} \right]} \times \left( \frac{R_F}{R_E} \times \frac{1}{1 + SC_F R_F} \right)$$

$$\frac{\Delta V_o(s)}{\Delta \omega_{in}(S)} = \frac{\frac{K_d}{C_2(R_1 + R_2)}}{\left( S^2 + S \left( \frac{1 + K_d K_V R_2 C_2}{C_2(R_1 + R_2)} \right) + \frac{K_d K_V}{C_2(R_1 + R_2)} \right)}{\left( S^2 + S \left( \frac{1 + K_d K_V R_2 C_2}{C_2(R_1 + R_2)} \right) + \frac{K_d K_V}{C_2(R_1 + R_2)} \right)}$$

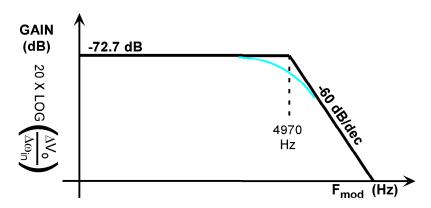
B)  $K_d = 0.68 \text{ V/r}$  if  $V_{in} > 100 \text{ mV}_{pp}$ , that is if LM565 phase detector O/P is saturated - see data sheets.

 $K_v = 49.4 F_{cen}/\Delta V_{cc} = 49.4*30 K/18 = 82333 r/s or 13.1 kHz/V.$ 

$$F_{n1} = \frac{1}{2\pi} \sqrt{\frac{K_{\nu}K_{d}}{C_{2}(R_{1} + R_{2})}} = \frac{1}{2\pi} \sqrt{\frac{82333 * 0.68}{12n * (3.6k + 1.2k)}} = 4962Hz \qquad \zeta_{1} = \frac{1 + K_{d}KC_{2}R_{2}}{2\omega_{n1}C_{2}(R_{1} + R_{2})} = 0.5029$$

$$F_{n2} = (2\pi R \ C_{F})^{-1} = 4977 \ Hz$$

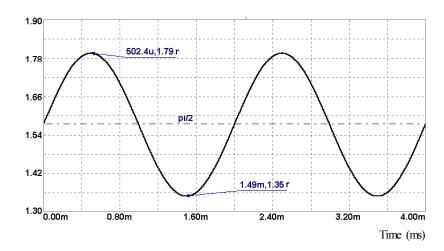
 $F_{n1}$  and  $F_{n2}$  are almost equal and  $\zeta_1 \approx 0.5$  which corresponds to a third order Butterworth response whose normalised poles are  $\overline{S}_1 = 1 \underline{/180^o}$  and  $\overline{S}_{2,3} = 1 \underline{/\pm 120^o}$ .  $\underline{\Delta V_o(s)}_o(S) = \frac{1}{K_V} \times \frac{R_F}{R_E} = 231.6 \mu$ 



C) 
$$\Delta F = 2 \text{ kHz}$$
,  $F_{\text{mod}} = 500 \text{ Hz}$ ,  $F_{\text{car}} = 30 \text{ kHz}$   $\omega_{n1}(\tau_1 + \tau_2) = 2\pi \times 4962 \left(12n \times \left(3.6k + 1.2k\right)\right) = 1.8$  and  $\frac{F_{\text{mod}}}{F_{n1}} = \frac{500}{4962} = 0.10076$ 

For all phase error graphs, the LF frequency value of  $\frac{\Delta\Phi_e}{\left(\Delta F_{in}/F_{n1}\right)} \approx \frac{1}{\omega_{n1}(\tau_1+\tau_2)} = \frac{1}{1.8}$ , therefore we have

 $\Delta\Phi_e(peak) \approx \frac{\left(\Delta F_{in}/F_{n1}\right)}{1.8} = \frac{2000/4962}{1.8} = 0.224 \; r_p \; \text{ Average phase error is } \pi/2 \; \text{if } \mathsf{F}_{\mathsf{cen}} = \mathsf{F}_{\mathsf{car}} \; \mathsf{for \; LM565}$ 



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D) 
$$\Delta F = 2 \text{ kHz}, F_{\text{mod}} = 5 \text{ kHz}, F_{\text{car}} = 30 \text{ kHz}$$

D) 
$$\Delta F = 2 \text{ kHz}, F_{\text{mod}} = 5 \text{ kHz}, F_{\text{car}} = 30 \text{ kHz}$$

$$\omega_{n1}(\tau_1 + \tau_2) = 2\pi \times 4962 \left(12n \times (3.6k + 1.2k)\right) = 1.8 \quad and \quad \frac{F_{\text{mod}}}{F_{n1}} = \frac{5000}{4962} = 1.0077$$

At  $\frac{F_{\rm mod}}{F}$  = 1, we read on the phase error graphs (pp 10-11)

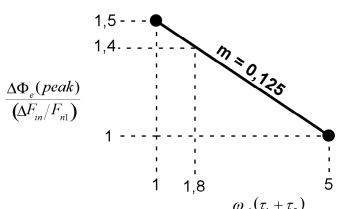
$$\frac{\Delta \Phi_e(peak)}{(\Delta F_{in}/F_{n1})} \approx 1.5 \quad for \quad \omega_{n1}(\tau_1 + \tau_2) = 1$$

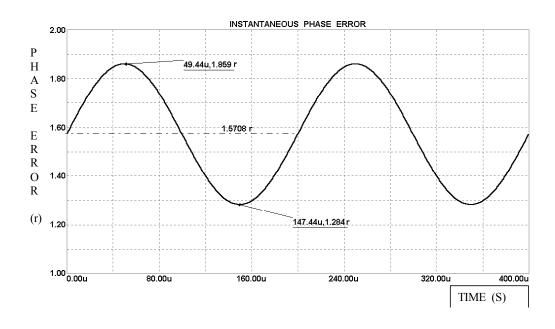
$$\frac{\Delta \Phi_e(peak)}{\left(\Delta F_{in}/F_{n1}\right)} \approx 1.0 \quad for \quad \omega_{n1}(\tau_1 + \tau_2) = 5$$

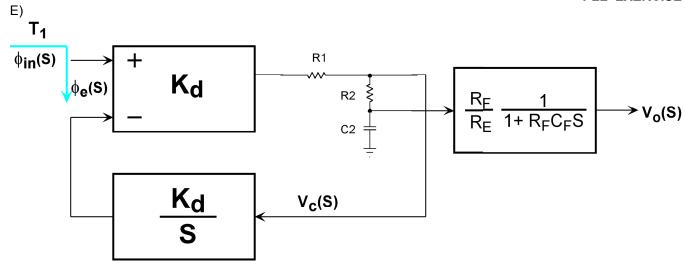
Using linear interpolation, we have

$$\Delta \Phi_e(peak) \approx \frac{\left(\Delta F_{in}/F_{n1}\right)}{1.4} = \frac{2000/4962}{1.4} = 0.288 \, r_p$$

Average phase error is  $\pi/2$  if  $F_{cen} = F_{car}$  for LM565







$$\frac{\Phi_{e}(S)}{\Phi_{in}(S)} = \frac{T_1 N_1}{1 - L(S)} = \frac{1 \times 1}{1 - \left[ -K_d \times \frac{R_2 + \frac{1}{SC_2}}{R_1 + R_2 + \frac{1}{SC_2}} \times \frac{K_V}{S} \right]} = \frac{1}{1 + \left[ K_d \times K_V \times \frac{1 + SC_2 R_2}{1 + SC_2 (R_1 + R_2)} \times \frac{1}{S} \right]}$$

$$\frac{\Phi_{e^{(S)}}}{\frac{\Theta_{o^{(S)}}}{S}} = \frac{\frac{S\left(1 + SC_{2}(R_{1} + R_{2})\right)}{C_{2}(R_{1} + R_{2})}}{\left(S^{2} + S\left(\frac{1 + K_{d}K_{V}R_{2}C_{2}}{C_{2}(R_{1} + R_{2})}\right) + \frac{K_{d}K_{V}}{C_{2}(R_{1} + R_{2})}\right)} \qquad \frac{\Delta\Phi_{e^{(S)}}}{\Delta\omega_{in}(S)} = \frac{\left(S + \frac{1}{C_{2}(R_{1} + R_{2})}\right)}{\left(S^{2} + S\left(\frac{1 + K_{d}K_{V}R_{2}C_{2}}{C_{2}(R_{1} + R_{2})}\right) + \frac{K_{d}K_{V}}{C_{2}(R_{1} + R_{2})}\right)}$$

$$\Delta \Phi_{e}(s) = \Delta \omega_{in}(S) \times \frac{\left(S + \frac{1}{C_{2}(R_{1} + R_{2})}\right)}{\left(S^{2} + S\left(\frac{1 + K_{d}K_{V}R_{2}C_{2}}{C_{2}(R_{1} + R_{2})}\right) + \frac{K_{d}K_{V}}{C_{2}(R_{1} + R_{2})}\right)}$$

$$\Delta \Phi_{e}(j\omega_{m}) = \Delta \omega_{in}(j\omega_{m}) \times \frac{\left(j\omega_{m} + \frac{1}{C_{2}(R_{1} + R_{2})}\right)}{\left((j\omega_{m})^{2} + \frac{K_{d}K_{V}}{C_{2}(R_{1} + R_{2})} + j\omega_{m}\left(\frac{1 + K_{d}K_{V}R_{2}C_{2}}{C_{2}(R_{1} + R_{2})}\right)\right)}$$

$$\Delta\Phi_{e}(j\omega_{m}) = \Delta\omega_{in}(j\omega_{m}) \times \frac{(j\omega_{m} + 17361)}{((j\omega_{m})^{2} + 972 \times 10^{6} + j\omega_{m}(31357.8))}$$

$$\Delta\Phi_{e}(peak) = \Delta\omega_{in}(peak) \times \frac{\left(j\omega_{m} + 17361\right)}{\left(\left(972 \times 10^{6} - \omega_{m}^{2}\right) + j\omega_{m}31357.8\right)}$$

For  $F_m = 500 \text{ Hz}$  we have

$$\Delta\Phi_{e}(peak) = 2\pi \times 2000 \times \frac{(j1000\pi + 17361)}{((972 \times 10^{6} - (1000\pi)^{2}) + j1000\pi \times 31357.8)} = 0,2258 \ r \ /0.179r$$

Compared to 0.224 r<sub>p</sub> found in step C. The phase angle of the phase error signal is the phase shift between the modulation signal and the phase error signal.

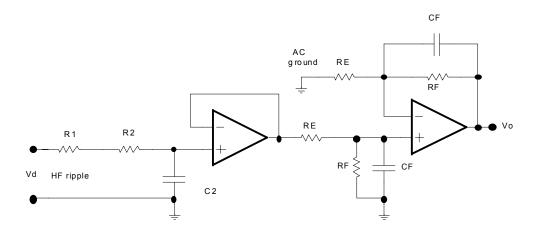
For  $F_m = 5 \text{ kHz}$  we have

$$\Delta\Phi_e(peak) = 2\pi \times 2000 \times \frac{(j10000\pi + 17361)}{((972 \times 10^6 - (10000\pi)^2) + j10000\pi \times 31357.8)} = 0,2302 \ r / 1.066r$$
Compared to 0.288 r. found in step D – I guess the graph was not that accurate

Compared to 0.288 r<sub>p</sub> found in step D – I guess the graph was not that accurate.

- For  $\omega_{n1}(\tau_1 + \tau_2)$  values of 5 and over the maximum phase error occurs at  $F_m = F_{n1} = 4962$  Hz. F) But for  $\omega_{n1}(\tau_1 + \tau_2) = 1$  the peak has shifted to about 0,8 F<sub>n1</sub>, so my estimate is that for  $\omega_{n1}(\tau_1 + \tau_2) = 1.8$  the maximum phase error will occur at approximately 0.9  $F_{n1}$  = 4466 Hz
- G) All yours to answer.
- H) Ripple calculation (see chart page 19 of theory section)

From phase detector O/P to final O/P, we have

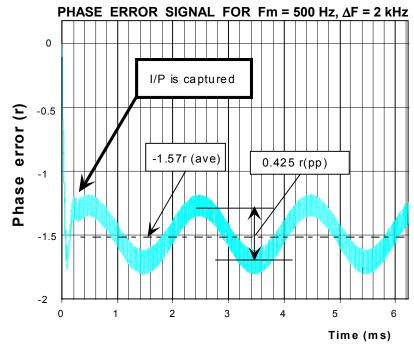


$$\frac{V_{o}(S)}{V_{d}(S)} = \frac{\frac{1}{SC_2}}{R_1 + R_2 + \frac{1}{SC_2}} \times \frac{R_F \left\| \frac{1}{SC_F} \right\|_{SC_F}}{R_E} \approx \frac{1}{SC_2(R_1 + R_2)} \times \frac{1}{SC_F R_E} = \frac{A_1}{S} \times \frac{A_2}{S}$$

$$\Delta V_o(pp) = \Delta V_o(pp) \times \frac{A_1 A_2}{4 \times 8 \times F_{rip}^2} = 2.16 \times \frac{1}{\left(4300 \times 390 \times 10^{-12}\right) \times \left(12 \times 10^{-9} \times 4800\right) \times 4 \times 8 \times 60K^2} = 194.1 \, mV_{PP}$$

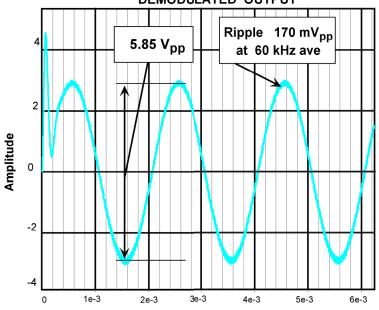
O/P signal level 
$$V_o(peak) = \frac{\Delta\omega_{in}}{K_V} \times \frac{R_F}{R_E} = \frac{2\pi \times 2000}{82333} \times \frac{82k}{4.3k} = 2.91 V_p$$
 % ripple = 194.1m / (2.91\*2) \*100 = 3.33%





**Time in Seconds** 

## **DEMODULATED OUTPUT**

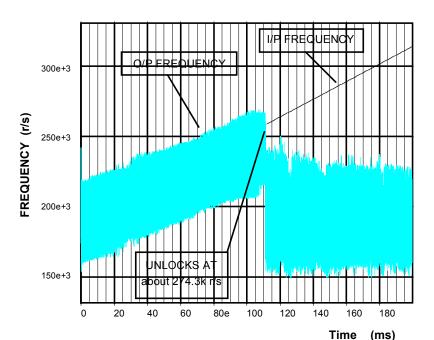


From the previous page, we obtained the following theoretical values:

 $V_o = 5.82 V_{pp}$  and ripple of 191.4 m $V_{pp}$ 

The ripple component is off because the duty cycle was assumed to be 50% at an average frequency of  $F_{\text{rip}} = 2F_{\text{car}}$  at the phase detector O/P. At  $F_{\text{min}}$  and  $F_{\text{max}}$  of the FM signal, the duty cycle is above and below 50% and  $F_{\text{rip}} = 2F_{\text{min}}$  and  $2F_{\text{max}}$  respectively.

Read ripple at top and bottom of demod O/P and average the two values which should come close to the predicted value for 50% duty cycle.



From the S/V simulation, the lock range is:

$$F_{\text{max}} = \frac{274.3k}{2\pi} = 43.656 \, kHz$$

$$F_{cen} = 30 \text{ kHz}$$

$$\Delta F_{\scriptscriptstyle L} = F_{\scriptscriptstyle \rm max} - F_{\scriptscriptstyle cen} = F_{\scriptscriptstyle cen} - F_{\scriptscriptstyle \rm min}$$

$$\Delta F_L = 43.656k - 30k = 13.66 \, kHz$$

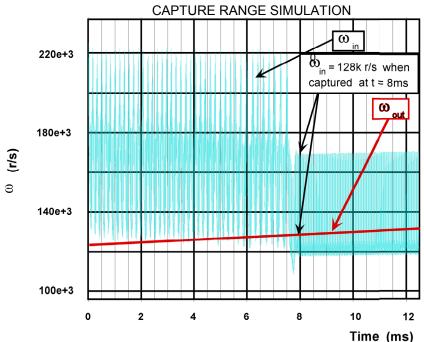
Theoretically we have:

$$\Delta F_{LOCK} \approx \pm \frac{8 \times F_{cen}}{\Delta V_{CC}}$$

$$\Delta F_{LOCK} = \pm \frac{8 \times 30k}{18} = 13.33kHz$$

**NOTE**: The simulation shown beside is not to scale – run sim file for a better look at results.

The above simulation was done starting at  $F_{in}$  = 30 kHz and then ramping up  $F_{in}$  at a rate of 100 kHz/sec. If the ramp rate is too fast then the lock rage will be less – lock range is defined for a slow variation of  $F_{in}$ . You can find  $F_{min}$  by starting at 30 kHz again but now ramping down  $F_{in}$  at –100 kHz/sec (Try it!)



From the S/V simulation, the capture range is:

$$F_{\min} = \frac{128k}{2\pi} = 20.37 \text{ kHz}$$

$$F_{cen} = 30 \text{ kHz}$$

$$\Delta F_L = F_{cen} - F_{\min} = F_{\max} - F_{cen}$$

$$\Delta F_L = 30k - 20.37k = 9.63 \, kHz$$

Theoretically we have:

$$\Delta F_{CAP} \approx \pm \frac{\sqrt{2}}{2\pi} \times \sqrt{2\zeta\omega_{n}K_{o}K_{d} - \omega_{n}^{2}} = \pm \frac{\sqrt{2}}{2\pi} \times \sqrt{2\times0.5\times(2\pi\times4962)\times82333\times0.68 - (2\pi\times4962)^{2}}$$

$$\Delta F_{CAP} = \pm 11.73 \ kHz$$

Nos.2, 3 and 5 you are on your own – see theory examples for help.

N0.4 A)  $F_o$  = 86 MHz to 108 MHz or  $F_o$  = 152 MHz to 174 MHz with steps of  $\Delta F_o$  = 200 kHz B, C and D are yours.