# PS 506 Intro to Game Theory

### (officially, "Seminar in Theories of Individual and Collective Choice II")

### Spring 2013

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Class meets Tuesday and Thursday 10:00-11:30 p.m. in Seigle 204;
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## Course Description

This course is intended as a serious introduction to noncooperative game theory for political science graduate students. It also covers some related topics (especially, connections with cooperative game theory) and important applications.

**Course prerequisites**

**Textbooks and reading assignments**

**Course Requirements and Grading**

**An outline of basic and special topics we may cover**

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# Prerequisites

Advisable, although not official, prerequisites for this course includes the basic mathematics found in either introductory calculus (Math 131 or equivalent) or the introductory graduate-level mathematical modeling course in Political Science (Pol Sci 5052).

More specifically, I will be assuming that students are comfortable with finding maxima or minima of a function using calculus, as well as with the concepts and notation of sets, graphs, functions, vectors, and probability.

No background in game theory is required. It would be good to have had some introduction to the notions of preferences, utility, and choice, although we will review this material briefly.

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# Textbooks and Reading Assignments

The course makes extensive use of one textbook, available from the campus bookstore:

- Martin J. Osborne, *An Introduction to Game Theory*

The latest printing is the 15th. It corrects a few errors in earlier printings, including some revisions to the statements of exercises.

Osborne maintains a website on this book, where corrections and some solutions are posted. And especially if you have a printing of the book earlier than the 15th, you should make use of the corrections pages, especially for all assigned homework problems. Also, the site offers a search facility for the textbook.

Links will be provided in the Course Details for various recommended and required readings. To use these links, you must be on campus or, in most cases, you may use the library's proxy server from off campus. For information on what the proxy server is and how to use it, look here.

**Required readings from Osborne**: you should make an effort to read required portions of Osborne before we cover them in class; that way, we can discuss questions you might have. You are responsible for all material in the required readings regardless of whether we discuss it in class.

In Osborne, "reading" always means working through his examples (aside, perhaps, from economics-oriented examples that use unfamiliar terms). Osborne's examples are one of the most important features of the book, covering a variety of applications and often representing simplified versions of game theory models found in notable journal articles.

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# Course Requirements and Grading

Your grades for the course will be based on three exams (60%), and on assigned exercises (40%). I will assign a constant diet of exercises for practice and to illustrate important basic models and problems in this field; when making exams, I will assume you have done these exercises and asked any questions you had about them. You will be required to hand in some of these exercises for grading, in order for me to more closely keep track of how you're doing. However, for exams you are equally responsible for all assigned material regardless of whether it was covered in a homework assignment to be turned in.

Virtually all assignments will be in the nature of problems to set up and solve.

To keep up with the course, it is critical to do the homework problems as they are asigned. Homework done a day or two late should be avoided but will be accepted on rare occasion.

Approximate **exam dates** and coverage are shown in Course Details. I will feel free to adjust these with at least two weeks' notice.

Exams are cumulative in the sense that tools and concepts learned early in the course will be used on subsequent problems in a cumulative fashion. However, there will be no questions aimed at earlier material just for the sake of coverage.

**Collaboration.** You are encouraged to work together on exercises, but every student should work on every assigned problem and fully understand it. You may NOT collaborate at all on exams.

**Help sessions.** We'll schedule regular, optional help sessions in which to discuss the assigned problems.

**Solution manual.** For exercises from the Osborne texbook, Osborne has provided a partial solution manual as a PDF file you can download.

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# Outline of basic and optional topics

The following table shows topics that may be covered in the course. Topics in the left column are the absolute basics, and we will cover them first. The right column lists important specialized topics and applications, and we will touch on as many of those as possible -- some of them as illustrations of the basic topics. Where topics are covered in the Osborne textbook, the chapter number is shown in parentheses.

<table>
<tr><th>Basic non-cooperative game theory</th><th>Other important topics</th></tr>
<tr><td>
<ol>
<li>Strategic games and Nash equilibrium (Osborne chapters 2, 3)</li>
<li>Mixed strategies (4)</li>
<li>Games in Sequential Form and Inductive or Rollback Equilibrium (5-7)</li>
<li>Sequential Games with Incomplete Information (10)</li>
</ol>
</td><td>
To be covered as time allows--details to follow
<ol>
<li>Comparative statics (with basic topics 1 and 2?)
<ul><li>implicit differentiation</li><li>monotone comparative statics</li></ul></li>
<li>Cooperative game theory
<ul><li>coalitional games and the core (8)</li>
<li>voting games
<ul><li>non-cooperative: sophisticated voting on agendas (with basic topic 4?)</li><li>cooperative: simple games</li></ul></li>
<li>Nash bargaining</li></ul></li>
<li>Rationalizability (12)</li>
<li>Evolutionary Games (13)</li>
<li>Repeated Games (14 on prisoner's dilemma; 15 if time)</li>
<li>Non-cooperative bargaining (16); legislative bargaining</li>
<li>Communication in games
<ul><li>signaling</li><li>cheap talk</li><li>mechanism design</li></ul></li>
<li>Behavioral game theory</li>
</ol>
</td></tr>
</table>

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# Course Outline

**(click to jump to details)**



# Course Details

The details below will change as the semester goes along to reflect changes in our schedule, exam dates, problems, and special items that come up along the way. You can set a bookmark directly to this course outline by clicking here and then using your browser's Add Bookmark commands. (Notice that there is also a link here and in the course website's opening screen to jump directly to the current topic in the course.

## 0. Introduction: Decisions and Games

**Jan 15**

Topics:

- preferences and utility
  - "weak order" relations: transitive, reflexive, and complete; the asymmetric (strict) and symmetric (indifference) parts of a weak order
  - ordinal and cardinal preferences
  - representation by a utility function
  - no interpersonal comparisons
  - rational choice
    - conditions for existence of a maximum
    - invariance under monotone transformations
- decisions under risk
  - representing likelihood via probability
  - lotteries, preferences over lotteries, and expected utility
    - von Neumann-Morgenstern ("VM" or "NM" or "vNM") utility
    - properties, especially invariance under linear transformations
    - risk aversion

Readings

- Osborne, chapter 1; sections 4.1.3, 4.12.
- references (not required):
  - R. Duncan Luce and Howard Raiffa, *Games and Decisions*. Dover Publications, 1989 (first published by Wiley, 1957). Chapter 2.
  - Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green, *Microeconomic Theory*. Oxford Univ. Press, 1995. Chapters 1 (all); 2F; 3A-3C; 6 (all).
  - David Kreps, *Notes on the Theory of Choice*. Westview Press, 1988.

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# 1. Strategic Games

## A. Simultneous actions and Nash equilibrium

**Jan. 17, 22, 24, 29**

(Chapters 2 and 3 in Osborne; specific readings given below)

**Reading** (always includes **working through all Examples** and Exercises except those exclusively oriented toward economics):

- chapter 2: all sections
- sections 3.3 and 3.4

**Solution manual:** Osborne supplies an on-line solution manual (pdf file) for all exercises marked with a bordered question-mark-circle symbol. These may be a help in working exercises NOT assigned to be handed in. However, not all of the hand-in problems assigned have solutions there.

Best-response analysis of a game of two-candidate competition

**Problems to hand in Tuesday Jan. 22:** 31.1, 34.2, 42.2, 44.1

**Literature notes (not required readings)**

The game of voter participation in exercise 34.2 is analyzed in detail in

- Thomas R. Palfrey and Howard Rosenthal, "A Strategic Calculus of Voting," *Public Choice* Vol. 41, issue 1 (1983), pp. 7-53. Click here to obtain online.

The Hotelling model of electoral competition springs from a suggestion by Hotelling himelf; its median voter result was exploited more thoroughly by Black as an application to committee decision making. At the time, neither really formalized the situation in terms of noncooperative game theory.

- Harold Hotelling, "Stability in Competition." *Economic Journal* vol. 39 (1929), pp. 41-57. Click here to obtain via JSTOR.
- Duncan Black. *The Theory of Committees and Elections*. Cambridge Univ. Press (1958).

Early studies of the equilibrium properties of elections with more than two candidates/parties include

- Gary W. Cox. "Multicandidate Spatial Competition." in J.M. Enelow and M.J. Hinich, eds., *Advances in the Spatial Theory of Voting* (Cambridge,England:Cambridge University Press, 1990).
- Thomas R. Palfrey, "Spatial Equilibrium with Entry." *Review of Economic Studies* vol. 51 (1984), pp. 139-56. Click here to obtain via JSTOR.

The citizen-candidate model first appeared, without that name, as a basis for Duverger's Law, in an article by Feddersen; later it was more extensively exploited by Osborne and Slivinski and by Besley and Coate:

- Timothy J. Feddersen, "A Voting Model Implying Duverger's Law and Positive Turnout." AJPS 36 (1992), pp. 938-62 Click here to obtain via JSTOR.
- Martin J. Osborne and Al Slivinski, "A Model of Political Competition with Citizen-Candidates." *Quarterly Journal of Economics*, Vol. 111, No. 1 (Feb., 1996), pp. 65-96. Click here to obtain via

JSTOR.
- Timothy Besley and Stephen Coate, "An Economic Model of Representative Democracy." *Quarterly Journal of Economics*, Vol. 112, No. 1 (Feb., 1997), pp. 85-114. Click here to obtain via JSTOR.

Candidates interested in the policy outcome of the election, rather than in winning office as such, were first explored by

- Donald Wittman, "Candidate Motivations: A Synthesis of Alternative Theories." *American Political Science Review*, vol. 77 (1983), pp. 142-57.
- Randall L. Calvert, "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* vol. 29 (1985), pp. 69-95.

**Problems to hand in Tuesday Jan. 29:**

- Electoral competition: 74.1 75.1, 75.2.
  - **NOTE:** In all but the latest printings of the textbook, there are some significant errors in the statements of exercises 74.1 (corrected in the 4th printing) and 75.2 (corrected in the 16th printing). See Osborne's errata for your printing, page 2.
- **Postpone due date until Tues. Feb. 5:** War of attrition: 80.2

## B. Mixed Strategies

##### Jan. 31, Feb. 5

(Chapter 4 in Osborne; specific readings given below)

Remark: why mixed strategies? The three Ps (none of these are required readings, but for your future reference only):

- **penalty kicks**, poker: to be successful, real competitors must act exactly as if they randomize. And they do -- sources:
  - Ignacio Palacios-Huerta, "Professionals Play Minimax." *Review of Economic Studies* vol 70 issue 2 (April 2003), pp. 395-415. To obtain the full article, click here and use the menu at upper right to click on "Full Text HTML" or "Full Text PDF".
  - P.-A. Chiappori, S. Levitt, and T. Groseclose, "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer." *American Economic Review* Vol. 92, No. 4. (Sept. 2002), pp. 1138-1151. Click here to obtain via JSTOR.
  - a contrary opinion: Michael Bar-Elia et al., "Action Bias among Elite Soccer Goalkeepers: The Case of Penalty Kicks." *Journal of Economic Psychology* Vol. 28, No. 5 (Oct. 2007), pp. 606-621. Click here to obtain online.
- **"polymorphic" equilibria** in "population games" -- especially in single pool, symmetric games, where sometimes the only stable equilibria correspond to mixed strategies.
- **purification** -- mixed strategy equilibrium as an approximation to frequencies of play in games in which each player has a small amount of uncertainty about other players' payoffs. Sources:

- John Harsanyi, "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points." *International Journal of Game Theory* vol. 2 (1973), pp. 1-23.
- Fudenberg and Tirole, *Game Theory*, pp. 233-234.

**Reading on mixed strategies**

- read sections 4.1 through 4.7 and 4.10
- skim sections 4.8, 4.9

**Problems due Tues. Feb. 5:** 80.2; 114.2, 121.2, 141.2, 142.1

- 80.2 left over from Chapter 3
- 114.2, 121.2, 141.2, 142.1
- You need not turn in 127.2 (Incompetent Experts), which was originally assigned here, since we we unable to get far enough on that example in class. It's still a good exercise.

Feb. 7: Review for Exam

**FIRST EXAM out Feb 7; due at beginning of class, Tues. Feb 12**

This exam will cover all material we covered in the first four chapters of Osborne, possibly requiring use of the minimal additional material we covered in class on decision theory and on maximization of payoff in continuous action sets.

## 2. Sequential Games

## A. Backward induction and subgame-perfect equilibrium

**Feb. 12, 14, 19, 21**

(Chapters 5-7 in Osborne)

Reading

- By Thurs. Feb. 14: read chapter 5 (all) and section 6.1.
- By Thurs. Feb. 21: read sections 7.1, 7.3, 7.4, 7.6; skim 7.7

**Problems to hand in:**

- On Tues. Feb 19:
  - Chapter 5, exercises 163.2, 173.2, 173.3
  - Chapter 6, exercise 187.1
- On Tues. Feb. 26:
  - Chapter 7, exercises 210.3, 216.1, 221.2, 227.3
    - Notice that 221.2 refers to the previous exercise 220.1*b*. In some early printings this appears incorrectly as simply "220.1".

**Imperfect equilibrium in the ultimatum game**

Are there any circumstances under which rational players might arrive at a subgame-imperfect equilibrium?

- John Gale, Kenneth G. Binmore, and Larry Samuelson, "Learning To Be Imperfect: The Ultimatum Game." *Games and Economic Behavior*, Volume 8, Issue 1 (1995), Pages 56-90. Click here to obtain online.

## B. Information Conditions

##### Feb. 26, 28, Mar. 5, 7

Reading:

- Read chapter 10 sections 1, 2, 4, 5.

**Problems:** complete and hand in the following exercises: **Exercise 331.1** from Osborne, plus the following problem:

- **Problem 2: Strangers bearing gifts.** With probability $p < 1/2$, Player 1 is a "Friend," and with probability $1 - p$, an "Enemy." Player 1 knows her own true type, but Player 2 knows only the value $p$. Player 1 may give a gift to Player 2, or do nothing. If Player 1 does nothing, the game ends and both players receive payoffs of 0. If Player 1 gives the gift, Player 2 may open the gift or leave it unopened. Player 1, whether Friend or Enemy, receives a payoff of 1 if the gift is opened, and -1 if the gift is left unopened. Player 2 receives 0 for leaving the gift unopened. If Player 2 opens the gift, he receives 1 if Player 1 is a Friend, and -1 if Player 1 is an Enemy. Draw the game tree, indicating any information sets with dotted line. Find two different weak sequential equilibria for this game, specifying, as always, both strategies and beliefs.
- Hint: Let $f$ = Pr(gift | Friend) and $e$ = Pr(gift | Enemy) describe Player 1's behavioral strategy. Is there a fully informative equilbrium ($f = 1$, $e = 0$)? Is there an uninformative equilibrium ($f = e$)? Is there a partially informative equilibrium? Try $f = 1$ and $0 < e < 1$.

SPRING BREAK Mar. 12, 14

Mar. 19, 21: slack and review

**SECOND EXAM out Mar. 21; due Mar. 26**

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## Partial List of Additional Topics

### 1. Repeated Games

assignments:

- Read Osborne chapter 14 at least sections 1-7
- Also: in case you're unfamiliar with it, Chapter 17 section 5 demonstrates how to calculate the sum of a series of discounted payoffs, a central tool in all work on repeated games.
- problems 426.1, 428.1, 429.1.

assignments:

- Readings: finish Chapter 14
- problem:
  1. Show that the strategy profile in which both players use Tit-for-Tat is NOT a subgame perfect equilibrium. (Hint: what happens following an uncalled-for defection by your opponent?)
  2. For the game shown in Figure 420.1, consider the modified Tit-for-Tat strategy defined as follows:
     - In the first period, play C
     - In subsequent periods, do whatever your opponent last did, EXCEPT:
     - If YOU ever Defect in violation of the previous instruction, then "apologize:" play C in the next two periods regardless of what your opponent does, and then return to using the previous instruction. (Naturally, your opponent will be Defecting in the first of these two "apology" periods.)

     Use the one-deviation property to show that the profile in which both players use this modified Tit-for-Tat is a subgame perfect equilibrium.

### 2. Coalitional Games and the Core

Read Osborne sections 8.1, 8.2, and 8.6

**Problems**: 243.1; 245.1; 247.1.

### 3. Bargaining

Read Chapter 16 section 1

**Problems:** 468.2, 473.1

### 4. Evolutionary Games

Reading: Osborne, sections 13.1, 13.2, 13.3

**Problems:** 409.2

last day of class: Thursday Apr.25

**THIRD EXAM out Apr. 25; due May 1**

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