TOPICS IN MEASUREMENT: MULTIDIMENSIONAL POVERTY AND

POLARIZATION

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To my mother, Indu.

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CHAPTER I

INTRODUCTION

The main focus of this thesis is the study of distributional aspects of income and other dimensions of well-being. The first two essays pertain to the use of multidimensional poverty measurement techniques. In the third essay I discuss a different distributional aspect, namely, the middle class as evaluated by income polarization measures.

The multidimensional framework has captured the interest of not only academics but also policy makers. Several measures of multidimensional poverty and inequality have been proposed. Also several governments including India and Mexico have expressed the need for a broader definition of poverty that incorporates aspects of well-being not captured fully by income or consumption measures alone.

While several multidimensional poverty measures are available, data availability restricts the choice of measures considerably. The Alkire and Foster (AF henceforth) (2010) measure is used extensively in this thesis. This class of measures is best suited for the problem at hand for several reasons. First, the data used have both ordinal and cardinal variables. For this reason alone, several of the other measures cannot be used. For instance, the measure proposed in Tsui (2002) is unsuitable for use with ordinal data. Secondly, we are often interested in poverty estimates across groups and a (population) decomposable measure is thus desirable. The measure proposed in Bourguignon and Chakravarty (2003) does not satisfy this, but the AF measure does. Lastly, this framework lends itself easily to statistical inference. In one of the essays discussed below I derive a simple test for comparing poverty across two groups given a set of cutoffs for the AF measure. Also, in related work (done jointly with Christopher Bennett) I explore the statistical inference procedures in

more detail allowing for testing multiple inequalities simultaneously. Before I proceed any further, a short discussion of the measure is advantageous. For reasons of brevity I will refer to multidimensional poverty measures as poverty measures and, to differentiate it from unidimensional measures, I will refer to the latter as income poverty measures.

Like its unidimensional counterpart, measurement of poverty in the multidimensional framework can also be divided into two steps-identification and aggregation. The identification step answers the question 'Who is poor?' The Alkire and Foster methodology uses a dual cutoff to identify the poor. For each dimension there is a poverty line which identifies individuals deprived in that specific dimension. The multidimensional case has a second cutoff which gives the minimum number of dimensions that an individual must be deprived to be considered poor. For example, if there are six dimensions being considered, dimension-specific poverty lines identify the dimensions in which an individual is deprived. If the second cutoff is chosen as four, then for the individual to be considered poor, she should be deprived in at least four of the six dimensions. The aggregation step leads to an overall measure of poverty accounting for the deprivation of all the individuals identified as poor. For the aggregation step, the AF methodology uses a dimension-adjusted Foster, Greer and Thorbecke (FGT) measure.

Two of the three essays use the AF methodology to make poverty comparisons across different groups. The first essay "Re-Assessing "Trickle-Down" Using a Multidimensional criterion: The Case of India" uses the multidimensional framework to gain a deeper understanding of the characteristics of poverty in India. Here I attempt to gauge the extent of "trickle-down" accompanying the uneven-growth process for a developing country[1]. Trickle-down has been addressed, so far, using income-based measures of inequality and

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[1] "trickle-down" is a term often used to describe the top-down effect of development policy. The idea is that effects of growth will gradually percolate down to all tiers of the society.

poverty. However concerns over inequality in access to other dimensions that are important for the quality of life such as education remains. Here I revisit trickle-down using a broad-based measure of poverty which incorporates these other dimensions. The AF class of measures is used to estimate multidimensional poverty in India utilizing National Sample Survey (NSS) data. The measures are modified to account for the complex survey design of the NSS and a test for differences in multidimensional poverty across subgroups of the population is also introduced. The AF poverty estimates are presented for the 16 major states of India and are compared to income-based measures. Incorporating additional dimensions such as education, availability of drinking water and others in poverty measurement results in the reversal of several income-based conclusions about poverty across regions. The findings suggest that equality across regions in terms of income is not synonymous with equality in standard of living. The paper also finds that contrary to income-based findings, Hindus are poorer than Muslims under the multidimensional criteria. To test the robustness of the results different weighting schemes (for the dimensions) are also used.

In the first essay I develop a test to check the statistical significance of the pairwise comparisons of poverty estimates. The test incorporates in its construction the complex NSS survey design and allows the use of the survey weights. Multidimensional poverty measures give rise to a host of statistical hypotheses (other than comparison of pairwise poverty estimates) which are of interest to applied economists and policy-makers alike. These questions necessitate a procedure that allows for testing of several claims simultaneously. The second essay in this thesis provides a general testing procedure that allows multiple hypothesis to be tested simultaneously. In the second essay (work done jointly with Christopher Bennett) "Multidimensional Poverty: Measurement, Estimation, and Inference" I develop a broader methodology for statistical inference for AF measures. I show that many hy-

potheses can be treated in a unified manner and tested simultaneously using the minimum p-value methodology of Bennett (2009). However it is only applicable to random sampling procedures. Incorporating other survey designs in the multiple testing framework is a topic of future research.

In the third essay I study a different aspect of the income distribution, the "middle class" as measured by income polarization. In any society, especially in the developing world, the middle-income group is often thought to be the main driver of economic growth. Unlike the poor, this group has resources to spend on consumption and also the ability and will to save and invest. There is a strong relationship between the size of the middle class and the degree of income polarization in society (see Esteban and Ray (2010) for a comprehensive discussion). A high degree of income polarization is suggestive of society dominated by two income groups — the "haves" and the "have-nots" and thus a smaller middle class.

The essay "Electoral Uncertainty and the Growth of the "Middle Class": Theory and Evidence from India" (work done jointly with Anirban Mitra) investigates how the presence of electoral uncertainty contributes to the rise of a "middle class" (middle-income group) in the context of a developing country. The theory developed here is based on the traditional Downsian (two-party) framework and predicts the following. Any increase in electoral uncertainty in a district not only increases the aggregate level of transfers to the district, but also leads to public expenditure that disproportionately benefits the poor as compared to the rich. This in turn leads to lowering of income inequality and more importantly lessening of income polarization. I test this hypothesis using data from the Indian parliamentary (national) elections which are combined with household-level consumption expenditure data rounds from National Sample Survey Organization (NSSO) (1987-88 and 2003-04) to yield a panel of Indian districts. The empirical exercise reveals that districts

that have experienced tight elections exhibit lower income polarization (alongside lesser income inequality) and hence a larger middle class, in support of the theoretical model.

CHAPTER II

RE-ASSESSING "TRICKLE-DOWN" USING A MULTIDIMENSIONAL CRITERION:
THE CASE OF INDIA

Introduction

A particularly serious concern, especially in the context of rapidly growing developing countries, involves the issue of "uneven growth". There have been many studies directed towards this issue (see Ray, 2010) and references therein) and the related one of "trickle-down" (see for example, Basu and Mallick, 2008). Most of them focus on the impact of economic reforms on changes in income inequality or income poverty (see Chaudhuri and Ravallion, 2006 , Ravallion and Dutta, 2002). Even if one sees little or no change in income inequality accompanying growth, can one infer that "trickle-down" has indeed occurred? Consider the case of rural India.[1] In the vast majority (if not all) of Indian villages, access to many public facilities is often determined not so much by income but by ethnic markers (like caste, religion, etc.), and so higher incomes are not always associated with better access to basic facilities. This clearly highlights the need to focus on direct access to public goods and consequent social capital formation. A closely related work in this context is Banerjee and Somanathan (2007).[2]

One way to incorporate achievements in the dimensions of social capital and human capital is to include them directly in a measure of well-being or rather, deprivation. This

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[1]Over 70% of India's population can be classified as rural.

[2]Banerjee and Somanathan (2007) look at the location of public goods between 1971 and 1991 in about 500 parliamentary constituencies in rural India to assess the differences in the allocation of public resources over the period for the various ethnic groups. They find that the allocation in areas with large Scheduled Caste populations has increased whereas the access is reduced in areas with Scheduled Tribes and Muslims.

paper uses a multidimensional poverty methodology to assess differences in access to basic facilities such as education by various regions and ethnic groups in India. Using this poverty approach ensures I can capture differences across regions and also focus on the deprived groups within these regions. If income poverty and multidimensional poverty estimates diverge then we may need to reinterpret findings regarding the "trickle-down" phenomena; as, income inequality following growth would not necessarily imply reduction in disparities across households.

Several studies have shown that the decline in income poverty in India from 36% in 1993-94 to 27% in 2003-04 has not been accompanied by similar improvements in other dimensions relevant for development. For example, according to a World Bank report[3], India is among the countries with the highest prevalence of underweight children. The performance in improving literacy has also been moderate compared to other nations in Asia.[4] The importance of dimensions of well-being in explaining differences in growth and development is well documented.[5] In this paper these dimensions of well-being are incorporated into the measurement of poverty in such a way that the extent of deprivation **across** dimensions by households can also be captured. I use a poverty measures that is sensitive to the joint distribution of achievements across the dimensions. Merely looking at deprivations in each of the dimensions separately does not inform us of the extent to which a family is multiply deprived.

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[3]See the World Bank report, India's Undernourished Children: Call for Reform and Action.

[4]57% of the population in India was literate according to the Census of India, 2000 which is well below Thailand's 96 percent, Sri Lanka's 92 percent, Indonesia's 87 percent, and China's 84 percent (See Barooah and Iyer, 2005).

[5]Chaudhari, Schneider and Chattopadhyay (2006) show that literacy and increased growth of newspapers translates into better governance. Gamper-Rabindran, Khan and Timmins(2009) discuss the effect of piped water on infant mortality in Brazil. In Datt and Ravallion(2002) initial conditions including indicators of health, education and standard of living is recognized as one of the factors explaining regional differences in pro-poor growth. Ferreira, Leite and Ravallion (2009) show that for Brazil impact of growth on poverty reduction is primarily explained by differences in macroeconomic factors however initial conditions have a significant albeit small effect.

Several organizations and governments including the UNDP and the Government of Mexico have adopted a multidimensional approach to the measurement of poverty.[6] In fact, the Multidimensional Poverty Index (MPI) developed by the UNDP is based on the Alkire and Foster (2010) methodology, which is also the methodology used in this paper. The Government of India has recognized the need to use a broader definition of poverty incorporating these other dimensions. Recently the Planning Commission of India has announced that it will use a broader definition to identify the poor, which will go beyond income to incorporate other dimensions.[7]

For reason described in the Introduction, the Alkire and Foster (2010) methodology is most suitable for the analysis done here. The data used is the NSS 60th round health care and morbidity survey, conducted during 2003-04. The choice of dimensions is always a challenge in any multidimensional analysis. The Human Development Index by the UNDP measures well-being along three dimensions-education, health and standard of living. In this essay dimensions are selected along similar lines. I use seven dimensions where sanitation, drainage facilities, source of drinking water and primary cooking medium are essential for better health outcomes; income and housing facilities are means to achieve a better standard of living; and the last dimension is education.

Using a particular calibration the AF approach I find that 50% of the population was poor in 2003-04 which is much higher than the official income poverty estimate of 27%.[8] This divergence in estimated headcount ratios for the income criteria and the multi-dimensional criteria may reflect the inability of the former to identify many who are truly

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[6]For the news release by UNDP-OPHI see
http://hdr.undp.org/en/reports/global/hdr2010/news/title,20523,en.html

[7]The Hindustan Times (a news daily from India) has reported the Deputy Planning Commissioner announcing, "In the new system poverty would be measured with reference to basic facilities like quality education, good health sectors and clean drinking water availability."

[8]The case of multidimensional poverty for the nation as a whole using a dimension cutoff of four.

deprived, not necessarily in terms of earnings but on other fronts. Among all the people regarded as poor according to the multidimensional criteria, a mere 45% are also income deprived; i.e. the other 55% are not income deprived but are multidimensionally poor.

This paper focuses on comparisons of poverty across different regions and groups of India. When making such comparisons, the differences in levels of poverty are sometimes large enough to leave no room for doubt. However when the poverty estimates happen to be very close to each other, when does it make sense to talk about differences in poverty? Consequently, in addition to providing estimates of poverty among different groups, this paper also tests whether these group-wise differences are statistically significant. For this purpose, a simple standard test is developed, which allows us to check differences in multi-dimensional poverty among groups. NSS is a multistage-stratified random sample; so first the AF measures are modified to estimate the poverty levels when all observations are not equally likely. Then the asymptotic properties of the test statistic for difference in poverty across groups are established keeping in mind the sampling design of the NSS data.[9]

Income-based measures have been used to discuss disparities in the growth and development experiences of the different Indian states. Datt and Ravallion (1998) find that success rates in reducing poverty vary significantly across the different states. Further, Deaton and Dreze (2002) have found that the regional differences in income poverty and inequality have increased over the 1990s. Is this disparity restricted to income alone? Kerala and Andhra Pradesh have similar levels of poverty according to the income headcount ratio (See Table 5). On the other hand, it is widely believed that Kerala has done better in improving the well being of its people. For instance, Kerala is the only state in India to have nearly 100% literacy whereas Andhra Pradesh does not have as good a record. So why

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[9]Bhattacharya (2005) has discussed the asymptotic properties for the estimation of the Gini coefficients using NSS data and the analysis here closely follows this work.

is it that income poverty estimates fail to capture these differences in well being? Could incorporating other dimensions in the poverty estimation technique better inform us? On the basis of the results presented here I argue that the answer is *yes*. In the multidimensional framework, Kerala exhibits much lower poverty than Andhra Pradesh. In fact, Kerala does so well in the multidimensional framework that it is next only to Punjab and Haryana. This, of course, should not come as a surprise since the efforts of the government of Kerala on improving the human capital have been documented in the literature (Sen,1999) .

Another special feature of the Indian economy and one that can detrimentally impact India's future performance is communalism - which often operates along religious lines.[10] In spite of the fact that the population is predominantly Hindu, India has a large Muslim population. Poverty analysis has shown that there is a higher percentage of income poor Muslims than Hindus (for example see Noland, 2005)). Interestingly, when other dimensions are added I find that Muslims are less poor than Hindus. Why is this the case? Which dimensions are responsible? Could it be that Muslims are concentrated in urban areas where health care, sanitation and other facilities are better provided? I first estimate the contributions of the specific dimensions to the overall multidimensional poverty estimates. I further investigate whether there are differences in the rural and urban comparisons of Hindu and Muslim poverty estimates. I also divide Hindus into sub-groups based on whether they belong to the Scheduled Castes and Tribe or not.[11] I find

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[10]Esteban and Ray (2008) refers to the linkages between economic inequality and ethnic conflict.

[11]In India, 85% of the population is Hindu and every Hindu is assigned a caste. Scheduled castes form about16% of the Indian population. Their economic backwardness is a direct result of the caste system which has been ingrained into the Indian social fabric for a long time. Caste membership is hereditary and is reflective of the person's traditional occupation choice. Particularly, members of the scheduled castes were traditionally assigned low-paying, labor-intensive jobs. In fact, they were denied education and barred from high-paying tasks.

Such an intra-group distinction does not apply to Muslims, since traditionally there were no castes associated with them and the Constitution of India does not recognize Muslims to be part of the Scheduled Castes.The following is a link to an extract from the Constitution of India available on the Ministry of Law and Justice (Government of India) web page, which clearly states that no Muslim group can be declared a member of the Scheduled Castes in India: http://lawmin.nic.in/ld/subord/rule3a.htm.

that the higher levels of Hindu poverty are driven by the extremely poor conditions of the underprivileged Hindu minorities-Scheduled Castes and Tribes. On average, high caste Hindus are less poor than the Muslims, whereas the low caste Hindus are considerably poorer than the Muslims.

The main results in the paper are presented using equal weights for all the seven dimensions. However the AF methodology allows for different weights to be used for the various dimensions. Therefore other weighting schemes are also considered to check the sensitivity of the results, namely progressively more weight is put on income relative to the other dimensions. An equal weights approach puts approximately 15% weight on each dimension. As the weight on income is increased above this, the main results of the paper apply until the weight on income is about 30% (with equal weights on the other six dimensions). Attributing more than 30% of the weight to income leads to results which mirror those from the income poverty estimates.[12]

The rest of the paper is organized as follows. Section 2 describes the AF measures in detail. Section 3 describes a modified estimator for use with complex samples and the corresponding test statistic used for inference.[13] Section 4 describes the data, the dimensions used and the dual cutoffs applied. Section 5 discusses the main empirical finding and Section 6 concludes.

The P_α class of measures

The measurement of poverty can be described as a two step procedure. The first stage involves the identification of the poor. The second stage is the aggregation step

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[12]However, the sets of individuals who are identified as poor by the two criteria may well be very different.
[13]By complex survey designs I mean multistage stratified sampling or other forms of complex sampling designs.

where all the data are combined to give an overall measure of poverty. For unidimensional poverty measurement, the identification step is characterized by setting a poverty line: anyone below the poverty line is identified as poor. For the aggregation step there are several poverty measures which can be used, includes for example the headcount ratio, the poverty-gap measure or the FGT measure. For multidimensional poverty, the identification and aggregation require modification.

The union approach, as described in Bourguignon and Chakravarty (2003) is one of the commonly used identification procedures for multidimensional poverty measurement. Under this approach a person is identified as poor if she is deprived in any one of the dimensions being considered. For example, if the analysis involves six dimensions, then a person is considered poor only if she is deprived in at least one of the six dimensions. This measure will have a tendency to identify sizeable sections of the population as poor; this could run contrary to our natural notion about poverty. On the other hand, the AF measure allows for more flexibility. The AF methodology uses a dual cutoff to identify the poor. The first set is the dimension specific cutoffs which identify whether the individual is deprived in that particular dimension. The second cutoff explains how widespread (across the dimensions) deprivation of the individual has to be for her to be considered poor. So the second cutoff (k) gives the minimum number (or weighted sum) of dimensions the individual has to be deprived in, to be considered poor. For aggregation Alkire and Foster (2010) suggests a class of dimension-adjusted FGT measures. Since the primary analysis uses equal weights (on all dimensions) the exposition of the AF measure presented below is for equal weights[14]. Towards formulating the AF class I introduce the following notation. Let $i = 1, ...n$ denote the n individuals in a random sample drawn from the population

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[14]A discussion of the more general case can be found in Alkire and Foster (2010).

of interest. There are d dimensions being considered, which are denoted by $j = 1, ..., d$.

Let y_{ij} denote individual $i's$ achievement in dimension j. Therefore, y_i is a $d \times 1$ vector of

individual $i's$ achievements in all the d dimensions and y is $n \times d$ matrix of achievements

of all individuals in society. The two sets of cutoffs are given by z that is made up of d

dimension specific cutoffs (so z_j is the deprivation cutoff for dimension j) and $k \in [1, d]$ is the

poverty cutoff which requires an individual to be deprived in at least k of the d dimensions

to be considered poor. Finally $I(\cdot)$ represents the indicator function which takes the value

of 1 if the statement inside the parentheses is true and the value of zero otherwise. Let

$$c_i = \sum_{j=1}^{d} I\left(y_{ij} < z_j\right).$$

Therefore c_i gives the total number of dimensions in which individual i is deprived. For the

purposes of comparison with the income headcount ratio, the multidimensional headcount

ratio will also be used. This can be represented as:

$$H = \frac{1}{n} \sum_{i=1}^{n} I\left(c_i \geq k\right),$$

and the AF class of multidimensional poverty measures can be expressed as the following:

Each individual's level of poverty is given by:

$$\pi_i\left(y_i; z, k\right) = I\left(c_i \geq k\right) \frac{1}{d} \left[\sum_{j=1}^{d} I\left(y_{ij} < z_j\right) \left(\frac{z_j - y_{ij}}{z_j}\right)^{\alpha} \right] \tag{1}$$

and multidimensional poverty for the society is given by:

$$\tilde{P}_\alpha\left(y; z, k\right) = \frac{1}{n} \sum_{i=1}^{n} \left[\pi_i\left(y_i; z, k\right)\right] = \frac{1}{nd} \sum_{i=1}^{n} \left[I\left(c_i \geq k\right) \left\{ \sum_{j=1}^{d} I\left(y_{ij} < z_j\right) \left(\frac{z_j - y_{ij}}{z_j}\right)^{\alpha} \right\} \right]. \tag{2}$$

For those with c_i less than k, Equation 1will take the value of zero. For the individuals identified as poor, i.e., c_i is greater than k, Equation 1 gives the average level of deprivation across dimensions. This is done by first calculating the normalized gap, given by $\frac{z_j - y_{ij}}{z_j}$, when an individual is deprived and zero otherwise. The normalized gap is raised to the power α, which for values of 0, 1 and 2 give the three most commonly used AF measures. For α equal to zero (P_0), an individual's poverty level reduces to:

$$\pi_i \left(y_i; z, k \right) = I \left(c_i \geq k \right) \frac{1}{d} \left[\sum_{j=1}^{d} I \left(y_{ij} < z_j \right) \right].$$

In this case, the information about the depth of deprivation in the dimensions is not used. For α equal to one, I obtain the poverty gap measure P_1 which incorporates the information about the depth of deprivation into the analysis. For α equal to two, I have the squared gap measure P_2 which puts more emphasis on the individuals who are severely deprived. However for ordinal data the dimension - adjusted FGT measure with α equal to zero (or P_0) is most suitable. The only information P_0 uses from the original data is whether or not the individual is deprived in that dimension. If the individual is deprived with respect to a particular dimension, she gets a value of one otherwise they get a value of zero.

Above I have described the AF measure when information is known about n individuals. If these n individuals constitute the entire society of interest, then we are done. However in most empirical work the n individuals are a sample drawn from the population of interest. In this case if the population had a joint distribution $G \left(y_1, y_2, ..., y_d \right)$ over the d dimensions of interest, then the AF measures can be represented in the following form:

$$P_\alpha \left(z, k \right) = \int \pi \left(y; z, k \right) dG \left(y \right).$$

Therefore $\tilde{P}_{\alpha}(y; z, k)$ described in Equation 2 is the estimate for this population poverty measure, when the sample is drawn randomly. Among others, Zheng (2001, 2004) describes the one dimensional counterpart for the FGT measures, which is similarly the expected value for the population.

However if the sample is not a simple random sample then $\tilde{P}_{\alpha}(y; z, k)$ should be modified to incorporate the complexity of the sample design. In the next section, I propose an alternative estimate for $P_{\alpha}(y; z, k)$. This new estimate is a weighted average of the deprivation of all observed individuals.

A New Estimator and Test Statistic

Estimate of P_{α} for complex samples

If the sample is random then each observations is weighted by $\frac{1}{n}$, where n is the sample size and the estimate of poverty for the sample is the simple average of the deprivation of each individual represented by $\pi_i(y_i; z, k)$. However when there is a non-random sample, then the weight on each observation will not be equal to $\frac{1}{n}$. Incorporating the non-randomness requires a good understanding of the weight that each observation should have. This is the approach used to derive the alternative estimate of the AF measures.

The notation used and the results closely follow Bhattacharya (2005). The sampling design is described as follows. Prior to sampling the population is divided into S first stage strata. In the population the stratum s contains H_s clusters. A sample of n_s (indexed by ψ_s) clusters is drawn via simple random sample with replacement from stratum s, for each s. The ψ_sth cluster has a total of $N_{s\psi_s}$ households. A simple random sample draws κ (equal across clusters and strata and indexed by h) households from each cluster for each

strata are drawn. The hth household in the ψ_sth cluster has $\nu_{s_\psi h}$ members.

The weight of every individual in the hth household who is part of the ψ_sth cluster of the sth stratum is given by

$$w_{s\psi_s h} = \frac{N_{s\psi_s} H_s}{\kappa n_s v_{s\psi_s h}}$$

Therefore the estimate for the AF measures incorporating the sample design is a simple weighted average of the deprivation of each individual, $\pi_i(y_i; z, k)$. The weight on each observation is $w_{s\psi_s h}$ and the new estimate, $\hat{P}_\alpha(y; z, k)$ is :

$$
\begin{aligned}
\hat{P}_\alpha(y; z, k) &= \sum_{s=1}^{S} \frac{H_s}{n_s} \sum_{\psi_s=1}^{n_s} \frac{N_{s\psi_s}}{\kappa} \sum_{h=1}^{\kappa} \frac{1}{v_{s\psi_s h}} \sum_{j_h=1}^{\nu_{s\psi_s h}} \pi_{j_h} \\
&= \sum_{s=1}^{S} \sum_{\psi_s=1}^{n_s} \sum_{h=1}^{\kappa} \sum_{j_h=1}^{\nu_{s\psi_s h}} w_{s\psi_s h} \pi_{j_h}.
\end{aligned}
$$

In the appendix I provide the asymptotic distribution of this new estimator. It is shown that this converges in probability to the population parameter P_α and is asymptotically normal. Therefore,

$$\sqrt{n}\left(\hat{P}_\alpha - P_\alpha\right) \longrightarrow^d N(0, V)$$

where $\hat{P}_\alpha$ is the estimated poverty measure, P_α is the population equivalent of the poverty estimate and V is the variance of the distribution. Using this information I propose a test for differences in poverty across groups.

The Test

The P_a measure is a sample average of the individual specific measures $\pi_i\left(y_i; z, k\right).$ Thus for a random sample the test would be a standard Wald test. However the NSS data is a multistage-stratified random sample. It has been shown in Bhattacharya (2005) that treating this sample as a random sample can give erroneous interpretation of results.

For a multistage-stratified sample (like NSS) I propose a test for the difference in poverty across groups. Let the two groups be denoted by 1 and 2 and samples of sizes n_1 and n_2, are drawn from the respective populations. The estimated multidimensional poverty for the two groups are $\hat{P}_\alpha^1$ and $\hat{P}_\alpha^2$ respectively. The corresponding population parameters of interest are denoted by P_α^1 and P_α^2. Note that these estimates depend on the dimension specific cutoff and the poverty cutoff that is used. For the analysis in this paper I will keep these cutoffs fixed.

The null hypothesis I am interested in is

$H_0 : P_\alpha^1 - P_\alpha^2 = 0$ (for a given k) and the alternative hypothesis is

$H_1 : P_\alpha^1 - P_\alpha^2 \neq 0$

Let $\theta\left(k\right) = P_\alpha^1 - P_\alpha^2$ and $\hat{\theta}\left(k\right) = \hat{P}_\alpha^1 - \hat{P}_\alpha^2$. Given that the samples for the two groups are independent from each other I get:

$$\sqrt{n_1}\left(\hat{\theta}\left(k\right) - \theta\left(k\right)\right) \longrightarrow^d N\left(0, V_1 + rV_2\right), \text{ where}$$

$$\sqrt{n_i}\left(\hat{P}_\alpha^i - P_\alpha^i\right) \longrightarrow^d N\left(0, V_i\right), \; i = 1, 2 \; \text{ and } r = \lim_{n_1, n_2 \to \infty} \frac{n_1}{n_2}.$$

Let $\sigma^2 = V_1 + rV_2$ and an estimate for σ^2, denoted by $\hat{\sigma}^2$, is given by respective estimates of V for the two groups as in Equation A.2 (in the appendix). So $\hat{\sigma}^2$ is equal to $\hat{V}_1 + r\hat{V}_2$.

Since I am only interested in making sure that the difference in the poverty estimates of the two groups is statistically significant the analysis becomes much more simpli-

fied. The null hypothesis I am testing is $\theta(k) = 0$, against the alternate $\theta(k) \neq 0$. This can be tested using the simple test statistic

$$t = \frac{\hat{\theta}(k)}{\hat{\sigma}}$$

which has an asymptotically standard normal distribution. At the $(1 - \alpha)$ level of significance the null hypothesis of equality in poverty levels across the two groups is rejected if $|t| > z_{1-\alpha}$, where $z_{1-\alpha}$ is the critical value from the standard normal distribution.

Description of Data

For this analysis, the NSS 60th round health and morbidity survey has been used. This survey was conducted during 2003-04. The data used here were collected from households across the country in both rural and urban areas. The total number of households surveyed is 47,302 and 26,566 for rural and urban areas respectively. The sampling procedure used in the NSS round is multi-stage stratified sampling. First, the entire nation is divided into strata based on the size of population of the area. Then these strata are further divided into clusters. The sampling proceeds in two steps. First, from each strata a number of clusters are chosen randomly. Then in the second stage, from each of these clusters a fixed number of households are chosen by random sampling without replacement. This constitutes the sample of households observed.

Dimensions and the respective deprivation cutoffs

From these data I am able to get information on seven aspects of an individual's living standard. These dimensions span the individual's income level, her education and

Table 1.: Dimension Specific Deprivation cutoffs

Dimension	Deprivation Cutoff
Income	As given by the Planning Commission
Education	Persons who have not obtained at least a primary education are deprived, others are not.
Sanitation	The non-availability of any sanitation facility indicates deprivation.
Drainage	The non-availability of any drainage facility indicates deprivation.
Structure of House	A person without any housing structure or a Kutcha unserviceable house is deprived, others are not.
Source of Drinking water	A person using a river/canal/pond/well is deprived, others are not.
Primary Cooking Medium	A person without any cooking arrangement or using firewood/dung cakes is deprived, others are not.

other indicators of her standard of living.

For the income poverty measure the poverty lines used are the rural and urban poverty lines for every state as given in the press release of the Planning Commission. For all the other dimensions, the thresholds that I have used are the basic minimum that a person should have. Most of the times, the cutoff just divides the population into two groups: people who have some access to the facility and those who have no access to the facility at all. For example, for type of sanitation facilities a person is not deprived if she has any kind of sanitation facility available. The precise definitions of the dimension specific cutoffs are given in Table 1 and discussed in more detail below.[15]

The dimensions of interest and the respective deprivation cutoffs are:

1. **Per Capita Expenditure**: The survey contains information on the total monthly expenditure of the household. This I have divided by the number of individuals in the household to arrive at the per capita figures. The poverty lines used are the official poverty lines given by the Planning Commission of India for the year 2003-04. The poverty lines are different for rural and urban areas and are also different for each of the states.

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[15]The choice of cutoffs here is somewhat arbitrary and represents the author's best estimate of *minimal* criteria.

2. **Educational attainment**: The last educational degree attained by each individual in a household is listed. I have information as to whether a person ever got any formal education, or did not finish primary school, or finished secondary school or higher degrees. A person is considered education deprived if she has not completed primary school. I do not use the official literacy definition[16] as the cutoff since it hardly provides any information about the individual's cognitive abilities. Completion of primary education implies that the individual is able to do simple math and follow simple instructions, and this justifies the use of primary education as the cutoff.

3. **Type of house structure**: The survey also notes the types of houses the household lives in. Information is furnished on whether it is a brick house or other type of mud house. A person with no housing facilities is of course deprived in the dimension. Also included in the deprived group are individuals who live in "kutcha" houses.[17] The NSS defines these houses as structures with walls made of material like grass, leaves, reeds, etc., and roofs made of similar materials.

4. **Type of sanitation facility and drainage**: Proper sanitation and drainage facilities can prevent the spread of diseases like diarrhea and malaria, etc. In some studies these two dimensions have been treated as one. This paper treats them as separate dimensions for the following reasons. First, the correlation coefficient between the dimensions of drainage and sanitation is around 0.35 (see Table 2). The second reason is conceptual - sanitation is a household characteristic whereas drainage typically concerns the living conditions of the entire neighborhood.

A person is deprived in the dimension of sanitation if she lives in a household without

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[16]A person is considered literate if she can sign her name.
[17]Kutcha in India means not firm/solid.

any sanitation facilities; so a person with access to a shared toilet is not deprived by this criteria. Even with such a minimalistic standard I find 62% of the population to be deprived. For drainage a person is deprived only if there is no drainage facility available in the area of her residence. This cutoff finds 47% of households to be deprived. In fact, this difference between deprivation in drainage and sanitation further justifies treating them as separate dimensions.

5. **Source of drinking water**: The household is asked about the primary source of drinking water. Since India is continually riddled with water shortages, access to drinking water cannot be taken for granted.[18] A person is considered to be deprived in this dimension if the primary source of drinking water is anything other than tap water or bottled water. By this cutoff almost 60% of the population are deprived in this dimension.

6. **Main cooking medium**: In India, even to this day, many households cook using firewood and dung-cakes. These individuals are more prone to respiratory problems.[19] These individuals are considered deprived in this dimension. Also, individuals without any arrangements for cooking (i.e. gatherers) are deprived.

Within a multidimensional framework, very closely related dimensions pose some problems. Table 2 contains the pair-wise correlations among the seven dimensions used here. It is readily checked that as such the correlations are low, with no individual correlation being higher than 0.55 thus allaying concerns of closely related dimensions. Note that the pair-wise correlation between income and the other dimensions is very low. This strengthens

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[18] It is shown in Jalan & Ravallion (2003) that access to piped drinking water reduces the chances of diarrhea among infants in India.

[19] Of course if the cooking is done in a separate room or in the open then the problem is less severe. However, the NSS is not able to give information on the arrangements of cooking beyond the cooking medium.

Table 2.: Correlation among dimensions.

Dimensions	Income	Education	Sanitation	Drainage	Housing	Drinking Water
Education	0.36					
Sanitation	0.28	0.33				
Drainage	0.37	0.35	0.35			
Housing	0.25	0.29	0.26	0.40		
Drinking Water	0.14	0.11	0.03	0.27	0.19	
Cooking Medium	0.43	0.44	0.42	0.55	0.39	0.25

This table gives the raw correlations between all pairs of dimensions.

the case for such a multidimensional perspective of deprivation.

The dual cutoff for the AF methodology requires us to specify the minimum number of dimensions an individual must be deprived in to be considered poor. The primary analysis in this paper is done using equal weights and so following the same idea a dual cutoff of four is also used. In other words, an individual is considered poor only if she is deprived in at least four of the seven dimensions. With this cutoff the percentage of households identified as poor is 50%.[20] The results in the next section are discussed in detail for equal weights on all dimensions and the dual cutoff of four. Results using other cutoffs (mainly 3 and 5) and weighting schemes are also discussed later.

Results

First, the multidimensional poverty estimates of the nation are presented and contrasted with the income poverty estimates. Next I discuss the disparity in the poverty levels across the states. These are contrasted with the income poverty estimates for the states. Finally, the difference in the deprivations across different religious groups are discussed.

The baseline results presented here impute equal weights for the dimensions. However several robustness exercises with respect to the weighting scheme have also been con-

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[20]This is much closer to the World Bank estimate of 42% of the population living below $1.25 a day in India than the official income poverty estimates for India.

ducted. For the comparisons of poverty across groups I find that varying the weight on income from 15% (equal weights) to 30% (and the remaining 70% divided equally among the other six dimensions) preserves the main findings of the paper. As the weight on income is increased further, the results of the multidimensional analysis closely resemble the results for income poverty alone. This is only natural since increasing the emphasis on income is tantamount to de-emphasizing the achievements in other dimensions. The fact remains that one can deviate a considerable amount from equal weighting of the dimensions and preserve the main findings of the paper.

Multidimensional poverty estimates for India

For the year 2003-04, the official income headcount ratio was 0.31, which implies that 31% of the population lives below the poverty line. When I take k as four (i.e. a person is identified as poor if she is deprived in four or more of the seven dimensions) around 50% of the people are multidimensional poor (see Table 3).[21] This is more in line with the World Bank finding of 42% poverty (using a $1.25 a day poverty-line) in India than the 31% income-poverty headcount.[22]

Table 3 also reveals that in the case of India, being multidimensional poor does not go hand in hand with being income poor. Among all the people who are poor in the multidimensional sense, using the union approach (that is k equal to one) only 31% are also

[21]When calculating the multidimensional poverty measure for the union approach (that is k=1) almost 90% of the population is identified as poor. In other words, 90% of the population is deprived in at least one of the seven dimensions. For the intersection approach, when k takes the value of seven, the number of people identified as poor is less than 3%. This suggests that looking at an intermediate dimension cutoff may yield more interesting insights.

[22]For income poverty the headcount ratio is the same as the FGT measure with alpha equal to zero. For the World Bank estimates see Poverty Data: A supplement to World Development Indicators 2008. Note for multidimensional poverty, the headcount ratio and the actual estimate of multidimensional poverty with α set to zero **diverge**. The multidimensional headcount is an indicator of the incidence of poverty but to gauge the depth of poverty in the multidimensional sense I look at the $P_0(k, z)$, the AF measure with alpha equal to zero. I have discussed the multidimensional headcount and how it varies with changes in k (Table 3). $P_0(k, z)$ for India is also described in Table 3. For the union approach (with k equal to 1) I have that $P_0(k, z)$ is equal to 0.466 and with k as four this estimate is 0.349.

Table 3.: Poverty Estimates for all k (All India).

Cutoff (k)	Headcount (in %)	P_0	Income Deprived (as a % of poor)
1	87.8	0.466	30.8
2	77.0	0.450	33.8
3	65.9	0.418	37.1
4	49.7	0.349	44.6
5	30.1	0.237	58.3
6	12.8	0.114	79.2
7	2.8	0.028	100.0

Notes: Column 2 gives the multidimensional Headcount measure. Column 3 gives the Multidimensional poverty measure and Column 4 gives the percentage of Multidimensional poor who are also deprived in the dimension of Income. These estimates use equal weights for all the dimensions.

Table 4.: Robustness checks with alternative weighting schemes with k fixed at 3.5: All India.

Weight on Income (in %)	Headcount	$P_0(k = 3.5)$	Percentage deprived in income
20	0.50	0.34	44.6%
25	0.52	0.34	47.0%
30	0.37	0.27	66.1%
35	0.37	0.27	66.1%
40	0.39	0.27	67.5%
45	0.29	0.23	90.7%

Notes: Each row uses a different weighting scheme. For example in the second row income has 20 % of the weight and the rest 80 % is equally divided among the rest of the six dimensions.

income deprived. Notice that the union approach identifies everyone who is income poor as multidimensional poor, but if one is not income poor but is deprived in any other dimension one is still counted as poor. At k equal to four, of all the people who are multidimensional poor only 45% are also income deprived (see Table 3). Thus this measure can identify people who lack access to basic facilities though they are not poor income-wise.

The results using alternative weighting schemes are presented in Table 4. Here the weight on income is progressively increased from 15% (under equal weights) to 45% (the rest 55% divided equally among the other 6 dimensions). Note that as the weight on income

increases the proportion of multidimensionally poor who are also income deprived rises from 45%(under equal weights) to 90%, when income itself accounts for 45% of the weight. This should not really come as a surprise since the correlation between multidimensional poverty and income poverty clearly rises when income starts to assume greater prominence among the dimensions.

Poverty estimates: Decomposition by State

There exist huge disparities among the Indian states with respect to dimensions such as growth, literacy and access to drinking water. The state of Kerala has achieved almost 100% literacy, whereas states like Bihar and Rajasthan have literacy rates below 50%. Bihar and Uttar Pradesh have gross state domestic product (GSDP) per capita which is half of that of Gujarat, Punjab, Haryana and Maharashtra. Disparities among the states have increased over the years with policies having differential impacts in the various states. For instance, the green revolution was initiated in India in 1965 to increase the yield in agriculture. The two states that benefitted the most were Punjab and Haryana. The success was muted in other parts of the nation; the BIMARU states (Bihar, Madhya Pradesh, Rajasthan and Uttar Pradesh) lagged behind.

There also exist differences among the states with respect to the success of poverty reduction schemes. This has been discussed in detail in Datt and Ravallion(2002). The poverty estimates for the 16 major states clearly reflect these disparities. For the analysis in this paper I will focus on these states, which together account for over 90% of India's population. I will exclude from the analysis the Union Territories and the states that are primarily mountainous.[23] Tables 5 and 6 give the income poverty and the multidimensional

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[23]This is done as these excluded regions have special features which need to be accounted for separately because of the terrain and the special treatment some of these regions receive from the central government. This, though interesting, is tangential to the main questions being addressed here.

poverty estimates for the 16 major states. I see from Table 5 that for Kerala, about 20% of the population have incomes below the poverty line. Secondly, Maharashtra is considerably worse than the other states. Table 5 clearly reveals that the states of Uttar Pradesh, Bihar, Madhya Pradesh and Orissa have the worst income-poverty levels in the country with 40% or more of the population making less than the subsistence minimum. On further exploration, it will become clear that the differences are not restricted to income alone.

Banerjee and Somanathan (2007) have found that over the 1970 and 1980s there has been a considerable equalization in the provision of public goods across states and ethnic groups. Despite this there exist differences at the state levels in access to basic facilities. In *Development as Freedom*, Amartya Sen gives a narrative of the accomplishments of Kerala's state government in providing basic amenities to its residents. Kerala has out-performed other states in reducing its mortality rates and increasing life expectancy at birth. These features are not (completely) captured by Kerala's average performance in reducing income poverty. The multidimensional measure better reflects the efforts of states like Kerala in improving the lot of its people. The Government of Rajasthan has taken measures to reduce the income poverty by initiatives to promote the area as a travel destination and putting emphasis on the marketing of the regional handicrafts industry. However other facilities required to improve well being have been neglected. As a consequence, people are more broadly deprived in dimensions like availability of drinking water, drainage, etc. Let us now turn to the specific results of the analysis incorporating these additional dimensions.

For the ease of exposition here I will use k of four (see Table 5). The results are similar for other values of k, specifically for k equal to three and five (see Table 6). Some remarks are in order here. First, though the poverty estimates for the states are very close to each other, the overall range is very large, from 0.118 for Punjab to 0.679

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Table 5.: Income vs Multidimensional poverty: States of India.

Income Based Measure		Multidimensional Measure	
State	Income Headcount	State	$P_0(k=4)$
Punjab	0.129	Punjab	0.118
Haryana	0.132	Haryana	0.150**
Gujarat	0.160	Kerala	0.156
Andhra Pradesh	0.172	Maharashtra	0.224***
Kerala	0.198	Tamil Nadu	0.261***
Tamil Nadu	0.237	Karnataka	0.275
Karnataka	0.279	Gujarat	0.289
Maharashtra	0.287	Andhra Pradesh	0.320*
Rajasthan	0.294	West Bengal	0.398***
West Bengal	0.346	Uttar Pradesh	0.408
Uttar Pradesh	0.373	Rajasthan	0.414
Madhya Pradesh	0.396	Madhya Pradesh	0.468***
Jharkhand	0.439	Jharkand	0.528***
Bihar	0.469	Bihar	0.538
Orissa	0.532	Chhattisgarh	0.572***
Chhattisgarh	0.543	Orissa	0.679***

Notes: Here the difference in poverty was tested between a particular state and the state immediately above it. For example, the difference in poverty between Haryana and Punjab was tested and was found to be significant at the 5% level. * represents significant at the 10% level and ** represents significant at the 5% level *** represents significant at the 1% level

for Orissa. Secondly, using the test developed in this paper I find that the differences in poverty estimates among the states are statistically significant for almost all pair wise comparisons.[24]

The states which perform very well using the income-based approach (like Punjab and Haryana) also happen to shine in the multidimensional framework; the same pattern exists for the bottom-ranking states. Notably, the maximum differences are among the middle-ranking states. There are states which greatly improve their relative standing vis-à-vis other states. The foremost example of this kind of change in ranking is the case of Kerala. I find that Keral is only out performed by Punjab, Haryana (the two agricultural success stories of the nation) in providing basic services to its people. Maharashtra improves its relative standing significantly when multidimensional poverty is measured rather than

[24]There are 128 combinations which are tested. The details of the test statistics are available from the author on request. Almost all difference are significant except a few including the following. It is not possible to statistically distinguish the poverty estimate of Tamil Nadu (0.261) from that of Karnataka (0.275). Also, for Jharkhand (0.528) and Bihar (0.538) the multidimensional poverty estimates can not be statistically distinguished from each other.

income poverty. In the multidimensional framework Maharashtra, in fact, does better than Gujarat (one of the most prosperous states in the nation) and all the states in the south.

Let us now turn to the states that have done worse in this new framework. Gujarat for instance is just below the two states with the lowest income poverty, namely, Punjab and Haryana. However it does considerably worse than Maharashtra, Kerala and several other states in terms of the multidimensional poverty measure P_0. Andhra Pradesh which is one of the more prosperous states also fails to impress in the multidimensional analysis. In fact, Andhra Pradesh's multidimensional poverty level is not far from those of the BIMARU states.[25]

Table 6 shows that results are similar if the k value is set at 3 or 5, and hence the results are largely robust to changes in the poverty cutoff. All these estimates give equal weights to all the seven dimensions. A second form of robustness involves the use of alternative weighting schemes for the dimensions. Table 20 (in the appendix) gives the estimates of poverty levels as the weight on income is progressively increased from 20% to 45% (and the other weights are proportionately decreased). Each column of this table represents a different weighting scheme. For ease of comparison, in this table the k value is fixed at 3.5. Table 21 (in the appendix) provides the rank of each state corresponding to the poverty levels in Table 20. What is notable is that the ranking of the states remains unaltered as one moves from equal weights to 30% weight on income alone.[26] In short, the results presented here are not particularly sensitive to the choice of k or the weighting scheme used.

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[25]This is perhaps not unexpected. Gujarat has witnessed severe outbreaks of Hindu-Muslim violence over the decades (which seems to have intensified in the last 20 years); this clearly results in destruction of social capital apart from creating widespread mistrust between religious groups. Also, successive state governments in Andhra Pradesh have been accused of favoring urban development (especially by promoting IT-led growth) at the expense of rural agriculture.

[26]When more than 30% weight is given to income, the ranking gradually start resembling the ranking generated by the income headcount. This is hardly unexpected.

Table 6.: Multidimensional poverty in the States of India: Alternative values of k.

State	$P_0(k=3)$	State	$P_0(k=5)$
Punjab	0.191	Haryana	0.051
Haryana	0.263***	Kerala	0.057
Maharashtra	0.289	Punjab	0.057
Kerala	0.305***	Maharashtra	0.126***
Tamil Nadu	0.334	Karnataka	0.145**
Gujarat	0.340	Tamil Nadu	0.145
Karnataka	0.362***	Gujarat	0.170
Andhra Pradesh	0.391***	Andhra Pradesh	0.201**
West Bengal	0.463***	Uttar Pradesh	0.265***
Rajasthan	0.473	Rajasthan	0.298*
Uttar Pradesh	0.486	West Bengal	0.306
Madhya Pradesh	0.517***	Madhya Pradesh	0.341***
Jharkand	0.566***	Bihar	0.417***
Bihar	0.591***	Jharkand	0.425***
Chhattisgarh	0.613	Chhattisgarh	0.456
Orissa	0.700***	Orissa	0.605***

Notes: Here the difference in poverty was tested between a particular state and the state immediately above it. For example, the difference in poverty between Haryana and Punjab was tested and was found to be significant at the 5% level. * represents significant at the 10% level and ** represents significant at the 5% level *** represents significant at the 1% level

Poverty estimates: By religious groups

Traditional income poverty analysis has found Muslims to be poorer than Hindus (on average) in India (see for example Noland, 2005). This result does not hold for multidimensional poverty measure P_0. Multidimensional poverty is unambiguously higher for Hindus than Muslims in India, this result holds for k values of 1 through 5. For k values of 6 and 7 there is very little difference in poverty between the two groups (see Table 7). The actual estimates for the two religions are numerically very close to each other. I use the test proposed in the previous Section and find the differences are not significant for all values of k. For the union approach (with k equal to 1) the difference between Hindu and Muslim poverty is not significant. Using the intersection approach, the estimates are very small as well. However for intermediate values of k, specifically k equal to 3 and 4 the difference is

Table 7.: Mutlidimensional Poverty for Hindus and Muslims.

Cutoff (k)	Headcount (Hindu)	Headcount (Muslim)	P-value	P_0 (Hindu)	P_0 (Muslim)	P-values
1	0.878	0.906	0.002	0.474	0.463	0.230
2	0.776	0.777	0.941	0.459	0.444	0.134
3	0.673	0.638	0.019	0.430	0.404	0.022
4	0.516	0.460	0.001	0.362	0.328	0.006
5	0.312	0.283	0.056	0.246	0.227	0.132
6	0.130	0.141	0.410	0.116	0.125	0.398
7	0.029	0.033	0.516	0.029	0.033	0.516

The p-values given are for the test of differences in the poverty levels between Hindus and Muslims for the given value of k.

highly significant.[27]
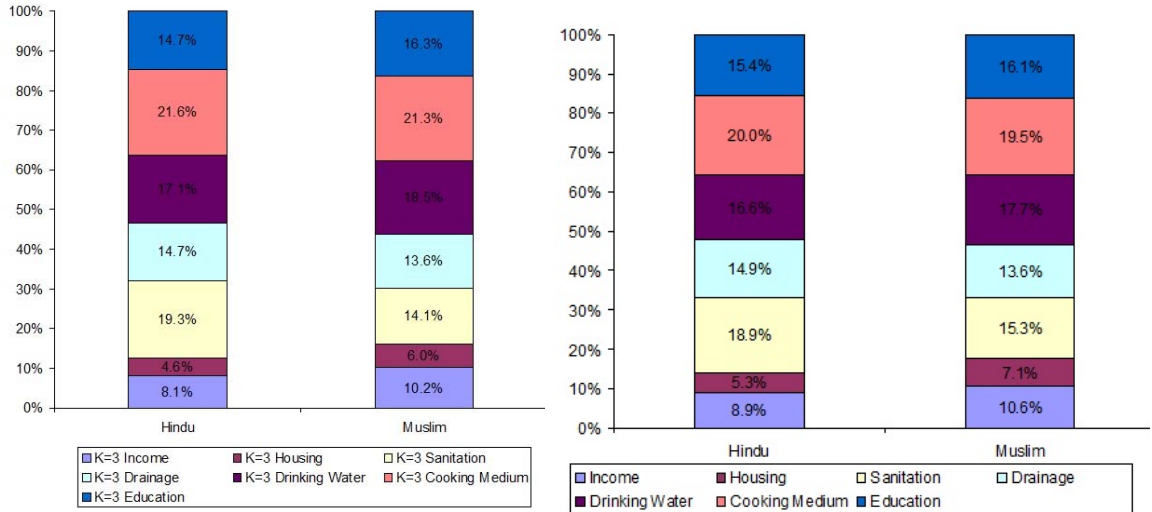
Figure 1.: Dimensional Decomposition of poverty. Notes The left panel gives the contributions of each of the dimensions in the poverty levels of Hindus and Muslims at k equal to 3. The right side panel gives the contributions of the dimensions when k is fixed at 4.

P_0 provides results contrary to previous findings so the following question naturally arises. Is it the case that some particular dimensions are driving these results? To answer this, I decompose the poverty estimates for each group in terms of the contributions of

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[27]Now looking at the multidimensional head count (in the left portion of the table), which gives the proportion of the population which is multidimensional poor, I see that again, a higher proportion of Hindus (compared to Muslims) are multidimensional poor. Again the difference in the values is not significant for all values of k. I find that a **significantly** higher proportion of Hindus are poor when k is equal to 3, 4, or 5. (also see Table 7). At k equal to one (which gives the union approach) I find a higher proportion of Muslims to be poor. However at this k value almost 90% of the population is poor, which seems very unrealistic according to standard notions of poverty.

each of the dimensions. Figure 1 gives the percentage contribution of each dimension in multidimensional poverty (for k equals 3 and 4) for the Hindus and Muslims respectively. It is clear from the figure that the contribution of the dimensions of sanitation and drainage to Hindu poverty (34%) is higher than that for Muslim poverty(28%). For Muslims, income and education contribute more to overall poverty than they do for the Hindus.

A related question concerns the condition of low caste Hindu households as compared to Muslims and high caste Hindus. It is important to bear in mind that the caste system applies only to the Hindu population of India. Figure 2 shows graphically the levels of poverty among the three groups - High caste Hindus, low caste Hindus, and Muslims. High caste Hindus have the lowest levels of poverty followed by Muslims; the low caste Hindus are the poorest. This result is very robust and holds for all values of k. The right-hand panel of Figure 2 gives the dimensional decomposition of poverty across the three groups. From the picture its is clear that the dimensional contributions of low caste Hindus and Muslims are similar.[28]
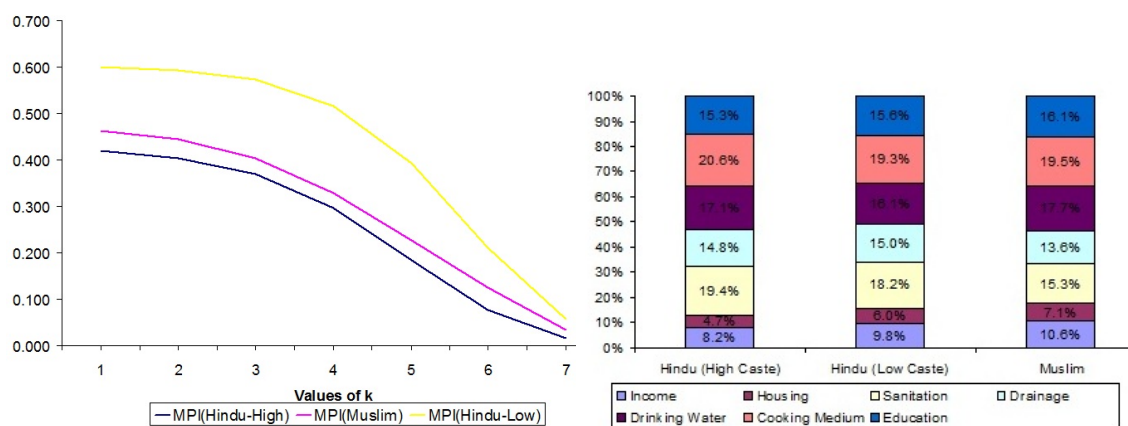
Figure 2.: Contribution of each dimension in Multidimensional Poverty For High Caste Hindus Low caste Hindus and Muslims.

[28]There is anecdotal evidence suggesting that in rural India low caste Hindus engage in more social interactions with Muslims than with high caste Hindus. Often they reside in contiguous neighborhoods which are some distance away from the neighborhoods populated by high caste Hindus.

The population can be split on the basis of different markers as well. For example I can compare Hindu and Muslim poverty across urban and rural areas. Table 22 in the appendix gives the estimates of rural and urban poverty of Hindus and Muslims separately. It should come as no surprise that for both religions the rural poverty is higher than urban poverty. However in rural India, poverty among Hindus and Muslims is indistinguishable (as can be seen by checking the corresponding p-values in the table). The overall differences (urban and rural) between Hindus and Muslims is probably coming from the fact that rural poverty is greater than urban poverty and a greater proportion of Muslims reside in urban areas than in rural areas as compared to Hindus. Once the Hindu population is decomposed along the caste dimension, it is evident that high caste Hindus are the least poor, followed by Muslims. Also, low caste Hindus are the poorest in both rural and urban areas. [29]

The analysis so far as assumed equal weights for all dimensions. I redo the robustness checks as before in the comparison of poverty across states. Tables 24, 26, 25, and 23 in the appendix represent the results for the alternate weighting schemes. It is clear from here that the main findings are preserved for a weight on income of up to 30%. A higher weight on income reverts the results back to the comparisons based on income alone.

Conclusion

If economic development is about improving living standards, broadly conceived, then one cannot neglect other achievements, that have a direct impact on well being such as direct access to public facilities. To understand whether income growth in developing countries has actually generated broad-based one has to look beyond poverty measures which are solely income-based. This paper focuses on a measure of multidimensional poverty

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[29]See table 23, in the appendix.

developed by Alkire and Foster (2010) that is based on an individual's access to basic public goods like education, sanitation and drainage as well as income to provide a fresh empirical perspective on the "trickle-down" issue.

Multidimensional measures of deprivation are computed for the 16 major states in India and their relative rankings by this measure are provided. Interestingly, this ranking turns out to be "quite different" from that provided by income poverty measures for the same 16 states. The maximum differences in ranks is among the states that are "middle-income" which resonates with the growth literature in cross-country studies (see for example Quah, 1997, Bulli, 2001).

Kerala does much better when using the multidimensional framework as compared to income poverty rankings. This is not surprising given the success of the literacy programs and other public policies implemented in the state (Sen, 1999). When using the income poverty approach Maharashtra does not do remarkably well. However, under the multidimensional framework only Punjab, Haryana and Kerala have less poverty than Maharashtra. Andhra Pradesh is one of the states that have done rather poorly under the multidimensional framework. This seems to give credence to the claim that the income growth the state had achieved over that past ten years has not benefitted the masses.[30] Similarly, Rajasthan, which is the success story for state initiative in industry, seems to do considerably worse in this framework.

The poverty estimates are also computed for the two predominant religious groups - Hindus and Muslims. I find that Muslims are less multidimensionally poor than Hindus irrespective of the value of k used, though the results were statistically significant only for k equal to 3 and 4. This is totally at odds with the income based approach which has always

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[30]The state government in Andhra Pradesh during that period stressed "IT (information technology)-led" growth which was primarily geared towards urban areas and possibly came at the expense of the rural sector.

found Muslims to be the poorer group. Looking at urban areas alone I see that Muslims are poorer than the Hindus. Access to facilities like sanitation, drainage, drinking water, etc. is more uniform in urban areas the it is in rural areas. Given that differences are less in the other dimensions, the income poor group (Muslims) are also the one who are poorer in the multidimensional sense. On the other hand in rural areas I cannot distinguish between Hindu and Muslim poverty levels. However low caste Hindus do exceptionally worse than high caste hindus and Muslims in general.

I have tested the robustness of all the results imposing progressively more weight on income compared to other dimensions. The findings are fairly robust to these alternative weighting schemes. Only for weighting schemes that allot more than 30% weight on income do these differences in Hindu-Muslim poverty or any other comparisons disappear. At this point the results mirror the income poverty estimates which does not come as a surprise since I am basically reducing the emphasis on the other dimensions.

By pointing out the divergence between rankings based on income poverty and multidimensional poverty this paper highlights the need to empirically re-examine the "trickle-down" issue for developing countries. The analysis here reiterates the importance of focusing on measures of direct access to public facilities as a means of improving living standards and capabilities. Continued complacency with income-based poverty measures may prevent us from successfully tapping the productive potential of the entire society.

CHAPTER III

MULTIDIMENSIONAL POVERTY: MEASUREMENT, ESTIMATION, AND
INFERENCE

Introduction

Multidimensional poverty measures give rise to a rich set of testable hypotheses. In this paper, I formulate a variety of these hypotheses - in the specific context of the measure of Alkire and Foster (2010) - which are likely to be of particular interest to applied economists and policy-makers alike. More importantly, I introduce a unified framework for developing statistical tests of these and other related hypotheses.

Governments of several nations, including those of India and Mexico, as well as numerous non-governmental agencies are in the process of adopting multidimensional measures of poverty to complement their traditional income (or consumption) analysis. The adoption of a multidimensional approach is largely in response to arguments that income alone does not completely identify the poor, and that there are other dimensions which are relevant to the well-being of individuals. The goal of a multidimensional approach to poverty analysis, therefore, is to move beyond the traditional univariate approach to incorporate additional relevant indicators of well-being.

Following Sen (1976), poverty measurement has been viewed as a two step procedure involving both an identification and an aggregation step. Identification grapples with the question: Who is poor? This involves the notion of poverty lines, whereby the individuals below a poverty line are identified as poor. In the multidimensional approach of Alkire and Foster (2010), however, two cutoffs must be considered for identification. First,

for each dimension, a dimension-specific poverty line identifies the individuals deprived in that particular dimension. The second cutoff determines the number of dimensions, k, in which one must be deprived before they are considered (multidimensionally) poor. The measures of Bourguingnon and Chakravarty (2005) and Tsui (2002), for example, adopt a union approach to identification whereby any individual who is deprived in at least one dimension is considered poor. In other words, their second cutoff is simply one dimension of deprivation. In practice, however, the union approach often identifies substantially essential high proportions of various populations as poor. In some instances, the union approach has been found to identify more than 90% of a population as poor (Mitra, 2009).

Alkire and Foster (2010) recently proposed a new class of multidimensional poverty measures based on the FGT class of unidimensional poverty measures. The AF measure is remarkably simple, both conceptually and computationally. In the identification stage, the AF measure involves selecting the second cutoff k to be any value between one (the union approach) and the maximum number of dimensions d (the intersection approach). The aggregation stage is then based on the FGT framework and thus retains many of the desirable properties of the FGT class of measures. Among these properties is decomposability of the overall poverty measures among sub-groups of the population. This property is essential, for example, when one wishes to compare poverty across sub-regions or ethnic groups.

The Alkire-Foster methodology has also recently been applied in several empirical studies; see, e.g., Alkire and Seth (2008), Santos and Ura (2008), and Betana (2008). However, these papers are primarily descriptive in nature due, largely, to a lack of available statistical testing procedures.[1] The present paper fills this void not only by formulating a variety of novel and interesting statistical hypotheses in this context, but also by contribut-

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[1] In contrast, statistical tests relating to the univariate approach to poverty analysis are well established; see, for example, Anderson(1996), Davidson and Duclos (2000), Barrett and Donald (2003), and Linton, Maasoumi and Whang (2005).

ing to the literature a general framework for developing statistical tests of these and related hypotheses. A distinguishing feature of this work is the emphasis on multiple testing procedures which enable users to identify from within a collection of hypotheses those which are not supported by the data. It is my contention that multiple testing procedures are of particular relevance in the context of multidimensional poverty analysis. Inferring, for instance, the specific range of poverty lines over which a poverty ordering holds, the sub-collection of measures over which a poverty ordering holds, or the specific dimensions (e.g. income, health, education) in which a country or region is under-performing, are of greater policy relevance than whether the ordering fails for some (possibly unidentified) poverty line, measure, or dimension. In contrast, most procedures currently applied in the context of poverty analysis are joint tests which permit us to draw less informative inferences. Betana (2008) for example, tests whether a poverty ordering based on the headcount ratio is consistent over a collection of poverty lines. Betana (2008) approach, which is based on the empirical likelihood ratio test developed in Davidson and Duclos (2006), allows him to infer only that the hypothesized ordering is violated without necessarily providing any compelling evidence concerning which poverty line(s) might suggest a reversal in the hypothesized ordering.

In contrast, I show that the recently introduced multiple testing procedures of Bennett (2010) are particularly well-suited to simultaneous testing of the hypotheses which arise naturally in the context of multidimensional measures.[2] The principle advantage of adopting multiple testing procedures is that, unlike the popular Wald-type tests (Wolak, 1989, Kodde and Palm,1986)e.g., for example, they offer compelling evidence concerning the source(s) of rejection whenever rejection of the joint intersection hypothesis occurs. The

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[2]For concreteness, I have chosen to frame the discussion in the context of the Alkire-Foster measure. However, the methodology may also be extended, for example, to test hypotheses that arise from the multidimensional orderings of Maasoumi and Lugo (2008) and Duclos, Sahn and Younger (2006). See also Kakwani and Silber (2008) for an overview of these and other approaches.

advantage of adopting the MinP procedure of Bennett (2010) , in particular, is that this test is shown capable of correctly identifying more false hypotheses (sources of rejection) than competing multiple testing procedures. Specific examples treated in this paper include (though are not limited to) simultaneous tests of the poverty ordering for various parameterizations of the Alkire-Foster measure (e.g. robustness to choice to poverty lines and/or k), simultaneous tests of poverty ordering of various populations relative to a benchmark population, and simultaneous tests of dimension-specific (e.g., health, income, education) poverty ordering.

To illustrate the methodology developed in this paper, I use the National Sample Survey (NSS) 60th round health and morbidity data to study the differences in multidimensional poverty among Hindus and Muslims in urban India. Two separate sets of hypotheses are tested. The first corresponds to a robustness check on the second cutoff (k). I find that for lower values of k, Muslims are poorer than Hindus. This is in accordance with income based poverty comparisons which have generally found Muslims to be more deprived. Interestingly, for higher values of k the results suggest that Hindus are, in fact, poorer than Muslims. In other words, a greater proportion of Hindus suffer from extreme poverty . To further the understanding of this reversal, I also investigate which of the dimensions may be responsible. Thus, the second set of tests correspond to a simultaneous test of the component ordering for fixed values of k. The results here suggest that for higher values of k, the difference in the contribution of income to Hindu and Muslim poverty is small (sometimes even insignificant), and that the reversal in the poverty ordering among the two ethnic groups is driven primarily by dimensions other than income. These results, while interesting in and of themselves, serve to highlight the rich empirical welfare analysis that can be conducted by coupling the statistical methodology with a multidimensional approach

to poverty.

The remainder of this paper is as follows. In the next section I formulate a generalized version of the recently proposed Alkire-Foster class of multidimensional poverty measures. Subsequently I discuss the formulation of a variety of statistical hypotheses and show that they may be treated in a unified manner. Section 4 develops suitable test statistics and the related asymptotics. Section 5 provides a discussion of the implementation of the minimum p-value methodology, which is followed by an empirical illustration in Section 6.

Formulation

Let $X = (X_1, \ldots, X_d)$ denote a random draw from a population with joint distribution of achievement F. The components of X may be ordinal or cardinal. Without loss of generality I assume that the first $d_1 \leq d$ components of the random vector X are ordinal whereas the remaining $d - d_1$ are cardinal. For a fixed k, $1 \leq k \leq d$, a pre-specified vector of poverty lines $\ell \in (0, \bar{\ell}]^d$, and a $d \times 1$ vector of "weights" denoted by ω, I formulate the multidimensional headcount ratio and generalized AF multidimensional poverty measures as

$$H(\ell, k, \omega, F) = \mathbb{E}_F \left[\mathbb{1} \left(\sum_{j=1}^{d} \omega_j \mathbb{1}(X_j \leq \ell_j) \geq k \right) \right], \tag{III.1}$$

and

$$P_\alpha(\ell, k, \omega, F) = \frac{1}{d} \mathbb{E}_F \left[\mathbb{1} \left(\sum_{j=1}^{d} \omega_j \mathbb{1}(X_j \leq \ell_j) \geq k \right) \left(\sum_{j=1}^{d_1} \omega_j \mathbb{1}(X_j \leq \ell_j) \right) \right]$$

$$+ \frac{1}{d} \mathbb{E}_F \left[\mathbb{1} \left(\sum_{j=1}^{d} \omega_j \mathbb{1}(X_j \leq \ell_j) \geq k \right) \left(\sum_{j=d_1+1}^{d} \omega_j \left(\frac{\ell_j - X_j}{\ell_j} \right)^\alpha \mathbb{1}(X_j \leq \ell_j) \right) \right].$$

For a given choice of k, ω, and ℓ, I see that under either measure an individual with observed vector of achievement $X = (X_1, \ldots, X_d)$ is identified as poor only if $\sum_{j=1}^{d} \omega_j \mathbb{1}(X_j \leq \ell_j) \geq k$. Identification thus involves a dual cut-off approach. In the first step, deprivation in dimension j is determined by comparing the level of achievement in dimension j to the corresponding poverty line. In the second stage, an individual is identified as being poor only if the weighted (by ω) sum of the indicators of dimension-specific poverty are at least equal to the multidimensional poverty threshold k.

When the dimensions are given equal weight (i.e. when ω equals the unit vector in $\mathbb{R}^d$), $H(\ell, k, \omega, F)$ is simply the proportion of the population that is deprived in k or more dimensions; or equivalently the probability that a randomly drawn person from population F is deprived in k or more dimensions. Alternatively, the measure $P_\alpha(\ell, k, \omega, F)$, for $\alpha > 0$, is a weighted sum of $H(\ell, k, \omega; F)$ where the individual weights correspond to FGT-type measures (Foster, Greer, and Thorbecke 1984) of the individual dimensions, thus allowing for the "depth" of deprivation to enter into the overall assessment of poverty. Greater values of α correspond to greater emphasis being placed on the "depth" of deprivation or equivalently greater emphasis being placed on the poorest of the poor. When $\alpha = 0$, $P_\alpha(\ell, k, \omega, F)$ reduces to a weighted sum of $H(\ell, k, \omega, F)$ where the weights are simply the probabilities of being deprived in each of the dimensions under consideration.[3]

Varying ω away from the unit vector amounts to a rescaling of the importance attributed to the various dimensions of poverty. For instance, if $\omega = (2, 0.5, 0.5, 0.5)$ and $k = 3$, then an individual is identified as poor only if they are deprived in the first dimension along

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[3]In some situations, it may be of interest to allow the value of α to be dimension-dependent. Although I have not formulated $P_\alpha(\cdot)$ to explicitly account for this possibility, I note that such an extension can easily be accommodated.

with being deprived in at least two other dimensions. Thus, deprivation in dimension one becomes a necessary condition for identification under this weighting scheme. In contrast, I see that under the equally weighted scheme the same individual would be identified as poor only if they are deprived in at least any three of the four dimensions. Thus, the choice of ω (and, of course, k) plays a crucial role in the identification of deprived individuals.

In addition to being intuitive and simple to compute, the Alkire-Foster measure also possesses the desirable properties of both subgroup and dimension-specific decomposability. For example, if Z is a discrete random variable with $Z = i$ denoting membership in subgroup i, then I may write the poverty measure as a weighted sum of the subgroup contributions to overall poverty, i.e.

$$P_\alpha(\ell, k, \omega, F) = \frac{1}{d} \sum_i \mathbb{E}_F \left[\mathbb{1} \left(\sum_{j=1}^d \omega_j \mathbb{1}(X_j \le \ell_j) \ge k \right) \times \right.$$
$$\left. \left(\sum_{j=1}^{d_1} \omega_j \mathbb{1}(X_j \le \ell_j) + \sum_{j=d_1+1}^d \omega_j \left(\frac{\ell_j - X_j}{\ell_j} \right)^\alpha \mathbb{1}(X_j \le \ell_j) \right) \mid Z = i \right] P(Z = i).$$

$$\text{(III.3)}$$

The values of $H(\ell, k, \omega, F)$ and $P_\alpha(\ell, k, \omega, F)$ are clearly influenced by the parameters ℓ, ω, α, and k, about whose values there may be considerable disagreement. Consequently, it may be of interest, for example, to test the robustness of a Alkire-Foster poverty ordering of two populations to changes in these parameter values. The formulation of such hypotheses is the subject of the next section.

Hypotheses

Let G denote the joint distribution of achievement of a population which is to be compared to that of F. Tests of multidimensional poverty ordering will invariably in-

volve hypotheses that are formulated based on the difference between $H(\ell_F, k, \omega, F)$ and

$H(\ell_G, k, \omega, G)$, $P_\alpha(\ell_F, k, \omega, F)$ and $P_\alpha(\ell_G, k, \omega, G)$, or the difference between several such

population parameters.[4] In this section, I outline the basic structure of the statistical hy-

potheses which are treated in this paper. I begin with a number of specific examples that

are likely to be of particular interest to practitioners.

Example 1 (Poverty Component Analysis) *Due to the composite nature of the measures, inferring, for example, that $P_\alpha(\ell_G, k, \omega, G) > P_\alpha(\ell_F, k, \omega, F)$ invariably leads to the question: "In which dimensions is the population G worse off?" Consequently, it may be of greater interest to consider both the P_α-ordering and the dimension specific ordering via a simultaneous test of the $d + 1$ hypotheses*

$$H_0 : P_\alpha(\ell_G, k, \omega, G) - P_\alpha(\ell_F, k, \omega, F) \leq 0$$

and

$$H_s : P_{\alpha,s}(\ell_G, k, \omega, G) - P_{\alpha,s}(\ell_F, k, \omega, F) \leq 0 \text{ for } 1 \leq s \leq d,$$

where the additional subscript "s" on the measure P_α denotes the sth dimension's contribution to the poverty measure.

Example 2 (Robustness) *In empirical work researchers often observe the poverty ordering reverse when the value of α or k is adjusted. When this does not occur and the ordering is consistent for all plausible values of α and k, the ordering is said to be robust. Along the lines of the previous example, robustness over (say) α may be tested via a simultaneous test of*

$$H_s : P_{\alpha_s}(\ell_G, k, \omega, G) - P_{\alpha_s}(\ell_F, k, \omega, F) \leq 0 \text{ for } 1 \leq s \leq S.$$

Clearly, testing for robustness over k is analogous, with the test being over various values of k as opposed to various values of α.

Example 3 (Poverty Orderings relative to a Benchmark) *For a given poverty measure, say $P_\alpha(\cdot)$, an analyst may wish to identify those populations which have less poverty than a benchmark population F_0. Letting $F_1, \ldots, F_S$ denote the various populations that have been chosen for comparison the testing problem can be formulated as a simultaneous test of the S hypotheses*

$$H_s : P_\alpha(\ell_{F_s}, k, \omega, F_s) - P_\alpha(\ell_{F_0}, k, \omega, F_0) \leq 0 \text{ for } 1 \leq s \leq S.$$

The theme which is common to these (and many other) examples is that the

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[4]The subscript on the poverty line vector highlights the fact that I allow for the pre-specified (exogenous) poverty lines to differ across any two populations.

hypotheses of interest may be written in the general form

$$E_P[m(X;\theta)] \leq 0$$

where m is a vector-valued function, X is a random vector with distribution P, and θ is a vector of (known) parameter values. This observation suggests that in the discussion of statistical testing I may treat these and other seemingly disparate tests in a unified manner; i.e., as simultaneous tests of multiple inequalities.

Estimation and Asymptotics

Fundamental to the testing procedures is the estimation of the multidimensional headcount ratio and generalized Alkire-Foster (AF) poverty measures for various configurations of the exogenous parameters α, ℓ, ω, and k. In this section I discuss the estimation strategy and I also establish the joint asymptotic distribution of the resulting estimators. Since the specific estimators of interest and the associated joint distribution will invariably depend upon the particular hypothesis under consideration, the asymptotic analysis here is most aptly handled by treating the empirical poverty measures as a stochastic process in the exogenous parameters and applying techniques from the empirical process literature for their analysis. I therefore begin this section by introducing an empirical process which nests many statistics, including for instance those pertinent to examples 1 through 3, as special cases. Then, by establishing the weak convergence of this process, the joint asymptotic normality of the statistics of interest may be obtained as simple corollaries.

In the analysis, I treat both the case of mutually dependent samples as well as the case of independent samples, the former being relevant in examining the evolution of poverty of a single group (e.g. changes in poverty over time), whereas the latter is relevant

in comparing poverty across any two groups (e.g. cross-national) where sampling is done independently within each group. For the sake of exposition I will assume, without loss of generality, that the number of populations under consideration in any given hypothesis is less than or equal to three. I begin the analysis with the dependent case.

Dependent Samples

Let $(X_1, Y_1, Z_1), \ldots, (X_n, Y_n, Z_n)$ be i.i.d. copies of a $3d \times 1$ random vector with distribution P and d-dimensional marginal cdfs F, G, and H. I denote by $\mathbb{P}_n$ the empirical measure based on a sample of size n from P, and I introduce the poverty vector functions $m_i : (x, y, z) \in \mathbb{R}^{3d} \to \mathbb{R}^{d+2}$, $i = 1, 2, 3$ which I define by

$$
m_1(x, y, z; \ell, k, \omega, \alpha) = \begin{pmatrix} \mathbb{1}\left(A(x)\right) \\ \mathbb{1}\left(A(x)\right) \frac{1}{d} \left[\sum_{j=1}^{d_1} \omega_j \mathbb{1}(x_j \leq \ell_j) + \sum_{j=1+d_1}^{d} \omega_j \left(\frac{\ell_j - x_j}{\ell_j}\right)^\alpha \mathbb{1}(x_j \leq \ell_j) \right] \\ \mathbb{1}\left(A(x)\right) \omega_1 \mathbb{1}(x_1 \leq \ell_1) \\ \vdots \\ \mathbb{1}\left(A(x)\right) \omega_d \left(\frac{\ell_d - x_d}{\ell_d}\right)^\alpha \mathbb{1}(x_d \leq \ell_d) \end{pmatrix},
$$

(III.4)

$$
m_2(x, y, z; \ell, k, \omega, \alpha) = m_1(z, x, y; \ell, k, \omega, \alpha),
$$

and

$$
m_3(x, y, z; \ell, k, \omega, \alpha) = m_1(y, z, x; \ell, k, \omega, \alpha),
$$

where $A(x) = \left\{ \sum_{j=1}^{d} \omega_j \mathbb{1}(x_j \leq \ell_j) \geq k \right\}$. Thus, m_i for $i = 2, 3$ is obtained from m_{i-1} through a cyclical permutation of the three $d \times 1$ arguments x, y, and z. For a fixed choice of parameters $(\ell, k, \omega, \alpha)$ the poverty vectors associated with the F, G, and H distribu-

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tions are simple population means which may be estimated in a straightforward manner as $\mathbb{P}_n m_1(x, y, z; \ell, k, \omega, \alpha)$, $\mathbb{P}_n m_3(x, y, z; \ell, k, \omega, \alpha)$, and $\mathbb{P}_n m_2(x, y, z; \ell, k, \omega, \alpha)$, respectively.[5]

In each of the examples considered in the previous section, appropriate test statistics of the individual hypotheses may be derived from

$$\sqrt{n}\mathbb{P}_n[m_i(x, y, z; \ell_i, k, \omega, \alpha) - m_j(x, y, z; \ell_j, k, \omega, \alpha)], \tag{III.5}$$

for some $i, j \in \{1, 2, 3\}$ and some configuration of the parameters (k, ω, α). Consequently, a treatment of the asymptotic behavior of the seemingly disparate cases may be handled in a uniform manner by viewing (III.5) as a stochastic process in the parameters and applying to it results from the empirical process literature. To this end, I begin by introducing the class of real-valued functions

$$\mathcal{F}_i = \{\langle m_i(x, y, z; \ell, k, \omega, \alpha), h \rangle : \ell \in [0, \bar{\ell}]^d,$$
$$k \in [\underline{k}, \bar{k}], \sum \omega_i = d, \omega_i \geq 0, h \in [0, 1]^{d+2}\} \tag{III.6}$$

where i is a fixed integer belonging to the set $\{1, 2, 3\}$ and $\langle \cdot, \cdot \rangle$ denotes the scalar product of two vectors. The goal is to establish that $\mathcal{F}_i$ is a Donsker class and hence that the empirical process $\{\sqrt{n}(\mathbb{P}_n - P)f : f \in \mathcal{F}_i\}$ converges weakly to a mean-zero Gaussian process in $\ell^\infty(\mathcal{F}_i)$. Establishing this result, which I state formally as Theorem 1 below, will enable us to obtain as corollaries a number of convergence results which will prove particularly useful in the development of various statistical tests of interest.

Theorem 1 *Suppose* $(X_1, Y_1, Z_1), \ldots, (X_n, Y_n, Z_n)$ *are i.i.d. copies of a* $3d \times 1$ *random vector with distribution* P. *Then, the class of functions* $\mathcal{F}_i$ *defined in (III.6) is* P-*Donsker for* $i \in \{1, 2, 3\}$.

Theorem 1 can be used to derive several important results. First, by defining the

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[5] For a given probability measure P, the notation Pf denotes the expectation with repect to P, i.e., $Pf := \int f dP$.

class of functions

$$\mathcal{F}'_i = \{ - \langle m_i(y, x, z; \ell, k, \omega, \alpha), h \rangle : z \in [0, \bar{z}]^d, $$

<div align="right">(III.7)</div>

$$k \in [\underline{k}, \bar{k}], \sum \omega_i = d, \omega_i \geq 0, h \in [0, 1]^{d+2}, 1 \leq \alpha \leq 3 \}$$

I obtain via Theorem 1 and Donsker preservation under addition (Kosorok 2008, p.173) that the empirical process

$$\{ \sqrt{n}(\mathbb{P}_n - P) f : f \in \mathcal{F}_1 + \mathcal{F}'_2 \}$$

converges weakly to a tight Gaussian process in $\ell^\infty(\mathcal{F}_1 + \mathcal{F}'_2)$. Since finite dimensional convergence is necessary for weak convergence of the empirical process, I immediately obtain, for example, the convergence of $\{ \sqrt{n}(\mathbb{P}_n - P)(f_1, \ldots, f_S) \}$ to a S-dimensional mean-zero normal distribution provided $f_s \in \mathcal{F}_1 + \mathcal{F}'_2$ for $s = 1, \ldots, S$. The connection to the testing problem is made upon noticing that an element, say f, of $\mathcal{F}_1 + \mathcal{F}'_2$ is of the form

$$f = \langle m_1(x, y, z; \ell_F, k, \omega, \alpha), h \rangle - \langle m_2(x, y, z; \ell_G, k, \omega, \alpha), h' \rangle,$$

and hence, for $h = h' = (1, 0, \ldots, 0)$ or $h = h' = (0, 1, 0, \ldots, 0)$, the scaled and centered random quantity $\sqrt{n}(\mathbb{P}_n - P)f$ is nothing other than the scaled and recentred difference between the estimates of $H(\ell_F, k, \omega, F)$ and $H(\ell_G, k, \omega, G)$, or $P_\alpha(\ell_F, k, \omega, F)$ and $P_\alpha(\ell_G, k, \omega, G)$, respectively.

Notice that Example 3 is a slight variation on the above themes in that it involves a comparison between several populations. In order to subsume Example 3, I introduce the class of functions $\mathcal{G}_j = \mathcal{F}_1 + \mathcal{F}'_j$ and denote by $\mathcal{H}$ the class of functions

$$\{ \langle f, \lambda \rangle : f \in \mathcal{G}_2 \times \mathcal{G}_3, \lambda \in [-1, 1]^2 \}.$$

which is also P-Donsker under the conditions of Theorem 1. The application of these results

to the testing problems are now made explicit by revisiting the earlier examples:

Example 4 (Example 1 continued) *Let h_i denote the ith standard basis vector in $\mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ satisfy $\sum \omega_i = d$, $\ell_G, \ell_F \in (0, \bar{\ell}]^d$, and α be a fixed positive integer. Then each member of the finite collection*

$$\{\langle m_1(x, y, z; \ell_F, k, \omega, \alpha), h_i \rangle - \langle m_2(x, y, z; \ell_G, k, \omega, \alpha), h_i \rangle : 2 \leq i \leq d+2\} \qquad \text{(III.8)}$$

belongs to $\mathcal{G}_2$. I therefore obtain the convergence of

$$\sqrt{n}(\mathbb{P}_n - P) \begin{pmatrix} m_{1,2}(x, y, z; \ell_G, k, \omega, \alpha) - m_{2,2}(x, y, z; \ell_F, k, \omega, \alpha) \\ m_{1,2}(x, y, z; \ell_G, k, \omega, \alpha) - m_{2,3}(x, y, z; \ell_F, k, \omega, \alpha) \\ \vdots \\ m_{1,d+2}(x, y, z; \ell_G, k, \omega, \alpha) - m_{2,d+2}(x, y, z; \ell_F, k, \omega, \alpha) \end{pmatrix}$$

to a mean-zero multivariate normal distribution.

Example 5 (Example 2 continued) *Let $h = (0, 1, 0, \ldots, 0) \in \mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ satisfy $\sum \omega_i = d$, $\ell_G, \ell_F \in (0, \bar{\ell}]^d$, and $\alpha(i) = i$ for $i = 1, 2, 3$. Then each member of the finite collection $\{\langle m_2(x, y, z; \ell_G, k, \omega, \alpha(i)), h \rangle - \langle m_1(x, y, z; \ell_F, k, \omega, \alpha(i)), h \rangle : 1 \leq i \leq 3\}$ belongs to $\mathcal{G}_2$. I therefore obtain the convergence of*

$$\sqrt{n}(\mathbb{P}_n - P) \begin{pmatrix} m_{1,2}(x, y, z; \ell_F, k, \omega, 1) - m_{2,2}(x, y, z; \ell_G, k, \omega, 1) \\ m_{1,2}(x, y, z; \ell_F, k, \omega, 2) - m_{2,2}(x, y, z; \ell_G, k, \omega, 2) \\ m_{1,2}(x, y, z; \ell_F, k, \omega, 3) - m_{2,2}(x, y, z; \ell_G, k, \omega, 3) \end{pmatrix}$$

to a mean-zero multivariate normal distribution.

Example 6 (Example 3 continued) *Let $h = (0, 1, 0, \ldots, 0) \in \mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ be a fixed vector satisfying $\sum \omega_i = d$, $\ell_G, \ell_F, \ell_H \in (0, \bar{\ell}]^d$, and α be a fixed positive integer. Then each member of the finite collection $\{\langle f, \lambda \rangle : f \in \mathcal{G}_2 \times \mathcal{G}_3, \lambda \in \{(1, 0), (0, 1)\}\}$ belongs to $\mathcal{H}$. I therefore obtain the convergence of*

$$\sqrt{n}(\mathbb{P}_n - P) \begin{pmatrix} m_{1,2}(x, y, z; \ell_F, k, \omega, \alpha) - m_{3,2}(x, y, z; \ell_H, k, \omega, \alpha) \\ m_{1,2}(x, y, z; \ell_F, k, \omega, \alpha) - m_{2,2}(x, y, z; \ell_G, k, \omega, \alpha) \end{pmatrix}$$

to a mean-zero bivariate normal distribution..

Independent Samples

I now specialize the above results to the case where $\mathcal{X} = (X_1, \ldots, X_{n_1})$, $\mathcal{Y} = (Y_1, \ldots, Y_{n_2})$, and $\mathcal{Z} = (Z_1, \ldots, Z_{n_3})$ are independent random samples with respective distributions P_X, P_Y and P_Z. To this end, let $\mathcal{F}$ denote the class of functions

$$\{\langle m(x; \ell, k, \omega, \alpha), h \rangle : \ell \in [0, \bar{\ell}]^d, k \in [\underline{k}, \bar{k}], \sum \omega_i = d, \omega_i \geq 0, h \in [0, 1]^{d+2}\}$$

where $m : \mathbb{R}^d \to \mathbb{R}^{d+2}$. Further, denote by $\mathbb{G}_{n_1,P_X}$ the signed measure $\sqrt{n_1}(\mathbb{P}_{n_1,X} - P_X)$ with analogous definitions for $\mathbb{G}_{n_2,P_Y}$ and $\mathbb{G}_{n_3,P_Z}$. For analyzing cases such as those presented in Examples 1 and 2 my interest centers on the asymptotic behavior of an empirical process of the form

$$\left\{ \left(\frac{n_1 n_2}{n_1 + n_2} \right)^{1/2} \left[n_1^{-1/2} \mathbb{G}_{n_1,P_X} f_1 - n_2^{-1/2} \mathbb{G}_{n_2,P_Y} f_2 \right] : (f_1, f_2) \in \mathcal{F} \times \mathcal{F} \right\} \qquad \text{(III.9)}$$

In order to establish the asymptotic behavior of the empirical process in (III.9) I will require the following assumption:

Assumption III.4.1 (Sampling Process) $\inf_{i \neq j}\{n_i/n_j\} \to (0,1)$ *as* $n \to \infty$.

From the independence assumption together with Assumption III.4.1 I obtain the following important result:

Theorem 2 *Suppose Assumption III.4.1 holds, then the empirical process in (III.9) converges to the limit process*

$$\left\{ \lambda_1^{1/2} \mathbb{G}_{P_X} f_1 - (1 - \lambda_1)^{1/2} \mathbb{G}_{P_Y} f_2 : (f_1, f_2) \in \mathcal{F} \times \mathcal{F} \right\}$$

for some $\lambda \in (0,1)$, *where* $\{\mathbb{G}_{P_X} f : f \in \mathcal{F}\}$ *and* $\{\mathbb{G}_{P_Y} f : f \in \mathcal{F}\}$ *are independent zero-mean Gaussian processes.*

The applications to the Examples 1 and 2 are immediate:

Example 7 (Example 1 continued) *Let* h_i *denote the ith standard basis vector in* $\mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ *satisfy* $\sum \omega_i = d$, $\ell_G, \ell_F \in (0, \bar{\ell}]^d$, *and* α *be a fixed positive integer. Then each member of the finite collection* $\{(\langle m(x; \ell_F, k, \omega, \alpha), h_i \rangle, \langle m(x; \ell_G, k, \omega, \alpha), h_i \rangle) : 2 \leq i \leq d+2\}$ *belongs to* $F \times F$. *I therefore obtain form Theorem 2 the convergence of*

$$\left(\frac{n_1 n_2}{n_1 + n_2} \right)^{1/2} \left[\mathbb{G}_{n_2,P_Y} \begin{pmatrix} m_2(x; \ell_G, k, \omega, \alpha) \\ m_3(x; \ell_G, k, \omega, \alpha) \\ \vdots \\ m_{(d+2)}(x; \ell_G, k, \omega, \alpha) \end{pmatrix} - \mathbb{G}_{n_1,P_X} \begin{pmatrix} m_2(x; \ell_F, k, \omega, \alpha) \\ m_3(x; \ell_F, k, \omega, \alpha) \\ \vdots \\ m_{d+2}(x; \ell_F, k, \omega, \alpha) \end{pmatrix} \right]$$

to a zero-mean multivariate normal distribution.

Example 8 (Example 2 continued) *Let* $h = (0,1,0,\ldots,0) \in \mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ *satisfy* $\sum \omega_i = d$, $\ell_G, \ell_F \in (0, \bar{\ell}]^d$, *and* $\alpha(i) = i$ *for* $i = 1, 2, 3$. *Then each member of the finite collection* $\{(\langle m(x; \ell_F, k, \omega, \alpha(i)), h \rangle, \langle m(x; \ell_G, k, \omega, \alpha(i)), h \rangle) : 1 \leq i \leq 3\}$ *belongs to* $\mathcal{F} \times \mathcal{F}$. *I therefore obtain the convergence of*

$$\left(\frac{n_1 n_2}{n_1 + n_2}\right)^{1/2} \left[\mathbb{G}_{n_2, P_Y} \begin{pmatrix} m_2(x; \ell_G, k, \omega, 1) \\ m_2(x; \ell_G, k, \omega, 2) \\ m_2(x; \ell_G, k, \omega, 3) \end{pmatrix} - \mathbb{G}_{n_1, P_X} \begin{pmatrix} m_2(x; \ell_F, k, \omega, 1) \\ m_2(x; \ell_F, k, \omega, 2) \\ m_2(x; \ell_F, k, \omega, 3) \end{pmatrix} \right]$$

to a zero-mean multivariate normal distribution.

Again, as in the dependent case, testing problems such as those encountered in Example 3 require a slight modification; namely, consider the process

$$\left\{ \eta^{1/2} \left[n_1^{-1/2} \mathbb{G}_{n_1, P_X} f_1 - n_2^{-1/2} \mathbb{G}_{n_2, P_Y} f_2 + (n_1^{-1/2} \mathbb{G}_{n_1, P_X} f_3 - n_3^{-1/2} \mathbb{G}_{n_3, P_Z} f_4) \right] \\ : (f_1, f_2, f_3, f_4) \in \mathcal{F}^4 \right\} \quad \text{(III.10)}$$

where $\eta = \left(\frac{n_1 n_2 n_3}{n_1 n_2 + n_1 n_3 + n_2 n_3}\right)$. In order to establish the asymptotic behavior of the empirical

process in (III.10) I require the following assumption:

Assumption III.4.2 (Sampling Process) $\inf_{(i,j) \neq (k,l)} \{(n_i n_j)/(n_k n_l)\} \to (0,1)$ *as* $n \to \infty$ *whenever* $i \neq j$ *and* $k \neq l$.

From the independence assumption together with Assumption III.4.2 I am able to establish the following important result:

Theorem 3 *Suppose Assumption III.4.2 holds, then (III.10) converges to the limit process*

$$\left\{ \lambda_1^{1/2} \mathbb{G}_{P_X} f_1 - \lambda_2^{1/2} \mathbb{G}_{P_Y} f_2 + \lambda_1^{1/2} \mathbb{G}_{P_X} f_3 - (1 - \lambda_3)^{1/2} \mathbb{G}_{P_Z} f_4 : (f_1, f_2, f_3, f_4) \in \mathcal{F}^4 \right\}$$

for some $\lambda_1, \lambda_2, \lambda_3 \in (0, 1)$ *with* $\sum \lambda_i = 1$, *where* $\{\mathbb{G}_{P_X} f : f \in \mathcal{F}\}$, $\{\mathbb{G}_{P_Y} f : f \in \mathcal{F}\}$, *and* $\{\mathbb{G}_{P_Z} f : f \in \mathcal{F}\}$ *are independent zero-mean Gaussian processes.*

I are now in a position to obtain the convergence result relevant to Example 3.

Example 9 (Example 3 continued) *Let* $h = (0, 1, 0, \dots, 0) \in \mathbb{R}^{d+2}$, $\omega \in \mathbb{R}_+^d$ *be a fixed vector satisfying* $\sum \omega_i = d$, $\ell_G, \ell_F, \ell_H \in (0, \bar{\ell}]^d$, *and* α *be a fixed positive integer. Then each member of the finite collection* $\{\langle f, \lambda \rangle : f \in \mathcal{G}_{(2)} \times \mathcal{G}_{(3)}, \lambda \in \{(1, 0), (0, 1)\}\}$ *belongs to* $\mathcal{H}$. *I therefore obtain the convergence of*

$$\sqrt{\eta}(\mathbb{P}_{n_1, n_2, n_3} - P) \begin{pmatrix} m_{3,2}(x, y, z; \ell_H, k, \omega, \alpha) - m_{1,2}(x, y, z; \ell_F, k, \omega, \alpha) \\ m_{2,2}(x, y, z; \ell_G, k, \omega, \alpha) - m_{1,2}(x, y, z; \ell_F, k, \omega, \alpha) \end{pmatrix}$$

to a mean-zero bivariate normal distribution as an immediate consequence of Theorem 3.

Testing Methodology

For a given collection $(f_1, \ldots, f_S)$ with f_s, $1 \leq s \leq S$, a member of the P-Donsker class $\mathcal{F}$ (c.f. Examples 4 through 9), my interest centers on a simultaneous test of the hypotheses

$$H_s : Pf_s \leq 0 \text{ against } H'_s : Pf_s > 0 \quad 1 \leq s \leq S.$$

It is well known that the classical Wald-type tests of (Wolak 1989) and (Kodde and Palm 1986), for example, can be applied here to test the joint intersection hypothesis

$$H_0 : Pf_s \leq 0 \text{ for all } 1 \leq s \leq S \quad \text{against} \quad H_A : Pf_s > 0 \text{ for some } 1 \leq s \leq S$$

Unfortunately, a rejection of H_0 based on the Wald-type test does not necessarily imply that H_s is rejected for some $1 \leq s \leq S$; indeed, I may reject the joint intersection hypothesis H_0 without finding compelling evidence against any individual hypothesis H_s. Thus, in the context of Example 1, for instance, policy makers who adopt a Wald-type procedure may infer that a country or region is underachieving and yet be unable to infer the specific dimensions (e.g. income, health, education, etc.) which are responsible for the finding. Clearly this is undesirable if policy makers wish to obtain compelling evidence regarding dimension-specific underachievement and design targeted efforts accordingly.

In contrast to Wald-type tests, minimum p-value (MinP) tests are designed specifically to allow one to identify the source(s) of rejection when rejection occurs. In order to provide some background on the MinP methodology, I begin first by describing a suitable procedure for the computation of bootstrap p-values. Towards this end, it is well known Kosorok (2008) (page 20) that the Donsker property of F implies not only that

$$\sqrt{n}(\mathbb{P}_n - P)(f_1, \ldots, f_S) \Rightarrow N_S(0, \Omega(P)), \tag{III.11}$$

but also that

$$\sqrt{n}(\hat{\mathbb{P}}_n - \mathbb{P}_n)(f_1, \ldots, f_S) \Rightarrow N_S(0, \Omega(P)), \tag{III.12}$$

in probability, where $\hat{\mathbb{P}}_n$ denotes the bootstrap empirical measure and $N_S(0, \Omega(P))$ denotes an S-dimensional normal distribution with covariance matrix $\Omega(P)$ (the notation here reflects the dependence of Ω on the underlying probability mechanism P). Letting $J_n(\cdot, \mathbb{P}_n)$ denote the bootstrap approximation (c.f. equation (III.12)) to the sampling distribution in (III.11) and denoting by $J_{n,s}(\cdot, \mathbb{P}_n)$ the sth marginal distribution, it is straightforward that the bootstrap p-values associated with each of the component statistics may be obtained from

$$\hat{p}_s = 1 - J_{n,s}(\sqrt{n}\mathbb{P}_n f_s, \mathbb{P}_n) \tag{III.13}$$

The bootstrap p-value $\hat{p}_s$ in (III.13) provides a measure of the strength of evidence against H_s, and it is tempting to reject H_s at the nominal level γ if $\hat{p}_s < \gamma$. This testing strategy, however, ignores the multiplicity of the hypotheses under test and will tend to reject true hypotheses too often in the sense that

$$Prob_P\Big\{\text{Reject at least one } H_s, s \in I(P)\Big\} > \gamma \tag{III.14}$$

whenever the collection of true hypotheses $I(P)$ contains two or more elements. For instance, if $S = 5$, $Pf_s = 0$ for every s (all H_s are true), and all tests are mutually independent, then,

at the 5% level of significance

$$Prob_P\Big\{\text{Reject at least one } H_s, s \in I(P)\Big\} = Prob_P\Big\{\min_{1 \le s \le S} \hat{p}_s < 0.05\Big\}$$

$$\overset{n \to \infty}{\Longrightarrow} Prob_P\Big\{\min_{1 \le s \le S} U_s < 0.05\Big\} \tag{III.15}$$

$$= 1 - (1 - 0.05)^5$$

$$= 0.226$$

where I have used the fact that the estimated p-values converge to mutually independent uniform random variates under the assumed conditions. If the number of hypotheses S is increased to 10 or the significance level of the test is increased to 10%, the corresponding error rates jump to 0.401 and 0.409, respectively.[6]

The essence behind the classical MinP procedure lies in appropriately adjusting the standard p-values so as to ensure, at least asymptotically, that

$$Prob_P\Big\{\text{Reject at least one } H_s, s \in I(P)\Big\} \le \gamma \tag{III.16}$$

With the bootstrap distribution $J_n(\cdot, P)$ already in hand, obtaining adjusted p-values satisfying (III.16) is rather straightforward. Indeed, for a random draw Y from the known distribution $J_n(\cdot, \mathbb{P}_n)$ I may compute

$$\hat{p}_{min} = \min_{1 \le s \le S}[1 - J_{n,s}(Y_s, \mathbb{P}_n)]. \tag{III.17}$$

The corresponding empirical distribution from B such draws, which I denote by $Q_n(\cdot, \mathbb{P}_n)$, constitutes an approximation to the distribution of the minimum p-values and hence may

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[6]The assumption of mutual independence is a worst case scenario with respect to error rate control and is made here for illustrative purposes. In practice, I can generally expect some degree of dependence among the hypotheses under test, however it is only in the case of perfect dependence that we can be guaranteed of appropriate error rate control if we adopt the strategy of independently testing several hypotheses on the basis of individual (unadjusted) p-values.

be used to obtain the MinP adjusted p-values

$$\hat{p}_s^{adj} = Q_n(\hat{p}_s, \mathbb{P}_n). \tag{III.18}$$

In contrast to the liberal procedure in which the individual hypotheses are rejected if their unadjusted p-values fall below the nominal level γ, it may be shown (Bennett 2010) that testing the individual hypotheses based on the modified decision rule

Reject H_s if $\hat{p}_s^{adj} < \gamma$

guarantees control of the error rate in (III.16), at least asymptotically. (Bennett 2010) also demonstrates that the ability of the MinP test to identify false hypotheses can be greatly enhanced by replacing the random draw $Y \sim J_n(\cdot, \mathbb{P}_n)$ which is subsequently evaluated in (III.17) with a random draw from the bootstrap distribution $J_n^{PC}(\cdot, \mathbb{P}_n)$ which is defined according to

$$\sqrt{n}(\hat{\mathbb{P}}_n - \mathbb{P}_n)(f_1, \ldots, f_S) - \sqrt{n}(|\mathbb{P}_n f_1| \mathbb{1}_{\{|\mathbb{P}_n f_1| > \delta_{1,n}\}}, \ldots, |\mathbb{P}_n f_S| \mathbb{1}_{\{|\mathbb{P}_n f_S| > \delta_{S,,n}\}}) \tag{III.19}$$

where the $S \times 1$ vector δ_n is selected by the practitioner in accordance with Assumption III.5.1 below:

Assumption III.5.1 *i.* $\|\delta_n\| = o_P(1)$;[7] *ii.* $\underset{n \to \infty}{} \inf_{1 \leq s \leq S} n^{1/2} \delta_{n,s} \to \infty$.

Remark 1 *An example of a sequence δ_n satisfying the conditions of Assumption III.5.1 above is given by*

$$\delta_{n,s} = \sqrt{\frac{2\hat{\sigma}_{n,s}^2 \log \log n}{n}},$$

where $\hat{\sigma}_{n,s}^2$ denotes a consistent estimator of the asymptotic variance of $\sqrt{n}\mathbb{P}_n f_s$.

To gain some intuition for the mechanics of this procedure first consider the case where all of the hypotheses are on the boundary, i.e. $Pf_s = 0$ for every s. In this case

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[7]$\|\cdot\|$ denotes the standard Euclidean norm.

$J_n^{PC}(\cdot, \mathbb{P}_n)$ and $J_n(\cdot, \mathbb{P}_n)$ both converge to $N_S(0, \Omega(P))$, and consequently $[1 - J_{n,s}(Y_s, \mathbb{P}_n)]$ where $Y \sim J_n^{PC}(\cdot, \mathbb{P}_n)$ converges to a uniform random variable for every $s \in \{1, \dots, S\}$. Thus, asymptotically, the minimum is over an $S \times 1$ vector random variable with uniform (univariate) marginals, as should be expected when all of the $Pf_s = 0$. In contrast, when $Pf_s \neq 0$ the sth marginal distribution $J_{n,s}^{PC}(\cdot, \mathbb{P}_n)$ converges in probability to a degenerate distribution at $-\infty$ (the term $\sqrt{n}(|\mathbb{P}_n f_s| \mathbb{1}_{\{|\mathbb{P}_n f_s| > \delta_{n,s}\}}$ in (III.19) tends to $-\infty$ with probability tending to 1 provided δ_n is chosen in accordance with Assumption III.5.1) in which case

$$[1 - J_{n,s}(Y_s, \mathbb{P}_n)] \to 1$$

in probability as $n \to \infty$, and the index set over which the minimum is computed is effectively reduced. Since the minimum p-value is generally decreasing in the number of indices over which the minimum is computed, the elimination of any index for which $Pf_s \neq 0$ generally reduces the adjusted p-values and ultimately enhances the test's ability to detect false hypotheses while still allowing us to maintain appropriate control over the error rate (c.f. equation (III.16)). In fact, not only does this modification lead to greater power while still maintaining appropriate error rate control, but it is also shown in (Bennett 2010) that this modified MinP procedure is capable of identifying more false hypotheses than related multiple testing procedures, including the iterative stepdown procedures of (Romano and Wolf 2005) and (Hsu, Hsu, and Kuan 2010).

The implementation of the MinP testing procedure as described above in the specific context of Example 1 and the case of dependent samples is conveniently summarized in Algorithm 1 below :

Algorithm 1 (Example 1 Cont'd: The Dependent Case)

1. *Draw a random sample of size n, i.e.* $\{(X_1^*, Y_1^*, Z_1^*), \dots, (X_n^*, Y_n^*, Z_n^*)\}$, *from*

$\{(X_1, Y_1, Z_1), \ldots, (X_n, Y_n, Z_n)\}$ *and compute the difference*

$$n^{-1} \sum_{i=1}^{n} \left[m_{(1)}(X_i^*, Y_i^*, Z_i^* : \ell, k, \omega, \alpha) - m_{(2)}(X_i^*, Y_i^*, Z_i^* : \ell, k, \omega, \alpha) \right] \qquad \text{(III.20)}$$

2. *Repeat Step 1 B times and compute the empirical bootstrap distribution $J_n(\cdot, \mathbb{P}_n)$ and the $B \times S$ matrix of partially recentred bootstrap statistics using equation (III.19).*

3. *Compute the p-values of the S original and $B \times S$ partially recentred bootstrap statistics by evaluating them in the appropriate marginal distributions $J_{n,s}(\cdot, \mathbb{P}_n)$ of $J_n(\cdot, \mathbb{P}_n)$.*

4. *Compute the empirical distribution of row-minimums from the $B \times S$ matrix of p-values obtained in Step 3.*

5. *Compute the adjusted p-values corresponding to each test by evaluating the p-values of the S original statistics (obtained in Step 2) in the empirical distribution obtained via Step 4.*

Aside from substituting for the appropriate statistics (i.e., in equation (III.20) of Step 1), the algorithms for Examples 2 and 3 are identical, and are thus omitted. Similarly, the modifications necessary for treating the case of independent samples are also straightforward and I omit the details of the respective bootstrap algorithms.

Empirical Illustration

In this section, I apply the proposed testing methodology to data from India's National Sample Survey (NSS). I am particularly interested in examining the relative state of poverty across two ethnic groups, namely Hindus and Muslims. India has a predominantly Hindu population however it has a sizeable proportion of Muslims as well. Traditional income poverty analysis has shown that a lesser proportion of Hindus are poor than the corresponding numbers for Muslims. However, it is of interest to examine whether these findings persist when relevant dimensions or indicators of poverty other than income (or consumption) are included in the analysis. For the pupose of illustration I focus on a comparison of Hindu-Muslim poverty. A more in-depth analysis of poverty in India using

this multidimensional framework and the same data has been done in Mitra (2009). In that paper she studies the differences in poverty levels across different religions and regions. She also compares poverty across Hindus and Muslims in rural and urban India. A simple test is also proposed to compare poverty across two groups which incorporates the complex survey design of the NSS. However the test is not extended to multiple inequality testing.

The data source is the National Sample Survey's (NSS) 60th round health and morbidity survey. This survey was conducted in the last 6 months of 2004. For the purposes of this illustration I restrict attention to urban poverty, for which there are 26,566 households included. Since I am looking only at Hindu and Muslim poverty all other households are dropped. In India, these two religious groups together account for more than 95% of the total population, and so the resulting sample of 20,243 Hindu households and 3,715 Muslim households consists of the majority of all urban households.

While the NSS is a multistage stratified random sample, for the purpose of this illustration I ignore the complications introduced by this particular sampling design and instead assume the observations to be generated through the process of simple random sampling. While ignoring the specific sampling design is likely to bias the findings,[8] a thorough consideration of the sampling design issue (e.g., providing a detailed discussion of the NSS sampling design, modifying the bootstrap accordingly, etc.) is beyond the scope of the current paper.

As for the dimensions of deprivation used in the analysis I include the following: Per capita monthly expenditure (PCME), level of educational attainment, source of drinking water, type of housing structure, type of sanitation, drainage facilities available and main cooking medium. Since I am measuring household poverty and not individual level poverty,

[8]Bhattacharya (2007), for example, discusses in detail the effect of ignoring the sampling design of the NSS on inequality measurement.

Table 8.: Correlation between the dimensions

	Income	Housing	Sanitation	Drainage	Water	Cooking Medium
Income	1					
Housing	0.2042	1				
Sanitation	0.1684	0.3012	1			
Drainage	0.3129	0.3841	0.2908	1		
Water	0.1160	0.1218	0.0187	0.2243	1	
Cooking Medium	0.3031	0.3754	0.3339	0.4270	0.1831	1
Education	0.3786	0.3063	0.2777	0.3330	0.0839	0.4748

I take for the education level the highest level of education earned by any member of the household. Except per capita expenditure and education, all variables used in the analysis are ordinal. I also implicitly treat all households equally in terms of size since the NSS weighs all households in a village/block equally and therefore does not explicitly account for household size.[9]

The dimensions are chosen to represent the standard of living and the capabilities of the households to improve their position. A notable omission is health. Unfortunately, reliable sources of data for health of individuals and households are not easily available for India. One source for data on health for India is the National Family Health Survey, however this survey does not ask about income or per capita expenditure. Researchers have used this data after computing an asset index. However for the purpose of this analysis I have chosen to use more standard measures of income at the cost of omitting the dimension of health.

Of the seven dimensions used, one might be concerned about a high degree of correlation and the inclusion of "redundant" dimensions. Surprisingly, I find correlations between the various dimensions to be rather low. Indeed, as can be seen in Table 8, no correlation coefficient exceeds 0.5.[10] Therefore, by incorporating all of these dimensions I

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[9]As pointed out by one of the referees, this is likely to bias the results since households typically have different sizes and household size is likely correlated with both poverty and religion.

[10]I thank an anonymous referee for suggesting that I investigate the correlation among dimensions.

Table 9.: Dimension Specific Poverty lines

Dimension	Poverty line
Income	As given by the Planning Commission
Education	Having not obtained at least a primary education
Sanitation	No sanitation facility available
Drainage	No accessible drainage system
Housing	Person does not reside in a pucca* structure
Source of Drinking water	Person used a river, canal, pond, or well
Primary Cooking Medium	Person had no cooking arrangement or used firewood or dung cakes

*Pucca refers to brick and mortar structures

am able to capture different forms of deprivation in urban India.

Table 9 gives the dimension specific poverty lines used. For PCME, I use the poverty line as established by the Planning Commission of India. The remaining cut-offs are chosen as to describe a minimum standard of living.

Table 10 summarizes the incidence of deprivation in each of the seven dimensions for Hindus and Muslims, respectively. Note that in every dimension, except sanitation and drainage, the incidence of poverty among Muslims is greater than among Hindus. Further, note that the largest disparities appear to be in the dimensions of income, main cooking medium, and education level.

Table 10.: Incidence of Deprivation expressed as a percentage

Dimension	Incidence(Hindu)	Incidence(Muslim)
Income	18.9%	30.8%
Housing	17.9%	19.5%
Sanitation	19.7%	15.8%
Drainage	16.8%	15.3%
Water	5.3%	8.8%
Cooking medium	26.2%	35.9%
Education	10.0%	17.1%

In addition to the dimension-specific poverty lines, the P_0 and H measures (or more generally the AF methodology) require us to set a second cut-off.[11] The second cutoff is the dimension cutoff k which in the analysis can take any value between 1 and 7. The

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[11]For the sake of brevity I consider only the P_0 measure and multidimensional headcount H

Table 11.: Level of Poverty: Multidimensional Headcount

	K=3	K=4	K=5	K=6
H for Hindus	0.187	0.113	0.054	0.016
H for Muslims	0.226	0.109	0.044	0.015
Adjusted p-values(Null: M-H$\geq$0)	1.000	0.902	0.061	**0.016**
Adjusted p-values (Null: H-M$\geq$0)	**0.002**	0.302	0.987	0.998

value of k may be set before the analysis is undertaken by governments or by the investigator given the objectives of the exercise. Once k is fixed I may compute the associated level of poverty. When k equals 5, for instance, I see that Hindus are poorer than Muslims under the P_0 measure of multidimensional poverty (see Table 12). This conclusion depends on both the dimension specific poverty lines (which I assume here to be exogenously determined) and the value of k (which may be set by the investigator). So a natural robustness check would entail checking the levels of poverty for various values for k. For example if we see that Hindus remain poorer than Muslims for k values ranging from, say, 3 through 6, then we may infer that the poverty ordering is robust to the choice of k. This robustness check corresponds to a multiple inequality test where the null hypotheses are given by:

$$H_k : H(\ell_F, k, \omega, F) \leq H(\ell_G, k, \omega, G), \ \text{k=3,4,5,6,}$$

and

$$H_k : P_\alpha(\ell_F, k, \omega, F) \leq P_\alpha(\ell_G, k, \omega, G), \ \text{k=3,4,5,6.}$$

The p-values from this test in fact suggest a reversal in the levels of poverty for Hindus and Muslims as k is varied. When k equals 3, for example, I am able to infer that poverty among Muslims is higher than poverty among Hindus. However for the higher k values of 5 and 6 I reach the opposite conclusion. At k equals 4 there is no significant difference between the levels of poverty for the two groups. It is important to emphasize

Table 12.: Level of poverty: Multidimensional poverty

	K=3	K=4	K=5	K=6
P_0 for Hindus	0.107	0.075	0.041	0.014
P_0 for Muslims	0.121	0.071	0.034	0.013
Adjusted p-values(Null: M-H$\geq$0)	1.000	0.709	**0.036**	**0.008**
Adjusted p-values (Null: H-M$\geq$0)	**0.000**	0.508	0.991	0.998

that the reported p-values are adjusted for multiplicity and thus permit us to draw valid inferences concerning the individual hypotheses under test. Consulting unadjusted p-values, on the other hand, would not protect against the multiplicity problem and generally lead one to find "too many" false positives.

The observed reversal in the poverty ordering raises questions about a plausible explanation. Perhaps this reversal is the result of the fact that Hindus can be divided further on the basis of the caste to which they belong. Traditionally, the lower castes have been found to be more deprived, for instance, being made to do menial labor for low wages, and at the expense of receiving education. Even in modern times these castes have lagged behind the rest of the population and constitute some of the poorest individuals in the society. I therefore offer the following plausible explanation for the observed reversal: at higher levels of k, I are primarily capturing the lower castes within the Hindu population. Perhaps what I am observing then is low caste Hindus facing greater hardships, on average, than the Indian Muslim population.[12] Another plausible explanation for the reversal is that for lower values of k, it may be the case that income contributes relatively more to multidimensional poverty than it does for higher values of k. In such a case I will see that for lower k, I have Hindus less poor, simply because they are less poor by any measure of income poverty. But as k increases the other dimensions become increasingly important in which case, I may see a reversal. A test of this second conjecture is pursued next.

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[12]Regional and religious disaggregation of poverty in India is explored in greater depth in Singh(2009)

For a given value of k, for example k equals 3 I decompose the P_0 measure into the contributions of each of the dimensions. I then test whether there is significant difference between the contribution of each dimension to Hindu and Muslim poverty. More precisely, I perform a simultaneous test of the $d+1$ hypotheses

$$H_0 : P_0(\ell_G, k, \omega, G) - P_0(\ell_F, k, \omega, F) \le 0$$

and

$$H_s : P_{0,s}(\ell_G, k, \omega, G) - P_{0,s}(\ell_F, k, \omega, F) \le 0 \text{ for } 1 \le s \le d,$$

where the additional subscript "s" on the measure P_α $(\alpha = 0)$ denotes the sth dimension's contribution to the poverty measure.

The results of the above test for $k \in \{3, 4, 5, 6\}$ are presented in the table at the end of the chapter. I have observed that poverty is higher among Muslims at $k = 3$. I now see from the decomposition that incidence in income, housing, water, cooking medium, and education are all lower for Hindus than for Muslims with k equal to 3. For k equals 4, there is no significant difference between Hindus and Muslim poverty and I also see that most of the dimensions do not have significantly different contributions among Hindus and Muslims. For k equal to 5, I find that Muslims are less poor than Hindus, and that there is no significant difference in the contribution of income to Hindu and Muslim poverty levels. The difference in overall poverty can be explained only by differences in the levels of deprivation in the other dimensions, namely housing, sanitation and drainage for which I may infer that there is more deprivation among Hindus than among Muslims. For k equal to 6 I find stronger evidence of higher poverty among Hindus than among Muslims. I find at this level of k I have that Hindu households are significantly more deprived in all

dimensions.

In summary, I find that as k increases beyond 4, income is no longer enough to differentiate between Hindu and Muslim poverty, and that only by including other dimensions are we able to distinguish between Hindu and Muslim households in extreme poverty. This is an interesting finding which lends empirical support to arguments advocating the use of a multidimensional approach to poverty analysis.

Concluding Remarks

I have shown that the AF multidimensional approach to poverty naturally gives rise to the consideration of multiple hypotheses. Specific examples include examining the robustness of the AF ordering to the choice of poverty lines and/or the number of dimensions of deprivation before one is considered poor, inferring poverty ordering of various populations relative to a benchmark population, and inferring the specific dimensions in which a population is underachieving. Additionally, I have shown how such hypotheses can be treated in a unified manner and also tested using the minimum p-value (MinP) methodology of Bennett (2010).

In applying the proposed methodology to study Hindu and Muslim poverty in India, I have illustrated the tremendous scope for examining a wide range of hypotheses and for revealing insights into the plight of the poor not otherwise captured by traditional univariate approaches to poverty analysis. The use of India's National Sample Survey in this illustrative example, however, motivates a thorough consideration of issues raised by the application of the methodology under various sampling designs. While beyond the scope of the current paper, research into sampling design related issues in currently in progress.

Finally, the focus in this paper has been on how to formulate and test rather general hypotheses in the specific context of the Alkire-Foster multidimensional poverty measure. However the proposed tests can be extended to test hypotheses that arise from alternative multidimensional poverty or inequality ordering. Obvious examples include the multidimensional ordering of Maasoumi and Lugo (2008) and Duclos, Sahn and Younger (2006). Further, the proposed testing procedures can be extended to allow for sample-dependent measurement parameters—e.g., estimated poverty lines—as opposed to the simpler case of exogenous parameters as treated herein.

Table 13.: Dimensional Decomposition of the Multidimensional poverty estimates

	P_0	Income	Housing	Sanitation	Drainage	Water	Cooking	Education
P_0 ($k=3$) Hindu	0.107	0.110	0.118	0.141	0.120	0.027	0.159	0.069
P_0 ($k=3$) Muslim	0.121	0.139	0.136	0.113	0.117	0.047	0.207	0.089
Adj. p-value ($Null : P_0^M - P_0^H \geq 0$)	1.000	1.000	0.992	**0.000**	0.535	1.000	1.000	1.000
Adj. p-value ($Null : P_0^H - P_0^M \geq 0$)	**0.000**	**0.000**	**0.008**	1.000	0.462	**0.000**	**0.000**	**0.000**
P_0 ($k=4$) Hindu	0.075	0.077	0.086	0.098	0.087	0.018	0.103	0.053
P_0 ($k=4$) Muslim	0.071	0.083	0.086	0.077	0.072	0.017	0.101	0.059
Adj. p-value ($Null : P_0^M - P_0^H \geq 0$)	0.906	1.000	0.952	**0.009**	0.081	0.973	0.973	1.000
Adj. p-value ($Null : P_0^H - P_0^M \geq 0$)	0.712	**0.028**	0.637	1.000	1.000	0.548	0.537	0.076
P_0 ($k=5$) Hindu	0.041	0.043	0.048	0.052	0.046	0.010	0.052	0.034
P_0 ($k=5$) Muslim	0.034	0.042	0.038	0.040	0.036	0.007	0.043	0.029
Adj. p-value ($Null : P_0^M - P_0^H \geq 0$)	**0.027**	0.621	**0.029**	**0.007**	**0.015**	0.292	0.123	0.285
Adj. p-value ($Null : P_0^H - P_0^M \geq 0$)	1.000	0.876	1.000	1.000	1.000	0.987	0.998	0.994
P_0 ($k=6$) Hindu	0.014	0.015	0.017	0.016	0.016	0.006	0.016	0.014
P_0 ($k=6$) Muslim	0.013	0.015	0.015	0.014	0.015	0.004	0.015	0.012
Adj. p-value ($Null : P_0^M - P_0^H \geq 0$)	**0.000**	**0.019**	**0.008**	**0.001**	**0.003**	**0.006**	**0.017**	**0.001**
Adj. p-value ($Null : P_0^H - P_0^M \geq 0$)	1.000	0.978	0.993	1.000	0.997	0.997	0.988	1.000

ELECTORAL UNCERTAINTY AND THE GROWTH OF THE "MIDDLE CLASS"
THEORY AND EVIDENCE FROM INDIA

Introduction

The "middle class" has resources to spend on consumption and also the ability

and will to save and invest. In the context of fast-growing developing nations such as

India,China and Brazil, the middle class has been posited as a driver of growth.[1] The rise

of India's "middle class" has been the subject of intense debate in recent times. There have

been several studies (see Topalova,2005, Bhagwati and Srinivasan, 2002) which have tried

to link the growth of the "middle class" to the policies of trade liberalization pursued since

the 1990s. Given the prominence of this group, it is important to identify the factors that

may impact the size of this group. This paper focuses on one such factor, namely the degree

of electoral uncertainty in society. Specifically, this paper studies the relationship between

electoral uncertainty and the degree of income polarization in society.

The degree of income polarization is a measure of the extent of clustering in society

along income lines (see Esteban and Ray, 2010for a comprehensive discussion). In particular,

a high degree of income polarization is suggestive of a society dominated by two income

groups — the "haves" and the "have-nots" and thus a smaller middle class.[2] I investigate

how the tightness of the elections can affect the performance of the parties; in particular,

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[1]The causes of the rise of the Indian middle class is highly-debated and is tied up with the evaluation of whether growth in India since the 1980s has been "balanced". In China, however, the high rates of growth is almost entirely state-led.

[2]The measure of polarization posited by Foster and Wolfson (1992, 2010) is well-disposed towards capturing the size of the middle class. In fact, this is the measure I use extensively in the paper.

how the parties target the low and middle-income groups. Any district in which a political party feels secure about victory will not only tend to get less attention from the party (in terms of public spending) but will also experience reduced redistribution (within a district) leading to low growth (if any) of the "middle class" in that district.

The literature so far has focused on how resources are targeted to districts that are "swing" or "non-partisan" as opposed to districts that are strongly inclined towards one party or another. Models of political competition which have directly addressed questions of this nature (see for e.g., Lindbeck and Weibull (1987), Dixit and Londregan (1996, 1998),) have generally concluded that "swing" districts get more targeted resources in the aggregate. Such theoretical findings have been empirically investigated. For instance, Arulampalam et al (2009) find evidence, in the case of India, of the central government making transfers to state governments on the basis of political considerations. They find that a state which is both aligned[3] and swing[4] in the last state election is estimated to receive 16% higher transfers than a state which is unaligned and non-swing.

Bardhan and Mookherjee (2010) investigate political determinants of land reform implementation in the Indian state of West Bengal since the late 1970s. Their findings suggest that land reforms were better implemented in districts where there was more electoral competition. Taking the discussion further, I investigate which groups within the swing districts get the larger share of the benefits. This paper also has some commonalities with the literature on "clientelism". Bardhan and Mookherjee (1999) provide a theoretical framework which connects electoral competitiveness to clientelism. However, their main focus is the relationship between the degree of decentralization and clientelism.

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[3]The state governtment and the central government have at least one party in common

[4]Swing is defined later in the theoretical model section

In principle, the relationship between electoral uncertainty and income polarization is complex as there are potentially several opposing forces at play. In an electorally secure district, the political party which enjoys the advantage tends to feel less insecure and can pursue investment in public infrastructure and broad-based development programmes in a relatively unhindered manner. On the other hand, this party could simply fall back on its inherent advantage and not pursue development with great zeal. After all, effective administration is costly and if the party can afford to get re-elected without working hard to improve the lot of the masses, it would like to exercise that option. Hence, overall the effect of heightened electoral uncertainty on income polarization could be ambiguous.

The model presented here attempts to incorporate some elements of the above intuition and is very much in the tradition of the standard two-party Downsian competition. Suppose there are three categories of public goods available in society — one that disproportionately benefits the poor, another which disproportionately benefits the rich and finally a pure public good which benefits all groups equally. One could think of these benefits as augmenting the incomes of the citizens. For simplicity, I assume that there are three income groups in society — the poor, the middle-income and the rich. I allow for a continuous distribution of incomes in society but only require that the different income-earners can be sorted into the three broad income groups; so within each group, there is some heterogeneity of incomes.[5]

Prior to elections, each of the two political parties can commit to a certain level of investment in each of these public goods. Of course, investment is costly — typically, it requires the party candidate to lobby the central government for funds, monitor the

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[5]There remains the question of whether the middle class in India is a significant chunk of the population or whether it is a small minority located in half a dozen urban centres of the country (see Banerjee and Piketty(2003)). The way I think of the "middle class" in any region is really in terms of a group which is spread around the median income-earner in the region — thus, in very relative terms.

progress of the projects and so on. Also, in the spirit of Lindbeck and Weibull (1987) and Dixit and Londregan (1996) there is a constituency-level bias in favor of one party which is drawn from some distribution known to all. This bias can be interpreted as the ideological preferences of the voters in the constituency.[6] Given that each party wishes to maximize plurality net of investment costs, in equilibrium both parties end up proposing identical investment platforms provided they face the same investment cost function.

The model also predicts that as the level of electoral uncertainty increases in the district, the equilibrium level of investment in each type of public good increases; in other words, there is greater transfer to the electorally competitive districts. This is in line with the findings in Lindbeck and Weibull (1987) and Dixit and Londregan (1996). Most importantly, I show that increased electoral uncertainty induces additional investment in a manner that the additional benefits to the poor exceed the additional benefits to the middle-income group, which in turn, exceed the additional benefits to the rich. The main intuition behind such allocation is the following. Increased electoral uncertainty forces each party to expend more effort into investment and so they allocate investment among the three goods in such a manner that gets them the highest possible return in terms of votes. The fact that the population mass of the poor exceeds that of the rich makes the former a more attractive option in each party's eyes when allocating additional investment. Thus, increased electoral uncertainty reduces all inter-person income differences in society and hence reduces inequality. Furthermore, by bringing both the poor and the rich "closer" to the middle class group, it reduces polarization.

As far as the behavior of political parties is concerned, the stress is more on the errors of "omission" rather than of "commission"; the intuition being that in secure districts,

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[6]Typically using any such probalistic-voting setup helps guarantee an equilibrium in pure strategies, which is something clearly desirable in this context.

politicians tend to expend less effort and resources rather than actively pursue rent-seeking. However, explicitly including clientelism in the spirit of Bardhan and Mookherjee (1999) — in allowing political parties to gain some additional payoff from tilting policy towards the rich in society — in the model would only reinforce the results. Increased electoral uncertainty would make it more costly for the parties to cater to the rich.

The main theoretical finding, namely, that electorally "swing" constituencies tend to exhibit lower degree of income polarization is supported by the empirical exercise. The main variable representing electoral "swing" is the actual margin of winning; in other words, I look at the difference between the percentage vote shares of the two parties that obtain the highest number of votes in any constituency. The two NSS consumer expenditure rounds I utilize have almost 16 years between them and these intervening years have been witness to several national elections.[7] In the baseline specification, I take an average of the winning margins over several elections prior to each of the NSS expenditure rounds to get a measure of the electoral volatility of the districts.

I also experiment with alternative variables for electoral swing; for example, I restrict attention to the most recent election that took place before the relevant NSS expenditure round (rather than take an average over several prior elections). The results I get are robust to such variations — more "swing" districts exhibit lower (expenditure) polarization. The pattern persists when I replace winning margin by simply the vote share of the winning party. There is evidence of a similar relationship between inequality (as measured by the Gini coefficient) and electoral uncertainty. Inter-quartile differences in expenditure (normalized by the average level of expenditure) are also positively associated with higher

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[7]The two rounds are the 43rd round (conducted in 1987-88) and the 61st round (conducted in 2003-04). Also, national elections take place once every 5 years. Sometimes, they are more frequent. For instance, when the incumbent government fails a "vote of confidence" (a sign that the ruling party has the support of the majority of the national legislators) and is forced to resign, fresh elections are called.

winning margins.

In sum, the empirical findings suggests a strong empirical relationship between electoral uncertainty and the growth of the middle class. In particular, the empirical exercise helps to identify at least one important channel which stifles the growth of the "middle class" in India; namely, the under-performance of political parties in districts which are electorally secure.

The remainder of the paper is organized as follows. Section 2 presents a simple model of electoral competition which describes the impact of uncertainty in election outcomes on equilibrium policy platforms and hence on the resulting income distribution in society. Section 3 describes the data, the empirical strategy and findings and Section 4 concludes.

The model

Suppose that society is composed of a unit mass of individuals who differ in terms of their incomes. For simplicity, assume that there are three distinct income groups in society — the poor (denoted by p), the middle-income (denoted by m) and the rich (denoted by r). Let $G(.)$ represent the cdf of incomes in society and let y_m and y_r be two income levels with $0 < y_m < y_r$ such that anyone with income lower than y_m falls into group p and anyone with income between y_m and y_r falls into group m. All individuals earning at least y_r constitute the group r. Also, let n_i denote the mass of group i for $i = p, m, r$. I assume that $n_p > n_r$. Note that the way the income groups (and their corresponding sizes) have been defined makes it clear that the median income earner in society could belong to any of the three groups.[8] However, it is reasonable to proceed with the presumption that the

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[8]The only restriction I impose is that n_p exceed n_r.

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median income-earner belongs to group m; in a sense, it provides a natural interpretation of the notion of a "middle class".

There are two political parties A and B who field candidates for election. Each candidate proposes some (non-negative) allocation of investment in public goods. I assume that there are three categories of public goods in society:

(i) Pro-poor public goods: Fix some level of investment in this good, say I_p. Any additional investment in this good generates some positive benefits for all income groups. However, the marginal benefit to group p outweighs that to group m which in turn exceeds that to group r.

(ii) Pro-rich public goods: Fix some level of investment in this good, say I_r. Any additional investment in this good generates some positive benefits for all income groups. However, the marginal benefit to group r outweighs that to group m which in turn exceeds that to group p.

(iii) Pure public goods: Fix some level of investment in this good, say I_m. Any additional investment in this good generates equal positive benefits for all income groups.

It is not difficult to cite examples of each type of public good particularly in the context of developing countries. Foster and Rosenzweig (2001) posit a model with three kinds of public goods — irrigation facilities (pumps, tanks, tubewells), roads and schools — which differentially affect the welfare of the landed (and hence better-off) and landless (and hence poor) households. Specifically, they show that public expenditure on road-construction programs primarily benefit landless households by increasing local labor demand and the public purchase of irrigation facilities increases agricultural production and thus raises land rents which boost the incomes of the landed households. In the model, I could think of public expenditure on schools as a pure public good.

I will now make precise how investment in each of the three public goods affects any citizen's payoff. Suppose the level of investment in the three goods are given by $I \equiv (I_p, I_m, I_r) \geq 0$.

For an individual in group p, this generates a payoff (over and above her initial exogenously given income) given by $w_p(I) = \lambda \beta(I_p) + \beta(I_m) + \mu \beta(I_r)$.

For an individual in group m, this generates a payoff (over and above her initial exogenously given income) given by $w_m(I) = \beta(I_p) + \beta(I_m) + \beta(I_r)$.

For an individual in group r, this generates a payoff (over and above her initial exogenously given income) given by $w_r(I) = \mu \beta(I_p) + \beta(I_m) + \lambda \beta(I_r)$.

By the nature of the public goods defined above, it must be that $\lambda > 1 > \mu > 0$. I assume that $\beta(0) = 0$, $\beta'(0) = \infty$, $\beta'(x) > 0$ for every $x > 0$ and $\beta'' < 0$. See Figure 3 for a useful illustration.
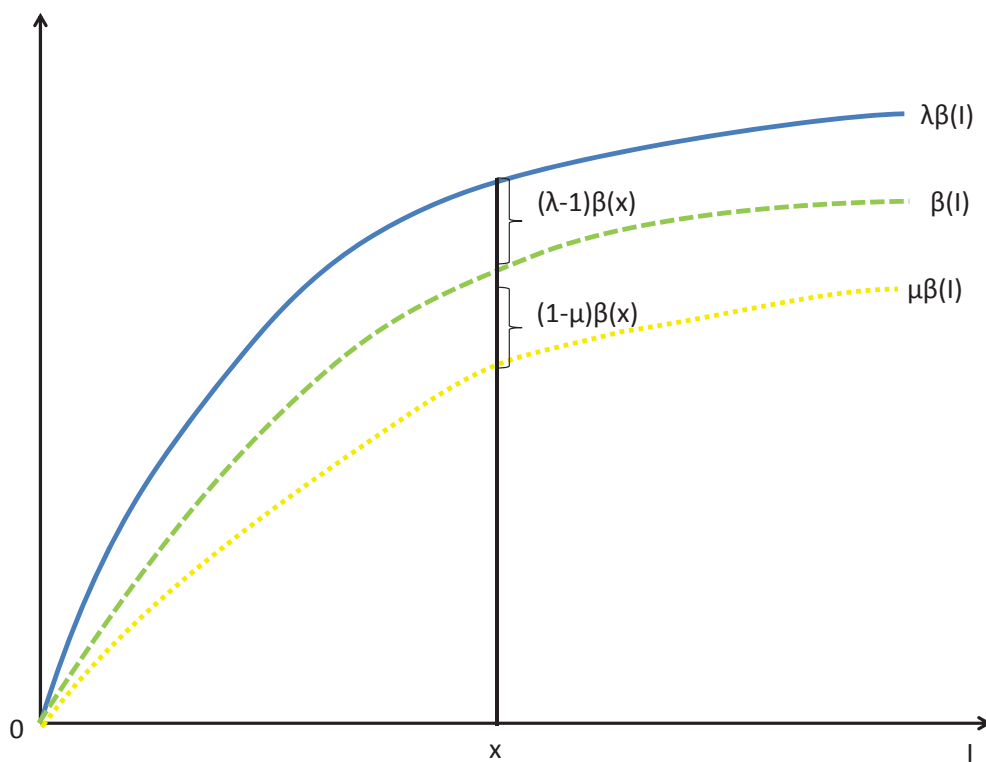
Figure 3.: The Returns-from-Investment curves.

Investment in any public good is costly in the sense that it requires effort by the party's candidate. One could think of this cost as lobbying costs for funds or monitoring costs of the projects, etc. I assume that the cost of investment by party j is given by $c(\hat{I}_j)$ where $\hat{I}_j = I_p^j + I_m^j + I_r^j$ for $j = A, B$. Also, I assume that $c(0) = c'(0) = 0$, $c'(I) > 0$ for every $I > 0$ and $c'' > 0$.

An individual's preferences over candidates (and their proposed policies) are described as follows. First, individual v exhibits a bias a_v, positive or negative, for party A. The corresponding payoff from B is normalized to be zero, so a_v is really a difference. This ideological bias can stem from many things, say the parties stand on issues other than public goods provision and so on. Moreover, I assume that every individual draws this bias from the same distribution with cdf $F(.)$ and corresponding density f positive everywhere on the real line. Thus, it is this F function which captures the ideological leanings or partisan nature of the constituency.

So the timing is as follows. Both parties A and B field their respective candidates each of whom proposes a vector of investments, i.e. $I_j \equiv (I_p^j, I_m^j, I_r^j) \geq 0$ for $j = A, B$. Each voter then draws her bias from F (note, F is public information but each individual's realization is observed by the individual alone) and then votes for the party who promises her higher utility. The party with the highest number of votes is declared the winner and the winner's proposed platform is implemented. Note, there is full-commitment from each party's side in keeping with the Downsian tradition.

Suppose (I_A, I_B) is offered by party A and party B, respectively. Consider a voter v who belongs to income group i where $i = p, m, r$. She will vote for $A's$ candidate if

$$w_i(\mathbf{I}_A) + a_v > w_i(\mathbf{I}_B).$$

Note, voter v will vote for $B's$ candidate if the opposite inequality holds and will be indifferent in case of equality.

From the perspective of the party, an individual's vote is stochastic. The probability that she will vote for party $A's$ candidate is given by

$$1 - F(w_i(\mathbf{I}_B) - w_i(\mathbf{I}_A)).$$

Call it p_i (note, it is the same for every voter v in group i). The expected plurality for party A is proportional to $\sum_i n_i p_i$.[9] Parties care about maximizing their respective expected plurality but are also sensitive to the cost of investment. In particular, given $B's$ platform I_B, party A will choose I_A to maximize:

$$\sum_i n_i p_i - c(\hat{I}_A).$$

Similarly, party B will take I_A as given and choose I_B to maximize:

$$1 - \sum_i n_i p_i - c(\hat{I}_B).$$

Assuming that $F(w_i(I_B) - w_i(I_A))$ is convex in I_A (for any I_B) and concave in I_B (for any I_A) for each group i — in the same vein as Lindbeck and Weibull (1987) — is sufficient to guarantee the existence of best-response functions for each of the two parties.

This sets the ground for the first result which is stated in Proposition 1 below.

Proposition 1 *There is a unique equilibrium of this game. Moreover, the equilibrium is symmetric with each party offering platform $I^* \equiv (I_p^*, I_m^*, I_r^*)$ where I^* satisfies the following equations (1)-(3):*

$$f(0)\beta'(I_p)[n_p\lambda + n_m + n_r\mu] = c'(\hat{I}) \tag{IV.1}$$

$$f(0)\beta'(I_m) = c'(\hat{I}) \tag{IV.2}$$

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[9]To be precise, the expected plurality is given by $\sum_i n_i[p_i - (1 - p_i)] = \sum_i n_i[2p_i - 1]$.

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$$f(0)\beta'(I_r)[n_p\mu + n_m + n_r\lambda] = c'(\hat{I}) \tag{IV.3}$$

Moreover, $I_p^* > I_r^*$.

Proof. The proof is established in a few steps. First I show that there exists a unique $I^* \equiv (I_p^*, I_m^*, I_r^*)$ which satisfies equations (1)-(3) and also $I_p^* > I_r^*$. Then I establish that that both parties offering I^* is an equilibrium of this game. The final step establishes the uniqueness of the equilibrium.

STEP 1 (existence of $I^* \equiv (I_p^*, I_m^*, I_r^*)$):

From equations (1) and (2), I get:

$$\beta'(I_p) = \beta'(I_m)/[n_p\lambda + n_m + n_r\mu].$$

This means I_p can be represented by some strictly increasing function of I_m given that $\beta'' < 0$. Similarly, equations (1) and (3) yield that I_r can be represented by some strictly increasing function of I_m. Moreover, both I_p and I_r approach 0 as I_m goes to 0 since $\beta'(0) = \infty$.

Now focus on equation (2). The RHS is simply an increasing function of $\hat{I}$. Note, $\hat{I}$ can be represented by some strictly increasing function of I_m given the discussion above. Hence, the RHS of equation (2) is a continuous and strictly increasing function of I_m given the strict convexity of c. The LHS of equation (2) is a continuous and strictly decreasing function of I_m owing to the strict concavity of β. Moreover, for values of I_m arbitrarily close to 0 LHS of (2) strictly exceeds RHS of (2) since $c'(0) = 0$ and $\beta' > 0$. Also, the RHS of (2) strictly exceeds LHS of (2) for exceedingly "large" values of I_m since β is strictly concave and c is strictly convex and increasing. This implies that there is a unique solution — call it $I^* = (I_p^*, I_m^*, I_r^*)$ — to equations (1)-(3).

Also, from equations (1) and (3), I get:

$$\beta'(I_p^*)[n_p\lambda + n_m + n_r\mu] = \beta'(I_r^*)[n_p\mu + n_m + n_r\lambda]$$

Re-arranging terms, I get:

$$\frac{\beta'(I_p^*)}{\beta'(I_r^*)} = \frac{n_p\mu + n_m + n_r\lambda}{n_p\lambda + n_m + n_r\mu}$$

Note that the RHS of the above equation is strictly less than unity since $\lambda > \mu$ and $n_p > n_r$. By the strict concavity of β I have that $I_p^* > I_r^*$.

STEP 2 (establishing that (I^*, I^*) constitutes an equilibrium):

Now I return to the basic problem each political party faces. Given I_B, Party A chooses I_A to maximize:

$$\sum_i n_i p_i - c(\hat{I}_A)$$

where

$$p_i = 1 - F(w_i(\mathbf{I}_B) - w_i(\mathbf{I}_A)).$$

Let

$$d_i \equiv w_i(\mathbf{I}_B) - w_i(\mathbf{I}_A).$$

The first order conditions are the following:

$$FOC(I_p^A) : f(d_p)\lambda\beta'(I_p^A)n_p + f(d_m)\beta'(I_p^A)n_m + f(d_r)\mu\beta'(I_p^A)n_r = c'(\hat{I}_A) \qquad \text{(IV.4)}$$

$$FOC(I_m^A) : f(d_p)\beta'(I_m^A)n_p + f(d_m)\beta'(I_m^A)n_m + f(d_r)\beta'(I_m^A)n_r = c'(\hat{I}_A) \qquad \text{(IV.5)}$$

$$FOC(I_r^A) : f(d_p)\mu\beta'(I_r^A)n_p + f(d_m)\beta'(I_r^A)n_m + f(d_r)\lambda\beta'(I_r^A)n_r = c'(\hat{I}_A) \qquad \text{(IV.6)}$$

Now, given I_A, Party B chooses I_B to maximize:

$$1 - \sum_i n_i p_i - c(\hat{I}_B)$$

It is easily checked that B's problem yields first-order conditions which are analogous to Party A's. They are:

$$FOC(I_p^B) : f(d_p)\lambda\beta'(I_p^B)n_p + f(d_m)\beta'(I_p^B)n_m + f(d_r)\mu\beta'(I_p^B)n_r = c'(\hat{I}_B) \qquad \text{(IV.7)}$$

$$FOC(I_m^B) : f(d_p)\beta'(I_m^B)n_p + f(d_m)\beta'(I_m^B)n_m + f(d_r)\beta'(I_m^B)n_r = c'(\hat{I}_B) \qquad \text{(IV.8)}$$

$$FOC(I_r^B) : f(d_p)\mu\beta'(I_r^B)n_p + f(d_m)\beta'(I_r^B)n_m + f(d_r)\lambda\beta'(I_r^B)n_r = c'(\hat{I}_B) \qquad \text{(IV.9)}$$

Suppose Party B offers I^*. Then using Party A's FOCs one can check that offering I^* constitutes a best-response for Party A. Similarly, when Party A offers I^*, offering I^* constitutes a best-response for Party B (using Party B's FOCs). Thus, I have established that both parties offering I^* is an equilibrium.

STEP 3 (uniqueness):

Suppose (I_A, I_B) is an equilibrium different from (I^*, I^*). There are two possibilities: (i) $\hat{I}_A = \hat{I}_B$ or (ii) $\hat{I}_A \neq \hat{I}_B$.

Suppose we are in case (i). From equations (5) and (8), note that $\hat{I}_A = \hat{I}_B$ implies $I_m^A = I_m^B$. Similarly, equations (4) and (7) yield $I_p^A = I_p^B$ thus implying $I_A = I_B$. Combined

with STEP 1, this means that $I_A = I_B = I^*$. This implies that case (i) is not a possibility. Therefore we must be in case (ii).

Now suppose $\hat{I}_A > \hat{I}_B$. From equations (5) and (8), note that $\hat{I}_A > \hat{I}_B$ implies $I_m^A < I_m^B$. Similarly, equations (4) and (7) yield $I_p^A < I_p^B$. Finally, comparing equations (6) and (9) yield $I_r^A < I_r^B$. This contradicts $\hat{I}_A > \hat{I}_B$. The case of $\hat{I}_B > \hat{I}_A$ can be analogously ruled out thus establishing that (I^*, I^*) is the only equilibrium of this game.

Combining STEPs (1)-(3) establishes the proposition. ∎

Like in the previous literature (for instance, see Arulampalam et al (2009)) I interpret the density of the bias evaluated at 0, namely $f(0)$, to be an index of how swing or non-partisan the constituency happens to be. To see why, consider two constituencies s and t where $f_s(0) > f_t(0)$. This is roughly equivalent to saying that constituency s, in relation to t, has a higher proportion of citizens who are ideologically equidistant (or detached) from either party. Thus, s is more swing than t and so the former constituency can be more unpredictable in terms of election results. This suggests that competition should be tighter in s as compared to t. In fact, in line with the findings of the previous literature, this is what is stated in Proposition 2 below.

Proposition 2 *Increased electoral uncertainty, as captured by a rise in $f(0)$, results in higher aggregate public good investment. Moreover, investment in every type of public good is increased.*

Proof. Let electoral uncertainty increase from $f(0)$ to $\overline{f}(0)$. Now suppose that the resulting equilibrium level of aggregate investment $\hat{\overline{I}}$ is such that $\hat{\overline{I}} \leq \hat{I}$. This implies, using equation (2), that $\overline{I}_m > I_m$. Similarly, equations (1) and (3) respectively imply that $\overline{I}_p > I_p$ and $\overline{I}_r > I_r$. This contradicts the initial supposition and establishes that $\hat{\overline{I}} > \hat{I}$.

I noted in the proof of Proposition 1 (see STEP 1) that both I_p and I_r can be represented by some strictly increasing function of I_m. This observation combined with

$\hat{\tilde{I}} > \hat{I}$ establishes that investment in every type of public good is increased. ∎

With these results in hand, I move on to the primary findings of the paper.

Proposition 3 *Suppose the returns-from-investment function, $\beta(x)$, is of the following generic CES functional form:*

$$\beta(x) = \frac{x^{1-\sigma}}{1-\sigma}.$$

Also, let $\lambda - 1 = 1 - \mu$.

Then for any $\sigma \in (0,1)$, increased electoral uncertainty, as captured by a rise in $f(0)$, leads to an unambiguous lowering of income inequality and income polarization. Hence for any $\sigma \in (0,1)$ and $\lambda - 1 = 1 - \mu$, any increase in the electoral competitiveness of a constituency promotes the rise of the "middle class" therein.

Proof. Let electoral uncertainty increase so that $f(0)$ rises to $\tilde{f}(0)$. Let $\tilde{I}$ represent the corresponding platform proposed by both parties in equilibrium.

Consider the change in the incomes of the members of group i for $i = p, m, r$.

$$\Delta y_p = \lambda[\beta(\tilde{I}_p) - \beta(I_p)] + [\beta(\tilde{I}_m) - \beta(I_m)] + \mu[\beta(\tilde{I}_r) - \beta(I_r)]$$

$$\Delta y_m = [\beta(\tilde{I}_p) - \beta(I_p)] + [\beta(\tilde{I}_m) - \beta(I_m)] + [\beta(\tilde{I}_r) - \beta(I_r)]$$

$$\Delta y_r = \mu[\beta(\tilde{I}_p) - \beta(I_p)] + [\beta(\tilde{I}_m) - \beta(I_m)] + \lambda[\beta(\tilde{I}_r) - \beta(I_r)]$$

This implies the following relationships:

$$\Delta y_p - \Delta y_m = (\lambda - 1)[\beta(\tilde{I}_p) - \beta(I_p)] + (\mu - 1)[\beta(\tilde{I}_r) - \beta(I_r)]$$

$$\Delta y_m - \Delta y_r = (1 - \mu)[\beta(\tilde{I}_p) - \beta(I_p)] + (1 - \lambda)[\beta(\tilde{I}_r) - \beta(I_r)]$$

Therefore,

$$\beta(\tilde{I}_p) - \beta(I_p) > \beta(\tilde{I}_r) - \beta(I_r)$$

is sufficient to guarantee $\Delta y_p > \Delta y_m$ and $\Delta y_m > \Delta y_r$ whenever $\lambda - 1 = 1 - \mu$.

For $\beta(x) = \frac{x^{1-\sigma}}{1-\sigma}$, equations (1) and (3) imply:

$$\frac{\beta'(I_p)}{\beta'(I_r)} = [I_r/I_p]^\sigma = \frac{n_p\mu + n_m + n_r\lambda}{n_p\lambda + n_m + n_r\mu} \equiv \rho < 1$$

Next I show that $\beta(\tilde{I}_p) - \beta(I_p) > \beta(\tilde{I}_r) - \beta(I_r)$ for any $\sigma \in (0,1)$.

Note, $\beta(\tilde{I}_i) - \beta(I_i) = \frac{1}{1-\sigma}[\tilde{I}_i^{1-\sigma} - I_i^{1-\sigma}]$ for $i = p, m, r$. Therefore,

$$\beta(\tilde{I}_p) - \beta(I_p) > \beta(\tilde{I}_r) - \beta(I_r) \Leftrightarrow \tilde{I}_p^{1-\sigma} - I_p^{1-\sigma} > \tilde{I}_r^{1-\sigma} - I_r^{1-\sigma}$$

Note,

$$\tilde{I}_p^{1-\sigma} - I_p^{1-\sigma} = \tilde{I}_r^{1-\sigma}[(\tilde{I}_p/\tilde{I}_r)^{1-\sigma} - (I_p/\tilde{I}_r)^{1-\sigma}]$$

$$= \tilde{I}_r^{1-\sigma}[\rho^{1-1/\sigma} - (I_p/I_r)^{1-\sigma}(I_r/\tilde{I}_r)^{1-\sigma}]$$

$$= \tilde{I}_r^{1-\sigma}[\rho^{1-1/\sigma} - \rho^{1-1/\sigma}(I_r/\tilde{I}_r)^{1-\sigma}]$$

$$= \tilde{I}_r^{1-\sigma}\rho^{1-1/\sigma}[1 - (I_r/\tilde{I}_r)^{1-\sigma}]$$

On the other hand,

$$\tilde{I}_r^{1-\sigma} - I_r^{1-\sigma} = \tilde{I}_r^{1-\sigma}[1 - (I_r/\tilde{I}_r)^{1-\sigma}].$$

Therefore,

$$\beta(\tilde{I}_p) - \beta(I_p) > \beta(\tilde{I}_r) - \beta(I_r) \Leftrightarrow \rho^{1-1/\sigma} > 1.$$

But $\rho^{1-1/\sigma} > 1$ for any $\sigma \in (0,1)$ since $\rho < 1$. This establishes $\Delta y_p > \Delta y_m > \Delta y_r$ whenever $\lambda - 1 = 1 - \mu$.

Intuitively, all income differences across individuals have been reduced in society. To see this more formally, suppose $\tilde{G}$ represents the new income distribution with all income normalized so that the mean income under the two distributions remain the same (mean-

normalization). Since $\Delta y_p > \Delta y_m > \Delta y_r$, one can construct G from $\tilde{G}$ using a set of regressive transfers; specifically, suitable transfers from the poorest n_p mass to the richest n_r mass will suffice. Therefore, income inequality as measured by any Lorenz-consistent measure must have undergone a reduction.

To see the effect on income polarization, suppose $\hat{G}$ represents the new income distribution with all income normalized so that the median income under the two distributions remain the same (median-normalization). Since $\Delta y_p > \Delta y_m > \Delta y_r$ and the median income-earner lies in group m, the mass of population earning between y_m and y_r is larger under $\hat{G}$ than under G. This clearly implies a growth of the middle class and reduced (income) polarization in terms of the Foster-Wolfson polarization measure (for a graphical demonstration, see Figure 4). ∎

The assumption $\lambda - 1 = 1 - \mu$ is really a kind of symmetry requirement on the returns-from-investment functions. Consider either the p or the r group. This restriction basically implies that the three different return functions (from the three different public goods) for this group (either p or r) are such that the return from the pure public good comes midway between the other two when the level of investment is held equal across the three public goods (see Figure 3). Of course, this restriction need not always hold; but it is useful to consider this at least as a benchmark case.

I will now explore the cases when $\lambda - 1 \neq 1 - \mu$. In particular, it is important to see how crucial this symmetry assumption (i.e. $\lambda - 1 = 1 - \mu$) really is.

It turns out that $\lambda - 1 = 1 - \mu$ is sufficient but not necessary for the main results. In other words, the presence of some amount of asymmetry in the sense of $\lambda - 1 \neq 1 - \mu$ does not affect the relationship between electoral competitiveness and polarization (or inequality). The following proposition attempts to outline the bounds on the asymmetry which can
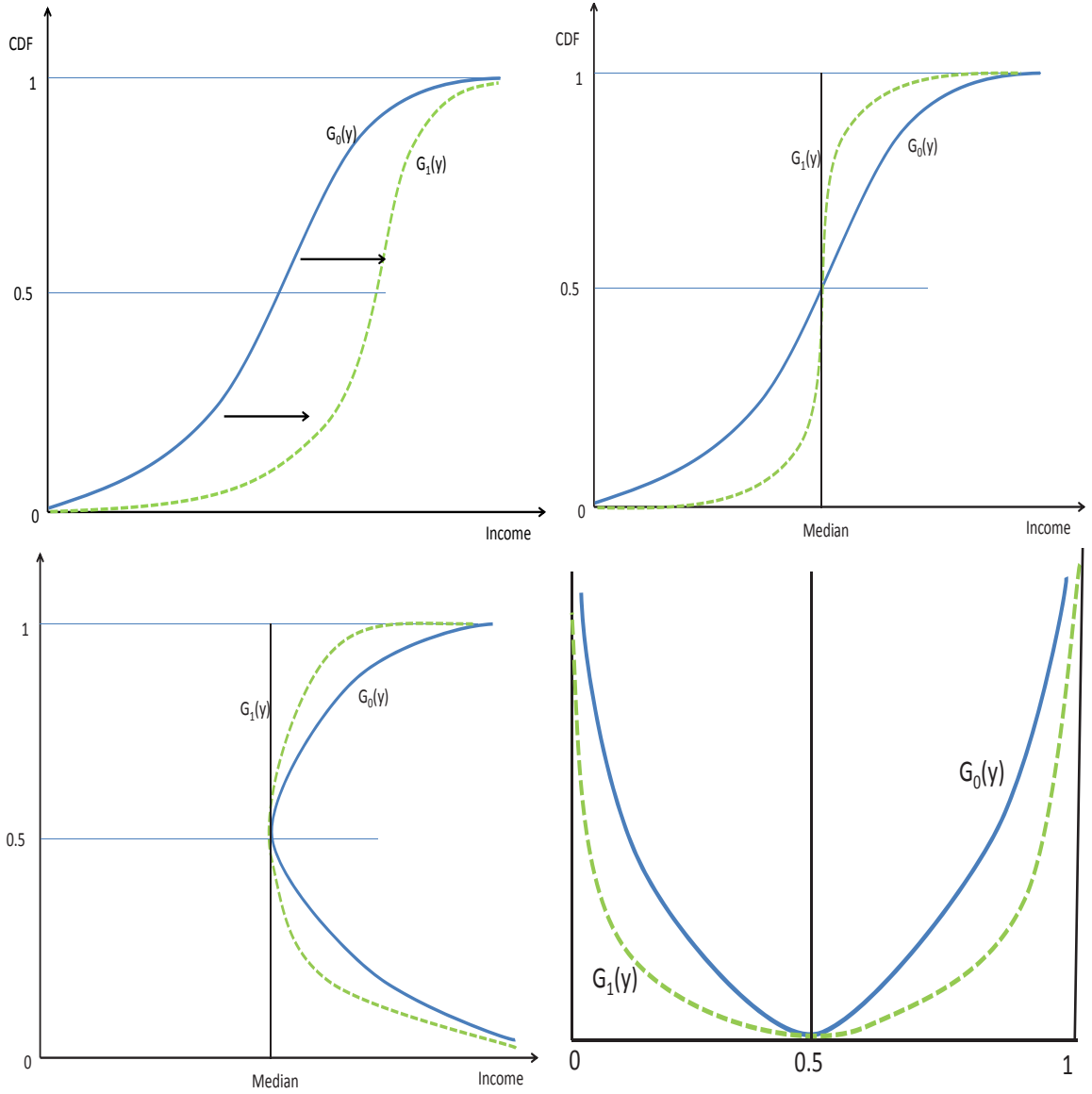
Figure 4.: Foster-Wolfson "Squeeze". Panel A shows the shift in the income distribution. Panel B shows the distributions once they are median normalized. In Panel C the image has been reflected on the axis of the median. Panel D shows the Polarization Curves as in Foster-Wolfson (2009)

sustain the main result.

Proposition 4 *Let the returns-from-investment function, $\beta(x)$, be as in Proposition 3 with $\sigma \in (0,1)$. Consider any $0 < \mu < 1 < \lambda$ such that $\lambda - 1 = 1 - \mu$. These values of λ, μ and the initial income distribution G determine the value of $\rho^{1-1/\sigma}(>1)$ where $\rho \equiv \frac{n_p\mu + n_m + n_r\lambda}{n_p\lambda + n_m + n_r\mu} < 1$. Any asymmetry induced by changing either λ or μ does not affect the main findings of Proposition 3 as long as the asymmetry is no higher than $\rho^{1-1/\sigma}$. Moreover, $\rho^{1-1/\sigma}$ is a lower bound on the maximum amount of asymmetry (i.e., either $\frac{\lambda-1}{1-\mu} > 1$ or $\frac{1-\mu}{\lambda-1} > 1$) that can sustain the main findings in Proposition 3.*

Proof. Note, $\Delta y_p \geq \Delta y_m \geq \Delta y_r$ with at least one inequality strict, is sufficient to generate the main findings of Proposition 3. The first inequality holds iff

$$(\lambda - 1)[\beta(\tilde{I}_p) - \beta(I_p)] \geq (1 - \mu)[\beta(\tilde{I}_r) - \beta(I_r)]$$

and the second inequality holds iff

$$(1 - \mu)[\beta(\tilde{I}_p) - \beta(I_p)] \geq (\lambda - 1)[\beta(\tilde{I}_r) - \beta(I_r)].$$

Now, from Proposition 3's proof I have

$$\frac{\beta(\tilde{I}_p) - \beta(I_p)}{\beta(\tilde{I}_r) - \beta(I_r)} = \rho^{1-1/\sigma} > 1.$$

Consider an increase in λ such that $\lambda - 1 > 1 - \mu$ (the argument for $\lambda - 1 < 1 - \mu$ is analogous) while keeping ρ at the initial value. Clearly, this leads to $\Delta y_p > \Delta y_m$. Also, as long as $\frac{\lambda-1}{1-\mu} \leq \rho^{1-1/\sigma}$, I have $\Delta y_m \geq \Delta y_r$.

It is easily checked that $\frac{\delta\rho}{\delta\lambda} < 0$ (and $\frac{\delta\rho}{\delta\mu} > 0$). Thus, I have $\frac{\delta\rho^{1-1/\sigma}}{\delta\lambda} > 0$ (and $\frac{\delta\rho^{1-1/\sigma}}{\delta\mu} < 0$).

For $\lambda - 1 > 1 - \mu$, this implies that the inequality concerning Δy_m and Δy_r is more easily satisfied in the desired direction. An analogous argument works for $\lambda - 1 < 1 - \mu$ (decreasing μ starting from the symmetry case) since $\frac{\delta\rho}{\delta\mu} > 0$.

This implies that $\rho^{1-1/\sigma}$, where ρ is evaluated at the initial values of λ and μ under the symmetry assumption, is a lower bound on the maximum amount of asymmetry that

can sustain the main findings in Proposition 3. ∎

Therefore, Proposition 4 demonstrates that the relationship between electoral competitiveness and income polarization (or inequality) is a robust one; it withstands some degree of asymmetry in the relative returns-from-investment functions.

In sum, the model above offers some interesting empirically testable predictions which I take to the data from India.

Empirical Analysis

The theoretical model offers several empirically testable hypotheses. As of now, I focus on the following:

[A] Any increase in electoral uncertainty leads to higher income polarization.

[B] Any increase in electoral uncertainty results in higher income inequality.

Data

To test these hypotheses I need to combine data on incomes with data on election outcomes. In the case of India, nationally representative data on personal incomes is hard to obtain since a vast majority of Indian households (primarily residing in rural parts) are exempt from payment of income taxes (see Banerjee and Pikkety, 2003). However, there is data on consumer expenditure in India which is publicly available; thus consumer expenditure serves as an excellent proxy for income in the analysis done here. These data are collected by the National Sample Survey Organization (NSSO) . The National Sample Survey (NSS) is a large-scale consumer expenditure survey which is conducted quinquennially and covers the entire nation; the unit of observation is a household. The recall period used is 30 days, i.e., the surveyed households are asked to provide information on consumption

expenditure incurred over the past 30 days. For the current study I use the 43rd and 61st rounds of the NSS. The 43rd round was conducted during July 1987-June 1988 and the 61st round was conducted during July 2004-June 2005. Alongside information on consumer expenditure, the survey also collects data on other socio-economic characteristics of the (surveyed) households such as religion, caste, education.

This information on household expenditure is combined with election data obtained from the Election Commission of India. I use the data for the parliamentary (or federal level) elections from 1977 to 2004. During this period, 11 such general elections took place in India. A proper test of the theory requires the use of some measure of the electoral competitiveness of the district — the "swing" nature, so to speak. I primarily utilize the difference in percentage vote shares of the two parties that obtain the highest number of votes in any constituency. This is in line with Arulampalam et al (2009). I use the winning margin and the vote share of the winning party — each averaged over the 3-4 elections prior to each expenditure round — in turn to capture the extent of electoral competition in the district.

I also use the information about whether the constituency had a shift away from or to a Congress Member of Parliament (MP). The use of a more refined measure of swing which takes into account movements to and from different parties is not possible for the following reason. There has been an immense proliferation of political parties at both the state and central levels, most of it arising from the splitting up of the main existing national or even regional parties. Moreover, various coalitions — ad hoc and otherwise — became popular from the 1980s onward. This makes it very difficult to say whether there really has been an effective shift of regime when say person X wins the same seat first as a candidate of party L and then as a candidate of party R. Given the way the nature of politics and

political parties evolved during this period, I chose to proceed with a rather conservative division of parties into "Congress" and "Non-Congress" camps and recorded the movements of a district between these camps over the different election periods.

A brief word about the Indian political system is in order. The Indian Parliament is bicameral in nature. However, the Lok Sabha is the popularly elected House and is de facto more powerful than the other House (Rajya Sabha). The popularly elected Members of Parliament (MP) enjoy a five-year term after which fresh Lok Sabha elections are held. There were 518 (Lok Sabha) constituencies in 1971. This went up to 542 after a Delimitation order in 1976 and then to 543 in 1991.

Population is the basis of allocation of seats of the Lok Sabha. As far as possible, every state gets representation in the Lok Sabha in proportion to its population as per census figures. Hence, larger and more populous states have more seats in the Lok Sabha as compared to their smaller and sparsely-populated counterparts. For example, Uttar Pradesh (a north Indian state) with a population of over 166 million has 80 Lok Sabha seats while the state of Nagaland with a population of less than 2 million has only one Lok Sabha seat.

The NSSO expenditure rounds allow identification of the surveyed household up to the district to which it belongs; no finer identification is possible. However, it is often the case that a single district houses more than one electoral constituency; this is especially true for more populous districts. Given the nature of the hypotheses, I have to restrict attention to only single-constituency districts, i.e. to those places where a district corresponds to just one single constituency.[10] In the sample, there are 179 such districts that are follow for two

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[10]In a district with several constituencies, the link between electoral competitivess and polarization (or inequality) cannot be clearly established. For example, any change in polarization (or inequality) in any one of the constituencies (presumably as a response to electoral competition in that constituency) does not necessarily reflect a similar change in polarization (or inequality) in the district overall.

Table 14.: Descriptive Statistics (1987-88).

Variable	Mean	Std Dev	Min	Max
Vote share of winner (in previous election)	54.347	8.299	35.910	81.080
Margin (in previous election)	22.055	14.025	0.030	64.080
Swing Congress	0.240	0.428	0.000	1.000
Average margin	23.587	8.837	3.930	51.370
Average Vote share of winner	55.356	5.126	41.907	74.405
Average pce	190.047	51.251	89.037	350.526
Literacy rate (%)	42.122	14.362	14.077	90.848
Population (%)	0.199	0.078	0.042	0.408
Rural population (%)	80.662	14.001	21.622	100.000
Headcount ratio (%)	34.209	17.831	4.023	83.317
Poverty gap ratio (%)	8.344	5.756	0.365	30.985
Gini (%)	30.038	5.065	15.816	47.209
Hindu population (%)	84.448	18.017	0.779	100.000
SC/ST (%)	29.877	15.195	0.261	88.176
Inter-quartile range/mean pce	0.531	0.083	0.272	0.810
Polarization (FW measure)	0.127	0.025	0.057	0.209

Notes: The information on electoral outcomes is from the Election Commission of India statistical reports. The national elections were held in 1977, 1980 and 1984-85. The data on the consumer expenditure and other demographic characteristics comes from the NSS 43rd round which was conducted during 1987-88.

time periods.[11]

Tables 14 and 15 provide the summary statistics of the variables used in the analysis. Comparing the election data across the two periods, I see that elections clearly became more competitive over the years. For instance, in the elections prior to 1988, the average margin of victory varied between 4% and 51%. On the other hand, in the elections between 1988 and 2004 the average margin was never higher than 31% for any constituency. Between the two periods, both poverty and inequality have fallen on average across the districts suggestive of a trend towards a secular balanced growth. Notably, polarization as measured by the Foster-Wolfson index registers a decline – on average – when comparing across the two periods; this is suggestive of the growth of the "middle class" over time. Altogether, these tables clearly indicate that there was dynamism both on the income distribution frontier and in the political scene in India during the period of this study.

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[11]These 179 districts are not systematically different from the others. See Table 28 in the Appendix.

Table 15.: Descriptive Statistics (2004-05).

Variable	Mean	Std Dev	Min	Max
Vote share of winner (in previous election)	48.794	8.189	26.540	69.830
Margin (in previous election)	11.270	8.970	0.190	40.660
Swing Congress	0.291	0.455	0.000	1.000
Average margin	11.433	5.820	2.573	30.433
Average Vote share of winner	46.211	6.183	29.837	62.823
Average pce	657.746	215.621	341.888	1,452.527
Literacy rate (%)	58.687	13.402	27.327	97.063
Population (%)	0.206	0.082	0.044	0.475
Rural population (%)	80.609	13.969	20.470	97.989
Headcount ratio (%)	23.024	16.060	0.000	67.986
Poverty gap ratio (%)	4.331	3.744	0.000	17.941
Gini (%)	26.162	5.514	11.621	43.083
Hindu population (%)	83.632	19.612	0.208	100.000
SC/ST (%)	29.757	15.920	0.000	91.977
Inter-quartile range/mean pce	0.473	0.104	0.202	0.954
Polarization (FW measure)	0.115	0.030	0.051	0.237

Notes: The information on electoral outcomes is from the Election Commission of India statistical reports. The national elections were held in 1991-92, 1996, 1998 and 1999. The data on the consumer expenditure and other demographic characteristics is from the NSS 61st round which was conducted during 2004-05.

I now move on to the details of the empirical strategy for the identification of the relevant parameters.

Empirical Specification

The data provides us with a two-period panel spanning 1987-88 and 2004-05. I use a linear fixed effects specification for the empirical exercise. Specifically, for every district d in time period t, I have:

$$y_{dt} = \alpha_d + \gamma_t + \beta X_{dt} + \rho Z_{dt} + \epsilon_{dt}$$

where y_{dt} is a measure of inequality or polarization, X_{dt} includes a vector of variables describing the political climate in the district, (like average margin in the last 3-4 elections, etc.). Z_{dt} is the set of demographic and geographical controls used such as the population

share of the district, percentage of Hindus in the district, literacy rates and average monthly per capita expenditure for the district. α_d represents the district fixed effects while γ_t captures the time effect. Also, ϵ_{dt} is the error term in this panel specification.

The primary results are collected below.

Results

I first turn to the prediction given under [A], namely that increased electoral uncertainty leads to lower income (in this case, proxied by consumer expenditure) polarization. As discussed briefly before, I construct several measures to capture the extent of political competition in a district. The primary proxy for electoral uncertainty exploits the difference in percentage vote shares of the two parties that obtain the highest number of votes in any constituency. This is in the spirit of Arulampalam et al (2009).

I use (i) the winning margin and (ii) the vote share of the winning party — each averaged over the 3-4 elections prior to each expenditure round — in turn to capture the extent of electoral competition in the district. The average margin in the previous 3 to 4 general elections is used as the primary variable to describe how closely the elections have been in a district. I use the average across the previous few elections to ensure that I am not capturing any effect particular to the previous election. The vote share of the winner is also used as a measure of electoral competition. Clearly, the higher the percentage of votes obtained by the winner, the lower the degree of electoral competitiveness in the district.

Main results

Table 16 gives the results for the benchmark case, the effect of average margin on polarization. I find that an increase in the political competition (a lower average margin)

Table 16.: Linear panel regression with average margin. Dependent variable is the Foster-Wolfson measure of Polarization.

	[1]	[2]	[3]	[4]	[5]
Average margin	0.006***	0.005***	0.005***	0.006***	0.006***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Population		0.267	0.256	0.261	0.250
		(0.361)	(0.360)	(0.352)	(0.351)
Rural percent		-0.006*	-0.006*	-0.005*	-0.005
		(0.003)	(0.003)	(0.003)	(0.003)
Hindu percent			0.001	0.001	0.001
			(0.003)	(0.003)	(0.003)
SC/ST percent			-0.002	-0.002	-0.002
			(0.002)	(0.002)	(0.002)
Headcount (poverty)				0.002	
				(0.002)	
Income gap (poverty)					0.009**
					(0.004)
Number of observations	358	358	358	358	358
Adjusted R2	0.197	0.213	0.212	0.217	0.233

Notes: Dependent variable is the Foster-Wolfson measure of Polarization. Average margin is constructed using the previous 3-4 general elections. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%.

is highly correlated with lower polarization or a larger middle class. This result is robust to adding controls including the poverty headcount and poverty gap measures. Table 17 reflects that the result is robust to using the average vote share of winner in place of the average margin.

Also, I use the normalized inter-quartile range as a proxy for the size of the middle class, with lower values of this variable implying a larger middle class. Even then I see that a higher average margin is congruent with greater difference between the two income quartiles thus normalized (see Table 18).

Next, I turn to the hypothesis [B], which is that increased electoral uncertainty leads to lower income (in this case, proxied by consumer expenditure) inequality. Table 29 shows that inequality is also higher when there is lesser political competition as measured by the average margin.

Table 17.: Linear panel regression with average vote. Dependent variable is the Foster-Wolfson measure of Polarization.

	[1]	[2]	[3]	[4]	[5]
Average vote share of winner	0.006**	0.005*	0.005*	0.005**	0.005**
	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)
Population		0.292	0.285	0.287	0.272
		(0.365)	(0.363)	(0.356)	(0.354)
Rural percent		-0.006**	-0.006**	-0.006*	-0.006*
		(0.003)	(0.003)	(0.003)	(0.003)
Hindu percent			0.001	0.001	0.001
			(0.003)	(0.003)	(0.003)
SC/ST percent			-0.002	-0.002	-0.002
			(0.002)	(0.002)	(0.002)
Headcount (poverty)				0.001	
				(0.002)	
Income gap (poverty)					0.009**
					(0.004)
Number of observations	358	358	358	358	358
Adjusted R2	0.167	0.190	0.189	0.192	0.210

Notes: Dependent variable is the Foster-Wolfson measure of Polarization. Average vote share of winning party is constructed using data from the previous 3-4 general elections. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% *** significant at 1%.

Table 18.: Linear panel regression.with average margin. Dependent variable is the log of the inter-quartile range normalized by the mean pce.

	[1]	[2]	[3]	[4]	[5]
Average margin	0.005***	0.004***	0.005***	0.005***	0.005***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Population		0.254	0.249	0.256	0.242
		(0.315)	(0.315)	(0.307)	(0.306)
Rural percent		-0.005**	-0.005*	-0.004*	-0.004
		(0.002)	(0.003)	(0.003)	(0.003)
Hindu percent			-0.001	0.000	-0.000
			(0.002)	(0.002)	(0.002)
SC/ST percent			-0.000	-0.001	-0.001
			(0.002)	(0.002)	(0.002)
Headcount (poverty)				0.002**	
				(0.001)	
Income gap (poverty)					0.012***
					(0.003)
Number of observations	358	358	358	358	358
Adjusted R2	0.265	0.280	0.277	0.294	0.322

Notes: Dependent variable is the log of the inter-quartile range normalized by the mean pce. Average margin is constructed using data from the previous 3-4 general elections. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%

Robustness checks

Rather than using the average values for the proxies of electoral competition in the previous 3-4 elections, one could also use the margin and vote share of winner from the most previous election. I do so and the results are similar to the earlier ones (see Table 30 and Table 31).

Another way to capture the idea of a swing district would be the following. One could possibly identify whenever there is a change in the political party which wins the election in the district. However in 1977 (the first election year I look at) there were only 20 recognized political parties which contested the elections. By 1999 the number of recognized political parties had risen to 47. This significant rise in the number of political parties was not merely a case of greater participation of the general populace in the political domain — it was more the case that several political parties were created by the splintering of existing political parties. Therefore, for the time horizon considered, I am unable to track whether there was a swing away from a particular political party or that merely a segment of the old party came back into power.

The only political party which has remained relatively "stable", in the sense of maintaining its core identity, is the Indian National Congress. Given the way the nature of politics and political parties evolved during this period, I chose to proceed with a rather conservative division of parties into "Congress" and "Non-Congress" camps and recorded the movements of a district between these camps over the different election periods. Therefore, as an additional measure of political regime change, I use whether or not the district moved away from towards a Congress MP. I create a dummy variable which takes the value of 1 if there was a change to or from a Congress MP in the district and 0 otherwise. Note, the swing congress variable is a very crude measure of the district's electoral volatility and it

Table 19.: Linear panel regression with swing away or towards Congress. Dependent variable is the Foster-Wolfson measure of Polarization.

	[1]	[2]	[3]	[4]	[5]
Swing congress	-0.081***	-0.073**	-0.074**	-0.074**	-0.073**
	(0.029)	(0.029)	(0.030)	(0.030)	(0.030)
Population		0.486	0.471	0.480	0.472
		(0.376)	(0.370)	(0.363)	(0.361)
Rural percent		-0.005*	-0.005*	-0.005*	-0.005
		(0.003)	(0.003)	(0.003)	(0.003)
Hindu percent			0.002	0.002	0.002
			(0.003)	(0.003)	(0.003)
SC/ST percent			-0.002	-0.002	-0.002
			(0.002)	(0.002)	(0.002)
Headcount (poverty)				0.001	
				(0.001)	
Income gap (poverty)					0.008**
					(0.004)
Number of observations	358	358	358	358	358
Adjusted R2	0.175	0.195	0.197	0.198	0.213

Notes: Dependent variable is the Foster-Wolfson measure of Polarization. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%.

exhibits much less variation vis-a-vis the other measures of electoral competition.

Table 19 contains some of the results using this Swing Congress variable. The first column in this table reveals a strong negative correlation between Swing Congress and polarization in accordance with the previous findings. This effect is robust to the inclusion of several controls; see columns (2) through (5).

The swing congress variable exhibits a similar effect on the degree of inequality in the district as captured by the gini coefficient. The results are collected in Table 32 shows that in all specifications the marginal effect of Swing Congress on inequality is significant and negative. Therefore, these results reiterate the basic findings.

Concerns

I discuss two of the main concerns that arise in an empirical exercise of the kind undertaken here. The first one is endogeneity due to reverse causality. One could argue

that the middle class votes in a certain way so as to make the political contest close. The second is the issue of migration as a result of political transfers or public goods provision. I will briefly discuss each issue in turn.

The first concern regarding the voting behavior of the middle class implies the following — it presumes that the members of the middle class vote in a markedly coordinated fashion, which is perhaps not plausible in the Indian context. Bardhan et al (2008) study political participation and targeting of public services in the Indian state of West Bengal. In their words "...the difference in reported registration rates and turnouts were modest, more similar to the European patterns rather than the steep asymmetries in the United States. With regard to voting disturbances, there was no clear correlation with socioeconomic status." They also find that attendance rates (in political meetings, such as rallies, election meetings called by political parties) did not exhibit any marked unevenness across different land classes. So this does not seem to pose a serious problem. Also, in all of the regressions presented so far, I look at the effect of elections on subsequent polarization (and inequality) — so that there is enough of a time lag with elections preceding the corresponding expenditure rounds.

As to the second concern — namely, migration as a response to political transfers/public goods provision — I can take some comfort in the fact that migration rates in India are rather low in comparison with other developing nations. In fact, Munshi and Rosenzweig (2009) explicitly state that "Among developing countries, India stands out for its remarkably low levels of occupational and spatial mobility." They delve into the proximate causes behind this phenomenon and using a unique panel dataset (identifying sub-caste (jati) membership) find that the existence of sub-caste networks that provide mutual insurance to their members play a key role in restricting mobility.

Taking stock of the entire empirical findings, one is lead to admit that there is a serious relationship between the degree of electoral competition in a district and the nature of redistribution pursued therein. More specifically, I find that districts which have experienced tighter elections also tend to be the ones with lower levels of inequality and polarization suggesting that the middle class thrives where political parties are perceived to be relatively balanced in the eyes of the voters.

Conclusion

This paper attempts to study how the degree of electoral competition affects the growth of a middle-income group — in other words, a middle class — in the context of a developing nation. Most developing countries are typically characterized by high levels of income inequality; the society is polarized with the affluent on one side and the destitute on the other. Of course, there exist income-earners somewhere between the two poles but for most developing nations, this intermediate group is quite small. However, it is this middle-income group which can boost the economy by both consuming and saving. A growing middle class is a healthy sign indicative of balanced growth and holds fewer threats of class cleavages. Therefore, it is important to focus on factors which affect the growth of the middle class. This paper studies the relationship between electoral uncertainty and the degree of income polarization in society with the help of a simple theoretical framework and then provides empirical findings in support of the theory.

The theory is based on the traditional two-party Downsian framework with ideological voters in the spirit of Lindbeck and Weibull (1987) and Dixit and Londregan (1996). Here, the political parties can a priori commit to some levels of investment in three different kinds of public goods — one that disproportionately benefits the poor, another which

disproportionately benefits the rich and finally a pure public good which benefits all groups equally. I assume that investment is costly and that each party wishes to maximize plurality net of investment costs. I show that as the level of electoral uncertainty increases in the district, the equilibrium level of investment in each type of public good increases; in other words, there is greater transfer to the electorally competitive districts (in line with the findings in the previous literature). Most importantly, I show that increased electoral uncertainty induces additional investment in a manner that the additional benefits to the poor exceed the additional benefits to the middle-income group, which in turn, exceed the additional benefits to the rich. Thus, increased electoral uncertainty reduces all inter-person income differences in society and hence reduces income inequality. Furthermore, by bringing both the poor and the rich "closer" to the middle class group, it reduces income polarization.

The model generates several empirically testable predictions which I then take to data from India. India has had a vibrant democracy since the nation's independence in 1947. Although there have been several political parties since the 1950s, the national elections had been by and large dominated by the Indian National Congress (INC) party. However, since the 1980s there have been a tremendous proliferation of political parties both at the state and the national levels. In fact, 1977 was witness to a non-Congress led government at the centre for the first time since India's independence.

Although the INC continues to be a major player in national elections till this day, it no longer enjoys the kind of monopoly it did till the mid-1960s. Moreover, a majority of elections in the 1990s resulted in "hung Parliaments" meaning that no single party obtained a clear majority of seats and thus began the era of coalitional politics in India. The period of study corresponds to the time after the INC had lost its quasi-monopoly in the political

arena. So the data is from the phase where national elections were more intensely fought. All of these factors contribute to making India an interesting candidate for testing the hypotheses.

The previous literature has stressed the role of political competition in directing transfers and have generally concluded that "swing" districts get more targeted resources in the aggregate. I subject the main theoretical finding — namely, that electorally "swing" constituencies tend to exhibit lower degree of income polarization — to rigorous empirical analysis. I use data from the Indian parliamentary (national) elections which are combined with household-level consumption expenditure data rounds from NSSO (1987-88 and 2003-04) to yield a panel of Indian districts. The main variable representing electoral "swing" is the actual margin of winning which is the difference between the percentage vote shares of the two parties that obtain the highest number of votes in any constituency. Using this variable as the baseline measure of electoral volatility of a district, I obtain that a district which has experienced close elections tends to exhibit a lower degree of income polarization.

I repeat the analysis with alternative variables for electoral swing; for example, I restrict attention to the most recent election that took place before the relevant NSS expenditure round (rather than take an average over several prior elections). The results I get are robust to such variations — more "swing" districts exhibit lower (expenditure) polarization. The pattern persists when I replace winning margin by simply the vote share of the winning party. There is evidence of a similar relationship between inequality (as measured by the Gini coefficient) and electoral uncertainty. Inter-quartile differences in expenditure (normalized by the average level of expenditure) also tend to be higher where winning margins are wider.

Overall the empirical findings clearly suggest that greater electoral uncertainty is

highly correlated with the growth of the middle class and reduces existing income disparities. In particular, the empirical analysis helps to identify at least one important channel which stifles the growth of the "middle class" in India; namely, the under-performance of political parties in districts which are electorally secure. It is important to point out that the notion of a middle class adopted here is fairly "local" in the following sense: the middle class in a district is some group whose earnings correspond to any given income band around the median income-earner in that district. Alternatively, one could think of a middle class at the level of the nation and then study the proportion of people in each district which falls in this "national middle class". One could investigate how district-level political competition affects the size of this "national middle class" in every district. I plan to explore this question in future work.

In a way the results seem to highlight some drawbacks of the electoral mechanism. The key issue here is the presence of people who are highly ideologically inclined towards some political party or the other. A party which rides to victory on the back of large popular support feels less inclined to cater to the toiling masses; after all, if the electorate likes the party anyhow why should the latter bother working hard to reduce existing disparities? However, if one extends this to a dynamic setting, the voters would potentially change their opinion over time about the inactive (and ineffective) incumbent party. The problem often is that the opposing party — the challenger, so to speak — may not be much of a viable alternative. However, the very realization that perhaps each political party is ex-ante as good as the other should drive this voter bias close to nil in expected terms thus inducing better promises (and action) from both parties in future. Hence, over time one could expect to see a convergence (towards zero) in voter biases.

The fact is that parties themselves change their stand and nature over time. This

makes any kind of convergence on part of voter biases quite unlikely. Incidentally, voter biases in regions tend to persist over time. For example, in the context of the US, New York has traditionally been Democrat. In India as well, this kind of party loyalty is fairly common — for e.g., West Bengal (a state in eastern India) had been under the rule of a Left-led coalitional government for over 30 years. There may be clientelistic relationships which develop between incumbents and certain sections of the voters (see Bardhan and Mookherjee (1999)) which create such long spells of governance by a party; perhaps longer than what a dynamic extension of the simple model (with updating of voter biases) would predict.

Finally, it would be interesting to explore how different political parties have re-invented themselves over time and what impact has this had on their loyalists — perhaps the conservatives of today would have been liberal half a century ago. A more holistic view of the interplay between party evolution and changing voter loyalties could provide meaningful insights to policy-making.

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Appendix to chapter II

Mathematical derivation of the distribution of P_α measures

The notation used and the results closely follow Bhattacharya (2005). The sampling design is described as follows. Prior to sampling the population is divided into S first stage strata. In the population the stratum s contains H_s clusters. A sample of n_s (indexed by ψ_s) clusters in drawn via simple random sample with replacement from stratum s, for each s. The ψ_sth cluster has a total of $N_{s\psi_s}$ households. A simple random sample draws κ (equal across clusters and strata and indexed by h) households from each cluster for each strata is drawn. The hth household in the ψ_sth cluster has $\nu_{s\psi h}$ members. The density of an individual characteristic π in the sth strata is given by $dF(\pi \mid s)$. Note that this joint density can differ across strata so that the sampled observations from different strata are independent but not identically distributed.

Let $n = \sum_{s=1}^{S} n_s$ and $n_s = na_s$ with $\sum_{s=1}^{S} a_s = 1$. The weight of every individual in the hth household in the ψ_sth cluster of the sth stratum is given by

$$w_{s\psi_s h} = \frac{N_{s\psi_s} H_s}{\kappa n_s v_{s\psi_s h}}$$

Now I am interested in the parameter P_a (the poverty estimate for the population), which solves the simple moment condition

$$0 = \sum_{s=1}^{S} H_s \int (P_\alpha - \pi) \ dF(\pi \mid s).$$

So the MoM estimate of P_α is based on the sample analog of the equation given above and can be written as :

$$\sum_{s=1}^{S} \frac{H_s}{n_s} \sum_{\psi_s=1}^{n_s} \frac{N_{s\psi_s}}{\kappa} \sum_{h=1}^{k} \frac{1}{v_{s\psi_s h}} \sum_{j_h=1}^{\nu_{s\psi_s h}} \left(\hat{P}_\alpha - \pi_{j_h} \right) \simeq 0 \tag{A.1}$$

However for the purpose of the asymptotic analysis it is beneficial to rewrite the above in the following way. I reindex the clusters so that they number from 1 to n. n denotes the total number of clusters in the sample. For every i there is s_i which indicates the stratum from which cluster i is drawn. Then, $\#(i \mid s_i = s) = n_s$ $for\ each\ 1 \leq s \leq S$. So, equation 1 can be rewritten as :

$$\frac{1}{n} \sum_{i=1}^{n} \sum_{s=1}^{S} \frac{H_s}{a_s} 1(s_i = s) \sum_{\psi_s=1}^{n_s} \frac{N_{s_s,i}}{\kappa} \sum_{h=1}^{k} \frac{1}{v_{s\psi_s h}} \sum_{j_{ih}=1}^{\nu_{s_i ih}} \left(\hat{P}_\alpha - \pi_{j_{ih}} \right) \simeq 0$$

Since the set up is identical to that in Bhattacharya (2005) I know that

$$p \lim_{n \to \infty} \left(\hat{P}_\alpha - P_\alpha \right) \quad = \quad 0,\ and$$

$$\sqrt{n} \left(\hat{P}_\alpha - P_\alpha \right) \quad \longrightarrow \quad {}^d N(0, V)$$

In this case V takes a simplified form since the parameter is simply a population average. Here

$$V = W_0 = \lim_{n \to \infty} W_n, \text{ and}$$

$$\lim_{n \to \infty} W_n = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} Var \left(\sum_{s=1}^{S} \frac{H_s}{a_s} 1 \left(s_i = s \right) \sum_{\psi_s=1}^{n_s} \frac{N_{s_s,i}}{\kappa} \sum_{h=1}^{\kappa} \sum_{j_{ih}=1}^{\nu_{s_i ih}} \left(\hat{P}_\alpha - \pi_{j_{ih}} \right) \right)$$

An estimate of W_n is given in the paper and is the same which will be used here.

It is the following

$$\hat{V} = \hat{W}_n = 4 \sum_{s=1}^{S} \frac{n_s - 1}{n_s} \left(\sum_{\psi_s=1}^{n_s} \sum_{h=1}^{\kappa} \sum_{j_{ih}=1}^{\nu_{s_i ih}} w_{sc_s h} \left(\hat{P}_\alpha - \pi_{j_{ih}} \right) \right)^2 \tag{A.2}$$

This expression is simpler than the one used in Bhattacharya (2005) since in this paper I don't want to disaggregate the stratum and cluster effects, so I can combine terms to get a simpler form for the variance.

Robustness checks for empirical results

Table 20.: Robustness Check with alternate weighting Schemes. Comparison of levels of poverty among states with k=3.5.

States	Weight on Income (Percent)					
	20%	25%	30%	35%	40%	45%
Andhra Pradesh	0.31	0.30	0.21	0.20	0.20	0.14
Bihar	0.53	0.53	0.45	0.45	0.45	0.40
Chhattisgarh	0.57	0.57	0.49	0.49	0.49	0.42
Gujarat	0.28	0.27	0.17	0.17	0.17	0.11
Haryana	0.14	0.15	0.08	0.08	0.09	0.08
Jharkand	0.52	0.51	0.44	0.44	0.44	0.37
Karnataka	0.27	0.28	0.20	0.20	0.21	0.18
Kerala	0.15	0.17	0.13	0.13	0.14	0.13
Madhya Pradesh	0.46	0.46	0.36	0.36	0.37	0.28
Maharashtra	0.22	0.23	0.18	0.18	0.19	0.17
Orissa	0.67	0.66	0.60	0.59	0.58	0.50
Punjab	0.11	0.12	0.08	0.08	0.09	0.08
Rajasthan	0.40	0.40	0.31	0.30	0.30	0.22
Tamil Nadu	0.25	0.26	0.18	0.18	0.19	0.16
Uttar Pradesh	0.40	0.40	0.31	0.31	0.32	0.27
West Bengal	0.39	0.39	0.33	0.33	0.33	0.28

Notes: Each column of this table gives the level of poverty in the state using a different weighting scheme of the dimensions. As one moves from left to right each column puts more weight on income and weighs all other dimensions equally. For example, column 5 puts 40 % weight on income and 10 % on each of the other 6 dimensions.

Table 21.: Robustness Check with alternate weighting Schems: Comparison of Ranks of states with k=3.5.

	Weight on Income (Percent)					
States	20 %	25 %	30 %	35%	40%	45%
Andhra Pradesh	8	8	8	8	7	5
Bihar	14	14	14	14	14	14
Chhattisgarh	15	15	15	15	15	15
Gujarat	7	6	4	4	4	3
Haryana	2	2	2	2	2	2
Jharkand	13	13	13	13	13	13
Karnataka	6	7	7	7	8	8
Kerala	3	3	3	3	3	4
Madhya Pradesh	12	12	12	12	12	12
Maharashtra	4	4	5	5	6	7
Orissa	16	16	16	16	16	16
Punjab	1	1	1	1	1	1
Rajasthan	11	10	9	9	9	9
Tamil Nadu	5	5	6	6	5	6
Uttar Pradesh	10	11	10	10	10	10
West Bengal	9	9	11	11	11	11

Notes: Each column of this table gives the relative rank of the state in a poverty comparison using a different weighting scheme of the dimensions. As one moves from left to right each column puts more weight on income and weighs all other dimensions equally. For example, column 5 puts 40 % weight on income and 10% on each of the other 6 dimensions.

Table 22.: Multidimensional poverty for Hindus and Muslims for Rural and Urban areas.

Cutoff (k)	P_0 (Hindu-Urban)	P_0 (Muslim-Urban)	P-values	P_0 (Hindu-Rural)	P_0 (Muslim-Rural)	P-values
1	0.197	0.249	0.000	0.572	0.583	0.274
2	0.159	0.209	0.000	0.565	0.577	0.286
3	0.122	0.156	0.004	0.539	0.544	0.656
4	0.086	0.094	0.475	0.460	0.460	0.978
5	0.050	0.040	0.174	0.315	0.332	0.361
6	0.021	0.014	0.181	0.150	0.188	0.028
7	0.005	0.002	0.059	0.037	0.051	0.206

Notes: The p-values given are for the test of the difference among Hindu and Muslim poverty in urban and rural areas separately.

Table 23.: Differences in poverty based on differences in Caste among the Rural and Urban populations.

Cutoff(k)	P_0 (Hindu-High Rural)	P_0 (Hindu-High Urban)	P_0 (Hindu-Low Rural)	P_0 (Hindu-Low Urban)	P_0 (Muslims-Rural)	P_0 (Muslims-Urban)
1	0.529	0.166	0.658	0.323	0.583	0.249
2	0.521	0.127	0.656	0.293	0.577	0.209
3	0.488	0.093	0.642	0.243	0.544	0.156
4	0.397	0.064	0.587	0.177	0.460	0.094
5	0.249	0.037	0.451	0.106	0.332	0.040
6	0.103	0.014	0.245	0.049	0.188	0.014
7	0.023	0.003	0.067	0.014	0.051	0.002

Notes: Hindu-High-Rural describes the poverty level of High caste Hindus living in Rural areas. Since Muslims do not have a caste system I only have poverty of Muslims in Rural and urban areas (the group is not further divided on the basis of caste)

Table 24.: Robustness checkes with alternative weighting schemes: Differences among Hindu and Muslim households.

Weight on Income (Percent)	P_0 (Hindu)	(Muslim)
20	0.35	0.32
25	0.35	0.34
30	0.27	0.28
35	0.27	0.29
40	0.28	0.30
45	0.23	0.27

Notes: Each row uses a different weighting scheme. For example in the second row income has 20 % of the weight and the rest 80 % is equally divided among the rest of the six dimensions

Table 25.: Robustness checkes with alternative weighting schemes(Differences among religious groups).

Weight on Income (Percent)	P_0 (Hindu-Rural)	P_0 (Hindu-Urban)	P_0 (Muslims-Rural)	P_0 (Muslim-Urban)
20	0.45	0.09	0.45	0.10
25	0.44	0.11	0.45	0.14
30	0.33	0.10	0.37	0.13
35	0.33	0.10	0.37	0.13
40	0.33	0.12	0.37	0.18
45	0.26	0.12	0.32	0.18

Notes: Each row uses a different weighting scheme. For example in the second row income has 20 % of the weight and the rest 80 % is equally divided among the rest of the six dimensions

Table 26.: Differences in poverty based on differences in Caste among the Rural and Urban populations.

Weight on Income (Percent)	P_0 (Hindu-High-Caste)	P_0(Hindu-Low-Caste)	P_0(Muslims)
20	0.29	0.51	0.32
25	0.29	0.51	0.34
30	0.21	0.42	0.29
35	0.21	0.42	0.29
40	0.22	0.42	0.30
45	0.17	0.35	0.27

Notes: Each row uses a different weighting scheme. For example in the second row income has 20 % of the weight and the rest 80 % is equally divided among the rest of the six dimensions

Table 27.: Robustness checks with alternative weighting schemes: Differences among religious groups in Rural and Urban areas.

Weight on Income (Percent)	P_0(Hindu-HighRural)	P_0(Hindu-HighUrban)	P_0(Hindu-LowRural)	P_0(Hindu-LowUrban)	P_0(Muslims-Rural)	P_0(Muslims-Urban)
20	0.39	0.06	0.57	0.18	0.45	0.09
25	0.38	0.08	0.56	0.21	0.45	0.14
30	0.27	0.07	0.46	0.20	0.37	0.13
35	0.27	0.07	0.46	0.20	0.37	0.14
40	0.27	0.09	0.46	0.23	0.37	0.18
45	0.21	0.09	0.37	0.23	0.32	0.19

Notes: Each row uses a different weighting scheme. For example in the second row income has 20 % of the weight and the rest 80 % is equally divided among the rest of the six dimensions

APPENDIX TO CHAPTER III

Proofs

Proof of Theorem 1. The proof turns out to be rather straightforward once I combine the fact that F can be built up from simple Donsker classes together with well established results on Donsker preservation. Thus, first introduce the classes

$$\mathcal{G}_1 = \left\{ \mathbb{1}\left(\sum_{j=1}^{d} \omega_j \mathbb{1}(x_j \leq \ell_j) \geq k\right) : \ell \in \mathbb{R}_{++}^d, k \in [\underline{k}, \bar{k}], \sum \omega_i = d, \omega_i \geq 0 \right\}, \qquad \text{(B.1)}$$

$$\mathcal{G}_2 = \left\{ \omega \left(\frac{\ell - x}{\ell}\right)^{\alpha} \mathbb{1}(x \leq \ell) : \ell \in [0, \bar{\ell}], 1 \leq \alpha \leq 3, \omega \in \mathbb{R} \right\},$$

and

$$\mathcal{G}_3 = \left\{ \omega \mathbb{1}(x \leq \ell) : \ell \in [0, \bar{\ell}], \omega \in \mathbb{R} \right\}.$$

That G_2 and G_3 are Donsker follows trivially from Theorem 9.23 and Lemma 9.8 of (Kosorok 2008), respectively. By appealing again to Lemma 9.8 (Kosorok 2008), it follows directly that G_1 is Donsker if the collection

$$\mathcal{A} = \{A(\ell, \omega, k) : k \in [\underline{k}, \bar{k}] \subset \mathbb{R}_{++}, \ell \in \mathbb{R}_{++}^d, \sum \omega_j = d, \omega \in \mathbb{R}_+^d\}, \qquad \text{(B.2)}$$

where $A(\ell, \omega, k) = \{x \in R^d : \sum_{j=1}^{d} \omega_j \mathbb{1}(x_j \leq \ell_j) \geq k\}$, forms a Vapnik-Červonenkis (VC) class of sets. Letting $D = \{1, \ldots, d\}$ and recognizing that $A \in A$ is always of the form

$$A = \prod_{j \in S \subseteq D} (-\infty, \ell_j] \times \prod_{j \in D \setminus S} (-\infty, \infty)$$

it follows that A is a subset of the collection of cells in R^d, and thus is VC with VC-index less than or equal to $d + 1$.

Given that G_1, G_2, and G_3 are (uniformly bounded) Donsker classes, the proof is completed upon repeated application of Corollary 9.32 together with Theorem 9.31 of (Kosorok 2008). ∎

Proof of Theorem 2. That F is a uniformly bounded Donsker class follows from the proof of Theorem 1 above. As an immediate consequence I obtain

$$\sqrt{n_1}(P_{n_1,X} - P_X)f \rightsquigarrow \mathbb{G}_{P_X}f, \tag{B.3}$$

and

$$\sqrt{n_2}(P_{n_2,Y} - P_Y)f \rightsquigarrow \mathbb{G}_{P_Y}f, \tag{B.4}$$

in $\ell^\infty(F)$, where $\rightsquigarrow$ denotes weak convergence. Then, noting that

$$\left(\frac{n_1 n_2}{n_1 + n_2}\right)^{1/2}\left[n_1^{-1/2}\mathbb{G}_{n_1,P_X}f_1 - n_2^{-1/2}\mathbb{G}_{n_2,P_Y}f_2\right] \tag{B.5}$$

may be written as

$$\left[\left(\frac{n_2}{n_1 + n_2}\right)^{1/2}\sqrt{n_1}(P_{n_1,X} - P_X)f_1 - \left(\frac{n_1}{n_1 + n_2}\right)^{1/2}\sqrt{n_2}(P_{n_2,Y} - P_Y)f_2\right], \tag{B.6}$$

where $f_1, f_2 \in F$, I obtain the desired result as a direct consequence of (B.3), (B.4), the assumed independence of the processes, and the convergence of the pre-multiplicative ratios as implied by Assumption III.4.1. ∎

Proof of Theorem 3. The proof is analogous to that of Theorem 2 and is therefore omitted. ∎

APPENDIX TO CHAPTER IV

Table 28.: District-level summary statistics:Comparing Single-constituency with Non-Single constituency districts.

Variables (district)	N	[A] Single Mean	N	[B] Non-Single Mean	Difference S–NonS
IQR	182	0.520	145	0.528	-0.008
		(0.089)		(0.099)	(0.010)
Q3/Q1	182	1.862	145	1.895	-0.033
		(0.223)		(0.267)	(0.027)
Gini	182	0.295	145	0.303	-0.008
		(0.052)		(0.057)	(0.006)
Per-capita exp. (in Rs.)	182	191.265	156	191.869	-0.605
		(50.985)		(54.656)	(5.751)
SC population (%)	182	18.565	156	17.400	1.165
		(9.644)		(8.599)	(1.001)
Population	182	0.203	156	0.374	-0.171
		(0.086)		(0.179)	(0.015)***

NOTES: Panel A corresponds to the 182 districts where each district has just one constituency while Panel B has data from the remaining 156 districts. Standard errors in parentheses for the columns indicating differences; standard deviation in parentheses for all others. T-test used for comparing differences. *significant at 10% **significant at 5% ***significant at 1%

Table 29.: Linear panel regression with average margin. Dependent variable is the Gini coefficient.

	[1]	[2]	[3]	[4]	[5]
Average margin	0.123***	0.106**	0.101**	0.110**	0.108**
	(0.044)	(0.045)	(0.043)	(0.044)	(0.043)
Population		6.783	6.208	6.329	6.067
		(8.366)	(8.255)	(8.141)	(8.110)
Rural percent		-0.172***	-0.171***	-0.162***	-0.150**
		(0.059)	(0.058)	(0.059)	(0.059)
Hindu percent			0.023	0.033	0.031
			(0.054)	(0.055)	(0.054)
SC/ST percent			-0.082*	-0.094**	-0.093**
			(0.042)	(0.044)	(0.042)
Headcount (poverty)				0.041	
				(0.025)	
Income gap (poverty)					0.216***
					(0.078)
Number of observations	358	358	358	358	358
Adjusted R2	0.338	0.366	0.378	0.385	0.399

Notes: Dependent variable is the Gini coefficient. Average margin is constructed using data from the previous 3-4 general elections. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. *significant at 10 % ** significant at 5% ***significant at 1%.

Table 30.: Linear panel regression with margin of winning in last election. Dependent variable is the Foster-Wolfson measure of Polarization.

	[1]	[2]	[3]	[4]	[5]
Margin in last election	0.003**	0.002**	0.003**	0.003**	0.003**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Population		0.466	0.449	0.463	0.455
		(0.375)	(0.369)	(0.362)	(0.360)
Rural percent		-0.005*	-0.005*	-0.005	-0.005
		(0.003)	(0.003)	(0.003)	(0.003)
Hindu percent			0.001	0.001	0.001
			(0.003)	(0.003)	(0.003)
SC/ST percent			-0.002	-0.003	-0.003
			(0.002)	(0.002)	(0.002)
Headcount (poverty)				0.001	
				(0.001)	
Income gap (poverty)					0.009**
					(0.004)
Number of observations	358	358	358	358	358
Adjusted R2	0.170	0.191	0.193	0.196	0.213

Notes: Dependent variable is the Foster-Wolfson measure of Polarization. All regressions contain district fixed effects and time dummies. Robust standard erros clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%.

Table 31.: Linear panel regression with winner's vote share in last election. Dependent variable is the Foster-Wolfson measure of Polarization.

	[1]	[2]	[3]	[4]	[5]
Vote share of winner last election	0.003*	0.003	0.003	0.003*	0.003*
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Population		0.415	0.398	0.408	0.400
		(0.369)	(0.364)	(0.358)	(0.355)
Rural percent		-0.006**	-0.006**	-0.006*	-0.005*
		(0.003)	(0.003)	(0.003)	(0.003)
Hindu percent			0.001	0.001	0.001
			(0.003)	(0.003)	(0.003)
SC/ST percent			-0.002	-0.002	-0.002
			(0.002)	(0.003)	(0.002)
Headcount (poverty)				0.001	
				(0.001)	
Income gap (poverty)					0.008**
					(0.004)
Number of observations	358	358	358	358	358
Adjusted R2	0.158	0.182	0.183	0.185	0.201

Notes: Dependent variable is the Foster-Wolfson measure of Polarization. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%.

Table 32.: Linear panel regression with swing away or towrds Congress. Dependent variable is the Gini coefficient.

	[1]	[2]	[3]	[4]	[5]
Swing Congress	-1.930***	-1.673***	-1.664***	-1.667***	-1.650***
	(0.631)	(0.615)	(0.629)	(0.622)	(0.612)
Population		11.320	10.568	10.827	10.589
		(8.529)	(8.310)	(8.208)	(8.167)
Rural percent		-0.161***	-0.164***	-0.158***	-0.146**
		(0.059)	(0.058)	(0.059)	(0.059)
Hindu percent			0.049	0.058	0.058
			(0.056)	(0.057)	(0.056)
SC/ST percent			-0.086**	-0.095**	-0.097**
			(0.043)	(0.046)	(0.044)
Headcount (poverty)				0.031	
				(0.025)	
Income gap (poverty)					0.197**
					(0.081)
Number of observations	358	358	358	358	358
Adjusted R2	0.331	0.361	0.375	0.378	0.392

Notes: Dependent variable is the Gini coefficient. All regressions contain district fixed effects and time dummies. Robust standard errors clustered by district. * significant at 10 % ** significant at 5% ***significant at 1%.