

# POLYA'S **FOUR** STEP PROBLEM SOLVING METHOD

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**NU** Nagoya  
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Mirai Education Project

名大みらい育成プロジェクト



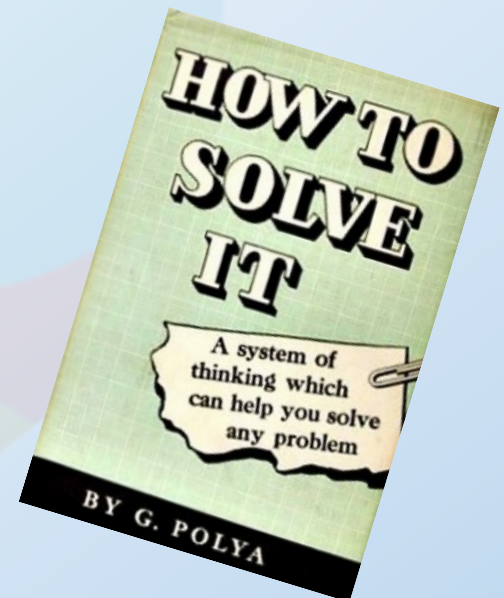
# WHO IS POLYA?

## George Pólya

1887-1985



- Was a teacher and mathematician.
- Published a book in 1945: “**How To Solve It**”, explaining how to become a better problem solver.



# WHAT IS A “PROBLEM”?

- A **problem** is a situation, condition, person, or thing that needs attention and needs to be dealt with or solved.
- Not solving a problem leads to undesirable consequences.

**“The solution to any problem lies in asking the right questions”**

**Polya’s four step method: A systematic way to answer/attack questions**

# POLYA'S **FOUR** STEPS

Polya's strategy to answer questions is given by the following four steps:

- 1 Understand the question**
- 2 Make a plan**
- 3 Carry out the plan**
- 4 Look back & Review**

# 1

## Understand the question

This seems so obvious that it is often ignored!

### Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the question in your own words?
- ❖ What part/information of the question is (un)important?
- ❖ Can you think of a picture or a diagram that might help you understand the question?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the question?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

## 2

## Make a plan

You must start somewhere so try something. How are you going to attack the question?

### Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.



# 3

## Carry out the plan

This is the step where you carry out the steps of your plan.

### Answering the question

- ❖ Try to use the strategy chosen in step 2.
- ❖ If this strategy does not work, try another one.



## 4

# Look back & Review

Finally, in this last step you look back reviewing and checking your results.

### Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the question or method be generalized to be useful for future problems?



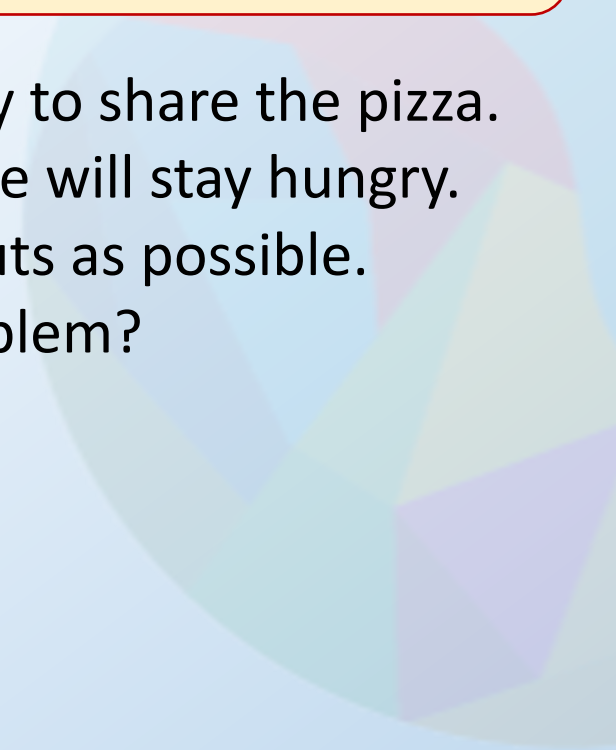


## EXAMPLE: Sharing a pizza

Let us try to use Polya's method to solve the following problem:

### Problem:

You are at a party with 11 people and you just have one pizza.

- This is a problem since you need to find a way to share the pizza.
  - Consequences if you do not share it: Someone will stay hungry.
  - Assume you want to cut the pizza with few cuts as possible.
  - What are possible questions to solve the problem?
- 

## EXAMPLE: Sharing a pizza

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- Consequences if you do not share it: Someone will stay hungry.
- Assume you want to cut the pizza with few cuts as possible.
- What are possible questions to solve the problem?

### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

# 1 Understand the question

## Pizza problem

### Question:

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# 1 Understand the question

## Pizza problem

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Can you cut a pizza into 11 pieces with just 4 cuts?

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# 1 Understand the question

## Pizza problem

### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

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- ❖ What are you asked to find or show?
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- ❖ Can you think of a picture or a diagram that might help you understand the problem?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the problem?
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# 1 Understand the question

## Pizza problem

### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

### Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the problem in your own words?
- ❖ **What part/information of the question is (un)important?**
- ❖ Can you think of a picture or a diagram that might help you understand the problem?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the problem?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

# 1 Understand the question

## Pizza problem

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Can you cut a pizza into 11 pieces with just 4 cuts?

### Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the problem in your own words?
- ❖ What part/information of the question is (un)important?
- ❖ Can you think of a picture or a diagram that might help you understand the problem?
- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in the problem?
- ❖ Do you need to ask a question to get the information you need?
- ❖ Why might this problem be difficult/easy?



# 1 Understand the question

## Pizza problem

### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

What kind of knife?      Straight cuts?      All pieces have some crust?  
Pieces of same size?      Can we stack pizza pieces?  
Does it always need to go through the center?

- ❖ Is there enough information to enable you to find a solution?
- ❖ Do you understand all the words used in stating the problem?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?



# 1 Understand the question

## Pizza problem

### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

If it is possible: Easy, since we can maybe give an explicit way of cutting the pizza.

If it is not possible: Maybe difficult because we need to argue why it is not possible.

- ❖ Is there a solution?
- ❖ Do you understand all the terms used in stating the problem?
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

**Possible Strategies**

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the question & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.

**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

**Possible Strategies**

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic. (If it is not possible)
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
- ❖ Make a list / Write down keywords.

**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

Strategies:

- ❖ Draw pictures or diagrams.
- ❖ Trial and error.



**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the problem or method be generalized to be useful for future problems?

# 4

## Look back & Review

### Pizza problem

#### Question:

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more efficient way to solve it?
- ❖ Is there a generalization to this problem?
- ❖ Can you find other similar problems?

Yes! We gave an explicit way of cutting a pizza into 11 pieces with 4 cuts.

**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem, with a different question?
- ❖ Can the problem be generalized to other numbers?

For example: If we restrict to “4 cuts which all go through the center” then there is no way to get 11 pieces.

**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?

❖ Is there another way of doing this?

❖ Can the

What is the maximum number of pieces you can obtain with 4 cuts?

What is the maximum number of (same sized) pieces (with crust) you can obtain with  $n$  cuts (which go through the center)?

What shape of knife would give more pieces?

Replace pizza (circle) by another shape.



**Question:**

Can you cut a pizza into 11 pieces with just 4 cuts?

Ask yourself the following questions

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
- ❖ Is there a better/more interesting version of the question?
- ❖ Is there another way of doing the problem which may be simpler?
- ❖ Can the problem or method be generalized to be useful for future problems?

# Pizza is not important...

- Of course there are more serious problems to solve.
- Most of them are not mathematical, i.e. maybe do not have only one “correct” solution.



8 External links

## Global catastrophic risks [ edit ]

*Main article: Global catastrophic risk*

- ~~Climate change~~
- **Artificial general intelligence** Last month
- Biotechnology risk
- Ecological collapse
- Molecular nanotechnology
- Nuclear holocaust
- Overpopulation
- Global pandemic

## UN list [ edit ]

The UN has listed issues that it deems to be the most pressing as of 2015:<sup>[5]</sup>

Top-level issue	Issues	Relevant UN di
	poverty, diseases, desertification,	

[https://en.wikipedia.org/wiki/List\\_of\\_global\\_issues](https://en.wikipedia.org/wiki/List_of_global_issues)

# Polya's method for general questions

- For general questions, especially Step 1 is important!

## Question:

Is it good to ...

How to ...

invest in solar energy      stop overpopulation  
stop eating meat      control the development of AI  
do more sports      decrease taxes  
...      ... ?

1

## Ask yourself the following questions

- ❖ What are you asked to find or show?
- ❖ Can you restate the question in your own words?
- ❖ What part/information of the question is (un)important?
- ❖ Can you think of a picture or a diagram that might help you understand the problem?
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Is it good to ...

How to ...

invest in solar energy      stop overpopulation  
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do more sports      decrease taxes  
...      ... ?

1

## Ask yourself the following questions

- ❖ What are you asked to find or show?

There might not be one single answer.

Maybe one can just give pro/contra arguments.

- ❖ Do you understand the problem?
- ❖ Why might this problem be difficult/easy?

# Polya's method for general questions

- For general question, especially Step 1 is important!

## Question:

Is it good to ...

How to ...

invest in solar energy      stop overpopulation  
stop eating meat      control the development of AI  
do more sports      decrease taxes  
...      ... ?

1

What does "good" mean? What would be "bad"?

Good for who? Economy? Humans? Animals? Me?

- ❖ Is there enough information to enable you to find a solution?
- ❖ **Do you understand all the words used in stating the problem?**
- ❖ Do you need to ask a question to get the answer?
- ❖ Why might this problem be difficult/easy?

# Polya's method for general questions

## Question:

Is it good to ...

How to ...

invest in solar energy      stop overpopulation  
stop eating meat      control the development of AI  
do more sports      decrease taxes  
...      ... ?

2

## Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases
- ❖ Guess and check. Trial and error. Guess and test.
- ❖ Look for a pattern or patterns.
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# Polya's method for general questions

## Question:

Is it good to ...  
How to ...

invest in solar energy      stop overpopulation  
stop eating meat      control the development of AI  
do more sports      decrease taxes  
...      ... ?

2

## Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic.
- ❖ Solve a simpler version of the problem & Consider special cases

Just focus on one aspect. For example, just consider the affects on economy and forget any other aspects (ethics etc.).

# Polya's method for general questions

## Question:

Is it good to ...  
How to ...

invest in solar energy      stop overpopulation  
stop eating meat      control the development of AI  
do more sports      decrease taxes  
...      ... ?

2

## Possible Strategies

- ❖ Draw pictures or diagrams.
- ❖ Eliminate possibilities
- ❖ Be systematic
- ❖ Make a mindmap or pro/contra list of arguments.
- ❖ Look for a pattern or patterns
- ❖ Make a list / Write down keywords.



# Review

**1**

**Understand the problem**

**2**

**Make a plan**

**3**

**Carry out the plan**

**4**

**Look back & Review**

- **Try to think about problems and come up with related questions.**
  - **How do the steps 1,2,3 and 4 look like for these questions?**

# Assignment: Problems of “new technology”

In the assignment you should think about problems related to **new technology** (of now or the future).

## new technology

(some examples)

Image & speech recognition

### Artificial intelligence

Self driving cars ...

### Virtual reality

.... and so on

TikTok

### Social media

LINE

Instagram ...

Time travel

Quantum computing

### Scientific breakthroughs

... CRISPR

- You are not restricted to the above topics. Choose whatever you are interested in!
- When introducing yourself to your group members give one example of “new technology” you are personally interested in.

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You can find all materials of today on my homepage

**Polyas's four step problem solving method** (Workshop Day 1, 21st August 2021)

- Talk slides **These slides**
- Assignment sheet **The assignment**

**Mathematics in daily life and the wonderful(?) world of artificial intelligence**

- Talk slides
- Abstract: This lecture was about mathematics and artificial intelligence, both the negative as well as positive effects of artificial intelligence. It was shown in which way (artificial intelligence) can be used to solve problems, which are hard to solve for humans (talking about "stupid machines", which are programs/algorithms which transform data into information). These machines are often hard to build but we understand their operation. Some problems are too complicated to be described by an algorithm. Therefore, we use machine learning. These machines can solve hard tasks (such as image recognition) by learning a function. This makes them unpredictable and maybe in some applications dangerous. Another aspect of the talk was to illustrate that these "smart machines" can be used in a bad way, for example, in automated driving cars, one should be aware of the risks and responsibilities. The lecture is the problem of deciding who is responsible when an automated



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# Assignment



Workshop Day 1, 21st August 2021

- 1) Decide together with your group members on a topic related to „new technology“.  
  
(This „new technology“ does not need to exist now. If you want, you can be creative and come up with some future technology)
- 2) Explain the new technology you choose in 1) and describe its positive aspects.
- 3) Formulate problems that might occur due to the use of the new technology you choose in 1).

*“The solution to any problem lies in asking the right questions”*

- 4) Come up with a question related to the problem(s) in 3) and answer it by using Polya’s four step problem solving method.

Create a presentation where you present your results of 1) - 4) .

Recall Polya's four step problem solving method:

1

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- ❖ Why might this question be difficult/easy?

2

## Make a plan

- ❖ Draw pictures or diagrams.
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- ❖ Be systematic.
- ❖ Solve a simpler version of the question & Consider special cases.
- ❖ Guess and check. Trial and error. Guess and test.
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## Carry out the plan

- ❖ Try to use the strategy chosen in step 2.
- ❖ If this strategy does not work, try another one.

4

## Look back & Review

- ❖ Did you answer the question? Is your result reasonable?
- ❖ What would change if you change the question a bit?
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- ❖ Is there another way of answering the question, which may be simpler?
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