Polynomial and Sinusoidal Functions Lesson #1:
Polynomial Functions of Degrees Zero, One, and Two

Answer Key

Overview

In this unit, we will describe the characteristics of polynomial functions and sinusoidal functions by analyzing their graphs and their equations. We will also determine the polynomial function or sinusoidal function that best approximates data and solve problems where a polynomial function or sinusoidal function can be used to model a situation.
In the first half of the unit we will focus on polynomial functions and in the second half of the unit we will focus on sinusoidal functions.

Polynomial Functions

A **polynomial function** consists of one or more **terms**, which are separated by + or − signs.

We have already met polynomial functions in one variable in previous mathematics courses. For example, the functions $f(x) = x^2 - 4x - 5$, $f(x) = 2x - 4$, and $f(x) = 3$ are polynomial functions we have studied in earlier courses.

The **degree** of a polynomial function is the value of the highest exponent in the function.
If a polynomial function includes a term with no variable, this term is called a **constant term.**

a) Complete the following.

- The degree of $f(x) = x^2 - 4x - 5$ is 2. The constant term is −5.
- The degree of $f(x) = 2x - 4$ is 1. The constant term is −4.
- Since $f(x) = 3$ can be written as $f(x) = 3x^0$, the degree of $f(x) = 3$ is 0.

A number that multiplies the variable in a polynomial is called a **coefficient.**
The **leading coefficient** is the number that multiplies the term with the highest power.

b) Complete the following.

- The leading coefficient of $f(x) = x^2 - 4x - 5$ is 1.
- The leading coefficient of $f(x) = 2x - 4$ is 2.

In a polynomial function, all the coefficients must be real numbers, and all the exponents must be whole numbers.

c) Write a polynomial function that satisfies the following conditions.

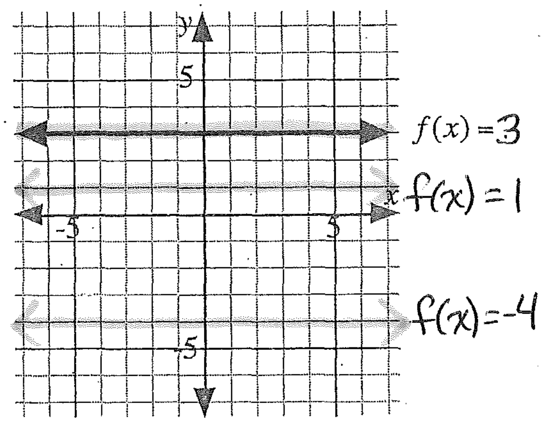
- degree 2, leading coefficient −3 $\quad f(x) = -3x^2$.
- degree 2, leading coefficient 7, two terms $\quad f(x) = 7x^2 + 1$.
- degree 1, leading coefficient 1 $\quad f(x) = x$.
- degree 0 $\quad f(x) = 10$.
- degree 3, constant term −8 $\quad f(x) = 3x^3 + x^2 - 3x - 8$

answers may vary

In the next three lessons, we will investigate the characteristics of polynomial functions of degrees 0, 1, 2, and 3.

Investigating Polynomial Functions of Degree Zero

1. The graph of the function $f(x) = 3$ is shown.



a) State the domain and range of $f(x) = 3$.

$x \in R$ range $\{y \mid y = 3\}$

b) Write two other functions of degree zero which can be graphed on the same grid.

answers may vary $\Big\{$ $f(x) = 1$ $f(x) = -4$

c) Sketch and label each of the functions in b) on the grid.

d) Describe the general shape of each graph.

horizontal line

e) State the number of y-intercepts of each graph. 1

f) Do any of your graphs have an x-intercept? If so, state the function. If not, can you determine a polynomial function of degree zero that does have x-intercepts?

none of above have an x intercept

$f(x) = 0$ has an infinite # of x intercepts

g) Describe the relationship between the equation of each function and the range of the function.

the fn $f(x) = k$ has a range $y = k$

* key ideas *

2. Generalize the above work to complete the following statements about **all** polynomial functions of degree zero in the form $f(x) = c$.

a) The graph is a __horizontal__ line with a slope of __0__.

b) The domain is __$x \in R$__.

c) The range is __$\{y \mid y = c\}$__.

d) There are __no__ x-intercepts, except for the function $f(x) =$ __0__.

e) There is __1__ y-intercept.

* **3. a)** Does the value of a polynomial function of degree zero depend on the value of x? no

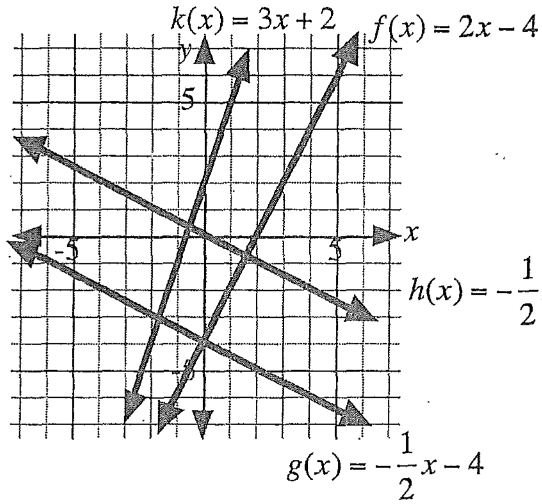
b) Since the values of a polynomial function of degree zero remain constant for all valu of x, this type of function is called a __constant__ function.

Investigating Polynomial Functions of Degree One

1. The graphs of four polynomial functions of degree one are shown.

a) The graphs shown have many characteristics in common. Make a list of the common characteristics in the space below.

→ straight line
→ 1 x-intercept
→ 1 y-intercept
→ domain $x \in R$
→ range $y \in R$



b) How does the constant term in the polynomial function relate to one of the characteristics of the graph?

constant term is the y-intercept

c) How can you tell from the equation of the polynomial function whether the line increases from quadrant 3 to quadrant 1 or decreases from quadrant 2 to quadrant 4?

→ if the leading coefficient is positive, the line increases from Q III to Q I
→ if the leading coeff is negative, the line decreases from Q II to Q IV

d) The polynomial functions $g(x)$ and $h(x)$ are represented graphically by parallel lines.

i) How can we tell from the equations of the functions that this is the case?

the leading coefficients of $g(x)$ and $h(x)$ are the same

ii) In general, what feature of the graph is represented by the leading coefficient of a polynomial function of degree one?

the slope of the graph

2. Generalize the above work to complete the following statements about **all** polynomial functions of degree one in the form $f(x) = mx + b$.

* key points *

a) The shape of the graph is a __line__ with a slope of __m__.

b) The domain is __$x \in R$__. **c)** The range is __$y \in R$__.

d) There is __1__ x-intercept and __1__ y-intercept. **e)** The y-intercept is __b__.

f) The direction of the line is determined by the value of the leading coefficient.

i) If the leading coefficient is __positive__, the line slopes up from quadrant 3 to quadrant 1.

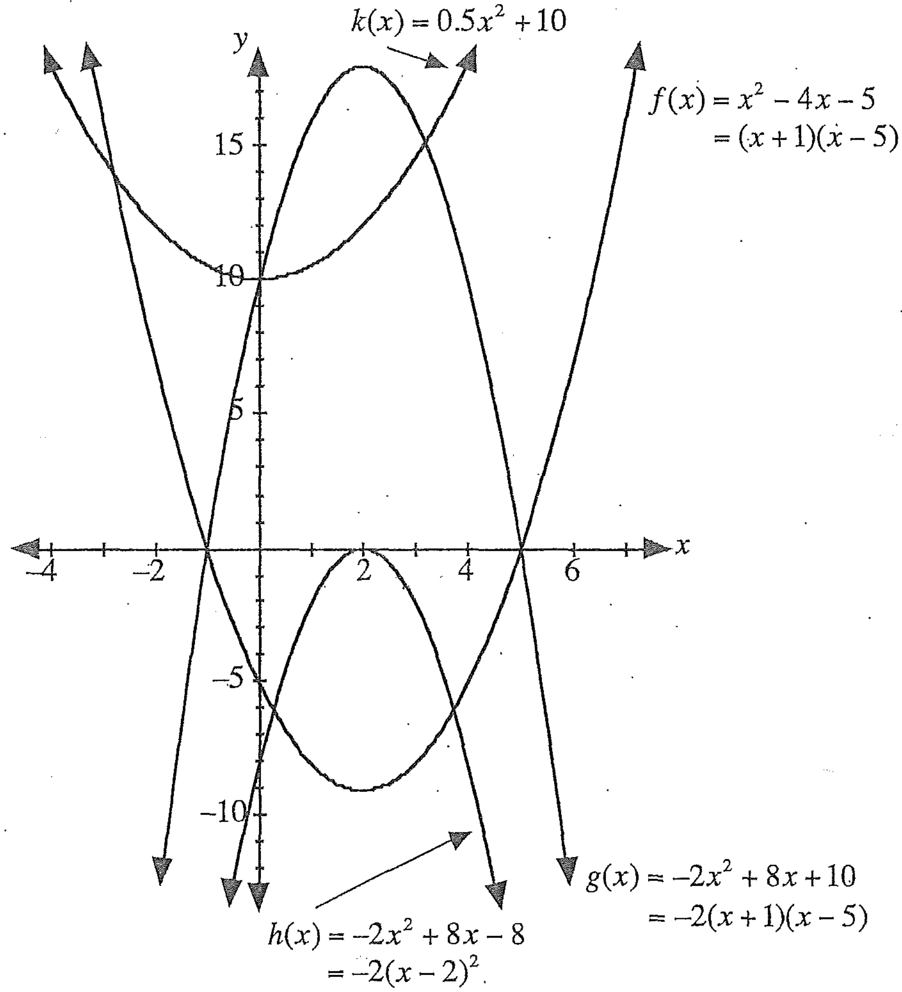
ii) If the leading coefficient is __negative__, the line slopes down from quadrant 2 to quadrant 4.

g) Since the graph of a polynomial function of degree one is a straight line, this type of function is called a __linear__ function.

Investigating Polynomial Functions of Degree Two

Polynomial functions of degree two are called **quadratic functions** and were studied in a previous mathematics course.

1. The graphs of four polynomial functions of degree two are shown.



a) From the list of characteristics below, circle the ones which are the same for all the graphs.

- Domain
- Range
- Number of x-intercepts
- Number of y-intercepts

b) How does the y-intercept of each graph relate to the equation of the quadratic function?

The y-int is the value of the constant term

c) The graphs are all **parabolas** opening up or down.

They all have one **turning point** which is either a **maximum** or **minimum**.

i) How can you tell from the equation of the polynomial function whether the parabola opens up or opens down?

- if the leading coefficient is positive, parabola opens up
- " " " " -negative, " opens down

ii) How can you tell from the equation of the polynomial function whether the turning point is a maximum or a minimum?

- if leading coefficient is positive, turning point is a min.
- " " " is negative, " " is a max

iii) The range can be written in the form $y \leq$ number, or $y \geq$ number. How can you tell from the equation of the polynomial function which form is appropriate?

- if leading coefficient is positive, range is $y \geq$ number
- " " " " negative, " " $y \leq$ number

2. The graphs on the previous page show that polynomial functions of degree two can have 0, 1, or 2 x-intercepts. The x-intercepts are the same as the **zeros** of the function.

Three of the functions on the previous page can be written in factored form as shown below. Complete the following:

a) The graph of $f(x) = x^2 - 4x + 5 = (x+1)(x-5)$ has 2 x-intercepts, at -1 and 5.

b) The graph of $g(x) = -2x^2 + 8x + 10 = -2(x+1)(x-5)$ has 2 x-intercepts, at -1 and 5.

c) The graph of $h(x) = -2x^2 + 8x - 8 = -2(x-2)^2$ has 1 x-intercept at 2.

Ex. #1

Consider the functions $P(x) = 7(x-1)(x+6)$ and $Q(x) = -(x+3)^2$.

a) State the zeros of each function.

zeros of $P(x)$ are 1 and -6 ; $Q(x)$ are -3

b) Complete the table below with the characteristics of the graph of each function.

	Direction of Opening	Number of x-intercepts	Value(s) of x-intercept(s)	Value of y-intercept
$P(x)$	up	2	-6, 1	-42
$Q(x)$	down	1	-3	-9

$P(0) = 7(0-1)(0+6) = -42$

$Q(0) = -(0+3)^2 = -9$

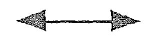
End Behaviour of the Graph of a Polynomial Function

The **end behaviour** of the graph of a polynomial function describes the appearance of the graph at the left and at the right tail, i.e. as the graph extends further and further to the left and further and further to the right.

Degree Zero

For the graph of a polynomial function of degree 0 (i.e. a constant function), the end behaviour at each tail is to remain at its constant value, as seen in Figure A. From left to right, the line extends from quadrant 2 to quadrant 1, or from quadrant 3 to quadrant 4, or is the x-axis.

Figure A



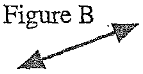
Degree One

For the graph of a polynomial function of degree 1 (a linear function), the end behaviour is determined by the leading coefficient.

Complete the following, inserting the word "up" or "down" in each blank space.

- If the leading coefficient is positive, as in Figure B, then the right tail of the graph goes __up__ and the left tail of the graph goes __down__.

Figure B



- If the leading coefficient is negative, as in Figure C, then the right tail of the graph goes __down__ and the left tail of the graph goes __up__.

Figure C

Degree Two

For the graph of a polynomial function of degree 2 (i.e. a quadratic function), the end behaviour is also determined by the leading coefficient.

Complete the following, inserting the word "up" or "down" in each blank space.

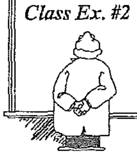
- If the leading coefficient is positive, as in Figure D, then the right tail of the graph goes __up__ and the left tail of the graph goes __up__.

Figure D

- If the leading coefficient is negative, as in Figure E, then the right tail of the graph goes __down__ and the left tail of the graph goes __down__.

Figure E

Class Ex. #2



Complete the table using the word "up" or "down" to describe the end behaviour of the graph of each function and the word "maximum", "minimum", or "none" to describe the nature of the turning point of each graph.

	Left Tail	Right Tail	Nature of Turning Point
$f(x) = -2x + 4$	up	down	none
$g(x) = 2x^2 - 7$	up	up	minimum
$h(x) = -5 + 3x$	down	up	none
$k(x) = 6 - 2x - x^2$	down	down	maximum

Complete Assignment Questions #1 - #9

Assignment

1. Write a polynomial function $f(x)$ of

   a) degree zero **b)** degree one **c)** degree two

2. Write a polynomial function $f(x)$ that satisfies the following conditions:

   a) degree 2, leading coefficient 5

   b) degree 0, y-intercept 5

   c) degree 1, constant term -3

3. **a)** From the list of characteristics below, circle the ones that are the same for the graphs of all polynomial functions of degree one.

   • Domain • Range • Number of x-intercepts • Number of y-intercepts

   b) From the list of characteristics below, circle the ones that are the same for the graphs of all polynomial functions of degree two.

   • Domain • Range • Number of x-intercepts • Number of y-intercepts

4. **a)** Complete the table below with the characteristics of the graph of each function.

	Direction of Opening	Number of x-intercepts	Value(s) of x-intercept(s)	Value of y-intercept
$P(x) = (x-1)(x+5)$				
$Q(x) = (x+4)^2$				
$R(x) = x(2-x)$				

   b) State the zeros of $P(x)$, $Q(x)$, and $R(x)$.