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Abstract

Two approaches were taken to make convenient spread sheet calculations of elastic constants from resonance data and the tables in ASTM C1259 and E1876: (1) polynomials were fit to the tables; (2) an automated spread sheet interpolation routine was generated. To compare the approaches, the resonant frequencies of circular plates made of glass, hardened maraging steel, alpha silicon carbide, silicon nitride, tungsten carbide, tape cast NiO-YSZ, and zinc selenide were measured. The elastic constants, as calculated via the polynomials and linear interpolation of the tabular data in ASTM C1259 and E1876, were found comparable for engineering purposes, with the differences typically being less than 0.5 percent. Calculation of additional ν values at t/R between 0 and 0.2 would allow better curve fits. This is not necessary for common engineering purposes, however, it might benefit the testing of emerging thin structures such as fuel cell electrolytes, gas conversion membranes, and coatings when Poisson's ratio is less than 0.15 and high precision is needed.

Introduction

The elastic constants of isotropic materials can be measured by using a variety of static and dynamic techniques such as strain gages, impulse excitation, or resonant ultrasound spectroscopy, and a recent comparison of the techniques has been made (ref. 1). Dynamic techniques have the advantage of simple test specimen geometry, good precision, and applicability over a wide range of temperatures. One convenient method is the impulse excitation technique in which a test specimen such as a rectangular beam or circular plate is excited by a brief mechanical impulse with a tool (see fig. 1) and allowed to vibrate naturally. The specimen geometry and support and impact locations are chosen to induce flexural or torsional modes of vibration. The vibrations are detected with a non-contacting microphone or contact accelerometer, and are analyzed by a signal processor to determine the fundamental resonant frequency. The fundamental frequency is used with the specimen geometry and mass to estimate dynamic elastic properties such as Young's modulus, Poisson's ratio, and shear modulus.

This method is nondestructive, sensitive, requires relatively little material, and can be used to test specimens designed for other purposes such as uniaxial or biaxial flexural strength

measurement (refs. 2 to 4). It also is useful for monitoring changes in newly developed materials that contain large amounts of porosity and undergo chemical changes such as reduction or oxidation during testing and use. Another advantage of the technique is the availability of American Society for Testing and Materials (ASTM) standards that give detailed guidance on the technique (refs. 5 and 6).

For a circular plate, the resultant first and second natural vibrations corresponding to torsion and flexure are measured and used to independently estimate Young's Modulus. The values are then used to calculate an average Young's modulus with the relation (ref. 5)

$$E = (E_1 + E_2) / 2 \quad (1)$$

where:

$$E_1 = \frac{37.6991 f_1^2 D^2 m (1 - \nu^2)}{K_1^2 t^3} \text{ and } E_2 = \frac{37.6991 f_2^2 D^2 m (1 - \nu^2)}{K_2^2 t^3} \quad (2)$$

with f_1 and f_2 being the first and second natural frequencies, D the disc diameter, t the thickness, m the mass, ν Poisson's ratio, and K_1 and K_2 the first and second natural geometric factors. The shear modulus, G , is calculated from the common relation

$$G = \frac{E}{2(1 + \nu)}. \quad (3)$$

Currently, the standards ASTM C1259 and E1876 (refs. 5 and 6) use linear interpolation of data compiled in tables to determine the constant ν as a function of t/R and f_1/f_2 where R is the disc radius. The constants K_1 and K_2 are then determined with linear interpolation as a function of t/R and ν . This is inconvenient as three manual interpolations are required per constant, and the accuracy is reduced because the functions are nonlinear and the interpolations of K_1 and K_2 depend on those of ν . Although extremely accurate values of E and ν are not needed for general engineering purposes, accurate constants are particularly useful in verification of test rigs and models. Further, rapid, convenient calculations are beneficial if elastic constants are used to nondestructively monitor material changes.

In order to allow accurate, convenient spread sheet calculations of elastic constants from plate data, two approaches were taken: (1) polynomials were fit to the data tables A1.1 to A1.5 of ASTM C1259 and E1876, which are identical and based on the data of Glandus and Martincek (refs. 7 and 8); and (2) an automated bilinear spread sheet interpolation routine was implemented (see the app.). The resultant polynomials exhibited residuals of less than 0.5 percent for K_1 and K_2 , and less than 1.5 percent for ν , and are sufficient for most engineering purposes. Comparisons of calculations made with linear interpolation and the polynomials were made for glass, steel, silicon carbide, silicon nitride, tungsten carbide, tape cast NiO-YSZ and zinc selenide, and indicate similar results.

Polynomial Functions

During fitting of the constants, an attempt was made to minimize the residuals while using a relatively simple polynomial function. This was possible for both K_1 and K_2 , however reasonable fits could not easily be made for ν . The most difficulty in fitting ν was encountered for values of $f_2/f_1 = 1.350$, for which ν varies by a factor of ~ 6 and changes from curvilinear behavior to very linear behavior at small ν as shown in figure 2. Thus three separate functions were generated to describe the range of ν given in table A1.1 of C1259. In addition, a wide range fit to the data of Glandus (ref. 7) that did not include values of ν for $f_2/f_1 = 1.350$ was generated. Other regions of the table that presented difficulty were additional columns containing several constant values of ν , as with the $f_2/f_1 = 1.350$ data, and columns containing linearly increasing values of ν as a function of t/R . Such regions of constant ν or linearly changing ν did not follow a smooth trend as might be expected for a physical phenomenon. This may have resulted from the numerical accuracy available when Martinec did the calculations and the use of a nomogram to present the data, and the fact that many of the columns and rows in table A1.1 were generated by interpolation of the data of Glandus (ref. 7), rather than from direct calculation. Refinement of the calculations for ν as a function of t/R and f_2/f_1 might allow better curve fits over a wider range.

An example of constant and linearly varying ν is shown in figure 2. Unfortunately, these complications tend to occur at small t/R ratios, which are commonly used in the mechanical testing of structural ceramics such as membranes, fuel cell elements, and coatings that are being manufactured in very thin sections ($t/R < 0.05$) and are required to carry thermal and structural loads. Fortunately, the complications occur for ν values (< 0.15) not usually exhibited by ceramics (typical $\nu = 0.16$ to 0.30) and the curves are relatively flat. For measurements of thin structures exhibiting low Poisson's ratio, improved values of ν between $t/R = 0$ and 0.2 might be beneficial.

For $0 \leq t/R \leq 0.500$, ν can be described for $1.375 \leq f_2/f_1 \leq 1.525$, $1.525 \leq f_2/f_1 \leq 1.725$, and $1.725 \leq f_2/f_1 \leq 1.900$ by the expression:

$$\begin{aligned} \nu = & a + b\left(\frac{t}{R}\right) + c\left(\frac{f_2}{f_1}\right)^{-1} + d\left(\frac{t}{R}\right)^2 + e\left(\frac{f_2}{f_1}\right)^{-2} + f\left(\frac{t}{R}\right)\left(\frac{f_2}{f_1}\right)^{-1} + g\left(\frac{t}{R}\right)^3 \\ & + h\left(\frac{f_2}{f_1}\right)^{-3} + i\left(\frac{t}{R}\right)\left(\frac{f_2}{f_1}\right)^{-2} + j\left(\frac{t}{R}\right)^2\left(\frac{f_2}{f_1}\right)^{-1} \end{aligned} \quad (4)$$

where the constants a through j are given in table 1. The fits are within 1 percent of the data in table A1.1 C1259. For the wider range of $0.00 \leq t/R \leq 0.500$, and $1.400 \leq f_2/f_1 \leq 1.900$, ν can be estimated within 1.3 percent of the data of Glandus (ref. 7) by using the constants in the last row of table 1 with equation (4). The ν values corresponding to $f_2/f_1 = 1.350$ were not included in the fit because they decreased the quality of fit and are not commonly encountered in experimental measurements of ceramics. Examples of fits to the data are shown in figures 3 and 4.

For fitting of K_1 and K_2 data, minor difficulties occurred around some data points. Although Glandus added many intermediate data points to the range of K_1 and K_2 given by Martinec, he appears to have re-used Martinec's data directly without adding precision: the data in Glandus's

tables are given to three places; however, many of the data points are identical to those of Martinec which were originally reported to two places. Evidently, Glandus simply added a third decimal (i.e., a zero) to Martinec's data rather than recalculating those points to a higher precision.

TABLE 1.—CONSTANTS FOR ESTIMATION OF ν FROM EQUATION (4)

Coefficient	f_2/f_1 range			
	1.375–1.525	1.525–1.725	1.725–1.900	1.400–1.900
<i>a</i>	0.7161	-4.3343	-8.0933	-4.30526
<i>b</i>	-0.6502	2.1017	3.4452	0.38999
<i>c</i>	-4.1377	5.6718	12.2471	6.10124
<i>d</i>	1.0459	0.5107	-2.2461	0.19582
<i>e</i>	4.2095	-2.2025	-6.1198	-2.76353
<i>f</i>	0.6056	-2.5965	-3.2070	-0.47553
<i>g</i>	-0.5148	-0.5109	-0.0872	-0.62008
<i>h</i>	-1.1313	0.2792	1.0725	0.45518
<i>i</i>	-0.1247	0.7924	0.7117	0.11816
<i>j</i>	-0.3197	0.0283	1.4525	0.31933

For $0 \leq t/R \leq 0.500$ and $0 \leq \nu \leq 0.500$, K_1 and K_2 can be described by the expression:

$$K_i = \frac{\left(a + b\left(\frac{t}{R}\right) + c\left(\frac{t}{R}\right)^2 + dv + ev^2 + fv^3 \right)}{\left(1 + g\left(\frac{t}{R}\right) + h\left(\frac{t}{R}\right)^2 + iv + jv^2 \right)} \quad (5)$$

where the constants a through j are given in table 2. The fits are within 0.5 percent of the data in table A1.2 and A1.4 of C1259. For the narrow range of $0.100 \leq t/R \leq 0.200$ and $0.14 \leq \nu \leq 0.34$ given in tables A1.3 and A1.5, K_1 and K_2 can be described by the expression:

$$K_i = a + b\left(\frac{t}{R}\right) + cv + d\left(\frac{t}{R}\right)^2 + ev^2 + f\left(\frac{t}{R}\right)v + g\left(\frac{t}{R}\right)^3 + hv^3 + i\left(\frac{t}{R}\right)v^2 + j\left(\frac{t}{R}\right)^2 v \quad (6)$$

The fits are within 0.1 percent of the data. Examples of fits to the data are shown in figures 5 to 8.

TABLE 2.—CONSTANTS FOR EQUATIONS (5) AND (6)

Coefficient	$0 \leq t/R \leq 0.500$ and $0 \leq \nu \leq 0.500$		$0.100 \leq t/R \leq 0.200$ and $0.14 \leq \nu \leq 0.34$	
	K_1	K_2	K_1	K_2
<i>a</i>	6.1528	8.2584	6.2305	8.3901
<i>b</i>	0.5085	-2.3794	-1.7997	-2.9434
<i>c</i>	0.8825	6.6164	-2.4825	2.0928
<i>d</i>	-3.9037	2.3540	3.6354	5.0895
<i>e</i>	-5.4739	-9.1109	-0.3022	1.9810
<i>f</i>	3.9297	-0.9923	3.6488	2.7693
<i>g</i>	0.1364	-0.2142	-9.3417	-18.3125
<i>h</i>	0.9120	1.7467	-2.2626	-3.0307
<i>i</i>	-0.3628	-0.0475	-3.3588	-3.1336
<i>j</i>	-0.3612	-0.9262	-10.7968	-15.1250

Experimental Results

In order to directly compare test results using linear interpolation and the polynomials, circular plates machined in accordance with the procedures of ASTM C1161 and C1499 (refs. 1 and 2) were tested. The dimensions and mass of all the test specimens were measured with a resolution of 0.001 mm and 0.0001 g, respectively. The plates were typically 50.1 mm in diameter and ranged in thickness from 1.54 to 4.53 mm ($t/R = 0.06$ to 0.18), and were intended for strength testing. The materials included glass, hardened maraging steel, alpha silicon carbide, silicon nitride, tungsten carbide, NiO-Yttria-stabilized zirconia (YSZ), and zinc selenide. The 75mol%NiO-YSZ was an unreduced anode material with ~32 percent porosity. It was manufactured by tape casting and sintered at 1200 °C for 2 hours as part of a fuel cell program being conducted at NASA Glenn Research Center (ref. 9). The zinc selenide was optical grade material made by chemical vapor deposition and intended for use on the International Space Station (ref. 10).

The test specimens were supported on the torsional or flexural nodal lines, as necessary, with foam rubber supports as shown in figure 1 and lightly impacted with a steel tipped hammer. The results are summarized in tables 3 and 4, and indicate that the differences in Poisson's ratio and Young's modulus as estimated from interpolation and the polynomials are usually less than 0.5 percent.

Conclusion

Polynomial fits and linear interpolation of the data tables of ASTM C1259 and E1876 produce comparable estimates of Poisson's ratio and Young's modulus for engineering purposes. For glass, hardened maraging steel, alpha silicon carbide, silicon nitride, tungsten carbide, NiO-YSZ, and zinc selenide the differences as calculated from polynomials and interpolation were typically less than 0.5 percent. Calculation of additional ν values at t/R between 0 and 0.2 would allow better curve fits and might benefit the testing of emerging thin structures such as fuel cell electrolytes, gas conversion membranes, and coatings when Poisson's ratio is less than 0.15 and high precision is needed. However, the current values are sufficient for common engineering purposes.

Appendix

The following Microsoft® Excel macro¹ was used to linearly interpolate values of ν , K_1 , and K_2 . The interpolated values were used with the physical data in tables 3 and 4 to estimate E_1 and E_2 which were then averaged to find the Dynamic Young's Modulus (E).

'USER DEFINED FUNCTION

'XYinterpolate

'Interpolates values from a named 2D table for row(x) and column(y) variables.

'The named table should include the row and column variables.

'Cell (1, 1) should not contain a numeric value or formula.

'Row variables should increase to the right. Column variables should increase downwards.

Static Function XYinterpolate(xyarray As Variant, x, y As Single) As Single

Dim n1, m1 as integer

Dim x1, x2, y1, y2, Ry1x1, Ry1x2, Ry1x1x2, Ry2x1, Ry2x2, Ry2x1x2 As Single

x1 = Application.HLookup(x, xyarray, 1)

n1 = Application.Match(x1, xyarray.Rows(1), 0)

x2 = xyarray.Cells(1, n1+1).Value

y1 = Application.VLookup(y, xyarray, 1)

m1 = Application.Match(y1, xyarray.Columns(1), 0)

y2 = xyarray.Cells(m1+1, 1).Value

Ry1x1 = xyarray.Cells(m1, n1)

Ry1x2 = xyarray.Cells(m1, n1+1)

Ry1x1x2 = (x-x1)/(x2-x1)*(Ry1x2-Ry1x1)+Ry1x1

Ry2x1 = xyarray.Cells(m1+1, n1)

Ry2x2 = xyarray.Cells(m1+1, n1+1)

Ry2x1x2 = (x-x1)/(x2-x1)*(Ry2x2-Ry2x1)+Ry2x1

XYinterpolate = (y-y1)/(y2-y1)*(Ry2x1x2-Ry1x1x2)+Ry1x1x2

End Function

¹This macro was courtesy of Dennis Kirk Engineering, denniskb@ozemail.com.au.

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TABLE 3.—COMPARISON OF *E* AND *v* VALUES FROM INTERPOLATION AND NARROW RANGE CURVE FITS. VALUES AFTER ± ARE ONE STANDARD DEVIATION. PERCENT DIFFERENCE IS FROM THE INTERPOLATED VALUE

Material	Diameter <i>D</i> = 2 <i>R</i> mm	Thickness <i>t</i> mm	Relative Thickness <i>t/R</i>	Frequency Ratio <i>f₂/f₁</i>	Interpolated		Curve Fit		% Difference	
					<i>E</i> MPa	<i>v</i>	<i>E</i> MPa	<i>v</i>	<i>E</i>	<i>v</i>
Silicon carbide ¹ (5)	50.60	2.040	0.081	1.509	427.2±1.09	0.17±0.0001	424.5±1.08	0.17±0.0003	-0.63	0.32
Silicon Nitride ² (3)	44.96	2.006	0.089	1.645	309.8±1.22	0.28±0.0005	310.0±1.21	0.28±0.0005	0.05	0.31
Glass (4)	57.01	2.481	0.087	1.583	71.8±0.33	0.23±0.0052	71.6±0.36	0.23±0.0051	-0.28	0.41
Mar 250 Steel (2)	50.68	1.541	0.061	1.707	184.2±1.28	0.32±0.0207	184.7±1.45	0.32±0.0201	0.26	0.03
Mar 250 Steel (2)	50.68	2.292	0.090	1.706	186.0±0.83	0.32±0.0047	186.6±0.78	0.32±0.0043	0.33	0.11
Mar 250 Steel (2)	50.68	4.526	0.179	1.682	186.0±0.81	0.31±0.0045	186.2±0.79	0.31±0.0046	0.11	-0.11
Tungsten Carbide ³ (35)	50.78	2.123	0.084	1.571	610.3±3.75	0.22±0.0023	608.1±3.72	0.22±0.0023	-0.36	0.48
NiO-YSZ (24)	30.11	0.710	0.047	1.631	66.5±2.53	0.27±0.0100	66.50±2.56	0.27±0.0100	0.03	-0.23
ZnSe ⁴ (25)	63.52	3.998	0.126	1.689	74.23±0.15	0.31±0.0013	74.49±0.15	0.31±0.0013	0.35	0.05

(Number of specimens tested)

1. Hexoloy SA alpha silicon carbide, Carborundum Corp.
2. AS 800 silicon nitride, Honeywell Corp.
3. KZ801, Kennametal Corp., Latrobe, PA.
4. CVD Zinc Selenide®, Rohm and Haas, Woburn, MA.

TABLE 4.—COMPARISON OF E AND V VALUES FROM INTERPOLATION AND WIDE RANGE CURVE FIT. VALUES AFTER ± ARE ONE STANDARD DEVIATION. PERCENT DIFFERENCE IS FROM THE INTERPOLATED VALUE

Material	Diameter $D = 2R$ mm	Thickness t mm	Relative Thickness t/R	Frequency Ratio f_2/f_1	Interpolated		Curve Fit		% Difference	
					E MPa	ν	E MPa	ν	E	ν
Silicon carbide ¹ (5)	50.60	2.040	0.081	1.509	427.2±1.09	0.17±0.0001	424.5±1.08	0.17±0.0003	-0.63	0.34
Silicon Nitride ² (3)	44.96	2.006	0.089	1.645	309.8±1.22	0.28±0.0005	310.1±1.21	0.28±0.0005	0.10	-0.46
Glass (4)	57.01	2.481	0.087	1.583	71.8±0.33	0.23±0.0052	71.6±0.36	0.23±0.0049	-0.25	-0.26
Mar 250 Steel (2)	50.68	1.541	0.061	1.707	184.2±1.28	0.32±0.0207	184.7±1.45	0.32±0.0202	0.28	-0.17
Mar 250 Steel (2)	50.68	2.292	0.090	1.706	186.0±0.83	0.32±0.0047	186.6±0.79	0.32±0.0046	0.35	-0.15
Mar 250 Steel (2)	50.68	4.526	0.179	1.682	186.0±0.81	0.31±0.0045	186.4±0.80	0.31±0.0047	0.17	-0.93
Tungsten Carbide ³ (35)	50.78	2.123	0.084	1.571	610.3±3.75	0.22±0.0023	608.1±3.72	0.22±0.0022	-0.34	-0.05
NiO-YSZ (24)	30.11	0.710	0.047	1.631	66.48±2.53	0.27±0.0100	66.52±2.56	0.27±0.0099	0.06	-0.68
ZnSe ⁴ (25)	63.52	3.998	0.126	1.689	74.32±0.15	0.31±0.0013	74.51±0.15	0.31±0.0014	0.26	-0.36

(Number of specimens tested)

1. Hexoloy SA alpha silicon carbide, Carborundum Corp.
2. AS 800 silicon nitride, Honeywell Corp.
3. KZ801, Kennametal Corp., Latrobe, PA.
4. CVD Zinc Selenide®, Rohm and Haas, Woburn, MA.

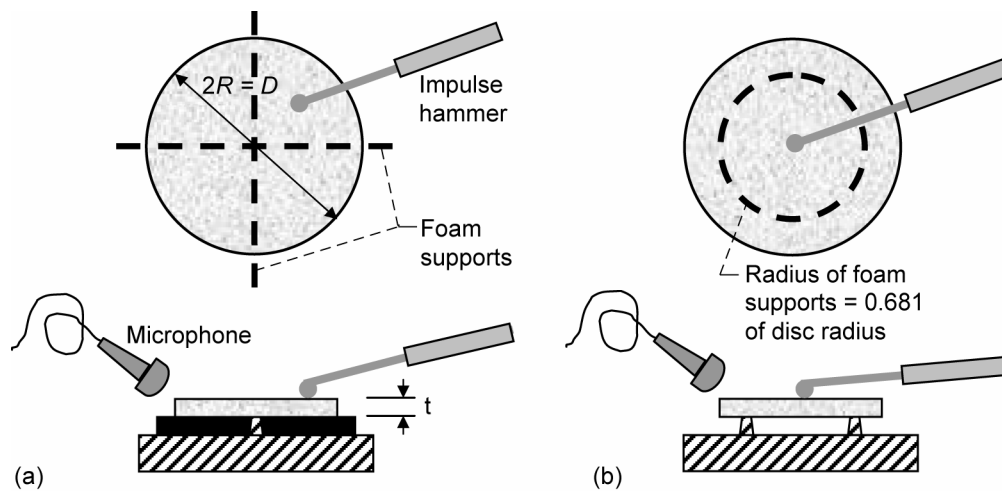


Figure 1.—Schematic of support setup for Impulse Excitation Technique: (a) torsional mode of vibration and (b) flexural mode of vibration.

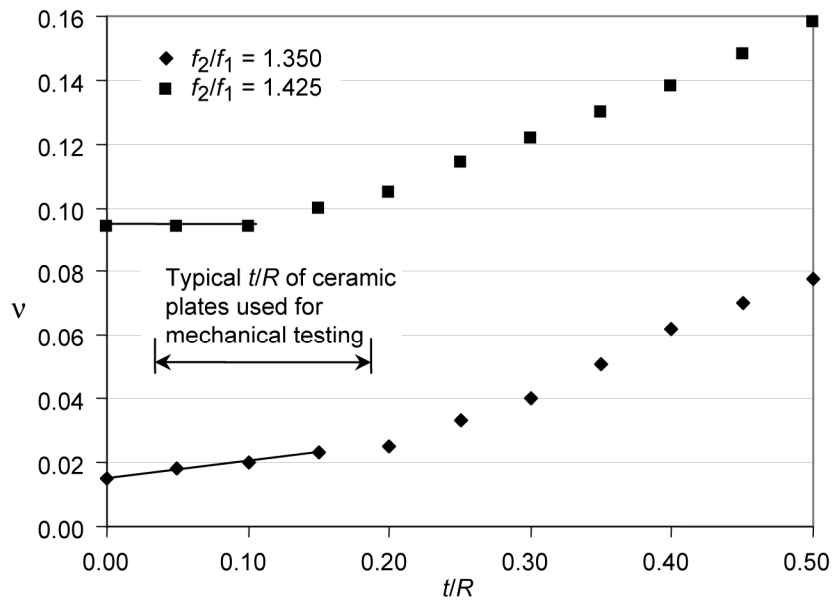


Figure 2.— ν as a function of t/R for regions exhibiting linear variation or no variation of ν with t/R . The data is from table A1.1 of ASTM C1259 (ref. 4).

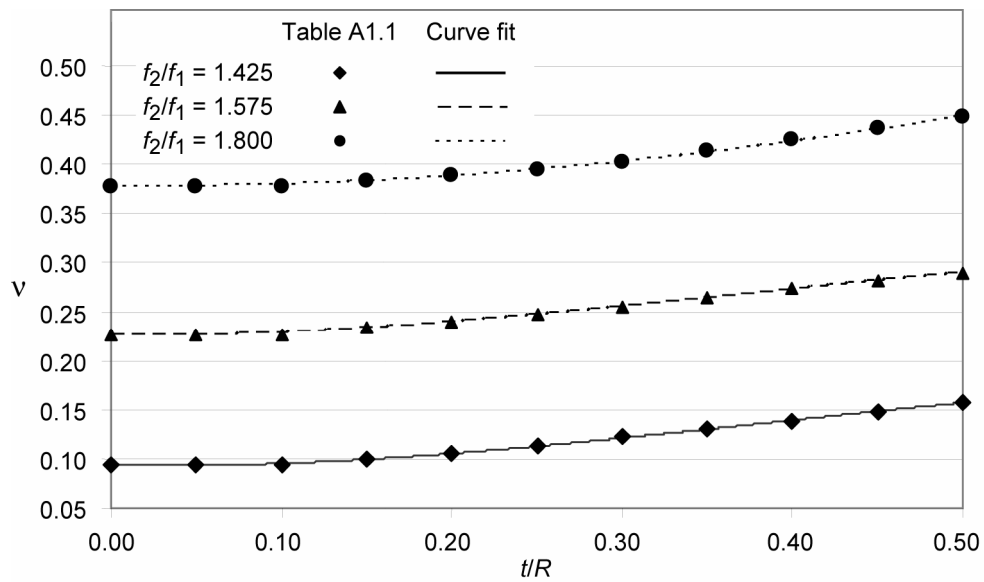


Figure 3.— ν as a function t/R for various f_2/f_1 . Solid lines are equation 4, solid symbols are data points from table A1.1 of ASTM C1259 (ref. 4) and references 6 and 7.

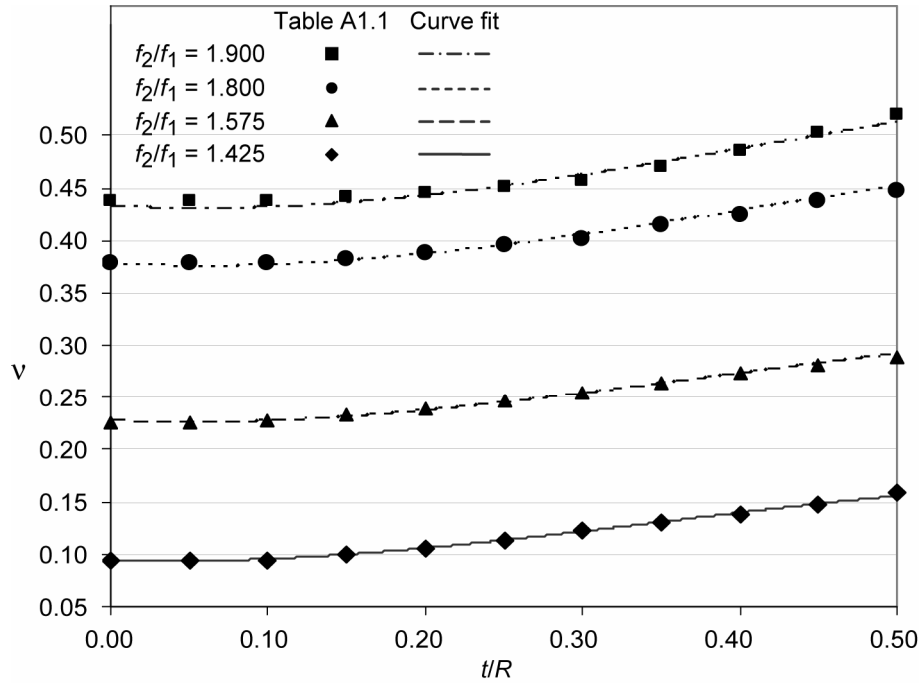


Figure 4.— ν as a function t/R for various f_2/f_1 . Solid lines are equation (4) wide range fit, solid symbols are data points from table A1.1 of ASTM C1259 (ref. 4) and references 6 and 7.

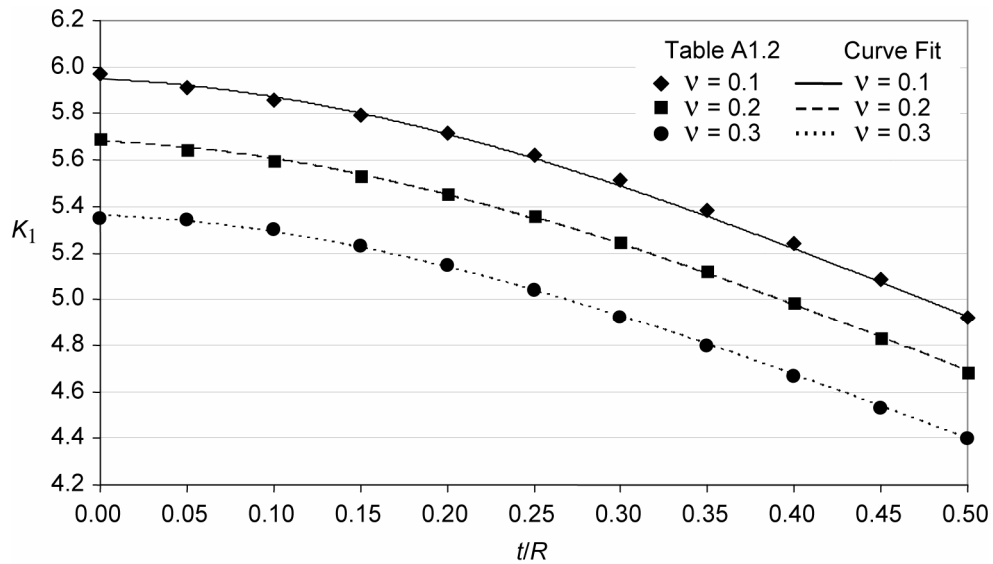


Figure 5.— K_1 as a function t/R for various ν . Solid lines are equation (5), solid symbols are data points from table A1.2 of ASTM C1259 (ref. 4) and references 6 and 7.

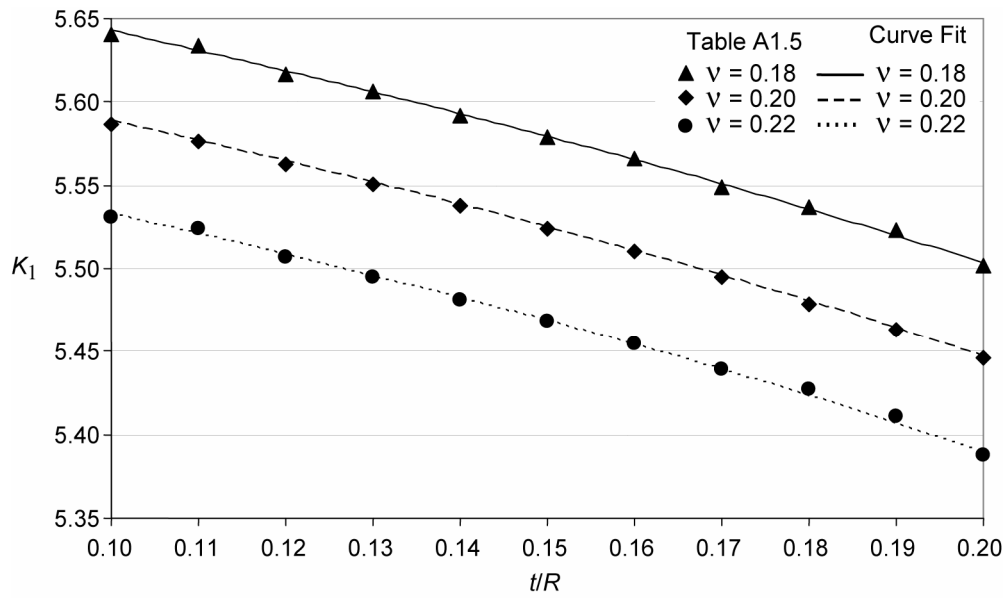


Figure 6.— K_1 as a function t/R for various ν . Solid lines are equation (6), solid symbols are data points from table A1.3 of ASTM C1259 (ref. 4) and references 6 and 7.

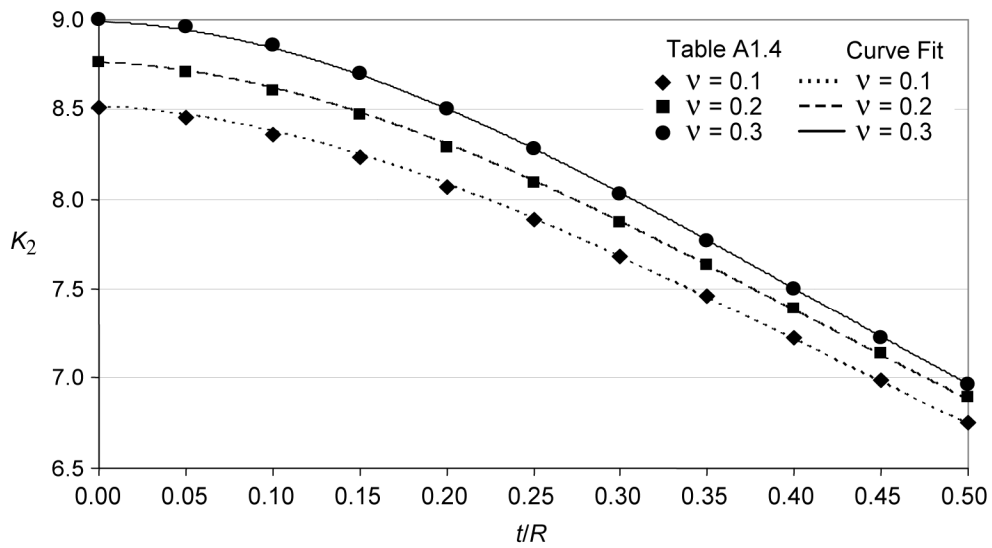


Figure 7.— K_2 as a function t/R for various ν . Solid lines are equation (5), solid symbols are data points from table A1.4 of ASTM C1259 (ref. 4) and references 6 and 7.

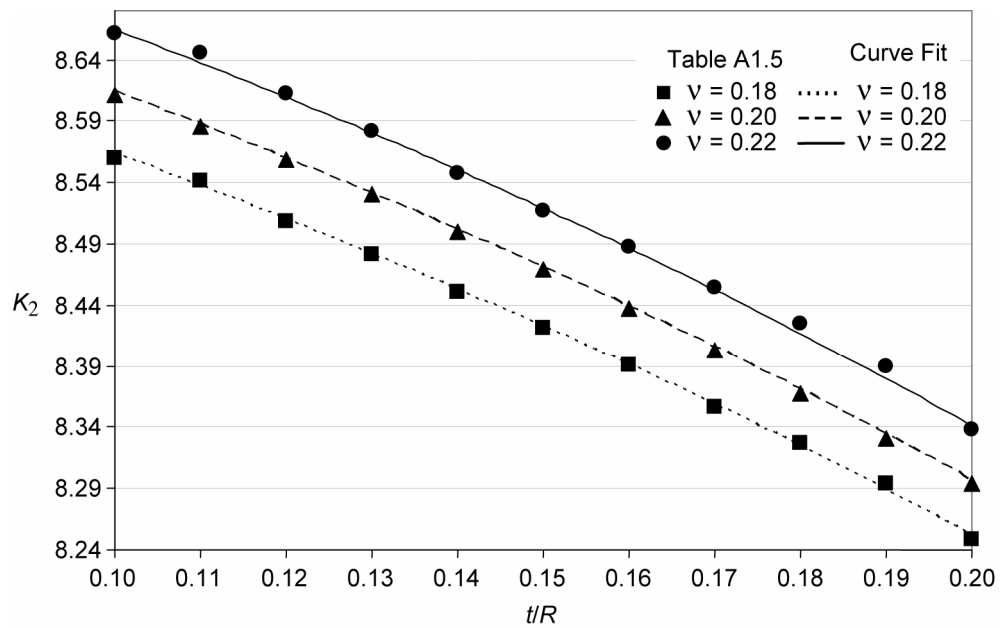


Figure 8.— K_2 as a function t/R for various ν . Solid lines are equation (6), solid symbols are data points from table A1.5 of ASTM C1259 (ref. 4) and references 6 and 7.

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13. ABSTRACT (<i>Maximum 200 words</i>) Two approaches were taken to make convenient spread sheet calculations of elastic constants from resonance data and the tables in ASTM C1259 and E1876: polynomials were fit to the tables; and an automated spread sheet interpolation routine was generated. To compare the approaches, the resonant frequencies of circular plates made of glass, hardened maraging steel, alpha silicon carbide, silicon nitride, tungsten carbide, tape cast NiO-YSZ, and zinc selenide were measured. The elastic constants, as calculated via the polynomials and linear interpolation of the tabular data in ASTM C1259 and E1876, were found comparable for engineering purposes, with the differences typically being less than 0.5 percent. Calculation of additional ν values at t/R between 0 and 0.2 would allow better curve fits. This is not necessary for common engineering purposes, however, it might benefit the testing of emerging thin structures such as fuel cell electrolytes, gas conversion membranes, and coatings when Poisson's ratio is less than 0.15 and high precision is needed.				
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