Multiplying polynomials:

- use the distributive property and the properties of exponents
- here the distributive property can be used to distribute one term or multiple terms
- after multiplying be sure to combine all like terms

Example 1: Multiply the polynomials and express your answers as simplified polynomials.

a. $(2x + 5)(3x - 7)$

F **O** **I** **L**

b. $(3x - 4)(3x + 4)$

F **O** **I** **L**

c. $(x^4 + 3y^2)(x^4 - 3y^2)$

F **O** **I** **L**

$(x^4)(x^4) + (x^4)(-3y^2) + (3y^2)(x^4) + (3y^2)(-3y^2)$

$x^8 - 3x^4y^2 + 3x^4y^2 - 9y^4$

$\boldsymbol{x^8 - 9y^4}$

d. $(3x + 5)(2x^2 + 9x - 5)$

e. $(x^2 - 2x + 2)(x^2 + 2x + 2)$

Since this is a trinomial times a trinomial I can't use FOIL, so instead I'll simply write each term from the first trinomial times the entire second trinomial.

$x^2(x^2 + 2x + 2) - 2x(x^2 + 2x + 2) + 2(x^2 + 2x + 2)$

$x^4 + 2x^3 + 2x^2 - 2x^3 - 4x^2 - 4x + 2x^2 + 4x + 4$

$\boldsymbol{x^4 + 4}$

f. $(5 - x)(x + 5)(x - 5)$

Since this is a binomial times a binomial times another binomial, I can only use FOIL to multiply two of the binomials. It makes no difference which two I choose to multiply first, so I'll just work from left to right and multiply the first two binomials first:

$\boldsymbol{(5 - x)(x + 5)}(x - 5)$

$(5x + 25 - x^2 - 5x)(x - 5)$

$(25 - x^2)(x - 5)$

Now that I've multiplied the first two binomials and simplified completely, I can take that product and multiply by the third binomial. And since I have two binomials, I can once again use FOIL.

$(25 - x^2)(x - 5)$

$\boldsymbol{25x - 125 - x^3 + 5x^2}$

As stated in the Lesson 2 lecture notes, there is a Product to a Power Rule and a Quotient to a Power Rule, but there is no Sum to a Power Rule or Difference to a Power Rule

- $(5x)^2 = 5^2x^2$, **BUT** $(5+x)^2 \neq 5^2 + x^2$
- $\left(\frac{6}{y}\right)^3 = \frac{6^3}{y^3}$, **BUT** $(6-y)^3 \neq 6^3 - y^3$
- when a sum or difference is raised to a power, the power is **NOT** distributive; if the power is a positive integer (such as 2 or 3), we simply write the sum or difference that number of times and then multiply the expressions
  - $(5+x)^2 = (5+x)(5+x)$
  - $(6-y)^3 = (6-y)(6-y)(6-y)$

Example 2: Multiply the polynomials and express your answers as simplified polynomials.

a. $(x-1)^2$

$(x-1)(x-1)$

F **O** **I** **L**

b. $(4x+1)^3$

$(4x+1)(4x+1)(4x+1)$

$\mathbf{(4x+1)(4x+1)}(4x+1)$

$(16x^2 + 4x + 4x + 1)(4x+1)$

$(16x^2 + 8x + 1)(4x+1)$

$16x^2(4x+1) + 8x(4x+1) + 1(4x+1)$

$64x^3 + 16x^2 + 32x^2 + 8x + 4x + 1$

$\mathbf{64x^3 + 48x^2 + 12x + 1}$

c. $-5(x^3 - y^3)^2$

$-5(x^3 - y^3)(x^3 - y^3)$

d. $(3 - x)^2(3 + x)^2$

$(3 - x)(3 - x)(3 + x)(3 + x)$

Once again, when you have a product containing more than two factors, it makes no difference which two factors you choose to multiply first; the order is irrelevant. In this example I'll re-arrange the factors so I can multiply $(3 - x)$ and $(3 + x)$.

$(3 - x)(3 + x)(3 - x)(3 + x)$

$(9 + 3x - 3x - x^2)(9 + 3x - 3x - x^2)$

$(9 - x^2)(9 - x^2)$

$81 - 9x^2 - 9x^2 + x^4$

$\boldsymbol{x^4 - 18x^2 + 81}$

The next example contains problem parts that are similar to some past exam problems that students have had trouble with. As I go through Example 3, please be sure you are paying attention and working through those problems with me, and please ask questions if you're unsure about anything.

Example 3: Multiply the polynomials and express your answers as simplified polynomials.

a. $5x^2 - 3x + 2 - (2x - 1)(3x + 4)$

b. $x^4 - (xy)^2 - (x^2 - y)^2$

$x^4 - (xy)(xy) - (x^2 - y)(x^2 - y)$

$x^4 - x^2y^2 - (x^4 - x^2y - x^2y + y^2)$

$x^4 - x^2y^2 - (x^4 - 2x^2y + y^2)$

$x^4 - x^2y^2 - x^4 + 2x^2y - y^2$

$\boldsymbol{-x^2y^2 + 2x^2y - y^2}$

c. $(x-3)(4-x)-(3x-1)(1-4x)$

$(4x-x^2-12+3x)-\ (3x-12x^2-1+4x)$

$(-x^2+7x-12)-\ (-12x^2+7x-1)$

$-x^2+7x-12+12x^2-7x+1$

$\mathbf{11x^2-11}$

d. $(x-y)^3-(x+y)^2+(x-y)$

$(\boldsymbol{x}-\boldsymbol{y})(\boldsymbol{x}-\boldsymbol{y})(x-y)-(x+y)(x+y)+(x-y)$

$(\boldsymbol{x^2}-\mathbf{2}\boldsymbol{xy}+\boldsymbol{y^2})(x-y)-(x^2+2xy+y^2)+x-y$

$x^2(x-y)-2xy(x-y)+y^2(x-y)-x^2-2xy-y^2+x-y$

$x^3-x^2y-2x^2y+2xy^2+xy^2-y^3-x^2-2xy-y^2+x-y$

$\boldsymbol{x^3-3x^2y+3xy^2-y^3-x^2-2xy-y^2+x-y}$

Once again, the expressions from Example 3 can be difficult to simplify completely, so be sure to spend some extra time working on problems like these not only in the homework, but also as you prepare for Exam #1. If you need assistance understanding how to simplify these types of expressions, please let me know.

The next example has two expressions that are not polynomials because the exponents are fractions $\left(\sqrt{x} = x^{\frac{1}{2}}\right)$ instead of nonnegative integers. However we can still multiply and combine like terms the same way we have with polynomials.

Example 4: Multiply the following expressions and simplify your answer as much as possible. Keep in mind that while these expressions are not polynomials, but they can still be multiplied using the same procedure.

a. $\left(\sqrt{x}+\sqrt{y}\right)^2 - \left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right) - \left(x-\sqrt{xy}+\sqrt{xy}-y\right)$

$x+\sqrt{xy}+\sqrt{xy}+y-(x-y)$

$x+2\sqrt{xy}+y-x+y$

$\boldsymbol{2\sqrt{xy}+2y}$

b. $\left(\sqrt{x}-\sqrt{y}\right)^2 - \left(\sqrt{x}+\sqrt{y}\right)^1 + \left(\sqrt{x}-\sqrt{y}\right)^0$

Answers to Examples:

1a $6x^2 + x - 35$; *1b.* $9x^2 - 16$; *1c* $x^8 - 9y^4$;
1d. $6x^3 + 37x^2 + 30x - 25$; *1e.* $x^4 + 4$; *1f.* $-x^3 + 5x^2 + 25x - 125$; *2a.* $x^2 - 2x + 1$; *2b.* $64x^3 + 48x^2 + 12x + 1$;
2c. $-5x^6 + 10x^3y^3 - 5y^6$; *2d.* $x^4 - 18x^2 + 81$;
3a. $-x^2 - 8x + 6$; *3b.* $-x^2y^2 + 2x^2y - y^2$; *3c.* $11x^2 - 11$;
3d. $x^3 - 3x^2y + 3xy^2 - y^3 - x^2 - 2xy - y^2 + x - y$;
4a. $2y + 2\sqrt{xy}$; *4b.* $x - 2\sqrt{xy} + y - \sqrt{x} - \sqrt{y} + 1$;