# Portfolio Performance Measurement and A New Generalized Utility-based N-moment Measure

September 2013

**Abstract**

Most of the performance measures proposed in the financial and academic literature are subject to be gamed in an active management framework (Goetzmann *et al.*, 2007). One of the main reasons of this drawback is due to an incomplete characterization by these measures of studied return distributions. We introduce a new flexible Generalized Utility-based N-moment measure of performance (GUN, in short), characterizing the whole return distribution, and hardly gamable. More precisely, it takes into account the first four moments of the return distribution and the associated sensitivities of the studied agent, reflecting his preferences and risk profile. The new performance measure is also well adapted for analyzing performance of hedge funds and more peculiarly in presence of derivative instruments associated with non-Gaussian return distributions. (JEL C16, G11, G23, G24).

## 1 Introduction

Fund performance measurement is an important question for both academics and practitioners, and a renewal of interest recently appeared in the literature (for instance, see Cherny and Madan, 2008; Capocci, 2009; Darolles *et al.*, 2009; Jha *et al.*, 2009; Jiang and Zhu, 2009; Zakamouline and Koekebakker, 2009; Darolles and Gouriéroux, 2010; Glawischnig and Sommersguter-Reichmann, 2010; Jones, 2010; Billio *et al.*, 2012a; Billio *et al.*, 2012b; Cremers *et al.*, 2012). Funds are generally ranked according to different *criteria* by investment banks and financial advisors. Such published rankings can have a significant impact on inflows and outflows (*Cf.* Hendricks *et al.*, 1993; Powell *et al.*, 2002) and, finally, on the allocation decisions of fund managers. Numerous measures have been proposed to evaluate the performance of active management since the introduction of the seminal one in 1966 by William Sharpe. In the following decades, a large financial literature has been dedicated to this subject.

But we can still wonder about which decision *criteria* will help an investor to prefer a certain measure of performance. As mentioned in Caporin *et al.* (2013), the choice between different performance measures, first, depends on the preferences of investors, and secondly, on the characteristics of underlying return distributions. Moreover, the wish to use a performance measure rather than another one is also related to the dimension considered (the evaluation of manager's abilities, the relevancy of investment strategies, the deviation of funds from their benchmarks, *etc.*).

Our article aims to contribute in two ways to the literature on performance measurement. First, we underline some weaknesses of the traditional performance measures in link with their structure. Secondly, it proposes a new flexible Generalized Utility-based N-moment measure of performance as an extension of the Mean-variance framework in order to better characterize the shape of return distributions to be evaluated and the preferences of investors.

From a theoretical point of view, the Sharpe (1966) ratio is a meaningful portfolio performance measure when risk can be adequately measured by standard deviation. Although this ratio remains a reference indicator for assessing the accuracy of investment strategies, its use is doubtful in presence of a non-null skewness or/and an excess *kurtosis*. Consequently, this article proposes to extend the Mean-variance analysis framework. We show that the new measure of performance can be defined as a generalization of some main performance measures: the Sharpe (1966) ratio, the Morningstar (2002) MRAR and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM measures.

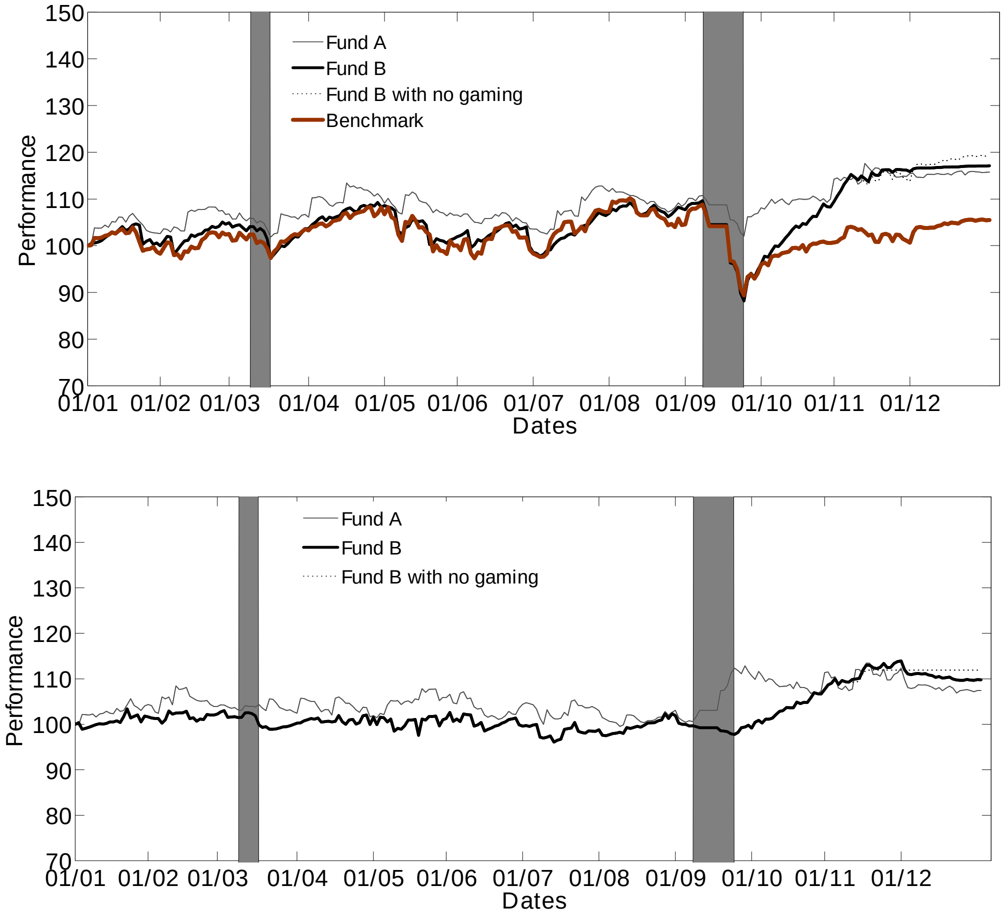
Let us first grasp the intuition of our approach by comparing two Funds, whose portfolio managers compete in the same market (*i.e.*, the US Equity Fund market), and where the Dow Jones Index is considered as the benchmark. Fund A is a portfolio managed by an informed agent, who continuously adapts the composition of his portfolio according to his forecasts about market conditions and expected future market returns. The second one, namely Fund B, is held by an uninformed but aggressive investor with a very consistent strong market exposure. Indeed, his portfolio returns are characterized by a (relative) high *beta* to the benchmark (1.10 in the long run *versus* .40 for Fund A).

Let us now assume in our preliminary illustration that over a one-year period, two major events happened. A first shock hurted the stock market in March (as the result of an earthquake as in March 2011) and the second turbulence was in September (another terrorist attack, similar to the one in 2001). In both cases, the manager of Fund A reacted, with some short delays, in a quite adequate way (presumably by cutting some of his positions, adding cash and bonds when possible, going defensive, flying to quality, buying some protective assets, rotating styles and sectors, diminishing exposure with options and futures...); on the contrary, the investor's behavior of Fund B - very determined in being aggressive - was rather inappropriate on these two occasions. Indeed, whilst the negative impact of these two market shocks on the performance of the first manager's portfolio was moderated (essentially concentrated over the first few days for each crisis), the performance of the second manager's portfolio strongly suffered in March and September, mainly due to his general driven investment strategy. This results in significant losses for each of these two risky sub-periods, with a high drawdown and a volatility of his portfolio returns.

Furthermore, a few weeks after the second crisis (and in the last six weeks of the year), the manager of Fund B - happy with the unexpected recent recovery of his performance in October and November - decided to secure his performance and "game" his Sharpe ratio (*Cf.* Goetzmann *et al.*, 2004 ; Brown *et al.*, 2005), putting in place a "split-strike conversion strategy" (adding to his long stock position, a long put and a short call - *Cf.* Bernard and Boyle, 2009) in order to almost neutralize the market effect for the rest of the year. He also proceeded, at the same time, to some "window dressing" for the end of the year (buying some of the hottest stocks on a year-to-date basis – *Cf.* Lakonishok *et al.*, 1991). These actions happened to cost globally, *ex post*, a couple of points of performance (2.14% in total) since he missed the final rebound in November-December, as a result of his "informationless" strategy.

If we first look at the NAV and related over-performance of the two Funds in the sample (see Figure 1 below), the final yearly performance of Fund B is slightly above those of Fund A (at the end of the sample – 100 basis points or so), and, accordingly for greedy investors, the MPPM concludes with the dominance of Fund B over Fund A.

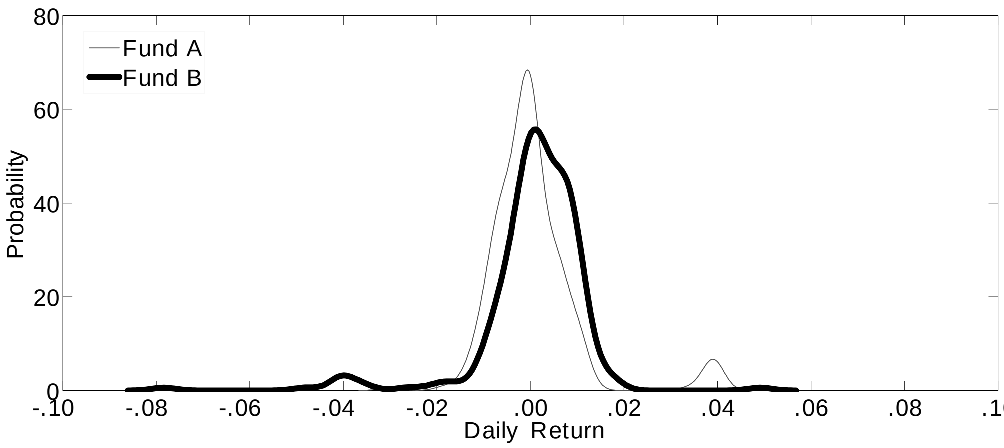
**Figure 1: Net Asset Values and Out-performance of the Fund A and B compared to Benchmark**




Source: Illustration of Net Asset Values of Fund A (thin grey line) and Fund B (bold black line) over a one-year period. Grey areas represent the market shocks. The x-axis corresponds to dates whilst the y-axis shows the performance of the two funds. Computations by the authors.

However, the comparisons of volatilities, skewness, *kurtosis*, Maximum Drawdowns (Peak-to-Valleys), Value-at-Risks at 95% and 99%, as well as Sharpe ratios (Sharpe, 1966), *Omega* measures (Keating and Shadwick, 2002), Kappa ratios (Kaplan and Knowles, 2004), and Sortino ratios (Sortino and Meer, 1991), lead us all, on the contrary, to prefer Fund A. Moreover, if we look at the return distributions of both Funds (see Figure 2 below), we observe two "bumpy" densities with two modes: a bump on the right (a good skewness and a better *kurtosis*) for Fund A, and a bump on the left for Fund B (bad skewness and *kurtosis*). In the sense of Kimball (1990), a risk averse, prudent and/or temperate investor would thus prefer Fund A, since they prefer – *ceteris paribus* - lower volatilities, higher skewness and lower *kurtosis.*

**Figure 2: "Bumpy" Probability Density Function Estimates of Fund A and B Daily Returns**

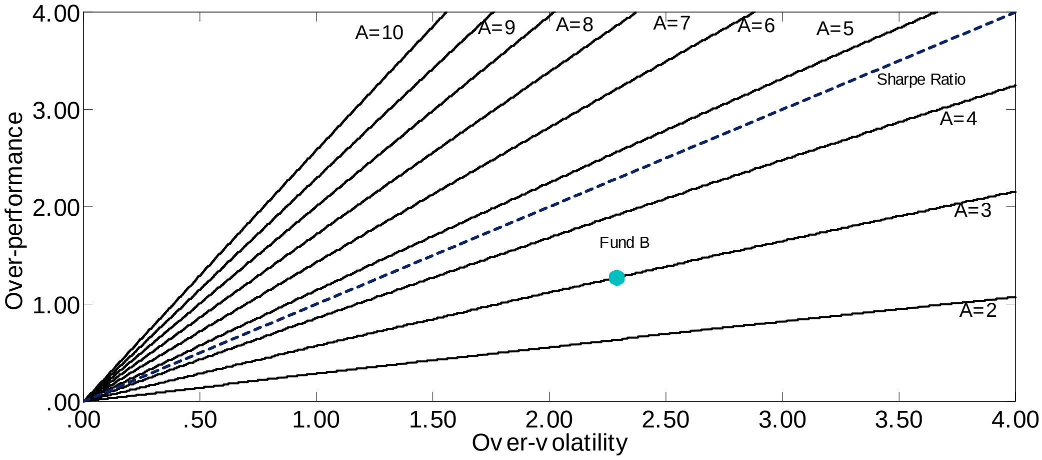



Source: This figure illustrates Kernel estimates of "bumpy" return distributions (using the cross-validation principle - see Silvermann, 1986), corresponding to quotes over a one-year period as represented in Figure 1 above: the return distribution of Fund A is characterized with a bump on the right (thin grey line) and the distribution of Fund B with a bump on the left (bold black line). The x-axis corresponds to daily returns and the y-axis represents the associated probabilities. Both axes are expressed in percentage. Computations by the authors.

This simple realistic example illustrates that most of the main performance measures contradicts the MPPM ranking, which indeed puts a lot of emphasis on the performance, and does not fully take into account all the features of return densities when considering several types of investors. In other words, we see here that MPPM might be focusing too much on performance in some circumstances as illustrated by the following figures. Using the simulation scheme presented in Goetzmann *et al.* (2007), we can distinguish, on the next Figure 3, two main areas for each line, corresponding to a given risk aversion coefficient considered in the MPPM computation (from 2 to 10): the first main area (at the bottom-right of each curve) implies that the ranking of MPPM concludes in the opposite direction to the performance, and the second one (at the upper-left of each curve) when the opposite is true, signaling an opposition between the performance and MPPM ranking.

For instance, when considering a risk aversion coefficient equal to 3 (respectively 5), a 50 basis point extra performance may compensate a supplementary limit over-volatility of .91% (respectively .45%), whilst (in a quasi-linear manner in this region) a 100 basis point extra performance may offset a surplus of volatility as large as 1.80% (respectively .90%), which has to be compared to the one-to-one performance-volatility relation with the Sharpe ratio. Moreover, as we can see in the following illustration, the MPPM is quite rather insensitive to the third (skewness) and the fourth (*kurtosis*) moments of the studied return distributions (*ceteris paribus*).

**Figure 3: *Iso*-MPPM Frontiers displaying the Quantity of Over-volatility required for a Given Over-performance for reversing the MPPM Ranking - in a Pure Simulation Case**




Source: Illustration of the over-performance (the y-axis) and over-volatility (the x-axis), both expressed in %. Fund B, compared to Fund A, is symbolized by a blue point. Ranking frontiers (solid lines) are computed from the Manipulation-Proof Performance Measure (see, Goetzmann *et al.*, 2007) when varying the risk aversion coefficient from 2 to 10 (A is equal to 3 in the original paper). They are realized by comparing 10,000 pairs of portfolios - each represented by 250 returns generated by a Gaussian law of parameters 17% for the annualized mean and 20% for the annualized standard deviation as in Goetzmann *et al.*, (2007). For each pair, one positive over-performance and the return series of the worst performer fund is distorted (by intensification of the volatility only) making equal both fund volatilities; then, volatility of the worst performer is once again distorted until an inversion of the ranking is obtained. The dashed bold blue line is the Sharpe (1966) ratio ranking frontier assuming that a unit of extra over-volatility - for a unit of a given over-performance - is required to inverse the ranking between two funds. Computations by the authors.

Actually, the MPPM is mainly related to the mean performance characteristics of fund histories (at least for low risk aversion coefficients), and not exactly in the way the performance is built (with more or less related losses and drawdowns), as mentioned by Brown *et al.* (2010) in their MPPM-corrected measure (called Doubt Ratio). As a result, it may miss some other characteristics, specifically when fund managers try to "game" their performance rankings with non-linear instruments, that may entail strong non-Gaussian features of densities. It is also shown (see below) that the MPPM ranking is linked to a specific preference setting (a particular set of moment preferences), depending on the financial characteristics of the studied return series (*i.e.* means and volatilities of funds compared). In other words, the relevance of the MPPM might be too specific to a class of agents, and not general enough for most investors.

All these results lead us to propose our Generalized Utility-based N-moment measure of performance (GUN, in short), grounded on an agent-preference setting and based on a more complete characterization of return distributions.

The new measure of performance presented in this article, and satisfying all the conditions to be characterized as a "good" measure, can be written such as:

$$\underset{[1\times 1]}{GUN_{n,i,p}} = \underset{[1\times n]}{\boldsymbol{\lambda}'_{n,i}} \times \underset{[n\times 1]}{\mathbf{C}_{n,p}}, \tag{1}$$

where the $GUN_{n,i,p}$ statistic, summarizing the performance of a portfolio $p$ held by an investor $i$, is expressed as a linear combination of the Conventional moments (C-moments in short) of a return distribution; $\boldsymbol{\lambda}_{n,i}$ is a column vector composed of the $n \times 1$ sensitivities of an investor $i$ to the $n$-th moment such as $0 \leq \lambda_{n,i} \leq 1$ for $n = [1, \ldots, 4]$ and $i = [1, \ldots, I]$; $\mathbf{C}_{n,p}$ is a column vector composed of the first four moments of the studied return distribution.

The rest of the article is structured as follows. Section 2 introduces our new flexible Generalized Utility-based N-moment measure (GUN, in short). Section 3 compares some of the main performance measures to our measure when using some realistic simulations. Section 4 proposes financial applications on Hedge Funds whilst Section 5 concludes.

# 2 The Generalized Utility-based N-moment Measure of Performance

In this section, we present the new flexible GUN measure, characterizing the whole return distribution. More precisely, this measure of performance takes into account the first four moments of the return distribution and the associated sensitivities of the studied agent, reflecting his preferences and risk profile.

We first propose hereafter to express the expected utility of an investor from the moments of a return distribution through a fourth-order Taylor expansion. Secondly, we introduce our measure that writes as a linear combination of the first four adjusted moments of the investor's return distribution.

## 2.1 Utility Functions with Higher-order Moments

In economics, agents' behavior is represented by utility functions which describe their preferences and risk profiles. The main objective of any agent (in the mainstream theoretical approach) is supposed to be the maximization of their expected utility, which can be represented by an indirect function that is strictly concave and decreasing with even moments and strictly concave and increasing with odd moments. Traditionally, only the first two moments, namely the mean and the variance, are considered to describe the preferences and the risk profiles of an investor in terms of asset allocation in a risky environment. We can establish, however, a link between the expected utility of an agent and higher-order moments of a return distribution through an expansion of Taylor to an infinite-order (Tsiang, 1972; Loistl, 1976; Lhabitant, 1997; Dávila, 2010). The utility of an investor, denoted $U(\cdot)$, can be formulated *via* a utility function that is arbitrarily continuously and differentiable in $\mathfrak{D}$ with $\mathfrak{D} \subset IR$. It represents the $n$-th order Taylor expansion, evaluated at the expected return on the investment, $\forall\, r_i \in \mathfrak{D}$, such as:

$$U_i(r) = \sum_{n=0}^{\infty} (n!)\, U^{(n)}[E(r)] \times [r - E(r)]^n + \widetilde{\varepsilon}_{n+1}(r), \tag{2}$$

where $E(r)$ are the expected returns, $n!$ is the n-factorial, $U^{(n)}(\cdot)$ is the $n$-th derivative of the utility function and $\widetilde{\varepsilon}_{n+1}(\cdot)$ is the Lagrange remainder. The latter can be decomposed as (with the previous notations):

$$\widetilde{\varepsilon}_{n+1}(r) = \frac{U^{(n+1)}(\xi)}{(N+1)!}[r - E(r)]^{n+1}, \tag{3}$$

where $n \in IN^*$ and $\xi \in [r, E(r)]$ if $r < E(r)$ otherwise $\xi \in ]r, E(r)[$.

In this analysis framework, we use a Taylor expansion in order to define the expected utility of any agent (see Jondeau and Rockinger, 2006; Jurczenko and Maillet, 2006), respecting accurate conditions (See Garlappi and Skoulakis, 2011) for the development to be exact (or approximative). To be valid, this approach requires that the Taylor approximation of the agent's utility function $U(\cdot)$ at the $n$-th order around $E(\cdot)$ absolutely converges towards $U(\cdot)$. Moreover, since the summand and integral operators are commutative, we assume that conventional moments for all orders exist and are unique to characterize the return distribution. Then, we can take, under some regularity conditions, the limit of $N$ towards infinity and the expected value on both sides in Equation (2), that lead us to (with the previous notations):

$$E[U_i(r)] = E\left\{ \underset{n\to\infty}{Lim} \left\{ \sum_{n=0}^{N} (n!)\, U^{(n)}[E(r)] \times [r - E(r)]^n + \widetilde{\varepsilon}_{n+1}(r) \right\} \right\}. \tag{4}$$

Through Equation (4), it is thus possible to express the expected utility of an economic agent from the first four moments of a return distribution. Considering an exact (or accurate approximate) Taylor expansion at the fourth-order of a general utility function (see, for details, Jurzcenko and Maillet, 2006), we have (with the previous notations):

$$\begin{aligned} E[U_i(r)] &= m_1(r) \times U^{(1)}[m_1(r)] + \frac{m_2(r)}{2!} U^{(2)}[m_1(r)] \\ &\quad + \frac{m_3(r)}{3!} U^{(3)}[m_1(r)] + \frac{m_4(r)}{4!} U^{(4)}[m_1(r)] + \widetilde{\epsilon}_{n+1}(r), \end{aligned} \tag{5}$$

where $m_n(.)$ corresponds the $n$-th moment with $n = [1, \ldots, N]$ that we can define more generally in the following way (with the previous notations, for $n > 1$):

$$m_n(r) = \int_{-\infty}^{+\infty} f(r)\,[r - m_1(r)]^n \, dr,$$

with:

$$m_1(r) = \int_{-\infty}^{+\infty} f(r)\, r dr,$$

where $f(.)$ is the probability density function.

## 2.2 A Generalized Expression of Traditional Utility Functions reflecting Investor's Preferences

The main characteristics of the previous development allow us to differentiate several investors according to their preferences and risk profiles. It is indeed possible to define these characteristics, assimilated to sensitivities, by studying moments of the return distribution, usually limited to the order four. The sensitivity of the first moment governs the so-called "greediness" of the investor, the sensitivity of the second moment represents his "risk aversion", whilst the third[1] and the fourth terms characterize respectively the "prudence" (see Kimball, 1990; Lajeri-Chaherli, 2004) and the "temperance" (see Kimball, 1992 and 1993; Eeckhoudt *et al.*, 1995; Menezes and Wang, 2005)[2]. We know that investor's preference functions determine what is the optimal combination between risky assets and the risk free rate an investor will hold, and how much this investor will consume and invest. The main restrictions existing on rational utility functions are: non satiation, risk aversion, absolute and relative risk aversion, prudence and temperance. Caballé and Pomansky (1996) analyze general utility functions exhibiting all derivatives of alternate signs and then propose an additional restriction on such utility functions, namely the Mixed Risk Aversion restriction. They formulate the property of Mixed Risk Aversion such as (with the previous notations):

$$-\frac{U^{(2)}(r_i)}{U^{(1)}(r_i)} \leq \frac{U^{(3)}(r_i)}{U^{(2)}(r_i)} \leq -\frac{U^{(4)}(r_i)}{U^{(3)}(r_i)} \leq \ldots \leq -\frac{U^{(n+2)}(r_i)}{U^{(n+1)}(r_i)}, \tag{6}$$

where $U^{(n)}(\cdot)$, for $n = [1, ..., N]$, corresponds to the $n$-th derivative of the utility function of an individual $i$ with respect to its return denoted $r_i$.

The concept of Mixed Risk Aversion can be linked to the other concepts in risk theory, namely the Proper Risk Aversion, the Standard Risk Aversion, and the Risk Vulnerability. The first one corresponds to utility functions for which successive derivatives alternate in sign, the first being positive. The second concept means that both Absolute Risk Aversion and Absolute Prudence are decreasing, whereas

[1] Some interesting recent works, however, also show that the ratio $[U^{(3)}(.)/U^{(1)}(.)]$ is also linked to a risk aversion characteristic of a rational agent, who makes an arbitrage between the first and the third moments (*Cf.* Crainich and Eeckhoudt, 2011).

[2] Lajeri-Chaherli (2004) extends the expansion to the order five, mentioning in reference the fifth-order risk "edginess", whilst Caballé and Pomansky (1996) refine even further the expansion to the $n$-th order, referring to the "risk aversion of order $n$", as an analogue to the traditional classical Absolute Risk Aversion (see also Eeckhoudt and Schlesinger, 2006).

the last one implies that Absolute Risk Aversion is decreasing and convex. A graphical observation of this condition is to say that the first, second, third and fourth derivative functions of such utility functions, increasing and convex, tend to an horizontal line. It is obvious that Mixed Risk Aversion implies Standardness, Properness and Risk Vulnerability (see Caballé and Pomansky, 1996). The advantage with the concept of Mixed Risk Aversion is that it allows us to deal with higher moments, while Risk Aversion is restricted to the second order moment and Standard Risk Aversion is directly related to the second and to the third order moments. Furthermore, most of the traditional utility functions which respects the property of Mixed Risk Aversion can be expressed according to a generalized form such as (with the previous notations):

$$E\left[U_i\left(r\right)\right] = \sum_{n=1}^{N} \lambda_{n,i} \times m_{n,p}\left(r\right) + \widetilde{\epsilon}_{n+1}\left(r\right), \tag{7}$$

where $\lambda_{n,i}$ is the sensitivity of an investor $i$ regarding the $n$-th moments, such as $\lambda_{n,i} = \omega_{n,i}\left(n!\right)^{-1} g_{n,i}\left(\delta_n\right)^{\tau_n}$ for $n = [1, \ldots, 4]$ and $i = [1, \ldots, N]$, where $\omega_{n,i}$ is a weight, $n!$ is the $n$-factorial, $g_{n,i}\left(\cdot\right)$ is a function, $\delta_n$ and $\tau_n$ are two constants (depending on the underlying utility function); so that our new measure reads:

$$GUN_{n,i,p} = E\left[U_i\left(r\right)\right]. \tag{8}$$

Then, we can express *via* the generalized *formulae* in equation (7), the most used utility functions. As an illustration, we sum up values associated to the different parameters defined above in order to obtain the main utility functions. We can see here there exists a link between the new Generalized Utility-based N-moment measure of performance and the Cumulative Prospect Theory when considering the investor's sensitivities as modified (subjective) probabilities associated with the distribution of non-distorted returns.

# 3 A Comparison with the Four Main Performance Measures

We present in this section an interpretation of the new flexible measure of performance and we explain how it can be seen as a generalization of some main performance measures.

First, we show that it is possible to express some of the main performance measures, namely the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM, as a linear combination of distorted moments from a lognormal transformation function. Secondly, using the simulation scheme presented in Goetzmann *et al.* (2007), we show how the GUN can identify good and bad performances by comparing four declinations of it, each measure characterizing a specific investor's profile. Moreover, we show that our

GUN can replicate the rankings of some of the main performance measures[3].

## 3.1 A Generalization of the Main Performance Measures

The new proposed measure of performance is based on the study of the first four moments of a return distribution. The main innovation of this new measure of performance is to consider the whole probability distribution. We show below that it is possible to express from the first two moments, some main performance measures as a linear function of distorted moments: the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM measures. Our goal hereafter is to show the flexibility of the new proposed measure of performance.

The Sharpe (1966) ratio is defined as a "Risk *Premium*" over the risk level of the studied portfolio such as (with the previous notations):

$$S_p = \left[E\left(r_p\right) - r_f\right] \times \left(\sigma_p\right)^{-1}, \tag{9}$$

where $S_p$ is the Sharpe (1966) ratio of the portfolio $p$, $r_f$ is the risk free rate and $\sigma_p$ is the standard deviation of the portfolio $p$. The numerator can be interpreted as the expectation of an individual in terms of returns and the denominator as the risk level of the studied portfolio (see Caporin *et al.*, 2012).

The Morningstar Risk-Adjusted Return, MRAR (Morningstar, 2002), is derived from a power-utility function and it is defined as the expected value of the certainty equivalent annualized geometric return. Then, it is defined such as:

$$MRAR_p = \begin{cases} E\left[\left(\frac{1+r_p}{1+r_f}\right)^{-A}\right]^{-\frac{12}{A}} - 1 & A > -1, A \neq 0 \\ \exp\left\{E\left[\ln\left(\frac{1+r_p}{1+r_f}\right)\right]\right\} - 1 & A = 0, \end{cases} \tag{10}$$

where $A$ is the risk aversion coefficient (which is set to 2 by Morningstar).

Finally, the MPPM measure introduced by Goetzmann *et al.* (2007) is written such as:

$$\Theta_p \equiv \left[\left(1-A\right)\Delta t\right]^{-1} \ln\left\{E\left\{\left[\left(1+r_p\right)\left(1+r_f\right)^{-1}\right]^{1-A}\right\}\right\}, \tag{11}$$

where the $\Theta_p$ statistic is the portfolio's *premium* return after adjusting for risk, $\Delta t$ is the length of time between observations, $r_p$ is the portfolio's (unannualized) rate of return, $r_f$ is the risk-free rate and $A$ is the risk aversion coefficient. The latter should be selected to make holding the benchmark optimal for an uninformed manager. Then, the portfolio has the same score as does a risk-free asset whose continuously-compounded return exceeds the interest rate by $\Theta_p$.

If we now apply the lognormal function to the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM measure, we obtain (with the previous notations):

[3]Those measures belong to the main four families (*Cf.* Caporin *et al.*, 2013).

$$\begin{cases} \ln(S_p) = \ln[E(r_x)] - \ln(\sigma_p) \\ \ln(MRAR_p) = a \ln\left\{E\left[(r^*)^b\right]\right\} \\ \ln(\Theta_p) = \ln\{\ln\{E[(r^*)^c]\}\} - \ln(d) \end{cases} \quad (12)$$

where $\sigma_p$ is the standard deviation of the portfolio $p$, $E(r_x)$ is the difference between the expected return of the portfolio $p$ and the risk-free rate $r_f$, $a = -12/A$ is the risk aversion level of the investor, $b = -A$, $r_p^*$ is equal to $(1 + r_p)(1 + r_f)^{-1}$, $c = 1 - A$, $d = (1 - A)\Delta t$ where $\Delta t$ is the time variation between two observations and $\ln(\cdot)$ is the logarithmic function.

If we restrict our GUN measure to the first two moments, we have (with the previous notations):

$$GUN_{2,i,p} = \lambda_{1,i} m_{1,p}(r) - \lambda_{2,i} m_{2,p}(r), \quad (13)$$

where $\lambda_{n,i}$ with $n = [1, 2]$ correspond to the sensitivities of an individual $i$ for the $n$-th moment denoted $m_{n,p}(\cdot)$ for $n = [1, \ldots, 2]$ that represents the first two moments of the return distribution of the portfolio $p$.

Let us now study the behavior of the Sharpe (1966) ratio, the Morningstar (2002) Risk-Adjusted Return, the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM and the GUN measure of performance when assuming (*ceteris paribus*) a positive variation of the mean, or a negative variation of the variance of the underlying return distribution: an increase of the mean or a decrease of the variance of the investor's portfolio return distribution will positively impact all the four measures.

If we now adjust the sensitivities of the GUN measure, to the category of individuals considered, we should be able to get a similar ranking with our measure of performance as those obtained with the Sharpe (1966) ratio, the Morningstar (2002) RAR and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM. To sum up, our main idea is to show our GUN measure is general and shares identical properties with other classes of performance measures.

## 3.2 A Special Focus on the Comparison with the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM

In this section, we first show how our GUN identifies good and bad performances by comparing four declinations of it, each characterizing a specific investor's profile, to some traditional measures, namely the Sharpe (1966) ratio, the Jensen (1968) *alpha*, the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM, the Henriksson-Merton (1981) and the Treynor-Mazuy (1966) measures. Secondly, we show that our GUN can replicate the rankings of the performance measures previously mentioned, and also, of the Darolles-Gouriéroux-Jasiak (2009) L-performance, the Keating-Shadwick (2002) *Omega* and the Morningstar (2002) Risk-Adjusted Return measures.

Obviously, we would like our GUN measure, just as such the MPPM, recognizes good performances as well as penalizes bad performances when they occur. The following Table 1 and Table 2 display the performances of portfolio managers, who provide, respectively, stock selection and market timing abilities. Moreover, we would wish our GUN to be flexible enough to replicate the rankings of most of the traditional performance measures. Table 3 and Table 4 present illustrations of the GUN ranking equivalent on 30 portfolio managers according to several measures of performance.

Based on the simulation scheme used in Goetzmann *et al.* (2007), Table 1 reports the average excess, standard deviation and frequencies of the difference between the managed and market portfolios according to the annualized Sharpe (1966) ratio, the Jensen (1968) *alpha*, the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM and four GUNs (on the various lines), each characterizing a specific investor's profiles[4], for both an informed (left columns) and an uninformed manager (right columns)[5]. The former informed is supposed to generate an annual *extra*-performance superior to 1% compared to that of the latter uninformed manager. The two traders hold similar underdiversified portfolios but the uninformed does not engage in any manipulation. For both managers, we distinguish three panels in Table 1, corresponding to different annual residual risk levels, equal, respectively, to 20.00% (Panel A), 2.00% (Panel B) and .20% (Panel C). As mentioned by Goetzmann *et al.* (2007), these specific risk levels could reflect a level of diversification of portfolios composed by a few, hundreds and thousands of component stocks.

When we look at the results, the Jensen (1968) *alphas*[6] average, as expected, 1.00% and .00% for the informed and uninformed managers in the three panels. Moreover, the uninformed investor's portfolios are significantly positive or negative just about the predicted 5.00% of the time. Indeed, the Jensen (1968) *alpha* does not penalize for under-diversification.

For Panel C (that corresponds to a large number of stock portfolios with a small idiosyncratic risk), all the seven measures show that the informed manager's portfolio is better than the market and the uninformed manager's portfolio is essentially identical to the market. For Panel B (that is associated with portfolios, composed by hundreds of stocks, with a reasonable specific risk), the Sharpe, MPPM and GUNs have almost the same results. For Panel A (that reflects portfolios, composed by few component stocks, with a high specific volatility), the MPPM as well as the four GUNs do substantially better in showing that the 1% *extra*-performance does not properly compensate for the lack of diversification. We also observe that the GUN, characterizing a risk-averse investor, is more penalized when the residual specific portfolio is high compared to the other agent's profiles. Results obtained with our

[4] We first start with the definition of a "neutral" agent for who the scalar products, respectively, between the first four sensitivities and the first four average moments of the studied sample are strictly identical. Thus, we secondly specify four different categories of investors characterized by a high sensitivity to only one of these four moments (*ceteris paribus*). More precisely, we have a greedy investor, denoted $GUN_G$, who is focused on the mean, a risk-averse agent, named $GUN_{RA}$, with a high sensitivity to the variance, a prudent one, called $GUN_P$, characterized by a significant preference to the third moment and a very temperate investor, alias $GUN_T$, who severely dislikes the fourth moment (*Cf.* Appendix A3 for more details).

[5] Table 1 is a replication of the simulation scheme provided in Goetzmann *et al.* (2007) - *Cf.* Table 5 on page 1534.

[6] Using the same hypothesis defined in Goetzmann *et al.* (2007), we compute the Jensen (1968) *alphas* assuming a systematic risk sensitivity of informed and uninformed managers' portfolios equals to 1.

**Table 1: The GUN Measure of Performance: Informed *versus* Uninformed Traders**

| | Informed Trader ($\gamma_p$ = 1.00%) | | | | | Uninformed Trader ($\gamma_p$ = .00%) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Residual Risk | Avg Excess | Std Dev. | Freq Won | Freq Signif + | Freq Signif - | Avg Excess | Std Dev. | Freq Won | Freq Signif + | Freq Signif - |
| Panel A: Annual logarithmic residual standard deviation = 20.00% | | | | | | | | | | |
| Sharpe | -.140 | .344 | 34.22% | 1.99% | 10.74% | -.176 | .344 | 30.47% | 1.56% | 12.75% |
| Jensen | 1.01% | 8.99% | 54.50% | 6.14% | 3.90% | .01% | 8.99% | 50.11% | 4.93% | 4.95% |
| MPPM | -4.99% | 8.99% | 29.04% | 1.37% | 13.76% | -5.99% | 8.99% | 25.31% | 1.03% | 16.30% |
| $GUN_G$ | -6.99% | 17.97% | 34.93% | 2.06% | 10.41% | -8.49% | 17.97% | 31.88% | 1.69% | 12.01% |
| $GUN_{RA}$ | -17.00% | 9.03% | 2.99% | .02% | 59.08% | 17.00% | 9.03% | 2.98% | .02% | 59.14% |
| $GUN_P$ | -7.94% | 8.99% | 18.88% | .57% | 22.20% | -8.44% | 8.99% | 17.37% | .48% | 23.90% |
| $GUN_T$ | -8.05% | 8.99% | 18.55% | .55% | 22.55% | -8.54% | 8.99% | 17.07% | .47% | 24.25% |
| Panel B: Annual logarithmic residual standard deviation = 2.00% | | | | | | | | | | |
| Sharpe | .047 | .045 | 85.15% | 26.29% | .35% | -.003 | .045 | 47.45% | 4.28% | 5.66% |
| Jensen | 1.00% | .90% | 86.65% | 29.67% | .29% | -.00% | .90% | 50.13% | 4.93% | 4.95% |
| MPPM | .94% | .90% | 85.17% | 27.37% | .34% | -.06% | .90% | 47.45% | 4.29% | 5.66% |
| $GUN_G$ | 1.82% | 1.80% | 84.39% | 26.31% | .38% | -.17% | 1.80% | 46.36% | 4.05% | 5.97% |
| $GUN_{RA}$ | .64% | .92% | 75.78% | 17.11% | .93% | -.34% | .91% | 35.78% | 2.18% | 10.02% |
| $GUN_P$ | .82% | .90% | 81.85% | 23.06% | .52% | -.17% | .90% | 42.77% | 3.32% | 7.13% |
| $GUN_T$ | .82% | .90% | 81.80% | 22.97% | .52% | -.17% | .90% | 42.66% | 3.29% | 7.17% |
| Panel C: Annual logarithmic residual standard deviation = .20% | | | | | | | | | | |
| Sharpe | .050 | .005 | 100.00% | 100.00% | .00% | .000 | .005 | 49.88% | 4.84% | 4.99% |
| Jensen | 1.00% | .09% | 100.00% | 100.00% | .00% | .00% | .09% | 50.13% | 4.93% | 4.99% |
| MPPM | 1.00% | .09% | 100.00% | 100.00% | .00% | .00% | .09% | 49.88% | 4.85% | 5.02% |
| $GUN_G$ | 2.00% | .18% | 100.00% | 100.00% | .00% | .00% | .18% | 49.76% | 4.82% | 5.04% |
| $GUN_{RA}$ | .99% | .09% | 100.00% | 100.00% | .00% | .00% | .09% | 48.63% | 4.53% | 5.33% |
| $GUN_P$ | 1.00% | .09% | 100.00% | 100.00% | .00% | .00% | .09% | 49.36% | 4.72% | 5.13% |
| $GUN_T$ | 1.00% | .09% | 100.00% | 100.00% | .00% | .00% | .09% | 49.37% | 4.72% | 5.13% |

Source: This table analyzes the effect of a variation of the portfolio residual risk according to four variants of the Generalized Utility-based N-moment measure of performance and three other measures (Sharpe, 1966; Jensen, 1968; Goetzmann *et al.*, 2007) for an (un-)informed trader who can create a portfolio with a positive *extra*-performance by taking on various levels of increased unsystematic risk. The Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM is defined such as: $\hat{\Theta}_p = ln\{1/T\sum_{t=1}^{T}[(1+r_f)^{-1} \times (1+r_{p,t})]^{1-A}\} \times [(1-A)\Delta_t]^{-1}$ and the Generalized Utility-based N-moment measure of performance - in short GUN - for an individual $i$ as follows: ${}_{4,i,p} = [\lambda_{1,i} \times m_{1,p}(r_p)] - [\lambda_{2,i} \times m_{2,p}(r_p)] + [\lambda_{3,i} \times m_{3,p}(r_p)] - [\lambda_{4,i} \times m_{4,p}(r_p)]$. The latter is declined according to four investor's profiles, namely $GUN_G$, $GUN_{RA}$, $GUN_P$ and $GUN_T$, which respectively refer to an investor strongly greedy, risk averse, prudent and temperate. The frequencies with which the investor's portfolio beats the market portfolio according to each measure are given along with the approximate frequencies with which the portfolio significantly (5.00%) outperforms or underperforms the market. These numbers are estimated as the frequency with which the performance measure was more than 1.65 standard deviations positive or negative. The computation is based on 350,000 managed portfolios with a 5-year return history, respecting the following market hypotheses (with a four-digit accuracy): risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00% and an investor's degree of risk aversion set to 3. Computations by the authors.

four variants of GUN lead us to think that the MPPM would correspond to a greedy investor.

In the main, we can here write that the GUN always coherently leads us to prefer the informed manager, whatever the quality of the signal, just as the MPPM does. Similarly, the more precise the signal, the better the performance (Panel A, B and C), for all the profiles of investors considered. However, the final impact on measures ultimately depends on the preferences of investors.

Grounded on the simulation parameters defined in Goetzmann *et al.* (2007), Table 2 reports the average, standard deviation and frequencies of timing coefficients and total contributed values for the Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures, of the differences between the managed and market portfolios according to the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM and the four GUNs, previously defined in Table 1, for both an informed (left columns) and a random market timer (right columns)[7]. The timing measures of Treynor and Mazuy

[7]Table 2 is a second replication of the simulation scheme used in Goetzmann *et al.* (2007) - *Cf.* Table 6 on page 1535.

(1966) and Henriksson and Merton (1981) are regressions that are based on an *extra* market factor to capture managers' timing abilities defined such as:

$$\widetilde{r}_p - r_f = \alpha_p + (\widetilde{r}_m - r_f) \times \beta_{1,p} + \widetilde{w}_m \times \beta_{2,p} + \widetilde{\varepsilon}, \tag{14}$$

where $\widetilde{r}_p$ are the managed portfolio returns, $r_f$ is the risk-free rate, $\alpha_p$ is the Jensen (1968) *alpha*, $\beta_{1,p}$ is the systematic risk sensitivity of the portfolio $p$ to the market portfolio $m$, $\beta_{2,p}$ is the market timing coefficient, $\widetilde{w}_m$ is equal to $\widetilde{w}_m = \max(r_f - \widetilde{r}_p, 0)$ and $\widetilde{w} = (\widetilde{r}_p - r_f)^2$ for, respectively, Henriksson and Merton (1981) and Treynor and Mazuy (1966).

The total contribution values corresponding to the money manager's contribution to timing and selectivity are respectively written as:

$$\begin{cases} HM_V = \alpha_p e^{-r_f \Delta t} + \beta_{2,p} P\left(1, \Delta t, e^{r_f \Delta t}\right) \\ TM_V = \alpha_p e^{-r_f \Delta t} + \beta_{2,p} e^{r_f \Delta t}\left(e^{\sigma_m^2 \Delta t} - 1\right), \end{cases} \tag{15}$$

where $P(1, \tau, K)$ is the value of a $\tau$-period put option on the market with a strike price of $K$.

The total contribution is the amount by which the value of the protective put exceeds its average "cost" measured by the lowered present value of the *extra* average return. The informed market timer optimally adjusts the systematic risk sensitivity of his portfolio according to his information that explains .10% (top part) or 1.00% (bottom part) of the market portfolio's variation. The uninformed trader wrongly believes that he has the same quality information and adjusts his leverage randomly to the same degree.

The Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures are built to only identify informed traders, but not to penalize the uninformed ones. Thus, these two models show results consistent with the null hypothesis for the uninformed timer just as they would for a manager not trying to time at all. Regarding the MPPM, we observe that the uninformed market timer, who incorrectly thinks to have better information, is definitely hurting the portfolio's performance. The Henriksson-Merton (1981) and Treynor-Mazuy (1966) models frequently recognize the informed traders because, unlike the MPPM, they do not penalize the portfolio's performance for the induced lack of intertemporal diversification. However, when we look at the four GUNs, only the greedy investor prefers the very informed trader compared to the three other specific profiles since he displays a significant preference to the mean. Consequently, those results lead us to think that the MPPM may be biased towards the mean.

As in Darolles *et al.* (2009), Table 3 (Panel A) reports the implied sensitivities, scores and rankings of 30 managed portfolios[8] according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and four variants of the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM when varying the risk aversion level. The second line displays "Implied sensitivities" corresponding to coefficients used in the computation of our GUN that allows us to exactly replicate the portfolios rankings obtained with each

[8]The 30 ranked portfolios correspond to 15 informed and 15 uninformed managers whom portfolio returns respect the simulation scheme defined in Table 1 (*Cf.* Goetzmann *et al.*, 2007).

**Table 2: The GUN Measure of Performance: Informed *versus* Random Market Timers**

| | Informed Timer ($\delta_p = .10\%$) | | | | | Random Timer | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Residual Risk | Avg | Std Dev. | Freq Won | Freq Signif + | Freq Signif - | Avg | Std Dev. | Freq Won | Freq Signif + | Freq Signif - |
| HM $\beta_{2,p}$ | .012 | .043 | 60.42% | 8.60% | 2.64% | .000 | .044 | 49.29% | 4.78% | 5.06% |
| HM $V$ | .10% | .35% | 60.68% | 8.66% | 2.62% | .00% | .36% | 49.22% | 4.74% | 5.02% |
| TM $\beta_{2,p}$ | .211 | .940 | 58.58% | 7.72% | 3.18% | -.001 | .955 | 49.94% | 4.64% | 4.66% |
| TM $V$ | .01% | .02% | 62.02% | 9.38% | 2.58% | .00% | .02% | 49.62% | 4.92% | 5.02% |
| MPPM | .42% | 2.37% | 57.22% | 6.80% | 3.52% | -.50% | 2.46% | 42.02% | 3.14% | 7.38% |
| $\text{GUN}_\text{G}$ | .51% | 4.76% | 54.28% | 5.92% | 3.94% | -1.28% | 4.95% | 39.42% | 2.90% | 8.20% |
| $\text{GUN}_\text{RA}$ | -1.45% | 3.28% | 33.18% | 1.62% | 11.42% | -2.33% | 2.56% | 18.16% | .32% | 22.52% |
| $\text{GUN}_\text{P}$ | -.27% | 2.56% | 46.30% | 3.62% | 6.22% | -1.17% | 2.55% | 31.88% | 1.92% | 11.38% |
| $\text{GUN}_\text{T}$ | -.33% | 2.53% | 45.44% | 3.36% | 6.50% | -1.22% | 2.51% | 30.76% | 1.74% | 12.08% |

| | Informed Timer ($\delta_p = 1.00\%$) | | | | | Random Timer | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Avg | Std Dev. | Freq Won | Freq Signif + | Freq Signif - | Avg | Std Dev. | Freq Won | Freq Signif + | Freq Signif - |
| HM $\gamma_2$ | .114 | .138 | 80.12% | 20.14% | .82% | .000 | .138 | 50.16% | 5.22% | 4.44% |
| HM $V$ | .95% | 1.14% | 80.42% | 20.38% | .74% | .00% | 1.14% | 50.24% | 5.20% | 4.50% |
| TM $\gamma_2$ | 2.057 | 3.014 | 75.68% | 16.14% | 1.10% | .006 | 2.99 | 49.70% | 5.26% | 4.94% |
| TM $V$ | .07% | .07% | 82.64% | 23.16% | .52% | .05% | .07% | 50.38% | 5.14% | 4.46% |
| MPPM | 4.09% | 7.50% | 70.34% | 13.38% | 1.16% | -4.16% | 7.67% | 29.04% | 1.62% | 13.48% |
| $\text{GUN}_\text{G}$ | 6.09% | 14.99% | 65.52% | 10.72% | 1.90% | -6.57% | 15.32% | 33.18% | 1.94% | 11.22% |
| $\text{GUN}_\text{RA}$ | -12.55% | 8.14% | 6.46% | .06% | 45.66% | -15.23% | 7.81% | 2.50% | .02% | 62.04% |
| $\text{GUN}_\text{P}$ | -2.06% | 7.62% | 38.94% | 2.92% | 8.30% | -7.20% | 7.76% | 17.40% | .58% | 22.96% |
| $\text{GUN}_\text{T}$ | -2.24% | 7.58% | 38.14% | 2.74% | 8.76% | -7.33% | 7.71% | 16.72% | .50% | 23.58% |

Source: This table compares the timing coefficient and the contributed value, denoted $\gamma_2$ and V, when using the Henriksson-Merton (1981) parametric model and the Treynor-Mazuy (1966) market timing model, along with the Ingersoll-Spiegel-Goetzmann-Welch (2007) MPPM and four variants of the GUN, for an informed market timer whose information about a changing mean explains .10% and 1.00% of the market's variance, and a random market timer who varies leverage randomly to the same degree. The Goetzmann *et al.* (2007) Manipulation-Proof Performance Measure - say MPPM - is defined such as: $\hat{\Theta_p} = ln\{1/T\sum_{t=1}^{T}[(1+r_f)^{-1}\times(1+r_{p,t})]^{1-A}\}\times[(1-A)\Delta_t]^{-1}$ and the GUN for an individual $i$ as follows: $_{4,i,p} = [\lambda_{1,i}\times m_{1,p}(r_p)]-[\lambda_{2,i}\times m_{2,p}(r_p)]+[\lambda_{3,i}\times m_{3,p}(r_p)]-[\lambda_{4,i}\times m_{4,p}(r_p)]$. The latter is declined according to four investor profiles, namely $\text{GUN}_G$, $\text{GUN}_{RA}$, $\text{GUN}_P$ and $\text{GUN}_T$, which respectively refer to an investor strongly greedy, risk averse, prudent and temperate. The frequencies with which the investor's portfolio beats the market portfolio according to each measure are given along with the approximate frequencies with which the portfolio significantly (5.00%) outperforms or underperforms the market. These numbers are estimated as the frequency with which the performance measure was more than 1.65 standard deviations positive or negative. The computation is based on a simulation of 350,000 managed portfolios, with a 5-year return history, respecting the following market hypotheses (with a four-digit accuracy): risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00% and an investor's degree of risk aversion set to 3. Computations by the authors.

measure of the six studied. We also present a summary statistics of Spearman's and Kendall's rank correlation coefficients computed from a large sample of portfolios when only varying the first two moments (Panel B) and the first four moments (Panel C).

**Table 3: GUN Ranking Equivalence**

Panel A: An illustration of *Iso*-GUN Rankings on 30 Portfolios

| Measures | Sharpe (1966) | | Jensen (1968) | | MPPM 2 (2007) | | MPPM 3 (2007) | | MPPM 4 (2007) | | MPPM 5 (2007) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An Implied Sensitivity Vector ( 1, 2 3, 4 ) | GUN (1.00, -.16 .00, .00) | | GUN (1.00, .00 .00, .00) | | GUN (1.00, -1.00 .00, .00) | | GUN (1.00, -1.50 .00, .00) | | GUN (1.00, -2.00 .00, .00) | | GUN (1.00, -2.50 .00, .00) | |
| | SR | Rank | JA | Rank | MPPM2 | Rank | MPPM3 | Rank | MPPM4 | Rank | MPPM5 | Rank |
| | .40 | 1 | 9.59% | 2 | 3.28% | 3 | -.75% | 15 | -4.16% | 19 | -6.15% | 1 |
| | .35 | 2 | 8.11% | 1 | 1.81% | 23 | -2.17% | 29 | -4.50% | 5 | -6.48% | 8 |
| | .29 | 3 | 6.59% | 20 | .19% | 27 | -2.21% | 11 | -4.80% | 9 | -6.87% | 24 |
| | .26 | 4 | 5.30% | 10 | -.18% | 29 | -2.52% | 9 | -4.87% | 21 | -7.01% | 3 |
| | .19 | 5 | 2.05% | 29 | -.54% | 10 | -2.86% | 19 | -5.04% | 12 | -7.05% | 18 |
| | .17 | 6 | 1.67% | 3 | -.54% | 2 | -3.07% | 6 | -5.08% | 8 | -7.14% | 14 |
| | .16 | 7 | 1.40% | 5 | -.86% | 5 | -3.11% | 5 | -5.17% | 10 | -7.21% | 27 |
| | .14 | 8 | 1.09% | 11 | -1.10% | 11 | -3.20% | 4 | -5.24% | 24 | -7.26% | 11 |
| | .14 | 9 | 1.05% | 15 | -1.14% | 15 | -3.26% | 23 | -5.26% | 11 | -7.28% | 9 |
| | .14 | 10 | .99% | 4 | -1.23% | 19 | -3.27% | 18 | -5.31% | 2 | -7.39% | 12 |
| | .14 | 11 | .96% | 19 | -1.26% | 4 | -3.34% | 2 | -5.42% | 6 | -7.81% | 2 |
| | .13 | 12 | .90% | 8 | -1.30% | 8 | -3.44% | 14 | -5.81% | 15 | -7.95% | 17 |
| | .13 | 13 | .83% | 12 | -1.37% | 12 | -3.81% | 3 | -5.98% | 20 | -8.01% | 26 |
| | .12 | 14 | .72% | 23 | -1.47% | 1 | -3.88% | 20 | -6.04% | 29 | -8.03% | 19 |
| | .11 | 15 | .44% | 6 | -1.81% | 6 | -4.01% | 27 | -6.06% | 1 | -8.07% | 16 |
| | .10 | 16 | .15% | 24 | -2.04% | 24 | -4.07% | 1 | -6.10% | 16 | -8.11% | 22 |
| | .09 | 17 | .09% | 30 | -2.10% | 30 | -4.09% | 26 | -6.15% | 22 | -8.16% | 10 |
| | .09 | 18 | .07% | 7 | -2.12% | 26 | -4.13% | 16 | -6.17% | 17 | -8.43% | 20 |
| | .09 | 19 | .05% | 27 | -2.16% | 16 | -4.18% | 30 | -6.23% | 14 | -8.48% | 13 |
| | .09 | 20 | .04% | 16 | -2.19% | 20 | -4.18% | 22 | -6.43% | 30 | -8.85% | 5 |
| | .09 | 21 | -.03% | 22 | -2.21% | 22 | -4.33% | 10 | -6.48% | 13 | -8.92% | 28 |
| | .08 | 22 | -.18% | 21 | -2.43% | 21 | -4.43% | 24 | -6.93% | 28 | -10.24% | 6 |
| | .08 | 23 | -.23% | 13 | -2.48% | 13 | -4.48% | 13 | -7.96% | 4 | -11.93% | 23 |
| | .05 | 24 | -.46% | 14 | -2.95% | 28 | -4.94% | 28 | -8.13% | 23 | -12.04% | 29 |
| | .04 | 25 | -.71% | 28 | -7.00% | 14 | -11.14% | 12 | -15.30% | 3 | -19.46% | 15 |
| | -.01 | 26 | -1.93% | 25 | -8.19% | 25 | -12.18% | 25 | -16.18% | 25 | -20.17% | 25 |
| | -.24 | 27 | -8.62% | 18 | -14.91% | 18 | -18.92% | 21 | -22.92% | 27 | -26.91% | 30 |
| | -.30 | 28 | -10.26% | 26 | -16.36% | 7 | -20.28% | 7 | -24.90% | 7 | -28.12% | 7 |
| | -.35 | 29 | -11.69% | 9 | -17.94% | 9 | -21.94% | 8 | -25.94% | 18 | -29.94% | 21 |
| | -.57 | 30 | -18.28% | 17 | -24.90% | 17 | -29.09% | 17 | -33.94% | 26 | -37.49% | 4 |
| Spearman $\rho$ | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 |
| Kendall $\tau$ | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 |

Panel B: Summary Statistics of 1000 Draws when only varying the First Two Moments

| | Sharpe | | Jensen | | MPPM2 | | MPPM3 | | MPPM4 | | MPPM5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ |
| Minimum | .37 | .26 | 1.00 | 1.00 | .76 | .79 | .76 | .54 | .75 | .69 | .68 | .46 |
| $1^{st}$ Quartile | .90 | 83 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .99 |
| Median | .98 | .94 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $3^{rd}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Panel C: Summary Statistics of 1000 Draws when varying the First Four Moments

| | Sharpe | | Jensen | | MPPM2 | | MPPM3 | | MPPM4 | | MPPM5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ |
| Minimum | .37 | .26 | 1.00 | 1.00 | .99 | .96 | 1.00 | .98 | .98 | .94 | 1.00 | .98 |
| $1^{st}$ Quartile | .94 | .87 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Median | 1.00 | .98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $3^{rd}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Source: This table compares the ranking of 30 simulated funds obtained according to six performance measures (respectively, Sharpe, 1966; Jensen, 1966; Goetzmann *et al.*, 2007). Similarly to Table 1 and to Goetzmann *et al.* (2007), managed portfolio return distributions are defined such as: $\tilde{r}_p = exp\{[\mu_m + \gamma_p - 0.5(\sigma_m^2 + \nu_m^2)]\Delta_t + (\sigma_m\tilde{\epsilon} + \nu_p\tilde{\eta})\sqrt{\Delta_t}\} - 1$ and market portfolio returns as: $\tilde{r}_p = exp\{[\mu_m - 0.5(\sigma_m^2)]\Delta_t + (\sigma_m\tilde{\epsilon})\sqrt{\Delta_t}\} - 1$, where $\mu_m$ is the market rate of return, $\gamma_p$ is the *extra*-performance of managed portfolios, $\sigma_m$ is the market standard deviation, $\nu_p$ is the residual standard deviation of the manager, $\tilde{\epsilon}$ and $\tilde{\eta}$ are Gaussian random variables. The 30 portfolios under study correspond to 15 informed traders and 15 uninformed managers when setting the annual residual risk to .20%, 2.00% and 20.00%. The second line displays the "Implied Sensitivities", varying from -1.00 to 1.00 with a step equals to .10, for each of the first four moments that allows us to obtain with the GUN exactly the same rankings as those obtained with the six other performance measures previously mentioned. The computation is based on 1,250 random series, equivalent to a 5-year return history, respecting the following market hypotheses: risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00%. Computations by the authors.

**Table 4: GUN Ranking Equivalence**

Panel A: An illustration of *Iso*-GUN Rankings on 30 Portfolios

| Measures | Henriksson-Merton (1981) | | Treynor-Mazuy (1966) | | MPPM 2 (2007) | | MPPM 3 (2007) | | MPPM 4 (2007) | | MPPM 5 (2007) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An Implied Sensitivity Vector ( 1, 2 3, 4 ) | GUN (1.00, .00 .00, .00) | | GUN (1.00, .00 .00, .00) | | GUN (1.00, -1.00 .00, .00) | | GUN (1.00, -1.58 .00, .00) | | GUN (1.00, -2.00 .00, .00) | | GUN (1.00, -2.44 .00, .00) | |
| | HM | Rank | TM | Rank | MPPM2 | Rank | MPPM3 | Rank | MPPM4 | Rank | MPPM5 | Rank |
| | -3.54 | 1 | -4.67 | 4 | 50.56% | 1 | 47.05% | 1 | 43.55% | 6 | 40.08% | 30 |
| | -3.61 | 2 | -4.75 | 13 | 43.42% | 13 | 40.35% | 27 | 37.29% | 27 | 34.25% | 27 |
| | -3.80 | 3 | -4.93 | 7 | 23.60% | 2 | 19.66% | 2 | 16.07% | 12 | 14.09% | 12 |
| | -3.84 | 4 | -4.97 | 6 | 20.30% | 15 | 18.06% | 12 | 15.73% | 2 | 11.83% | 2 |
| | -3.87 | 5 | -5.01 | 20 | 20.04% | 27 | 16.79% | 15 | 13.29% | 15 | 11.02% | 1 |
| | -3.89 | 6 | -5.02 | 15 | 17.80% | 6 | 15.54% | 6 | 13.28% | 1 | 10.34% | 5 |
| | -3.90 | 7 | -5.03 | 17 | 16.81% | 3 | 14.65% | 5 | 12.50% | 5 | 9.78% | 15 |
| | -3.91 | 8 | -5.04 | 9 | 14.94% | 23 | 12.73% | 3 | 10.53% | 3 | 8.33% | 3 |
| | -3.92 | 9 | -5.05 | 23 | 13.88% | 9 | 10.46% | 9 | 7.67% | 10 | 5.59% | 10 |
| | -3.93 | 10 | -5.07 | 8 | 11.81% | 10 | 9.74% | 10 | 7.05% | 9 | 3.98% | 19 |
| | -3.95 | 11 | -5.09 | 10 | 10.51% | 21 | 8.33% | 19 | 6.16% | 19 | 3.64% | 16 |
| | -3.96 | 12 | -5.10 | 25 | 10.33% | 8 | 7.23% | 16 | 4.96% | 16 | 2.70% | 8 |
| | -3.96 | 13 | -5.10 | 21 | 9.50% | 16 | 6.52% | 24 | 2.72% | 14 | -1.08% | 14 |
| | -3.97 | 14 | -5.11 | 16 | 8.60% | 25 | 5.36% | 25 | 2.12% | 25 | -1.12% | 25 |
| | -4.01 | 15 | -5.14 | 18 | 4.92% | 12 | 2.77% | 14 | .67% | 24 | -1.42% | 24 |
| | -4.02 | 16 | -5.15 | 27 | 4.87% | 14 | 2.48% | 8 | .42% | 8 | -1.65% | 9 |
| | -4.02 | 17 | -5.15 | 26 | 4.54% | 24 | 2.10% | 11 | -.11% | 29 | -2.31% | 29 |
| | -4.02 | 18 | -5.16 | 14 | 4.30% | 11 | 2.00% | 13 | -.41% | 13 | -2.46% | 13 |
| | -4.03 | 19 | -5.16 | 11 | 3.70% | 20 | 1.65% | 17 | -.93% | 17 | -3.27% | 22 |
| | -4.03 | 20 | -5.16 | 24 | 3.70% | 5 | 1.13% | 23 | -1.07% | 22 | -3.53% | 20 |
| | -4.03 | 21 | -5.16 | 2 | 3.33% | 28 | .93% | 18 | -1.30% | 18 | -3.87% | 17 |
| | -4.03 | 22 | -5.17 | 22 | 3.16% | 17 | .76% | 28 | -2.03% | 4 | -4.09% | 4 |
| | -4.04 | 23 | -5.17 | 3 | 2.52% | 4 | .03% | 4 | -2.17% | 28 | -5.12% | 28 |
| | -4.04 | 24 | -5.17 | 5 | 2.40% | 26 | -.53% | 7 | -3.58% | 7 | -6.64% | 7 |
| | -4.05 | 25 | -5.18 | 30 | 2.09% | 7 | -1.03% | 29 | -4.47% | 11 | -7.91% | 11 |
| | -4.07 | 26 | -5.21 | 19 | -2.20% | 19 | -5.07% | 26 | -7.22% | 26 | -9.36% | 26 |
| | -4.10 | 27 | -5.23 | 29 | -2.93% | 29 | -5.20% | 21 | -7.78% | 30 | -9.98% | 6 |
| | -4.10 | 28 | -5.24 | 1 | -3.38% | 30 | -5.58% | 30 | -8.22% | 21 | -11.25% | 21 |
| | -4.11 | 29 | -5.25 | 28 | -6.74% | 22 | -10.10% | 20 | -13.42% | 20 | -16.74% | 23 |
| | -4.11 | 30 | -5.25 | 12 | -6.79% | 18 | -10.12% | 22 | -13.51% | 23 | -16.89% | 18 |
| Spearman $\rho$ | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 |
| Kendall $\tau$ | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | .99 |

Panel B: Summary Statistics of 1000 Simulations when only varying the First Two Moments

| | Henriksson-Merton | | Treynor-Mazuy | | MPPM2 | | MPPM3 | | MPPM4 | | MPPM5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | .80 | .81 | .72 | .59 | .68 | .66 | .58 | .48 |
| $1^{st}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .98 | .95 | .96 | .91 |
| Median | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .98 |
| $3^{rd}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Panel C: Summary Statistics of 1000 Simulations when varying the First Four Moments

| | Henriksson-Merton | | Treynor-Mazuy | | MPPM2 | | MPPM3 | | MPPM4 | | MPPM5 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ | $\rho$ | $\tau$ |
| Minimum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | .97 | 1.00 | .98 | 1.00 | .97 | 1.00 | .97 |
| $1^{st}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Median | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $3^{rd}$ Quartile | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

Source: This table compares the ranking of 30 simulated funds obtained according to six performance measures (respectively, Henriksson and Merton, 1981; Treynor and Mazuy, 1966; Goetzmann *et al.*, 2007). Similarly to Table 2 and to Goetzmann *et al.* (2007), market portfolio return distributions are defined such as: $\tilde{r}_m = exp[(\mu_m + \tilde{s} - 0.5\sigma_m^2)\Delta_t + (\sigma_m\tilde{\epsilon})\sqrt{(1-\delta_p^2)\Delta_t}] - 1$, where $\mu_m$ is the market rate of return, $\tilde{s}$ is the signal, $\sigma_m$ is the market standard deviation, $\delta_p$ is the information, $\tilde{\epsilon}$ is a Gaussian random variable. The market's unconditional expected rate of return and logarithmic variance *per* unit time are $\mu_m$ and $\sigma_m^2$, respectively. The information about the changing mean is in the signal, $\tilde{s}$, which is normally distributed with mean zero and variance $\delta_p^2\sigma_m^2\Delta_t$, where $\delta_p$ is the fraction of variation known to the informed trader. In the simulations, $\delta_p$ is set to .10% and 1.00%. The optimal market holding conditional on a signal, $s$, is equal to the conditional risk *premium* divided by the relative risk aversion times the conditional variance, $(\mu_m + s - r)[\rho(1-\delta_p^2)\sigma_m^2]^{-1}$. Since the unconditional risk *premium* is equal to the relative risk aversion times the unconditional variance, the optimal leverage conditional on a signal s is $[1 + s(\rho\sigma_m^2)^{-1}](1-\delta_p^2)^{-1}$. The 30 portfolios under study correspond to 15 informed market timers and 15 random managers as defined in Table 2. The Implied Sensitivities display the average sensitivities, varying from -1.00 to 1.00 with a step equals to .10, for each of the first four moments that allows us to obtain with the GUN exactly the same rankings as those obtained with the six other performance measures previsouly mentioned. The computation is based on 1,250 random series, equivalent to a 5-year return history, respecting the following market hypotheses: risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00%. Computations by the authors.

As expected, Table 3 shows that the rankings of the 30 managed portfolios obtained according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and four variants of the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM can be exactly replicated – as shown by the Spearman's and Kendall's rank correlation coefficients equals to 1.00 for each of these six measures - when adjusting the sensitivities applied to the first four moments when computing GUN measures. Those results lead us to conclude that our GUN is flexible enough to replicate, with a fine accuracy, the rankings of the main traditional performance measures.

Table 4 (Panel A) reports the implied sensitivities, scores and rankings of 30 managed portfolios[9] according to the Henriksson-Merton (1981) and the Treynor-Mazuy (1966) measures, and four variants of the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM when varying the risk aversion level. The second line displays "Implied sensitivities", corresponding to coefficients used in the computation of our GUN that allows us to exactly replicate the portfolios rankings obtained with the six studied performance measures. We also present a summary statistics of the Spearman's and Kendall 's rank correlation coefficients computed from a large sample of portfolios when only varying the first two moments (Panel B) and the first four moments (Panel C).

As anticipated, this second illustration of the flexibility of our measure is able to exactly reproduce the rankings obtained according to the Henriksson-Merton (1981) and Treynor-Mazuy (1966) measures, as shown by the value of Spearman's and Kendall's rank correlation coefficients equals to 1.00 for each of these six measures.

# 4 Financial Applications on Hedge Funds

In this section, we compare the rankings of hedge funds according to several performance measures on a sample as in Darolles *et al.* (2009). We use the HFR database including 2,294 pure hedge funds and 1,926 funds of funds expressed in US dollars. At inception, the hedge funds are self-declared in one or several categories, called styles (see e.g. Das and Das (2004) for a description of styles in various databases). These categories describe either the type of assets in the portfolio (styles "Currencies", "Distressed Securities"), or the type of portfolio management (styles "Global Macro", "Merger Arbitrage"), or both (styles "Fixed Income Arbitrage", "Equity Long/Short"). A majority of pure hedge funds belong in 9 categories, that are "Equity Long/Short", "Fixed Income", "Global Macro", "Currency", "Futures", "Equity Long Short Equally Weighted", "Fixed Income Arbitrage", "Merger Arbitrage" and "Distressed Securities". We select 30 hedge funds that represent various styles and report the information on the management company, the self-declared strategy and the assets under management.

[9]The 30 ranked portfolios correspond to 15 informed and 15 random market timers whom portfolio returns respect the simulation scheme defined in Table 2 (*Cf.* Goetzmann *et al.*, 2007).

**Table 5: Hedge Funds classified by Style**

| N° | Fund Name | Style | Company Name |
|---|---|---|---|
| 1 | Exane Investors Gulliver Fund | Equity Hedge | Exane Structured Asset Management |
| 2 | Ibis Capital, LP | Equity Hedge | Ibis Management, LLC |
| 3 | Odey European Inc. | Equity Hedge | Odey Asset Management Limited |
| 4 | Platinum Fund Ltd. | Equity Hedge | Optima Fund Management |
| 5 | Permal U.S. Opportunities Ltd. | Equity Hedge | Permal Investment Management Services Ltd |
| 6 | RAB Europe Fund | Equity Hedge | RAB Capital PLC |
| 7 | Pioneer Long Short European Equity | Equity Hedge | Pioneer Asset Management |
| 8 | Emerging Value Opportunities Fund Ltd. | Equity Hedge | Value Line, Inc |
| 9 | Robbins Capital Partners, L.P. | Equity Hedge | T. Robbins Capital Management, LLC |
| 10 | Invesco QLS Equity | Equity Market Neutral | Invesco Structured Products Group |
| 11 | Thames River European Fund | Equity Non-Hedge | Thames River Capital LLP |
| 12 | Craigmillar Partners L.P. | Equity Non-Hedge | Craigmillar Ltd. |
| 13 | SSI Long/Short Equity Market Neutral L.P. | Long/Short | SSI Investment Management, Inc. |
| 14 | Friedberg Global Macro Hedge Fund Ltd. | Macro | Friedberg Mercantile Group Ltd. |
| 15 | Sunrise Capital Diversified, Ltd. | Macro | Sunrise Capital Partners |
| 16 | FX Concepts Global Currency Program | Macro | FX Concepts, Inc. |
| 17 | Haidar Jupiter International Ltd. | Macro | Haidar Capital Management, LLC |
| 18 | GLC Directional Fund, Ltd. | Macro | Glc Directional Fund, L.P. |
| 19 | R.G. Niederhoffer Diversified Fund II, Ltd. | Macro | R.G. Niederhoffer Diversified Fund II, Ltd. |
| 20 | QM Premier Fund USD Share Class | Macro | QM Premier Fund USD Share Class |
| 21 | Alternative Treasury Strategy, LLC | Macro | Alternative Treasury Strategy, LLC |
| 22 | Forest Multi Strategy Fund LLC | Relative Value Arbitrage | Forest LLC |
| 23 | Aristeia International, Ltd. | Convertible Arbitrage | Aristeia Capital LLC |
| 24 | Paulson International Ltd. | Merger Arbitrage | Paulson & Co., Inc. |
| 25 | Schultze Offshore Fund, Ltd. | Event-driven | Schultze Offshore Ltd. |
| 26 | York European Opportunities Fund, L.P. | Event-driven | York European Opportunities L.P |
| 27 | Lion Fund Limited | Event-driven | Lion Fund Limited |
| 28 | Fletcher Income Arbitrage Fund, Ltd. | Fixed Income Arbitrage | Fletcher Income Arbitrage Fund, Ltd. |
| 29 | Coast Arbitrage Fund II, Ltd. | Fixed Income Arbitrage | Coast Arbitrage Fund II, Ltd. |
| 30 | Global Distressed Fund | Distressed | Global Investment House |

Source: 30 Hedge Funds have been extracted from the HFR database and 18 are also present in Darolles *et al.* (2009). They are all expressed in US dollars and the period of interest goes from June 2004 to July 2007. Historical return data are on a monthly basis.

As in Darolles *et al.* (2009), Table 6 reports the implied sensitivities, scores and rankings of 30 Hedge Funds according to the Sharpe (1966) ratio, the Jensen (1968) *alpha* and four variants of the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM. Implied sensitivities are coefficients that make equal the ranking obtained with the GUN and the six studied measures.

As anticipated, the implied sensitivities of the Jensen (1968) *alpha* are equal to 1 for the first moment and null for the three others.

As in Darolles *et al.* (2009), Table 7 reports the implied sensitivities, scores and rankings of 30 Hedge Funds according to the Henriksson-Merton (1981) and the Treynor-Mazuy (1966) measures, two variants of the Darolles-Gouriéroux-Jasiak (2009) L-performance, the Morningstar (2002) RAR and the Keating-Shadwick (2002) *Omega* measure. The second line displays "Implied Sensitivities" that are coefficients that make equal the ranking obtained with the GUN and each measure of the six studied.

**Table 6: *Iso*-GUN Rankings on 30 Hedge Funds**

| Measures | Sharpe (1966) | | Jensen (1968) | | MPPM2 (2007) | | MPPM3 (2007) | | MPPM4 (2007) | | MPPM5 (2007) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An Implied Sensitivity Vector ( 1, 2 3, 4 ) | GUN (1.00, -4.84 .00, .00) | | GUN (1.00, .00 .00, .00) | | GUN (1.00, -1.00 .00, .00) | | GUN (1.00, -1.65 .00, .00) | | GUN (1.00, -2.26 .00, .00) | | GUN (1.00, -2.67 .00, .00) | |
| | SR | Rank | JA | Rank | MPPM2 | Rank | MPPM3 | Rank | MPPM4 | Rank | MPPM5 | Rank |
| | 1.90 | 1 | -6.35% | 16 | 34.41% | 27 | 31.08% | 27 | 27.83% | 27 | 24.63% | 27 |
| | 1.82 | 2 | -7.94% | 19 | 19.90% | 19 | 18.50% | 19 | 17.13% | 19 | 16.22% | 19 |
| | 1.69 | 3 | -7.81% | 7 | 18.75% | 7 | 17.88% | 7 | 17.04% | 20 | 15.80% | 20 |
| | 1.57 | 4 | -8.13% | 22 | 17.17% | 22 | 16.56% | 22 | 15.96% | 22 | 15.36% | 22 |
| | 1.55 | 5 | -8.31% | 4 | 16.18% | 4 | 15.27% | 4 | 14.36% | 30 | 13.46% | 30 |
| | 1.49 | 6 | -8.55% | 14 | 14.39% | 14 | 13.78% | 16 | 13.16% | 16 | 12.55% | 16 |
| | 1.47 | 7 | -8.60% | 27 | 13.93% | 17 | 13.34% | 17 | 12.75% | 17 | 12.16% | 17 |
| | 1.44 | 8 | -8.81% | 20 | 11.36% | 13 | 10.93% | 13 | 10.52% | 13 | 10.10% | 13 |
| | 1.36 | 9 | -8.82% | 2 | 11.00% | 23 | 10.41% | 3 | 9.87% | 6 | 9.34% | 4 |
| | 1.23 | 10 | -8.84% | 15 | 10.95% | 2 | 10.24% | 21 | 9.52% | 21 | 8.80% | 21 |
| | 1.12 | 11 | -8.85% | 6 | 10.95% | 3 | 10.23% | 23 | 9.45% | 23 | 8.66% | 23 |
| | 1.11 | 12 | -8.92% | 26 | 9.81% | 26 | 9.10% | 26 | 8.43% | 9 | 8.15% | 9 |
| | 1.03 | 13 | -8.93% | 24 | 9.67% | 11 | 9.00% | 11 | 8.39% | 26 | 7.69% | 11 |
| | 1.01 | 14 | -8.94% | 11 | 9.63% | 24 | 8.87% | 24 | 8.34% | 11 | 7.67% | 26 |
| | .99 | 15 | -9.06% | 9 | 9.00% | 9 | 8.71% | 9 | 8.12% | 24 | 7.37% | 25 |
| | .98 | 16 | -9.21% | 21 | 6.63% | 1 | 6.33% | 1 | 6.02% | 7 | 5.72% | 2 |
| | .80 | 17 | -9.23% | 25 | 5.98% | 25 | 5.21% | 20 | 4.79% | 2 | 4.36% | 5 |
| | .79 | 18 | -9.25% | 12 | 5.91% | 12 | 5.18% | 12 | 4.61% | 18 | 4.31% | 18 |
| | .75 | 19 | -9.26% | 1 | 5.81% | 21 | 5.14% | 25 | 4.45% | 12 | 4.11% | 15 |
| | .74 | 20 | -9.32% | 29 | 5.63% | 29 | 4.90% | 18 | 4.37% | 15 | 3.94% | 8 |
| | .68 | 21 | -9.38% | 18 | 5.20% | 18 | 4.79% | 29 | 4.31% | 25 | 3.71% | 12 |
| | .68 | 22 | -9.41% | 13 | 4.90% | 15 | 4.64% | 15 | 4.22% | 8 | 3.50% | 10 |
| | .66 | 23 | -9.42% | 8 | 4.78% | 8 | 4.50% | 8 | 3.81% | 10 | 3.46% | 24 |
| | .63 | 24 | -9.45% | 10 | 4.42% | 10 | 4.11% | 10 | 3.76% | 28 | 3.44% | 28 |
| | .59 | 25 | -9.46% | 23 | 4.31% | 20 | 4.02% | 2 | 3.73% | 5 | 2.73% | 1 |
| | .37 | 26 | -9.56% | 28 | 2.57% | 28 | 2.00% | 28 | 1.43% | 1 | .85% | 7 |
| | .16 | 27 | -9.76% | 30 | .61% | 6 | .30% | 6 | .00% | 4 | - .31% | 6 |
| | .09 | 28 | -9.77% | 3 | - .78% | 30 | -1.78% | 30 | -2.77% | 3 | -3.76% | 3 |
| | .03 | 29 | -9.83% | 5 | -1.85% | 5 | -3.00% | 5 | -4.14% | 29 | -5.27% | 29 |
| | -.07 | 30 | -9.98% | 17 | -5.23% | 16 | -7.16% | 14 | -9.10% | 14 | -11.05% | 14 |
| Spearman $\rho$ | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | | 1.00 | |
| Kendall $\tau$ | 1.00 | | 1.00 | | 1.00 | | 1.00 | | .99 | | .99 | |

Source: Monthly quotes in US dollars from June 2004 to July 2007. This table compares the ranking of 30 Hedge Funds obtained according to the Sharpe (1966) ratio, the Jensen (1966) *alpha* and four variations of the MPPM (Goetzmann *et al.*, 2007). The second line displays the "implied sensitivities" for each of the four moments allowing us to obtain with the GUN exactly the same rankings as those obtained with the six studied performance measures. Sensitivities with an asterix have been divided by 100. Computations by the authors.

**Table 7: *Iso*-GUN Rankings on 30 Hedge Funds**

| Measures | Henriksson-Merton (1981) | | Treynor-Mazuy (1966) | | L-performance 1 (2009) | | L-performance 3 (2009) | | Morningstar (2002) | | Keating-Shadwick (2002) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An Implied Sensitivity Vector ( 1, 2 3, 4 ) | GUN (1.00, -2.01 7.08*, -12.77*) | | GUN (1.00, -1.00, 6.81, -9.78) | | GUN (1.00, -.10, .08, -12.57) | | GUN (1.00, -0.15, 7.88*, -9.08*) | | GUN (1.00, -1.66 .00, .00) | | GUN (1.00, -4.89, .01, -.61*) | |
| | HM | Rank | TM | Rank | L1 | Rank | L3 | Rank | MRAR | Rank | *Omega* | Rank |
| | 41.53% | 1 | 30.17% | 18 | 25.68% | 17 | 33.50% | 30 | 36.46% | 27 | 3.55 | 1 |
| | 20.84% | 2 | 17.10% | 23 | 25.43% | 2 | 33.30% | 12 | 20.32% | 19 | 3.42 | 2 |
| | 17.44% | 3 | 13.71% | 30 | 24.85% | 4 | 32.83% | 30 | 19.58% | 7 | 3.21 | 3 |
| | 13.70% | 4 | 11.75% | 17 | 23.64% | 1 | 31.31% | 1 | 18.01% | 22 | 2.75 | 4 |
| | 12.47% | 5 | 10.33% | 16 | 23.37% | 3 | 31.01% | 16 | 16.50% | 4 | 2.67 | 5 |
| | 11.32% | 6 | 9.92% | 20 | 22.17% | 6 | 30.40% | 13 | 14.77% | 16 | 2.66 | 6 |
| | 11.17% | 7 | 9.88% | 14 | 22.17% | 7 | 30.30% | 23 | 14.27% | 17 | 2.64 | 7 |
| | 9.16% | 8 | 7.75% | 9 | 21.79% | 5 | 28.60% | 9 | 11.56% | 13 | 2.61 | 8 |
| | 7.46% | 9 | 6.67% | 13 | 21.53% | 13 | 27.76% | 20 | 10.97% | 3 | 2.59 | 9 |
| | 6.33% | 10 | 5.61% | 2 | 20.51% | 23 | 27.06% | 14 | 10.78% | 21 | 2.40 | 10 |
| | 6.15% | 11 | 5.45% | 3 | 19.90% | 16 | 26.93% | 3 | 10.77% | 23 | 2.27 | 11 |
| | 5.94% | 12 | 5.27% | 29 | 19.44% | 29 | 24.83% | 29 | 9.53% | 26 | 2.14 | 12 |
| | 5.80% | 13 | 5.19% | 25 | 19.28% | 25 | 24.81% | 25 | 9.42% | 11 | 2.05 | 13 |
| | 5.80% | 14 | 5.10% | 5 | 18.85% | 9 | 24.43% | 22 | 9.28% | 24 | 1.95 | 14 |
| | 5.13% | 15 | 4.56% | 12 | 18.48% | 12 | 24.13% | 2 | 9.10% | 9 | 1.91 | 15 |
| | 4.76% | 16 | 4.13% | 8 | 18.37% | 8 | 23.34% | 8 | 6.53% | 1 | 1.87 | 16 |
| | 3.66% | 17 | 3.42% | 1 | 17.49% | 14 | 22.91% | 18 | 5.35% | 20 | 1.72 | 17 |
| | 2.87% | 18 | 2.43% | 15 | 17.11% | 15 | 20.79% | 15 | 5.32% | 12 | 1.70 | 18 |
| | 2.23% | 19 | 1.83% | 7 | 17.10% | 18 | 19.70% | 7 | 5.28% | 25 | 1.66 | 19 |
| | 1.94% | 20 | 1.76% | 26 | 16.61% | 26 | 19.33% | 26 | 5.02% | 18 | 1.66 | 20 |
| | 1.82% | 21 | 1.69% | 10 | 16.61% | 10 | 18.47% | 10 | 4.91% | 29 | 1.57 | 21 |
| | 1.37% | 22 | 1.55% | 19 | 14.90% | 19 | 18.14% | 21 | 4.75% | 15 | 1.56 | 22 |
| | 1.32% | 23 | 1.33% | 6 | 14.39% | 20 | 17.27% | 11 | 4.60% | 8 | 1.56 | 23 |
| | 1.11% | 24 | 1.05% | 24 | 13.21% | 24 | 17.21% | 24 | 4.20% | 10 | 1.55 | 24 |
| | 1.11% | 25 | .88% | 22 | 12.15% | 22 | 16.80% | 5 | 4.11% | 2 | 1.52 | 25 |
| | .89% | 26 | .47% | 28 | 12.10% | 28 | 15.38% | 19 | 2.02% | 28 | 1.27 | 26 |
| | -2.84% | 27 | -2.42% | 28 | 9.89% | 11 | 13.55% | 6 | .30% | 6 | 1.12 | 27 |
| | -2.91% | 28 | -2.71% | 4 | 3.33% | 30 | 4.00% | 4 | -1.76% | 30 | 1.06 | 28 |
| | -7.11% | 29 | -6.53% | 21 | .68% | 21 | .53% | 28 | -2.95% | 5 | 1.02 | 29 |
| | -11.66% | 30 | -11.18% | 27 | - .19% | 27 | -1.04% | 27 | -6.91% | 14 | .95 | 30 |
| Spearman $\rho$ | | .92 | | .85 | | .81 | | .89 | | 1.00 | | 1.00 |
| Kendall $\tau$ | | .94 | | .88 | | .83 | | .91 | | 1.00 | | .99 |

Source: Monthly quotes in US dollars from June 2004 to July 2007. This table compares the ranking of 30 Hedge Funds obtained according to the Henriksson- Merton (1981) and Treynor-Mazuy (1966) measures, two variants of the L-performance (Darolles *et al.*, 2009), the MRAR (Morningstar, 2002) and the *Omega* (Keating and Shadwick, 2002). The second line displays the "Implied Sensitivities" for each of the four moments allowing us to obtain with the GUN exactly the same ranking as those obtained with the six studied performance measures. Computations by the authors.

As in the previous examples (*Cf.* Tables 3 and 4), Table 6 and Table 7 show that we can almost reproduce any ranking based on main performance measures when assets are non-Gaussian.

# 5 Conclusion

Portfolio performance measurement is a topic of interest within both academic and practitioner communities, as well as financial authorities. Funds are generally ranked according to different *criteria* by investment banks and financial advisors. Such published rankings can have a significant impact on allocation decisions of fund managers. Numerous measures have been proposed to evaluate the performance of an active management since the introduction of the main one in 1966 by William Sharpe. However, a reasonable concern among those who use a particular measure is whether or not the manager being evaluated might react by attempting to manipulate it. More formally, manipulation is the action taken to increase a fund's performance measure that does not actually add value for the fund's investor. Several articles have noted that even when the evaluator knows the moments of the return distribution, it is still possible to use informationless trades to boost the expected Sharpe ratio. If most of the performance measures can be manipulated, can we then find one that is not gamable for investors?

This article is organized into three sections: the introduction of our new flexible Generalized Utility-based N-moment measure of performance, a comparison with some of the main performance measures and financial applications on Hedge Funds.

The new measure of performance, built as a generalization of the Sharpe (1966) ratio, nicely compete with the Morningstar (2002) measure and the Goetzmann-Ingersoll-Spiegel-Welch (2007) MPPM. It is founded on an extension of the Mean-variance analysis in a first four moment framework. The GUN measure is a simple basic generalization of the well-known Sharpe (1966) ratio, making flexible the first four sensitivities in the utility function of the investor applied to the mean, the variance, the skewness and the *kurtosis*. The main objective of this adjustment is to be able in a near future to adapt the proposed measure of performance for each category of individuals by taking into account their preferences and risk profiles. For instance, the sensitivities applied to the first four moments of the Generalized Utility-based N-moment measure of performance will be different between an individual who mainly wants to maximize the excess return, another one who prefers to minimize his portfolio risk or a last one who mainly considers a constraint on the fourth moment adopting a "safety first" behavior. This is the main point for justifying the proposed measure of performance.

Furthermore, we can distinguish two main potential uses of the Generalized Utility-based N-moment measure of performance for financial regulators and financial advisors. First, the GUN can be used as any other common ranking *criteria*, but also considered as an optimization function in some asset allocation problems or used for recovering risk aversion parameters from a specific ranking. Secondly, the GUN can be viewed as a potential fraud indicator (see Bernard and Boyle, 2009; Brown *et al.*, 2010) that could be very helpful for investors and financial regulators. We have also planned to complement our intuition that our measure respects all the axioms that have to be satisfied by a "good" performance measure (*Cf.* Chen and Knez, 1996; Pedersen and Satchell, 1998; Hübner, 2006; Eberlein and Madan, 2008).

But this is left for a further research.

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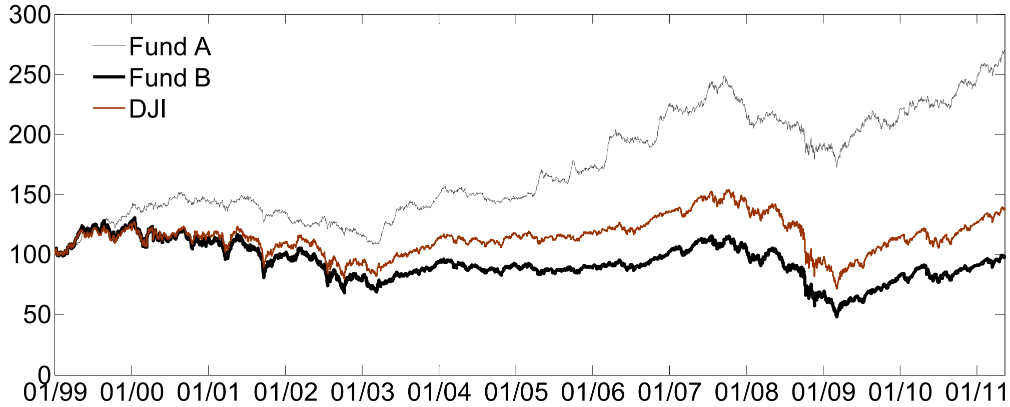
## Appendix A1

This first appendix presents the performance of Funds A and B on a 12-year period (see Figure A1.1) as well as the main descriptive statistics over a long (Table A1.1) and a short term period (Table A1.2). We also detail the sketch of the algorithm used for building Figure 3.

Figure A1.1: Performance of the Fund A (Informed Investor), the Fund B (Uninformed Agent) and the DJI (Benchmark) on a 12-year Period




Source: USD Daily quotes, from 01/01/1999 to 05/13/2011. We compare in this illustration the performance of the Fund A and the Fund B – the uninformed agent – to their benchmark which is the DJI. Fund A corresponds to the informed investor who is characterized by an *alpha* set to .01% and a sensitivity of his portfolio to the benchmark equals to .39, while the Fund B, namely the uninformed investor, has respectively an *alpha* -.01% sets to and a *beta* equals to 1.10. Computations by the authors.

Table A1.1: Main Descriptive Statistics on the Fund A and B on a 12-year Period

| | Fund A | Fund B | Ranking |
|---|---|---|---|
| Daily Mean Return | .02% | .01% | A |
| Standard Deviation | .65% | 1.36% | A |
| Ann. Performance | 4.36% | -.21% | A |
| Ann. Volatility | 10.39% | 21.60% | A |
| Skewness | -.08 | .18 | B |
| *Kurtosis* | 5.81 | 11.26 | A |
| Max Drawdown | -32.00% | -63.00% | A |
| Value-at-Risk 95% | -1.05% | -2.15% | A |
| Value-at-Risk 99% | -1.75% | -3.80% | A |
| Sharpe | -.01 | -.13 | A |
| *Omega* | -.31 | -6.30 | A |
| Sortino | -.01 | -.17 | A |
| Kappa 3 | -.12 | -1.96 | A |
| Calmar | -.01 | -.05 | A |
| Burke | -.00 | -.01 | A |
| Sharpe-*Omega* | .05 | .01 | A |
| Treynor | -.03 | -.05% | A |
| Treynor-Black | -.01 | -.02 | A |
| Graham-Harvey | .01 | -.02 | A |
| Cornell | 4.03% | -2.79% | A |
| RAP | 4.86% | 2.41% | A |
| MRAP | 1.78% | .26 | A |
| SRAP | .37% | -2.08% | A |
| Ulcer | -.01 | -.09 | A |
| Ziemba | -.01 | -.13 | A |
| MPPM (A = 2) | -1.15% | -7.42% | A |
| MPPM (A = 3) | -1.69% | -9.75% | A |
| MPPM (A = 4) | -2.23% | -12.09% | A |
| MPPM (A = 5) | -2.77% | -14.43% | A |
| GUN Greedy | 9.97% | 4.32% | A |
| GUN Risk Averse | -9.50% | -4.23% | B |
| GUN Prudent | .16% | .04% | A |
| GUN Temperate | .08% | .01% | A |

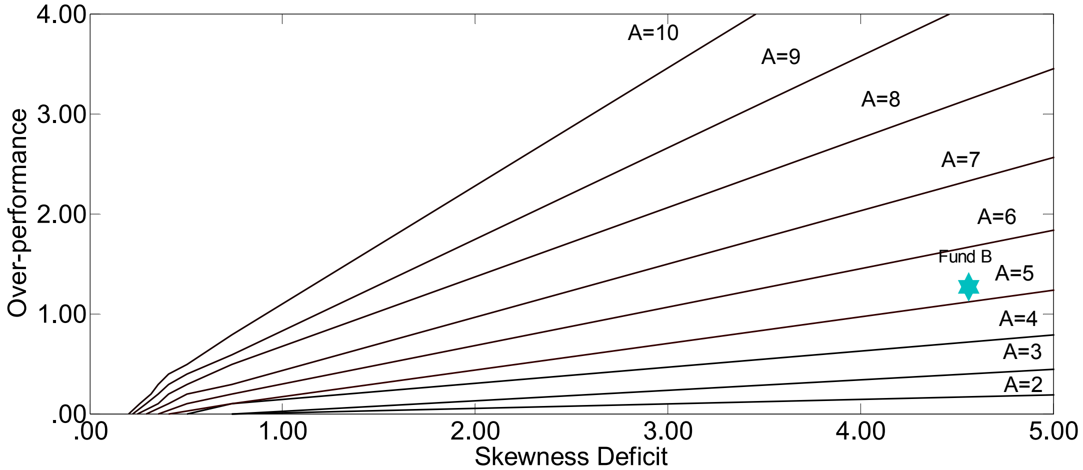
Source: All these figures have been computed for Fund A and Fund B on a 12-year period (from 01/01/1999 to 05/13/2011). Fund A corresponds to the informed investor who is characterized by an *alpha* set to .01% and a sensitivity of his portfolio to the benchmark equals to .39, while the Fund B, namely the uninformed investor, has respectively an *alpha* -.01% sets to and a *beta* equals to 1.10. *Cf* Caporin *et al.* (2013) for the definition of the performance measures used in this table. Computations by the authors.

Table A1.2: Main Descriptive Statistics of Returns on Fund A and B over the one-year period

| | Fund A | Fund B | Ranking |
|---|---|---|---|
| Daily Mean Return | .06% | .07% | B |
| Standard Deviation | .99% | 1.14% | A |
| Ann. Performance | 15.53% | 16.47% | B |
| Ann. Volatility | 15.79% | 18.09% | A |
| Skewness | 2.12 | -2.44 | A |
| *Kurtosis* | 9.86 | 16.06 | A |
| Max Drawdown | -10.00% | -20.00% | A |
| Value-at-Risk 95% | -1.06% | -1.68% | A |
| Value-at-Risk 99% | -1.54% | -4.09% | A |
| Sharpe | .99 | .96 | A |
| *Omega* | 57.16 | 55.45 | A |
| Sortino | 2.30 | .83 | A |
| Kappa 3 | 23.51 | 10.87 | A |
| Calmar | 1.56 | .89 | A |
| Burke | .54 | .37 | A |
| Sharpe-*Omega* | .13 | .11 | A |
| Treynor | .32 | .22 | A |
| Treynor-Black | .28 | .18 | A |
| Graham-Harvey | .13 | .12 | A |
| Cornell | 13.41% | 13.29% | A |
| RAP | 19.99% | 19.50% | A |
| MRAP | 33.00% | 22.82% | A |
| SRAP | 12.72% | 12.22% | A |
| Ulcer | 1.92 | 1.58 | A |
| Ziemba | 1.62 | .59 | A |
| MPPM (A = 2) | 11.95% | 12.56% | B |
| MPPM (A = 3) | 10.76% | 10.83% | B |
| MPPM (A = 4) | 9.58% | 9.07% | A |
| MPPM (A = 5) | 8.42% | 7.27% | A |
| GUN Greedy | 17.86% | 19.93% | B |
| GUN Risk Averse | -14.14% | 15.41% | A |
| GUN Prudent | 1.30% | 1.58% | B |
| GUN Temperate | 1.18% | 1.44% | B |

Source: All these figures have been computed for Fund A and Fund B on the year 2010. Fund A corresponds to the informed investor who is characterized by an *alpha* set to .05% and a sensitivity of his portfolio to the benchmark equals to .36, while the Fund B, namely the uninformed investor, has respectively an *alpha* .03% sets to and a *beta* equals to .86. *Cf* Caporin *et al.* (2013) for the definition of the performance measures used in this table. Computations by the authors.

Figure A1.2: *Iso*-MPPM Frontiers displaying the Quantity of Skewness required for a Given Over-performance for reversing the MPPM Ranking - in a Pure Simulation Case




Source: Illustration of the over-performance (the y-axis) - expressed in % - and over-skewness (the x-axis). Fund B, compared to Fund A, is symbolized by a blue point. Ranking frontiers (solid lines) are computed from the Manipulation-Proof Performance Measure (see, Goetzmann *et al.*, 2007) when varying the risk aversion coefficient from 2 to 10 (A is equal to 3 in the original paper). They are realized by comparing 10,000 pairs of portfolios - each represented by 250 returns generated with a Hansen (1994) *t*-skew Student law with mean performance de.ned on the y-axis, volatility of Fund B and various skewness-governing parameters. For each comparison, for a given over-performance and the return series of the worst performer fund is distorted (by intensification of the skewness only) making equal both fund skewnesses; then the skewness of the worst performer is once again distorted until an inversion of the ranking is obtained. Please see the sketch of the algorithm in Appendix A2. Computations by the authors.

## Appendix A2: Sketch of the Algorithm for building Figure 3

Parameters:

$N = 10,000$ (number of return series), $\delta = .01\%$ (increment for distortion), $[\mathbf{r}_i]$ (time-series of returns on fund *i*), $j$ (an increment), $\mathbf{o\sigma} = [o\sigma_t]$ (vector of over-volatilities), $\mathbf{o}\mathrm{E} = [oE_t]$ (vector of over-performances), $\tau = [.00,...,.10]$ (the threshold is fixed).

Choose one $\tau$.

Main Program:

$j = 0$; $j = j + 1$ ;

1. Choose $i = [1,...,N]$ and pick the fund $i$;

2. Compute $[\mathbf{r}_i]_{A_j} = [\mathbf{r}_i] + (j \times \delta)$ ;

/* *Distort the series such as* $E(r_i)_j = E(r_i) + (j \times \delta)$ *(with same volatility)* */

3. Compute $[\mathbf{r}_i]_{B_j} = [\mathbf{r}_i] \times (j \times \delta_j) - E(r_i)$;

/* *Distort the series such as* $\sigma(r_i)_j = \sigma(r_i) - (j \times \delta)$ *(with the same mean when rescaled)* */

4. Compute $\Omega_{A_j}(.)$ and $\Omega_{B_j}(.)$ for one specific threshold, $\tau$;

5. If $\Omega_{A_j}(.) = \Omega_{B_j}(.)$ stop, then:

$$\mathbf{o\sigma} = \mathbf{o\sigma} \Big| \left\{ \sigma\left([\mathbf{r}_i]_{B_j}\right) - \sigma\left([\mathbf{r}_i]\right) \right\} ;$$

/* *We add here the over-volatility of fund B in the vector of over-volatilities* $\mathbf{o\sigma}$ */

$$\mathbf{oE} = \mathbf{oE} \Big| \left\{ E\left([\mathbf{r}_i]_{A_j}\right) - E\left([\mathbf{r}_i]\right) \right\} ;$$

/* *We add here the over-performance of fund A in the vector of over-performances* $\mathbf{o}\mathrm{E}$ */

If not (*i.e.* $\Omega_{A_j}(.) \neq \Omega_{B_j}(.)$) then repeat (next *j*)…

### Appendix A3: Algorithm for the Simulation of the Table 2

The computation of the table 2 follows the simulation scheme defined in Goetzmann *et al.* (2007). It is based on return distributions corresponding to a 10,000 random return series, recalibrated in order to respect the following market hypotheses (with a four-digit accuracy): risk free rate 5.00% *per* year, market *premium* 12.00%, market standard deviation 20.00% and an investor's degree of risk aversion set to 3 to be consistent. The frequencies, at which the portfolio beats the market according to each, are given along with the approximate frequencies with which the portfolio significantly (5.00%) outperformed or underperformed the market. These numbers are estimated as the frequency with which the measure was more than 1.65 standard deviations positive or negative.

Return distributions of the timer's portfolio and those of the market portfolio are simulated according to following *formula*:

$$\tilde{r}_p = \exp\left\{\left(\mu_m + \tilde{s} - 0.5\sigma_m^2\right)\Delta t + \sigma_m\sqrt{\left(1-\delta_p^2\right)\Delta t}\,\tilde{\varepsilon}\right\} - 1, \qquad \text{(A3.1)}$$

and:

$$\tilde{r}_m = \exp\left\{\left(\mu_m - 0.5\sigma_m^2\right)\Delta t + \sigma_m\tilde{\varepsilon}\right\} - 1, \qquad \text{(A3.2)}$$

where $\tilde{s}$ is the signal characterizing the information about the changing mean, $\sigma_m$ is the total risk of the market portfolio $m$, $\delta_p$ is a constant and $\tilde{\varepsilon}$ is a Gaussian random variable.

The table below summarized the different parameters used to compute the market and random timer's portfolio return distributions.

**Table A3.1: Parameters characterizing the Random, the Informed Timer's and the Market Portfolio Returns**

| | | Portfolio Returns | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\mu_m$ | $\tilde{s}$ | $\sigma_m$ | $\delta_p$ | $\Delta t$ | $\tilde{\varepsilon}$ |
| Timer | *Informed* | 7.00% | $N\left(0,\delta_p^2\sigma_m^2\Delta t\right)$ | 20.00% | $\left(.01\%,1.00\%\right)$ | 1.00 | $N\left(0,1\right)$ |
| | *Random* | 7.00% | $N\left(0,\delta_p^2\sigma_m^2\Delta t\right)$ | 20.00% | $\left[.01\%,1.00\%\right]$ | 1.00 | $N\left(0,1\right)$ |
| Market | | 7.00% | - | 20.00% | - | 1.00 | $N\left(0,1\right)$ |

Source: the table displays the different market hypotheses used in Equation (A3.1) and (A3.2) for the computation of the market timer's and the market portfolio returns.

The table 2 then compares respectively the market timing coefficient and the contribution value obtained with the Henriksson and Merton (1981) parametric model and the Treynor and Mazuy (1966) market timing model, the Goetzmann *et al.* (2007) Manipulation-Proof Performance Measure and the Generalized Utility-based N-moment measure of performance for both, an informed timer whose information about a changing mean explains .10% or 1.00% of the market's variance and a random market timer who varies leverage randomly to the same degree (see Table 6 in Goetzmann *et al.*, 2007, on page 1,535). The Generalized Utility-based N-moment measure of performance (in short GUN) is declined according to four investor profiles, namely $GUN_G$, $GUN_{RA}$, $GUN_P$ and $GUN_T$, which respectively refer to an agent strongly greedy, risk averse, prudent or temperate. More precisely, these different performance measures are defined such as:

$$\begin{cases} HM_p = \left[E\left(r_p\right)-r_f\right]-\gamma_{1,p}\times\left[E\left(r_m\right)-r_f\right]-\gamma_{2,p}\times Max\left[0,r_f-E\left(r_m\right)\right] \\ TM_p = \left[E\left(r_p\right)-r_f\right]-\gamma_{1,p}\times\left[E\left(r_m\right)-r_f\right]-\gamma_{2,p}\times\left[E\left(r_m\right)-r_f\right]^2 \\ \Theta_p = \ln\left\{E\left[\left(1+r_f\right)^{-1}\times\left(1+r_f+r_p\right)^{1-\rho}\right]\right\}\times\left[\left(1-\rho\right)\times\Delta t\right]^{-1} \\ GUN_{4,G,p} = \left[10\times m_{1,p}\left(r_p\right)\right]-\left[.5\times m_{2,p}\left(r_p\right)\right]+\left[.33\times m_{3,p}\left(r_p\right)\right]-\left[.25\times m_{4,p}\left(r_p\right)\right] \\ GUN_{4,RA,p} = \left[m_{1,p}\left(r_p\right)\right]-\left[10\times m_{2,p}\left(r_p\right)\right]+\left[.33\times m_{3,p}\left(r_p\right)\right]-\left[.25\times m_{4,p}\left(r_p\right)\right] \\ GUN_{4,P,p} = \left[m_{1,p}\left(r_p\right)\right]-\left[.5\times m_{2,p}\left(r_p\right)\right]+\left[10\times m_{3,p}\left(r_p\right)\right]-\left[.25\times m_{4,p}\left(r_p\right)\right] \\ GUN_{4,T,p} = \left[m_{1,p}\left(r_p\right)\right]-\left[.5\times m_{2,p}\left(r_p\right)\right]+\left[.33\times m_{3,p}\left(r_p\right)\right]-\left[10\times m_{4,p}\left(r_p\right)\right] \end{cases} \quad \text{(A3.3)}$$

where $HM_p$ is the Henriksson-Merton (1981) ratio, $TM_p$ is the Treynor and Mazuy (1966) market timing model, $\Theta_p$ is the Goetzmann *et al.* (2007) MPPM, $GUN_{4,G,p}$, $GUN_{4,RA,p}$, $GUN_{4,P,p}$ and $GUN_{4,T,p}$ are the four declinations of the GUN measures, $r_p$ and $r_m$ are the returns of the investor's portfolio $p$ and the market portfolio $m$, $r_f$ is the risk-free rate, $\gamma_{1,p}$ is the systematic risk sensitivity of the portfolio $p$, $\gamma_{2,p}$ is the market timing coefficient of the investor's portfolio $p$, $m_{n,p}(.)$ with $n=[1,\ldots,4]$ is the *n-th* (centered) C-moment.