Potential Energy and Conservation of Energy

Lecture 11-12

Chapter 8

(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)

Chapter 8

Potential Energy and Conservation of Energy

In this chapter we will introduce the following concepts:

Potential Energy
Conservative and non-conservative forces
Mechanical Energy
Conservation of Mechanical Energy

The conservation of energy theorem will be used to solve a variety of problems

As was done in Chapter 7 we use scalars such as work ,kinetic energy, and mechanical energy rather than vectors. Therefore the approach is mathematically simpler.

Gravitational Potential Energy

• **An object in motion has kinetic energy**

• **An object may possess energy by virtue of its position relative to the earth – such an object is said to have gravitational potential energy**

$$\Delta y = h_o - h_f \qquad mg$$
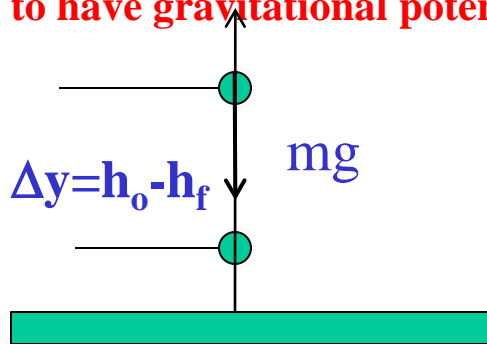
$$W_{gravity} = mgh_0 - mgh_f$$

$$\boxed{\Delta U = -W}$$

Initial gravit.
potential energy
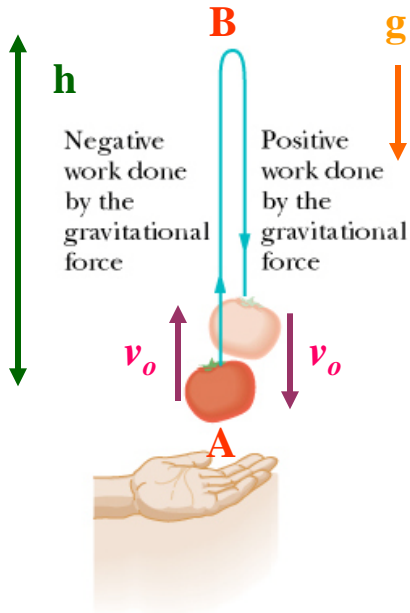$PE_0 = U_0$

Final gravit.
potential energy
$PE_f = U_f$

• **The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level.**

$$PE = U = mgh \qquad \text{SI unit: joule (J)}$$

• **The gravitational potential energy belongs to the object and the earth as a system although one often speaks of the object alone as possessing the gravitational potential energy.**

Negative work done by the gravitational force

Positive work done by the gravitational force

h

B

g

v_o

v_o

A

Work and Potential Energy:

Consider the tomato of mass m shown in the figure. The tomato is taken together with the earth as the system we wish to study. The tomato is thrown upwards with initial speed v_o at point A. Under the action of the gravitational force it slows down and stops completely at point B. Then the tomato falls back and by the time it reaches point A its speed has reached the original value v_o.

Below we analyze in detail what happens to the tomato-earth system. During the trip from A to B the gravitational force F_g does negative work $W_1 = -mgh$. Energy is transferred by F_g **from** the kinetic energy of the tomato **to** the gravitational potential energy U of the tomato-earth system. During the trip from B to A the transfer is reversed. The work W_2 done by F_g is positive ($W_2 = mgh$). The gravitational force transfers energy **from** the gravitational potential energy U of the tomato-earth system **to** the kinetic energy of the tomato. The change in the potential energy U is defined as:

$$\Delta U = -W$$

(a)

(b)
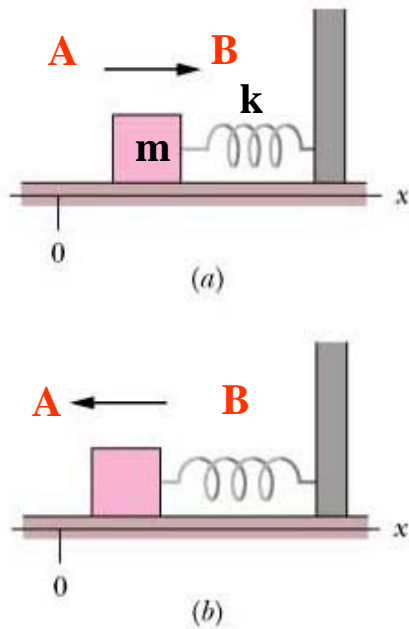
Consider the mass m attached to a spring of spring constant k as shown in the figure. The mass is taken together with the spring as the system we wish to study. The mass is given an initial speed v_o at point A. Under the action of the spring force it slows down and stops completely at point B which corresponds to a spring compression x. Then the mass reverses the direction of its motion and by the time it reaches point A its speed has reached the original value v_o.

As in the previous example we analyze in detail what happens to the mass-spring system . During the trip from A to B the spring force F_s does negative work $W_1 = -kx^2/2$. Energy is transferred by F_s **from** the kinetic energy of the mass **to** the potential energy U of the mass-spring system.
During the trip from B to A the transfer is reversed. The work W_2 done by F_s is positive ($W_2 = kx^2/2$). The spring force transfers energy **from** the potential energy U of the mass-spring system **to** the kinetic energy of the mass. The change in the potential energy U is defined as:

$$\Delta U = -W$$

Conservative and non-conservative forces.
The gravitational force as the spring force are called **"conservative"** because they can transfer energy from the kinetic energy of part of the system to potential energy and vice versa.
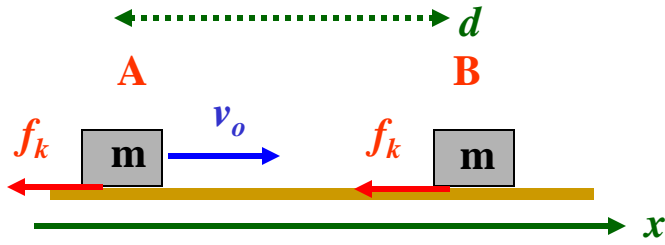
Frictional and drag forces on the other hand are called **"non-conservative"** for reasons that are explained below.

Consider a system that consists of a block of mass m and the floor on which it rests. The block starts to move on a horizontal floor with initial speed v_o at point A. The coefficient of kinetic friction between the floor and the block is μ_k. The block will slow down by the kinetic friction f_k and will stop at point B after it has traveled a distance d. During the trip from point A to point B the frictional force has done work $W_f = -\mu_k mgd$. The frictional force transfers energy from the kinetic energy of the block to a type of energy called **thermal** energy. This energy transfer **cannot be reversed**. Thermal energy cannot be transferred back to kinetic energy of the block by the kinetic friction. This is the hallmark of non-conservative forces.

(a)   (b)
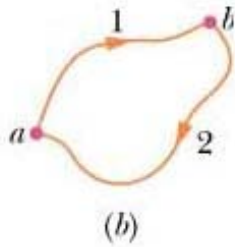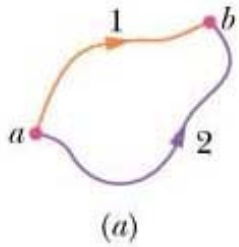
Path Independence of Conservative Forces

In this section we will give a test that will help us decide whether a force is conservative or non-conservative.

A force is conservative if the net work done on a particle during a round trip is always equal to zero (see fig.b).

$$W_{net} = 0$$

Such a round trip along a closed path is shown in fig.b. In the examples of the tomato-earth and mass-spring system $W_{net} = W_{ab,1} + W_{ba,2} = 0$

We shall prove that if a force is conservative then the work done on a particle between two points a and b does **not** depend on the path.

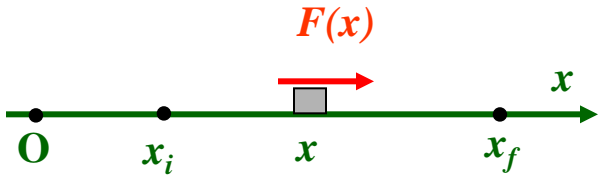From fig. b we have: $W_{net} = W_{ab,1} + W_{ba,2} = 0$ □ $W_{ab,1} = - W_{ba,2}$ **(eqs.1)**

From fig.a we have: $W_{ab,2} = - W_{ba,2}$ **(eqs.2)**

If we compare eqs.1 and eqs.2 we get: $$W_{ab,1} = W_{ab,2}$$

Determining Potential Energy Values:

In this section we will discuss a method that can be used to determine the difference in potential energy ΔU of a conservative force F between points x_f and x_i on the x-axis if we know $F(x)$
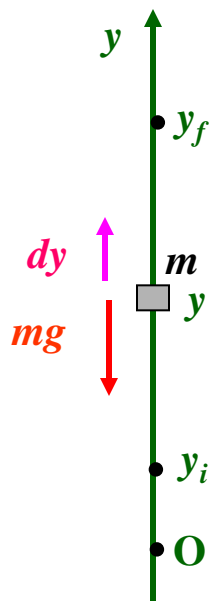
A conservative force F moves an object along the x-axis from an initial point x_i to a final point x_f. The work W that the force F does on the object is given by :

$$W = \int_{x_i}^{x_f} F(x)dx$$ The corresponding change in potential energy ΔU was defined as:

$\Delta U = -W$ Therefore the expression for ΔU becomes:

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

Gravitational Potential energy:

Consider a particle of mass m moving vertically along the y-axis from point y_i to point y_f. At the same time the gravitational force does work W on the particle which changes the potential energy of the particle-earth system. We use the result of the previous section to calculate ΔU

$$\Delta U = -\int_{y_i}^{y_f} F(x)\,dy \quad F = -mg \quad \rightarrow \quad \Delta U = -\int_{y_i}^{y_f} (-mg)\,dy = mg\int_{y_i}^{y_f} dy = mg\left[y\right]_{y_i}^{y_f}$$

$\Delta U = mg\left(y_f - y_i\right) = mg\Delta y$ We assign the final point y_f to be the "generic"

point y on the y-axis whose potential is $U(y)$. $\rightarrow$ $U(y) - U_i = mg\left(y - y_i\right)$

Since only changes in the potential are physically menaingful, this allows us

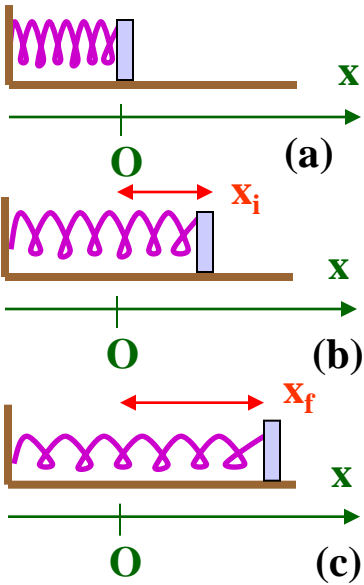to define arbitrarily y_i and U_i The most convenient choice is:

$y_i = 0$, $U_i = 0$ This particular choice gives:

$$\boxed{U(y) = mgy}$$

Potential Energy of a spring:

Consider the block-mass system shown in the figure.

The block moves from point x_i to point x_f. At the same time the spring force does work W on the block which changes the potential energy of the block-spring system by an amount

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx = -k\int_{x_i}^{x_f} xdx \qquad \Delta U = -W \rightarrow$$

$$\Delta U = k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = \frac{kx_f^2}{2} - \frac{kx_i^2}{2}$$

We assign the final point x_f to be the "generic"

point x on the x-axis whose potential is $U(x)$. $\rightarrow$ $U(x) - U_i = \dfrac{kx^2}{2} - \dfrac{kx_i^2}{2}$

Since only changes in the potential are physically menaingful, this allows us to define arbitrarily x_i and U_i The most convenient choice is:

$y_i = 0$, $U_i = 0$ This particular choice gives:

$$\boxed{U = \frac{kx^2}{2}}$$

Problem 6. A .50 kg snowball is fired from a cliff 12.5 m high. The snowball initial velocity is 14.0 m/s, directed 41.0° Above the horizontal. (a) How much work is done on the snowball by the gravitational force during its flight to the flat ground below the cliff? (b) What is the change in the gravitational potential energy of the snowball-Earth system during the flight? (c) If that gravitational potential energy is taken to be zero at the height of the cliff, what is its value when the snowball reaches the ground?

(a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50 \text{ kg})(9.80 \text{ m/s}^2)(12.5 \text{ m}) = 184 \text{ J} .$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184$ J.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184$ J when the snowball hits the ground.

Problem 7. A thin rod of length L=2.00 m and negligible mass is pivoted about one end and is capable to rotate in a vertical circle. A ball of mass 5.00 kg is attached to the other end. The rod is pulled aside to angle 30.0° and released with initial velocity v_0=0. AS the ball descends to the lowest point, (a) how much work does the gravitational force do on it, and (b) what is the change in the gravitational potential energy of the ball-Earth system? (c) if the gravitational potential energy is taken to be zero at the lowest point, what is its value just as the ball is released? (d) Do the magnitudes of the answers to (a) through © increase, decrease, or remain the same if angle θ is increased?

Lcosθ

θ

L

h=L-Lcosθ

The main challenge in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L \cos \theta$ (for angle θ measured from vertical). Once this relation is established, then the principal results of this problem follow from $W_g = mgd\cos\varphi$ and $U = mgy$

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL(1 - \cos \theta)$$
$$= (5.00 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})(1 - \cos 30°) = 13.1 \text{ J}$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos \theta) = -13.1 \text{ J}$.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos \theta) = 13.1 \text{ J}$.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos \theta$ decreases so that $1 - \cos \theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b also increases.
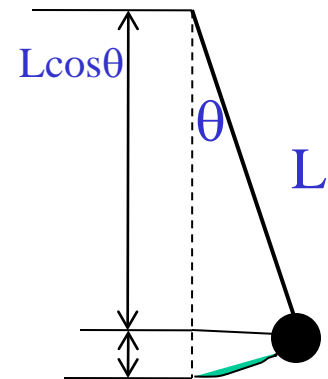
Conservation of Mechanical Energy:

Mechanical energy of a system is defined as the sum of potential and kinetic energies $E_{mech} = K + U$ We assume that the system is isolated i.e. no external forces change the energy of the system. We also assume that all the forces in the system are conservative. When an interal force does work W on an object of the system this changes the kinetic energy by $\Delta K = W$ (eqs.1)   This amount of work also changes the potential energy of the system by an amount $\Delta U = -W$ (eqs.2) If we compare equations 1 and 2 we have: $\Delta K = -\Delta U \rightarrow$ $K_2 - K_1 = -\left(U_2 - U_1 \right) \rightarrow K_1 + U_1 = K_2 + U_2$ This equation is known as the principle of conservation of mechanical energy. It can be summarized as:

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$$

For an isolated system in which the forces are a mixture of conservative and non conservative forces the principle takes the following form

$$\boxed{\Delta E_{mech} = W_{nc}}$$

Here, W_{nc} is defined as the work of all the non-consrvative forces of the system

The Work-Energy Theorem

$$W_c + W_{nc} = 1/2mv_f^2 - 1/2mv_0^2$$

$$W_c = W_{gravity} = mg(h_0 - h_f)$$

$$mg(h_0 - h_f) + W_{nc} = 1/2mv_f^2 - 1/2mv_0^2$$

$$W_{nc} = (1/2mv_f^2 - 1/2mv_0^2) + (mgh_f - mgh_0)$$

$$W_{nc} = (KE_f - KE_0) + (PE_f - PE_0)$$

Net work done by nonconserv. forces

Change to kinetic energy

Change in potential energy

$$W_{nc} = \Delta KE + \Delta PE$$

The conservation of mechanical energy:

$$E = KE + PE \quad - \quad \text{total mechanical energy}$$

Conservation of Mechanical Energy

• $E = KE + PE$ total mechanical energy

• $W_{nc} = (KE_f - KE_0) + (PE_f - PE_0) = (KE_f + PE_f) - (KE_0 + PE_0)$

The net work done by external nonconservative forces changes the total mechanical energy from an initial value E_0 to a final value of E_f

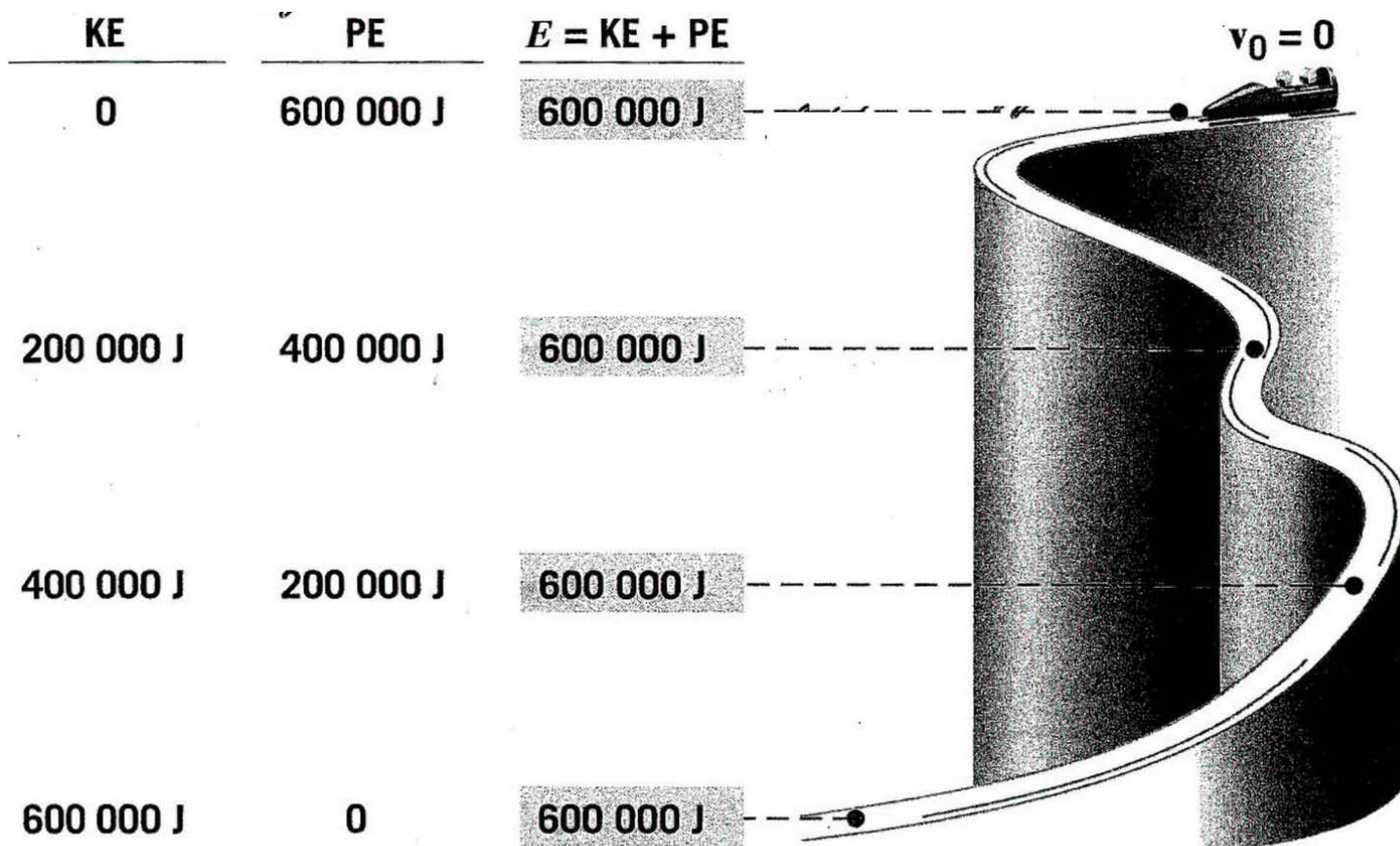• Suppose $W_{nc} = 0$ => $E_f = E_0$ $1/2 mv_f^2 + mgh_f = 1/2 mv_0 + mgh_0$

total mechanical energy conserved

The total mechanical energy ($E = KE + PE$) of an object remains constant as the object moves provided that the net work done by external nonconservative forces is zero, $W_{nc} = 0$ J

Conservation of Mechanical Energy

$E = KE + PE = $ constant if $W_{nc} = 0$

If friction and wind resistance are ignored, a bobsled run illustrates how kinetic and potential energy can be interconverted while the total mechanical energy remains constant. The total mechanical energy is all potential energy at the top and all kinetic energy at the bottom.

KE	PE	$E = KE + PE$	$v_0 = 0$
0	600 000 J	600 000 J	
200 000 J	400 000 J	600 000 J	
400 000 J	200 000 J	600 000 J	
600 000 J	0	600 000 J	

An example of the principle of conservation of mechanical energy is given in the figure. It consists of a pendulum bob of mass m moving under the action of the gravitational force
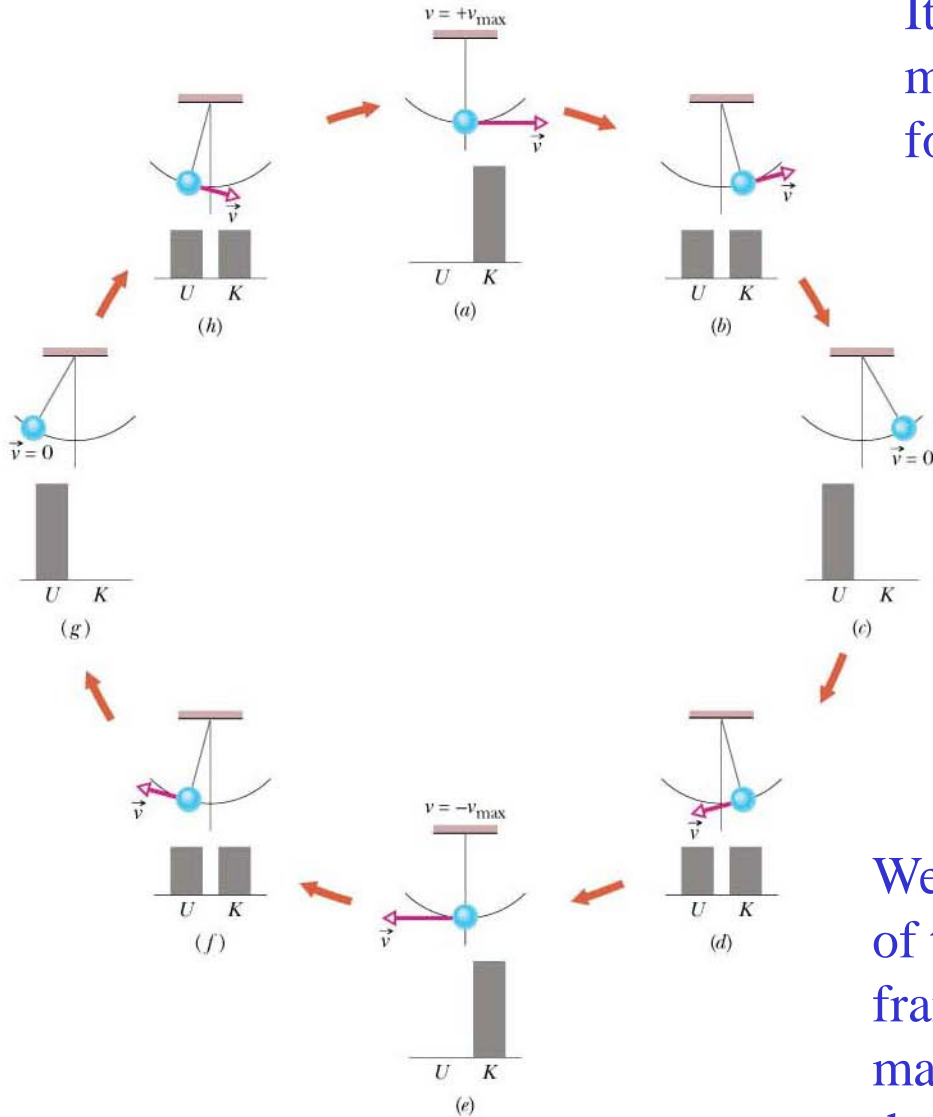
The total mechanical energy of the bob-earth system remains constant. As the pendulum swings, the total energy E is transferred back and forth between kinetic energy K of the bob and potential energy U of the bob-earth system

We assume that U is zero at the lowest point of the pendulum orbit. K is maximum in frame a, and e (U is minimum there). U is maximum in frames c and g (K is minimum there)

A golf ball of mass 50g released from a height of 1.5m above a concrete floor bounces back to a height of 1.0m

a) **What is the kinetic energy of the ball just before contact with the floor begins? Ignore air friction.**

b) **What is the kinetic energy of the ball just after contact with the floor ends?**

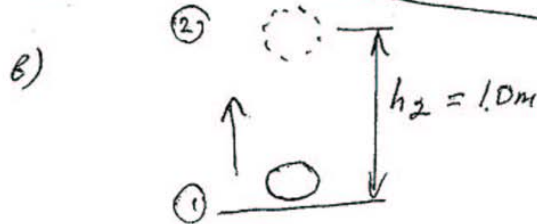c) **What is the loss of energy during contact with the floor?**

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a) Given:
$m = 0.05 \, kg$
$h_1 = 1.5 \, m$
$h_2 = 1$

h_1
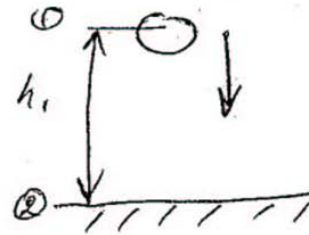
① $K_1 = 0$ $U_i = mgh_1$
② $K_2 = \frac{mv^2}{2}$ $U_2 = 0$
③ $K_1 + U_1 = K_2 + U_2$ $\Rightarrow K_2 = U_1 = mgh_1 =$
$= 0.05 \times 9.8 \times 1.5 = \boxed{0.74 \, J}$

b)

$h_2 = 1.0 \, m$

① $K_1 = ?$ $U_i = 0$
② $K_2 = 0$ $U_2 = mgh_2$
③ $K_2 + U_2 = K_1 + U_1 \Rightarrow K_1 = U_2 = mgh_2 =$
$= (0.05)(9.8)(1.0) = \boxed{0.49 \, J}$

c) $E_{loss} = K_{2a} - K_{2b} = 0.74 - 0.49 = \boxed{0.25 \, J}$

A pendulum consists of a mass "m" tied to one end of a string of length "l". The other end of the string is attached to a fixed point on the wall. Suppose that the pendulum is initially held at an angle of 90° with the vertical. If the pendulum is released from this position, what will be the speed of the pendulum at the instant it passes through its lowest point?

Given:

$m, \ell, K_1 = 0$

Find v?
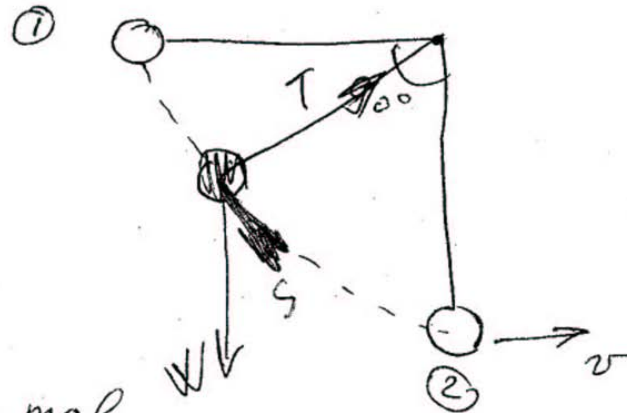
① $K_1 = 0; \quad U_1 = mg\ell$

② $K_2 = \frac{mv^2}{2}; \quad U_2 = 0$

③ $K_2 + U_2 = K_1 + U_1$

$\frac{mv^2}{2} + 0 = 0 + mg\ell$

$v = \sqrt{2g\ell}$

In a rollercoaster, a car starts on the top of a 30m high mountain. It rolls down into a valley and then up a 20m high mountain. What is the speed of the car at the bottom of the valley, at ground level? What is the speed of the car at the top of the second mountain?
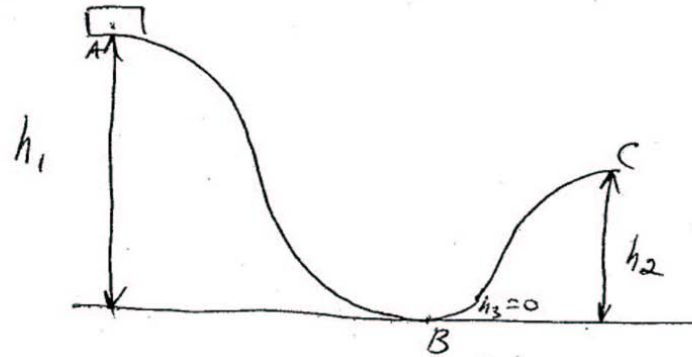
Given:
$h_1 = 30m$
$h_2 = 20m; h_3 = 0$

Find

1) $v_B = ?$

2) $v_C = ?$

1) $A: \quad K_A = 0; \quad U_A = mg\,h_1$

        $B: \quad K_B = \dfrac{m v_B^2}{2}; \quad U_B = mg\,h_3 = 0$

$$K_A + U_A = K_B + U_B \quad \Rightarrow \quad K_B = mg\,h_1$$

$$\frac{m v_B^2}{2} = mg\,h_1 \quad ; \quad \Rightarrow \quad v_B = \sqrt{2g h_1} =$$

$$= \sqrt{2 \cdot 9.8 \cdot 30} = \boxed{24 \ ^m/_s}$$

2) $A: \quad K_A = 0; \quad U_A = mg\,h_1$

        $C: \quad K_C = \dfrac{m v_C^2}{2}; \quad U_C = mg\,h_2$

$$K_A + U_A = K_C + U_C \quad ; \quad \Rightarrow \quad K_C = U_A - U_C = mg(h_1 - h_2)$$

$$\Rightarrow \frac{m v_C^2}{2} = mg(h_1 - h_2); \quad v_C = \sqrt{2g(h_1 - h_2)} = \boxed{14 \ ^m/_s}$$

A block of mass m slides down an inclined plane into a loop of radius R. a) Neglecting friction, what is the minimum speed the block must have at the highest point of the loop to stay in the loop. b) At what vertical height on the inclined plane (in terms of radius of the loop) must the block be released if it is to have the required minimum speed at the top of the loop?

1)

$$N + mg = \frac{mv^2}{R}$$

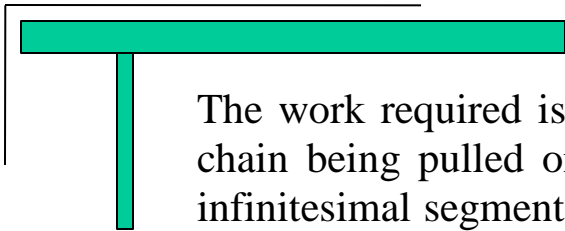$$N = 0 \quad \text{for} \quad V_{min} \Rightarrow mg = \frac{mV_{min}^2}{R}$$

$$\boxed{V_{min} = \sqrt{Rg}}$$

2) $mgh = E_0 \qquad E_0 = E_t$
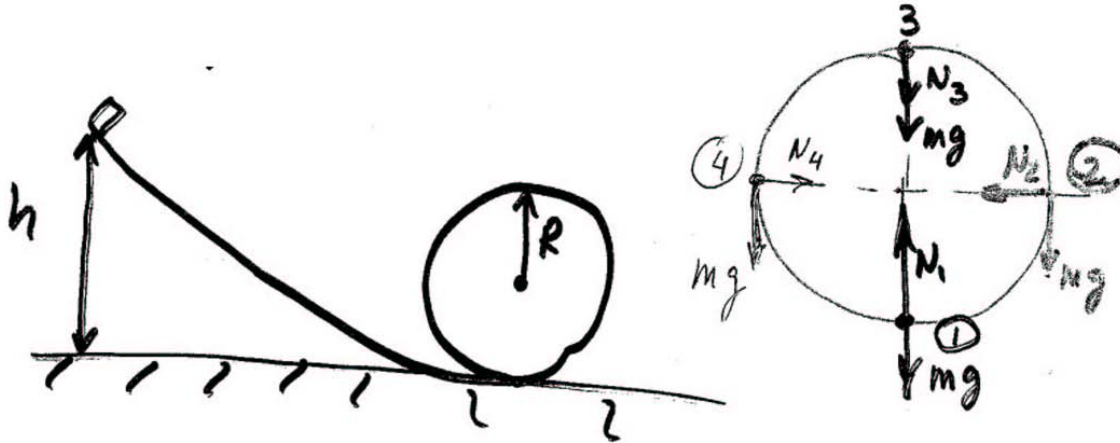
$$\frac{mv^2}{2} + mg \, 2R = E_t$$

$$mgh = \frac{m Rg}{2} + mg \, 2R \quad \boxed{h = 2.5R}$$

Problem 32. A chain is held on a frictionless table with one forth of its length hanging over the edge. If the chain has length L=28 cm and mass m=0.012 kg, how much work is required to put the hanging part back into the table.

The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is
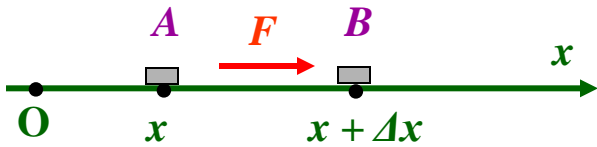
$$dU = (m/L)g|y|\,dy = -(m/L)gy\,dy$$

since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Box U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32.$$

The work required to pull the chain onto the table is therefore

$$W = \Delta U = mgL/32 = (0.012\text{ kg})(9.8\text{ m/s}^2)(0.28\text{ m})/32 = 0.0010\text{ J}.$$

Finding the Force $F(x)$ analytically from the potential energy $U(x)$

Consider an object that moves along the x-axis under the influence of an unknown force F whose potential energy U(x) we know at all points of the x-axis. The object moves from point A (coordinate x) to a close by point B (coordinate x + Δx). The force does work W on the object given by the equation:

$W = F\Delta x$ eqs.1

The work of the force changes the potential energy U of the system by the amount:

$\Delta U = -W$ eqs.2   If we combine equations 1 and 2 we get:

$F = -\dfrac{\Delta U}{\Delta x}$ We take the limit as $\Delta x \to 0$ and we end up with the equation:

$$F(x) = -\dfrac{dU(x)}{dx}$$

The potential Energy Curve

If we plot the potential energy U versus x for a force F that acts along the x-axis we can get a wealth of information about the motion of a particle on which F is acting. The first parameter that we can determine is the force $F(x)$ using the equation:

$$F(x) = -\frac{dU(x)}{dx}$$
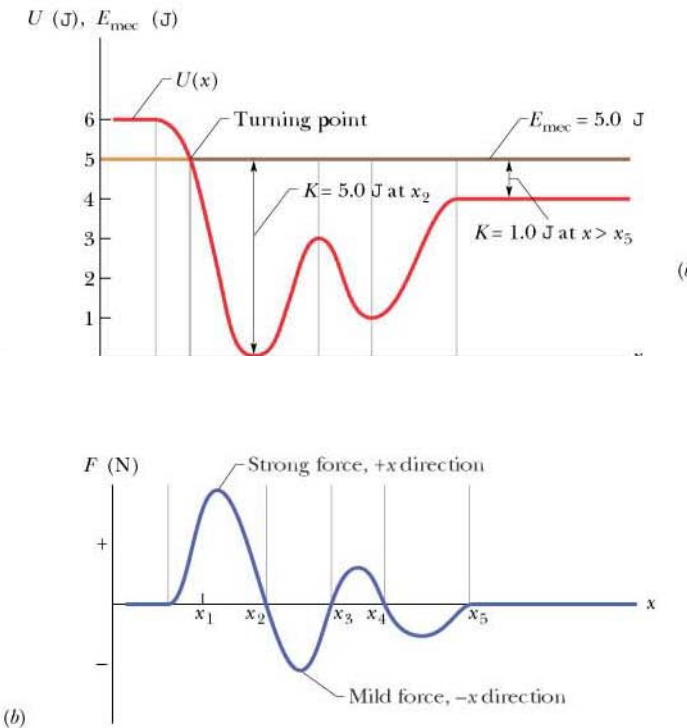
An example is given in the figures below.
In fig.a we plot $U(x)$ versus x.
In fig.b we plot $F(x)$ versus x.

For example at x_2, x_3 and x_4 the slope of the $U(x)$ vs x curve is zero, thus $F = 0$.

The slope dU/dx between x_3 and x_4 is negative; Thus $F > 0$ for the this interval.
The slope dU/dx between x_2 and x_3 is positive; Thus $F < 0$ for the same interval
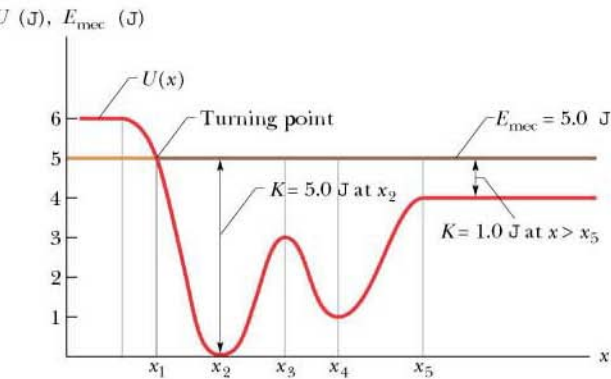
Turning Points:

The total mechanical energy is $E_{mec} = K(x) + U(x)$

This energy is constant (equal to 5 J in the figure) and is thus represented by a horizontal line. We can solve this equation for $K(x)$ and get:

$K(x) = E_{mec} - U(x)$ At any point x on the x-axis we can read the value of $U(x)$. Then we can solve the equation above and determine K

From the definition of $K = \dfrac{mv^2}{2}$ the kinetic energy cannot be negative.

This property of K allows us to determine which regions of the x-axis motion is allowed. $K(x) = K(x) = E_{mec} - U(x)$

If $K > 0 \rightarrow E_{mech} - U(x) > 0 \rightarrow U(x) < E_{mec}$ Motion is allowed

If $K < 0 \rightarrow E_{mech} - U(x) < 0 \rightarrow U(x) > E_{mec}$ Motion is forbidden

The points at which: $E_{mec} = U(x)$ are known as turning points for the motion. For example x_1 is the turning point for the U versus x plot above. At the turning point $K = 0$

U (J), E_{mec} (J)

6
5
4
3
2
1

x_1 x_2 x_3 x_4 x_5

x

(c)
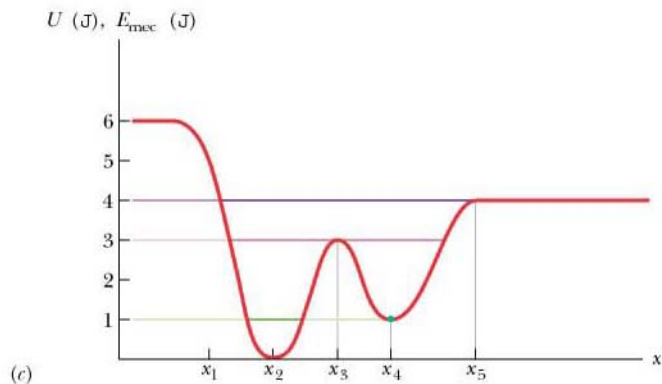
Given the $U(x)$ versus x curve the turning points and the regions for which motion is allowed depends on the value of the mechanical energy E_{mec}

In the picture to the left consider the situation when $E_{mec} = 4$ J (purple line) The turning points ($E_{mec} = U$) occur at x_1 and $x > x_5$. Motion is allowed for $x > x_1$ If we reduce E_{mec} to 3 J or 1 J the turning points and regions of allowed motion change accordingly.

Equilibrium Points: A position at which the slope $dU/dx = 0$ and thus $F = 0$ is called an equilibrium point. A region for which $F = 0$ such as the region $x > x_5$ is called a region of **neutral equilibrium**. If we set $E_{mec} = 4$ J the kinetic energy K = 0 and any particle moving under the influence of U will be stationary at any point with $x > x_5$

Minima in the U versus x curve are positions of **stable equilibrium**
Maxima in the U versus x curve are positions of **unstable equilibrium**

(c)
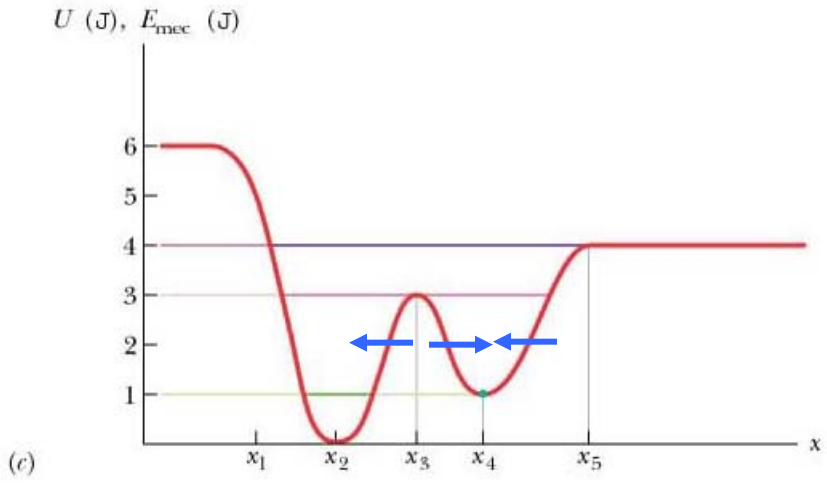
Note: The blue arrows in the figure indicate the direction of the force F as determined from the equation:

$$F(x) = -\frac{dU(x)}{dx}$$

Positions of Stable Equilibrium. An example is point x_4 where U has a minimum. If we arrange $E_{mec} = 1$ J then $K = 0$ at point x_4. A particle with $E_{mec} = 1$ J is stationary at x_4. If we displace slightly the particle either to the right or to the left of x_4 the force tends to bring it back to the equilibrium position. This equilibrium is stable.

Positions of Unstable Equilibrium. An example is point x_3 where U has a maximum. If we arrange $E_{mec} = 3$ J then $K = 0$ at point x_3. A particle with $E_{mec} = 3$ J is stationary at x_3. If we displace slightly the particle either to the right or to the left of x_3 the force tends to take it further away from the equilibrium position. This equilibrium is unstable
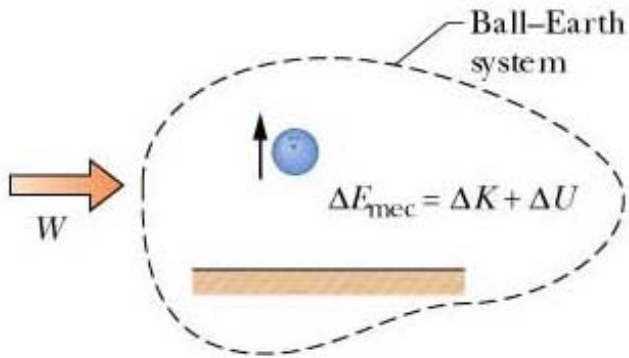
Work done on a System by an External Force (no friction involved)

Ball–Earth system

$$\Delta E_{mec} = \Delta K + \Delta U$$

W

Up to this point we have considered only isolated systems in which no external forces were present. We will now consider a system in which there are forces external to the system

The system under study is a bowling ball being hurled by a player. The system consists of the ball and the earth taken together. The force exerted on the ball by the player is an external force. In this case the mechanical energy E_{mec} of the system is not constant. Instead it changes by an amount equal to the work W done by the external force according to the equation:

$$W = \Delta E_{mec} = \Delta K + \Delta U$$

Work done on a System by an External Force (friction involved)
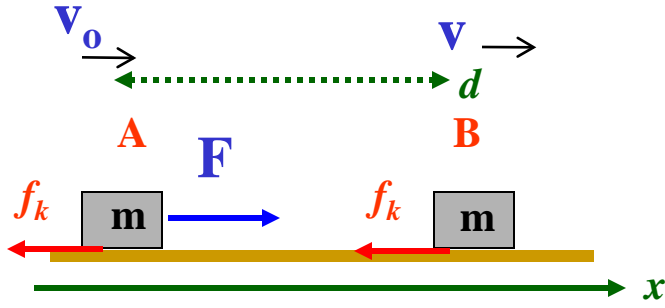
- 2nd Newton's Law for x component $F-f_k=ma$

- Since a=const, $v^2 = v_o^2 +2ad$, solving for a and substituting the result in Newton's Eq.

$$Fd=1/2mv^2-1/2mv_o^2+f_kd=\Delta K+f_kd$$

- Through experiment, we find that the increase in thermal energy by sliding : $\mathbf{\Delta E_{th}=f_kd}$

- Fd is the work W done by external force F on the block floor system

- Hence, $\mathbf{W=\Delta E_{mec}+\Delta E_{th}}$ energy statement for the work done on a system by an external force when friction is involved.

Problem 42. A worker pushed a 27 kg block 9.2 m along a level floor at constant speed with a force directed 32 degree below the horizontal. If the coefficient of kinetic friction between block and floor was 0.20, what were (a) the work done by the worker's force and (b) the increase in thermal energy of the block-floor system?

. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where $\theta = 32°$) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, implying

$$F_N = F\sin\theta + mg, \text{ on the other hand } F\cos\theta = \mu_k F_N$$

which is solved to find $F = 71$ N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd \cos\theta = (71\,\text{N})(9.2\,\text{m})\cos 32° = 5.6 \times 10^2\,\text{J} .$$

(b) Since $f_k = \mu_k (mg + F \sin \theta)$, we find $\Delta E_{th} = f_k d = (60\,\text{N})(9.2\,\text{m}) = 5.6 \times 10^2\,\text{J}.$

Conservation of Energy

Energy cannot magically appear or disapear. It can be transferred to or from objects and systems by means of work done on a system.

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W=\Delta E=\Delta E_{mec}+\Delta E_{th}+\Delta E_{int}$$

The law of conservation of energy is not something we have derived from basic physics principles. Rather it is a law based on countless experiments. We have never found an exception to it.

Isolated System

If a system is isolated from its environment, there can be no energy transfers to or from it. **The total energy E of an isolated system cannot change.**

Problem 61. A stone with a weight of 5.29 N is launched vertically from ground level with an initial speed of 20.0 m/s, and the air drag on it is 0.265 N throughout the flight. What are (a) the maximum height reached by the stone and (b) its speed just before it hits the ground?

(a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{th} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{th} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with $m = (5.29 \text{ N})/(9.80 \text{ m/s}^2)=0.54$ kg,

$$h = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(1+0.265/5.29)} = 19.4 \text{ m}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$\frac{2fv_0^2}{2g(1 + f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1 + f/w)} = v_0^2 - \frac{2fv_0^2}{w(1 + f/w)} = v_0^2\left(1 - \frac{2f}{w + f}\right) = v_0^2\frac{w - f}{w + f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w - f}{w + f}} = (20.0 \text{ m/s})\sqrt{\frac{5.29 \text{ N} - 0.265 \text{ N}}{5.29 \text{ N} + 0.265 \text{ N}}} = 19.0 \text{ m/s}.$$