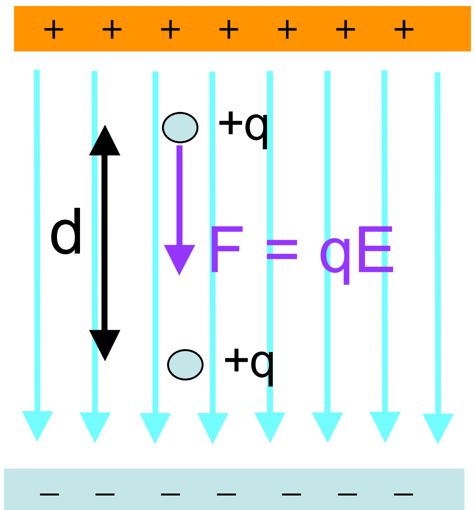
Potential Energy of a system of charges



Potential Energy PE (scalar):

ΔPE = – Work done by the Electric field

$$\Delta PE = -W = -Fd = -qEd$$

(units = J)

Work done by the E-field (to move the +q closer to the negative plate) REDUCES the P.E. of the system

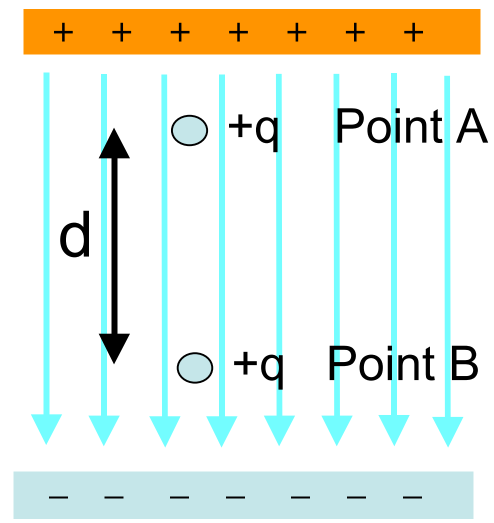
If a positive charge is moved AGAINST an E-field (which points from + to -), the charge-field system gains Pot. Energy. If a negative charge is moved against an E-field, the system loses potential energy

Electric Potential Difference, ΔV

$\Delta V = V_B - V_A = \Delta PE / q$

Units: Joule/Coulomb = VOLT

Scalar quantity



Relation between ΔV and E:

$\Delta V = Ed$

E has units of V/m = N/C

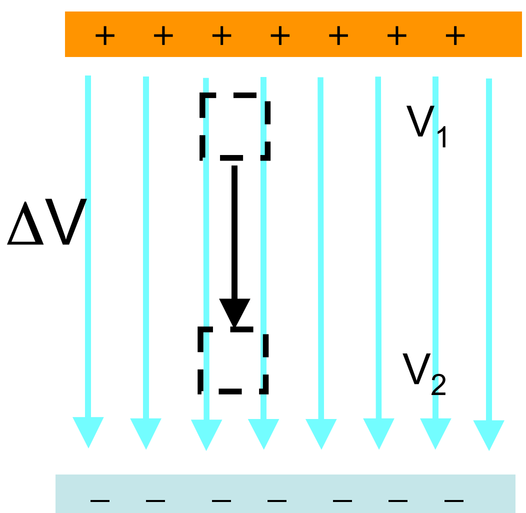
(V / m = J / Cm = Nm / Cm = N / C)

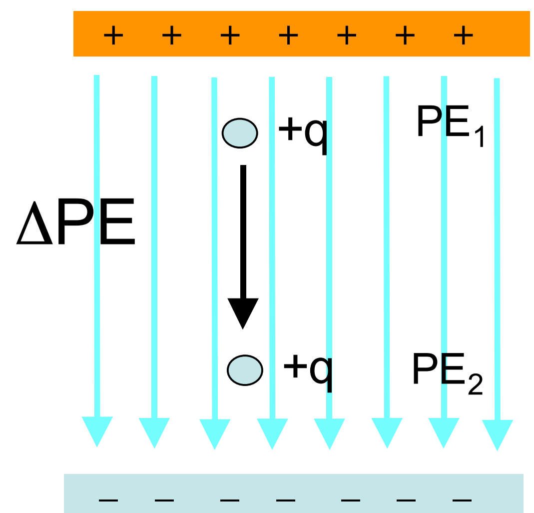
Potential vs. Potential Energy

POTENTIAL: Property of space due to charges; depends only on location

Positive charges will accelerate towards regions of low potential.

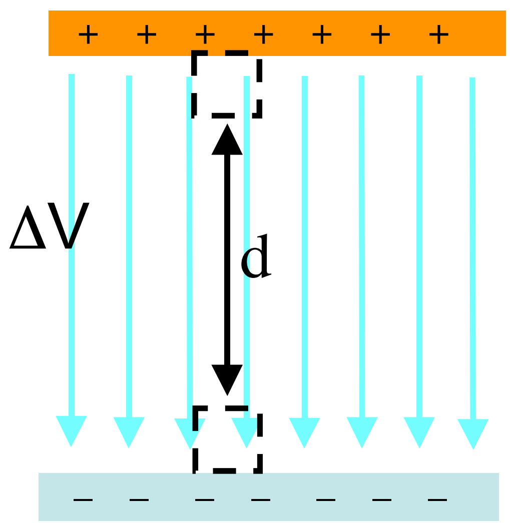
POTENTIAL ENERGY: due to the interaction between the charge and the electric field





Example of Potential Difference

A parallel plate capacitor has a constant electric field of 500 N/C; the plates are separated by a distance of 2 cm. Find the potential difference between the two plates.



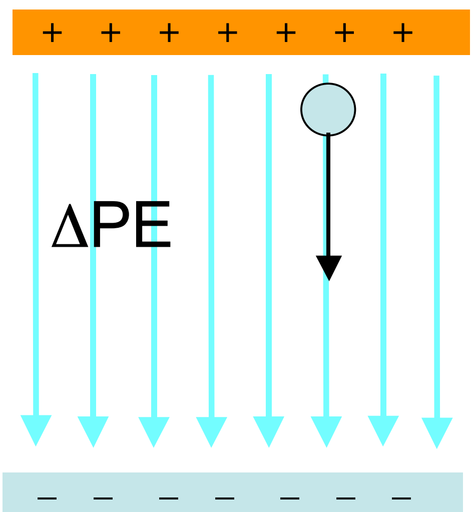
E-field is uniform, so we can use
$\Delta V = Ed = (500V/m)(0.02m) = 10V$

Remember: potential difference ΔV does not depend on the presence of any test charge in the E-field!

Example of Potential Difference

Now that we've found the potential difference ΔV, let's take a molecular ion, CO_2^+ (mass = 7.3×10^{-26} kg), and release it from rest at the anode (positive plate). What's the ion's final velocity when it reaches the cathode (negative plate)?

Solution: Use conservation of energy: $\Delta PE = \Delta KE$



$\Delta PE = \Delta V\, q$

$\Delta KE = 1/2\ m\ v_{final}^2 - 1/2\ m\ v_{init}^2$

$\Delta V\, q = 1/2\ m\ v_{final}^2$

$v_{final}^2 = 2\Delta Vq/m = (2)(10V)(1.6\times10^{-19}C)/7.3\times10^{-26}$ kg

$v_{final} = 6.6\times10^3$ m/s

Thunderstorms:

From ground to cloud base:
$\Delta V = 10^8$ V, $E \sim 10^{4\text{-}5}$ V/m

Lightning: $E = 3 \times 10^6$ V/m is electric field strength at which air becomes ionized enough to act as a conductor.

Fair weather: $E \sim 10^2$ V/m

Example for Potential Difference:
BATTERIES: the potential difference across terminals is kept at a constant value, e.g., 9 volts

16.2: V and PE due to point charges

Recall that $\vec{E} = \frac{k_e q}{r^2}$ for a single point charge.

Potential $V = \frac{k_e q}{r}$

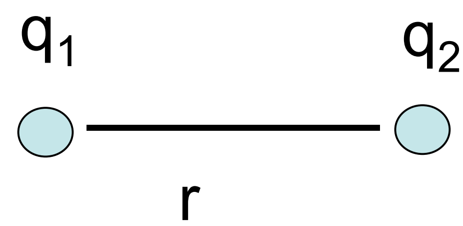
V is defined such that $V=0$ at $r=\infty$

Dimensional arguments: For a constant field, $V = Ed$

For a point charge, V has units of $E \times$ distance

P.E. of two point charges

Define V_{21} as the potential due to the presence of charge q_2 at the location of charge q_1.



$$V_{21} = \frac{k_e q_2}{r}$$

$$PE = q_1 V_{21} = q_2 V_{12}$$
$$PE = 0 \quad \text{at} \quad \mathrm{r} = \infty$$

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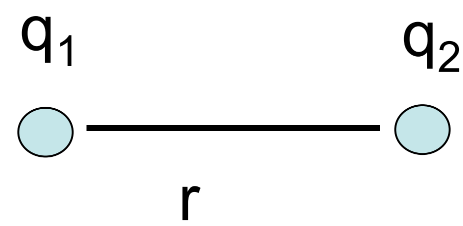


$$V_{21} = \frac{k_e q_2}{r}$$

$$PE = q_1 V_{21} = q_2 V_{12}$$
$$PE = 0 \quad \text{at} \quad \mathrm{r} = \infty$$

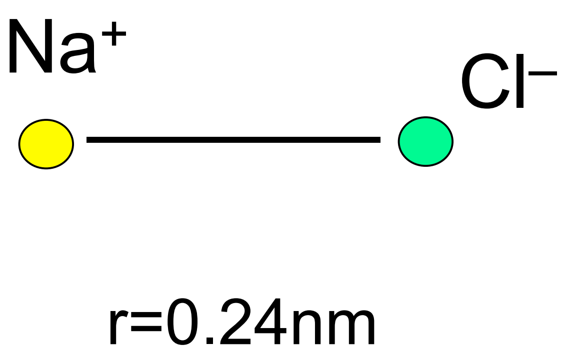
Reminder: V & PE are scalar (not vector) quantities

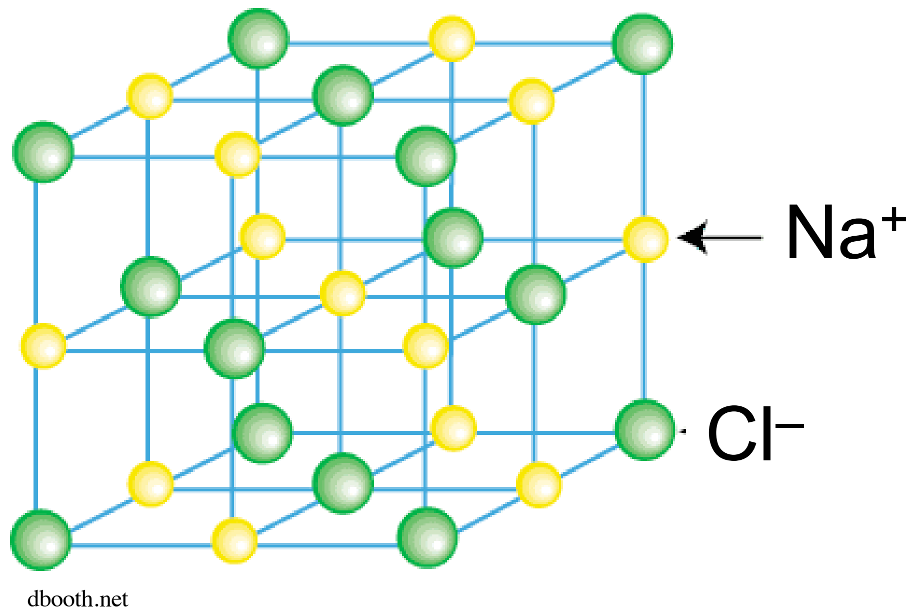
Point charge q_1:	$\vec{E} = k_e q_1 / r^2$	
Point charge q_1:	$V = k_e q_1 / r$	$(= \vec{E}\ r)$
Point charge q_1, introduce q_2:	$\mathrm{PE} = k_e q_1 q_2 / r$	$(= V\ q_2\ )$
Point charge q_1, introduce q_2:	$\vec{F_E} = k_e q_1 q_2 / r^2$	$(= \vec{E}\ q_2)$

Note:
When two LIKE charges are close together, the potential energy is positive (the higher the PE, the more likely the system is to come apart)
When two UNLIKE charges are close together, the potential energy is negative (the lower the PE, the more stable the system is)

V & PE of atoms in a crystal lattice

In a crystal of salt (Na^+ & Cl^-) the distance between the ions is 0.24 nm. Find the potential due to Cl^- at the position of the Na^+ ion. Find the electrostatic energy of the Na^+ due to the interaction with Cl^-.





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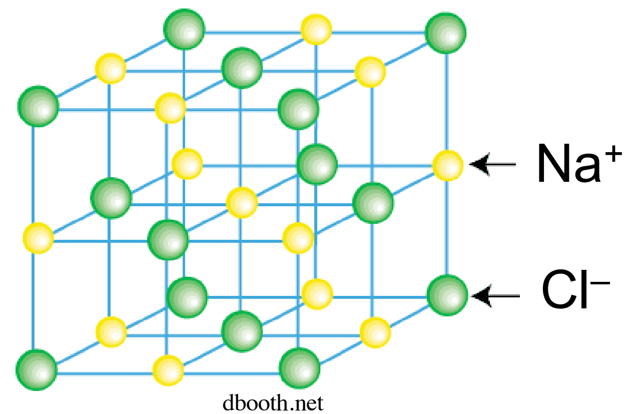
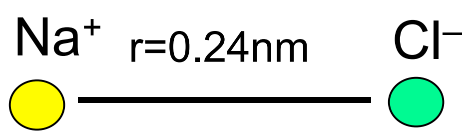




$V = k_e q/r = 9\times10^9 Nm^2/C^2\,(-1.6\times10^{-19}C) / (0.24\times10^{-9}\ m) = -6.0\ V$

$PE = qV = (1.6\times10^{-19}C)(-6.0V) = -9.6\times10^{-19}\ J$

ELECTRON VOLT (convenient unit for atomic physics)

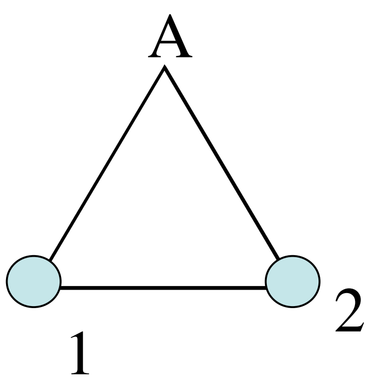
$1eV = 1.6\times10^{-19}\ J$

So PE = –6.0 eV   (energy in eV is V × the charge in units of *e*)

V for a distribution of charges

Potential is a scalar: Total V at point A dur to other charges = $V_{1A} + V_{2A} + V_{3A} + \ldots$

Two charges of +q each are placed at corners of an equilateral triangle, with sides of 10 cm. If the electric field due to each charge at point A is 100 V/m, find the total potential at A.



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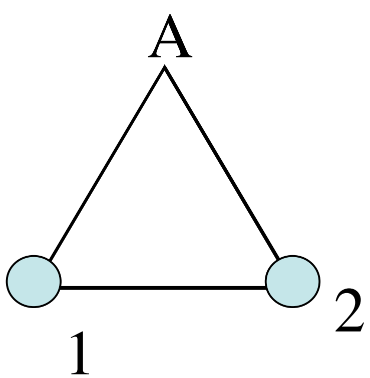


$\vec{E} = k_e q / r^2$ generated by each charge

V due to each: $k_e q / r = Er = (100\text{V/m})(0.10\text{m}) = 10\text{ V}$

$V_{total} = V_{1A} + V_{2A} = 10\text{ V} + 10\text{ V} = 20\text{ V}$

Total Energy of a charge distribution

Suppose you have two charges, each +q, held fixed a distance d apart. When released, they'll want to fly away from each other and head out to infinity.

$PE_{init} = \frac{k_e q_1 q_2}{d}$ (positive value)

$PE_{final} = \frac{k_e q_1 q_2}{\infty}$

Note: PE_{init} = high ; PE_{final} = zero.

$\Delta PE = PE_{final} - PE_{init}$ = negative

Work = $-\Delta PE$ = positive:

(this is work done BY the E-fields to separate the charges)

Total energy of system = amount of work needed to assemble the system

Total Energy of a charge distribution

Now let’s do the opposite situation: We will bring in two charges, each of +1q, from r=∞ to r=d. How much work will be required (by us) to overcome the repelling E-fields?

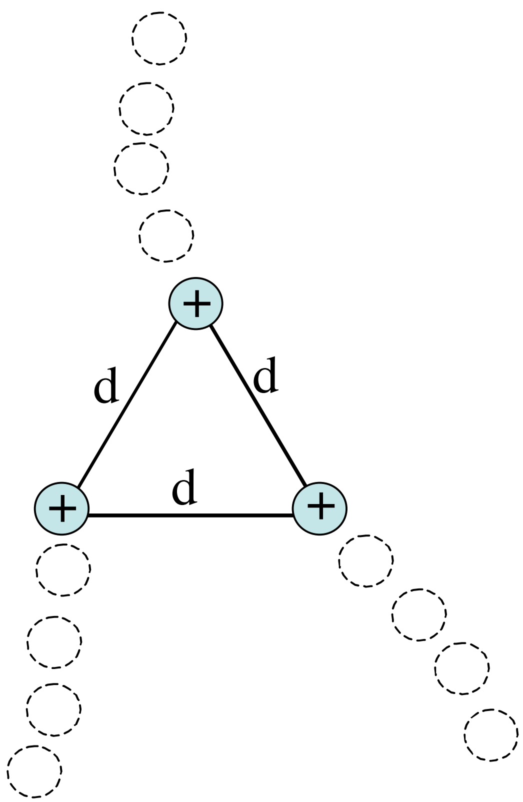
$$\Delta PE = PE_{final} - PE_{init} = \frac{k_e q_1 q_2}{d} - \cancelto{0}{\frac{k_e q_1 q_2}{\infty}}$$

(work done by E-field = –ΔPE is negative because the E-fields made “negative progress” in trying to separate the charges)

Total energy of system= amount of work needed (by us) to assemble the system = amount of energy stored in a chemical bond, for instance

Total energy of a charge distribution

Suppose you wish to bring in THREE protons, from infinity to the corners of an equilateral triangle with sides having length 1 nm. How much work is required (by us) to accomplish this?



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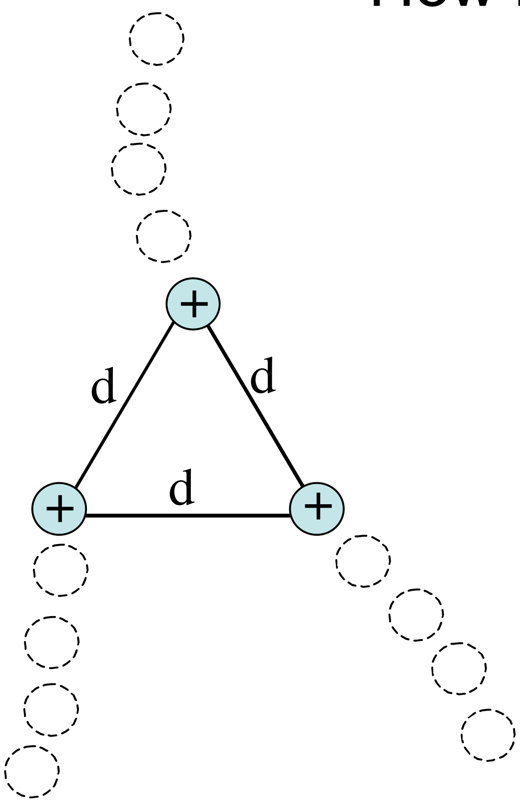
Suppose you wish to bring in THREE protons, from infinity to the corners of an equilateral triangle with sides having length 1 nm. How much work is required (by us) to accomplish this?

How many interactions? 3



Total energy of a charge distribution

Suppose you wish to bring in THREE protons, from infinity to the corners of an equilateral triangle with sides having length 1 nm. How much work is required (by us) to accomplish this?

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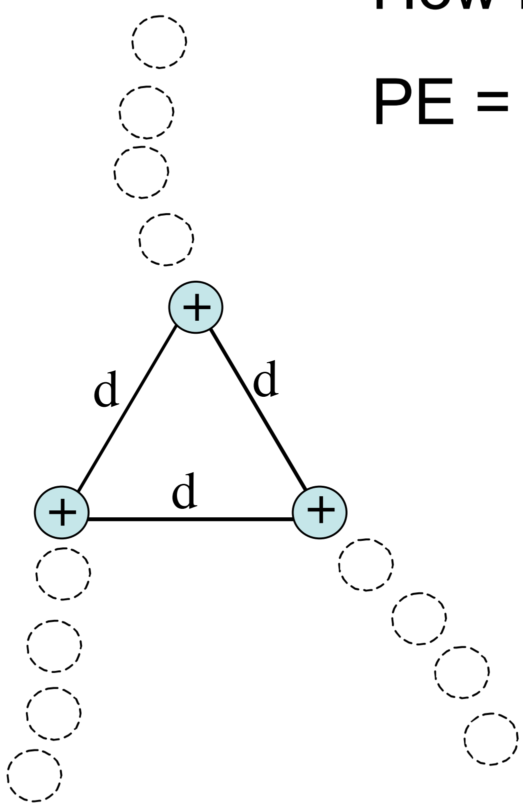
$$PE = PE_{12}+PE_{13}+PE_{23} = 3\,(k_e\, q_1\, q_2 / r) = 3k_e q^2 / r$$



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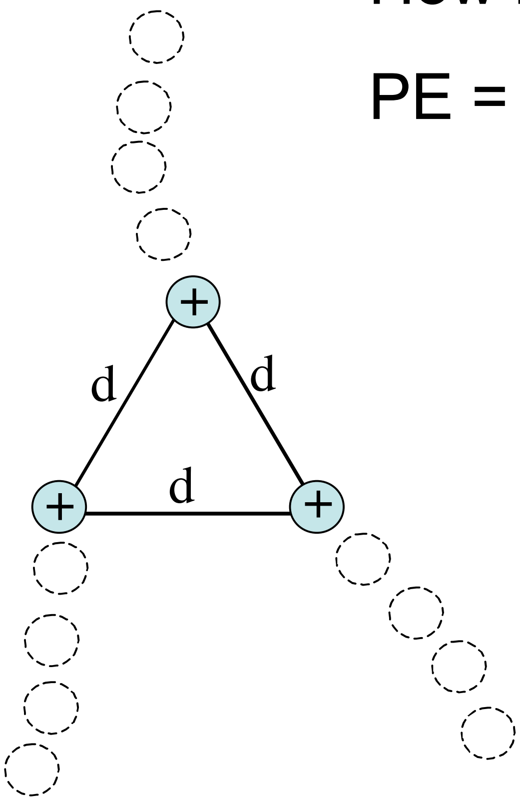


$$PE = \frac{3\,(9 \times 10^{9}\ \mathrm{Nm^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})^2}{10^{-9}\ \mathrm{m}}$$

$$PE = 6.9 \times 10^{-19}\ \mathrm{J}$$

($1\mathrm{eV} = 1.6 \times 10^{-19}$ J, so this PE = 4.32 eV)

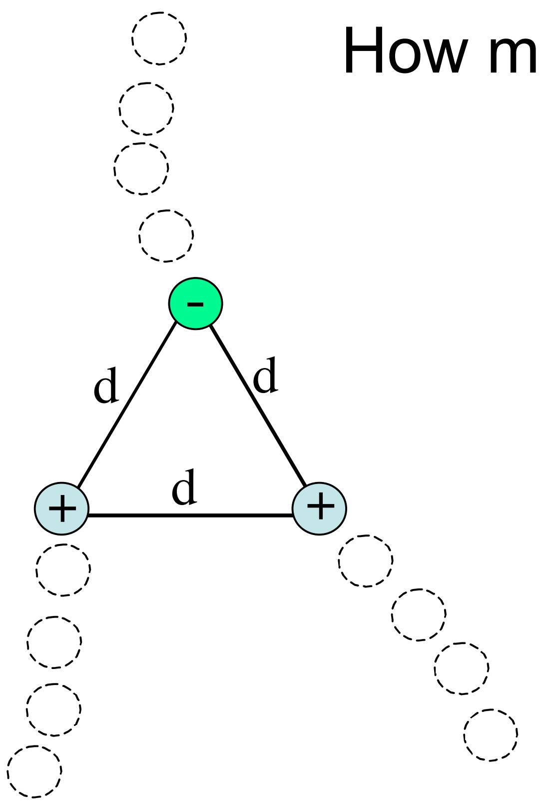
Total energy of a charge distribution

Suppose instead of 3 protons, you have 2 protons and 1 electron.

Now how much work would be required by us?

Note: pay attention to signs of each charge!

How many interactions? 3



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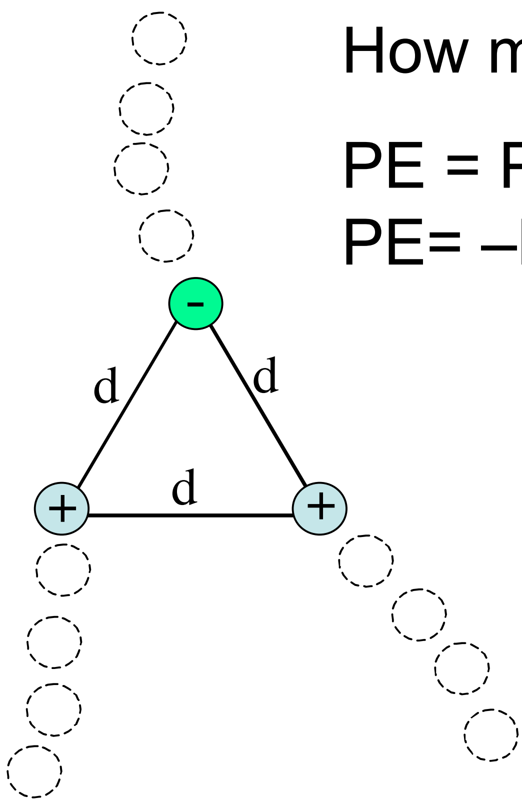

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$$PE = PE_{12}+PE_{13}+PE_{23} = +(k_e q^2/r) - (k_e q^2/r) - (k_e q^2/r)$$

$$PE = -k_e\, q^2/r$$

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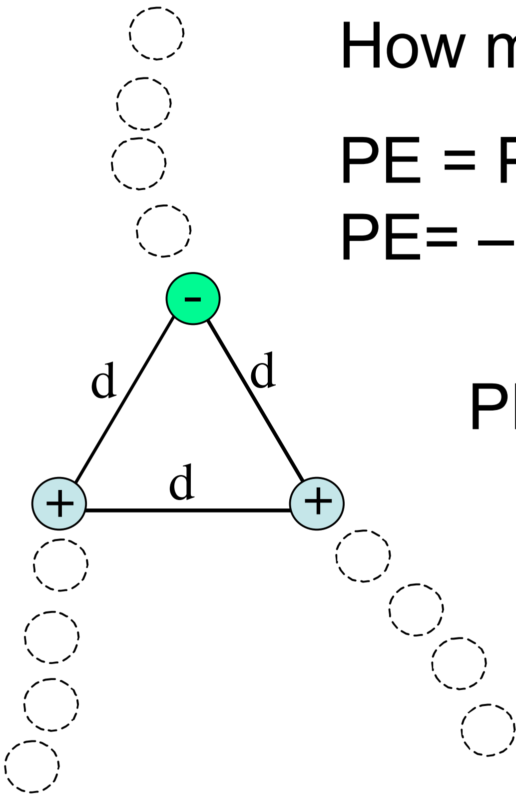

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$$PE = -k_e q^2/r$$

$$PE = \frac{-1\ (9 \times 10^9\ Nm^2/C^2)(1.6 \times 10^{-19}\ C)^2}{10^{-9}\ m}$$

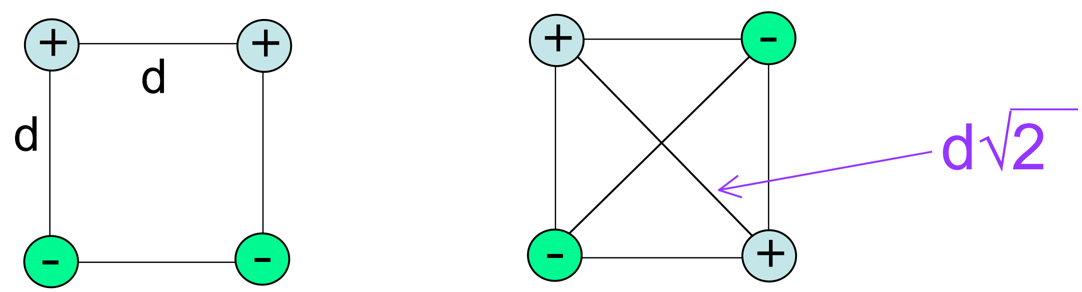
$$PE = -2.3 \times 10^{-19}\ J = -1.44\ eV$$

Total P.E. is less than before (in fact, negative): distribution is MUCH more stable!

Which is more stable?

That is, which has the lower total P.E.?

(closer to –infty →more stable)



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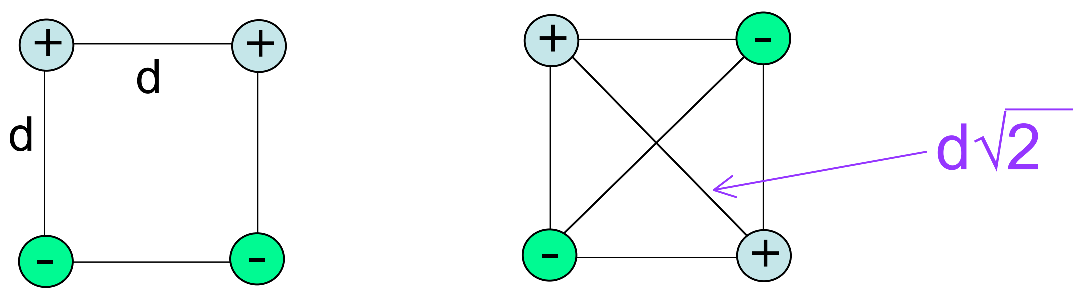

Total PE = PE of each side + PE of each diagonal

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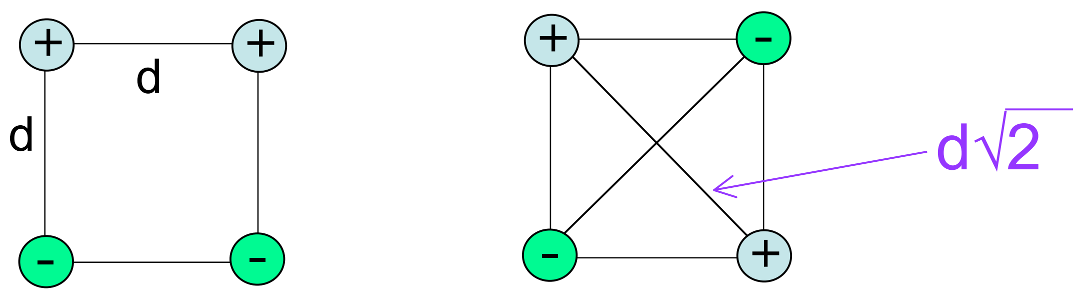
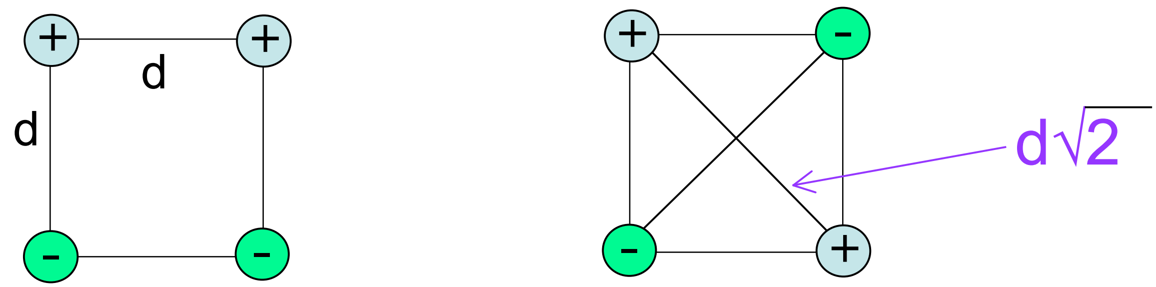


Total PE = PE of each side + PE of each diagonal

$PE_{side} = k_e q_1 q_2 / d$ *(pay attention to signs of charges!!!!)*

$PE_{diagonal} = k_e q_1 q_2 / (d\sqrt{2})$

Which is more stable?



Define $PE_0 = k_e q^2/d$

Sides: PE_0	+2	-2	-4
Diag.: $PE_0/\sqrt{2}$		-2	+2
Total PE	$(-2/\sqrt{2})\ PE_0 = -1.41PE_0$		$(-4 + 2/\sqrt{2})PE_0 = -2.59PE_0$

Yes, this distribution is stable….

… but this one is MORE stable!

The Size of Atomic Nuclei

Ernest Rutherford et al.'s scattering experiments, 1911

Goal: Probe structure of atoms: How are the + and – charges distributed, and what's their size?

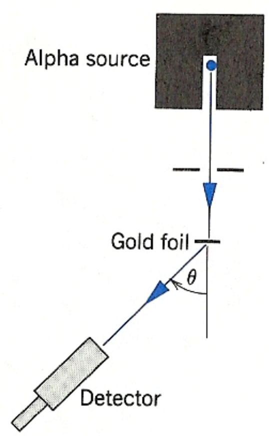


Figure 1 The experimental arrangement used in Rutherford's laboratory to study the scattering of α particles by thin metal foils. The detector can be rotated to various scattering angles θ.

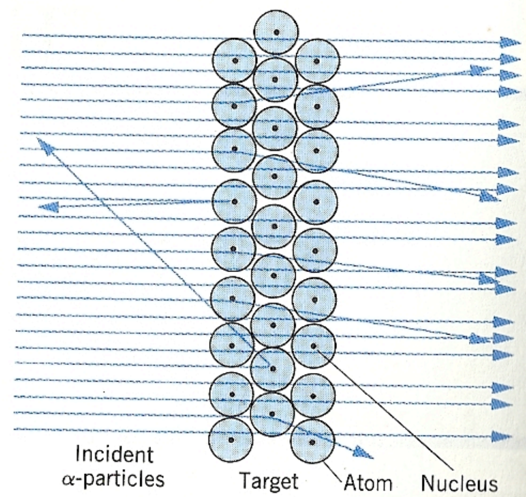
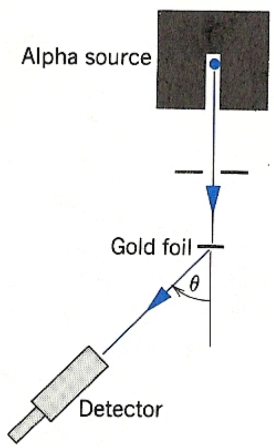


Figure 3 The angle through which an α particle is scattered depends on how close its extended incident path lies to the nucleus of an atom. Large deflections result only from very close encounters.

Method: Fire positively charged alpha-particles (ionized He nuclei, Z=2) at a very thin metal (Au, Z=79) foil
Most passed through, but a few were deflected through large angles-- including up to 180º!
(See Ch 28-29 of text)



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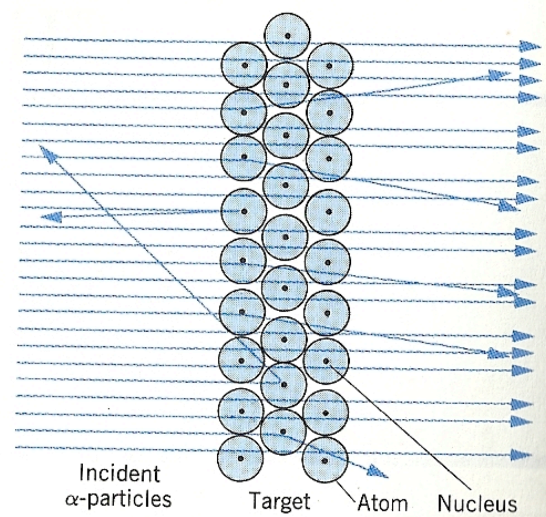
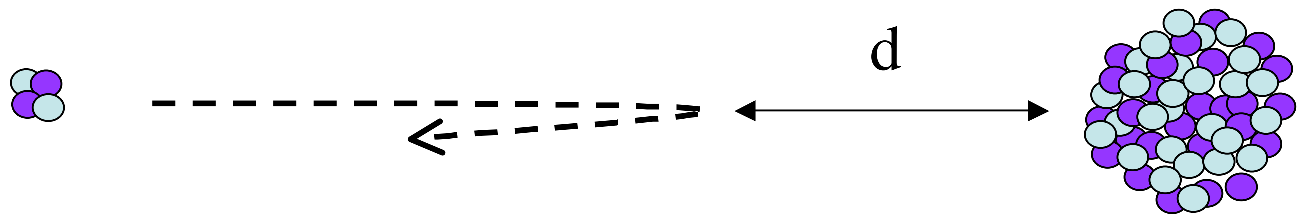
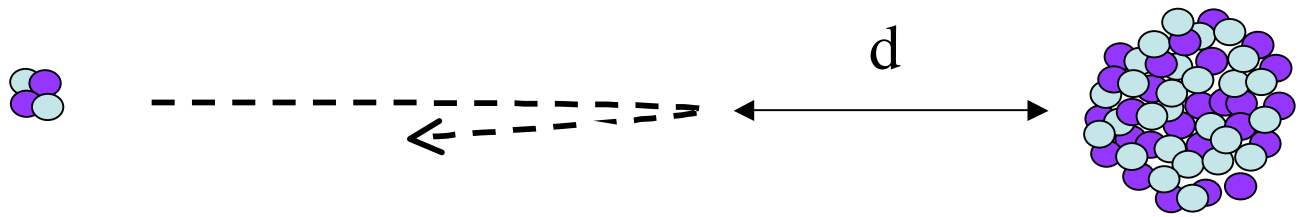


Figure 3 The angle through which an α particle is scattered depends on how close its extended incident path lies to the nucleus of an atom. Large deflections result only from very close encounters.



An alpha particle (He^{2+} = 2p + 2n, total mass = $4*1.67*10^{-27}$ kg) is fired at v= $1.0x10^7$ m/s and happens to be headed directly for the nucleus of a gold atom (79 p) at rest. How close does it get to the gold nucleus before the electric force brings it to a momentary stop and reverses its course? Neglect the recoil of the Au nucleus; neglect the Au atom's electrons.

The Size of Atomic Nuclei



Initially, total energy = K.E. of He^{+2} (P.E. = zero since d = ∞)

At closest interaction, total energy = P.E. = $k_eQ_1Q_2$ / d

$d = k_eQ_1Q_2$ / K.E.

K.E. = 1/2 $m_{He}v^2$ = 1/2 $(4*1.67\text{x}10^{-27}\text{kg})(1\text{x}10^{7}\text{m/s})^2 = 3.3\text{x}10^{-13}$ J

$d = (9\text{x}10^{9}\ \text{Nm}^2/\text{C}^2)(2)(79)(1.6\text{x}10^{-19}\text{C})^2 / (3.3\text{x}10^{-13}\text{J}) = 1.1\text{x}10^{-13}$ m = 110 fm

Size of nucleus must be smaller than this -- VERY compact compared to size of atom (~10^{-11} m)

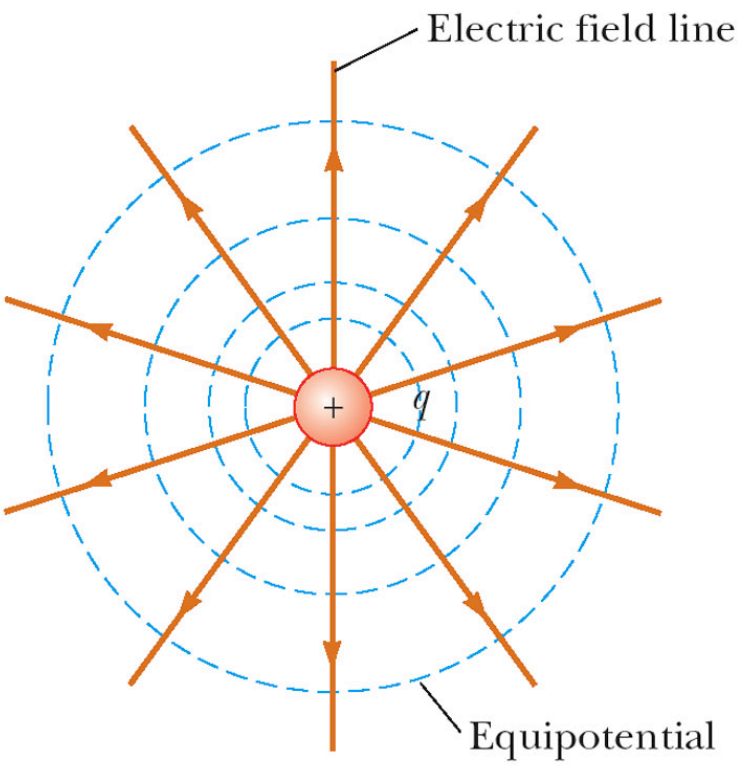
110 fm is small by atomic standards, but not by nuclear standards

Equipotential Surfaces

Surface at which all points have the same potential

Potential difference between any 2 points on the same equipotential surface is zero

Potential is a scalar, not a vector: so no arrows drawn

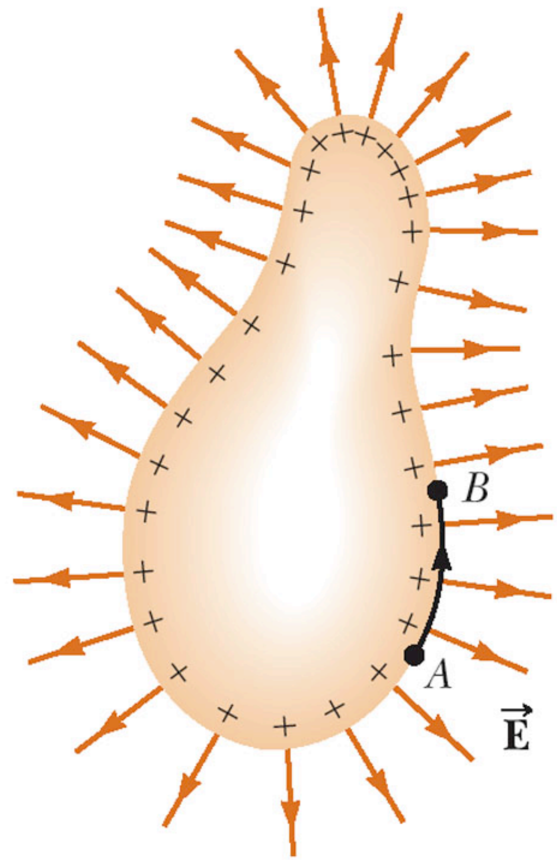


Note that E-field lines and equipotential surfaces are perpendicular

Ex.: Surface of a Charged Conductor

Potential is same at all points on conductor's surface

E-field is ⊥ to surface at all points



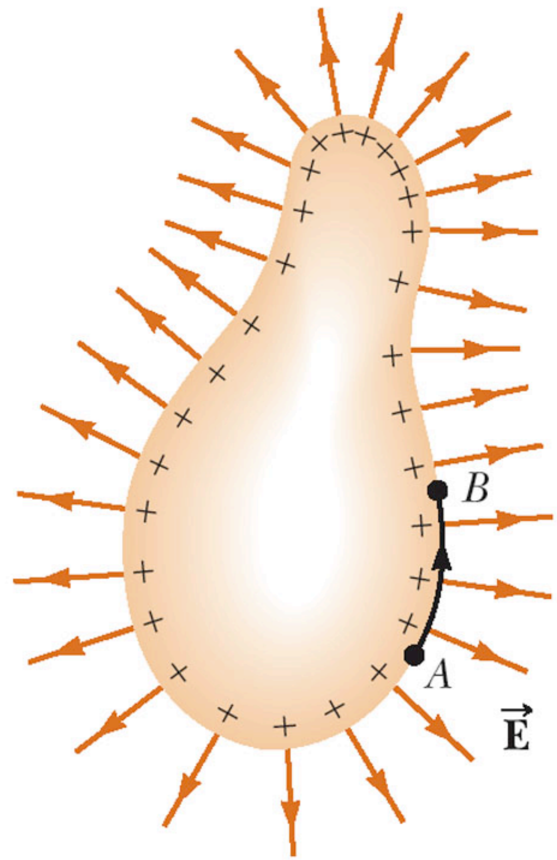
Ex.: Surface of a Charged Conductor

Potential is same at all points on conductor's surface

E-field is ⊥ to surface at all points

No net work required to move a charge along surface

$W = -\Delta PE$
$\Delta PE = q(V_b - V_a)$
If $V_a = V_b$, then $W = 0$!

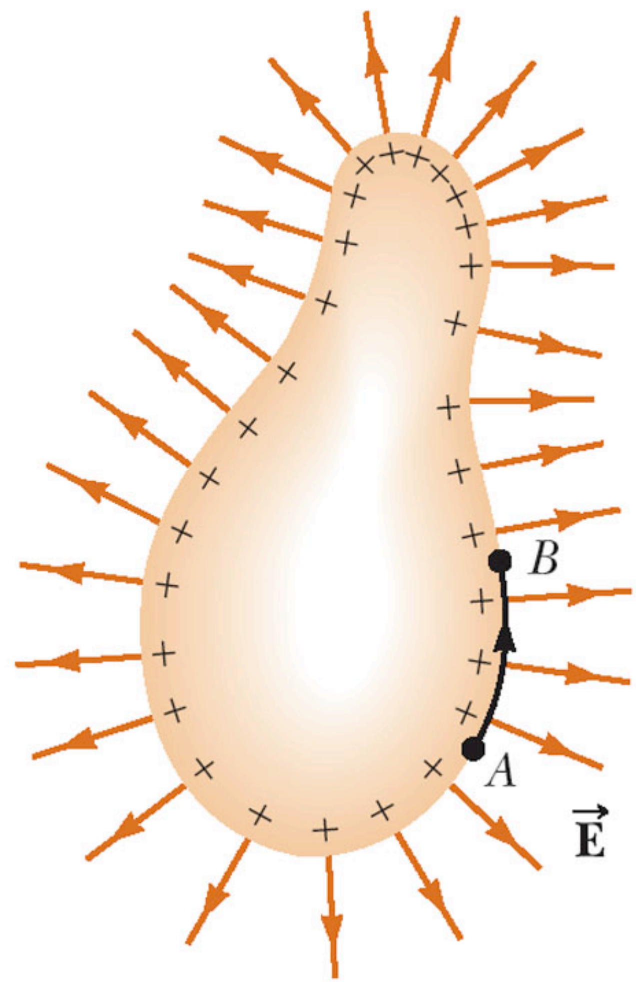


Interior of a Charged Conductor

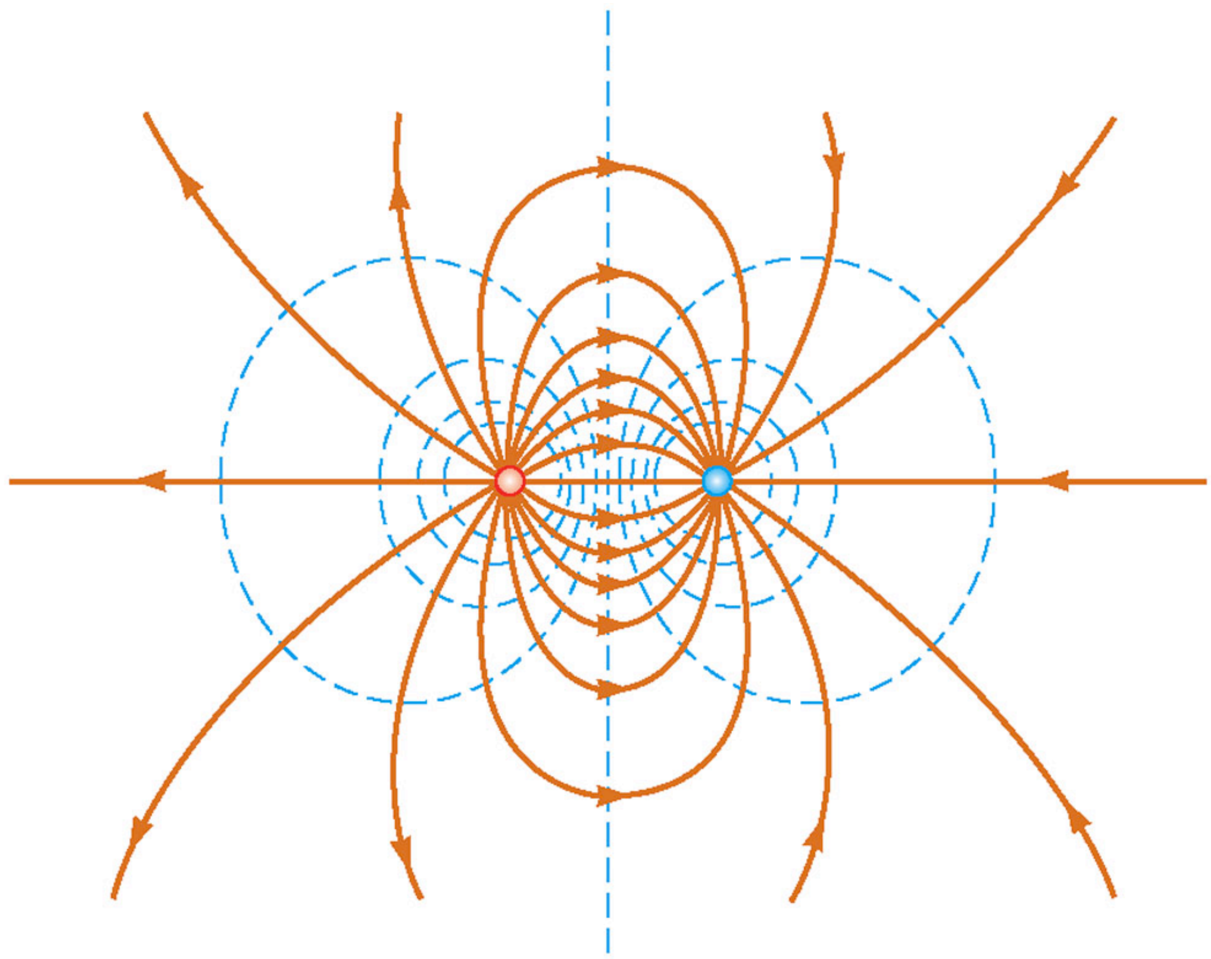
At all points inside a conductor, the potential is the same as at the surface

Reminder: E = 0 inside the conductor

$\Delta V = Ed = 0d$



A Dipole’s Equipotential Surfaces



2 Oppositely-Charged Planes

Equipotential surfaces are parallel to the planes and ⊥ to the E-field lines