Poverty Measurement: History and Recent Developments

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Outline

Motivation

Historical Background
- Early History
- Late 20th Century Consensus

Recent Developments
- Developments within framework
- Multiple Dimensions of Poverty
- Time: Chronic and Intertemporal Poverty

A Taste of My Research
- Representation of a Separable Preorder
- Framework and Information
- Two (or Three) Fundamental Principles
- A Theorem

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Motivation

Summary measures and indicators:

- Guide policy.
- Impact on resource allocation
- Embody assumptions:
  - Information
  - Ethical principles
- Dangerous! *Does it do what it says on the tin?*

Examples:

- World Bank/MDG 1 'Dollar a Day' Poverty Measure.
- UK Child Poverty Measure.
- MDG 5 Maternal Mortality.

Motivation

Does it do what it says on the tin?

- Opportunity for analysis.
- Information:
  - Explicit analytical framework.
  - Should reflect information content of data.
- Ethical Principles:
  - Perhaps not for the economist to decide!
  - What principles does the policymaker choose?
  - What do desired principles entail for form of the measure?
  - Exactly which measures embody such principles?

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Early History

- Non-bureaucratic support (or not) for the destitute (family, community, local religious institutions)
- Europe: bureaucratisation in 16th and 17th centuries (UK: dissolution of the monasteries under Henry VIII $\rightarrow$ social problems $\rightarrow$ Old Poor Law mandates parishes of Church of England to provide for the poor).
- Information gathered and utilised locally but determined liability for taxation: 1691 William and Mary's four shilling Quarterly Poll instituted by act of Parliament 'for raiseing money by a Poll payable quarterly for One year for the carrying on a vigorous War against France'.
- 1696: Gregory King: 55% of the population of England and Wales found to be insolvent (excused from William and Marys Quarterly Poll)

Early History

- 1895, Charles Booth: Poverty Maps of London

Early History

- 1902, Benjamin Rowntree, census in York
- 1920s: statistics! so we can use survey data
- General approach headcount (number of individuals/proportion of population below ‘poverty line’). Still used: World Bank (Ravallion), Millennium Development Goals.

Late 20th Century Consensus

- Vector of individual incomes $x = (x_1, x_2, \ldots, x_n)$, poverty line z.
- The framework: Sen (1976) distinguished *identification* and *aggregation*.
- Many measures suggested 1976–1984; some have nice properties, some do not.
- FGT (1984) introduced P_α family: nice properties and conceptually straightforward $\rightarrow$ gold standard
- Meanwhile Foster and Shorrocks (1991) characterised *entire class* of unidimensional measures with nice properties:

$$P(x; z) = \frac{1}{n} \sum_{i=1}^{n} \phi(x_i)$$

  where $\phi(x_i)$ is non-increasing, zero above z and continuous except possibly at z.

Late 20th Century Consensus

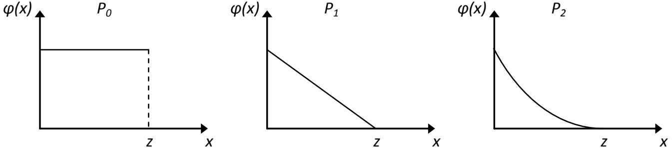
- Class of unidimensional measures with nice properties:

$$P(x; z) = \frac{1}{n}\sum_{i=1}^{n} \phi(x_i)$$

  where $\phi(x_i)$ is non-increasing, zero above z and continuous except possibly at z.
- Nice properties **plus**
  - Monotonicity if $\phi(x_i)$ is decreasing below z (e.g. P_1).
  - Transfer if if $\phi(x_i)$ is convex below z (e.g. P_2).
- P_α measures belong to this class but do not exhaust it! – but well-established.
- Little further exploration of this class...

Late 20th Century Consensus

ϕ functions for P_α measures:



Illustrate implicit interpersonal tradeoffs.

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Developments within framework

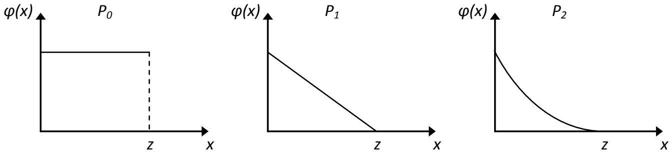
General form:

$$P(x;z) = \frac{1}{n}\sum_{i=1}^{n}\phi(x_i)$$

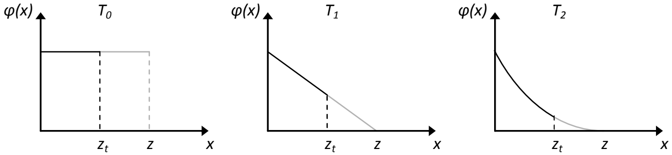
where the x_is are real-valued indicators of individual/household wellbeing.

- Consumption vs income data (Ravallion 1994)
- Individual vs household indicators (intra-HH distribution)
- 'Targeted' poverty measures focussing on the 'poorest of the poor' (Alkire and Foster 2012); within Foster and Shorrocks (1991) framework, new functional forms for $\phi(x_i)$

- ϕ functions for P_α measures:



- ϕ functions for targeted P_α measures:



Multiple Dimensions of Poverty

Rationale:

- If we lived in a world of complete and perfect markets (first fundamental welfare theorem) then individual command over income can be argued to be a sufficient measure of wellbeing.
- But we do not! Consumption of health, education etc. . .

Approaches:

- Dashboard (MDGs etc)
- Aggregate: over society/within dimension first (Human Poverty Index: HDR 1997 – 2009)
- Aggregate: over dimensions/within individual-first (Tsui 2002, Bourguignon and Chakravarty 2003, Alkire and Foster 2010, Multidimensional Poverty Index: HDR 2010 onward).

Multiple Dimensions of Poverty

Aggregating over dimensions/within individual-first retains the general functional form:

$$P(x; z) = \frac{1}{n} \sum_{i=1}^{n} \phi(x_i)$$

but now the x_i's are vectors of individual indicators in multiple dimensions; requires detailed, representative household survey

Example MPI: Data from DHS, ϕ is an indicator function (0,1) of {a weighted average of indicator functions representing 'poverty' according to the following indicators} being greater than 1/3:

- Health (nutrition, child mortality)
- Education (years of schooling, enrollment)
- Living standards (6 standard DHS indicators)

Time: Chronic and Intertemporal Poverty

General functional form:

$$P(x; z) = \frac{1}{n} \sum_{i=1}^{n} \phi(x_i)$$

Now x_i is a trajectory of wellbeing indicators.

Literature

'Spells' Approach:

- 'Still poor after x years'; compare headcount.
- Chronic Poverty Reports (CPRC, 2005 and 2009)

'Components' Approach:

- Poverty of permanent component of (or average) income; transient fluctuations.
- Rodgers and Rodgers (1993; US); Jalan and Ravallion (2000). Both based on poverty-gap-squared (Foster, Greer and Thorbecke, 1984).

Literature

More recent proposals (all indices aggregating over individuals and time):

- Calvo and Dercon (2009), Foster (2009), Gradin, Del Rio and Canto (2011), Hoy and Zheng (2011), Bossert, Chakravarty and D'Ambrosio (2012), Foster and Santos (2013), Porter and Quinn (2008, 2014).

None combine *all* of the properties that we might want a chronic poverty measure to embody:

- *Either*: Not sensitive to chronicity/persistence (so more appropriate to measure 'total' intertemporal poverty)
- *Or*: Discontinuities lead to counter-intuitive ordering of trajectories

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Analytical Approach

Clearest method of analysis: characterisation of poverty and social welfare measures.

- Within a certain framework, characterise the class of exactly those measures that satisfy certain properties (axioms).

Limitations in the literature (even the most elegant papers):

- Information framework too restrictive in relation to data.
- Limited by topological assumptions.
  - Continuity: what about poverty lines and more complex extensions?
  - Connected domain: what about categorical or discrete information?
- Properties imposed without good normative motivation.
  - *The poverty measure is twice continuously differentiable...*
  - *The poverty measure has a particular, rather odd, functional form...*

Background: Key Literature

- Foster and Shorrocks (1991), *Subgroup Consistent Poverty Indices*
- Relies on Gorman (1968), *The Structure of Utility Functions*
- Relies on Debreu (1960), *Topological Methods in Cardinal Utility Theory*

Limiting assumptions:

- Continuity of the ordering (typically in Euclidean topology but clearly generalisable – dependent on topology)
- Connectedness of the domain

Similar issues:

- Characterisation of generalised utilitarian social welfare functions (Blackorby, Bossert and Donaldson, 2005)
- Dutta, Pattanaik and Xu (2003)

Representation of a Separable Preorder

The relationship between separability of a preorder and existence of an additively separable representation is well known:

- Leontief (1947) and Samuelson (1947) require continuous differentiability of the representing function; this imposes restrictions on the structure of the domain and the preorder.
- Debreu (1960), extended by Gorman (1968) relax differentiability, but require:
  - Connectedness of the domain.
  - Continuity of the preorder.

The main result of this paper:

- Relax topological conditions to point of necessity.
- Introduce symmetry.

Framework and Information

The poverty analyst:

- Has information $x_i \in X$ relating to each individual i in a population of size $n \in \mathbb{N}$, $n \geq 3$.
- Note: no restriction on X.
  - Continuous, discrete, categorical data
  - Individual, social, environmental characteristics
  - Multidimensional, intertemporal...
  - (Implicitly comparable across individuals – see later)
- So: domain of analysis is

$$\mathcal{X} = \bigcup_{n=3}^{\infty} X^n.$$

Framework and Information

Domain of analysis

$$\mathcal{X} = \bigcup_{n=3}^{\infty} X^n.$$

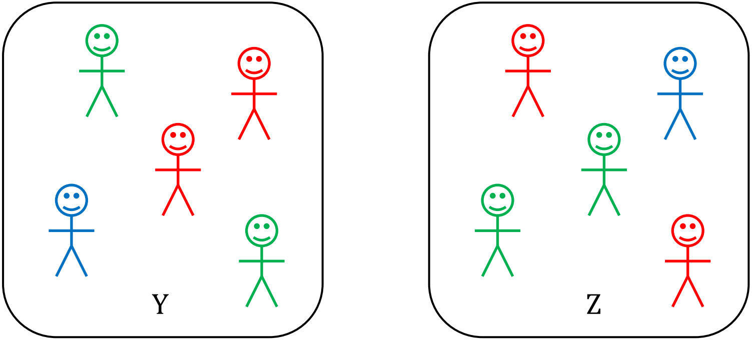
The poverty analyst:

- Evaluates poverty according to some binary relation $\precsim$ on $\mathcal{X}$, the poverty ordering.
- For profiles $Y, Z \in \mathcal{X}$ such that $Y \precsim Z$, reads 'Z contains more poverty than Y'.

Two (or Three) Fundamental Principles

- Anonymity
- Subset Consistency
- Representability

1: Anonymity



$$Y \sim Z$$

1: Anonymity

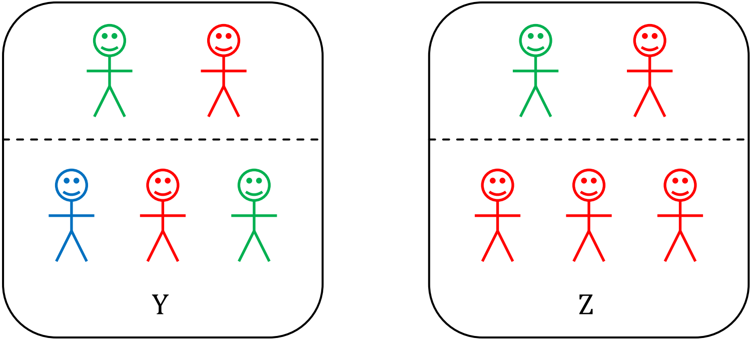
Informally:

- The poverty analyst evaluates as equivalent profiles which differ only by a permutation of characteristics among individuals.

Formally equivalent to permutation-symmetry of the poverty ordering:

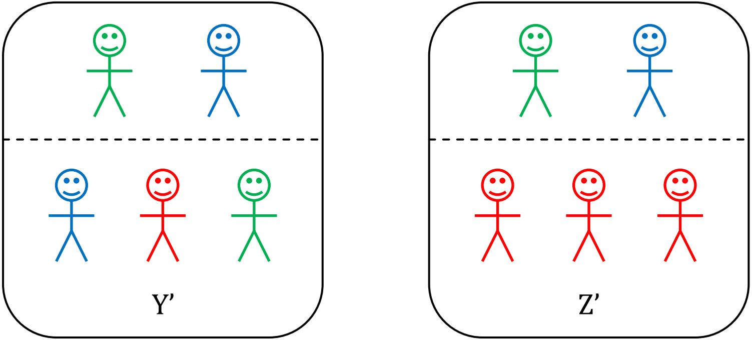
- A **permutation on** n is a bijective function $p : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$. Define a function $f_p : X^n \to X^n$ such that $f_p : x \mapsto f_p(x)$ where $[f_p(x)]_i = x_{p(i)}$ for each $i \in \{1, 2, \ldots, n\}$.
- A binary relation R on a symmetric product space X^n is a **permutation-symmetric relation** if, for every permutation on n, p, and for every $x, y \in X^n$, $f_p(x) R f_p(y) \Leftrightarrow xRy$.

2: Subset Consistency



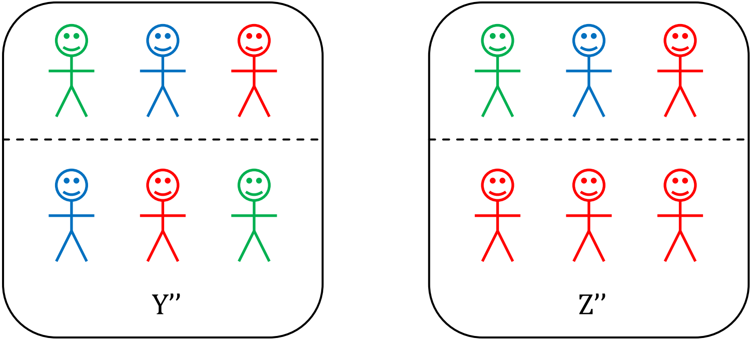
If $Y \precsim Z$

2: Subset Consistency



then $Y' \precsim Z'$

2: Subset Consistency



and $Y'' \precsim Z''$

2: Subset Consistency

Informally:

- If the measure of poverty increases in a subset of the population while the profile of individual characteristics remains unchanged in the rest of the population then overall poverty must increase.
- (Regardless of the number of individuals and the profile of their characteristics in the unchanging part of the population.)

Formally equivalent to full separability of the poverty ordering:

- Too much notation?

2: Full Separability: Notation

- For $n \geq 2$, let A be any proper subset of $\{X_1, \ldots, X_n\}$ (neither $\{X_1, \ldots, X_n\}$ nor the empty set) and let $\bar{A} = \{X_1, \ldots, X_n\} \setminus A$. Let $\mathbf{X}_A$ be the Cartesian product of the elements of A, $\mathbf{X}_A = \prod_{i|X_i \in A} X_i$. $\mathbf{X}_A$ is a **subspace** of $\mathbf{X}$.
- Let $\mathbf{X}_{\bar{A}}$ be the Cartesian product of the elements of $\bar{A}$, $\mathbf{X}_{\bar{A}} = \prod_{i|X_i \in \bar{A}} X_i$. $\mathbf{X}_A$ and $\mathbf{X}_{\bar{A}}$ are **complementary subspaces** of $\mathbf{X}$.

2: Full Separability: Definition

- Let $\precsim$ be a partial preorder on a product space $\mathbf{X}$ with complementary subspaces $\mathbf{X}_A$ and $\mathbf{X}_{\bar{A}}$.
- Given an element $\bar{a} \in \mathbf{X}_{\bar{A}}$, define a **conditional order** $\precsim_{\bar{a}}$ on $\mathbf{X}_A$ such that for all $a, b \in \mathbf{X}_A$, $a \precsim_{\bar{a}} b$ if and only if $x \precsim y$ where $x = x(a, \bar{a}) \in \mathbf{X}$ and $y = x(b, \bar{a}) \in \mathbf{X}$.
- We say that the subspace $\mathbf{X}_A$ is **separable under** $\precsim$ if, for all $\bar{a}, \bar{b} \in \mathbf{X}_{\bar{A}}$ and for all $a, b \in \mathbf{X}_A$, $a \precsim_{\bar{a}} b \Leftrightarrow a \precsim_{\bar{b}} b$.
- We say that the partial preorder $\precsim$ is **fully separable on $\mathbf{X}$** if $\mathbf{X}_A$ is separable under $\precsim$ for all subspaces $\mathbf{X}_A$ of $\mathbf{X}$.

3: Representability

Given a non-empty set A and a binary relation $\precsim$ on A:

- The real valued function $u : A \to \mathbb{R}$ represents $\precsim$ on A if for all $x, y \in A$, $x \precsim y \Leftrightarrow u(x) \leq u(y)$.
- Alternatively u is order-preserving.

A question:

Precisely which binary relations on A may be represented by a real valued function $u : A \to \mathbb{R}$?

- Well known that completeness and transitivity of $\precsim$ on A are necessary for existence of u. So $\precsim$ is a total preorder.
- But not sufficient: Debreu (1954) gives the classic counterexample of the lexicographic ordering of $\mathbb{R}^2$.

3: Representability

- If A is finite or countable, completeness and transitivity of $\precsim$ are sufficient. (So the lexicographic ordering of $\mathbb{Q}^2$ is representable.)
- If A is $\mathbb{R}^n$, add Euclidean continuity: sufficient but not necessary.
- Debreu (1954) established general necessary and sufficient conditions; some debate over the validity of his proof but Jaffray (1975) gives an elegant – and correct – proof.

Debreu's representation theorem (paraphrased)

Given a set A and a total preorder $\precsim$ on A, there exists on A a real function $u : A \to \mathbb{R}$ representing $\precsim$ if and only if the preorder topology is second countable.

The Theorem

Informally: For fixed population size, subset consistency and anonymity are necessary and sufficient for representation of a (representable) poverty ordering by a symmetric additive function.

Theorem

Given a set X, a natural number $n \geq 3$ and a binary relation $\precsim$ on X^n, there exists a real function $u : X^n \to \mathbb{R}$ representing $\precsim$ of the form

$$u : x \mapsto \sum_{i=1}^{n} \phi(x_i),$$

where $\phi : X \to \mathbb{R}$, if and only if $\precsim$ is a fully separable and permutation-symmetric total preorder whose preorder topology is second countable.

Sketch of Proof

'Only if' is straightforward (necessity of properties).

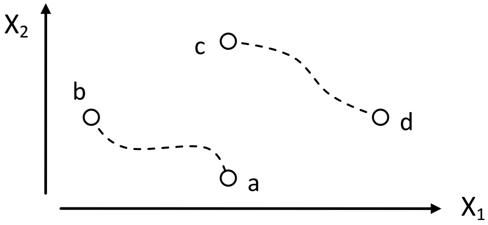
'If' (sufficiency of properties) is less straightforward:

- Lemma 1: Establish Hexagon Condition for $n = 3$.
- Lemma 2: Establish sufficiency for $n = 3$.
- Extend to all natural numbers $n > 3$ by induction.

Lemma 1

Lemma 1

Given a non-empty set X, let $X^3 = X \times X \times X$. Let $\precsim$ be a fully separable p-symmetric partial preorder on X^3 with derived symmetric relation $\sim$. For all $a, b, c, d \in X^3$ such that $a \sim b$, $c \sim d$, $a_i = c_i$, $b_j = d_j$ and $a_k = b_k = c_k = d_k$ for distinct $i, j, k \in \{1, 2, 3\}$, there exist $e, f \in X^3$ such that $e_i = b_i$, $e_j = c_j$, $f_i = d_i$, $f_j = a_j$ and $e_k = f_k = a_k$, and furthermore, $e \sim f$.



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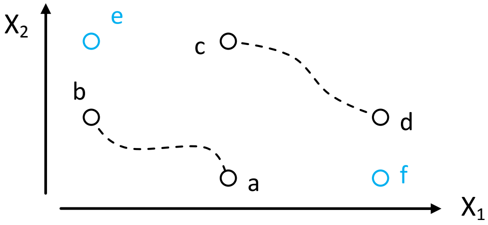

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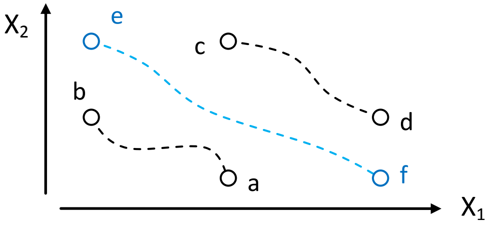

Lemma 1

Without loss of generality write $x = (x_i, x_j, x_k)$ for all $x \in X^3$.

a) First demonstrate that e and f are elements of X^3. Consider $a, b, c, d \in X^3$ such that $a_i = c_i = \alpha$, $b_j = d_j = \beta$ and $a_k = b_k = c_k = d_k = \gamma$ for distinct $i, j, k \in \{1, 2, 3\}$. Let $a_j = \delta, b_i = \epsilon, c_j = \zeta$ and $d_i = \eta$. It follows from symmetry of X^3 that $\{\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta\} \subseteq X$; they need not all be distinct. $X^3 = X \times X \times X$, therefore $e = (\epsilon, \zeta, \gamma) \in X^3$ and $f = (\eta, \delta, \gamma) \in X^3$.

Existence of e and f arises directly from symmetry.

b) Now show that $e \sim f$.

i) Let $a \sim b$ and $c \sim d$. It follows from full separability of $\precsim$ and thus $\sim$ that $(\alpha, \delta, x_k) \sim (\epsilon, \beta, x_k)$ and $(\alpha, \zeta, x_k) \sim (\eta, \beta, x_k)$ for all $x_k \in X$. In particular, $(\alpha, \delta, \beta) \sim (\epsilon, \beta, \beta)$ and $(\alpha, \zeta, \beta) \sim (\eta, \beta, \beta)$.

ii) By p-symmetry of $\precsim$ and thus $\sim$ we may permute j and k to obtain $(\alpha, \beta, \delta) \sim (\epsilon, \beta, \beta)$ from $(\alpha, \delta, \beta) \sim (\epsilon, \beta, \beta)$. It follows from full separability of $\precsim$ and thus $\sim$ that $(\alpha, x_j, \delta) \sim (\epsilon, x_j, \beta)$ for all $x_j \in X$. In particular, $(\alpha, \zeta, \delta) \sim (\epsilon, \zeta, \beta)$.

iii) Recall from part (i) that $(\alpha, \delta, \beta) \sim (\epsilon, \beta, \beta)$. Recall from part (ii) that $(\alpha, \beta, \delta) \sim (\epsilon, \beta, \beta)$. By transitivity of $\sim$, therefore, we have $(\alpha, \beta, \delta) \sim (\alpha, \delta, \beta)$. It follows from full separability of $\precsim$ and thus $\sim$ that $(x_i, \beta, \delta) \sim (x_i, \delta, \beta)$ for all $x_i \in X$. In particular, $(\eta, \beta, \delta) \sim (\eta, \delta, \beta)$.

Lemma 1

iv) Recall from part (i) that $(\alpha, \zeta, x_k) \sim (\eta, \beta, x_k)$ for all $x_k \in X$. In particular, $(\alpha, \zeta, \delta) \sim (\eta, \beta, \delta)$. From part (ii) $(\alpha, \zeta, \delta) \sim (\epsilon, \zeta, \beta)$ and from part (iii) $(\eta, \beta, \delta) \sim (\eta, \delta, \beta)$ therefore by transitivity (applied twice) $(\epsilon, \zeta, \beta) \sim (\eta, \delta, \beta)$.

v) It follows from full separability that $(\epsilon, \zeta, x_k) \sim (\eta, \delta, x_k)$ for all $x_k \in X$ and in particular $(\epsilon, \zeta, \gamma) \sim (\eta, \delta, \gamma)$. But $(\epsilon, \zeta, \gamma) = e$ and $(\eta, \delta, \gamma) = f$, therefore $e \sim f$.

Lemma 2

Lemma 2

Given a non-empty set X, let $X^3 = X \times X \times X$. Let $\precsim$ be a fully separable p-symmetric total preorder on X^3 whose preorder topology is second countable. There exists a function $u : X^3 \to \mathbb{R}$, which represents $\precsim$, such that $u : x \mapsto \phi(x_1) + \phi(x_2) + \phi(x_3)$ for some function $\phi : x \to \mathbb{R}$.

Steps in proof:

- Existence of representation for induced preorder on X.
- Map into $\mathbb{R}^2$.
- Invoke Lemma 1 and Thomsen-Blaschke Theorem (cf Debreu 1960) to obtain additive representation.
- Invoke symmetry and separability to extend to $\mathbb{R}^3$.
- Map back to X^3.

Application to Poverty Measurement

- Information $y_i \in X$ (X unrestricted) relating to each individual i in a population of size $n \in \mathbb{N}$, $n \geq 3$.
- Domain of analysis

$$\mathcal{X} = \bigcup_{n=3}^{\infty} X^n$$

Informally: an real-valued poverty measure $\mathcal{P} : \mathcal{X} \to \mathbb{R}$ represents a poverty ordering with the properties anonymity and subset consistency if and only if it has the form

$$\mathcal{P} : Y \mapsto g\left(\sum_{i=1}^{n(Y)} \phi(y_i)\right).$$

where $\phi : X \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ is strictly increasing.