POWER POINT PRESENTATION

ON

MECHANICAL VIBRATION AND STRUCTURAL DYNAMICS

IV B. Tech I semester (JNTUH-R15)

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Mechanical Vibration and Structural Dynamics

Unit 1: Introduction - Single degree-of-freedom system

Contents

1.0 Some historical background

- Historically studies on **vibration** (acoustics) started long ago (around 4000BC)
- Musicians and philosophers have sought out the rules and laws of sound production, used them in improving musical instruments, and passed them on from generation to generation
- Music had become highly developed and was much appreciated by Chinese, Hindus, Japanese, and, perhaps, the Egyptians.
- These early peoples observed certain definite rules in connection with the art of music, although their knowledge did not reach the level of a science.
- Early applications (by Egyptian) to single or multiple string instruments known as Harps
- Our present system of music is based on ancient Greek civilization.
- The Greek philosopher and mathematician Pythagoras (582-507 B.C.) is considered to be the first person to investigate musical sounds on a **scientific basis** [later on we will be talking about **Mathematical Basis** as well]

1.1 Introductory Remarks

- Most human activities involve vibration in one form or other. For example, we hear because our eardrums vibrate and see because light waves undergo vibration

- Any motion that repeats itself after an interval of time is called *vibration or oscillation.*

- The general terminology of “Vibration” is used to describe oscillatory motion of mechanical and structural systems

- The Vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately

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1.1 Introductory Remarks

- Any object in this world having mass and elasticity is capable of vibration
- We are mainly interested in vibration of mechanical system
- When subjected to an oscillating load, this system undergoes a vibratory behavior
- Vibrations are an engineering concern in these applications because they may cause a catastrophic failure (complete collapse) of the machine or structure because of excessive stresses and amplitudes (resulting mainly from resonance) or because of material fatigue over a period of time

Example: - Failure of Tacoma Narrows Bridge in 1940 due to 42-mile-per-hour wind undergoing a torsional mode resonance
- Vibration of machine components generate annoying noise
- Vibration of string generate pleasing music (already discussed before)

- Vibrations in mechanical system (or more preciously flight vehicles) is dissipated by inherent damping of the material
- Vibration of mechanical system is model as a combination of spring-mass-damper

1.1 Introductory Remarks

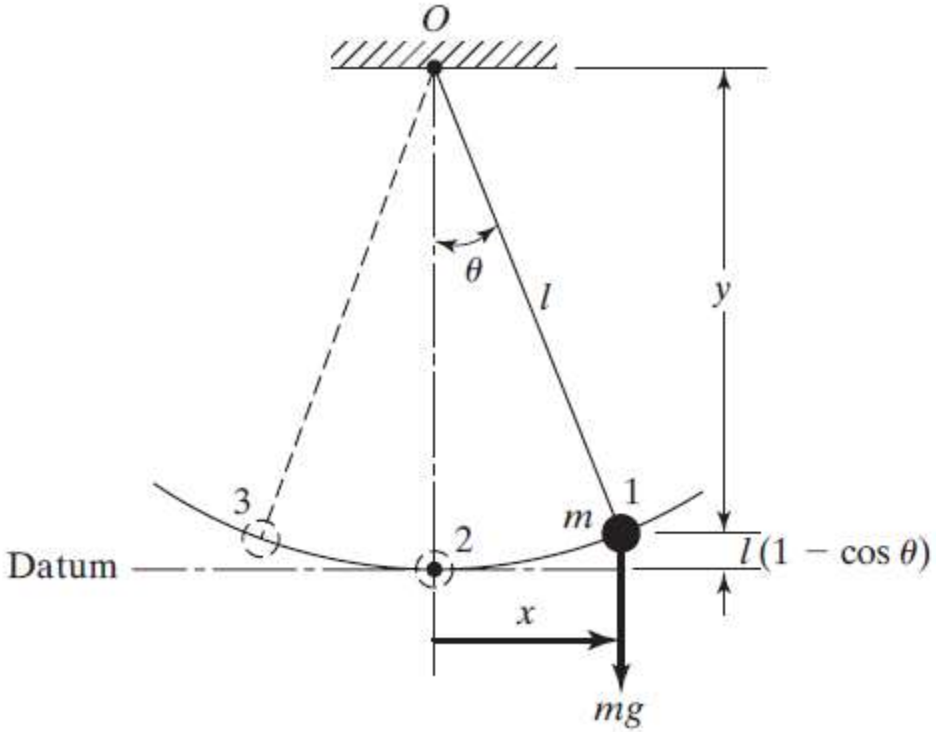
- In some system it may be clearly visible – for example vibration of automobiles

  - The body mass represented by concentrated mass m
  - The Stiffness of suspension system is represented by linear/nonlinear spring k
  - The shock absorber is represented by damper c

- In most of the cases (like in continuous system) it may not be possible clearly identify spring-mass-damper system

  - Vibration of flight vehicle
  - Vibration of machine component etc

1.2 Degrees of freedom



"Period of vibration" is the time that it takes to complete one cycle. It is measured in seconds.
"Frequency" is the number of cycles per second. It is measured in Hz (1 cycle/second). It could be also measured in radians/second.

Period of vibration: T
Frequency of vibration: $f = (1/T)$ Hz or $\omega = (2\pi/T)$ radians/s $T=(2\pi/\omega) = (1/T)$

Types of Vibratory Motion

Oscillatory motion may repeat itself regularly, as in the case of a simple pendulum, or it may display considerable irregularity, as in the case of ground motion during an earthquake.

If the motion is repeated after equal intervals of time, it is called *periodic motion. The simplest type of periodic motion is harmonic motion.*

Harmonic motion

It is described by sine or cosine functions.

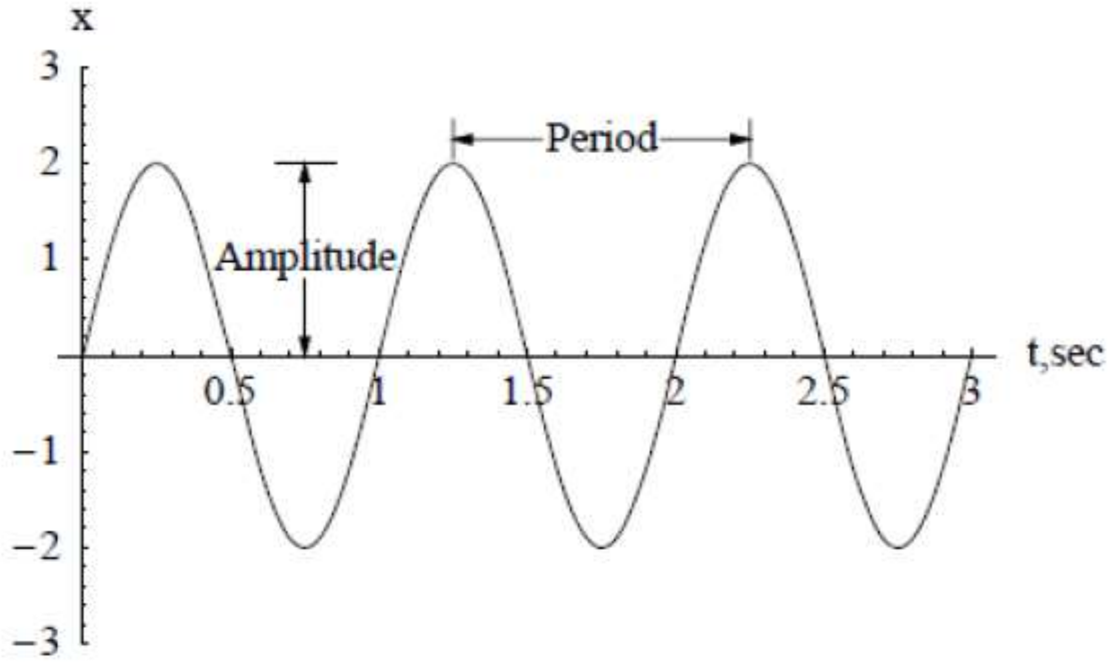
$x(t) = A\sin(\omega\, t)$

A is the amplitude while ω is the frequency (radians/sec)

$$\dot{x}(t) = \omega\ A\cos(\omega\ t)$$

$$\ddot{x}(t) = -\omega^2 A\sin(\omega\ t) = -\omega^2 x(t)$$

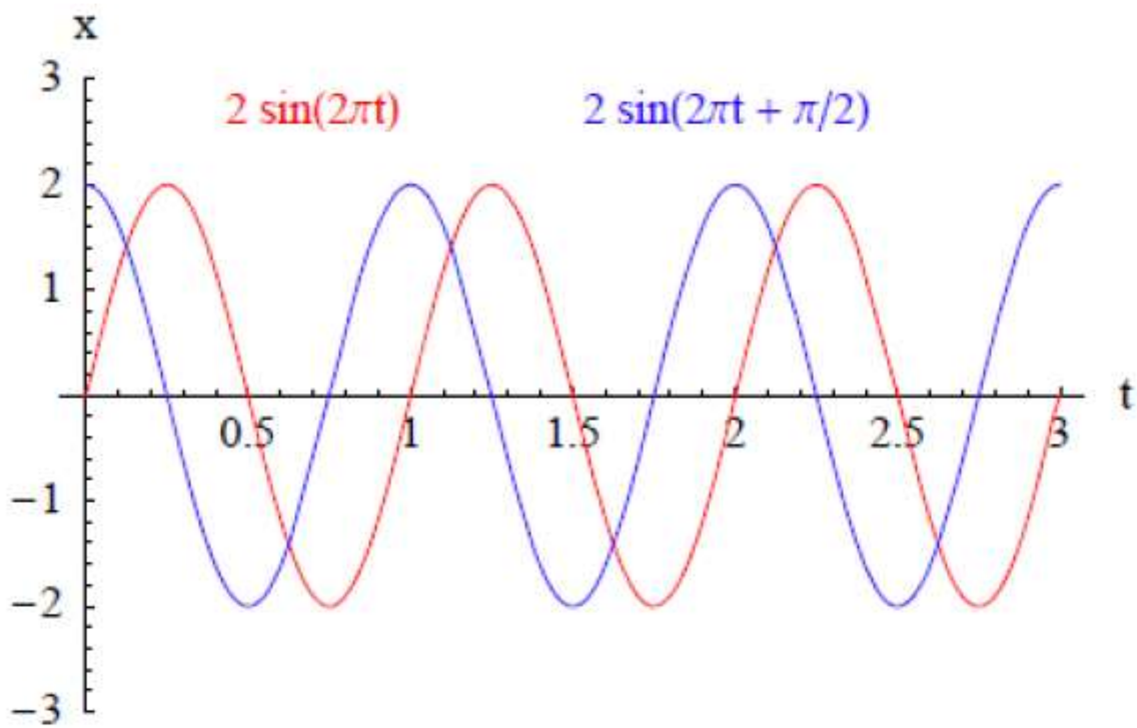
Types of Vibratory Motion



Plot of $x(t) = 2\sin(2\pi t)$

Types of Vibratory Motion

Two harmonic motions having the same period and/or amplitude could have different phase angle



Plot of two harmonic functions $2\sin(2\pi t)$ and $2\sin(2\pi t + \pi/2)$

Types of Vibratory Motion

- A harmonic motion can be written in terms of exponential functions.

$$\sin \omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}; \quad \cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

so that

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

A harmonic motion could be written as

$$x(t) = a\, e^{i\omega t}$$

- Alternative forms for harmonic motion

Generally, a harmonic motion can be expressed as a combination of sine and cosine waves.

$$y(t) = A \cos \omega t + B \sin \omega t \quad \Longleftrightarrow \quad y(t) = Y \sin(\omega t + \theta)$$

Types of Vibratory Motion

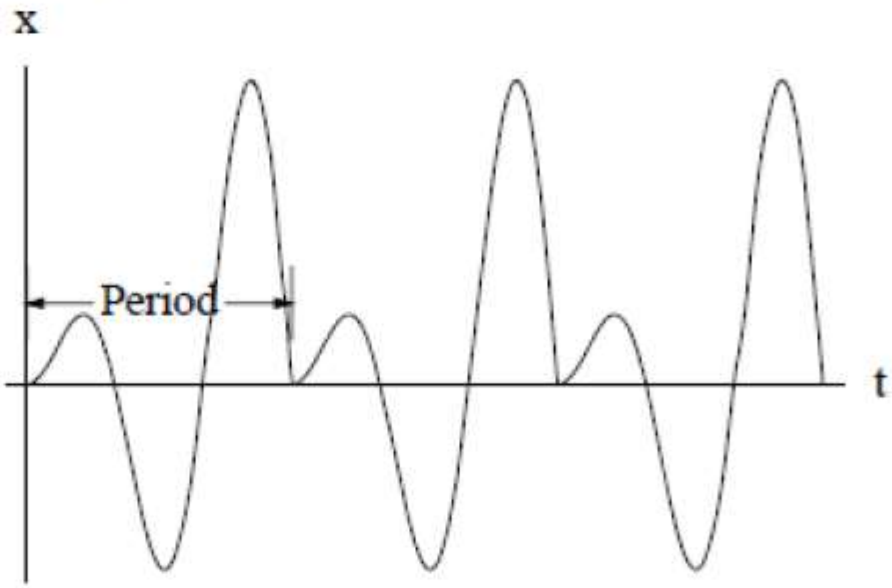
$$Y = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1}(A/B)$$

or

$$y(t) = A\cos\omega t - B\sin\omega t \iff y(t) = -Y\sin(\omega t - \theta) = Y\cos(\omega t - \theta)$$

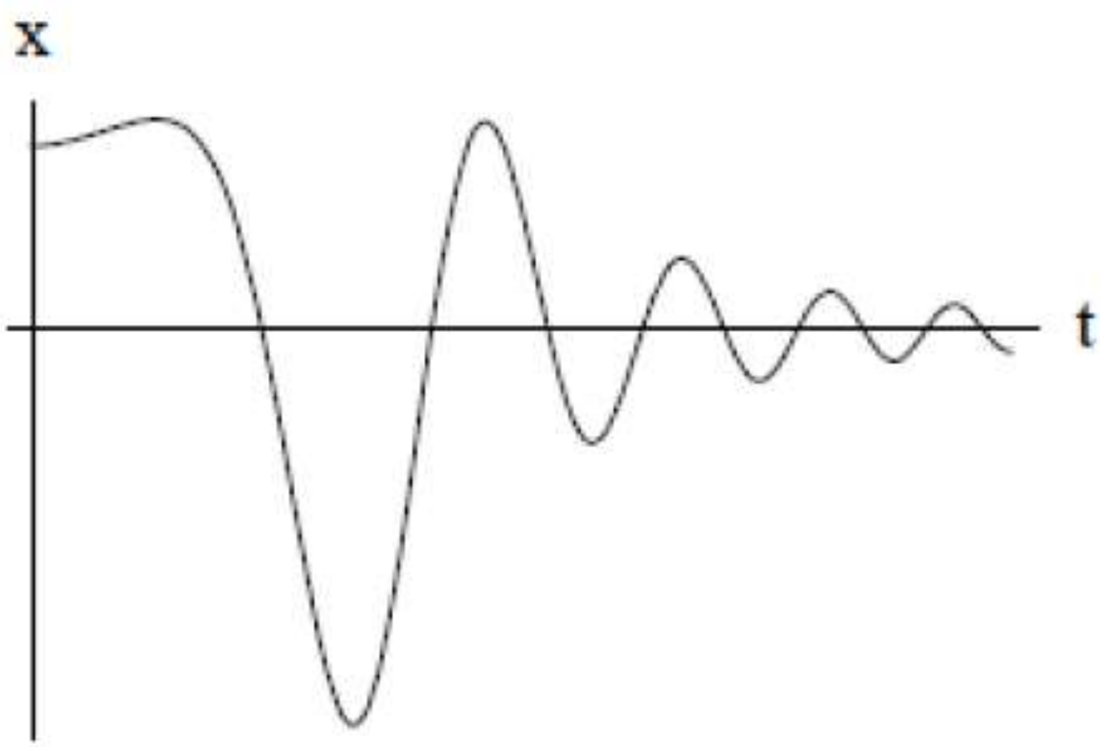
- Periodic motion

The motion repeats itself exactly.

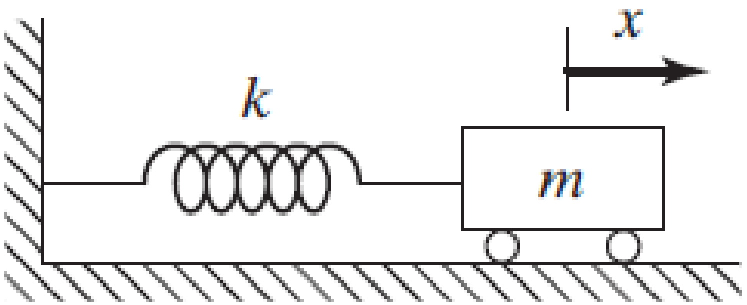


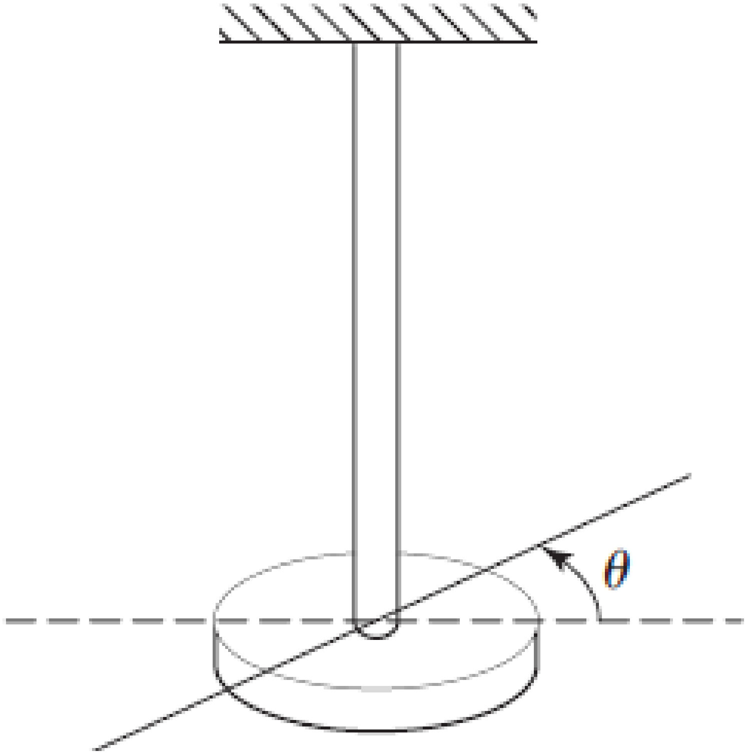
Types of Vibratory Motion

A general vibratory motion doesn't have a repeating pattern.

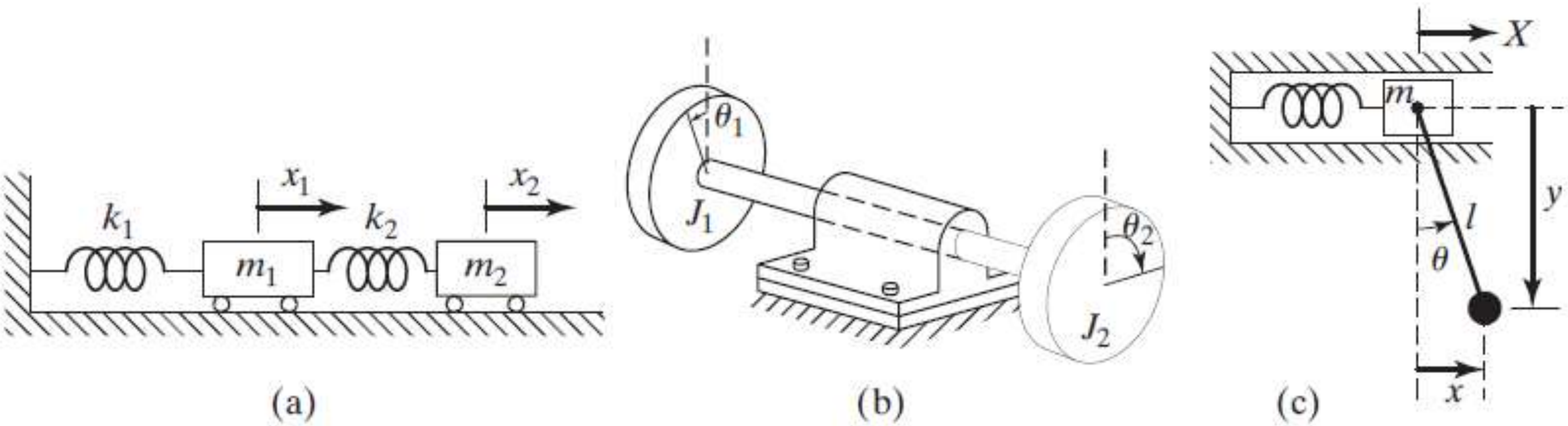


1.2 Degrees of freedom (cont…)

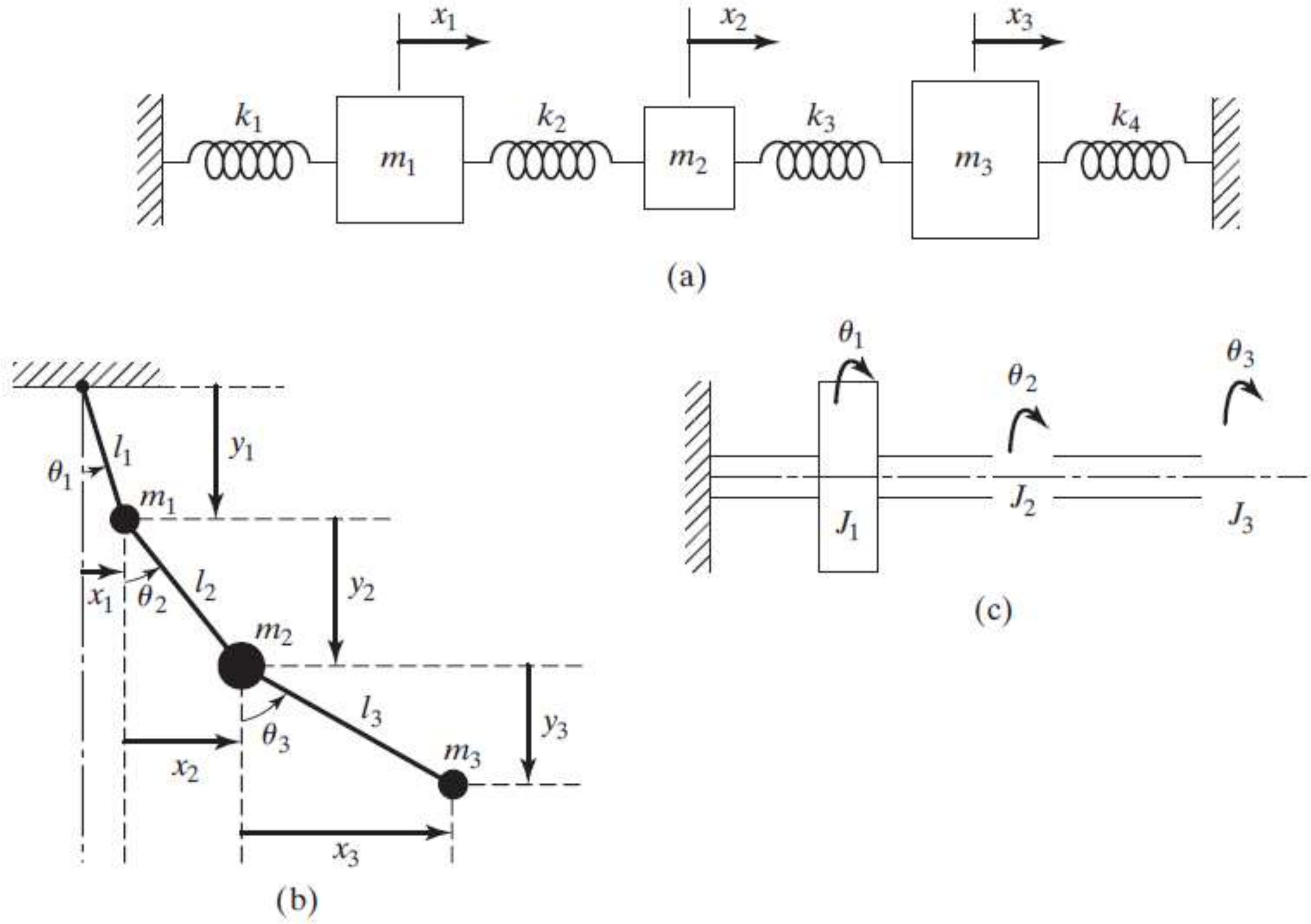




1.2 Degrees of freedom (cont...)



1.2 Degrees of freedom (cont…)



1.2 Degrees of freedom (cont…)

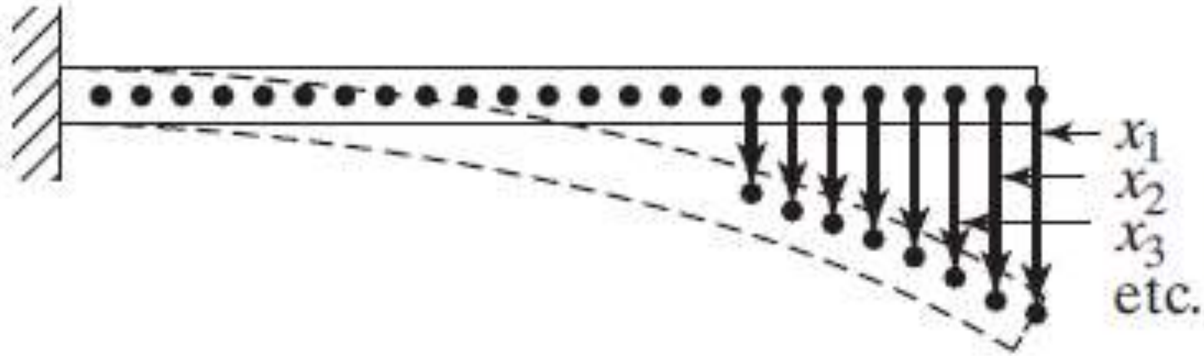


FIGURE 1.14 A cantilever beam (an infinite-number-of-degrees-of-freedom system).

1.3 Classification Vibration

Vibration can be classified in several ways. Some of the important classifications are as
follows.

a) Free and forced vibration
b) Undamped and damped vibration
c) Linear and nonlinear vibrations
d) Deterministic and random vibration

The terminology of **"Free Vibration"** is used for the study of natural vibration modes in the absence external loading.

The terminology of **"Forced Vibration"** is used for the study of motion as a result of loads that vary rapidly with time. Loads that vary rapidly with time are called dynamic loads.

1.3 Classification Vibration

If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as “**undamped vibration**”.

If any energy is lost in this way, however, is called “**damped vibration”.**

If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

Importance of Dynamic Analysis

Load magnification and Fatigue effects

A static load is constant and is applied to the structure for a considerable part of its life. For example, the self weight of building. Loads that are repeatedly exerted, but are applied and removed very slowly, are also considered static loads.

Fatigue phenomenon can be caused by repeated application of the load. The number of cycles is usually low, and hence this type of loading may cause what is known as low-cycle fatigue.

Quasi-static loads are actually due to dynamic phenomena but remain constant for relatively long periods.

Most mechanical and structural systems are subjected to loads that actually vary over time. Each system has a characteristic time to determine whether the load can be considered static, quasi-static, or dynamic. This characteristic time is *the fundamental period of free vibration of the system.*

Importance of Dynamic Analysis

Dynamic Load Magnification factor (DLF) is the ratio of the maximum dynamic force experienced by the system and the maximum applied load.

The small period of vibration results in a small DLF.

Fatigue phenomenon can be caused by repeated application of the load. The number of cycles and the stress range are important factors in determining the fatigue life.

1.3 Classification Vibration

1.4 Spring, inertia and damping elements

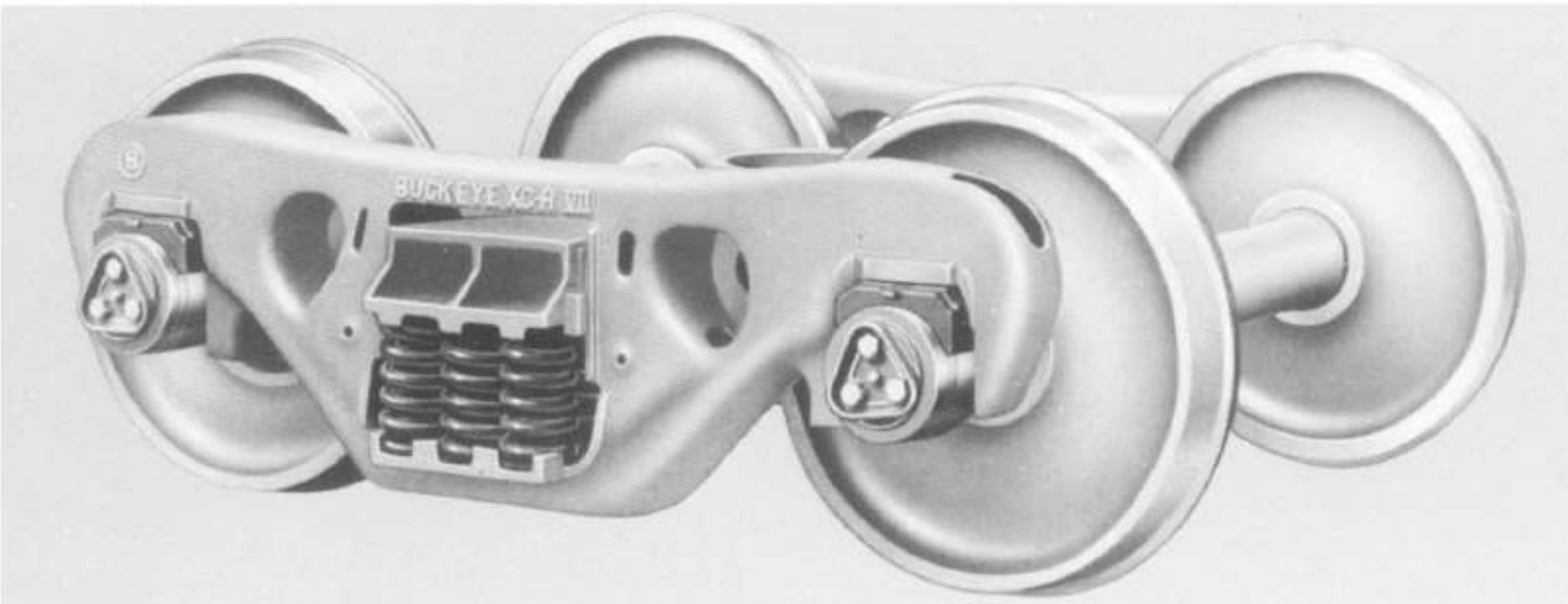
A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom (DOF) of the system.

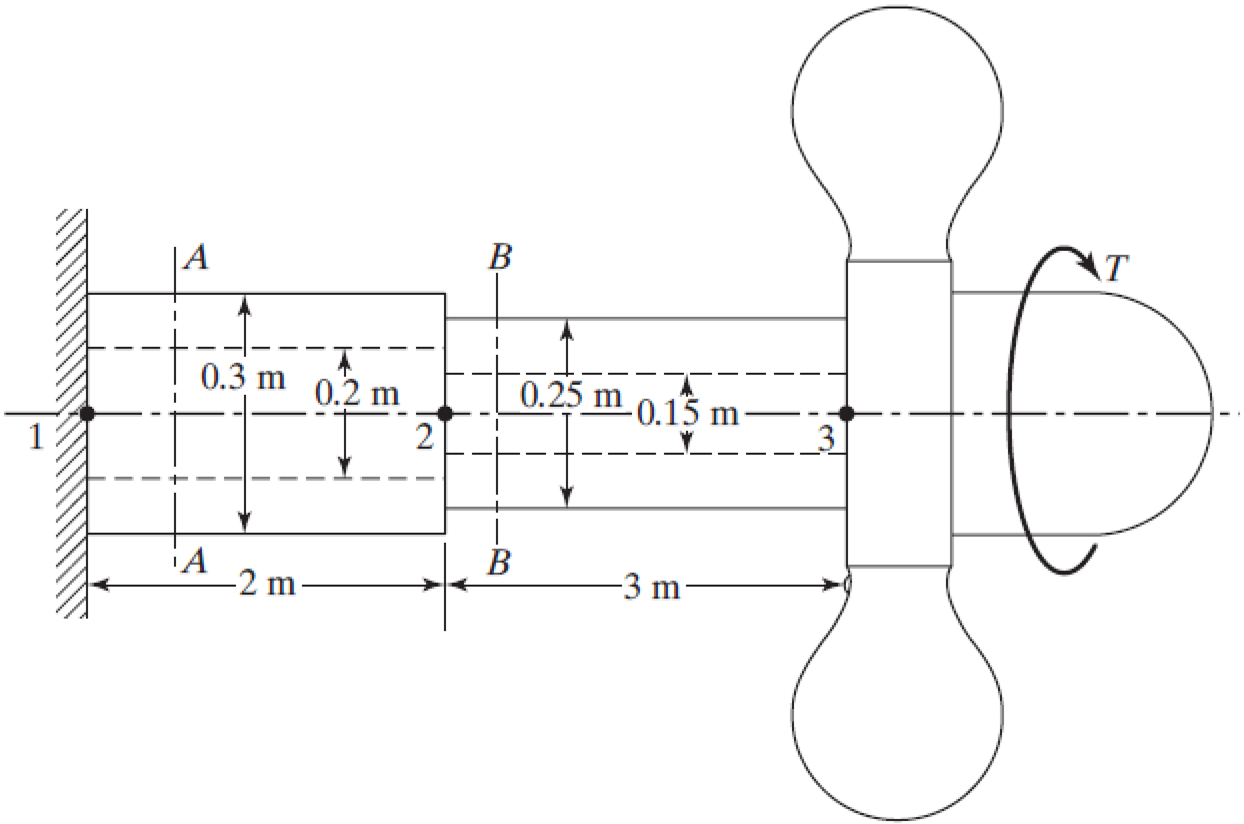
A large number of practical systems can be described using a finite number of DOFs. Systems with a finite number of DOFs are called *discrete* or *lumped parameter systems.*

Some systems, especially those involving continuous elastic members, have an infinite number of DOFs. Those systems are called *continuous or distributed systems.*

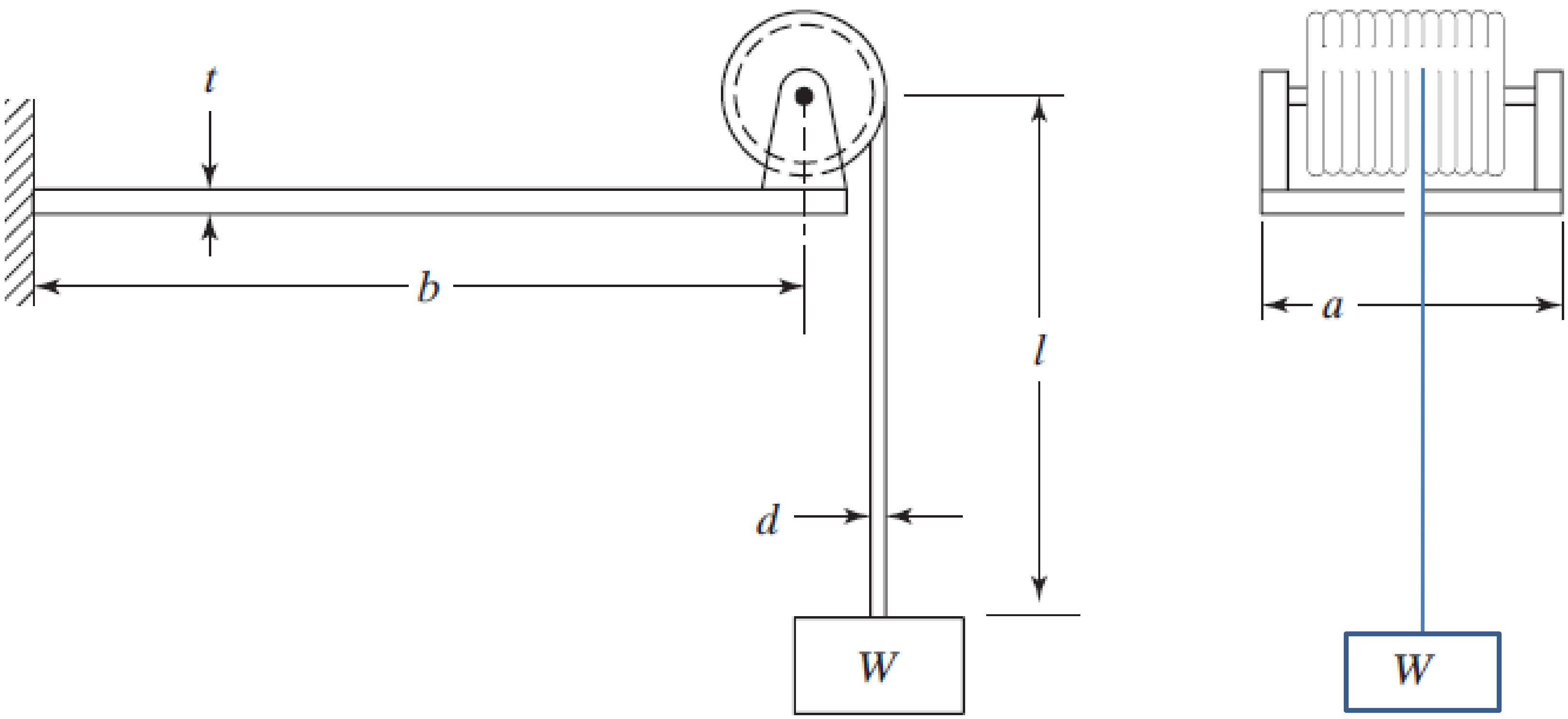
Parallel arrangement of springs in a freight truck



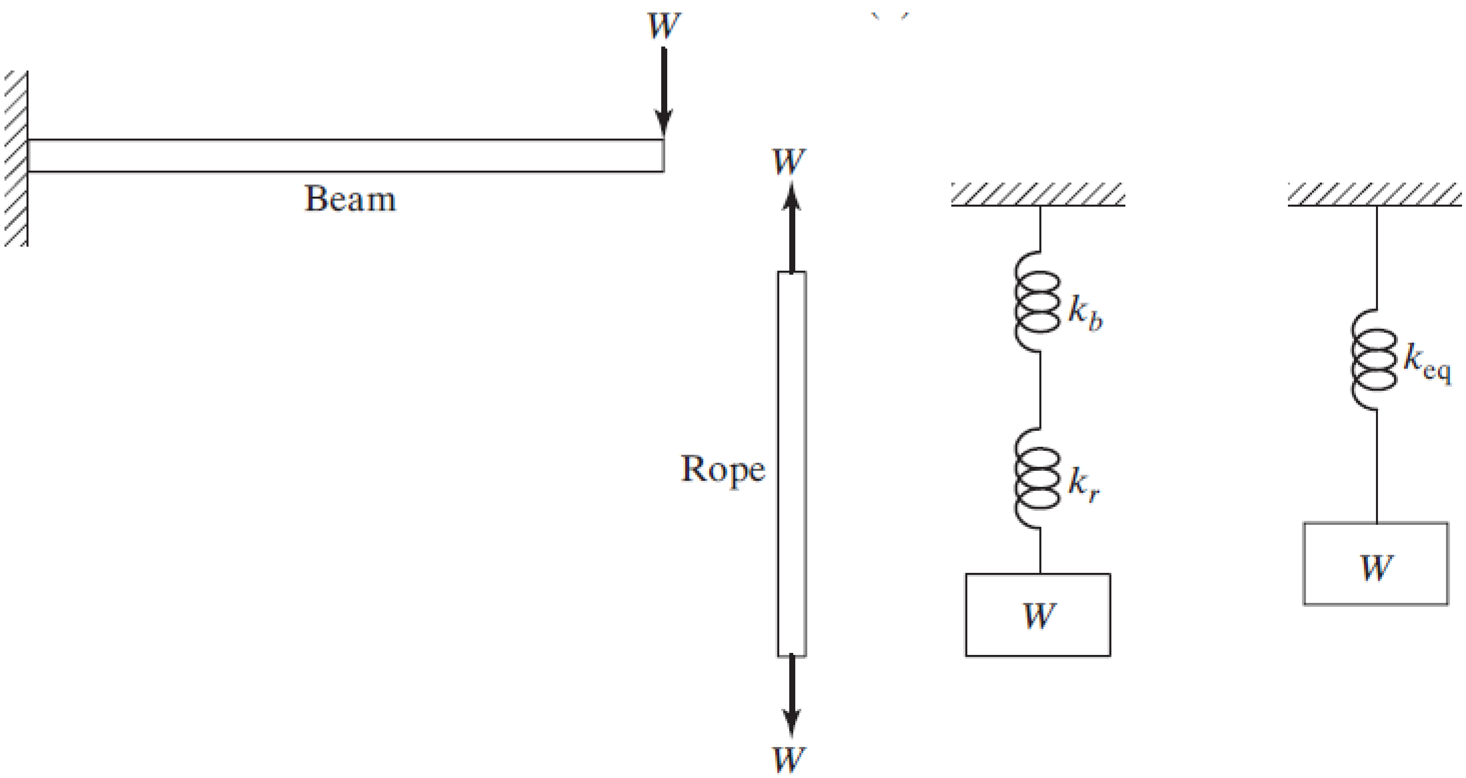
Torsional Spring Constant of a Propeller Shaft



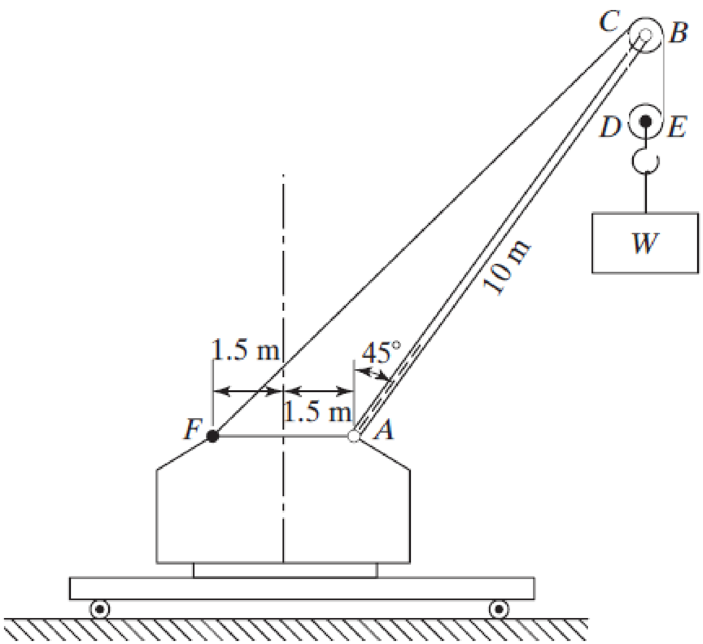
Equivalent k of Hoisting Drum

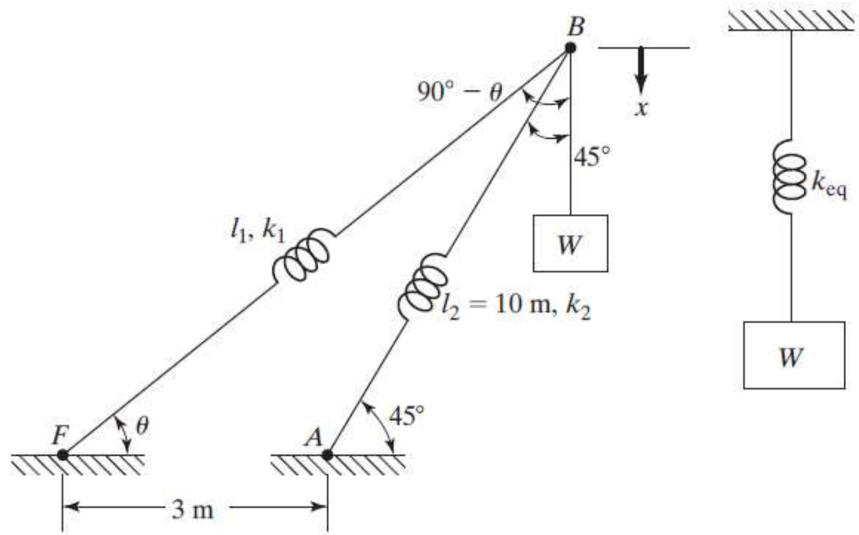


Equivalent k of Hoisting Drum



Equivalent k of a Crane





1.4 Dynamic Loads on Flight Vehicle Structures

Unsteady air loads – Atmospheric turbulence, gust, engine vibration

Pilots input to control surfaces for manoeuver

Landing impact

Runway unevenness'

Blast pressure

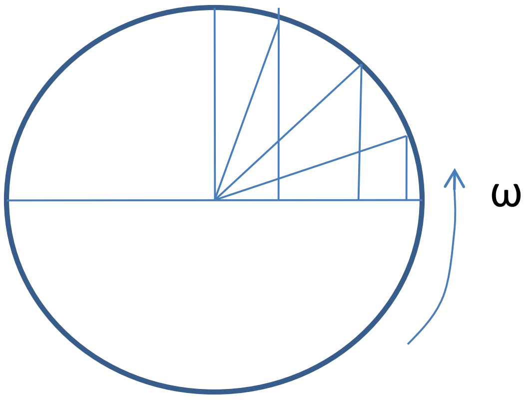
Acoustic loads

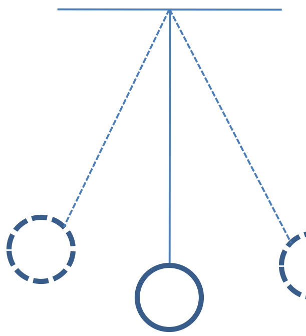
1.4 Spring, Damper and Mass elements

FIGURE 9.17 (a) Undamped spring mount; (b) damped spring mount; (c) pneumatic rubber mount. (Courtesy of *Sound and Vibration.*)

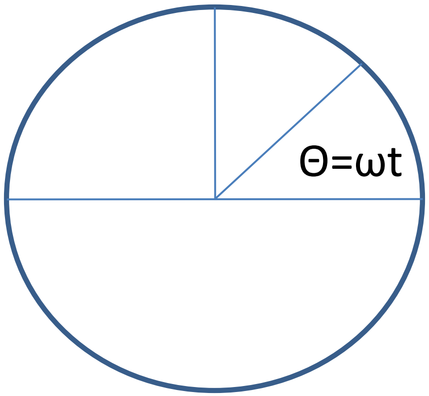
1.4.1 Simple Harmonic Motion (SHM)

A particle moves to and fro in such a way that the acceleration is always proportional to the displacement and directed towards origin, the motion is called SHM





A particle is moving along a circular path with constant velocity ω rad/sec



1.4.1 Simple Harmonic Motion (SHM)

$$x(t) = A\sin \omega t$$

$$\omega = \frac{2\pi}{\tau} = 2\pi f$$

$$\dot{x} = \omega A\cos \omega t$$

$$\ddot{x} = -\omega^2 A\sin \omega t = -\omega^2 x$$

$$\ddot{x} = -\omega^2 x$$

$$\ddot{x} + \omega^2 x = 0 \qquad (1.1)$$

1.4.2 Energy Method

- Application of conservation of energy
- For free vibration of undamped system, the energy is partly potential and partly kinetic
- Their sum is always constant

$$T + U = \text{constant} \qquad (1.2)$$

$$\frac{d}{dt}\left(T + U\right) = 0 \qquad (1.3)$$

- From principle of conservation of energy we can write

$$T_1 + U_1 = T_2 + U_2 \qquad (1.4)$$

- Let 1 and 2 are two instances of time
- Let 1 corresponds to equilibrium position, $U_1 = 0$
- Let 2 corresponds to maximum displacement, $T_2 = 0$
- Therefore, $T_1 + 0 = 0 + U_2 \qquad (1.5)$

1.4.2 Energy Method

- Since system is undergoing harmonic motion, then T_1 and U_2 are maximum values, hence

$$T_{max} = U_{max} \qquad (1.6)$$

- For a spring-mass system, kinetic energy is given by

$$T = \tfrac{1}{2} m\dot{x}^2$$

- Potential energy is given by

$$U = \tfrac{1}{2} kx^2$$

- Let $x = A\sin\omega t$, then one can write $\dot{x} = A\omega;\quad \dot{x}^2 = A^2\omega^2$

- Substituting for x and dx/xt in the expression for U and T one can write

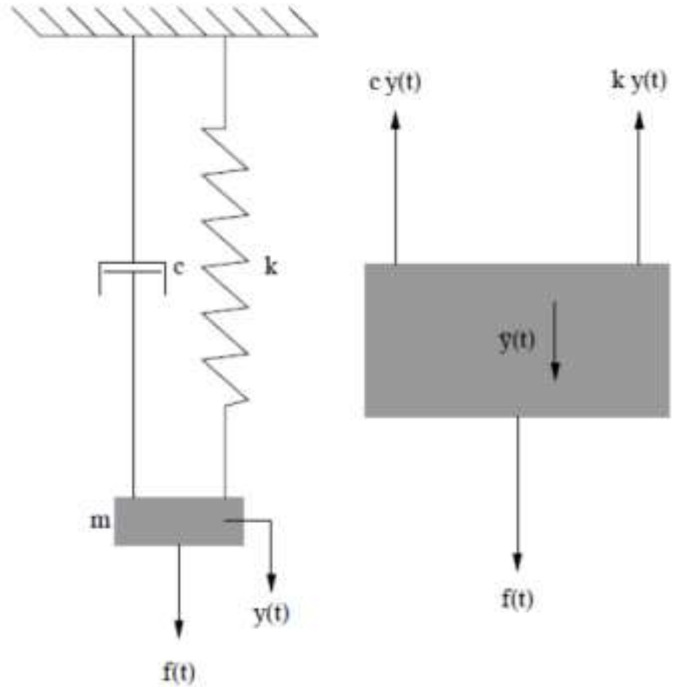
$$T_{max} = \tfrac{1}{2} mA^2\omega^2$$

$$U_{max} = \tfrac{1}{2} kA^2$$

$$\tfrac{1}{2} mA^2\omega^2 = \tfrac{1}{2} kA^2$$

$$\omega = \sqrt{\tfrac{k}{m}} \qquad (1.7)$$

1.5 Equations of motion



$$y(t) = A_1\, e^{p_1 t} + A_2\, e^{p_2 t} = e^{-(c/2\,m)\,t}\left(A_1\, e^{\left(\sqrt{(c/2\,m)^2-\omega^2}\right)t} + A_2\, e^{-\left(\sqrt{(c/2\,m)^2-\omega^2}\right)t}\right)$$

(a) Critical damping: $(c\,/\,2\,m)^2 = \omega^2 \quad \Longrightarrow \quad c_c = 2\,m\,\omega$

(b) Overdamped system: $(c\,/\,2\,m)^2 > \omega^2$

(c) Underdamped or lightly damped system: $(c\,/\,2\,m)^2 < \omega^2$

1.5 Equations of motion

Introducing the damping ratio,

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega}$$

Therefore,

$$p_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega^2} = -\xi\,\omega \pm \sqrt{(\xi\,\omega)^2 - \omega^2} = \omega\left(-\xi \pm \sqrt{\xi^2 - 1}\right)$$

$$y(t) = e^{-\xi\omega t}\left(A_1\, e^{\left(\omega\sqrt{\xi^2-1}\right)t} + A_2\, e^{-\left(\omega\sqrt{\xi^2-1}\right)t}\right)$$

Finally, we have

a) Critical damping: $\xi = 1$

b) Overdamped system: $\xi > 1$

c) Underdamped or lightly damped system: $0 < \xi < 1$

1.5 Equations of motion

The above can be classified as critically damped motion; nonoscillatory motion; and oscillatory motion.

- Underdamped or lightly-damped motion: $0 < \xi < 1$

$$y(t) = e^{-\xi\omega t}\left(u_0 \cos\omega_d t + \frac{\xi\omega u_0 + v_0}{\omega_d} \sin\omega_d t\right)$$

$$y(t) = e^{-\xi\omega t}(Y \sin\theta \cos\omega_d t + Y \cos\theta \sin\omega_d t) \equiv e^{-\xi\omega t}\, Y \sin(\omega_d t + \theta)$$

where

$$Y = \sqrt{u_0^2 + \left(\frac{\xi\omega u_0 + v_0}{\omega_d}\right)^2}$$

$$\theta = \tan^{-1}\left(u_0 \Big/ \left(\frac{\xi\omega u_0 + v_0}{\omega_d}\right)\right)$$

1.5 Equations of motion

Overdamped (Nonoscillatory) motion: $\xi > 1$

$$y(t) = e^{-\xi\omega t}\left(\frac{\xi\omega u_0+\sqrt{\xi^2-1}\;\omega u_0+v_0}{2\sqrt{\xi^2-1}\;\omega}e^{\left(\omega\sqrt{\xi^2-1}\right)t} - \frac{\xi\omega u_0-\sqrt{\xi^2-1}\;\omega u_0+v_0}{2\sqrt{\xi^2-1}\;\omega}e^{-\left(\omega\sqrt{\xi^2-1}\right)t}\right)$$

Critically damped motion: $\xi = 1$

Logarithmic Decrement

Logarithmic decrement: If there are the displacements at two consecutive peaks at t_1 and t_1+T_d

$$y(t_1) \equiv y_1 = e^{-\xi \omega t_1} Y \sin(\omega_d t_1 + \theta)$$

$$y(t_2) \equiv y_2 = e^{-\xi \omega (t_1+T_d)} Y \sin(\omega_d (t_1 + T_d) + \theta)$$

The *logarithmic decrement* is defined as

$$\delta = \ln\left(\frac{y_1}{y_2}\right) = \ln\left(\frac{e^{-\xi \omega t_1} Y \sin(\omega_d t_1 + \phi)}{e^{-\xi \omega (t_1+T_d)} Y \sin(\omega_d (t_1+T_d)+\phi)}\right)$$

$$\delta = \ln\left(\frac{e^{-\xi \omega t_1}}{e^{-\xi \omega (t_1+T_d)}}\right) = \ln\left(\frac{1}{e^{-\xi \omega T_d}}\right) = \ln\left(e^{\xi \omega T_d}\right) \equiv \xi \omega T_d$$

$$\delta = \xi \omega \left(\frac{2\pi}{\omega \sqrt{1-\xi^2}}\right) = \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

Logarithmic Decrement

The relationship between the logarithmic decrement and the damping ratio

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

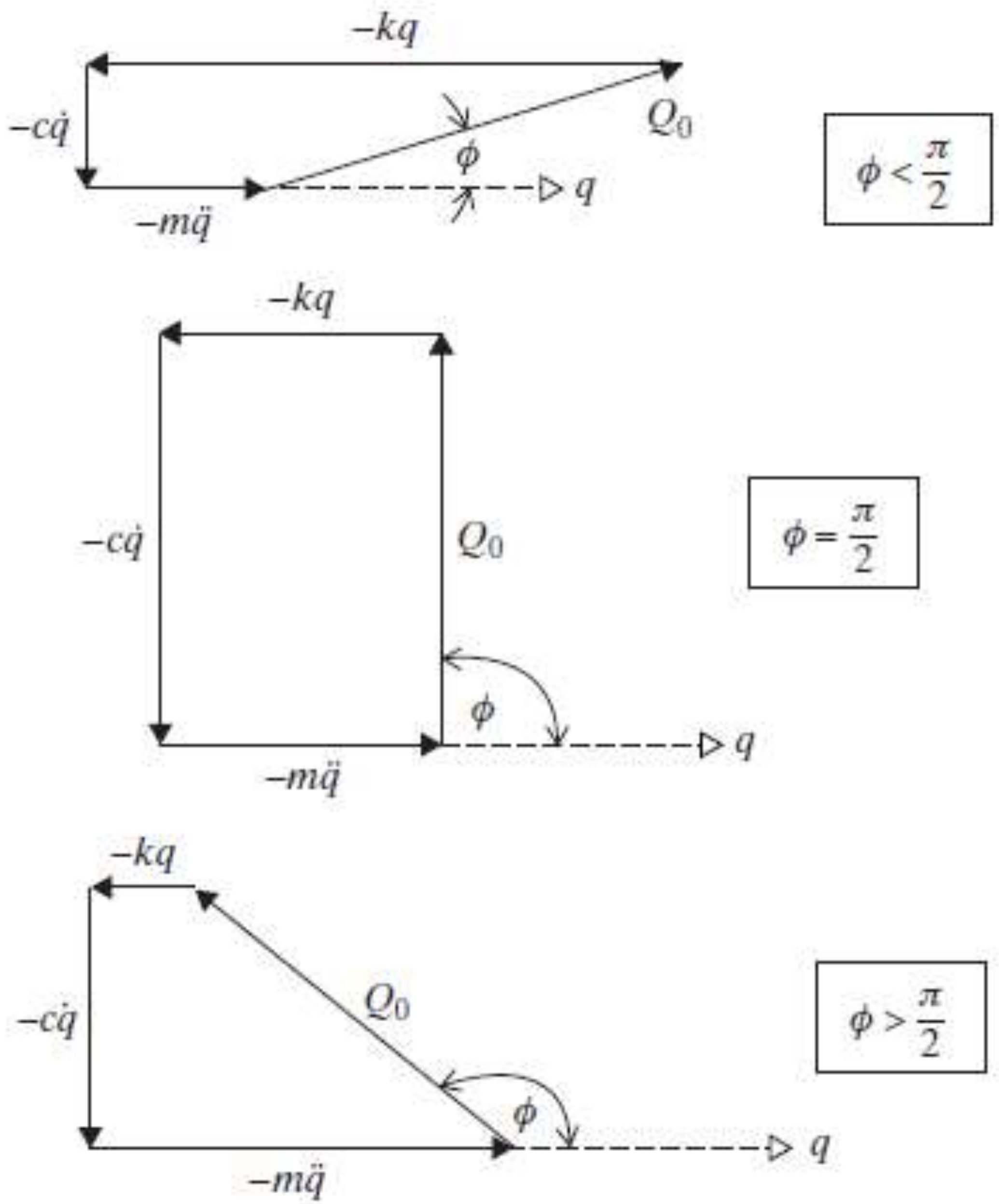
For lightly damped systems, the difference between two successive peaks may be too small to measure accurately. Since the logarithmic decrement between any two successive peaks is constant, we can determine the decrement from the first peak and the peak n cycles later.

$$\delta = \frac{1}{n}\ln\left(\frac{y_0}{y_n}\right)$$

1.7 Damped forced vibration

1.7.1 Resonance

Phase relationships among the applied, spring, damping, and inertia forces for harmonic motion for frequency ratio values less than one-half, equal to one, and equal to one and a half.



Modeling Mechanical Systems

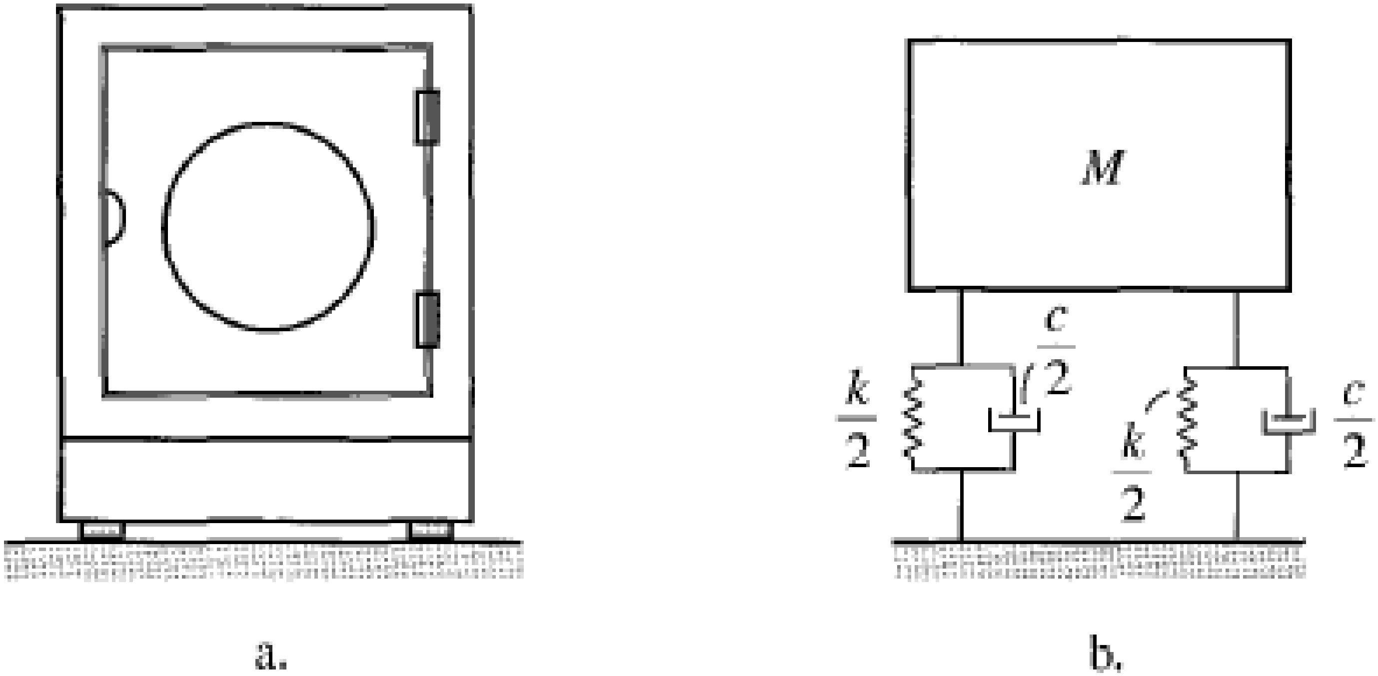


FIGURE 1.28
a. Washing machine, b. Model of washing machine

Modeling Structural Dynamic Systems

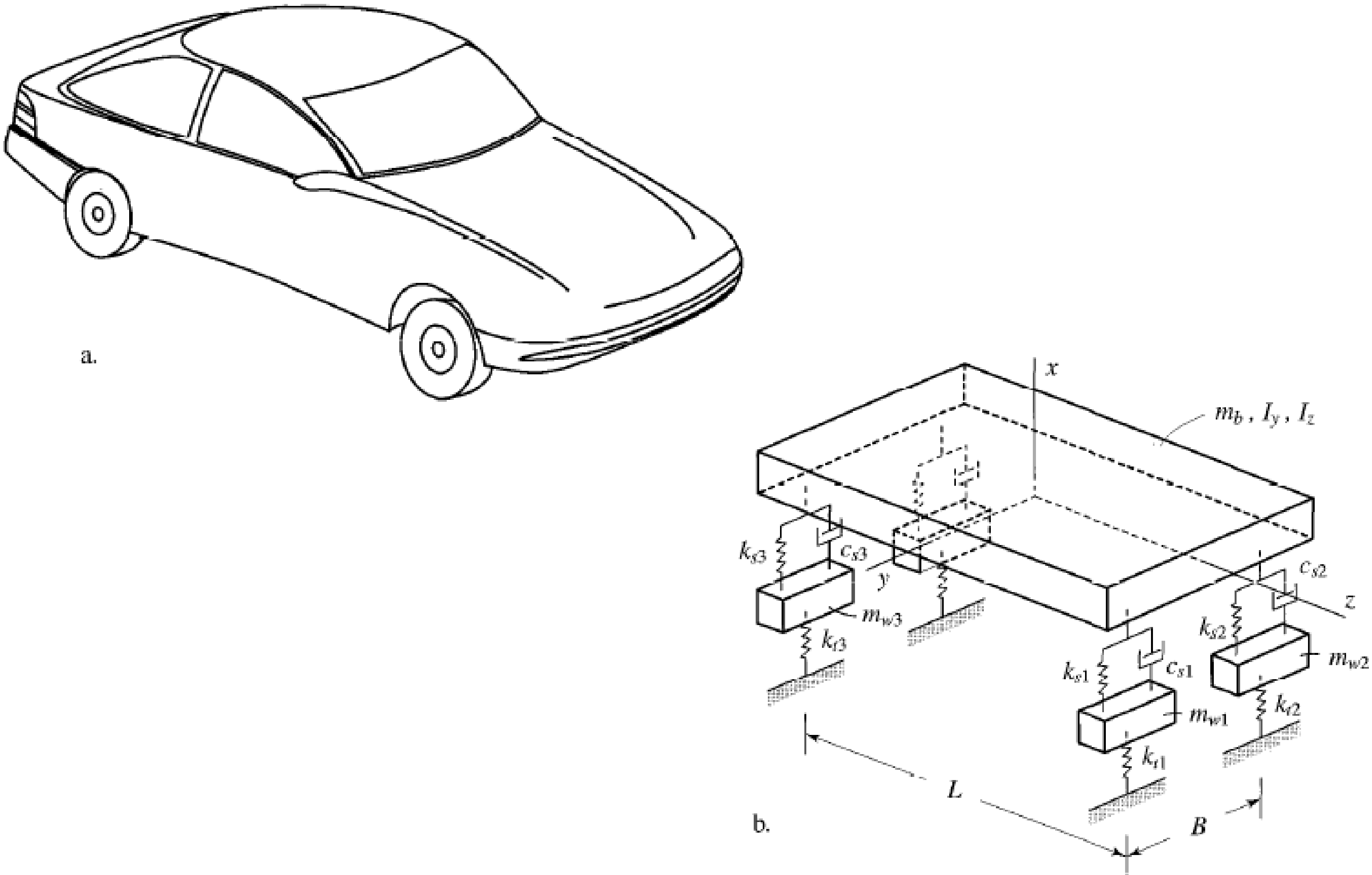


FIGURE 1.29
a. Automobile, b. Model of automobile

Modeling Structural Dynamic Systems

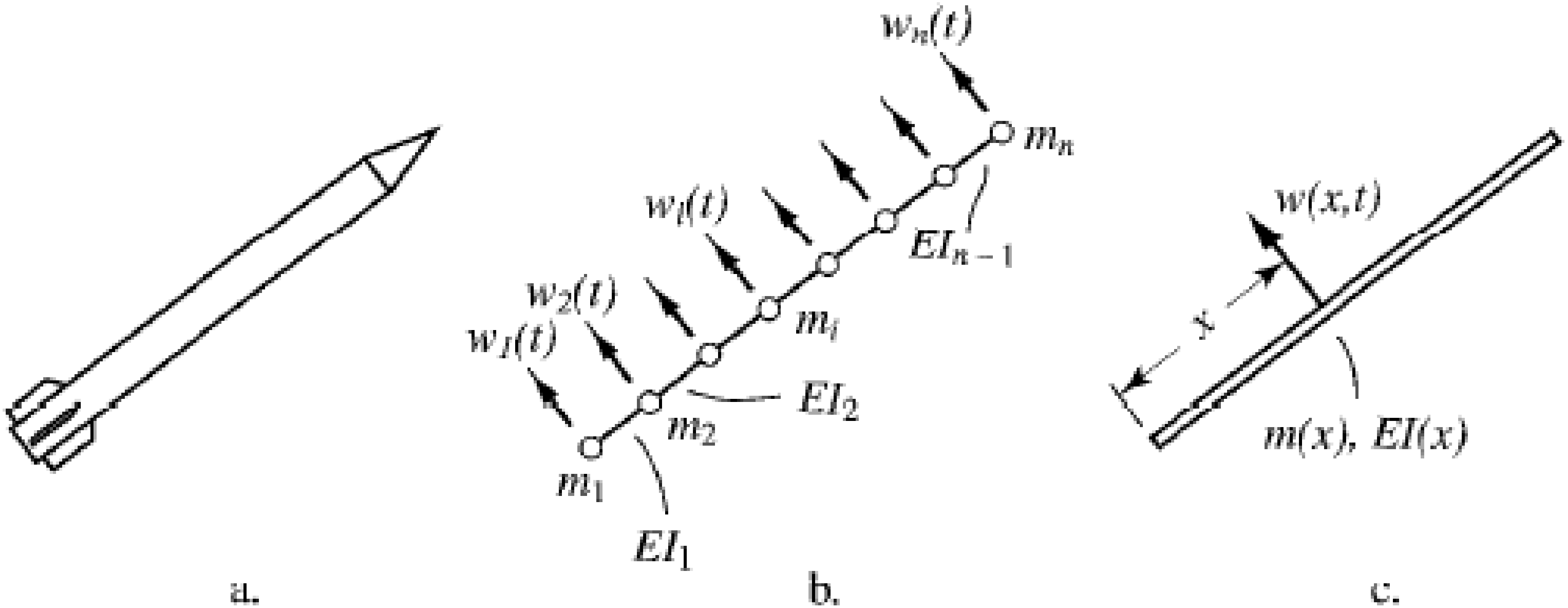


FIGURE 1.30

a. Missile in free flight, b. Discrete model, c. Distributed-parameter model

Modeling Structural Dynamic Systems

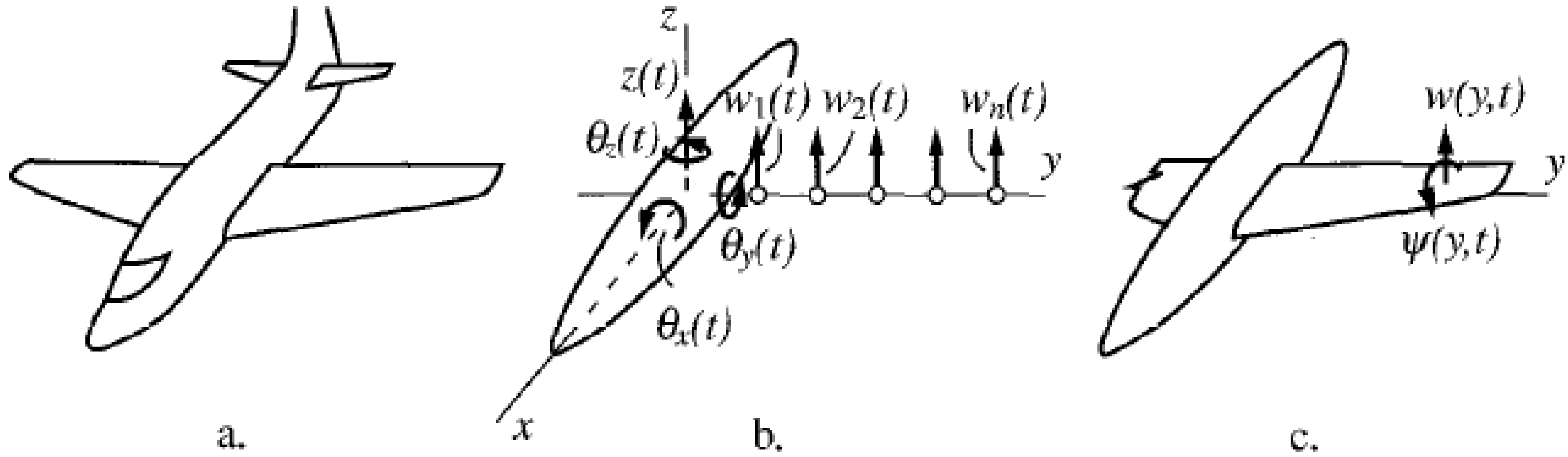


FIGURE 1.31
a. Aircraft in flight, b. Discrete model, c. Distributed-parameter model

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Mechanical Vibration and Structural Dynamics

Unit 2: Vibration of discrete System

Contents

Lecture No.	Date	UNIT	TOPIC	Reference	Pages
		II	**Vibration of discrete systems**		
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		2.2	Static and dynamic coupling		
			Examples		
		2.3	Principle coordinates		
			Principle modes		
		2.4	Orthogonality conditions		
		2.5	Extension to multiple-degrees-of-freedom systems		
		2.6	Vibration absorber		

2.0 Discrete and continuous system

- A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple systems shown in slides 5 to 7.

- Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom.

- As a simple example, consider the cantilever beam shown in slide 8.

- Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration.

- The infinite number of coordinates defines its elastic deflection curve.

- Thus the cantilever beam has an infinite number of degrees of freedom.

- Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom

- Systems with a finite number of degrees of freedom are called *discrete or lumped parameter systems,* and those with an infinite number of degrees of freedom are called *continuous or distributed systems.*

2.0 Discrete and continuous system (cont…)

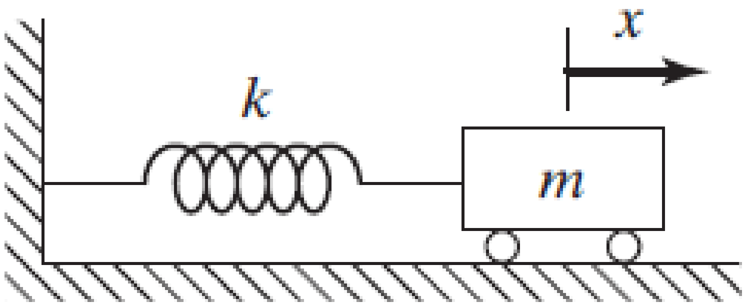
- Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner.

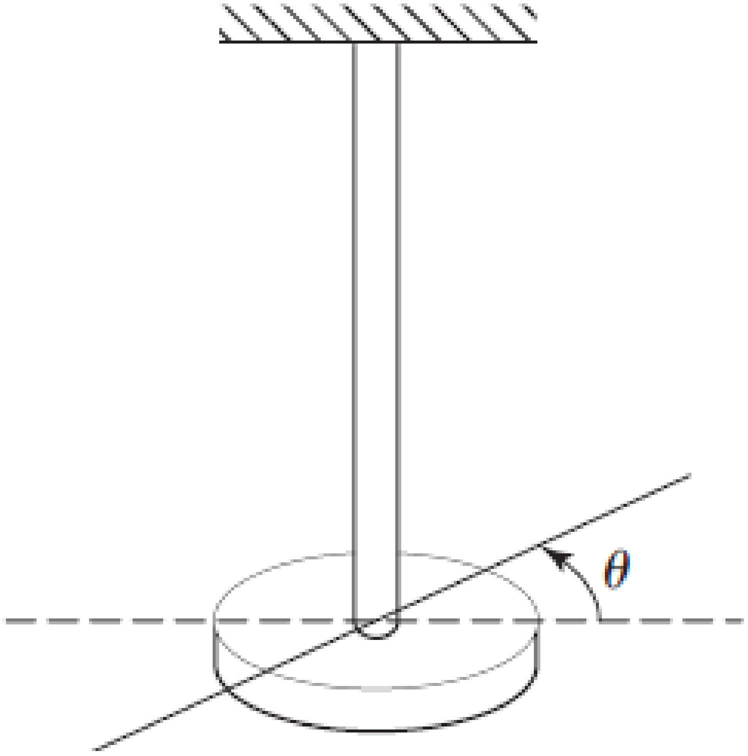
- Although treatment of a system as continuous gives exact results, the analytical methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods, and thin plates.

- Hence most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers.

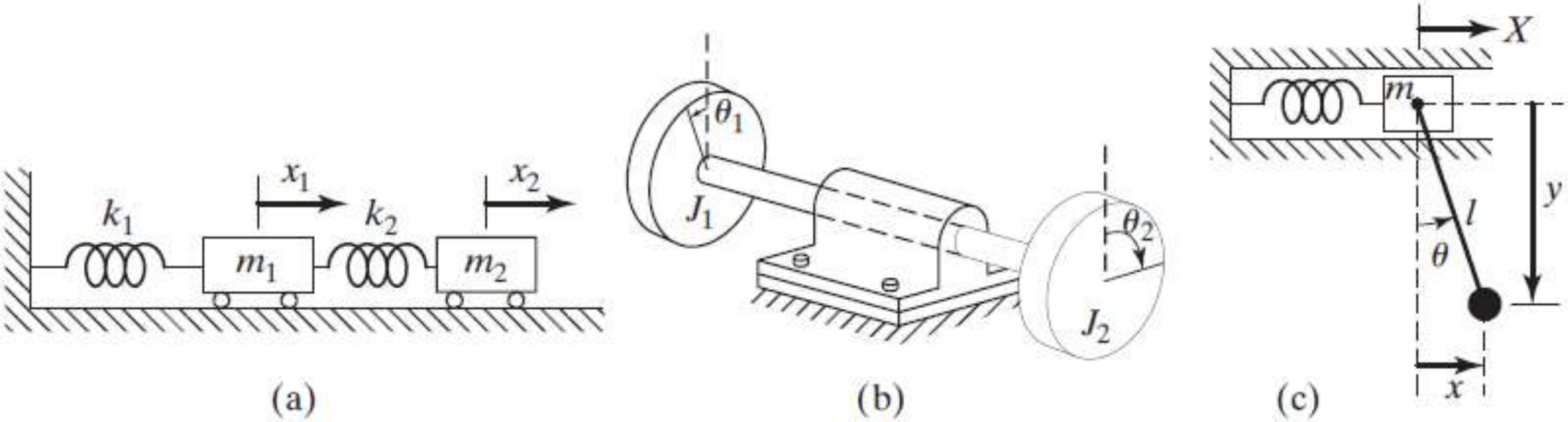
- In general, more accurate results are obtained by increasing the number of masses, springs, and dampers - that is, by increasing the number of degrees of freedom.

2.1 Two/Three-degree-of-freedom (MDOF) system

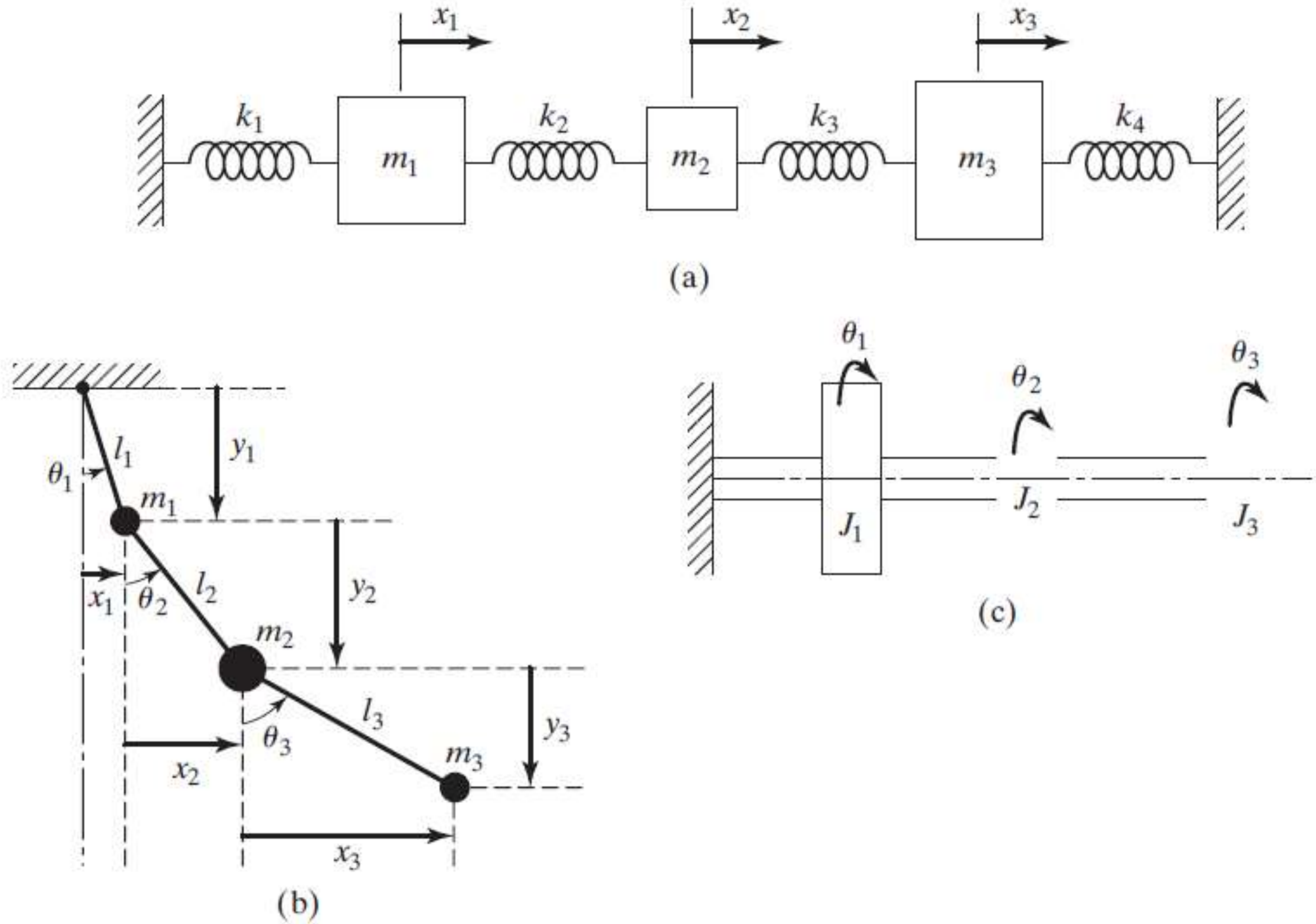




2.1 Two/Three-degree-of-freedom (MDOF) system



2.1 Two/Three-degree-of-freedom (MDOF) system



2.1 Two/Three-degree-of-freedom (MDOF) system

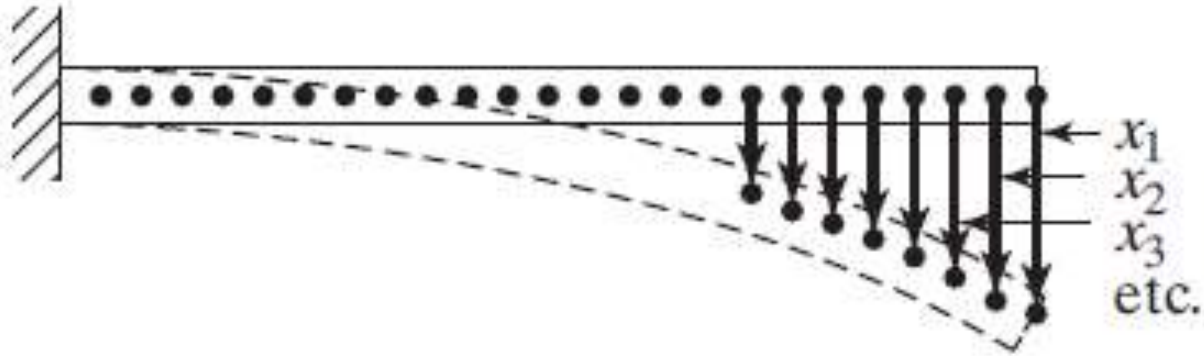
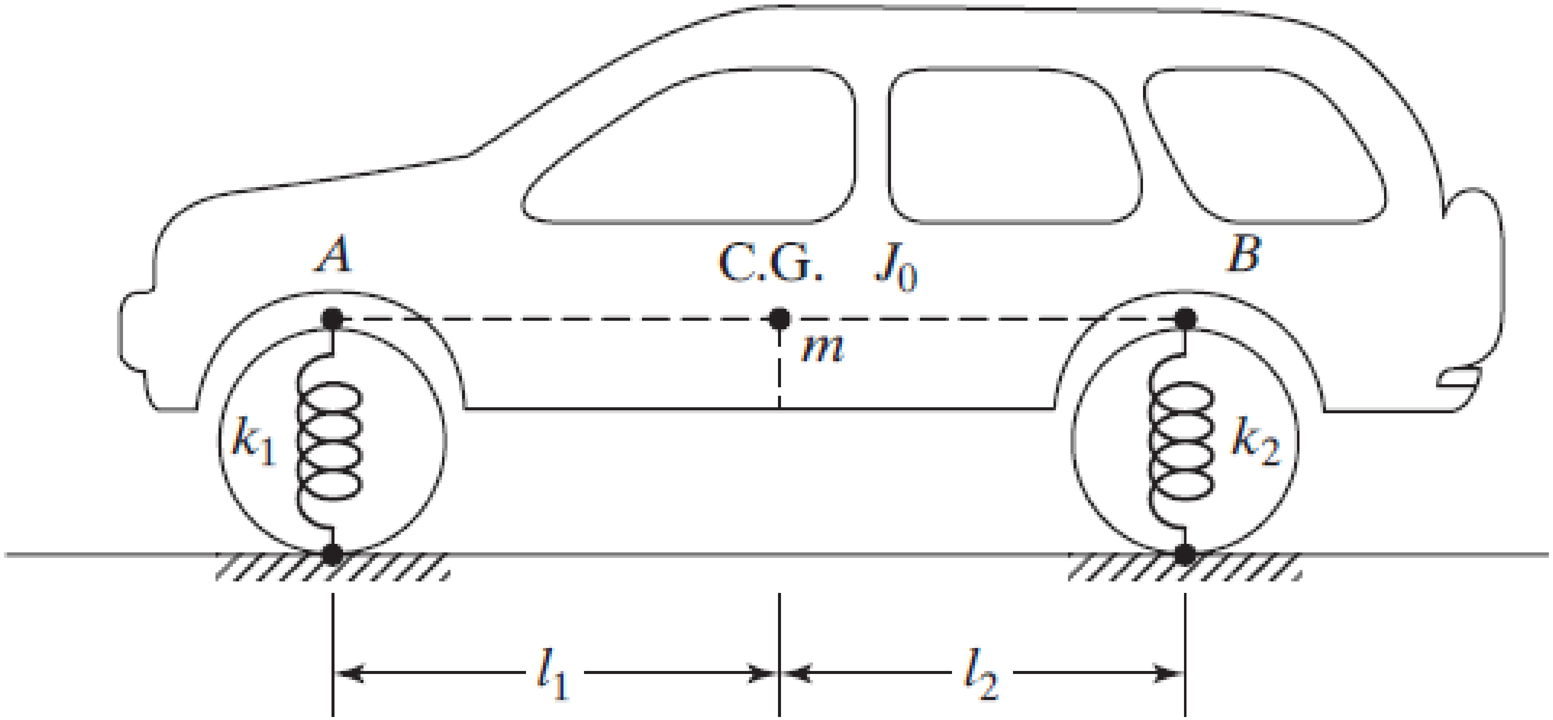
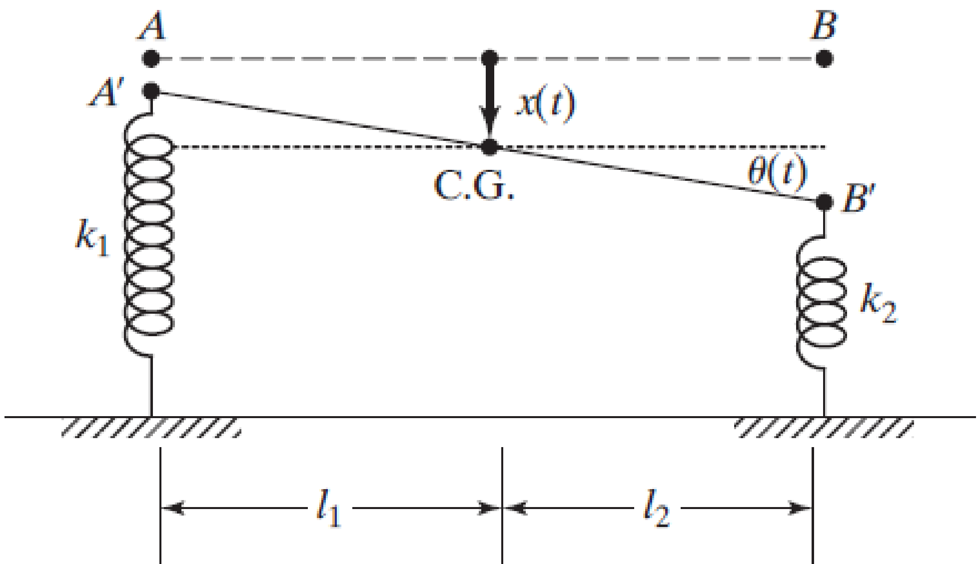


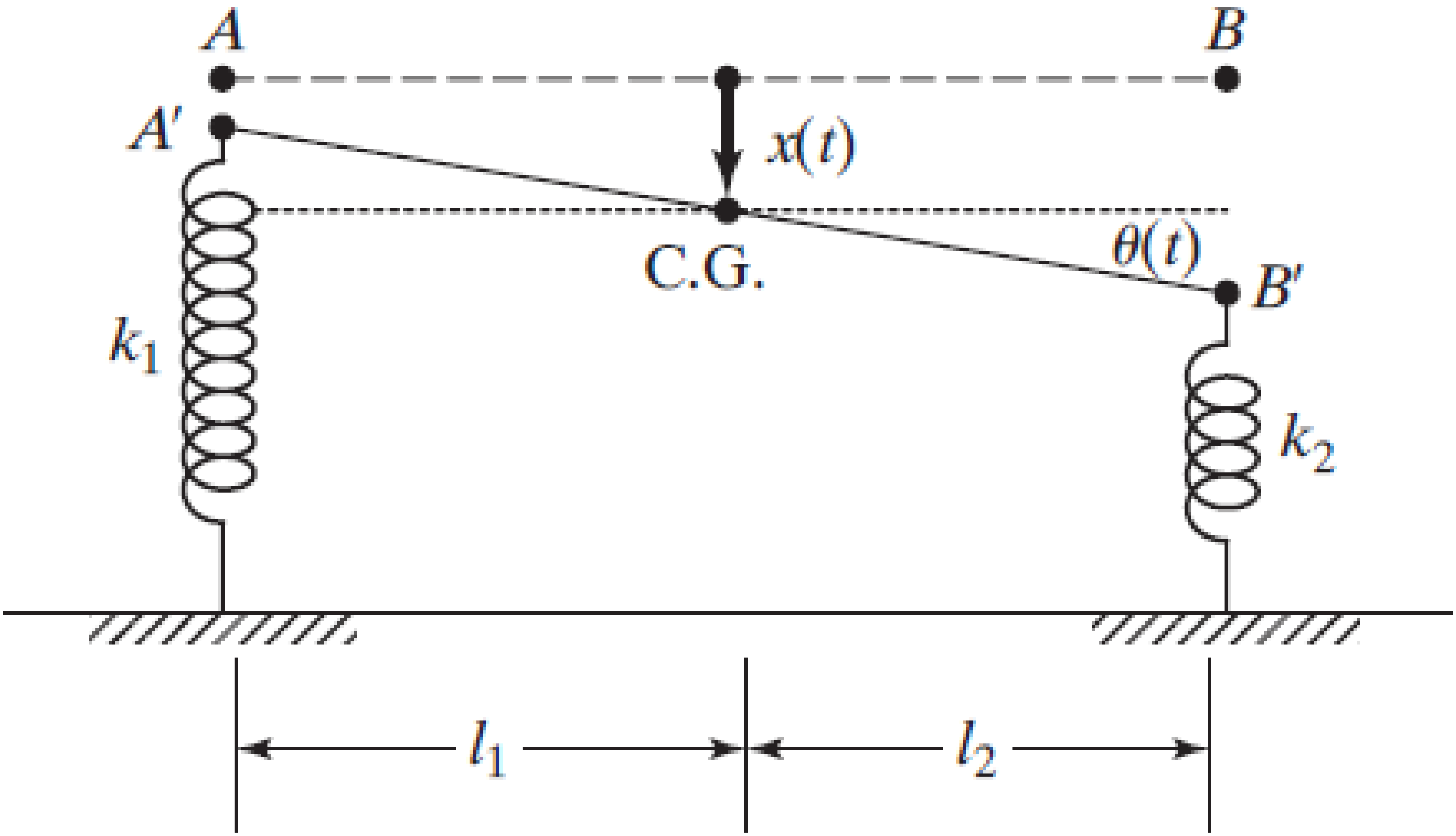
FIGURE 1.14 A cantilever beam (an infinite-number-of-degrees-of-freedom system).

2.2 Static and Dynamic couplings

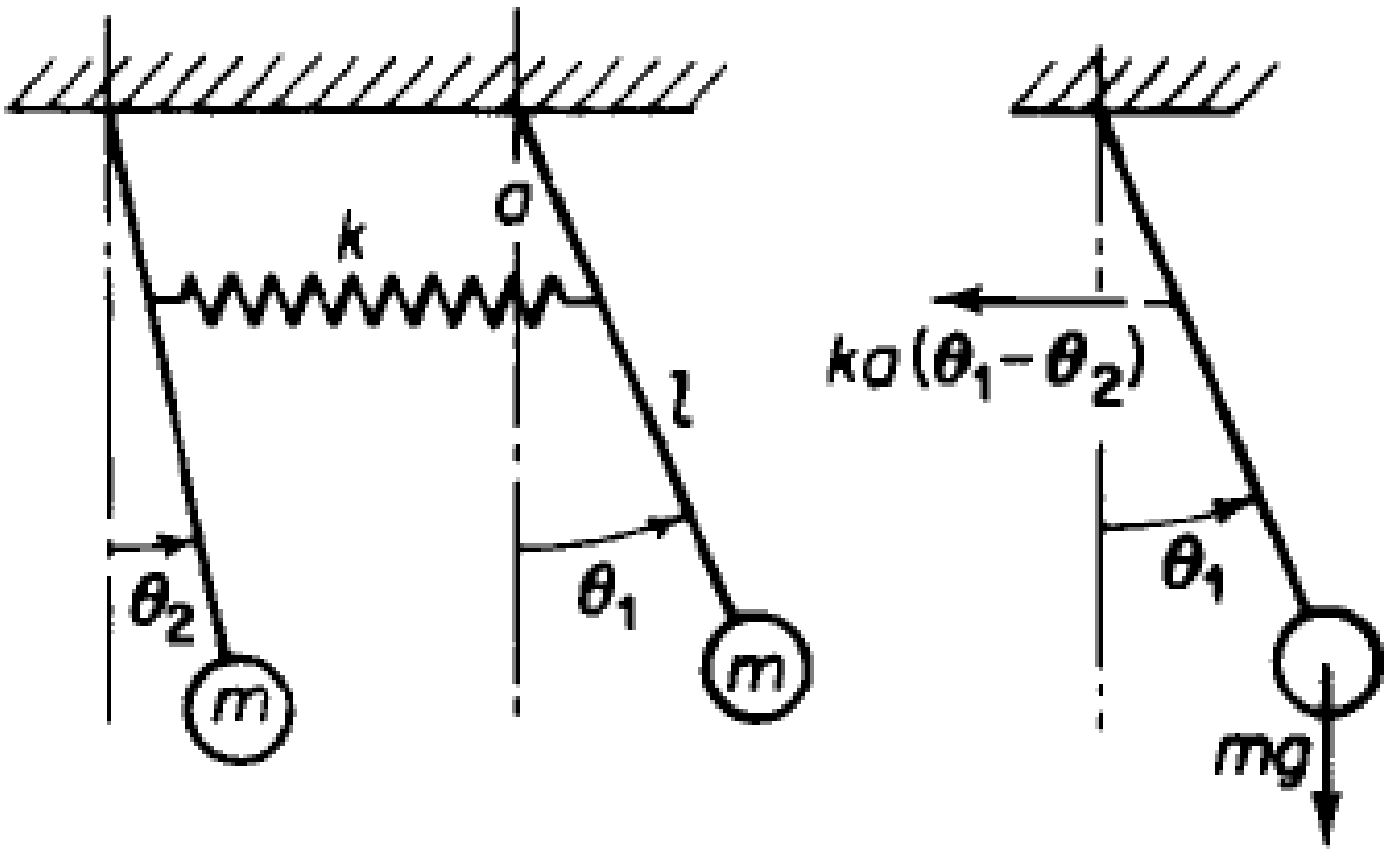




2.2 Static and Dynamic couplings



2.2 Static and Dynamic couplings



2.2 Static and Dynamic couplings

$$ml^2\ddot{\theta}_1 = -mgl\theta_1 - ka^2(\theta_1 - \theta_2)$$

$$ml^2\ddot{\theta}_2 = -mgl\theta_2 + ka^2(\theta_1 - \theta_2)$$

Assuming the normal mode solutions as

$$\theta_1 = A_1 \cos \omega t$$

$$\theta_2 = A_2 \cos \omega t$$

the natural frequencies and mode shapes are found to be

$$\omega_1 = \sqrt{\frac{g}{l}} \qquad \omega_2 = \sqrt{\frac{g}{l} + 2\frac{k}{m}\frac{a^2}{l^2}}$$

$$\left(\frac{A_1}{A_2}\right)^{(1)} = 1.0 \qquad \left(\frac{A_1}{A_2}\right)^{(2)} = -1.0$$

2.2 Static and Dynamic couplings

Figure below shows a rigid bar with its centre of mass not coinciding with its geometric centre, ie, $l_1 \neq l_2$, and supported by two springs, k_1 and k_2.

It represents a two degree of freedom since two coordinates are necessary to describe its motion

The choice of the coordinates will define the type of coupling which can be immediately determine from the mass and stiffness matrices.

Mass or **dynamic coupling** exists if the mass matrix is non-diagonal, whereas stiffness or **static coupling** exists if the stiffness matrix is non-diagonal.

It is possible to have both forms of coupling.

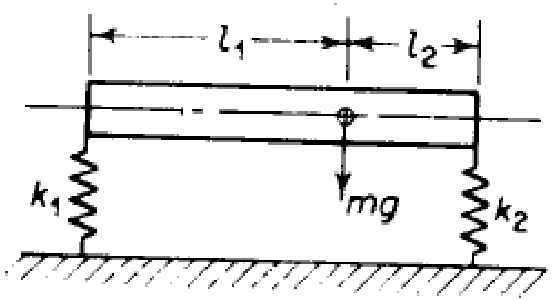


Figure 5.2-1.

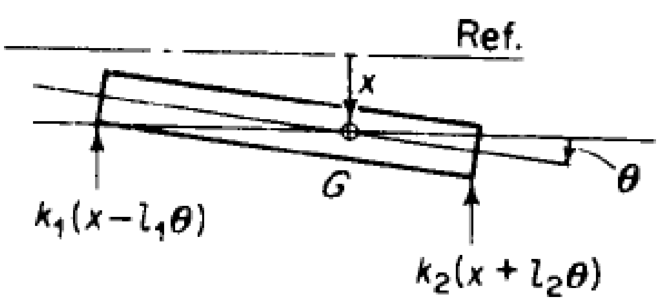


Figure 5.2-2. Coordinates leading to static coupling.

2.2 Static and Dynamic couplings

Static Coupling

Choosing coordinates x and θ shown in the figure below, where x is the linear displacement of the center of mass, the system will have static coupling as shown by the matrix equation

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & (k_2 l_2 - k_1 l_1) \\ (k_2 l_2 - k_1 l_1) & (k_1 l_1^2 + k_2 l_2^2) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

If $k_1 l_1 = k_2 l_2$, the coupling disappears, and we obtain uncoupled x and θ vibrations

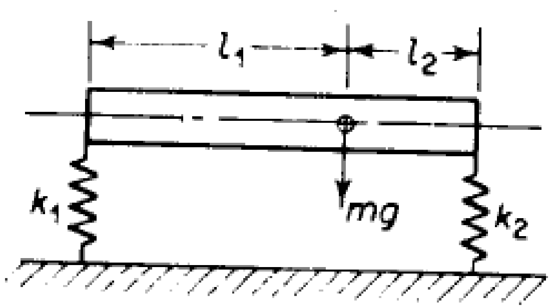


Figure 5.2-1.

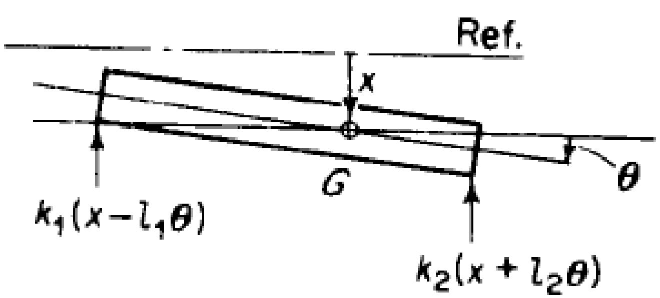


Figure 5.2-2. Coordinates leading to static coupling.

2.2 Static and Dynamic couplings

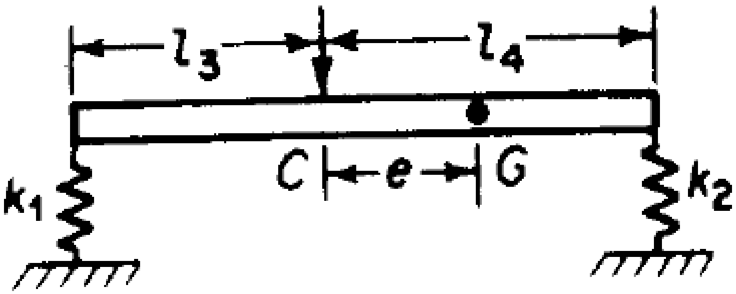
Dynamic Coupling

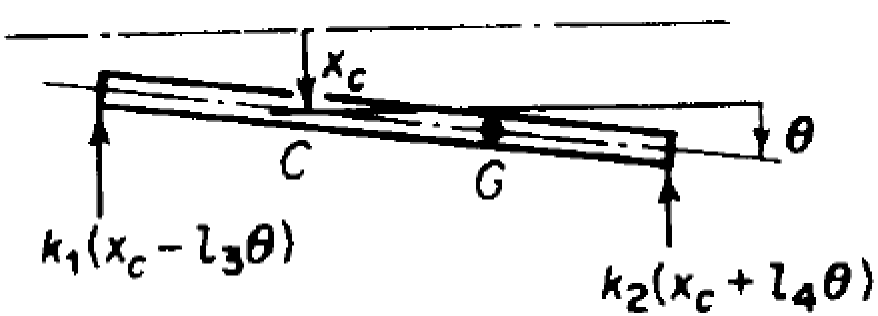
There is some point C along the bar where a force applied normal to the bar produces pure translation; i.e.,

The equations of motion in terms of x_c and θ can be shown to be

$$\begin{bmatrix} m & me \\ me & J \end{bmatrix}\begin{Bmatrix} \ddot{x}_c \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} (k_1+k_2) & 0 \\ 0 & (k_1 l_3^2+k_2 l_4^2) \end{bmatrix}\begin{Bmatrix} x_c \\ \theta \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Which shows that the coordinates chosen eliminated the static coupling and introduced dynamic coupling





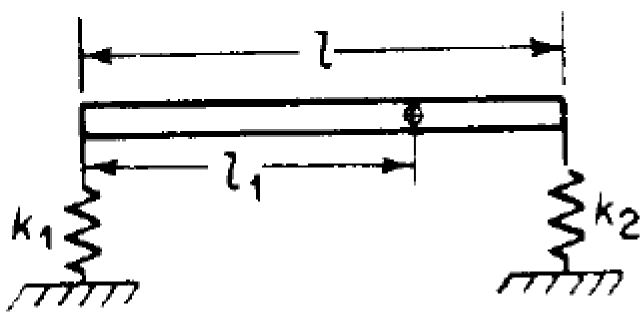
2.2 Static and Dynamic couplings

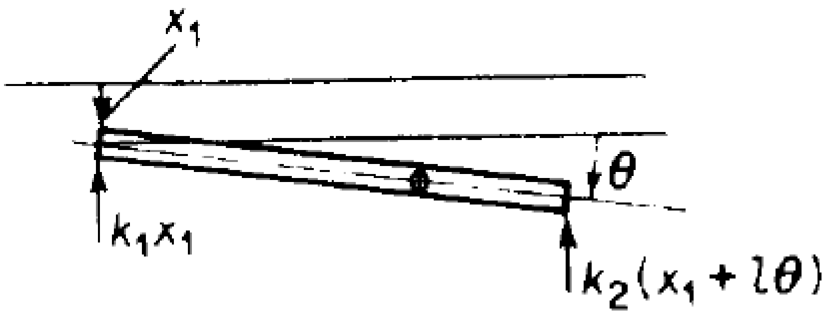
Static and Dynamic Coupling

If we choose x=x1 at the end of the bar, as shown in figure below, the equations of motion become

$$\begin{bmatrix} m & ml_1 \\ ml_1 & J_1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & k_2 l \\ k_2 l & k_2 l^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

and both static and dynamic coupling are now present





2.2 Static and Dynamic couplings

$$T = \frac{1}{2} m\dot{x}^2$$

2.2 Forced vibration of 2-DOF System

The equations of motion of a general two-degree-of-freedom system under external forces can be written as

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (2.3)$$

We shall consider the external forces to be harmonic:

$$F_j(t) = F_{j0} e^{i\omega t} \qquad j = 1,2 \qquad (2.4)$$

where ω is the forcing frequency.

We can write the steady-state solution as

$$x_j(t) = X_j e^{i\omega t} \qquad j = 1,2 \qquad (2.5)$$

where X_1 and X_2 are, in general, complex quantities that depend on and the system parameters.

2.2 Forced vibration of 2-DOF System

Substitution of Eqs. (2.4) and (2.5) into Eq. (2.3) leads to

$$\begin{bmatrix} \left(-\omega^2 m_{11} + i\omega c_{11} + k_{11}\right) & \left(-\omega^2 m_{12} + i\omega c_{12} + k_{12}\right) \\ \left(-\omega^2 m_{12} + i\omega c_{12} + k_{12}\right) & \left(-\omega^2 m_{22} + i\omega c_{22} + k_{22}\right) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} \qquad (2.6)$$

we define the mechanical impedance, $Z_{rs}(i\omega)$ as

$$Z_{rs}(i\omega) = -\omega^2 m_{rs} + i\omega c_{rs} + k_{rs} \qquad r, s = 1,2 \qquad (2.7)$$

and write Eq. (2.6) as

$$[Z(i\omega)]\vec{X} = \vec{F}_0 \qquad (2.8)$$

2.2 Forced vibration of 2-DOF System

where

$$[Z(i\omega)] = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{12}(i\omega) & Z_{22}(i\omega) \end{bmatrix} = \text{Impedance} \quad \text{matrix}$$

$$\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

and

$$\vec{F}_0 = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix}$$

Equation (5.32) can be solved to obtain

$$\vec{X} = [Z(i\omega)]^{-1} \vec{F}_0 \tag{2.9}$$

2.2 Forced vibration of 2-DOF System

where the inverse of the impedance matrix is given by

$$[Z(i\omega)]^{-1} = \frac{1}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)} \begin{bmatrix} Z_{22}(i\omega) & -Z_{12}(i\omega) \\ -Z_{12}(i\omega) & Z_{11}(i\omega) \end{bmatrix} \qquad (2.10)$$

Equations (2.9) and (2.10) lead to the solution

$$X_1(i\omega) = \frac{Z_{22}(i\omega)F_{10} - Z_{12}(i\omega)F_{20}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)}$$

$$X_2(i\omega) = \frac{-Z_{12}(i\omega)F_{10} - Z_{11}(i\omega)F_{20}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}^2(i\omega)} \qquad (2.11)$$

By substituting Eq. (2.11) into Eq. (2.5) we can find the complete solution.

2.4 Multiple-degree-of-freedom Linear System

Equations of Motion

2.4.1 Position Vector

Let P_0 be the space coordinates of a point of an elastic mechanical system at a time t_o.

Because of the application of an external force at $t = t_o$, the point in consideration will occupy a new position P at a time t.

The vector PP_o will thus represent the displacement of the point with initial position P_0.

If we now consider a discrete system, or a continuum that has been approximated as a discrete system using a set of generalized coordinates q, we can write

$$P = F(q) \tag{2.5}$$

where q is the set of the generalized coordinates that define completely the mechanical system and F is the transformation operator.

For a linear system, the transformation operator F does not depend on the generalized coordinates q, and thus we can write for any point j of the mechanical system

2.4 Multiple-degree-of-freedom Linear System

$$P_j = \begin{bmatrix} \dfrac{\partial P_j}{\partial q_1} & \dfrac{\partial P_j}{\partial q_2} & \cdots & \dfrac{\partial P_j}{\partial q_n} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{Bmatrix} \qquad (2.6)$$

where $\partial P_j / \partial q_i$ are constants that do not depend on the generalized coordinates for a linear system and that represent the variation in the displacement at the point in consideration due to a unit variation in the generalized coordinate q_i.

In this section, to simplify the notation, we will use Einstein's summation notation for repeated indices, and we write Eq. (2.6) as

$$P_j = \sum_{i=1}^{n} \left[\frac{\partial P_j}{\partial q_i} \right] q_i = \frac{\partial P_j}{\partial q_i} q_i \qquad (2.7)$$

2.4 Multiple-degree-of-freedom Linear System

2.4.2 Velocity Vector

The velocity at any point j of the mechanical elastic system at a time t can be written as

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

Using Eq. (2.6), we can write the velocity vector as

$$V_j = \frac{dP_j}{dt} = \frac{dP_j}{\partial q_i}\frac{dq_i}{dt} = \frac{dP_j}{\partial q_i}q_i^{'} \qquad (2.9)$$

where $q_i^{'} = dq_i / dt$

2.4.3 Kinetic Energy Functional

The kinetic energy functional of the elastic mechanical system reads

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

2.4 Multiple-degree-of-freedom Linear System

Where ρ(P) is the material density at a point P, V(P) is the velocity vector at point P, and v is the volume of the elastic mechanical system.

For a discrete system we can use Eqs. (2.9) and (2.10) and write kinetic energy functional as

$$T = \frac{1}{2} q_j^{'} \left[\int_v \rho \frac{\partial P}{\partial q_j} \cdot \frac{\partial P}{\partial q_i} dv \right] q_i^{'} \qquad (2.11)$$

Or, in matrix notation, we can write

$$T = \tfrac{1}{2} \{q'\}^T [M] \{q'\} \qquad (2.12)$$

We call [M] the mass matrix of the mechanical system.

The elements of the mass matrix are given by

$$M_{ij} = \int_v \rho \frac{\partial P}{\partial q_i} \frac{\partial P}{\partial q_j} dv \qquad (2.13)$$

2.4 Multiple-degree-of-freedom Linear System

We conclude from Eq. (2.13) that the mass matrix is a symmetrical real matrix and because the expression $\{q'\}^T[M]\{q'\}$ represents an energy expression for any vector $\{q'\}$ different from the null vector, we further conclude that

$$\{x\}^T[M]\{x\} > 0 \qquad \forall\{x\} \neq \{0\} \qquad (2.14)$$

Therefore, [M] is a positive definite matrix

2.4.4 Strain Energy Functional

The stress-strain relationship for an elastic linear continuum can be written as

$$\{\sigma\} = [C]\{\varepsilon\} \qquad (2.15)$$

where [C] is the material constitutive matrix and is a symmetric matrix because the stress and strain tensors are symmetric tensors.

Writing now the strain-displacement relationship as

$$\{\varepsilon\} = [d]\{P\} \qquad (2.16)$$

2.4 Multiple-degree-of-freedom Linear System

where [d] is the differential operator relating the strains to the displacements, and substituting Eq. (2.7) into Eq. (2.16), we obtain

$$\{\varepsilon\} = [d][N]\{P\} \tag{2.17}$$

where [N] has been used to denote the transformation matrix of the displacements to the generalized coordinates. The strain energy functional of the elastic mechanical system reads

$$U = \frac{1}{2}\int_v \{\sigma\}^T \{\varepsilon\} dv \tag{2.18}$$

Using now the relation of Eqs. (2.15) and (2.17) and Eq. (2.18), we can write the strain energy functional as

$$U = \tfrac{1}{2}\{q\}^T \int_v [N]^T [d]^T [C][d][N] dv \{q\} \tag{2.19}$$

or

$$U = \tfrac{1}{2}\{q\}^T [K]\{q\} \tag{2.20}$$

where

$$[K] = \int_v [N]^T [d]^T [C][d][N] dv \tag{2.21}$$

2.4 Multiple-degree-of-freedom Linear System

We call [K] the stiffness matrix of the elastic mechanical system.

Again, we observe that [K] is a real symmetrical matrix because the constitutive material matrix is a symmetric matrix and is real.

Furthermore, from energy consideration concepts, we conclude from Eq. (2.20) that [K] is a positive definite matrix for a constrained mechanical elastic system or a semi-positive definite matrix for an elastic mechanical free body.

2.4.5 Expression of the Dissipation Function

We consider in this section that the damping forces of the elastic mechanical system are of viscous nature and are linearly related to the velocity vector, and we write

$$F_D(P) = \frac{\partial F_D(P)}{\partial q_i'} q_i' \qquad (2.22)$$

where $F_D(P)$ is the damping force of the elastic mechanical system at point P.

The variation in the virtual work of the damping forces in a virtual displacement δP reads

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_v \rho(P)V(P).V(P)dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_{v} \rho(P) V(P).V(P) dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.4 Multiple-degree-of-freedom Linear System

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2.4 Multiple-degree-of-freedom Linear System

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2.4 Multiple-degree-of-freedom Linear System

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2.4 Multiple-degree-of-freedom Linear System

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2.4 Multiple-degree-of-freedom Linear System

$$T = \frac{1}{2}\int_v \rho(P) V(P).V(P) dv \qquad (2.10)$$

$$V_j = \frac{dP_j}{dt} \qquad (2.8)$$

2.5 Coordinate Coupling and Principle coordinates

As stated earlier, an n-degree-of-freedom system requires n independent coordinates to describe its configuration.

Usually, these coordinates are independent geometrical quantities measured from the equilibrium position of the vibrating body.

However, it is possible to select some other set of n coordinates to describe the configuration of the system.

The latter set may be, for example, different from the first set in that the coordinates may have their origin away from the equilibrium position of the body.

There could be still other sets of coordinates to describe the configuration of the system. Each of these sets of n coordinates is called **the generalized coordinates**

2.7 Vibration Absorber

The vibration absorber, also called dynamic vibration absorber, is a mechanical device used to reduce or eliminate unwanted vibration.

It consists of another mass and stiffness attached to the main (or original) mass that needs to be protected from vibration.

Thus the main mass and the attached absorber mass constitute a two-degree-of-freedom system, hence the vibration absorber will have two natural frequencies.

The vibration absorber is commonly used in machinery that operates at constant speed, because the vibration absorber is tuned to one particular frequency and is effective only over a narrow band of frequencies.

Common applications of the vibration absorber include reciprocating tools, such as sanders, saws, and compactors, and large reciprocating internal combustion engines which run at constant speed (for minimum fuel consumption).

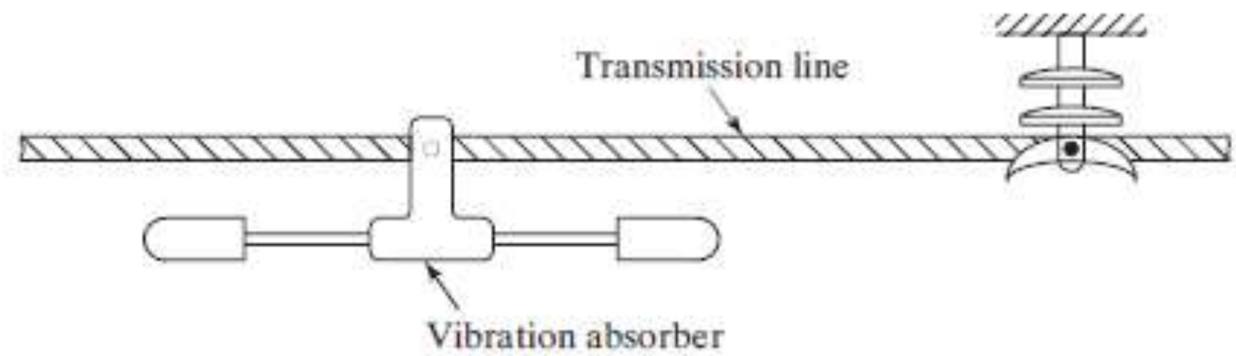
2.7 Vibration Absorber

In these systems, the vibration absorber helps balance the reciprocating forces.

Without a vibration absorber, the unbalanced reciprocating forces might make the device impossible to hold or control.

Vibration absorbers are also used on high-voltage transmission lines.

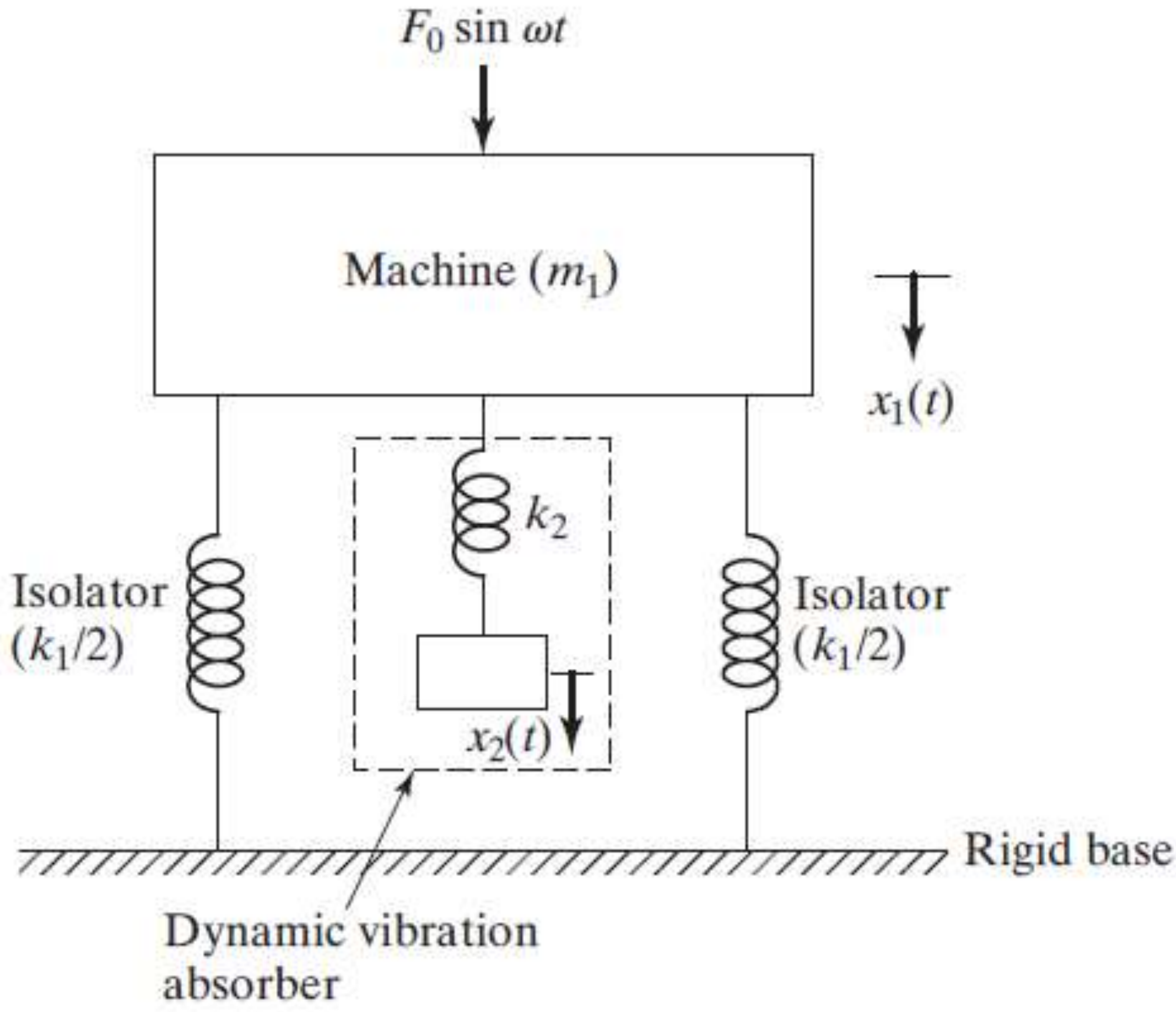
In this case, the dynamic vibration absorbers, in the form of dumbbell-shaped devices (Figure below), are hung from transmission lines to mitigate the fatigue effects of wind induced vibration.



2.7 Vibration absorber

2.7 Vibration absorber

2.7 Dynamic Vibration Absorber



2.7 Dynamic Vibration Absorber

When we attach an auxiliary mass m_2 to a machine of mass m_1 through a spring of stiffness k_2 the resulting two-degree-of-freedom system will look as shown in Figure in next slide.

The equations of motion of the masses m_1 and m_2 are

$$m_1\ddot{x}_1 + k_1 x_1 + k_2\left(x_1 - x_2\right) = F_0 \sin \omega t$$

$$m_2\ddot{x}_2 + k_2\left(x_2 - x_1\right) = 0 \qquad (2.30)$$

By assuming harmonic solution,

$$x_j\left(t\right) = X_j \sin \omega t \qquad j = 1,2 \qquad (2.31)$$

2.7 Dynamic Vibration Absorber

we can obtain the steady-state amplitudes of the masses m_1 and m_2 as

$$X_1 = \frac{\left(k_2 - m_2\omega^2\right)F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2} \tag{2.32}$$

$$X_2 = \frac{k_2 F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2} \tag{2.33}$$

We are primarily interested in reducing the amplitude of the machine (X_1)

In order to make the amplitude of m1 zero, the numerator of Eq. (2.32) should be set equal to zero.

This gives

$$\omega^2 = \frac{k_2}{m_2} \tag{2.34}$$

2.7 Dynamic Vibration Absorber

If the machine, before the addition of the dynamic vibration absorber, operates near its resonance, $\omega^2 \cong \omega_1^2 = k_1 / m_1$

Thus if the absorber is designed such that

$$\omega^2 = \frac{k_2}{m_2} = \frac{k_1}{m_1} \qquad (2.35)$$

the amplitude of vibration of the machine, while operating at its original resonant frequency, will be zero. By defining

$$\delta_{st} = \frac{F_0}{k_1}; \qquad \omega_1 = \left(\frac{k_1}{m_1}\right)^{\frac{1}{2}}$$

as the natural frequency of the machine or main system, and

$$\omega_2 = \left(\frac{k_2}{m_2}\right)^{\frac{1}{2}} \qquad (2.36)$$

2.7 Dynamic Vibration Absorber

as the natural frequency of the absorber or auxiliary system, Eqs. (2.32) and (2.33) can be rewritten as

$$\frac{X_1}{\delta_{st}} = \frac{1-\left(\dfrac{\omega}{\omega_2}\right)^2}{\left[1+\dfrac{k_2}{k_1}-\left(\dfrac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\dfrac{\omega}{\omega_2}\right)^2\right]-\dfrac{k_2}{k_1}} \qquad (2.37)$$

$$\frac{X_2}{\delta_{st}} = \frac{1}{\left[1+\dfrac{k_2}{k_1}-\left(\dfrac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\dfrac{\omega}{\omega_2}\right)^2\right]-\dfrac{k_2}{k_1}} \qquad (2.38)$$

2.7 Dynamic Vibration Absorber

Figure in next slide shows the variation of the amplitude of vibration of the machine (X_1/δ_{st}) with the machine speed (ω/ω_1).

The two peaks correspond to the two natural frequencies of the composite system.

As seen before, $X_1 = 0$ at $\omega = \omega_1$

At this frequency, Eq. (2.38) gives

$$X_2 = -\frac{k_1}{k_2}\delta_{st} = -\frac{F_0}{k_2} \qquad (2.39)$$

This shows that the force exerted by the auxiliary spring is opposite to the impressed force and neutralizes it, thus reducing to zero.

The size of the dynamic vibration absorber can be found from Eqs. (9.142) and (9.138):

2.7 Dynamic Vibration Absorber

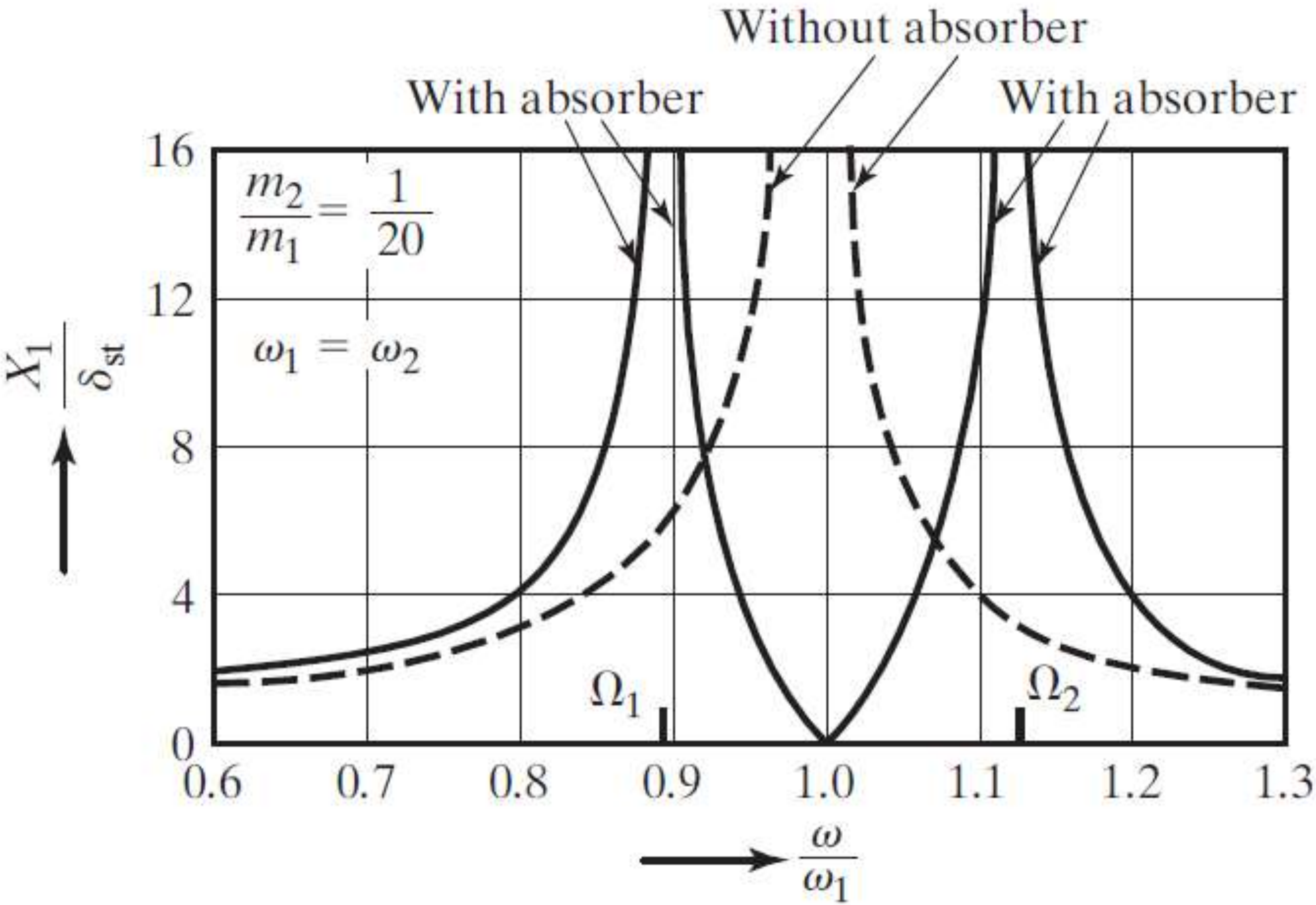
This shows that the force exerted by the auxiliary spring is opposite to the impressed force ($k_2X_2 = -F_0$) and neutralizes it, thus reducing X_1 to zero.

The size of the dynamic vibration absorber can be found from Eqs. (2.39) and (2.35):

$$k_2 X_2 = m_2 \omega^2 X_2 = -F_0 \qquad (2.40)$$

Thus the values of k_2 and m_2 depend on the allowable value of X_2.

2.7 Dynamic Vibration Absorber



Effect of undamped vibration absorber on the response of machine

2.7 Dynamic Vibration Absorber

It can be seen from Figure in previous page that the dynamic vibration absorber, while eliminating vibration at the known impressed frequency ω, introduces two resonant frequencies Ω_1 and Ω_2 at which the amplitude of the machine is infinite.

In practice, the operating frequency ω must therefore be kept away from the frequencies Ω_1 and Ω_2.

The values of Ω_1 and Ω_2 can be found by equating the denominator of Eq. (2.37) to zero.

Noting that

$$\frac{k_2}{k_1} = \frac{k_2}{m_2}\frac{m_2}{m_1}\frac{m_1}{k_1} = \frac{m_2}{m_1}\left(\frac{\omega_2}{\omega_1}\right)^2 \tag{2.41}$$

and setting the denominator of Eq. (2.37) to zero leads to

$$\left(\frac{\omega}{\omega_2}\right)^4\left(\frac{\omega_2}{\omega_1}\right)^2 - \left(\frac{\omega}{\omega_2}\right)^2\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right]+1=0 \tag{2.42}$$

2.7 Dynamic Vibration Absorber

The two roots of this equation are given by

$$\left.\begin{matrix}\left(\dfrac{\Omega_1}{\omega_2}\right)^2 \\ \left(\dfrac{\Omega_2}{\omega_2}\right)^2\end{matrix}\right\} = \frac{\left\{\left[1+\left(1+\dfrac{m_2}{m_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right]\mp\left\{\left[1+\left(1+\dfrac{m_2}{m_1}\right)\left(\dfrac{\omega_2}{\omega_1}\right)^2\right]^2-4\left(\dfrac{\omega_2}{\omega_1}\right)^2\right\}^{\frac{1}{2}}\right\}}{2\left(\dfrac{\omega_2}{\omega_1}\right)^2} \qquad (2.43)$$

which can be seen to be functions of (m_2/m_1) and (ω_2/ω_1).

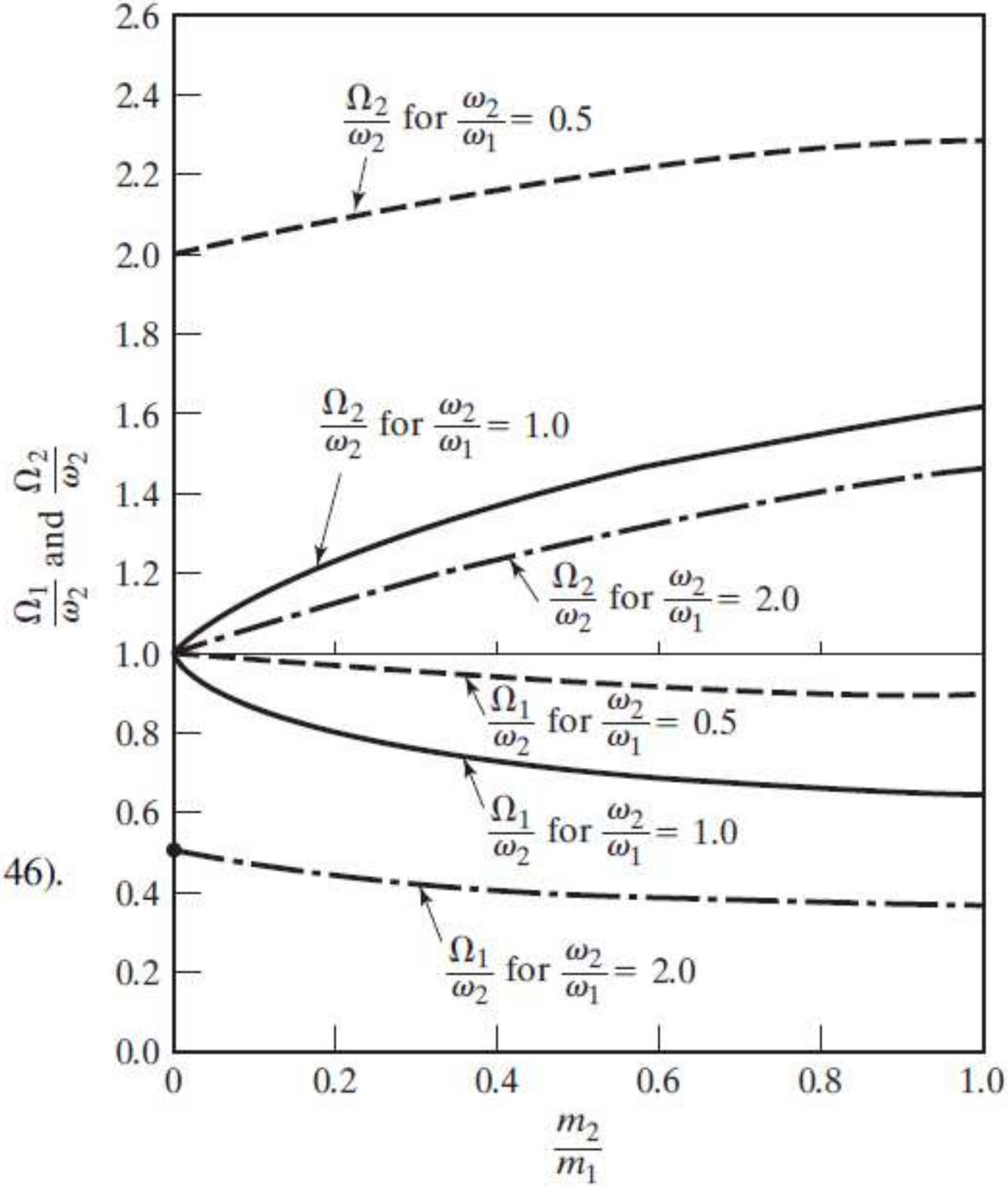
2.7 Dynamic Vibration Absorber

1. It can be seen, from Eq. (9.146), that is less than and is greater than the operating speed (which is equal to the natural frequency,) of the machine. Thus the machine must pass through during start-up and stopping. This results in large amplitudes.

2. Since the dynamic absorber is tuned to one excitation frequency the steady-state amplitude of the machine is zero only at that frequency. If the machine operates at other frequencies or if the force acting on the machine has several frequencies, then the amplitude of vibration of the machine may become large.

3. The variations of and as functions of the mass ratio are shown in Fig. 9.35 for three different values of the frequency ratio It can be seen that the difference between and increases with increasing values of m2/m1.

2.7 Dynamic Vibration Absorber



Variations of Ω_1 and Ω_2 given by Eq. (9.146).

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Mechanical Vibration and Structural Dynamics

Unit 3: Vibration of continuous system

Contents

III	**Vibration of Continuous system**
3.1	Introduction to Hamilton Principle
3.2	Longitudinal, transverse and torsional vibration of cylindrical shaft - extension to taper shaft
3.3	Dynamic equations of equilibra of general elastic body

3.1 What is continuous system?

A structural member consisting of a single piece of a particular material(s) without any visible discontinuity is a continuous structure or continuous system

Example: Rods, Beams, shafts, panels/plates, and shells

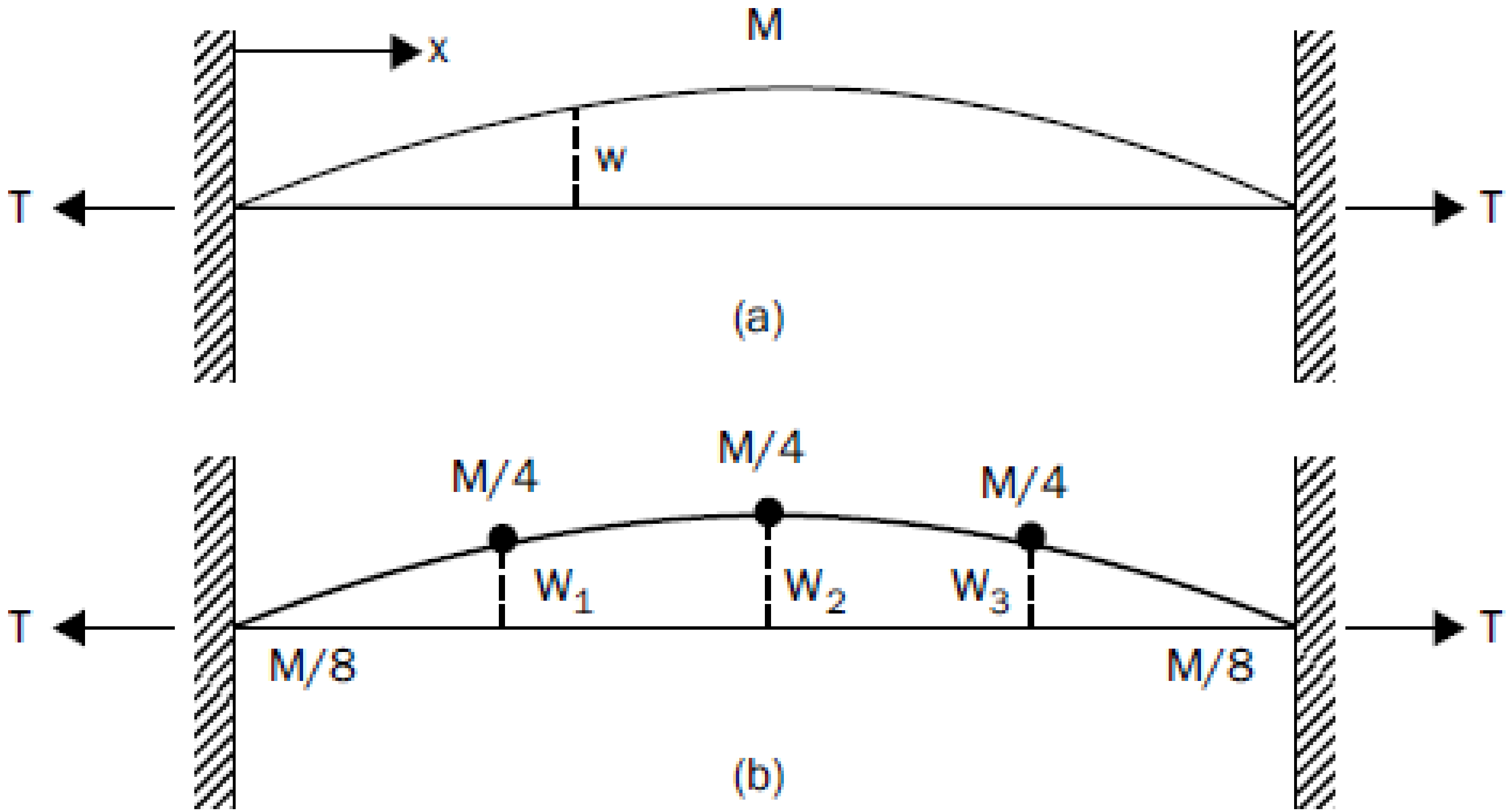
A single piece of above kind of continuous structure made of composites materials is essentially a continuous system

Smart structures are also modeled a continuous structures

Sometimes discontinuous structure, behaves like continuous structure when properly joined with bolts, rivets or weld

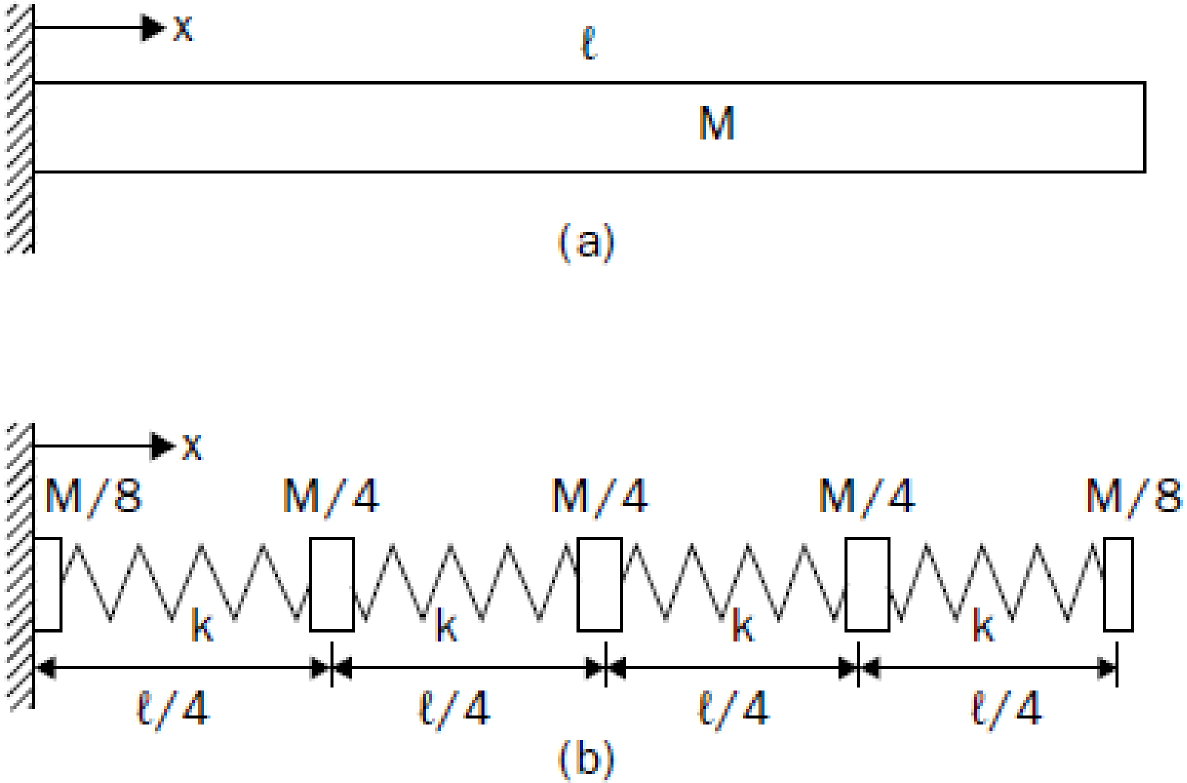
Vehicle structures (surface, air and space) appear and behave like a continuous structures

3.1 What is continuous system?



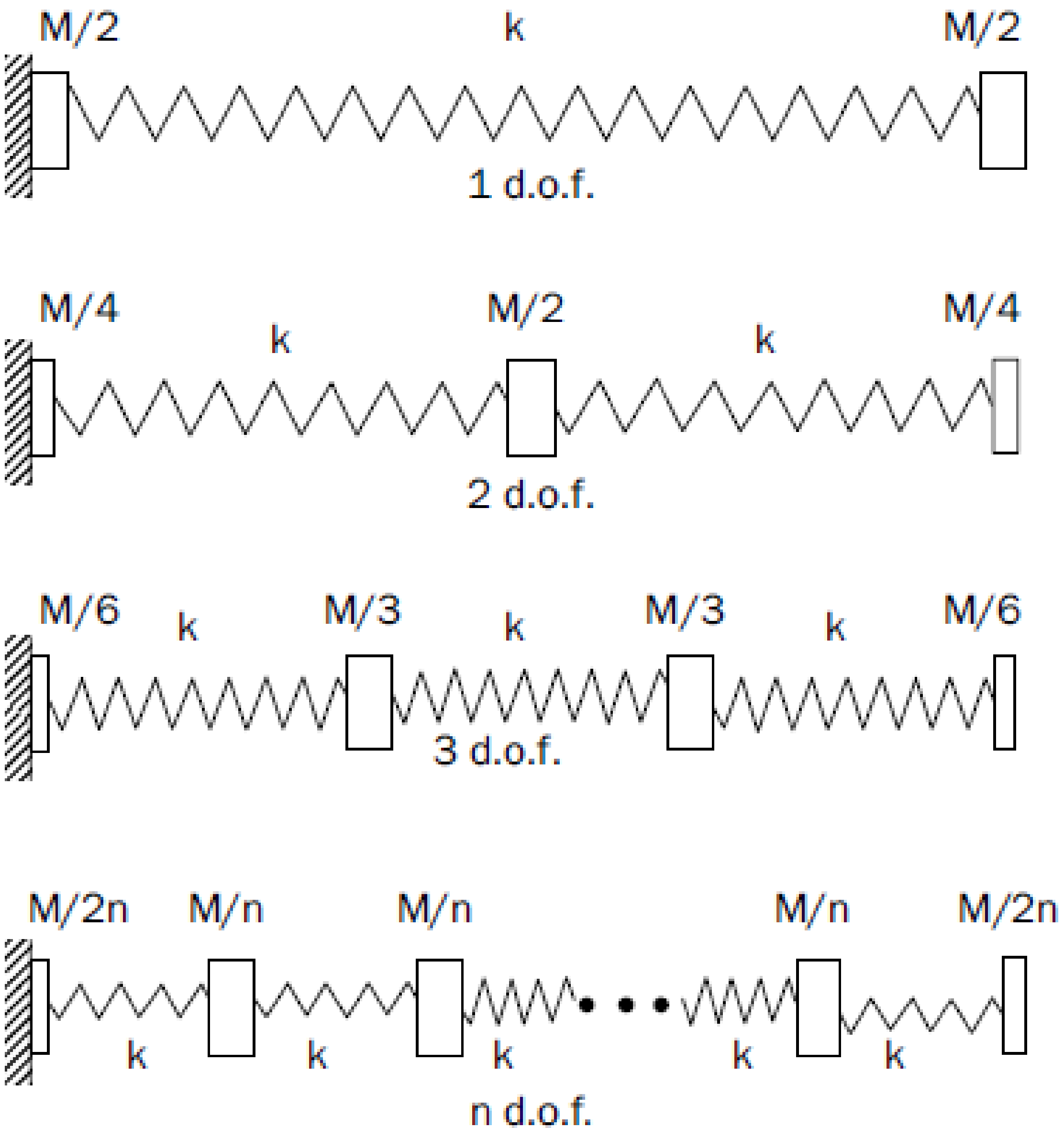
(a) A continuous string of mass M, displaced transversely;
(b) a discrete model of the string.

3.1 What is continuous system? (cont…)



(a) A continuous bar of mass M;
(b) A discrete model of the bar.

3.1 What is continuous system?



3.1 What is continuous system?

n	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	1.4142	–	–	–	–
2	1.5307	3.6955	–	–	–
3	1.5529	4.2426	5.7956	–	–
4	1.5607	4.4446	6.6518	7.8463	–
5	1.5643	4.5399	7.0711	8.9101	9.8769
7	1.5675	4.6239	7.4484	9.8995	11.8541
10	1.5692	4.6689	7.6537	10.4500	12.9890
15	1.5701	4.6930	7.7646	10.7510	13.6197
20	1.5704	4.7015	7.8036	10.8576	13.8447
∞ (exact)	1.5708	4.7124	7.8540	10.9956	14.1372

Nondimensional Frequencies $\omega^* = \omega \sqrt{(Ml/AE)}$ *for n d.o.f. Discrete* Models of Longitudinal Vibrations of a Fixed-Free Bar, as Described in Figure in the previous slide

3.1 Introduction to continuous system

- The displacement, velocity and acceleration are describe as a function of space (x,y,z) and time (t)
- Coordinate System (rectangular, cylindrical and spherical)
- In analytical dynamics generalized coordinate system
- Application of variation principles
- Derivation of energy expressions (KE, PE , Virtual work, etc)
- Application of Lagrange's equation or Hamilton's principle

Continuous systems	Dimensionality	Differential order
String	1	2
Bar	1	2
Beam	1	4
Membrane	2	2
Plate	2	4
Shell	2	8
Three dimensional	3	6

3.2 Hamilton's Principle

Hamilton's Principle is used for the development of equations of motion in vectorial form using scalar energy quantities in a variational form

$$\int_{t_1}^{t_2} \delta(T - V)\,dt + \int_{t_1}^{t_2} \delta W_{nc}\,dt = 0$$

Where T = total kinetic energy of system
V=potential energy of system, including both strain energy and potential of any conservative external forces
W_{nc}= work done by non-conservative forces acting on system, including damping and any arbitrary external loads
δ = variation taken during indicated time interval

Hamilton's principle states that the variation of kinetic and potential energy plus the variation of the work done by the non-conservative forces considered during interval t_1 to t_2 must equal to zero

The application of this principle leads directly to the equations of motion for any given system

3.3 Solutions of vibration problems using Variational Principles

3.1 Introduction to continuous system
3.2 Discreatize models of continuous systems
3.3 Solutions of vibration problems using Variational Principles
3.4 Vibrations of strings, bars, shafts and beams

3.3.1 Rayleigh – Ritz Method

Table 9.6 Scalar products for Rayleigh-Ritz method

Structural element	Case	$(u, v)_T$	$(u, v)_V$
Torsional shaft	No added disks or springs	$\int_0^L \rho J u(x) v(x)\, dx$	$\int_0^L GJ \frac{du}{dx} \frac{dv}{dx} dx$
	Added disk at $x = \tilde{x}$	$\int_0^L \rho J u(x) v(x)\, dx + I_D u(\tilde{x}) v(\tilde{x})$	$\int_0^L GJ \frac{du}{dx} \frac{dv}{dx} dx$
	Torsional spring at $x = \tilde{x}$	$\int_0^L \rho J u(x) v(x)\, dx$	$\int_0^L GJ \frac{du}{dx} \frac{dv}{dx} dx + k_t u(\tilde{x}) v$
Longitudinal bar	No added masses or springs	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EA \frac{du}{dx} \frac{dv}{dx} dx$
	Added mass at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + m u(\tilde{x}) v(\tilde{x})$	$\int_0^L EA \frac{du}{dx} \frac{dv}{dx} dx$
	Spring at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EA \frac{du}{dx} \frac{dv}{dx} dx + k u(\tilde{x}) v$

3.3.1 Rayleigh – Ritz Method

Structural element	Case	$(u, v)_T$	$(u, v)_V$
Beam	No added masses, disks, or springs	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EI \frac{d^2u}{dx^2} \frac{d^2v}{dx^2} dx$
	Added mass at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + m u(\tilde{x}) v(\tilde{x})$	$\int_0^L EI \frac{d^2u}{dx^2} \frac{d^2v}{dx^2} dx$
	Added spring at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx$	$\int_0^L EI \frac{d^2u}{dx^2} \frac{d^2v}{dx^2} dx + k u(\tilde{x}$
	Added disk (I_D) at $x = \tilde{x}$	$\int_0^L \rho A u(x) v(x)\, dx + I_D \frac{du(\tilde{x})}{dx} \frac{dv(\tilde{x})}{dx}$	$\int_0^L EI \frac{d^2u}{dx^2} \frac{d^2v}{dx^2} dx$

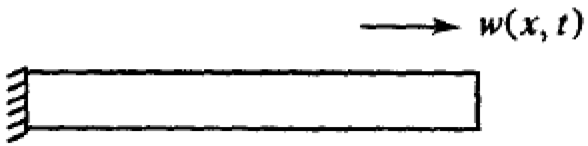
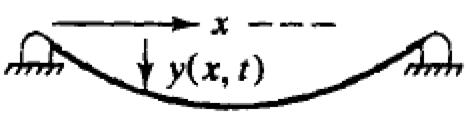
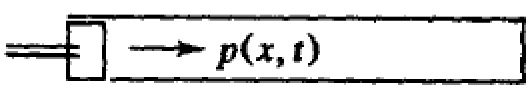
3.4.3 Torsional Vibrations of shafts

Table 9.1 Boundary conditions for torsional ascillations of a circular shaft

End condition	Boundary condition	Remarks
Fixed, $x = 0$ or $x = 1$	$\theta = 0$	
Free, $x = 0$ or $x = 1$	$\dfrac{\partial\theta}{\partial x} = 0$	
Torsional spring, $x = 0$	$\dfrac{\partial\theta}{\partial x} = \beta\theta$	$\beta = \dfrac{k_t L}{JG}$
Torsional spring, $x = 1$	$\dfrac{\partial\theta}{\partial x} = -\beta\theta$	$\beta = \dfrac{k_t L}{JG}$
Torsional damper, $x = 0$	$\dfrac{\partial\theta}{\partial x} = \beta\dfrac{\partial\theta}{\partial t}$	$\beta = c_t\sqrt{\dfrac{J}{\rho G}}$
Torsional damper, $x = 1$	$\dfrac{\partial\theta}{\partial x} = -\beta\dfrac{\partial\theta}{\partial t}$	$\beta = c_t\sqrt{\dfrac{J}{\rho G}}$
Attached disk, $x = 0$	$\dfrac{\partial\theta}{\partial x} = \beta\dfrac{\partial^2\theta}{\partial t^2}$	$\beta = \dfrac{I_D}{\rho JL}$
Attached disk, $x = 1$	$\dfrac{\partial\theta}{\partial x} = -\beta\dfrac{\partial^2\theta}{\partial t^2}$	$\beta = \dfrac{I_D}{\rho JL}$

3.4.3 Torsional Vibrations of shafts

Table 9.2 Physical problems governed by the wave equation

Problem	Schematic	Nondimensional wave equation	Wave speed
Torsional oscillations of circular cylinder		$\frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2\theta}{\partial t^2}$	$c = \sqrt{\frac{G}{\rho}}$ G = shear modulus ρ = mass density
Longitudinal oscillations of bar		$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial t^2}$	$c = \sqrt{\frac{E}{\rho}}$ E = elastic modulus ρ = mass density
Transverse vibrations of taut string		$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$	$c = \sqrt{\frac{T}{\mu}}$ T = tension μ = linear density
Pressure waves in an ideal gas		$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2}$	$c = \sqrt{kRT}$ k = ratio of specific heats R = gas constant T = temperature
Waterhammer waves in rigid pipe		$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial t^2}$	$c = \sqrt{\frac{k}{\rho}}$ k = bulk modulus of fluid ρ = mass density

3.4.4 Vibrations of beams

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = f(x, t) \qquad [9.68]$$

Equation (9.68) is nondimensionalized by introducing

$$x^* = \frac{x}{L} \qquad t^* = t\sqrt{\frac{EI}{\rho A L^4}} \qquad w^* = \frac{w}{L} \qquad f^* = \frac{f}{f_m} \qquad [9.69]$$

where f_m is the maximum value of f. The resulting nondimensional form of Eq. (9.68) is

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} = \frac{f_m L^3}{EI} f(x, t) \qquad [9.70]$$

3.4.4 Vibrations of beams

Table 9.3 Boundary conditions for transverse vibrations of a beam

End condition	Boundary condition A	Boundary condition B	Remarks
Free, $x = 0$ or $x = 1$	$\dfrac{\partial^2 w}{\partial x^2} = 0$	$\dfrac{\partial^3 w}{\partial x^3} = 0$	
Pinned, $x = 0$ or $x = 1$	$w = 0$	$\dfrac{\partial^2 w}{\partial x^2} = 0$	
Fixed, $x = 0$ or $x = 1$	$w = 0$	$\dfrac{\partial w}{\partial x} = 0$	
Linear spring, $x = 0$	$\dfrac{\partial^2 w}{\partial x^2} = 0$	$\dfrac{\partial^3 w}{\partial x^3} = -\beta w$	$\beta = \dfrac{kL^3}{EI}$
Linear spring, $x = 1$	$\dfrac{\partial^2 w}{\partial x^2} = 0$	$\dfrac{\partial^3 w}{\partial x^3} = \beta w$	$\beta = \dfrac{kL^3}{EI}$

3.4.4 Vibrations of beams

End condition	Boundary condition A	Boundary condition B	Remarks
Viscous damper, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = -\beta \frac{\partial w}{\partial t}$	$\beta = \frac{cL}{\sqrt{\rho EIA}}$
Viscous damper, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = \beta \frac{\partial w}{\partial t}$	$\beta = \frac{cL}{\sqrt{\rho EIA}}$
Attached mass, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = -\beta \frac{\partial^2 w}{\partial t^2}$	$\beta = \frac{m}{\rho AL}$
Attached mass, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial x^3} = \beta \frac{\partial^2 w}{\partial t^2}$	$\beta = \frac{m}{\rho AL}$
Attached inertia element, $x = 0$	$\frac{\partial^2 w}{\partial x^2} = -\beta \frac{\partial^3 w}{\partial x \partial t^2}$	$\frac{\partial^3 w}{\partial x^3} = 0$	$\beta = \frac{J}{\rho AL^3}$
Attached inertia element, $x = 1$	$\frac{\partial^2 w}{\partial x^2} = \beta \frac{\partial^3 w}{\partial x \partial t^2}$	$\frac{\partial^3 w}{\partial x^3} = 0$	$\beta = \frac{J}{\rho AL^3}$

3.4.4 Vibrations of beams

Frequency equations and eigenfunctions for each of the six cases are summarized below.

Clamped–clamped:

$$\cos\beta \bullet \cosh\beta = 1 \tag{4.30a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.30b}$$

$\gamma = 0.98250,\ 1.00078,\ 0.99997,\ 1.00000,\ \ldots$

Free–free:

$$\cos\beta \bullet \cosh\beta = 1 \tag{4.31a}$$

$$X = (\cosh\beta\xi + \cos\beta\xi) - \gamma(\sinh\beta\xi + \sin\beta\xi) \tag{4.31b}$$

$\gamma =$ same as clamped-clamped

3.4.4 Vibrations of beams

Clamped–SS:

$$\tan\beta = \tanh\beta \tag{4.32a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.32b}$$

$\gamma = 1.00078, 1.00000, \ldots$

Free–SS:

$$\tan\beta = \tanh\beta \tag{4.33a}$$

$$X = (\cosh\beta\xi + \cos\beta\xi) - \gamma(\sinh\beta\xi + \sin\beta\xi) \tag{4.33b}$$

$\gamma =$ same as clamped-SS

3.4.4 Vibrations of beams

Clamped–free:

$$\cos\beta \cdot \cosh\beta = -1 \tag{4.34a}$$

$$X = (\cosh\beta\xi - \cos\beta\xi) - \gamma(\sinh\beta\xi - \sin\beta\xi) \tag{4.34b}$$

$\gamma = 0.73410,\ 1.01847,\ 0.99922,\ 1.00003,\ 1.00000, \ldots$

SS–SS:

$$\sin\beta = 0 \tag{4.35a}$$

$$X = \sin\beta\xi \tag{4.35b}$$

In the above equations, $\xi = x/\ell$ is measured in each case from the left end of the beam. The values of β are the square roots of the frequency parameters listed in Table in next slide. More accurate values of β and γ are available in the classical study of Young and Felgar .

3.4.4 Vibrations of beams

m	C–C	C–SS	C–F	SS–SS	SS–F	F–F
1	22.373	15.418	3.5160	9.8696	0	0
2	61.673	49.965	22.034	39.478	15.418	0
3	120.903	104.248	61.697	88.826	49.965	22.373
4	199.859	178.270	120.902	157.914	104.248	61.673
5	298.556	272.031	199.860	246.740	178.270	120.903
>5	$(2m+1)^2\pi^2/4$	$(4m+1)^2\pi^2/16$	$(2m-1)^2\pi^2/4$	$m^2\pi^2$	$(4m-3)^2\pi^2/16$	$(2m-3)^2\pi^2/4$

Table 4.1 Frequency Parameters $\beta^2 = \omega \ell^2 \sqrt{\rho A / EI}$ for Beams

3.4.4 Vibrations of beams

Table 9.4 Natural frequencies and mode shapes for beams.

End conditions $X = 0 \quad X = 1$	Characteristic equation	Five lowest natural frequencies $\omega_k = \sqrt{\lambda_k}$	Mode shape	Kinetic energy scalar product $(X_j(x), X_k(x))$
Fixed-fixed	$\cos\lambda^{1/4}\cosh\lambda^{1/4} = 1$	$\omega_1 = 22.37$ $\omega_2 = 61.66$ $\omega_3 = 120.9$ $\omega_4 = 199.9$ $\omega_5 = 298.6$	$C_k\left[\cosh\lambda_k^{1/4}x - \cos\lambda_k^{1/4}x - \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cosh\lambda_k^{1/4} - \cos\lambda_k^{1/4}}{\sinh\lambda_k^{1/4} - \sin\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Pinned-pinned	$\sin\lambda^{1/4} = 0$	$\omega_1 = 9.870$ $\omega_2 = 39.48$ $\omega_3 = 88.83$ $\omega_4 = 157.9$ $\omega_5 = 246.7$	$C_k \sin\lambda_k^{1/4}x$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-free	$\cos\lambda^{1/4}\cosh^{1/4} = -1$	$\omega_1 = 3.51$ $\omega_2 = 22.03$ $\omega_3 = 61.70$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$C_k\left[\cosh\lambda_k^{1/4}x - \cos\lambda_k^{1/4}x - \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Free-free	$\cosh\lambda^{1/4}\cos\lambda^{1/4} = 1$	$\omega_1 = 0$ $\omega_2 = 22.37$ $\omega_3 = 61.66$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$1, \sqrt{3}x\,(k = 1)$ $C_k\left[\cosh\lambda_k^{1/4}x + \cos\lambda_k^{1/4}x + \alpha_k(\sinh\lambda_k^{1/4}x + \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cosh\lambda_k^{1/4} - \cos\lambda_k^{1/4}}{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-linear spring	$\lambda^{3/4}(\cosh\lambda^{1/4}\cos\lambda^{1/4} + 1)$ $-\beta(\cos\lambda^{1/4}\sinh\lambda^{1/4} - \cosh\lambda^{1/4}\sin\lambda^{1/4})$ $= 0$	For $\beta = 0.25$ $\omega_1 = 3.65$ $\omega_2 = 22.08$ $\omega_3 = 61.70$ $\omega_4 = 120.9$ $\omega_5 = 199.9$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x - \alpha_k(\sin\lambda^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$

3.4.4 Vibrations of beams

Pinned-linear spring	$\cot\lambda^{1/4} - \coth\lambda^{1/4} = -\dfrac{2\beta}{\lambda^{3/4}}$	For $\beta = 0.25$ $\omega_1 = 0.8636$ $\omega_2 = 15.41$ $\omega_3 = 49.47$ $\omega_4 = 104.25$ $\omega_5 = 178.27$	$C_k\left[\sin\lambda_k^{1/4}x + \dfrac{\sin\lambda_k^{1/4}}{\sinh\lambda_k^{1/4}}\sinh\lambda_k^{1/4}x\right]$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-attached mass	$\lambda^{1/4}(\cos\lambda^{1/4}\cosh\lambda^{1/4}+1)$ $+\beta(\cos\lambda^{1/4}\sinh\lambda^{1/4} - \cosh\lambda^{1/4}\sin\lambda^{1/4})$ $=0$	For $\beta = 0.25$ $\omega_1 = 3.047$ $\omega_2 = 21.54$ $\omega_3 = 61.21$ $\omega_4 = 120.4$ $\omega_5 = 199.4$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x + \alpha_k(\sinh\lambda_k^{1/4}x - \sin\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} + \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$ $+\beta X_j(1)X_k(1)$
Pinned-free	$\tan\lambda^{1/4} = \tanh\lambda^{1/4}$	$\omega_1 = 0$ $\omega_2 = 15.42$ $\omega_3 = 49.96$ $\omega_4 = 104.2$ $\omega_5 = 178.3$	$\sqrt{3}x,\ (k=1)$ $C_k\left[\sin\lambda_k^{1/4}x + \dfrac{\sin\lambda_k^{1/4}}{\sinh\lambda_k^{1/4}}\sinh\lambda_k^{1/4}x\right]$ $(k>1)$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-pinned	$\tan\lambda^{1/4} = \tanh\lambda^{1/4}$	$\omega_1 = 15.42$ $\omega_2 = 49.96$ $\omega_3 = 104.2$ $\omega_4 = 178.3$ $\omega_5 = 272.0$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x - \alpha_k(\sin\lambda_k^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\cos\lambda_k^{1/4} - \cosh\lambda_k^{1/4}}{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$
Fixed-attached inertia element	$\cos\lambda^{1/4}\cosh\lambda^{1/4}$ $+\beta(\sin\lambda^{1/4}\cosh\lambda^{1/4} + \cos\lambda^{1/4}\sinh\lambda^{1/4})$ $=-1$	For $\beta = 0.25$ $\omega_1 = 4.425$ $\omega_2 = 27.28$ $\omega_3 = 71.41$ $\omega_4 = 135.4$ $\omega_5 = 219.2$	$C_k\left[\cos\lambda_k^{1/4}x - \cosh\lambda_k^{1/4}x + \alpha_k(\sin\lambda_k^{1/4}x - \sinh\lambda_k^{1/4}x)\right]$ $\alpha_k = \dfrac{\sin\lambda_k^{1/4} - \sinh\lambda_k^{1/4}}{\cos\lambda_k^{1/4} + \cosh\lambda_k^{1/4}}$	$\int_0^1 X_j(x)X_k(x)\,dx$ $+\beta X_j(1)X_k(1)$

I The dimensional natural frequencies are obtained by multiplying the given nondimensional natural frequencies by $\sqrt{EI/\rho AL^4}$; for a given beam β is as defined in Table 9.3.

3.4.4 Vibrations of beams

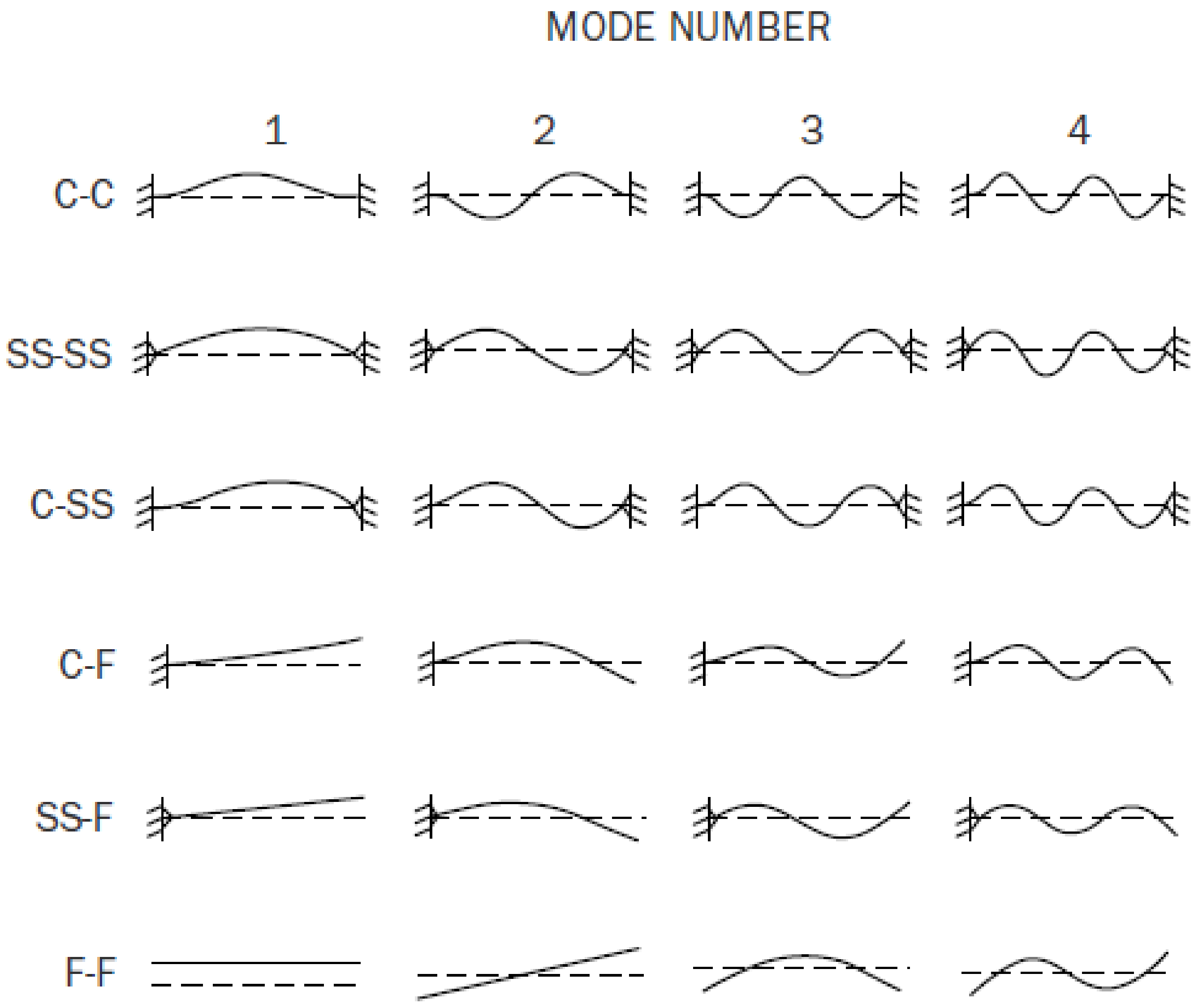


Figure 4.3 The first four mode shapes for beams with different boundaries.

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Mechanical Vibration and Structural Dynamics

Unit 4: Determination of natural frequencies and mode shapes

Contents

4.2 Solution Methods for Eigenproblems

We concentrate on the solution of the eigenproblem

$$K\phi = \lambda M\phi \qquad (1)$$

and., in particular, on the calculation of the smallest eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_p$ and corresponding eigenvectors $\phi_1, \phi_2, \phi_3, \ldots, \phi_p$.

The solution methods that we considered here first can be subdivided into four groups, corresponding to which basic property is used as the basis of the solution algorithm (Ref. J.H. Wilkinson)

1. Vector Iteration Method

$$K\phi_i = \lambda_i M\phi_i \qquad (2)$$

2. Transformation Method

   First we have to determine mode shapes matrix Φ, such that

$$\Phi^T K\Phi = \Lambda \qquad (3)$$

$$\Phi^T M\Phi = I \qquad (4)$$

4.2 Solution Methods for Eigenproblems

where
$$\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \end{bmatrix}$$
$$\Lambda = diag(\lambda_i), \quad i = 1,2,\cdots,n$$

3. Polynomial Iteration

$$p(\lambda_i) = 0 \tag{5}$$
$$\text{where } p(\lambda) = \det(K - \lambda M) \tag{6}$$

4. Sturm Sequence Property of the Characteristic Polynomials

$$p(\lambda) = \det(K - \lambda M) \tag{7}$$
$$\text{where} \quad p^{(r)}\left(\lambda^{(r)}\right) = \det\left(K^{(r)} - \lambda^{(r)} M^{(r)}\right) \tag{8}$$

n=1,2,3,…,(n-1)

4.2 Solution Methods for Eigenproblems

$p^{(r)}\left(\lambda^{(r)}\right)$ is the characteristic polynomial of r^{th} associated constraint problem corresponding to $\mathrm{K}\phi = \lambda \mathrm{M}\phi$

5. Lanczos Method and Subspace Iteration Method used combination of above 4 methods

4.3.1 Eigenvalue Extraction Methods in MSC/NASTRAN

In MSC/NASTRAN following Methods are Available for Real Eigenvalue Extraction

1. Transformation Methods

- ❑ Givens Method
- ❑ Householder Method
- ❑ Modified Givens Method
- ❑ Modified Householder Method

2. Tracking Methods

- ❑ Inverse Power Method
- ❑ Sturm Modified Inverse Power Method

Lanczos Method combines the best characteristics of both the tracking and transformation methods.

4.3.1Eigenvalue Extraction Methods in MSC/NASTRAN

Table 3-1 Comparison of Eigenvalue Methods

	Method				
	Givens, Householder	Modified Givens, Householder	Inverse Power	Sturm Modified Inverse Power	Lanczos
Reliability	High	High	Poor (can miss modes)	High	High
Relative Cost: Few Modes Many Modes	 Medium High	 Medium High	 Low High	 Low High	 Medium Medium
Limitations	Cannot analyze singular [M] Expensive for problems that do not fit in memory	Expensive for many modes Expensive for problems that do not fit in memory	Can miss modes Expensive for many modes	Expensive for many modes	Difficulty with massless mechanisms
Best Application	Small, dense matrices that fit in memory Use with dynamic reduction (Chapter 11)	Small, dense matrices that fit in memory Use with dynamic reduction (Chapter 11)	To detemine a few modes	To detemine a few modes Backup method	Medium to large models

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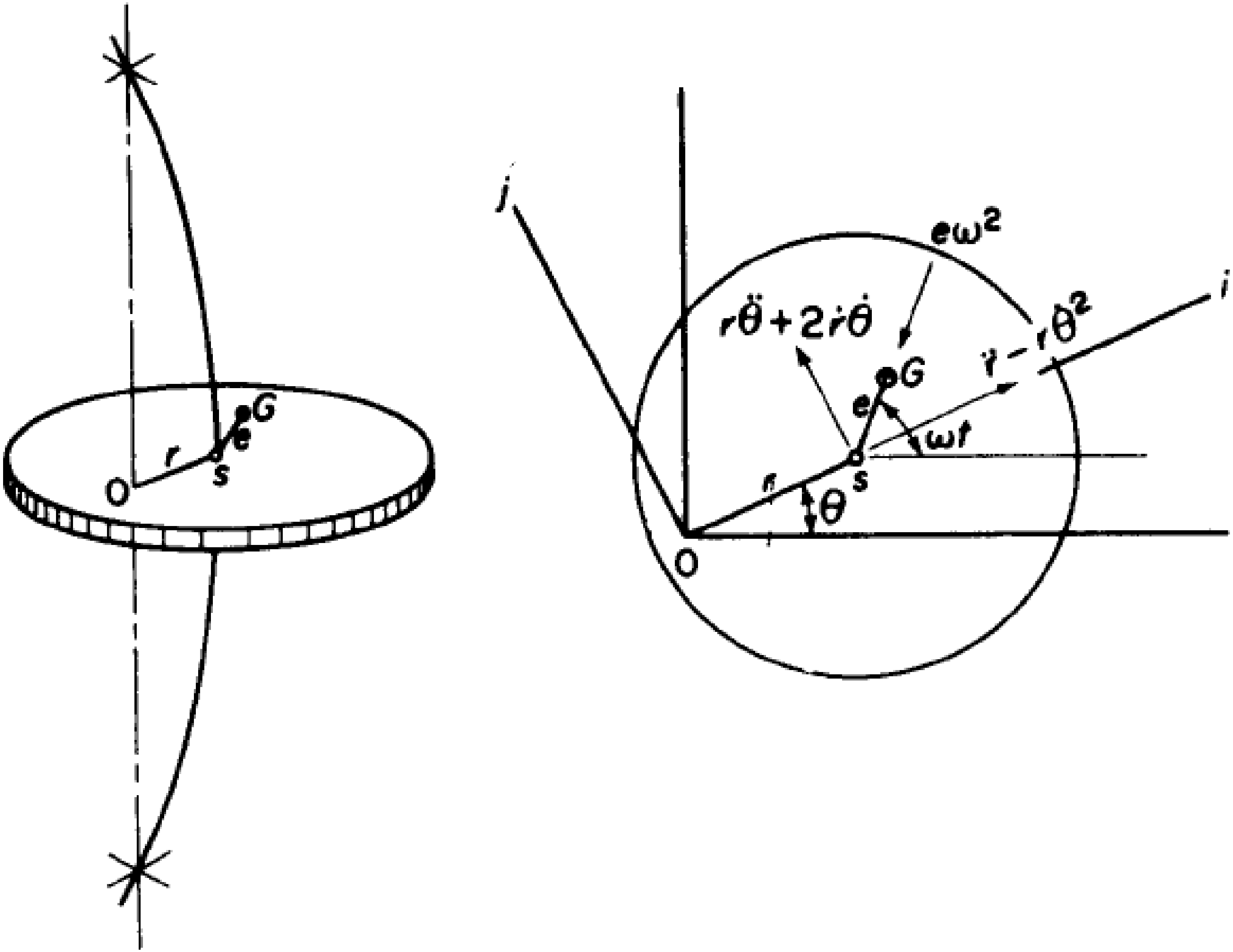
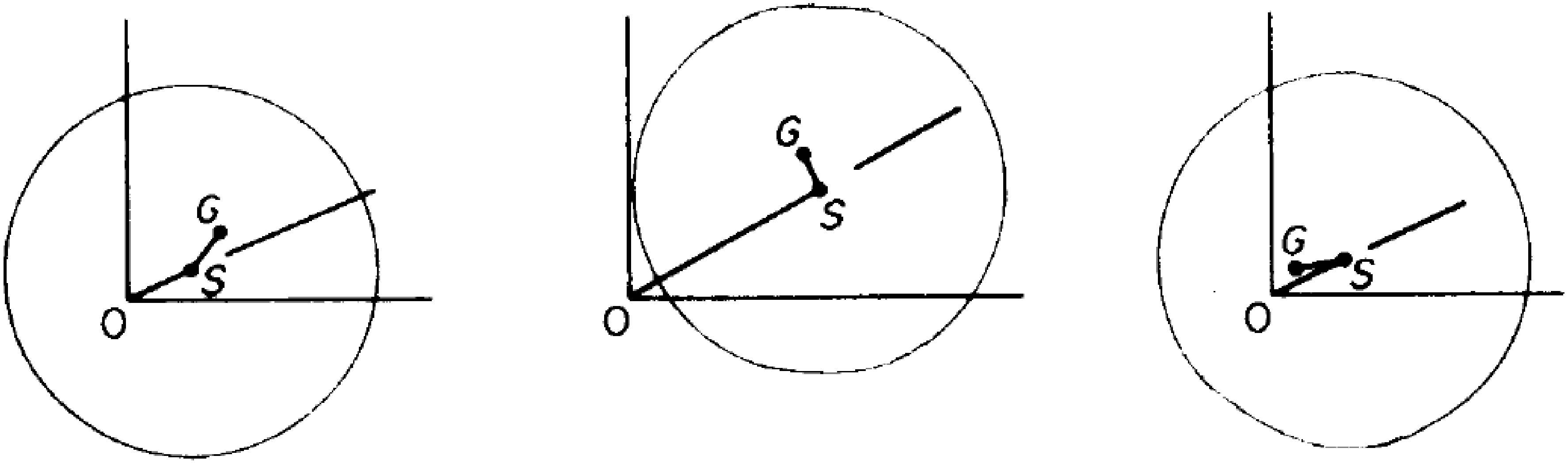
Unit 5:

Contents

5.2 Whirling of shafts



5.2 Whirling of shafts



$\omega << \omega_n$ $\omega = \omega_n$ $\omega >> \omega_n$

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1. Clough, R.W., and Penzien, J., *Dynamics of Structures,* McGraw-Hill, Inc., 1975.
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