Power Scheduling for Energy Harvesting Wireless Communications With Battery Capacity Constraint

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***Abstract*—Power scheduling is an important issue for energy harvesting systems. In this work, we study the power control policy for minimizing the weighted sum of the outage probabilities under a set of predetermined transmission rates over a finite horizon. This problem is challenging in that the objective function is non-convex. To make the analysis tractable, we apply the approximation at high signal-to-noise ratios and obtain a near-optimal offline solution. In the case of infinite battery capacity, we demonstrate that the allocated power has a piecewise structure, i.e., each power scheduling cycle should be divided into disjoint segments and the normalized power should remain constant within each segment. An iterative algorithm is developed to obtain the power solution. In the case of finite battery capacity, we show that the piecewise structure still holds true, and we develop a divide-and-conquer algorithm to recursively solve the power allocation problem. Finally, we obtain a simple online power control policy that is fairly robust to prediction errors of the harvested energy. Simulations demonstrate that the proposed power solution has better performance than other strategies such as best-effort, fixed-ratio and random allocation.**

***Index Terms*—Energy harvesting, outage probability, power scheduling.**

I. Introduction

In conventional sensor networks, sensors are equipped with batteries of limited capacity. When the stored energy is exhausted, it could be inconvenient or even impossible to refill the energy when, for example, the sensors are scattered in the broad space or embedded in the human body. Energy harvesting (EH) technology, which allows the sensors to collect energy from ambient environments, is an efficient solution to address this issue [2]–[6]. When the harvested energy is persistent, the life time of the entire sensor network could be extended significantly.

One big challenge to EH technology is the time-varying behavior of the harvested energy. For example, there could be some drained periods during which there is almost no energy to harvest. This may happen when the solar radiation level is very low in the rainy days, or during nights when there is almost no solar energy. Such time-varying energy supply would degrade the system performance, and makes it hard to maintain good operation quality over long durations. To tackle this problem, a good practice is to have a smart power management policy that dynamically schedules the power according to real-time system states.

Power scheduling plays an important role in the communication systems. For conventional systems with constant energy supply, it is well known that water-filling policy can maximize the channel capacity [7]. However, water-filling is no longer optimal for EH systems because of the EH causality constraint and the batter capacity (BC) constraint. EH causality constraint means that only the harvested energy that is currently available could be used, even if an unlimited amount of energy might be harvested in the future. In practice, the unused energy could be stored in the local battery for future use. However, the stored energy shall never exceed the battery capacity, which is known as BC constraint. Those two types of constraints are specific to EH systems and complicate the design of power management policy.

Depending on the knowledge of energy state information (ESI) to the power scheduler, there are two main approaches to managing power usage. The ESI involves information of the energy arrival time and the amount of harvested energy. For *online* methods, the scheduler only knows causal ESI. Typically, the online power scheduling problem can be solved by dynamic programming, but the computational complexity could be very high [8]. For this reason, the low-complexity *offline* solutions have been widely studied in the literature. The offline solutions generally require non-causal knowledge of ESI, which nearly holds true when the harvested energy could be accurately predicted based on historical data and advanced modeling techniques [9]–[11]. The importance of offline methods is two-fold. On the one hand, offline solutions provide performance upper bounds for the corresponding online solutions. On the other hand, offline solutions can usually be obtained through some fast algorithms, which may provide some guidelines for designing efficient online solutions. In this work, we focus mainly on offline power scheduling policy, based on which we shall also develop a low-complexity online policy.

A. Related Work

Power scheduling in EH systems has been an active research area in the past decade. The established power scheduling policies usually differ quite a lot. For different applications, the design goals and system models may also vary accordingly.

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In applications with variable-rate transmission, the scheduler may jointly decide the transmission power and transmission rates over time according to channel state information (CSI) and ESI. As transmission rates could be adjusted dynamically, the allocated power might be arranged in such away for maximizing the system throughput. For single-user point-to-point channels, the throughput maximization problem is respectively investigated in [12], [13] under continuous-time model and in [14] under discrete-time model. It is demonstrated that the throughput-optimal offline solutions could be obtained by the modified water-filling algorithm. In [12], [14], [15] the authors solve the problem of minimizing the transmission completion time given some fixed number of information bits. Interestingly, this problem can be mapped to an equivalent throughput maximization problem, and the mapping could be established through the maximum departure curve [12]. The same problem is later studied in the multi-user context of broadcast channels [16], [17] and multiple-access channels [18]. Yet there is also some work on network optimization. For example, in [19] the authors study the joint power, rates and subcarriers allocation policy for maximizing the weighted energy efficiency of the cellular downlink using orthogonal frequency-division multiplexing (OFDM). In [20], the goal of network utility maximization is pursued by jointly scheduling the power and sampling rate at all sensor nodes in the network. While all the aforementioned work assumes that the information is ready in the data buffer upfront, the random information arrival model is investigated in [21], [22]. In [21], the authors seek to minimize the mean transmission delay. Surprisingly, it is shown that the greedy policy is delay-optimal in the low signal-to-noise ratio (SNR) regions. The constraint of finite data buffer size is introduced in [22], and it is demonstrated that there is a basic tradeoff between battery discharge probability and buffer overflow probability.

Variable-rate transmission could improve throughput by dynamically adjusting the coding and modulation scheme (MCS), but devices must be equipped with powerful baseband processors. Moreover, variable-rate transmission requires extra power and channel use, since the transmitter and the receiver must repeatedly exchange the MCS information. Therefore, variable-rate transmission might not be friendly towards the low-cost and power-limited EH sensors. In practice, fixed-rate transmission could be a better choice especially for large-scale deployments.

For fixed-rate applications, data rates and MCS are predetermined. As a result, the transmission quality is an important performance measure. The average channel outage probability minimization problem is addressed in [23] by assuming constant-rate transmission and ignoring BC constraint. In [24], the authors develop a simple save-then-transmit protocol that takes into account both the channel outage and circuit outage events. The problem of minimizing the cost of energy use is investigated in [25] under the constraint that the outage probability must be below a target threshold. In [26], a probabilistic ON-OFF power control policy is investigated for maximizing a general reward function, where the harvested energy is supposed to be a binary-state Markov process. The truncated channel inversion policy and constant power policy are studied in [27]. A two-stage approach is developed for maximizing the network utility under an energy neutrality constraint. Another work on optimizing network utility is [8], in which the authors obtain an asymptotically optimal power control policy over infinite horizon by mapping to an equivalent non-EH power allocation problem.

B. Scope of This Work

In this work, we investigate the problem of minimizing the weighted sum of the outage probabilities over a finite horizon. We focus on fixed-rate transmission because it has lower implementation complexity compared to variable-rate transmission. Our problem formulation is very general in that it could be translated to a family of design goals. For example, depending on the choice of weights, the optimization objective could be maximizing the throughput or minimizing the average outage probability. The most related work is [23], in which the authors seek to minimize the average outage probability. However, in that work the BC constraint is ignored and it is assumed that data is transmitted at constant rate. On the contrary, we incorporate the BC constraint into our problem formulation, and we consider a more general framework in which the transmission rates could have arbitrary values. Another related work is [8], which also optimizes a general objective function. However, that work ignores the BC constraint as well. Moreover, the proposed scheme in [8] is only asymptotically optimal over infinite horizon, whereas in this work we focus on performance optimization over a finite horizon.

The formulated power scheduling problem is challenging in that the objective function is non-convex. As a result, there is no simple solution. To make the analysis tractable, we approach the original objective function by its high-SNR approximation, which is convex and thus much easier to deal with. We then study the structure of the optimal offline solutions. In the first stage, we ignore the BC constraint and consider the EH causality constraint alone. We show that the entire power scheduling cycle should be divided into small segments, and within each segment the normalized power should remain constant. We demonstrate that the boundaries of those segments are the slots in which the harvested energy should be depleted, and we develop an iterative algorithm to determine those segments. In the second stage, we consider the more general problem with BC constraint. We show that the piecewise structure of optimal power still holds true. However, there is no closed-form formula to determine those segments. To address this issue, we design a divide-and-conquer algorithm which divides the original problem into a couple of independent and solvable subproblems. Finally, from the offline algorithm we also develop a simple online policy that is fairly robust to prediction errors of the harvested energy.

The rest of this paper is organized as follows. We first describe the system model and formulate the power scheduling problem in Section II. Then in Section III and Section IV, we study the power scheduling policy without and with BC constraint, respectively. Simulation results are given in Section V, and conclusions are given in Section VI.

II. System Model and Problem Formulation

A. System Model

In this work, we consider a point-to-point communication channel with a single pair of transmitter and receiver. The transmitter is supposed to be an EH sensor node. We are interested in a single power scheduling cycle that consists of T time slots. For notational convenience, we assume that the duration of each time slot is one second, such that the value of energy is equal to the value of power within a single time slot.

Each time slot consists of two phases, i.e., the transmission phase and the energy harvesting phase. In the i-th slot for $i = 1, 2, \cdots, T$, the transmitter first sends data to the receiver at a transmission rate R_i. The transmission rates are predetermined but may vary in different slots. In practice, sensor nodes may need to wake up periodically and report different types of information to data center. The information could be packed into different packets. For example, the signalling packets may involve some control information such as the device status, and the data packets may contain the measurement data. Practically, those packets could be transmitted with different MCS configuration and as a result, the data rates of those packets may vary in a predetermined manner.

During the transmission phase of the i-th slot, the received signal is given by

$$y_i = h_i \sqrt{P_i} x_i + n_i, \tag{1}$$

where x_i is the transmitted signal with normalized power, h_i is the Rayleigh fading channel coefficient with zero mean and variance γ, P_i is the transmitted power, and n_i is the complex additive white Gaussian noise with zero mean and variance σ^2. The instantaneous mutual information of the channel is given by

$$I(x_i; y_i) = \log_2 \left(1 + \frac{|h_i|^2 P_i}{\sigma^2}\right). \tag{2}$$

As the transmission rate R_i is predetermined, the outage probability is given by

$$\mathcal{O}(P_i, R_i) = \Pr\left[I(x_i; y_i) < R_i\right] = 1 - \exp\left(-\frac{\eta_i}{P_i}\right), \tag{3}$$

where $\eta_i = \frac{(2^{R_i}-1)\sigma^2}{\gamma}$. At moderate-to-high SNR (i.e., $\frac{\eta_i}{P_i} \ll 1$), the outage probability could be approximated by

$$\mathcal{O}(P_i, R_i) \approx \frac{\eta_i}{P_i}, \tag{4}$$

Our objective is to minimize the weighted sum of the outage probabilities over the entire horizon via proper power scheduling. Details will be given in the next subsection.

The transmission phase is followed by the energy harvesting phase, in which the transmitter collects energy from the surrounding environment. The harvested energy in the i-th slot is denoted by E_i for $i = 0, 1, \cdots, T$, where E_0 is the initial energy stored in the battery before the entire transmission cycle starts. The harvested energy is first stored in the local battery before being consumed in the future. In this work, we assume that the battery has limited capacity, which is denoted by B. The harvested energy that exceeds this limit would be abandoned. For this reason, we focus only on the scenario $E_i \leq B$ for $i = 0, 1, \cdots, T$. Apart from transmitted power, we ignore all other kinds of energy consumption.

In this work, it is assumed that the harvested energy E_i is known *a priori* to the scheduler, and we mainly seek to find a sound offline power scheduling policy. Having non-causal knowledge of ESI may correspond to the scenario where the harvested energy could be accurately predicted according to historical data and advanced modeling techniques [9]–[11]. The deterministic behavior of the harvested energy is a widely used study assumption in the community, which has been justified in the references [12], [13], [15]–[20], [23], [25]. In practice, it is likely that the scheduler knows only causal ESI and expects an online policy. A suboptimal yet efficient online policy will be given at the end of Section IV to deal with this scenario.

B. Problem Formulation

In this subsection, we formulate the power scheduling problem over a finite horizon. Let B_i be the energy stored in the battery at the beginning of the i-th slot for $i = 1, 2, \cdots, T$, where $B_1 = E_0$ is the initial energy that is available before the entire power allocation cycle starts. As the energy harvested in the i-th slot is available after the transmission phase, the transmitted power cannot exceed the initial energy B_i, i.e., $P_i \leq B_i$ for $i = 1, 2, \cdots, T$. Once the power P_i is determined, the unused energy after the transmission phase is given by $B_i - P_i \geq 0$. In the special case of $P_i = B_i$, all available energy would be depleted after the transmission phase, and we shall refer to the i-th time slot as an energy depletion slot. The unused energy, together with the newly harvested energy E_i, becomes the initial energy of the next slot, which is given by

$$B_{i+1} = \min\{B_i - P_i + E_i, B\} \tag{5}$$

for $i = 1, 2, \cdots, T-1$. Because of the BC constraint, some harvested energy could be abandoned along the way if $B_i - P_i + E_i > B$. As increasing the transmitted power always helps reduce the outage probability, we impose the additional constraint that no energy should be abandoned along the way, i.e., $B_i - P_i + E_i \leq B$ for $i = 1, 2, \cdots, T-1$. This constraint specifies the minimum amount of power that should be consumed in each slot to avoid battery overflow. After some manipulation, the afore-mentioned two types of constraints could be rewritten as

$$\begin{aligned}(\textbf{EHconstraint}) : \sum_{i=1}^{t} P_i \leq \sum_{i=0}^{t-1} E_i, t = 1, 2, \cdots, T \\ (\textbf{BCconstraint}) : \sum_{i=1}^{t} P_i \geq \sum_{i=0}^{t} E_i - B, t = 1, 2, \cdots, T-1.\end{aligned} \tag{6}$$

The EH constraint requires that the total amount of transmitted power should not exceed the total amount of energy harvested up to slot t. On the other hand, the BC constraint indicates that during the energy harvesting phase of slot t, no energy should be abandoned due to overflow.

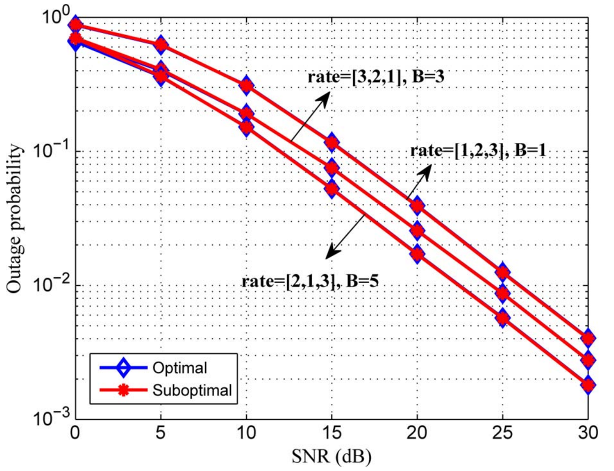

Fig. 1. Outage probability under optimal and suboptimal power solution.

Our design goal is to minimize the weighted sum of the outage probabilities over T time slots through proper power through proper power scheduling. This optimization problem could be formulated as

$$\begin{aligned}(\text{P1}) : \min_{\{P_i\}} \sum_{i=1}^{T} w_i \mathcal{O}(P_i, R_i) \approx \sum_{i=1}^{T} \frac{w_i \eta_i}{P_i} \triangleq \sum_{i=1}^{T} \frac{\alpha_i}{P_i} \\ \text{s.t.} \sum_{i=1}^{t} P_i \leq \sum_{i=0}^{t-1} E_i, t = 1, 2, \cdots, T \\ \sum_{i=1}^{t} P_i \geq \sum_{i=0}^{t} E_i - B, t = 1, 2, \cdots, T-1 \\ P_t \geq 0, \ t = 1, 2, \cdots, T.\end{aligned} \quad (7)$$

Here, the weight w_i reflects the relative importance of the i-th packet, and $\alpha_i \triangleq w_i \eta_i$ is constant. Note that this objective function may represent different performance metrics depending on the choice of weights $\{w_i\}$. For example, $w_i \equiv \frac{1}{T}$ means all packets are equally important and the average outage probability will be minimized. As only one packet is transmitted in each time slot, this is also an equivalent way to maximize the expected number of packets that can be successfully delivered over the entire horizon. Alternatively, we may also choose $w_i = R_i$, in which case we are essentially maximizing the expected throughput.

Note that the original objective function (i.e., weighted sum of the exact outage probabilities) is a non-convex function of power, which makes it hard to solve this optimization problem directly. To make the analysis tractable, we apply the high-SNR approximation given by (4). It is easy to show that after replacing the objective function, the new optimization problem is convex, which is easier to deal with. It should be pointed out that although the optimal power solution to the new problem is strictly suboptimal to the original problem, the performance loss is actually very small. To give an example, in Fig. 1 we compare the performance under optimal and suboptimal power solution for $T = 3$. The harvested energy is uniformly distributed in the range [0.1, 5], and the system SNR is defined as $\rho = \frac{1}{\sigma^2}$ where σ^2 is noise power. For simplicity, the channel variance γ is normalized to 1, and the weight $w_i \equiv \frac{1}{T}$ such that the objective function represents the average outage probability. The optimal power solution is obtained through exhaustive search, and we use the exact outage probability (3) as objective function. On the other hand, the suboptimal power solution corresponds to when the approximated outage probability (4) is used as the objective function. It can be observed that there is almost no performance difference at all SNR values. So, in the sequel, we will instead use the approximated outage probability as objective function.[1]

As a quick overview, in Section III and Section IV we will respectively study the scenarios with infinite and finite battery capacity. Along the way we will derive a couple of properties that help demonstrate how the optimal power solution should look like. Readers could safely skip those derivations and find the general algorithm to obtain the optimal power solution at the end of Section IV.

III. Power Scheduling With Infinite Battery Capacity

In this section, we first study power scheduling with infinite battery capacity, i.e., $B = \infty$. The original problem (P1) can thus be simplified as

$$\begin{aligned}(\text{P2}) : \min_{\{P_i\}} \sum_{i=1}^{T} \frac{\alpha_i}{P_i} \\ \text{s.t.} \sum_{i=1}^{t} P_i \leq \sum_{i=0}^{t-1} E_i, P_t \geq 0, t = 1, 2, \cdots, T.\end{aligned} \quad (8)$$

Problem (P2) is still a convex optimization problem, which can be solved by Lagrange multiplier method. The Lagrangian function is given by

$$\mathcal{L} = \sum_{i=1}^{T} \frac{\alpha_i}{P_i} + \sum_{i=1}^{T} \lambda_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i-1} E_k \right) - \sum_{i=1}^{T} \omega_i P_i, \quad (9)$$

where $\lambda_i, \omega_i \geq 0$ are Lagrange multipliers. The Karush-Kuhn Tucker (KKT) conditions are given by

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\frac{\alpha_i}{P_i^2} + \sum_{k=i}^{T} \lambda_k - \omega_i = 0 \quad (10)$$

for $i = 1, 2, \cdots, T$. The complementary slackness conditions could be written as

$$\lambda_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i-1} E_k \right) = 0, \quad (11)$$

$$\omega_i P_i = 0 \quad (12)$$

for $i = 1, 2, \cdots, T$. From (10), the optimal power could be expressed as

$$P_i^* = \sqrt{\frac{\alpha_i}{\sum_{k=i}^{T} \lambda_k - \omega_i}}. \quad (13)$$

[1] In the following sections, whenever we refer to optimal power solution, it is with respect to the problem which uses the approximated outage probability as objective function.

Combining (12) and (13), we have $\omega_i = 0$ for $i = 1, 2, \cdots, T$, such that

$$P_i^* = \sqrt{\frac{\alpha_i}{\sum_{k=i}^{T} \lambda_k}}. \quad (14)$$

To obtain the optimal power, we need to find the values of Lagrange multipliers λ_i, which is quite difficult. Instead, we first study some structural properties of the optimal power solution, based on which we may deduce the optimal power via an iterative algorithm.

Property 1: Suppose the optimal power sequence is given by $\{P_k^*\}_{k=1}^T$. If the i-th EH constraint is not binding, i.e., $\sum_{k=1}^{i} P_k^* < \sum_{k=0}^{i-1} E_k$, then the optimal power P_i^* and P_{i+1}^* have the relationship $\frac{P_i^*}{\sqrt{\alpha_i}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}$ When the i-th EH constraint is binding, then $\lambda_i \geq 0$ and the energy should be depleted after the transmission phase of the i-th slot, i.e., $\sum_{k=1}^{i} P_i^* = \sum_{k=0}^{i-1} E_i$.

Proof: According to (11), there could be two outcomes for the value of λ_i. If $\sum_{k=1}^{i} P_k^* < \sum_{k=0}^{i-1} E_k$, then $\lambda_i = 0$ and from (14), we have

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i+1}^{T} \lambda_k}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}. \quad (15)$$

The second part is a direct result of complementary slackness. ■

The above property indicates an iterative way to find the optimal power sequence. Specifically, suppose somehow we already know the energy depletion slots t_k in which the EH constraint is binding for $k = 1, 2, \cdots, N$, then we can divide the entire power allocation cycle into a set of disjoint segments $[t_k + 1, t_{k+1}]$ for $k = 0, 1, \cdots, N-1$ with $t_0 = 0$ and $t_N = T$. Within each segment, according to the first part of Property 1 the normalized power should be a constant, i.e.,

$$\frac{P_j^*}{\sqrt{\alpha_j}} \equiv \sqrt{\frac{1}{\sum\limits_{i=t_{k+1}}^{T} \lambda_i}} \quad (16)$$

for $j \in [t_k + 1, t_{k+1}]$. Besides, according to the second part of Property 1 the sum power is equal to

$$\sum_{i=t_k+1}^{t_{k+1}} P_i^* = \sum_{i=1}^{t_{k+1}} P_i^* - \sum_{i=1}^{t_k} P_i^* = \sum_{i=t_k}^{t_{k+1}-1} E_i. \quad (17)$$

From those two equations, we can solve for

$$P_j^* = \frac{\sqrt{\alpha_j}}{\sum_{i=t_k+1}^{t_{k+1}} \sqrt{\alpha_i}} \sum_{j=t_k}^{t_{k+1}-1} E_j \triangleq \sqrt{\alpha_j} f(t_k+1, t_{k+1}, E_{t_k}) \quad (18)$$

for $j \in [t_k + 1, t_{k+1}]$, where for notational convenience we define the function

$$f(i, j, x) = \frac{1}{\sum\limits_{k=i}^{j} \sqrt{\alpha_k}} \left(\sum_{k=i}^{j-1} E_k + x \right) \quad (19)$$

for $1 \leq i \leq j \leq T$. From the above result, we observe that the optimal power solution has a *piecewise* structure, i.e., within each segment the normalized power is a constant, and the optimal power could be calculated once we know all the energy depletion slots $\{t_k\}$. Yet we still need a few more properties to determine those energy depletion slots.

Property 2: In each segment $[t_k + 1, t_{k+1}]$, we have $f(t_{k+1}, t_{k+1}, E_{t_k}) < f(t_k + 1, j, E_{t_k})$ for $j = t_{k+1}, \cdots, t_{k+1} - 1$.

Proof: In each segment $[t_{k+1}, t_{k+1}]$, EH constraints are not binding in all slots except the t_{k+1}-th slot, i.e.,

$$\sum_{m=1}^{j} P_m^* < \sum_{m=0}^{j-1} E_m \quad (20)$$

for $j = t_{k+1}, \cdots, t_{k+1} - 1$. Because t_k is an energy depletion slot, we also have

$$\sum_{m=1}^{t_k} P_m^* = \sum_{m=0}^{t_k-1} E_m. \quad (21)$$

Subtracting (21) from (20), we can solve for

$$\begin{aligned} \sum_{m=t_k+1}^{j} P_m^* &= \left[\sum_{m=t_k+1}^{j} \sqrt{\alpha_m} \right] f(t_k + 1, t_{k+1}, E_{t_k}) \\ &< \sum_{m=t_k}^{j-1} E_m, \end{aligned} \quad (22)$$

where the equality is due to (18). The above result can be rewritten as

$$f(t_k+1, t_{k+1}, E_{t_k}) < \frac{\sum_{m=t_k}^{j-1} E_m}{\sum_{m=t_k+1}^{j} \sqrt{\alpha_m}} = f(t_k+1, j, E_{t_k}), \quad (23)$$

which completes the proof. ■

Property 3: The sequence $\{f(t_{k-1} + 1, t_k, E_{t_{k-1}})\}_{k=1}^N$ is non-decreasing, i.e.,

$$\begin{aligned} f(t_0 + 1, t_1, E_{t_0}) &\leq f(t_1 + 1, t_2, E_{t_1}) \leq \cdots \\ &\leq f\left(t_{N-1} + 1, t_N, E_{t_{N-1}}\right). \end{aligned} \quad (24)$$

Proof: From Property 1, we learn that the Lagrange multipliers λ_i are non negative in the energy depletion slot t_k for $k = 1, 2, \cdots, N$, and $\lambda_i = 0$ for $i \in [t_k + 1, t_{k+1} - 1]$. Thus we have

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i}^{T} \lambda_k}} \leq \sqrt{\frac{1}{\sum_{k=i+1}^{T} \lambda_k}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}} \quad (25)$$

for $i = 1, 2, \cdots, T - 1$. That is, the normalized power sequence $\left\{ \frac{P_i^*}{\sqrt{\alpha_i}} \right\}$ is non-decreasing. Within each segment, the power is given by (18), based on which we can deduce that

$$\begin{aligned} f\left(t_{k-1} + 1, t_k, E_{t_{k-1}}\right) &= \frac{P_{t_k}^*}{\sqrt{\alpha_{t_k}}} \leq \frac{P_{t_{k+1}}^*}{\sqrt{\alpha_{t_{k+1}}}} \\ &= f\left(t_k + 1, t_{k+1}, E_{t_k}\right) \end{aligned} \quad (26)$$

for $k = 1, 2, \cdots, N - 1$. ■

Using the above two properties, we can determine all energy depletion slots and then obtain the optimal power solution from (18). We summarize the major results of this section in the following theorem.

Theorem 1: The optimal solution to Problem (P2) is given by

$$P_k^* = \sqrt{\alpha_k} f\left(t_j + 1, t_{j+1}, E_{t_j}\right) \quad (27)$$

for $k \in [t_j + 1, t_{j+1}]$ and $j = 0, 1, \cdots, N-1$, where[2]

$$t_j = \arg\min_{t_{j-1}+1 \le k \le T} f(t_{j-1} + 1, k, E_{t_{j-1}}) \quad (28)$$

for $j = 1, 2, \cdots, N$, with $t_0 = 0$ and $t_N = T$.

Proof: Please refer to Appendix A. ■

Corollary 1: If the sequence $\left\{\frac{E_{i-1}}{\sqrt{\alpha_i}}\right\}_{i=1}^{T}$ is non-decreasing, the optimal power solution is the *best-effort* strategy, i.e., $P_i^* = E_{i-1}$ for $i = 1, 2, \cdots, T$. On the other hand, if the sequence $\left\{\frac{E_{i-1}}{\sqrt{\alpha_i}}\right\}_{i=1}^{T}$ is non increasing, the optimal power solution is $P_i^* = \sqrt{\alpha_i} f(1, T, E_0)$ for $i = 1, 2, \cdots, T$.

Proof: When $\left\{\frac{E_{i-1}}{\sqrt{\alpha_i}}\right\}_{i=1}^{T}$ is non increasing, it is easy to show that the normalized power should be a constant over the entire horizon. If $\left\{\frac{E_{i-1}}{\sqrt{\alpha_i}}\right\}_{i=1}^{T}$ is non-decreasing, each slot is a separate segment. As a result, the stored energy should be depleted in each slot. ■

Note that the coefficients α_i play an important role in the power allocation. Indeed, α_i is monotonically increasing with the transmission rate R_i. So, roughly speaking, α_i is a kind of information measure, and the normalized power $\frac{P_i}{\sqrt{\alpha_i}}$ can be regarded as the allocated power per information measure in the i-th slot. Theorem 1 reveals that we should allocate the total power over the entire information measure as evenly as possible. However, this may violate some EH constraints. As a result, the best we can do is to divide the entire horizon into disjoint segments, and within each segment to make the normalized power constant.

IV. Power Scheduling With Finite Battery Capacity

So far, we have found the optimal power solution when the battery capacity is infinite. In this section, we take into account both EH and BC constraints. Again, we start by studying the necessary optimality conditions and discuss the piecewise structure of the optimal power solution. Then, we discuss how to divide the entire cycle into a set of disjoint segments through a divide-and conquer algorithm.

In this section, we consider the general problem (P1) that is subject to both EH and BC constraints. Problem (P1) is a convex optimization problem, which can be solved by Lagrange multiplier method. The Lagrangian function is given by

$$\mathcal{L} = \sum_{i=1}^{T} \frac{\alpha_i}{P_i} + \sum_{i=1}^{T} \lambda_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i-1} E_k \right) - \sum_{i=1}^{T-1} \mu_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i} E_k + B \right) - \sum_{i=1}^{T} \omega_i P_i, \quad (29)$$

[2]Throughout this work, if there exist multiple minima (maxima), it always refers to the first minima (maxima).

where $\lambda_i, \mu_i, \omega_i \ge 0$ are Lagrange multipliers. The KKT conditions are given by

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\frac{\alpha_i}{P_i^2} + \sum_{k=i}^{T} \lambda_k - \sum_{k=i}^{T-1} \mu_k - \omega_i = 0 \quad (30)$$

for $i = 1, 2, \cdots, T$. The complementary slackness conditions can be written as

$$\lambda_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i-1} E_k \right) = 0, i = 1, 2, \cdots, T \quad (31)$$

$$\mu_i \left(\sum_{k=1}^{i} P_k - \sum_{k=0}^{i} E_k + B \right) = 0, i = 1, 2, \cdots, T-1 \quad (32)$$

$$\omega_i P_i = 0, i = 1, 2, \cdots, T. \quad (33)$$

From (30), we can solve for

$$P_i^* = \sqrt{\frac{\alpha_i}{\sum_{k=i}^{T} \lambda_k - \sum_{k=i}^{T-1} \mu_k - \omega_i}}. \quad (34)$$

From (33) and (34), we can deduce that $\omega_i = 0$ for $i = 1, 2, \cdots, T$. Thus, the optimal transmitted power can be rewritten as

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i}^{T} \lambda_k - \sum_{k=i}^{T-1} \mu_k}}. \quad (35)$$

It is very difficult to obtain the Lagrange multipliers directly. Again, we start by studying the structural properties of the optimal power solution. For notational convenience, we define two Boolean vectors $\mathbf{e} = (e_1, e_2, \cdots, e_T)$ and $\mathbf{b} = (b_1, b_2, \cdots, b_{T-1})$. The value of e_i indicates whether the i-th EH constraint is binding under the optimal power solution, i.e., $e_i = 1$ when $\sum_{k=1}^{i} P_k^* = \sum_{k=0}^{i-1} E_k$ and $e_i = 0$ otherwise for $i = 1, 2, \cdots, T$. Due to complementary slackness condition (31), we can deduce that $e_i = 1$ if $\lambda_i > 0$ and $\lambda_i = 0$ if $e_i = 0$. It should be pointed out that the last EH constraint is always binding because $\lambda_T > 0$. Likewise, $b_i = 1$ means that the i th BC constraint is binding under the optimal power solution (i.e., $\sum_{k=1}^{i} P_k^* = \sum_{k=0}^{i} E_k - B$) and $b_i = 0$ otherwise for $i = 1, 2, \cdots, T-1$. From (32), we can also deduce that $b_i = 1$ if $\mu_i > 0$ and $\mu_i = 0$ if $b_i = 0$. In the following, we will say k is an energy depletion slot if $e_k = 1$, and k is a battery overflow slot if $b_k = 1$.

Property 4: Depending on the values of e_i and b_i for $i = 1, 2, \cdots, T-1$, we have the following properties:

(i) $\frac{P_i^*}{\sqrt{\alpha_i}} \ge \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}$ if $e_i = 0$ and $b_i = 1$.
(ii) $\frac{P_i^*}{\sqrt{\alpha_i}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}$ if $e_i = 0$ and $b_i = 0$.
(iii) $\frac{P_i^*}{\sqrt{\alpha_i}} \le \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}$ if $e_i = 1$ and $b_i = 0$.
(iv) $E_i = B$ if and only if $e_i = 1$ and $b_i = 1$.

Proof: When $e_i = 0$ and $b_i = 1$, we have $\lambda_i = 0$, $\mu_i \geq 0$ and

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i+1}^{T}\lambda_k - \sum_{k=i}^{T}\mu_k}} \geq \sqrt{\frac{1}{\sum_{k=i+1}^{T}\lambda_k - \sum_{k=i+1}^{T}\mu_k}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}. \tag{36}$$

When $e_i = 0$ and $b_i = 0$, we have $\lambda_i = 0$, $\mu_i = 0$ and

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i+1}^{T}\lambda_k - \sum_{k=i+1}^{T}\mu_k}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}. \tag{37}$$

When $e_i = 1$ and $b_i = 0$, we have $\lambda_i \geq 0$, $\mu_i = 0$ and

$$\frac{P_i^*}{\sqrt{\alpha_i}} = \sqrt{\frac{1}{\sum_{k=i}^{T}\lambda_k - \sum_{k=i+1}^{T}\mu_k}} \leq \sqrt{\frac{1}{\sum_{k=i+1}^{T}\lambda_k - \sum_{k=i+1}^{T}\mu_k}} = \frac{P_{i+1}^*}{\sqrt{\alpha_{i+1}}}. \tag{38}$$

For the last case, when $e_i = b_i = 1$ both the i-th EH constraint and the i-th BC constraint are binding, and we can solve for $E_i = B$. Conversely, if $E_i = B$ the two constraints $\sum_{k=1}^{i} P_k^* \leq \sum_{k=0}^{i-1} E_k$ and $\sum_{k=1}^{i} P_k \geq \sum_{k=0}^{i-1} E_k$ must hold true simultaneously, which indicates that both inequalities must be binding. ■

The above property indicates that the optimal power sequence still has a piecewise structure. Specifically, denote all energy depletion slots by $t_1, t_2, \cdots, t_N$, where $t_N = T$ because the last EH constraint must be binding. In particular, we also define $t_0 = 0$ for notational convenience. Suppose there are $L_k \geq 0$ battery overflow slots $s_{k,1}, s_{k,2}, \cdots, s_{k,L_k}$ in the segment $[t_k + 1, t_{k+1}]$ for $k = 0, 1, \cdots, N-1$, and $t_k < s_{k,1} < s_{k,2} < \cdots < s_{k,L_k} < t_{k+1}$. Both energy depletion slots and battery overflow slots are boundary slots that divide the entire power scheduling cycle into disjoint segments. Between any two consecutive boundary slots, both EH constraints and BC constraints are not binding. So according to Property 4(ii), the normalized power should be a constant between any two consecutive boundary slots. In other words, once all the boundary slots are given, we can easily deduce the optimal power sequence as follows.

Theorem 2: If the energy depletion slots and the battery overflow slots are known and are given by

$$0 = t_0 < s_{0,1} < s_{0,2} < \cdots < s_{0,L_0} < t_1 < s_{1,1} < s_{1,2} < \cdots < s_{1,L_1} < t_2 < \cdots < t_N = T,$$

then the optimal power scheduling strategy is given by (39), shown at the bottom of the page.

Proof: Suppose $j \in [t_k + 1, s_{k,1}]$ and $L_k \neq 0$, then according to Property 4(ii) we have $\frac{P_m^*}{\sqrt{\alpha_m}} \equiv \text{const}$ for $m \in [t_k + 1, s_{k,1}]$. Because t_k and $s_{k,1}$ are energy depletion slot and battery overflow slot, respectively, we can deduce that

$$\sum_{m=t_k+1}^{s_{k,1}} P_m^* = \sum_{i=t_k}^{s_{k,1}} E_i - B. \tag{40}$$

Combining the above two equations we can solve for

$$P_j^* = \sqrt{\alpha_j} f\left(t_k + 1, s_{k,1}, E_{t_k} + E_{s_{k,1}} - B\right) \tag{41}$$

for $j \in [t_k + 1, s_{k,1}]$. We can follow the similar steps to prove the other cases. ■

The work left is to determine all the boundary slots. Unfortunately, there are no closed-form solutions to determine those boundary slots directly. So, in the sequel, we design a divide-and-conquer algorithm to recursively solve the original problem (P1). The important thing to note is that the problem (P1) actually consists of a couple of similar subproblems of smaller size. Specifically, let us define a family of problems

$$\begin{aligned} \text{Prob}(i, j, x, y) :&= \min_{\{P_m\}} \sum_{m=i}^{j} \frac{\alpha_m}{P_m} \\ \text{s.t.} \ & \sum_{m=i}^{t} P_m \leq \sum_{m=i}^{t-1} E_m + x + y \times 1_{\{t=j\}}, t = i, i+1, \cdots, j \\ & \sum_{m=i}^{t} P_m \geq \sum_{m=i}^{t} E_m + x - B, t = i, i+1, \cdots, j-1 \\ & P_t \geq 0, \ t = i, i+1, \cdots, j \end{aligned} \tag{42}$$

where $1_{\{.\}}$ is the indicator function that takes value 1 if the argument is true and 0 otherwise. Then the original problem (P1) can be represented as $\text{Prob}(1, T, E_0, 0)$. Any boundary slot may divide the original problem into some subproblems having a similar form as (42), and the global optimal solution (39) also solves those subproblems. For example, given an arbitrary energy depletion slot t_k, the original problem (P1) can be divided into two independent subproblems $\text{Prob}(1, t_k, E_0, 0)$ and $\text{Prob}(t_k + 1, T, E_{t_k}, 0)$, the solutions of which still have the structure given by (39). Similarly, each battery overflow slot $s_{k,l}$ also provides a way to divide the original problem (P1) into

$$\frac{P_j^*}{\sqrt{\alpha_j}} = \begin{cases} f\left(t_k + 1, s_{k,1}, E_{t_k} + E_{s_{k,1}} - B\right), j \in [t_k + 1, s_{k,1}], L_k \neq 0 \\ f\left(s_{k,l} + 1, s_{k,l+1}, E_{s_{k,l+1}}\right), j \in [s_{k,l} + 1, s_{k,l+1}], L_k \neq 0 \\ f\left(s_{k,L_k} + 1, t_{k+1}, B\right), j \in [s_{k,L_k} + 1, t_{k+1}], L_k \neq 0 \\ f\left(t_k + 1, t_{k+1}, E_{t_k}\right), j \in [t_k + 1, t_{k+1}], L_k = 0 \end{cases} \tag{39}$$

two independent subproblems $\text{Prob}(1, s_{k,l}, E_0, E_{s_{k,l}} - B)$ and $\text{Prob}(s_{k,l}+1, T, B, 0)$. If some boundary slots can be determined, we can first divide the original problem into a couple of subproblems and then solve those subproblems separately. The advantage of such divide-and-conquer algorithm is that the subproblems are of smaller size and thus are much simpler to solve. Under some special conditions, we can solve those subproblems directly. Otherwise, we can continue to divide the subproblems into even smaller subproblems until they are solvable.

In the sequel, we discuss how to divide the original problem into subproblems, and study the special condition under which we can solve the subproblem directly. Without loss of generality, we focus on the general problem $\text{Prob}(i, j, x, y)$. Using the Lagrange method, we can show that the optimal solution to the problem $\text{Prob}(i, j, x, y)$ is still given by (35) after replacing T with j, and Property 4 and Theorem 2 still hold true. Note that if there exists $t \in [i, j]$ such that $E_t = B$, then according to Property 4(iv) both EH constraint and BC constraint must be binding in the t-th slot. So t should be both energy depletion slot and battery overflow slot, and we can divide the original problem $\text{Prob}(i, j, x, y)$ into $\text{Prob}(i, t, x, 0)$ and $\text{Prob}(t+1, j, B, y)$. In the sequel, we assume such simple partition has been done and $E_t < B$ for all $t \in [i, j]$. We first give some results that would be used repeatedly later. The proof is very straightforward and is thus omitted.

Lemma 1: Consider any $x_i, y_i > 0$ for $i = 1, 2$. If $\frac{x_1}{y_1} \le \frac{x_2}{y_2}$, or $\frac{x_1}{y_1} \le \frac{x_1+x_2}{y_1+y_2}$, or $\frac{x_1+x_2}{y_1+y_2} \le \frac{x_2}{y_2}$, then $\frac{x_1}{y_1} \le \frac{x_1+x_2}{y_1+y_2} \le \frac{x_2}{y_2}$.

Corollary 2: Consider any $x_i, y_i > 0$ for $i = 1, 2, \cdots, N$. If $\frac{x_1}{y_1} \le \frac{x_2}{y_2} \le \cdots \le \frac{x_N}{y_N}$, then for any $k = 1, 2, \cdots, N$ we have $\frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{k} y_i} \le \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} y_i} \le \frac{\sum_{i=k+1}^{N} x_i}{\sum_{i=k+1}^{N} y_i}$.

Corollary 3: Consider any $x_i, y_i > 0$ for $i = 1, 2, 3$. If $x_2 \le x_3$ and $y_2 \le y_3$, then

(i) $\frac{x_1+x_2}{y_1+y_2} \le \frac{x_1+x_3}{y_1+y_3}$ if $\frac{x_1}{y_1} \le \frac{x_3}{y_3}$ and $\frac{x_2}{y_2} \le \frac{x_3}{y_3}$.
(ii) $\frac{x_1+x_2}{y_1+y_2} \ge \frac{x_1+x_3}{y_1+y_3}$ if $\frac{x_1}{y_1} \ge \frac{x_3}{y_3}$ and $\frac{x_2}{y_2} \ge \frac{x_3}{y_3}$.

The next few properties specify how to divide the original problem into subproblems.

Property 5: Consider the general problem $\text{Prob}(i, j, x, y)$, where $0 \le x \le B$, $-B \le y \le 0$, and $E_k < B$ for all $k \in [i, j]$. Denote the optimal solution by $\{P_k^*\}_{k=i}^{j}$.

(i) If t_1 is the first energy depletion slot, i.e.,

$$t_1 = \min \left\{ \begin{array}{l} k : k \in [i, j], \\ \sum_{m=i}^{k} P_m^* = \sum_{m=i}^{k-1} E_m + x + y \times 1_{\{k=j\}} \end{array} \right\}, \quad (43)$$

then t_1 must satisfy the condition

$$t_1 = \arg\min_{i \le m \le t_1} f\left(i, m, x + y \times 1_{\{m=j\}}\right). \quad (44)$$

(ii) If s_1 is the first battery overflow slot, i.e.,

$$s_1 = \min \left\{ \begin{array}{l} k : k \in [i, j-1], \\ \sum_{m=i}^{k} P_m^* = \sum_{m=i}^{k} E_m + x - B \end{array} \right\}, \quad (45)$$

then s_1 must satisfy the condition

$$s_1 = \arg\max_{i \le m \le s_1} f(i, m, x + E_m - B). \quad (46)$$

Proof: Please refer to Appendix B. ■

Property 6: Consider the general problem $\text{Prob}(i, j, x, y)$, where $0 \le x \le B$, $-B \le y \le 0$ and $E_k < B$ for all $k \in [i, j]$. Denote the optimal solution by $\{P_k^*\}_{k=i}^{j}$, and let $\hat{t} = \arg\min_{i \le k \le j} f(i, k, x + y \times 1_{\{k=j\}})$. Then $\hat{t}$ must be an energy depletion slot, i.e., $\sum_{m=i}^{\hat{t}} P_m^* = \sum_{m=i}^{\hat{t}-1} E_m + x + y \times 1_{\{\hat{t}=j\}}$.

Proof: Please refer to Appendix C. ■

The above property indicates that the slots obtained iteratively through (28) are all energy depletion slots. This is because for the original problem $\text{Prob}(t_k+1, T, E_{t_k}, 0)$, the minimum of $f(t_k+1, m, E_{t_k})$ as a function of m is achieved at t_{k+1}. As a result, we can conclude that if the stored energy should be depleted in the t_k-th slot when ignoring the BC constraints, in that specific slot the stored energy should still be depleted in the presence of BC constraints. Moreover, we can divide the original problem $\text{Prob}(1, T, E_0, 0)$ into a couple of subproblems $\text{Prob}(t_k+1, t_{k+1}, E_{t_k}, 0)$, where t_k are given by (28). For each subproblem, it has the property that $t_{k+1} = \arg\min_{t_k+1 \le m \le t_{k+1}} f(t_k+1, m, E_{t_k})$. In the sequel, we develop another important property to solve subproblems of this kind.

Property 7: Consider the general problem $\text{Prob}(i, j, x, y)$, where $0 \le x \le B$, $-B \le y \le 0$ and $E_k < B$ for all $k \in [i, j]$. Suppose $j = \arg\min_{i \le m \le j} f(i, m, x + y \times 1_{\{m=j\}})$, and the optimal solution is denoted by $\{P_k^*\}_{k=i}^{j}$. Let $\hat{s} = \arg\max_{i \le m \le j-1} f(i, m, x + E_m - B)$. If $i = j$ or $f(i, j, x+y) \ge f(i, \hat{s}, x + E_{\hat{s}} - B)$, then all BC constraints are not binding and the optimal power is given by $P_m^* = \sqrt{\alpha_m} f(i, j, x+y)$ for $m \in [i, j]$. Otherwise, $\hat{s}$ must be a battery overflow slot, i.e., $\sum_{m=i}^{\hat{s}} P_m^* = \sum_{m=i}^{\hat{s}} E_m + x - B$.

Proof: Please refer to Appendix D. ■

Based on Property 6 and Property 7, we can design a divide-and-conquer algorithm to solve the original problem $\text{Prob}(1, T, E_0, 0)$. Specifically, for the original problem we first find $\hat{t} = \arg\min_{1 \le k \le T} f(1, k, E_0)$ and divide the original problem into two separate subproblems $\text{Prob}(1, \hat{t}, E_0, 0)$ and $\text{Prob}(\hat{t}+1, T, E_{\hat{t}}, 0)$. For each subproblem, we can follow the similar procedure and divide the subproblems into smaller subproblems of the similar form. The division continues until all subproblems $\text{Prob}(i, j, x, y)$ satisfy $j = \arg\min_{i \le k \le j} f(i, k, x + y \times 1_{\{k=j\}})$. For each subproblem $\text{Prob}(i, j, x, y)$, the optimal power is $P_m^* = \sqrt{\alpha_m} f(i, j, x+y)$ for $m \in [i, j]$ if $i = j$ or $f(i, j, x+y) \ge \max_{i \le k \le j-1} f(i, k, x + E_k - B)$. Otherwise, let $\hat{s} = \arg\max_{i \le k \le j-1} f(i, k, x + E_k - B)$ and divide the subproblem $\text{Prob}(i, j, x, y)$ into two smaller subproblems $\text{Prob}(i, \hat{s}, x, E_{\hat{s}} - B)$ and $\text{Prob}(\hat{s}+1, j, B, y)$. This divide-and-conquer procedure continues until all subproblems can be solved. A detailed description of the algorithm is summarized in *Algorithm 1*.

Algorithm 1 Divide-and-conquer algorithm to solve the problem Prob(i, j, x, y)

If $\exists k \in [i, j-1]$ such that $E_i = B$
  Solve the subproblems Prob$(i, k, x, 0)$ and Prob$(k+1, j, B, y)$ separately;
  Return;
End
Denote $\hat{t} = \arg\min_{i \le k \le j} f(i, k, x + y \times 1_{\{k=j\}})$;
If $\hat{t} < j$
  Solve the subproblems Prob$(i, \hat{t}, x, 0)$ and Prob$(\hat{t}+1, j, E_{\hat{t}}, y)$ separately;
  Return;
End
Denote $\hat{s} = \arg\max_{i \le k \le j-1} f(i, k, x + E_k - B)$;
If $i = j$ or $f(i, j, x+y) \ge f(i, \hat{s}, x + E_{\hat{s}} - B)$
  The optimal solution is $P_m^* = \sqrt{\alpha_m} f(i, j, x+y)$ for $m \in [i, j]$;
Else
  Solve the subproblems Prob$(i, \hat{s}, x, E_{\hat{s}} - B)$ and Prob$(\hat{s}+1, j, B, y)$ separately;
End
Return;

A. *Online Policy*

Note that *Algorithm 1* is an offline policy in that the scheduler needs to know the exact value of the harvested energy E_i for $i = 1, 2, \cdots, T$. Such offline policy could be obtained by running *Algorithm 1* once at the beginning of each power scheduling cycle. In practice, the power scheduler has only causal knowledge of ESI and the exact energy E_i is unknown. Nevertheless, the scheduler could still make prediction of the harvested energy and apply *Algorithm 1* in an online manner.

Suppose in the t-th time slot for $t = 1, 2, \cdots, T$, the scheduler knows only the expected energy $\{\bar{E}_k\}_{k=t}^{T}$ that will arrive in the future slots, where $\bar{E}_k = E[E_k]$ and $E[\cdot]$ stands for expectation. The available energy at the beginning of the t-th time slot is still given by B_t, and this quantity is known to the scheduler. To calculate the power P_t for the t-th time slot, the scheduler needs to run *Algorithm 1* and solve the problem Prob$(t, T, B_t, 0)$ by plugging in $\bar{E}_k$ instead of E_k for $k = t, t+1, \cdots, T$. Suppose the actual energy harvested in the t-th slot is E_t, which may be different from its expected value $\bar{E}_t$. Then the energy available at the beginning of the $(t+1)$-th slot is computed based on the allocated power P_t and the actual harvested energy E_t according to $B_{t+1} = \min\{B_t - P_t + E_t, B\}$. Afterwards, the scheduler could repeat the same process and calculate the power P_{t+1} for the $(t+1)$-th time slot by solving the problem Prob$(t+1, T, B_{t+1}, 0)$, again by plugging in $\bar{E}_k$ for $k = t+1, t+2, \cdots, T$. This procedure is repeated until the last time slot.

The aforementioned scheme is an online policy in that only the predicted value of the harvested energy would be used to calculate the power. However, as the prediction could be wrong, the scheduler needs to dynamically adjust the allocated power whenever any new energy is harvested. If the prediction is accurate enough, such online policy would achieve nearly the same performance as the offline solution. In practice, prediction errors would inevitably degrade the performance. Fortunately, simulations shall demonstrate that this online policy is fairly robust to prediction errors.

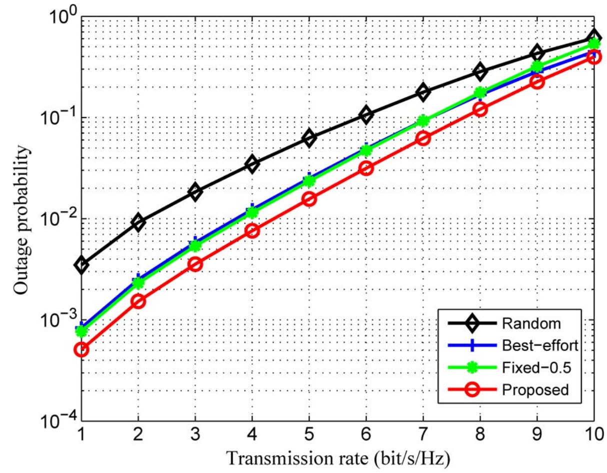

Fig. 2. Outage probability versus transmission rate for SNR $\rho = 30$ dB.

V. Simulations

In this section, we evaluate the performance of the proposed power scheduling algorithm and compare with other alternative approaches. In simulations, we adopt the real-world solar radiation data provided by National Renewable Energy Laboratory [28] after proper normalization. It is assumed that the transmitter is a sensor node which needs to wake up periodically. During each wake-up cycle, the sensor node shall report data to the data center for $T = 100$ time slots. The range of harvested energy is normalized to $E_i \in [0.1, 5]$, and the battery capacity is assumed to be $B = 3$ if not mentioned otherwise. The transmission rates are uniformly distributed in the range [1, 3] if not mentioned otherwise. We use the path loss model $\gamma = d^{-3}$, where d is the distance between the transmitter and the receiver. By default, distance is normalized to 1. System SNR is defined as $\rho = \frac{1}{\sigma^2}$, where σ^2 is the noise power. For simplicity, we choose the weight $w_i \equiv \frac{1}{T}$ such that the objective function represents the average outage probability.

We compare the proposed power scheduling algorithm with several other strategies: 1) Best-effort power scheduling strategy, which allocates all the power within each slot (i.e., $P_i = E_{i-1}$). 2) Fixed-ratio power scheduling strategy, which consumes a fixed portion (denoted by β) of the available energy in each slot, i.e., $P_i = \beta \times B_{i-1}$ for $i = 1, 2, \cdots, T$. (3) Random power allocation strategy, which allocates a random portion (denoted by η) of energy in each slot, where η is uniformly distributed in the interval [0, 1].

We first show the outage probability versus transmission rate in Fig. 2. In those simulations, the transmission rates in all time slots are assumed to be constant, i.e., $R_i \equiv R$. It is observed that the outage probability is an increasing function of transmission rate, because higher transmission rates tend to cause channel outage more often. The proposed power scheduling strategy can

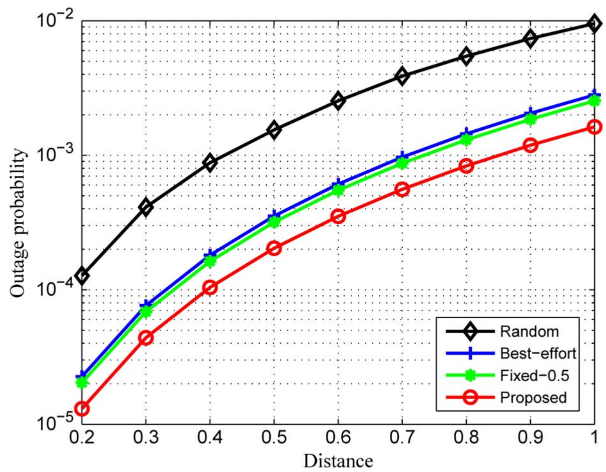

Fig. 3. Outage probability versus distance for SNR $\rho = 30$ dB.

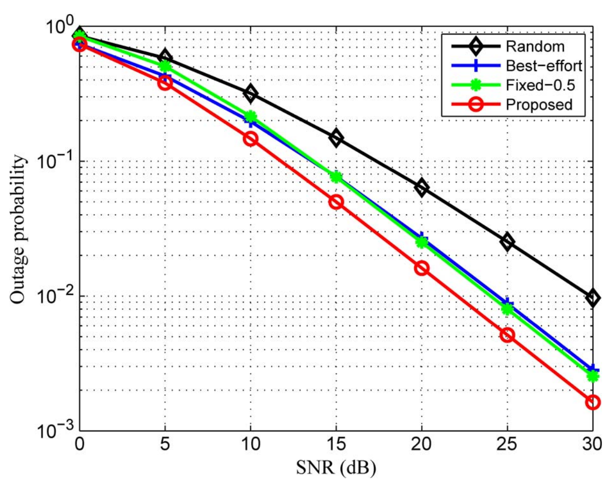

Fig. 4. Outage probability versus system SNR.

improve the outage performance compared to other strategies. The proposed strategy achieves much better performance than the best-effort strategy especially when the transmission rate is low. This is because when the harvested energy is extremely low in certain slots, the following slots would become the system bottleneck under best-effort strategy. On the contrary, in the proposed strategy some power in the previous slots would be saved for those bottleneck slots as compensation. The proposed strategy is also superior to fixed-0.5 strategy, since it adaptively allocates the harvested energy in different time slots.

Then in Figs. 3 and 4 we show the outage probability versus the distance between the transmitter and the receiver and the outage probability versus system SNR, respectively. It is observed that the proposed power scheduling scheme is superior to other schemes in all scenarios. When SNR is fairly high, the proposed scheme achieves around 2 dB SNR gain against best-effort strategy and fixed-0.5 scheme. It is observed that at low SNRs, best-effort scheme is near-optimal. This is because when the channel conditions are poor on average, the harvested energy may hardly support the fixed-rate transmission in most time slots. As a result, most stored energy should be consumed as is the case under best-effort scheme. However, as channel conditions get better on average, the bottleneck slots are

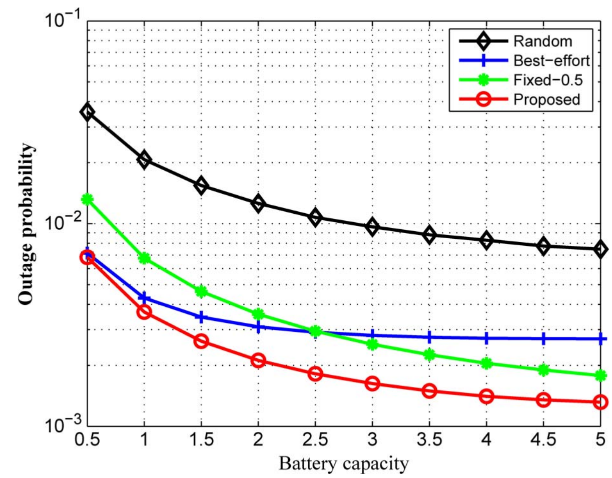

Fig. 5. Outage probability versus battery capacity for SNR $\rho = 30$ dB.

unlikely to appear quite often. In this case, some power should be reserved for the bottleneck slots that may occur later. That is why fixed-0.5 scheme is slightly better at high SNRs.

To better understand the impact of battery capacity, we then show in Fig. 5 the outage probability versus battery capacity. It is observed that the outage probability is a decreasing function of battery capacity for all schemes. This is because smaller battery capacity would lower the saved power available for later slots. Besides, lower battery capacity may also cause some harvested energy to be abandoned along the way. Indeed, when battery capacity is low most power should be consumed in each slot. Otherwise, the saved power may be unnecessarily abandoned due to battery overflow. That is why best-effort scheme is nearly optimal at low battery capacity. On the contrary, when the battery capacity is high fixed-0.5 scheme is much better. This is because as more power could be reserved, it can get better compensation for the bottleneck slots in which only little energy can be harvested. Nevertheless, the proposed scheme still demonstrates noticeable gain due to adaptive power scheduling rather than fixed-ratio power allocation.

Finally, we study the performance of online policy given in Section IV.A. For simulation purpose, we assume that only the expected energy $\{\bar{E}_k\}_{k=1}^{T}$ is known. The actual harvested energy may vary uniformly around its expected value, i.e., $E_k = \bar{E}_k \times (1 + \tau_k)$. Here τ_k stands for the relative prediction error and is uniformly distributed in the range $(-\theta, \theta)$, where θ is the maximum relative prediction error. We show the outage performance versus maximum relative prediction error in Fig. 6. It is observed that the prediction error would degrade the outage performance. However, the performance loss is almost negligible when the maximum relative prediction error is below 0.2, which demonstrates that the proposed scheme is fairly robust to prediction error.

VI. Conclusion

In this work, we have designed a power scheduling strategy to minimize the weighted sum of the outage probabilities for energy harvesting systems with infinite/finite battery capacity. We demonstrated that the entire power scheduling cycle can be divided into disjoint segments, within which the normalized

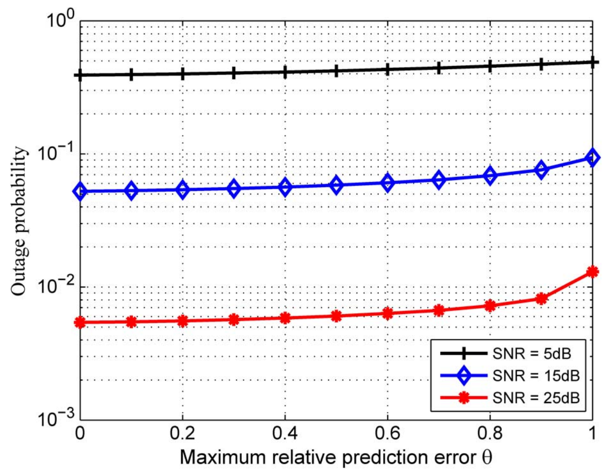

Fig. 6. Outage probability versus prediction error.

power level is a constant. In the case of infinite battery capacity, we developed an efficient algorithm to iteratively search for the optimal power allocation. In the case of finite battery capacity, we designed a divide-and-conquer algorithm that recursively divides the original problem into smaller solvable subproblems. We also developed an efficient online solution that is fairly robust to prediction errors of the harvested energy.

APPENDIX

A. Proof of Theorem 1

The first part has been proved in (18). For the second half, we start with the case $j = 1$. We are to prove that

$$t_1 = \arg\min_{1\leq k\leq T} f(1, k, E_{t_0}). \tag{47}$$

According to Property 2, we have

$$f(1, t_1, E_{t_0}) < f(1, j, E_{t_0}) \tag{48}$$

for $j = 1, 2, \cdots, t_1 - 1$. Next, we consider the case $j \in [t_{k-1} + 1, t_k]$ for some $k > 1$. According to (19), we have (49), shown at the bottom of the page, where the first inequality is due to Property 2, and the second inequality is due to Property 3. Combining the above two results completes the proof.

If $t_1 \neq T$, we could find the following energy depletion slots $t_2, t_3, \cdots, t_N$ in a similar manner. This is because t_1 is an energy depletion slot, in which all the stored energy would be depleted after the transmission phase of the t_1-th slot. Consequently, since slot $t_1 + 1$ a new power allocation sub-cycle starts with the initial energy being E_{t_1}. The optimal power scheduling in the sub-cycle is actually a subproblem that mimics the original one, and the structure of the solution should be similar after properly changing the initial condition. This justifies the iterative formula (28) for $j = 2, 3, \cdots, N$. The iteration terminates until $t_N = T$ for some $N \geq 1$. Since then one entire power cycle terminates.

B. Proof of Property 5

We first prove the first part. Without loss of generality, suppose there are L battery overflow slots $s_1, s_2, \cdots, s_L$ within $[i, t_1]$, and $i < s_1 < s_2 < \cdots < s_L < t_1 \leq j$. If $L = 0$, then according to (39) we have

$$\frac{P_m^*}{\sqrt{\alpha_m}} = f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) \tag{50}$$

$$\begin{aligned}
f(1, j, E_{t_0}) &= \frac{\sum_{m=0}^{t_1-1} E_m + \sum_{m=t_1}^{t_2-1} E_m + \cdots + \sum_{m=t_{k-1}+1}^{j-1} E_m}{\sum_{m=1}^{j} \sqrt{\alpha_m}} \\
&= \frac{f(1, t_1, E_{t_0})\left(\sum_{m=1}^{t_1}\sqrt{\alpha_m}\right) + f(t_1+1, t_2, E_{t_1})\left(\sum_{m=t_1+1}^{t_2}\sqrt{\alpha_m}\right) + \cdots + f(t_{k-1}+1, j, E_{t_{k-1}})\left(\sum_{m=t_{k-1}+1}^{j}\sqrt{\alpha_m}\right)}{\sum_{m=1}^{j}\sqrt{\alpha_m}} \\
&\geq \frac{f(1, t_1, E_{t_0})\left(\sum_{m=1}^{t_1}\sqrt{\alpha_m}\right) + f(t_1+1, t_2, E_{t_1})\left(\sum_{m=t_1+1}^{t_2}\sqrt{\alpha_m}\right) + \cdots + f(t_{k-1}+1, t_k, E_{t_{k-1}})\left(\sum_{m=t_{k-1}+1}^{j}\sqrt{\alpha_m}\right)}{\sum_{m=1}^{j}\sqrt{\alpha_m}} \\
&\geq \frac{f(1, t_1, E_{t_0})\left(\sum_{m=1}^{t_1}\sqrt{\alpha_m}\right) + f(1, t_1, E_{t_0})\left(\sum_{m=t_1+1}^{t_2}\sqrt{\alpha_m}\right) + \cdots + f(1, t_1, E_{t_0})\left(\sum_{m=t_{k-1}+1}^{j}\sqrt{\alpha_m}\right)}{\sum_{m=1}^{j}\sqrt{\alpha_m}} \\
&= f(1, t_1, E_{t_0})
\end{aligned} \tag{49}$$

for $m \in [i, t_1]$. Because all EH constraints are not binding for $m \in [i, t_1 - 1]$, we have

$$\sum_{k=i}^{m} P_k^* = f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) \sum_{k=i}^{m} \sqrt{\alpha_k} < \sum_{k=i}^{m-1} E_k + x, \quad (51)$$

which implies that

$$f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) < \frac{\sum_{k=i}^{m-1} E_k + x}{\sum_{k=i}^{m} \sqrt{\alpha_k}} = f(i, m, x) \quad (52)$$

for $m \in [i, t_1 - 1]$.

Next we consider the case $L > 0$. According to (39), the optimal power is

$$\frac{P_m^*}{\sqrt{\alpha_m}} = \begin{cases} f(i, s_1, x + E_{s_1} - B), m \in [i, s_1] \\ f\left(s_k + 1, s_{k+1}, E_{s_{k+1}}\right), m \in [s_k + 1, s_{k+1}] \\ f\left(s_L + 1, t_1, B + y \times 1_{\{t_1=j\}}\right), m \in [s_L + 1, t_1] \end{cases} \quad (53)$$

From Property 4(i), we can deduce that

$$f(i, s_1, x + E_{s_1} - B) \geq f(s_1 + 1, s_2, E_{s_2}) \geq \cdots \geq f\left(s_L + 1, t_1, B + y \times 1_{\{t_1=j\}}\right). \quad (54)$$

Using Corollary 2, we can conclude that

$$f(i, s_1, x + E_{s_1} - B) \geq f(i, s_2, x + E_{s_2} - B) \geq \cdots \geq f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right). \quad (55)$$

Since $f(i, s_k, x + E_{s_k} - B) < f(i, s_k, x)$ for $k = 1, 2, \cdots, L$, we have

$$f(i, s_k, x) > f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right). \quad (56)$$

for $k = 1, 2, \cdots, L$. Next we consider a specific slot $m \in [i, s_1 - 1]$. Because all EH constraints are not binding for $k \in [i, s_1 - 1]$, we have

$$\sum_{k=i}^{m} P_k^* = f(i, s_1, x + E_{s_1} - B) \sum_{k=i}^{m} \sqrt{\alpha_k} < \sum_{k=i}^{m-1} E_k + x. \quad (57)$$

As a result, we have

$$f(i, m, x) = \frac{\sum_{k=i}^{m-1} E_k + x}{\sum_{k=i}^{m} \sqrt{\alpha_k}} > f(i, s_1, x + E_{s_1} - B) > f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) \quad (58)$$

for $m \in [i, s_1 - 1]$. Next we consider a specific slot $m \in \left[s_k + 1, s_{k+1} - 1\right]$. Note that the optimal power $\{P_n^*\}_{n=s_k+1}^{s_{k+1}}$ is also the solution to the problem Prob $\left(s_k + 1, s_{k+1}, B, E_{s_{k+1}} - B\right)$. Using the similar argument, we can show that $f(s_k + 1, m, B) > f\left(s_k + 1, s_{k+1}, E_{s_{k+1}}\right)$ for $m \in [s_k + 1, s_{k+1} - 1]$. As a result, we have (59), shown at the bottom of the page, for $m \in [s_k + 1, s_{k+1} - 1]$. Using the similar argument, we can also show that the above inequality also holds true for $m \in [s_L + 1, t_1 - 1]$. To conclude, we have shown that

$$f(i, m, x) > f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) \quad (60)$$

for any $m \in [i, t_1 - 1]$, no matter $L = 0$ or $L > 0$. The second part can be proved in a similar way and the proof is omitted.

C. Proof of Property 6

The conclusion is true when $\hat{t} = j$, because all the stored energy should be depleted in the last slot. In the sequel, we prove by contradiction that this should also be true when $\hat{t} < j$. Suppose now that $\hat{t}$ is not an energy depletion slot. Denote the largest energy depletion slot before $\hat{t}$ by t_p, and the smallest energy depletion slot after $\hat{t}$ by t_n, such that $t_p < \hat{t} < t_n$ and there are no other energy depletion slots in $[t_p + 1, t_n - 1]$. Note that for problem Prob $\left(t_p + 1, t_n, E_{t_p}, y \times 1_{\{t_n=j\}}\right)$, t_n is the first and the unique energy depletion slot. From Property 5(i), we must have

$$f\left(t_p + 1, \hat{t}, E_{t_p}\right) > f\left(t_p + 1, t_n, E_{t_p} + y \times 1_{\{t_n=j\}}\right). \quad (61)$$

On the other hand, from the definition of $\hat{t}$ we have

$$\min\left(f(i, t_p, x), f(i, t_n, x + y \times 1_{\{t_n=j\}})\right) \geq f\left(i, \hat{t}, x\right). \quad (62)$$

$$\begin{aligned} f(i, m, x) &= \frac{f(i, s_1, x + E_{s_1} - B) \sum_{n=i}^{s_1} \sqrt{\alpha_n} + f(s_1 + 1, s_2, E_{s_2}) \sum_{n=s_1+1}^{s_2} \sqrt{\alpha_n} + \cdots + f(s_k + 1, m, B) \sum_{n=s_k+1}^{m} \sqrt{\alpha_n}}{\sum_{n=i}^{m} \sqrt{\alpha_n}} \\ &> \frac{f(i, s_1, x + E_{s_1} - B) \sum_{n=i}^{s_1} \sqrt{\alpha_n} + f(s_1 + 1, s_2, E_{s_2}) \sum_{n=s_1+1}^{s_2} \sqrt{\alpha_n} + \cdots + f\left(s_k + 1, s_{k+1}, E_{s_{k+1}}\right) \sum_{n=s_k+1}^{m} \sqrt{\alpha_n}}{\sum_{n=i}^{m} \sqrt{\alpha_n}} \\ &> \frac{f(i, s_1, x + E_{s_1} - B) \sum_{n=i}^{s_1} \sqrt{\alpha_n} + f(s_1 + 1, s_2, E_{s_2}) \sum_{n=s_1+1}^{s_2} \sqrt{\alpha_n} + \cdots + f\left(s_k + 1, s_{k+1}, E_{s_{k+1}}\right) \sum_{n=s_k+1}^{s_{k+1}} \sqrt{\alpha_n}}{\sum_{n=i}^{s_{k+1}} \sqrt{\alpha_n}} \\ &= f\left(i, s_{k+1}, x + E_{s_{k+1}} - B\right) > f\left(i, t_1, x + y \times 1_{\{t_1=j\}}\right) \end{aligned} \quad (59)$$

Using Lemma 1, we can deduce that

$$f\left(t_p+1,\hat{t},E_{t_p}\right)\leq f\left(i,\hat{t},x\right)\leq f\left(\hat{t}+1,t_n,E_{\hat{t}}+y\times 1_{\{t_n=j\}}\right). \quad (63)$$

Using Lemma 1 once more, we can conclude that

$$f\left(t_p+1,\hat{t},E_{t_p}\right)\leq f\left(t_p+1,t_n,E_{t_p}+y\times 1_{\{t_n=j\}}\right), \quad (64)$$

which contradicts (61). Consequently, $\hat{t}$ must be an energy depletion slot.

D Proof of Property 7

When $i=j$, only one slot is considered and there is no BC constraint. As a result, the EH constraint should be binding and $P_i^*=x+y=\sqrt{\alpha_i}f\left(i,i,x+y\right)$.

Next we consider the case $i<j$ and $f\left(i,j,x+y\right)\geq f\left(i,\hat{s},x+E_{\hat{s}}-B\right)$. If we ignore all BC constraints, then according to Theorem 1 the optimal power is $P_m^*=\sqrt{\alpha_m}f\left(i,j,x+y\right)$ for $m\in[i,j]$. This solution happens to satisfy all BC constraints automatically, because

$$\begin{aligned}\sum_{m=i}^{k}P_m^*&=f(i,j,x+y)\sum_{m=i}^{k}\sqrt{\alpha_m}\\&\geq f(i,\hat{s},x+E_{\hat{s}}-B)\sum_{m=i}^{k}\sqrt{\alpha_m}\\&\geq f(i,k,x+E_k-B)\sum_{m=i}^{k}\sqrt{\alpha_m}\\&=\sum_{m=i}^{k}E_m+x-B\end{aligned} \quad (65)$$

for $k\in[i,j-1]$. Consequently, the optimal solution remains the same even in the presence of BC constraints.

Finally, we consider the case $i<j$ and $f\left(i,j,x+y\right)<f\left(i,\hat{s},x+E_{\hat{s}}-B\right)$, which implies that there must exist at least one battery overflow slot within $[i,j]$. Denote the first battery overflow slot by s_1. According to Property 5(ii), $s_1=\arg\max_{i\leq m\leq s_1}f\left(i,m,x+E_m-B\right)$. This implies that $s_1\leq\hat{s}$, otherwise we would have $f\left(i,\hat{s},x+E_{\hat{s}}-B\right)>f\left(i,s_1,x+E_{s_1}-B\right)$ which leads to contradiction. Next, we prove by contradiction that $\hat{s}$ must be an battery overflow slot. Suppose now that $\hat{s}$ is not a battery overflow slot. We first show that this implies that there would be no battery overflow slot within $[\hat{s},j]$. Otherwise, denote the largest battery overflow slot before $\hat{s}$ by s_p, and the smallest battery overflow slot after $\hat{s}$ by s_n, such that $s_p<\hat{s}<s_n$ and there are no other battery overflow slots within $[s_p+1,s_n-1]$. Note that for problem Prob(s_p+1,j,B,y), s_n is the first battery overflow slot. From Property 5(ii), we can deduce that

$$f\left(s_p+1,s_n,E_{s_n}\right)>f\left(s_p+1,\hat{s},E_{\hat{s}}\right). \quad (66)$$

On the other hand, from the definition of $\hat{s}$ we have

$$\max\left(f\left(i,s_p,x+E_{s_p}-B\right),f\left(i,s_n,x+E_{s_n}-B\right)\right)\leq f\left(i,\hat{s},x+E_{\hat{s}}-B\right). \quad (67)$$

Using Lemma 1, we can deduce that

$$f(s_p+1,\hat{s},E_{\hat{s}})\geq f(i,\hat{s},x+E_{\hat{s}}-B)\geq f(\hat{s}+1,s_n,E_{s_n}). \quad (68)$$

Using Lemma 1 once more, we can conclude that

$$f\left(s_p+1,\hat{s},E_{\hat{s}}\right)\geq f\left(s_p+1,s_n,E_{s_n}\right), \quad (69)$$

which contradicts (66).

Consequently, if $\hat{s}$ is not a battery overflow slot, we have shown that there must exist at least one battery overflow slot before $\hat{s}$, and there can not exist any battery overflow slot after $\hat{s}$. However, we are going to show that this can not happen too by contradiction. Suppose now that the largest battery overflow slot before $\hat{s}$ is denoted by s_p, and there are no other battery overflow slots after $\hat{s}$. Those conditions imply that for problem Prob (s_p+1,j,B,y), there would be no battery overflow slots. Besides, we must have

$$f\left(s_p+1,\hat{s},E_{\hat{s}}\right)>f\left(s_p+1,j,B+y\right). \quad (70)$$

Otherwise, according to Corollary 3(i) we would have $f\left(i,\hat{s},x+E_{\hat{s}}-B\right)\leq f\left(i,j,x+y\right)$, which leads to contradiction. Suppose there are N energy depletion slots within $[s_p+1,j]$, which are denoted by t_k for $k=1,2,\cdots,N$ and satisfy $s_p=t_0<t_1<t_2<\cdots<t_N=j$. From Property 4(iii) and Corollary 2, we can deduce that

$$\begin{aligned}&f\left(s_p+1,t_1,B\right)\leq f\left(s_p+1,t_2,B\right)\leq\cdots\\&\quad\leq f\left(s_p+1,j,B+y\right)<f\left(s_p+1,\hat{s},E_{\hat{s}}\right).\end{aligned} \quad (71)$$

Suppose now that $\hat{s}\in[t_k+1,t_{k+1}]$ and $k>0$. Using Lemma 1, we can conclude that

$$\begin{aligned}&f\left(s_p+1,t_k,B\right)\\&\quad\leq\min\left\{\begin{array}{l}f\left(t_k+1,\hat{s},E_{t_k}+E_{\hat{s}}-B\right),\\f\left(t_k+1,t_{k+1},E_{t_k}+y\times 1_{\{t_{k+1}=j\}}\right)\end{array}\right\}.\end{aligned} \quad (72)$$

Note that this implies that

$$\begin{aligned}&f\left(t_k+1,\hat{s},E_{t_k}+E_{\hat{s}}-B\right)\\&\quad>f\left(t_k+1,t_{k+1},E_{t_k}+y\times 1_{\{t_{k+1}=j\}}\right).\end{aligned} \quad (73)$$

Otherwise, from Corollary 3 we would have

$$f\left(s_p+1,\hat{s},E_{\hat{s}}\right)\leq f\left(s_p+1,t_{k+1},B+y\times 1_{\{t_{k+1}=j\}}\right), \quad (74)$$

which contradicts (71). However, (73) indicates that there must exist some battery overflow slot within $[t_k+1,t_{k+1}]$, which is a contradiction to the assumption that there are no battery overflow slots within $[s_p+1,j]$. Consequently, the only possibility is that $\hat{s}\in[s_p+1,t_1]$. However, this can not occur either because the inequalities

$$\begin{aligned}&f\left(s_p+1,t_1,B+y\times 1_{\{t_1=j\}}\right)\\&\quad\leq f\left(s_p+1,j,B+y\right)<f\left(s_p+1,\hat{s},E_{\hat{s}}\right)\end{aligned} \quad (75)$$

also imply that there must exist some battery overflow slot within $[s_p+1,t_1]$, which is a contradiction as well. To conclude, we have proved that there is no way that $\hat{s}$ is not a battery overflow slot.

REFERENCES

[1] S. Wei, W. Guan, and K. J. R. Liu, "Outage probability optimization with equal and unequal transmission rates under energy harvesting constraints," in *Proc. IEEE GLOBECOM*, Atlanta, GA, USA, Dec. 2013, pp. 2530–2525.

[2] X. Jiang, J. Polastre, and D. Culler, "Perpetual environmentally powered sensor networks," in *Proc. 4th Int. Symp. Inf. Process. Sens. Netw.*, 2005, pp. 463–468.

[3] J. Taneja, J. Jeong, and D. Culler, "Design, modeling, and capacity planning for micro-solar power sensor networks," in *Proc. 7th Int. Conf. Inf. Process. Sens. Netw.*, 2008, pp. 407–418.

[4] V. Raghunathan, A. Kansal, J. Hsu, J. Friedman, and M. Srivastava, "Design considerations for solar energy harvesting wireless embedded systems," in *Proc. 4th Int. Symp. Inf. Process. Sens. Netw.*, 2005, pp. 457–462.

[5] C. Park and P. Chou, "Ambimax: Autonomous energy harvesting platform for multi-supply wireless sensor nodes," in *Proc. 3rd Annu. IEEE Commun. Soc. Sens. Ad Hoc Commun. Netw.*, 2006, pp. 168–177.

[6] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 443–461, Sep. 2011.

[7] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[8] N. Zlatanov, Z. Hadzi-Velkov, and R. Schober, "Asymptotically optimal power allocation for point-to-point energy harvesting communication systems," in *Proc. IEEE GLOBECOM*, Dec. 2013, pp. 2502–2507.

[9] M. Hassan and A. Bermak, "Solar harvested energy prediction algorithm for wireless sensors," in *Proc. ASQED*, Jul. 2012, pp. 178–181.

[10] K. B. Joshi, J. H. Costello, and S. Priya, "Estimation of solar energy harvested for autonomous jellyfish vehicles," *IEEE J. Ocean. Eng.*, vol. 36, no. 4, pp. 539–551, Oct. 2011.

[11] A. Cammarano, C. Petrioli, and D. Spenza, "Pro-energy: A novel energy prediction model for solar and wind energy-harvesting wireless sensor networks," in *Proc. IEEE 9th Int. Conf. MASS*, Las Vegas, NV, USA, Oct. 2012, pp. 75–83.

[12] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732–1743, Sep. 2011.

[13] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, pp. 1180–1189, Mar. 2012.

[14] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications withe energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[15] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system,"*IEEE Trans. Commun.*, vol. 60, no.1, pp. 220–230, Jan. 2012.

[16] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1721–1731, Sep. 2011.

[17] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, no.6, pp. 2193–2203, Jun. 2012.

[18] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *J. Commun. Netw.*, vol. 14, no. 2, pp. 140–150, Apr. 2012.

[19] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation in ofdma systems with hybrid energy harvesting base station," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3412–3427, Jul. 2013.

[20] Y. Zhang, S. He, J. Chen, Y. Sun, and X. Shen, "Distributed sampling rate control for rechargeable sensor nodes with limited battery capacity," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 3096–3106, Jun. 2013.

[21] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Commun.*, vol. 9, no. 4, pp. 1326–1336, Apr. 2010.

[22] R. Srivastava and C. E. Koksal, "Basic performance limits and tradeoffs in energy-harvesting sensor nodes with finite data and energy storage," *IEEE/ACM Trans. Netw.*, vol. 21, no.4, pp. 1049–1062, Aug. 2013.

[23] C. Huang, R. Zhang, and S. Cui, "Optimal power allocation for outage probability minimization in fading channels with energy harvesting constraints," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 1074–1087, Feb. 2014.

[24] S. Luo, R. Zhang, and T. J. Lim, "Optimal save-then-transmit protocol for energy harvesting wireless transmitters," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1196–1207, Mar. 2013.

[25] X. Kang, Y. K. Chia, C. K. Ho, and S. Sun, "Cost minimization for fading channels with energy harvesting and conventional energy," *IEEE Trans. Wireless Commun.*, vol. 13, no. 8, pp. 4586–4598, Aug. 2014.

[26] N. Michelusi, K. Stamatiou, and M. Zorzi, "Transmission policies for energy harvesting sensors with time-correlated energy supply," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2988–3001, Jul. 2013.

[27] S. Reddy and C. R. Murthy, "Dual-stage power management algorithms for energy harvesting sensors," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1434–1445, Apr. 2012.

[28] "Solar radiation resource information," Nat. Renew. Energy Lab., Golden, CO, USA, Feb. 2012. [Online]. Available: http://www.nrel.gov/rredc/

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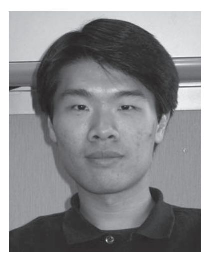

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