

2

Power System Network Matrices – I

2.1 INTRODUCTION

The various terms used in Graph Theory are presented in this chapter. Formulation of different network matrices are discussed. Primitive impedance and admittance matrices are explained. This chapter also deals with the formulation of Y_{BUS} .

2.2 IMPORTANT DEFINITIONS IN GRAPH THEORY

A **graph** shows the geometrical interconnection of the elements of a network.

A **subgraph** is any subset of elements of the graph.

A **path** is a subgraph of connected elements with not more than two elements connected to any one node.

A graph is connected if and only if there is a path between every pair of nodes.

If each element of the connected graph is assigned a direction then it is an **oriented graph**.

A representation of a power system and the corresponding graph and oriented graph are shown in Figs. 2.1, 2.2 and 2.3.

A **tree** is a connected subgraph of a connected graph having all the nodes of the graph but without any closed path (or) loop. The elements of a tree are called tree branches (or) twigs and are denoted by thick lines.

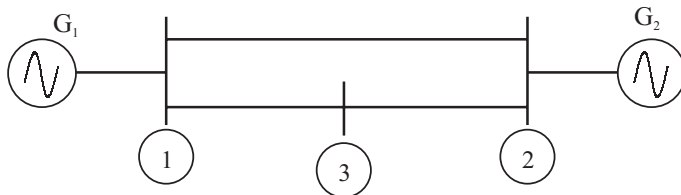


Fig. 2.1: Single-line diagram of power system.

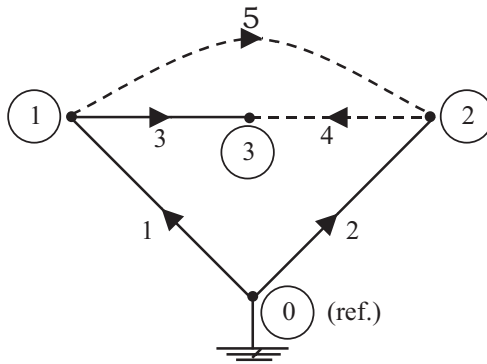


Fig. 2.4: Tree of power system, with tree branches $T [1, 2, 3]$.

2.3 BASIC LOOPS (OR) FUNDAMENTAL LOOPS

Loops which contain only one link are independent and are called basic loops as shown in Fig. 2.5. In other words, whenever a link element is added to the existing tree, basic loops or fundamental loops can be obtained.

No. of fundamental loops = No. of links

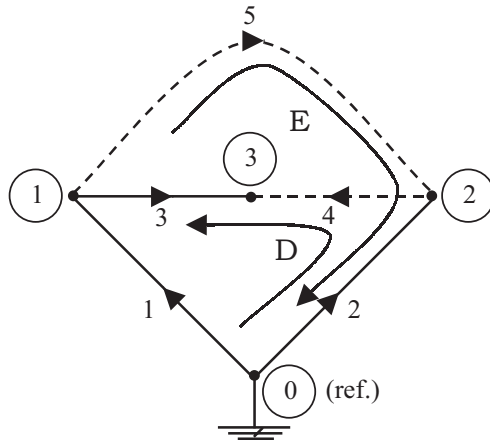


Fig. 2.5: Basic loops, i^{th} tree branches $T [1, 2, 3]$.

2.4 BASIC CUT SETS (OR) FUNDAMENTAL CUT SETS

A cut set is a set of elements that, if removed, divides a connected subgraphs. In other words, a basic or fundamental cut set of the graph is the set of elements consisting of only one branch (or) twig and minimal number of links (or) chords as shown in Fig. 2.6.

No. of basic cut sets = No. of twigs

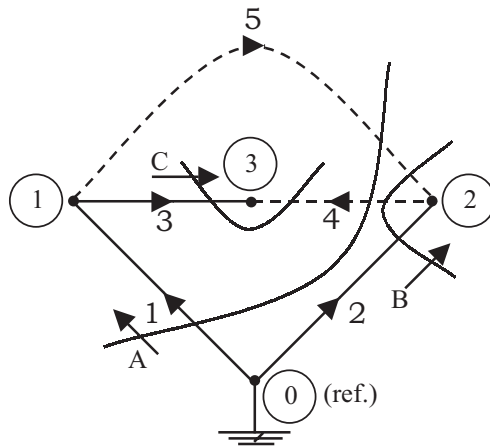


Fig. 2.6: Basic cut set, with tree branches, T [1, 2, 3].

2.5 INCIDENCE MATRICES

Every element of a graph is incident between any two nodes. Incidence matrices give the information about incidence of elements—may be incident to loops, cut sets etc. and this information is furnished in a matrix, known as incidence matrix as follows:

2.5.1 Element-Node Incidence Matrix (\hat{A})

The incidence matrix (A) describes whether an element is incident to a particular node (or) not. The elements of the matrix are as follows:

- $a_{ij} = 1$ If the i^{th} element is incident to and oriented away from the j^{th} node.
- $a_{ij} = -1$ If the i^{th} element is incident to and oriented towards from the j^{th} node.
- $a_{ij} = 0$ If the i^{th} element is not incident to the j^{th} node.

The dimension of the matrix is $e \times n$ where e is the number of elements and n is the number of nodes in the graph. The element-node incidence matrix (\hat{A}) for the graph is shown in Fig. 2.7.

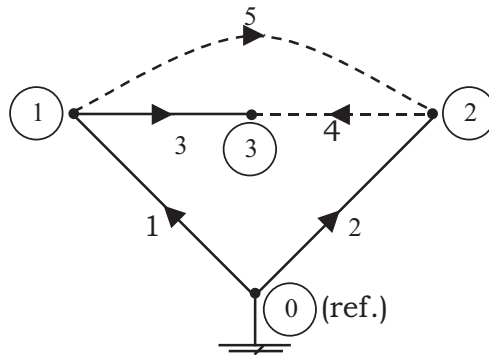


Fig. 2.7: Tree of power system, with tree branches, T [1, 2, 3].

The transpose of this matrix is

$$\therefore K^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

The branch-path incidence matrix and the submatrix A_b relate the branches to paths and branches to buses respectively. Since, there is a one-to-one correspondence between paths and buses, we can prove that the relation $A_b K^T$ is an unity matrix.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $A_b K^T = U$... (2.2)

where $U \rightarrow$ is the unity matrix

\therefore we can write $K^T = A_b^{-1}$

2.5.4 Basic Cut Set Incidence Matrix (B)

The incidence of elements to basic cut sets of a connected graph is shown by the basic (or) fundamental cut set incidence matrix (B).

The elements of this matrix are as follows:

- $b_{ij} = 1 \rightarrow$ If the i^{th} element is incident to and oriented in the same direction as the j^{th} basic cut set.
- $b_{ij} = -1 \rightarrow$ If the i^{th} element is incident to and oriented in the opposite direction as the j^{th} basic cut set.
- $b_{ij} = 0 \rightarrow$ If the i^{th} element is not incident to the j^{th} basic cut set.

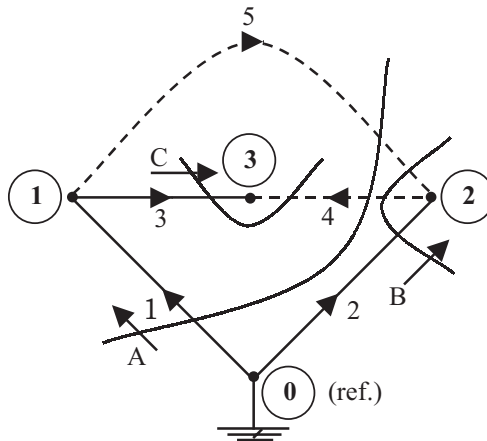


Fig. 2.9: Basic cut set.

The basic cut set incidence matrix, of dimension $e \times B$, for the graph is shown in Fig. 2.9.

Matrix B can be partitioned into submatrices U_b and B_ℓ where the rows of U_b correspond to branches and the rows of B_ℓ to links. The partitioned matrix is shown above.

$$[B] = \begin{array}{c|ccc} e \backslash \text{Basic cut sets} & A & B & C \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ \hline 4 & 1 & -1 & 1 \\ 5 & -1 & 1 & 0 \end{array} = \begin{bmatrix} U_b \\ \dots \\ B_\ell \end{bmatrix} \quad \dots (2.3)$$

Fig. 2.9

The identity matrix U_b shows the one-to-one correspondence of the branches and basic cut sets.

The submatrix B_ℓ can be obtained from the bus incidence matrix A . The incidence of the links to buses is shown by the submatrix, A_ℓ and the incidence of branches to buses is shown by the sub-matrix A_b . But $B_\ell A_b$ shows the incidence of links to buses, i.e.,

$$\begin{aligned} B_\ell A_b &= A_\ell \Rightarrow B_\ell = A_\ell A_b^{-1} \\ B_\ell &= A_\ell K^T \quad \because A_b^{-1} = K^T \end{aligned} \quad \dots (2.4)$$

2.5.5 Augmented Cut Set Incidence Matrix (\hat{B})

The basic cut set incidence matrix is of the size $e \times b$, therefore, a non-square matrix and hence, inverse does not exist. In other words, B is a singular matrix. In order to make the matrix B a non-singular matrix, we augment the number of columns equal to the number of links by adding fictitious cut sets known as tie-cut sets, which contain only links. The tie-cut sets are added for the graph in Fig. 2.10 and are shown as,

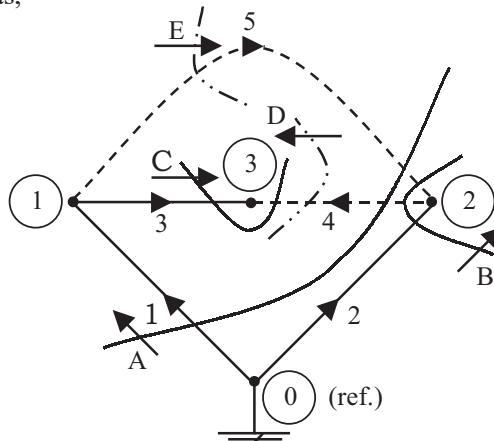


Fig. 2.10: Augmented basic cut sets.

		Basic			Tie			
		cut sets			cut sets			
$\hat{[B]} =$	e \ e	A	B	C	D	E		
	1	1	0	0	0	0	}	Branches
	2	0	1	0	0	0		
	3	0	1	1	0	0		
		4	1	-1	1	1	0	}
5	-1	1	0	0	1			

$= \left[\begin{array}{c|c} U_b & 0 \\ \cdots & \cdots \\ B_\ell & U \end{array} \right] \quad \dots (2.5)$

This is a square matrix of dimension $e \times e$ and is non-singular. Matrix $\hat{[B]}$ can be partitioned as above.

2.5.6 Basic Loop Incidence Matrix (C)

The incidence of elements to basic loops of a connected graph is shown by the basic loop incidence matrix (C). The elements of this matrix are as follows:

- $c_{ij} = 1 \rightarrow$ If the i^{th} element is incident to and oriented in the same direction as the j^{th} basic loop.
- $c_{ij} = -1 \rightarrow$ If the i^{th} element is incident to and oriented in the opposite direction as the j^{th} basic loop.
- $c_{ij} = 0 \rightarrow$ If the i^{th} element is not incident to the j^{th} basic loop.

The basic loop incidence matrix, of dimension $e \times \ell$, for the graph shown in Fig. 2.11 is,

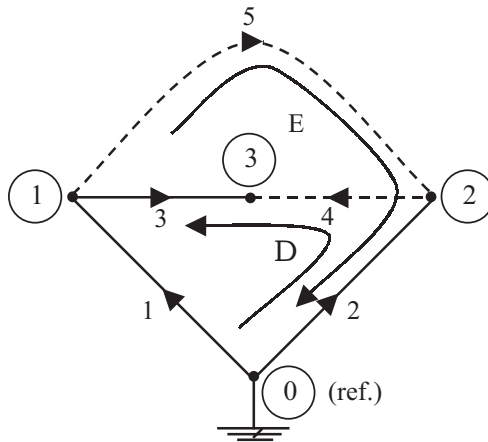


Fig. 2.11: Basic loops.

where v , e and i are the column matrices of size $e \times 1$ and e is the number of elements. z is a square matrix of size $e \times e$. The matrix z is known as primitive impedance matrix. A diagonal element of the matrix z of the primitive network is the self-impedance $z_{ik\ ik}$. An off-diagonal element is the mutual impedance $z_{ik\ ps}$ between the elements ik and ps .

2.6.2 Primitive Network in Admittance Form

Let the element $i - k$ connected between the two nodes i and k . This is shown in Fig. 2.14.

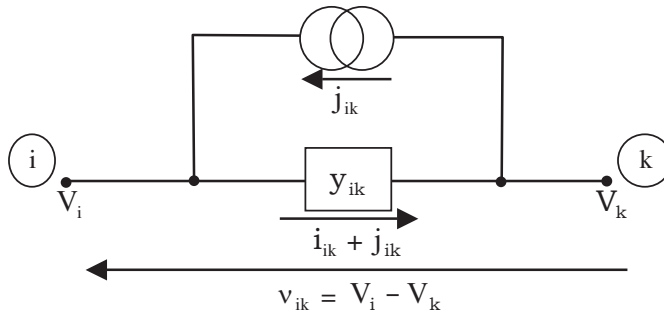


Fig. 2.14: Primitive network.

where $V_i, V_k \rightarrow i^{th}$ and k^{th} node voltages respectively
 $v_{ik}, V_i - V_k \rightarrow$ Voltage across the element $i - k$
 $j_{ik} \rightarrow$ Source current in parallel with element $i - k$
 $y_{ik} \rightarrow$ Self-admittance of the element $i - k$
 $i_k \rightarrow$ Current flowing through the element $i - k$
 Hence, current flowing through the element,

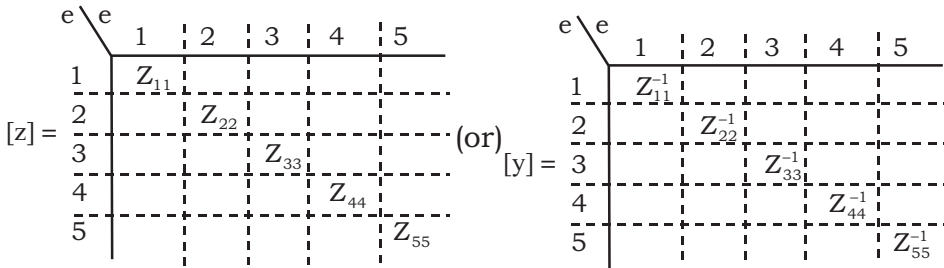
$$i_{ik} + j_{ik} = y_{ik} v_{ik} \quad \dots (2.10)$$

The above Eq. (2.10), for all the elements in a condensed form can be written as,

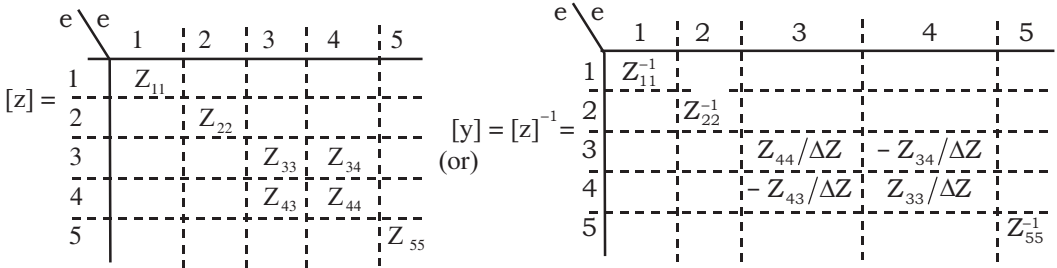
$$i + j = y v \quad \dots (2.11)$$

Where i, j and v are the column matrices of size $e \times 1$ and matrix y is a square matrix of size $e \times e$. The matrix y is known as primitive admittance matrix. The diagonal elements of the matrix $y_{ik\ ik}$ represents self-admittances and off-diagonal elements of the matrix $y_{ik\ ps}$ represents the mutual admittances of the elements ik and ps .

If there is no mutual coupling between the elements, the primitive admittance matrix y can be obtained by inverting the primitive impedance matrix z . The matrices z and y are diagonal matrices. In the case, the self-impedances are equal to the reciprocals of the corresponding self-admittances.



If the buses 3 and 4 have mutual element then the corresponding primitive impedance matrix is shown below:



2.7 NETWORK EQUATIONS AND NETWORK MATRICES

A power system is a big complex network. Therefore, we require transforming the primitive network matrices to be developed either in the bus frame, branch frame or loop frame of reference. In these frames of references network matrices can be written as,

$$V_{Bus} = Z_{Bus} I_{Bus} \quad (\text{or}) \quad V_{BR} = Z_{BR} I_{BR} \quad (\text{or}) \quad V_{Loop} = Z_{Loop} I_{Loop} \quad (2.12)$$

- where V_{Bus} → Matrix contains bus voltages
- I_{Bus} → Matrix represents the injected currents into the buses
- Z_{Bus} → Bus impedance matrix

For an n-bus power system, the dimensions of these matrices are $n \times 1$, $n \times 1$, and $n \times 1$, respectively.

The network equations in admittance form can be written as,

$$I_{Bus} = Y_{Bus} V_{Bus} \quad \text{where} \quad Y_{Bus} = Z_{Bus}^{-1} \quad \dots (2.13)$$

The branch frame of reference performance equations can be written as,

$$I_{BR} = Y_{BR} V_{BR} \quad \text{where} \quad Y_{BR} = Z_{BR}^{-1} \quad \dots (2.14)$$

Here, V_{BR} and I_{BR} represent branch voltages and currents. Z_{BR} and Y_{BR} represent branch impedance and admittance matrices respectively. The dimensions of these matrices depend upon the number of branches in a graph of a given power system.

Finally, the loop frame of reference V_{Loop} denotes the basic loop voltages, I_{Loop} represents the basic loop currents and Z_{Loop} is the loop impedance matrix. In the admittance form, the network equations can be written as,

$$I_{\text{Loop}} = Y_{\text{Loop}} V_{\text{Loop}} \quad \text{where} \quad Y_{\text{Loop}} = Z_{\text{Loop}}^{-1} \quad \dots (2.15)$$

The size of the matrices in the network equation based on loop frame depends on the number of basic links or loops in a graph of a given power system.

2.8 BUS ADMITTANCE MATRIX

Bus admittance matrix (Y_{Bus}) for an n-bus power system is square matrix of size $n \times n$. The diagonal elements represent the self or short circuit driving point admittances with respect to each bus. The off-diagonal elements are the short circuit transfer admittances (or) the admittances common between any two number of buses. In other words, the diagonal element y_{ii} of the Y_{Bus} is the total admittance value with respect to the i^{th} bus and y_{ik} is the value of the admittance that is present between i^{th} and k^{th} buses.

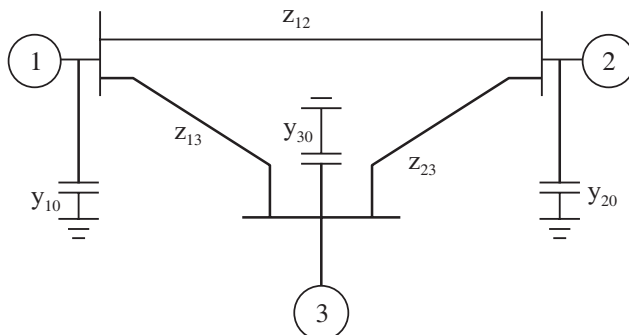
Y_{Bus} can be obtained by the following methods:

1. Direct inspection method
2. Step-by-step procedure
3. Singular transformation
4. Non-singular transformation

2.9 PROCEDURE OF FORMULATION OF Y_{Bus}

2.9.1 By Direct Inspection Method

Formulation of Y_{Bus} by direct inspection method is suitable for the small size networks. In this method the Y_{Bus} matrix is developed simply by inspecting structure of the network without developing any kind of equations. Let us consider a 3-bus power system shown in figure below:



Since $[B]^T * i$ is zero because, algebraic sum of all the currents meeting at a node is zero. The source current matrix $[j]$ can be partitioned into,

$$[j] = \begin{bmatrix} j_b \\ j_\ell \end{bmatrix} \quad \dots (2.28)$$

Where j_b is the source acting in parallel across the branches

$$\therefore [B]^T * [j] = [B]^T * [j_b] = I_{BR} \quad \dots (2.29)$$

Therefore, Eq. (2.20) is modified as,

$$I_{BR} = [B]^T * y * [B] * V_{BR} \quad \dots (2.30)$$

or
$$I_{BR} = [Y_{BR}] * V_{BR} \quad (\text{OR}) \quad I_{BR} = Y_{BR} * V_{BR} \quad \dots (2.31)$$

where

$$Y_{BR} = B^T * [y] * B, \text{ and}$$

$$Z_{BR} = Y_{BR}^{-1} = \{ B^T * [y] * B \}^{-1} \quad \dots (2.32)$$

2.9.4 Loop Impedance Matrix

The relation between element currents and the loop currents can be written as

$$i = [C] * I_{Loop} \quad \dots (2.33)$$

The primitive equation in the impedance form is

$$v + e = z i \quad \dots (2.34)$$

Multiplying both sides by $[C]^T$, where C is the basic loop incidence matrix,

$$[C]^T * v + [C]^T * e = [C]^T * z * i \quad \dots (2.35)$$

Since $[C]^T * v$ is zero because, algebraic sum of all voltages in a closed loop is zero. Similarly, $[C]^T * e$ gives the algebraic sum of series source voltages around each basic loop.

$$\therefore E_{Loop} = [C]^T * e \quad \dots (2.36)$$

Therefore, Eq.(2.35) is modified as,

$$E_{\text{Loop}} = [C]^T * Z * [C] * I_{\text{Loop}} \quad \dots (2.37)$$

or
$$E_{\text{Loop}} = [Z_{\text{Loop}}] * I_{\text{Loop}} \quad (\text{or}) \quad E_{\text{Loop}} = Z_{\text{Loop}} * I_{\text{Loop}} \quad \dots (2.38)$$

where

$$Z_{\text{Loop}} = C^T * [Z] * C, \text{ and}$$

$$Y_{\text{Loop}} = Z_{\text{Loop}}^{-1} = \{ C^T * [Z] * C \}^{-1} \quad \dots (2.39)$$

2.10 NUMERICAL PROBLEMS

P. 2.1. For the power system network shown in Fig. 2.15: (i) draw the oriented graph, (ii) formulate element node incidence matrix, bus incidence matrix, basic cut set incidence matrix, augmented cut set incidence matrix, basic loop incidence matrix, augmented loop incidence matrix and path incidence matrix, (iii) formulate primitive admittance matrix, and (iv) formulate Y_{Bus} , Y_{Br} and Z_{Loop} . Take reactance of each element $j0.2$ pu.

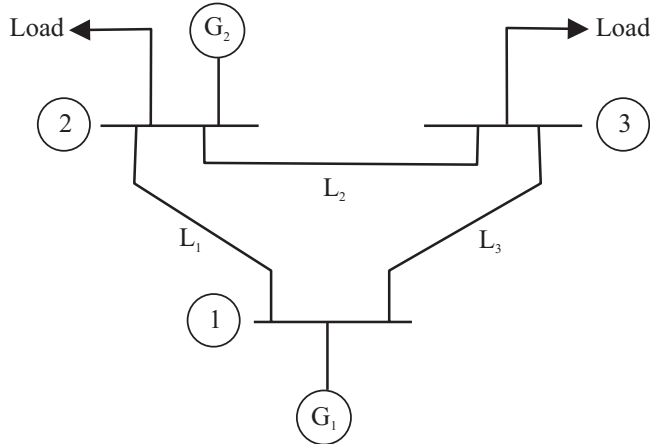


Fig. 2.15: Power system network.

Solution: For the given power system network graph, oriented graph and tree graph are shown in Figs (2.16), (2.17) and (2.18) respectively.

No. of elements, $e = 5$

No. of nodes, $n = 4$

No. of buses $= n - 1 = 4 - 1 = 3$

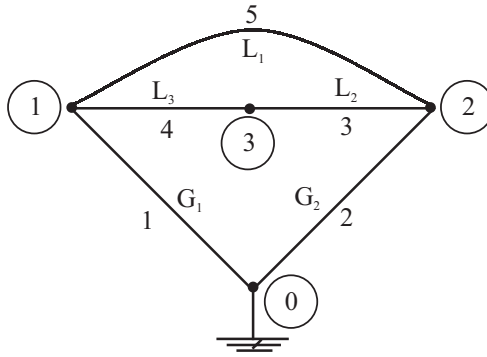


Fig. 2.16: Connected graph of power system.

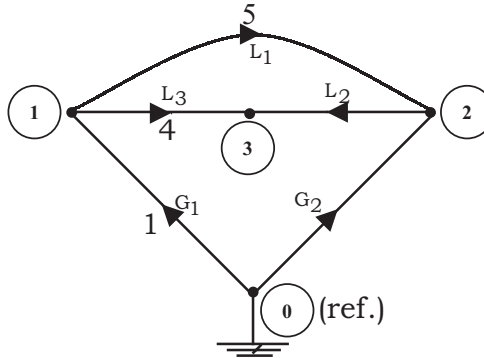


Fig. 2.17: Oriented graph of power system.

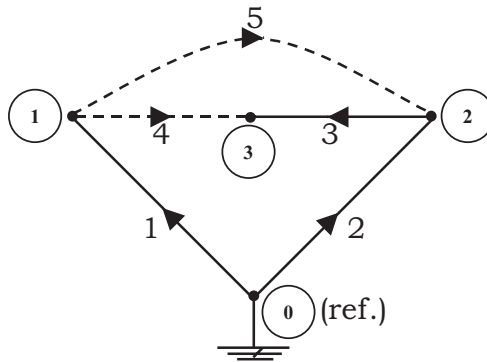


Fig. 2.18: Tree of power system with tree branches, $T [1, 2, 3]$.

No. of branches, $b = n - 1 = 4 - 1 = 3$

No. of links, $\ell = e - b = e - (n - 1) = e - n + 1 = 5 - 3 = 2$

Element node incidence matrix, (\hat{A})

$$[\hat{A}] = \begin{array}{c|cccc} e \backslash n & 0 & 1 & 2 & 3 \\ \hline 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 \\ 3 & 0 & 0 & 1 & -1 \\ 4 & 0 & 1 & 0 & -1 \\ 5 & 0 & 1 & -1 & 0 \end{array}$$

Bus incidence matrix, (A)

$$[A] = \begin{array}{c|ccc} e \backslash (n-1) & 1 & 2 & 3 \\ \hline 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 1 & -1 \\ \hline 4 & 1 & 0 & -1 \\ 5 & 1 & -1 & 0 \end{array} \left. \begin{array}{l} \text{Branches} \\ (A_b) \end{array} \right\} = \begin{bmatrix} A_b \\ \dots \\ A_l \end{bmatrix}$$

Links (A_l)

Bus cut set incidence matrix, (B)

$$[B] = \begin{array}{c|ccc} e \backslash \text{Basic cut sets} & A & B & C \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ \hline 4 & -1 & 1 & 1 \\ 5 & -1 & 1 & 0 \end{array} \left. \begin{array}{l} \text{Branches} \\ (B_b \text{ or } U_b) \end{array} \right\} = \begin{bmatrix} U_b \\ \dots \\ B_l \end{bmatrix}$$

Links (B_l)

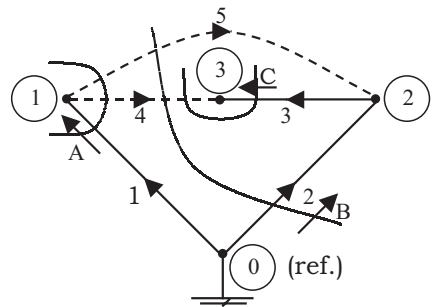


Fig. 2.18a: Tree of power system with tree branches $T [1, 2, 3]$.

Augmented cut set incidence matrix, $\hat{[B]}$

e	e	Basic cut sets			Tie cut sets		
		A	B	C	D	E	
1	1	1	0	0	0	0	} Branches
	2	0	1	0	0	0	
	3	0	1	1	0	0	
2	4	-1	1	1	1	0	} Links
	5	-1	1	0	0	1	

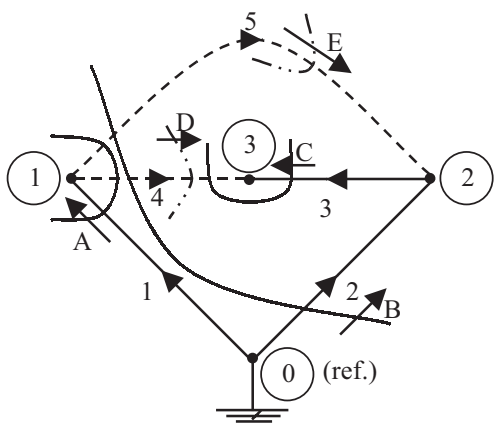
$$= \left[\begin{array}{c|c} U_b & O \\ \dots & \dots \\ B_\ell & U_\ell \end{array} \right]$$


Fig. 2.18b: Tree of power system with tree branches, $T [1, 2, 3]$.

Basic loop incidence matrix, $[C]$

e	Basic loops			
		D	E	
1	1	1	1	} Branches
	2	-1	-1	
	3	-1	0	
2	4	1	0	} Links
	5	0	1	

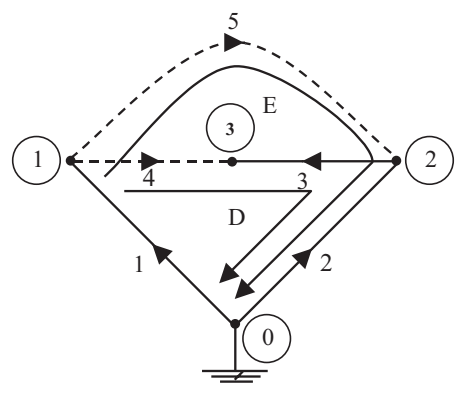
$$= \left[\begin{array}{c} C_b \\ \dots \\ U_\ell \end{array} \right]$$


Fig. 2.18c: Tree of power system with tree branches, $T [1, 2, 3]$.

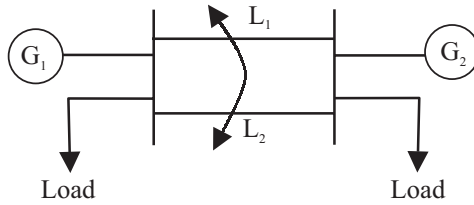


Fig. 2.19: Power system network.

Solution: For the given power system network graph, oriented graph and tree graph are shown in Figs. 2.20, 2.21 and 2.22 respectively.

No. of elements, $e = 4$

No. of nodes, $n = 3$

No. of buses = $n - 1 = 2$

No. of branches, $b = n - 1 = 2$

No. of links, $\ell = e - n + 1 = 2$

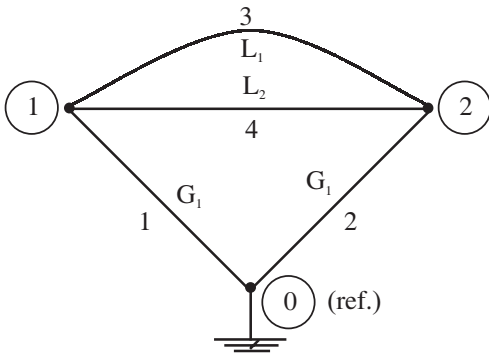


Fig. 2.20: Connected graph.

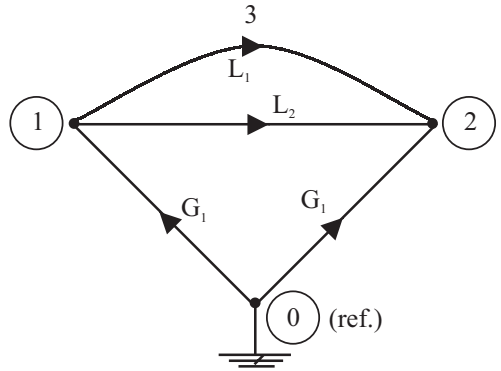


Fig. 2.21: Oriented graph.

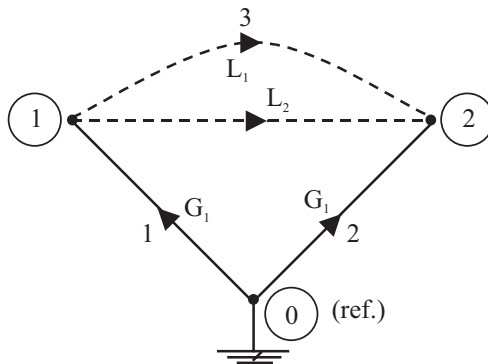


Fig. 2.22: Tree graph.

From given data, the self-impedances are

$$Z_{11} = Z_{22} = j0.2 \quad Z_{33} = Z_{44} = j0.3$$

The mutual impedances are

$$Z_{12} = Z_{13} = Z_{14} = Z_{21} = Z_{23} = Z_{24} = 0;$$

$$Z_{34} = Z_{43} = j0.04$$

$$Y_{11} = \frac{1}{Z_{11}} = \frac{1}{j0.2} = -j5 = Y_{22} \quad (\because Z_{11} = Z_{22})$$

$$\begin{bmatrix} Y_{33} & Y_{34} \\ Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} Z_{33} & Z_{34} \\ Z_{43} & Z_{44} \end{bmatrix}^{-1} = \begin{bmatrix} j0.3 & j0.04 \\ j0.04 & j0.3 \end{bmatrix}^{-1} = \begin{bmatrix} -j3.3937 & j0.4525 \\ j0.4525 & -j3.3937 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} j0.2 & 0 & 0 & 0 \\ 0 & j0.2 & 0 & 0 \\ 0 & 0 & j0.3 & j0.04 \\ 0 & 0 & j0.04 & j0.3 \end{bmatrix}$$

and

$$[Y] = \begin{bmatrix} -j5 & 0 & 0 & 0 \\ 0 & -j5 & 0 & 0 \\ 0 & 0 & -j3.3937 & j0.4525 \\ 0 & 0 & j0.4525 & -j3.3937 \end{bmatrix}$$

P. 2.3. Consider the power system as shown in Fig. 2.23. Each generator and the line impedance of $(0.2 + j0.2)$ pu and $(0.5 + j0.5)$ pu respectively. Neglecting line charging admittances, form Y_{Bus} by direct inspection method.

Solution: The admittance of each generator is,

$$Y_{G1} = Y_{G2} = Y_{G3} = \frac{1}{(0.2 + j0.2)} = 2.5 - j2.5$$

The admittance of each line is

$$Y_{L1} = Y_{L2} = Y_{L3} = \frac{1}{(0.5 + j0.5)} = 1 - j1$$

Since, power system has three buses ($n = 3$), the size of the Y_{Bus} is 3×3 . The elements of Y_{Bus} by direct inspection is as follows:

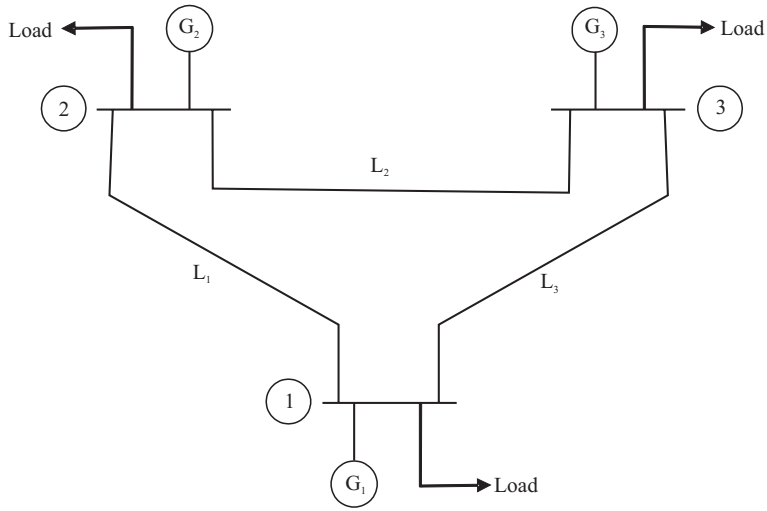


Fig. 2.23: Power system network.

Diagonal elements:

$$Y_{11} = y_{G1} + y_{L1} + y_{L3} = 2.5 - j2.5 + 1 - j1 + 1 - j1 = 4.5 - j4.5$$

$$Y_{22} = y_{G2} + y_{L1} + y_{L2} = 2.5 - j2.5 + 1 - j1 + 1 - j1 = 4.5 - j4.5$$

$$Y_{33} = y_{G3} + y_{L2} + y_{L3} = 2.5 - j2.5 + 1 - j1 + 1 - j1 = 4.5 - j4.5$$

Off-diagonal elements:

$$Y_{12} = Y_{21} = -y_{L1} = -(1 - j1) = -1 + j1$$

Similarly,

$$Y_{23} = Y_{32} = -y_{L2} = -(1 - j1) = -1 + j1$$

$$Y_{13} = Y_{31} = -y_{L3} = -(1 - j1) = -1 + j1$$

$$\therefore [Y_{\text{Bus}}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} 4.5 - j4.5 & -1 + j1 & -1 + j1 \\ -1 + j1 & 4.5 - j4.5 & -1 + j1 \\ -1 + j1 & -1 + j1 & 4.5 - j4.5 \end{bmatrix}$$

P. 2.4. Using data in the above problem, obtain Y_{Bus} by singular transformation method.

Solution: For the given power system network, the tree graph with tree branches T[1,2,3] is shown in Fig. 2.24.

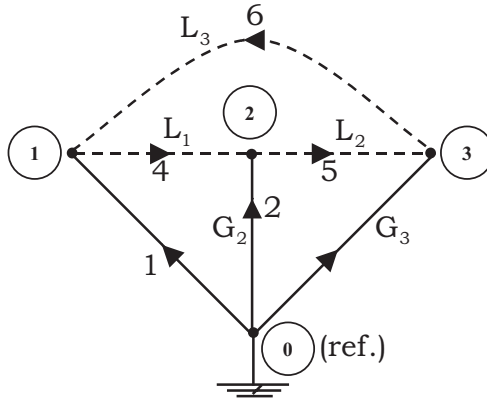


Fig. 2.24: Tree of power system.

$$\begin{array}{c|cccc}
 & \begin{matrix} n \\ e \end{matrix} & 0 & 1 & 2 & 3 \\
 \hline
 1 & & 1 & -1 & 0 & 0 \\
 2 & & 1 & 0 & -1 & 0 \\
 3 & & 1 & 0 & 0 & -1 \\
 4 & & 0 & 1 & -1 & 0 \\
 5 & & 0 & 0 & 1 & -1 \\
 6 & & 0 & -1 & 0 & 1
 \end{array}
 =
 \begin{bmatrix}
 1 & -1 & 0 & 0 \\
 1 & 0 & -1 & 0 \\
 1 & 0 & 0 & -1 \\
 0 & 1 & -1 & 0 \\
 0 & 0 & 1 & -1 \\
 0 & -1 & 0 & 1
 \end{bmatrix}$$

$$[A] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow [A]^T = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix}$$

The primitive impedance matrix can be written as,

$$[z] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0.2 + j0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 + j0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 + j0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 + j0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 + j0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 + j0.5 \end{bmatrix} \end{matrix}$$

Primitive admittance matrix can be written as,

$$[y] = [z]^{-1} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 2.5 - j2.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.5 - j2.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.5 - j2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 - j1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - j1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - j1 \end{bmatrix} \end{matrix}$$

Therefore, the Y_{Bus} is

$$\therefore [Y_{\text{Bus}}] = [A]^T * [y] * [A] = \begin{bmatrix} 4.5 - j4.5 & -1 + j1 & -1 + j1 \\ -1 + j1 & 4.5 - j4.5 & -1 + j1 \\ -1 + j1 & -1 + j1 & 4.5 - j4.5 \end{bmatrix}$$

P. 2.5. For the power system shown in Fig. 2.25, build Y_{Bus} matrix using: (i) by direct inspection, and (ii) singular transformation. The branch impedances of the lines are as follows:

Line	Impedances (Ω)
1 – 2	$15 + j40$
1 – 4	$30 + j50$
2 – 3	$15 + j25$
2 – 4	$45 + j20$
3 – 4	$20 + j30$

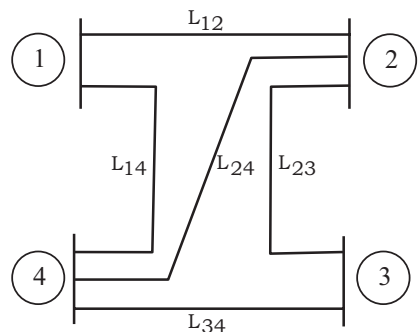


Fig. 2.25: Power system network.

$$Y_{01} = Y_{10} = Y_{02} = Y_{20} = Y_{03} = Y_{30} = Y_{13} = Y_{31} = 0$$

Number of buses are 4, therefore, Y_{Bus} is 4×4 matrix.

$$\therefore [Y_{\text{Bus}}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$= \begin{bmatrix} 0.0170 - j0.0366 & -0.0082 + j0.0219 & 0 & -0.0088 + j0.0147 \\ -0.0082 + j0.0219 & 0.0444 - j0.0596 & -0.0176 + j0.0294 & -0.0186 + j0.0082 \\ 0 & -0.0176 + j0.0294 & 0.0330 - j0.0525 & -0.0154 + j0.0231 \\ -0.0088 + j0.0147 & -0.0186 + j0.0082 & -0.0154 + j0.0231 & 0.0522 - j0.0594 \end{bmatrix}$$

(ii) By using singular transformation

For the given power system network, the tree graph with tree branches $T[1,2,3,4]$ is shown in Fig. 2.26.

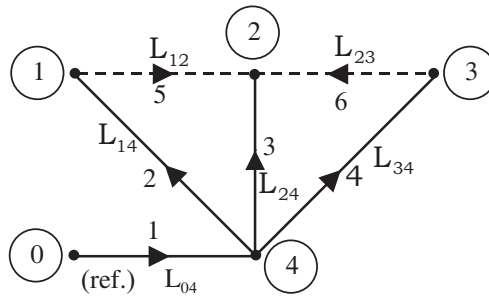


Fig. 2.26: Tree graph.

$$[\hat{A}] = \begin{array}{c|ccccc} & n & & & & \\ e & 0 & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 0 & -1 \\ 2 & 0 & -1 & 0 & 0 & 1 \\ 3 & 0 & 0 & -1 & 0 & 1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 & 0 \\ 6 & 0 & 0 & -1 & 1 & 0 \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \Rightarrow [A]^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The primitive impedance matrix can be written as,

$$[z] = \begin{array}{c} \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \end{array} \begin{bmatrix} 35 + j50 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 + j50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 45 + j20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 + j30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 + j40 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 + j25 \end{bmatrix}$$

$$[y] = [z]^{-1} = \begin{array}{c} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \end{array} \begin{bmatrix} 0.0094 - j0.0134 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0088 - j0.0147 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0186 - j0.0082 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0154 - j0.0231 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0082 - j0.0219 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0176 - j0.0294 \end{bmatrix}$$

$$\therefore [Y_{\text{Bus}}] = [A]^T * [y] * [A] =$$

$$\begin{bmatrix} 0.0170 - j0.0366 & -0.0082 + j0.0219 & 0 & -0.0088 + j0.0147 \\ -0.0082 + j0.0219 & 0.0444 - j0.0596 & -0.0176 + j0.0294 & -0.0186 + j0.0082 \\ 0 & -0.0176 + j0.0294 & 0.0330 - j0.0525 & -0.0154 + j0.0231 \\ -0.0088 + j0.0147 & -0.0186 + j0.0082 & -0.0154 + j0.0231 & 0.0522 - j0.0594 \end{bmatrix}$$

P. 2.6. Form Y_{Bus} for the network, using: (i) direct inspection method and (ii) singular transformation.
JNTU (Nov 2008)

Element	Positive sequence reactance
E – A	0.04
E – B	0.05
A – B	0.04
B – C	0.03
A – D	0.02
C – F	0.07
D – F	0.10

Solution:**(i) By direct inspection method:**

For the given data,

$$y_{EA} = \frac{1}{z_{EA}} = \frac{1}{j0.04} = -j25$$

$$y_{EB} = \frac{1}{z_{EB}} = \frac{1}{j0.05} = -j20$$

$$y_{AB} = \frac{1}{z_{AB}} = \frac{1}{j0.04} = -j25$$

$$y_{BC} = \frac{1}{z_{BC}} = \frac{1}{j0.03} = -j33.3333$$

$$y_{AD} = \frac{1}{z_{AD}} = \frac{1}{j0.02} = -j50$$

$$y_{CF} = \frac{1}{z_{CF}} = \frac{1}{j0.07} = -j14.2857$$

$$y_{DF} = \frac{1}{z_{DF}} = \frac{1}{j0.10} = -j10$$

Assume that node E as reference,

Diagonal elements:

$$Y_{AA} = y_{AE} + y_{AB} + y_{AD} = -j100$$

$$Y_{BB} = y_{BE} + y_{BA} + y_{BC} = -j78.3333$$

$$Y_{CC} = y_{CB} + y_{CF} = -j47.619$$

$$Y_{DD} = y_{DA} + y_{DF} = -j60$$

$$Y_{FF} = y_{FC} + y_{FD} = -j24.2857$$

Off-diagonal elements:

$$Y_{AB} = Y_{BA} = -y_{AB} = j25$$

$$Y_{AC} = Y_{CA} = 0$$

$$Y_{AD} = Y_{DA} = -y_{AD} = j50$$

$$Y_{AF} = Y_{FA} = 0$$

$$Y_{BC} = Y_{CB} = -y_{BC} = j33.3333$$

$$Y_{BD} = Y_{DB} = 0$$

$$Y_{BF} = Y_{FB} = 0$$

$$Y_{CD} = Y_{DC} = 0$$

$$Y_{CF} = Y_{FC} = -y_{CF} = j14.2857$$

$$Y_{DF} = Y_{FD} = -y_{DF} = j10$$

$$\therefore [Y_{\text{Bus}}] = \begin{bmatrix} -j100 & j25 & 0 & j50 & 0 \\ j25 & -j78.33 & j33.33 & 0 & 0 \\ 0 & j33.33 & -j47.62 & 0 & j14.29 \\ j50 & 0 & 0 & -j60 & j10 \\ 0 & 0 & j14.29 & j10 & -j24.29 \end{bmatrix}$$

P. 2.7. From Y_{Bus} for the network, using: (i) by direct inspection method and (ii) singular transformation. JNTU (2008)

Element	Positive sequence reactance
1 – 2	j1.0
2 – 3	j0.4
2 – 4	j0.2
3 – 4	j0.2
3 – 1	j0.8
4 – 5	j0.08

Solution:

(i) By direct inspection method:

For the given data,

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{j1.0} = -j1.0 = y_{21}$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{j0.4} = -j2.5 = y_{32}$$

$$y_{24} = \frac{1}{z_{24}} = \frac{1}{j0.2} = -j5 = y_{42}$$

$$y_{34} = \frac{1}{z_{34}} = \frac{1}{j0.2} = -j5 = y_{43}$$

$$y_{31} = \frac{1}{z_{31}} = \frac{1}{j0.8} = -j1.25 = y_{13}$$

$$y_{45} = \frac{1}{z_{45}} = \frac{1}{j0.08} = -j12.5 = y_{54}$$

Diagonal elements:

$$Y_{11} = y_{12} + y_{13} = -j2.25$$

$$Y_{22} = y_{21} + y_{23} + y_{24} = -j8.5$$

$$Y_{33} = y_{31} + y_{32} + y_{34} = -j8.75$$

$$Y_{44} = y_{42} + y_{43} + y_{45} = -j22.5$$

Off-diagonal elements:

$$Y_{12} = Y_{21} = -y_{12} = j1.0$$

$$Y_{13} = Y_{31} = -y_{13} = j1.25$$

$$Y_{14} = Y_{41} = 0$$

$$Y_{23} = Y_{32} = -y_{23} = j2.5$$

$$Y_{24} = Y_{42} = -y_{24} = j5$$

$$Y_{34} = Y_{43} = -y_{34} = j5$$

$$\therefore [Y_{\text{Bus}}] = \begin{bmatrix} -j2.25 & j1.0 & j1.25 & 0 \\ j1.0 & -j8.5 & j2.5 & j5.0 \\ j1.25 & j2.5 & -j8.75 & j5.0 \\ 0 & j5.0 & j5.0 & -j22.5 \end{bmatrix}$$

(ii) Singular transformation:

For the given data, power system network tree graph is shown in Fig. 2.28.

Assume node 5 as reference,

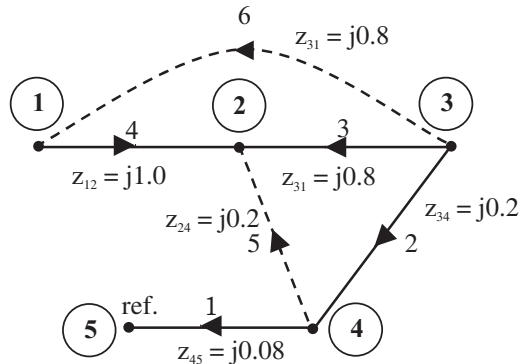


Fig. 2.28: Tree graph.

	1	2	3	4	5
\hat{A}	1	2	3	4	5

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \Rightarrow [A]^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[z] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} j0.08 & 0 & 0 & 0 & 0 & 0 \\ 0 & j0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & j0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & j1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & j0.8 \end{bmatrix} \end{matrix}$$

$$[y] = [z]^{-1} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} -j12.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j1.25 \end{bmatrix} \end{matrix}$$

$$\therefore [Y_{\text{Bus}}] = [A]^T * [y] * [A] = \begin{bmatrix} -j2.25 & j1.0 & j1.25 & 0 \\ j1.0 & -j8.5 & j2.5 & j5.0 \\ j1.25 & j2.5 & -j8.75 & j5.0 \\ 0 & j5.0 & j5.0 & -j22.5 \end{bmatrix}$$

P. 2.8. Form the Y_{Bus} for the given network.

JNTU (2008, 2009)

Bus code	Z_{pq}	$\frac{y'_{pq}}{2}$
1 – 2	$0.02 + j0.08$	$0.00 + j0.03$
1 – 3	$0.08 + j0.24$	$0.00 + j0.025$
2 – 3	$0.06 + j0.18$	$0.00 + j0.02$
2 – 4	$0.06 + j0.18$	$0.00 + j0.02$
2 – 5	$0.04 + j0.12$	$0.00 + j0.015$
3 – 4	$0.01 + j0.03$	$0.00 + j0.01$
4 – 5	$0.08 + j0.24$	$0.00 + j0.025$

Solution: The admittance to ground is obtained by adding the admittances to ground at that bus

Bus code, p	Admittance to ground, y_p
1	$j0.03 + j0.025 = j0.055 = y_1$
2	$j0.3 + j0.02 + j0.02 + j0.015 = j0.085 = y_2$
3	$j0.025 + j0.02 + j0.01 = j0.055 = y_3$
4	$j0.02 + j0.01 + j0.025 = j0.055 = y_4$
5	$j0.015 + j0.025 = j0.04 = y_5$

The primitive admittances are:

$$y_{12} = y_{21} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.08} = 2.9412 - j11.7647$$

$$y_{13} = y_{31} = \frac{1}{z_{13}} = \frac{1}{0.08 + j0.24} = 1.25 - j3.75 = y_{45} = y_{54}$$

$$y_{23} = y_{32} = \frac{1}{z_{23}} = \frac{1}{0.06 + j0.18} = 1.6667 - j5 = y_{24} = y_{42}$$

$$y_{25} = y_{52} = \frac{1}{z_{25}} = \frac{1}{0.04 + j0.12} = 2.5 - j7.5$$

$$y_{34} = y_{43} = \frac{1}{z_{34}} = \frac{1}{0.01 + j0.03} = 10 - j30$$

The diagonal elements are:

$$Y_{11} = y_{12} + y_{13} + y_1 = 4.1912 - j15.5697$$

$$Y_{22} = y_{21} + y_{23} + y_{24} + y_{25} + y_2 = 8.7746 - j29.3497$$

$$Y_{33} = y_{31} + y_{32} + y_{34} + y_3 = 12.9167 - j38.805$$

$$Y_{44} = y_{42} + y_{43} + y_{45} + y_4 = 12.9167 - j38.805$$

$$Y_{55} = y_{52} + y_{54} + y_5 = 3.75 - j11.29$$

The off-diagonal elements are:

$$Y_{12} = Y_{21} = -y_{12} = -2.9412 + j11.7647$$

$$Y_{13} = Y_{31} = -y_{13} = -1.25 + j3.75 = Y_{45} = Y_{54}$$

$$Y_{23} = Y_{32} = -y_{23} = -1.6667 + j5 = Y_{24} = Y_{42}$$

$$Y_{25} = Y_{52} = -y_{25} = -2.5 + j7.5$$

$$Y_{34} = Y_{43} = -y_{43} = -10 + j30$$

$$Y_{14} = Y_{41} = Y_{15} = Y_{51} = Y_{35} = Y_{53} = 0$$

Therefore, the Y_{Bus} is:

$$[Y_{\text{Bus}}] = \begin{bmatrix} 4.1912 - j15.5697 & -2.9412 + j11.7647 & -1.25 + j3.75 & 0 & 0 \\ -2.9412 + j11.7647 & 8.7746 - j29.3497 & -1.6667 + j5 & -1.6667 + j5 & -2.5 + j7.5 \\ -1.25 + j3.75 & -1.6667 + j5 & 12.9167 - j38.805 & -10 + j30 & 0 \\ 0 & -1.6667 + j5 & -10 + j30 & 12.9167 - j38.805 & -1.25 + j3.75 \\ 0 & -2.5 + j7.5 & 0 & -1.25 + j3.75 & 3.75 - j11.29 \end{bmatrix}$$

P. 2.9. For the power system network shown in Fig. 2.29, formulate Y_{Bus} by singular transformation method. The self and mutual reactances in pu are indicated. Take bus (1) as reference.

Solution: The oriented network is shown in Fig. 2.30 and tree graph is shown in Fig. 2.31.

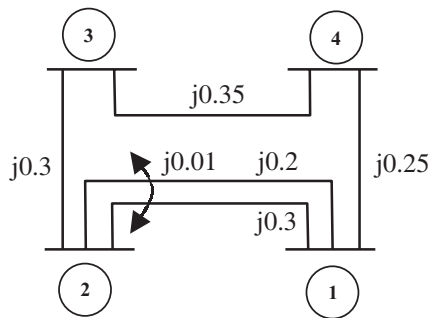


Fig. 2.29: Tree graph.

$$\left[\hat{A} \right] = \begin{array}{c|cccc} e^n & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & -1 \\ 2 & 0 & 0 & -1 & 1 \\ 3 & 0 & -1 & 1 & 0 \\ 4 & -1 & 1 & 0 & 0 \\ 5 & -1 & 1 & 0 & 0 \end{array}$$

$$\left[A \right] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow \left[A \right]^T = \begin{bmatrix} 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[Y_{\text{Bus}} \right] = \left[A \right]^T * \left[y \right] * \left[A \right] = \begin{bmatrix} -j11.3467 & j3.3333 & 0 \\ j3.3333 & -j6.1905 & j2.8571 \\ 0 & j2.8571 & -j6.8571 \end{bmatrix}$$

P. 2.10. Fig. 2.32 shows a 3-bus power system network. The line impedances are given below:

Line (bus to bus):	1 – 2	1 – 3	2 – 3(1)	2 – 3(2)
Impedance in pu:	$0.02 + j0.1$	$0.02 + j0.1$	$0.05 + j0.25$	$0.05 + j0.3$

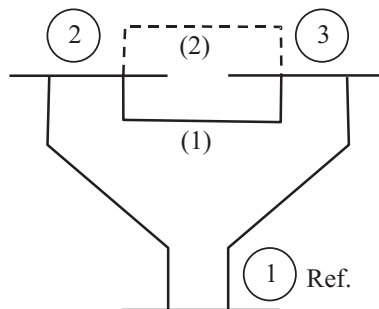


Fig. 2.32: Tree graph.

Formulate Y_{Bus} for the following cases:

1. Assume that the line shown dotted between bus 2 and bus 3 i.e., line 2–3(2) is not present.
2. A new line 2–3(2) is connected and no mutual coupling with other lines.
3. When line 2–3(2) which has mutual impedance of $(0.00 + j0.01)$ pu with line 2 – 3(1) is connected.

Solution: Case (1):

By direct inspection:

$$y_{12} = y_{21} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.1} = 1.9231 - j9.6154 = y_{13} = y_{31}$$

$$y_{23} = y_{32} = \frac{1}{z_{23}} = \frac{1}{0.05 + j0.25} = 0.7692 - j3.8462$$

$$Y_{22} = y_{21} + y_{23} = 2.6923 - j13.4616$$

$$Y_{33} = y_{31} + y_{32} = 2.6923 - j13.4616$$

$$Y_{23} = Y_{32} = -y_{23} = -0.7692 + j3.8462$$

$$[Y_{\text{Bus}}] = \begin{bmatrix} 2.6923 - j13.4616 & -0.7692 + j3.8462 \\ -0.7692 + j3.8462 & 2.6923 - j13.4616 \end{bmatrix}$$

By singular transformation:

$$[\hat{A}] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \end{matrix}$$

$$[A] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow [A]^T = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$[z] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.02 + j0.1 & 0 & 0 \\ 0 & 0.02 + j0.1 & 0 \\ 0 & 0 & 0.05 + j0.25 \end{bmatrix} \end{matrix}$$

$$[y] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1.9231 - j9.6154 & 0 & 0 \\ 0 & 1.9231 - j9.6154 & 0 \\ 0 & 0 & 0.7692 - j3.8462 \end{bmatrix} \end{matrix}$$

$$[Y_{\text{Bus}}] = [A]^T * [y] * [A] = \begin{bmatrix} 2.6923 - j13.4616 & -0.7692 + j3.8462 \\ -0.7692 + j3.8462 & 2.6923 - j13.4616 \end{bmatrix}$$

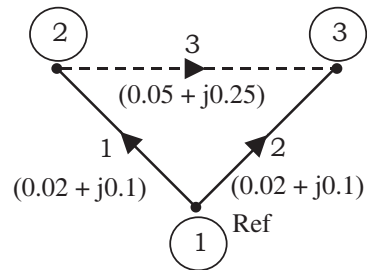


Fig. 2.32(a): Tree graph, T[1, 2].

Case (2):

By direct inspection:

$$y_{12} = y_{21} = \frac{1}{z_{12}} = \frac{1}{0.02 + j0.1} = 1.9231 - j9.6154 = y_{13} = y_{31}$$

$$y_{23,1} = y_{32,1} = \frac{1}{z_{23,1}} = \frac{1}{0.05 + j0.25} = 0.7692 - j3.8462$$

$$y_{23,2} = y_{32,2} = \frac{1}{z_{23,2}} = \frac{1}{0.05 + j0.3} = 0.5405 - j3.2432$$

$$Y_{22} = y_{21} + y_{23,1} + y_{23,2} = 3.2328 - j16.7048$$

$$Y_{33} = y_{31} + y_{32,1} + y_{32,2} = 3.2328 - j16.7048$$

$$Y_{23} = Y_{32} = -y_{23,1} - y_{23,2} = -1.3098 + j7.0894$$

$$\therefore [Y_{\text{Bus}}] = \begin{bmatrix} 3.2328 - j16.7048 & -1.3098 + j7.0894 \\ -1.3098 + j7.0894 & 3.2328 - j16.7048 \end{bmatrix}$$

By singular transformation:

$$\begin{matrix} & 1 & 2 & 3 \\ \begin{bmatrix} \hat{A} \end{bmatrix} = & 1 \begin{bmatrix} 1 & -1 & 0 \\ 2 \begin{bmatrix} 1 & 0 & -1 \\ 3 \begin{bmatrix} 0 & 1 & -1 \\ 4 \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{matrix}$$

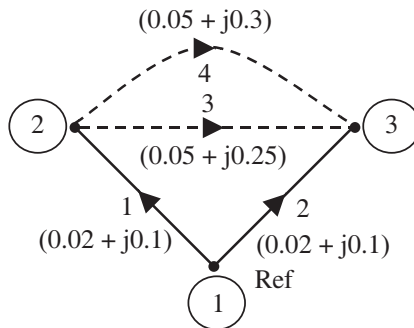


Fig. 2.32(b): Tree graph, T[1, 2].

$$[z] = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 0.02 + j0.1 & 0 & 0 & 0 \\ 0 & 0.02 + j0.1 & 0 & 0 \\ 0 & 0 & 0.05 + j0.25 & j0.01 \\ 0 & 0 & j0.01 & 0.05 + j0.3 \end{bmatrix}$$

$$[z_{3,4}] = \begin{bmatrix} 0.05 + j0.25 & j0.01 \\ j0.01 & 0.05 + j0.3 \end{bmatrix}$$

$$\Rightarrow [y_{3,4}] = [z_{3,4}]^{-1} = \begin{bmatrix} 0.05 + j0.25 & j0.01 \\ j0.01 & 0.05 + j0.3 \end{bmatrix}^{-1} = \begin{bmatrix} 0.7719 - j3.8504 & -0.0458 + j0.1207 \\ -0.0458 + j0.1207 & 0.5427 - j3.2469 \end{bmatrix}$$

$$[y] = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 1.9231 - j9.6154 & 0 & 0 & 0 \\ 0 & 1.9231 - j9.6154 & 0 & 0 \\ 0 & 0 & 0.7719 - j3.8504 & -0.0458 + j0.1207 \\ 0 & 0 & -0.0458 + j0.1207 & 0.5427 - j3.2469 \end{bmatrix}$$

$$[Y_{\text{Bus}}] = [A]^T * [y] * [A] = \begin{bmatrix} 3.1460 - j16.4713 & -1.2229 + j6.8559 \\ -1.2229 + j6.8559 & 3.1460 - j16.4713 \end{bmatrix}$$

P. 2.11. Form Y_{Bus} , Y_{Br} and Z_{Loop} by singular transformation for the network connections given below: JNTU(2009)

Element	p-r
1	1 – 2 (1)
2	1 – 2 (2)
3	1 – 3
4	2 – 4
5	3 – 4

SELF		MUTUAL	
Bus code	Impedance	Bus code	Impedance
1 – 2 (1)	0.6		
1 – 3	0.5	1 – 2 (1)	0.1
3 – 4	0.5		
1 – 2 (2)	0.4	1 – 2 (1)	0.2
2 – 4	0.2		

Solution: The power network is shown in Fig. 2.33.

Element	SELF		MUTUAL	
	Bus code	Impedance	Bus code	Impedance
1	1 – 2 (1)	0.6		
2	1 – 3	0.5	1 – 2 (1)	0.1
3	3 – 4	0.5		
4	1 – 2 (2)	0.4	1 – 2 (1)	0.2
5	2 – 4	0.2		

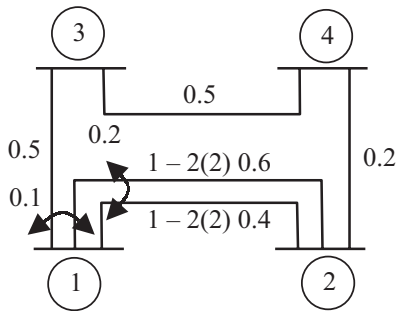


Fig. 2.33: Power system network.

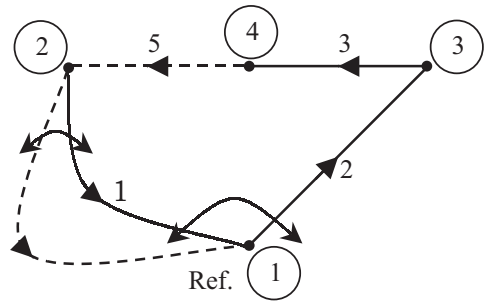


Fig. 2.33(a): Tree graph, T [1, 2, 3].

$$[z] = \begin{matrix} 1-2(1) \\ 1-3 \\ 3-4 \\ 1-2(2) \\ 2-4 \end{matrix} \begin{bmatrix} 0.6 & 0.1 & 0 & 0.2 & 0 \\ 0.1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.2 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \quad (\text{or}) \quad \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} 0.6 & 0.1 & 0 & 0.2 & 0 \\ 0.1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.2 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}$$

$$[y] = [z]^{-1} = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} 2.0833 & -0.4167 & 0 & -1.0417 & 0 \\ -0.4167 & 2.0833 & 0 & 0.2083 & 0 \\ 0 & 0 & 2.0 & 0 & 0 \\ -1.0417 & 0.2083 & 0 & 3.0208 & 0 \\ 0 & 0 & 0 & 0 & 5.0 \end{bmatrix}$$

$$\begin{bmatrix} \hat{A} \end{bmatrix} = \begin{matrix} & 1 & 2 & 3 & 4 \\ 1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 \\ 3 & 0 & 0 & 1 & -1 \\ 4 & -1 & 1 & 0 & 0 \\ 5 & 0 & -1 & 0 & 1 \end{matrix}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow [A]^T = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$[Y_{Bus}] = [A]^T * [y] * [A] = \begin{bmatrix} 8.0208 & -0.2083 & -5.0000 \\ -0.2083 & 4.0833 & -2.0000 \\ -5.0000 & -2.0000 & 7.0000 \end{bmatrix}$$

$$[Z_{Bus}] = [Y_{Bus}^{-1}] = \begin{bmatrix} 0.2713 & 0.1264 & 0.2299 \\ 0.1264 & 0.3437 & 0.1885 \\ 0.2299 & 0.1885 & 0.3609 \end{bmatrix}$$

$$[B] = \begin{matrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ 4 & 1 & 0 & 0 \\ 5 & -1 & -1 & -1 \end{matrix}$$

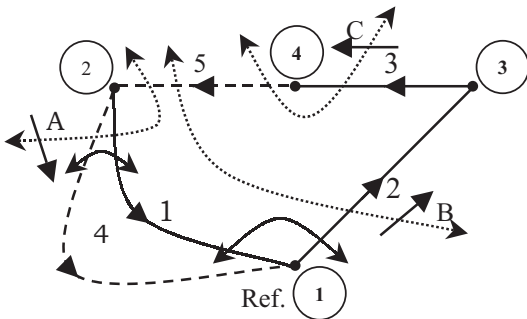


Fig. 2.33(b): Tree graph, T [1, 2, 3].

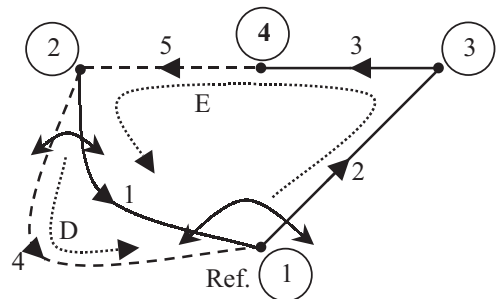


Fig. 2.33(c): Tree graph, T [1, 2, 3].

$$[Y_{Br}] = [B]^T * [y] * [B] = \begin{bmatrix} 8.0208 & 4.7917 & 5.0000 \\ 4.7917 & 7.0833 & 5.0000 \\ 5.0000 & 5.0000 & 7.0000 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \\ 4 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$$

$$[Z_{Loop}] = [C]^T * [z] * [C] = \begin{bmatrix} 0.6000 & -0.5000 \\ -0.5000 & 2.0000 \end{bmatrix}$$

P. 2.12. Find the bus admittance matrix of the following power system.

Bus code	Line data impedance
1-2	$10 + j20$
2-3	$0.96154 + j4.80769$
3-5	$0.82192 + j2.19178$
3-4	$5 + j5$
4-5	$10 + j20$
1-5	$1.72414 + j 4.31034$

Solution:

The primitive line admittances are obtained as

$$y_{12} = y_{21} = \frac{1}{(10 + j20)} = y_{45} = y_{54} = 0.02 - j0.04$$

$$y_{23} = y_{32} = \frac{1}{(0.96154 + j4.80769)} = 0.04 - j0.02$$

$$y_{34} = y_{43} = \frac{1}{(5 + j5)} = 0.1 - j0.1$$

$$y_{35} = y_{53} = \frac{1}{(0.82192 + j2.19178)} = 0.15 - j0.04$$

$$y_{15} = y_{51} = \frac{1}{(1.72414 + j4.31034)} = 0.08 - j0.2$$

The diagonal elements are given by

$$y_{11} = y_{12} + y_{15} = 0.1 - j0.24$$

$$y_{22} = y_{21} + y_{23} = 0.6 - j0.06$$

$$y_{33} = y_{32} + y_{34} + y_{35} = 0.29 - j0.16$$

$$y_{44} = y_{43} + y_{45} = 0.12 - j0.14$$

$$y_{55} = y_{51} + y_{54} = 0.1 - j0.24$$

$$y_{12} = y_{21} = -y_{12} = -0.02 + j0.04$$

$$y_{23} = y_{32} = -y_{23} = -0.04 + j0.02$$

$$y_{34} = y_{43} = -y_{34} = -0.1 + j0.1$$

$$y_{35} = y_{53} = -y_{35} = -0.15 + j0.04$$

$$y_{45} = y_{54} = -y_{45} = -0.02 + j0.04$$

$$y_{15} = y_{51} = -y_{15} = -0.08 + j0.2$$

By direct inspection, the Y_{Bus} is

$$Y_{\text{Bus}} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0.1 - j0.24 & -0.02 + j0.04 & 0 & 0 & -0.08 + j0.2 \\ -0.02 + j0.04 & 0.06 - j0.06 & -0.04 + j0.02 & 0 & 0 \\ 0 & -0.04 + j0.02 & 0.29 - j0.16 & -0.1 + j0.1 & -0.15 + j0.04 \\ 0 & 0 & -0.1 - j0.1 & 0.12 - j0.14 & -0.02 + j0.04 \\ -0.08 + j0.2 & 0 & -0.15 + j0.04 & -0.02 + j0.04 & 0.1 - j0.24 \end{bmatrix} \end{matrix}$$

P. 2.13. Form the Y_{Bus} for the given network

GATE

Element no.	Positive sequence reactance
1-2	$j1.00$
2-3	$j0.40$
2-4	$j0.20$
3-4	$j0.20$
3-1	$j0.80$
4-5	$j0.08$

Solution:

The primitive admittances are

$$y_{12} = y_{21} = \frac{1}{j1.0} = -j1.0, \quad y_{23} = y_{32} = \frac{1}{j0.40} = -j2.5, \quad y_{42} = y_{24} = \frac{1}{j0.2} = -j5 = y_{34} = y_{43}$$

$$y_{31} = y_{13} = \frac{1}{j0.8} = -j1.25, \quad y_{45} = y_{54} = \frac{1}{j0.08} = -j12.5$$

The diagonal elements are given by

$$Y_{11} = (y_{12} + y_{13}) = -j1.0 - j1.25 = -j2.25$$

$$Y_{22} = (y_{21} + y_{23} + y_{24}) = -j1.00 - j2.5 - j5 = -j8.50$$

$$Y_{33} = (y_{31} + y_{32} + y_{34}) = -j1.25 - j2.5 - j5 = -j8.75$$

$$Y_{44} = (y_{42} + y_{43} + y_{45}) = -j5 - j5 - j12.5 = -j22.5$$

$$y_{55} = (y_{45}) = -j12.5$$

The off-diagonal elements are given by

$$y_{12} = y_{21} = -y_{21} = j1.0$$

$$y_{23} = y_{32} = -y_{23} = j2.5$$

$$y_{24} = y_{42} = -y_{24} = j5 = y_{34} = y_{43}$$

$$y_{31} = y_{13} = -y_{31} = j1.25$$

$$y_{45} = y_{54} = -y_{45} = j12.5$$

The Y_{BUS} matrix is

$$Y_{\text{BUS}} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} -j2.25 & j1 & j1.25 & 0 & 0 \\ j1 & -j8.5 & j2.5 & j5 & 0 \\ j1.25 & j2.5 & -j8.75 & j5 & 0 \\ 0 & j5 & j5 & -j22.5 & j12.5 \\ 0 & 0 & 0 & j12.5 & -j12.5 \end{bmatrix} \end{matrix}$$

P. 2.14. The single line-diagram of a network is shown below. The line reactance is 0.001 pu/km and shunt capacitance is 0.0016 pu km. Assemble the bus admittance matrix neglecting the line resistance.

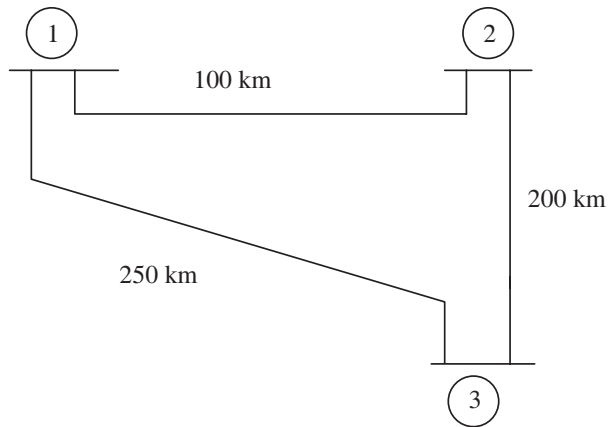


Fig. 2.34: Single-line diagram of a 3-bus system.

Solution:

The calculated line parameters are

Line	Series impedance in pu In pu	Shunt admittance
1-2	$0.001(100) = j0.10$	$0.0016(100) = j0.60$
2-3	$0.001(200) = j0.20$	$0.0016(200) = j0.32$
3-1	$0.001(250) = j0.25$	$0.0016(250) = j0.40$

The pu series admittance is

$$y_{12} = y_{21} = 1/(j0.1) = -j10, y_{13} = y_{31} = 1/(j0.25) = -j4 \text{ and } y_{23} = y_{32} = 1/(j0.2) = -j5$$

Shunt admittances at the nodes are

$$y_{11} = \frac{1}{2}(j0.16 + j0.40) = j0.28$$

$$y_{22} = \frac{1}{2}(j0.32 + j0.16) = j0.24$$

$$y_{33} = \frac{1}{2}(j0.40 + j0.32) = j0.36$$

The elements of bus admittance matrix are calculated as

$$Y_{11} = (y_{11} + y_{12} + y_{13}) = (j0.28 - j10 - j4) = -j13.72$$

$$Y_{22} = (y_{22} + y_{21} + y_{23}) = (j0.24 - j10 - j5) = -j14.76$$

$$Y_{33} = (y_{33} + y_{31} + y_{32}) = (j0.36 - j4 - j5) = -j8.64$$

$$Y_{12} = Y_{21} = -y_{12} = j10$$

$$Y_{31} = Y_{13} = -y_{13} = j4$$

$$Y_{23} = Y_{32} = -y_{23} = j5$$

Then the bus admittance matrix is denoted by

$$Y_{\text{BUS}} = \begin{bmatrix} -j13.72 & j10 & j4 \\ j10 & -j14.6 & j5 \\ j4 & j5 & -j8.64 \end{bmatrix}$$

P. 2.15. The line data of a 4-bus system is given below, obtain the Y_{BUS} .

GATE

Bus code	Line data impedance
1-2	j0.20
1-3	j0.15
1-4	j0.10
2-3	j0.10
2-4	j0.15

Solution: By inspection the Y_{BUS} is calculated. The values of the system are

$$y_{12} = y_{21} = \frac{1}{j0.2} = -j5.0, y_{13} = y_{31} = y_{24} = y_{42} = \frac{1}{j0.15} = -j6.667,$$

$$y_{14} = y_{41} = y_{23} = y_{32} = \frac{1}{j0.1} = -j10.0$$

The value of $Y_{11} = Y_{22} = (-j5 - j6.667 - j10) = -j21.667$

$$Y_{33} = Y_{44} = (-j6.667 - j10) = -j16.667$$

$$Y_{12} = Y_{21} = -y_{12} = j5, Y_{13} = Y_{31} = -y_{13} = j6.667,$$

$$Y_{14} = Y_{41} = -y_{14} = j10$$

The Y_{BUS} is shown as

$$(Y_{\text{BUS}}) = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} -j21.667 & j5.00 & j6.667 & j10.00 \\ j5.00 & -j21.667 & j10.00 & j6.667 \\ j6.667 & j10.00 & -j16.667 & j0.00 \\ j10.00 & j6.667 & j0.00 & -j16.667 \end{bmatrix} \end{matrix}$$

Solution: Admittance = $1/(3 + j4) = (3 - j4)/5$, the conductance is $3/5$.

P. 2.19. The z matrix of a two-port network is given by $\begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$. The element Y_{22} of the corresponding Y matrix is

- (a) 1.2 (b) 0.4 (c) -0.4 (d) 1.8

Solution:

The determinant of z is

$$\begin{vmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{vmatrix} = 0.5$$

$$[Y] = [Z]^{-1} = \frac{1}{0.5} \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.9 \end{bmatrix} = \begin{bmatrix} 1.2 & -0.4 \\ -0.4 & 1.8 \end{bmatrix}$$

hence

$$Y_{22} = 1.8$$

P. 2.20. The network shown in Fig. 2.36 has impedance in pu as indicated, the shunt admittance is $-j20$ at each bus. The diagonal element of Y_{22}

GATE

- (a) $-j19.8$ (b) $j20$ (c) $j0.2$ (d) $-j19.95$

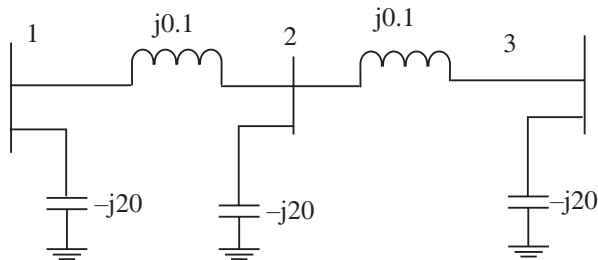


Fig. 2.36: Power system network.

Solution:

$$y_{21} = \frac{1}{j0.1} = -j10, \quad y_{23} = \frac{1}{j0.1} = -j10, \quad y_{22} = \frac{1}{-j20} = j0.05$$

$$Y_{22} = (y_{22} + y_{21} + y_{23}) = (j0.05 - j10 - j10) = -j19.95$$

P. 2.21. The bus admittance matrix of the network shown in the Fig. 2.37 is