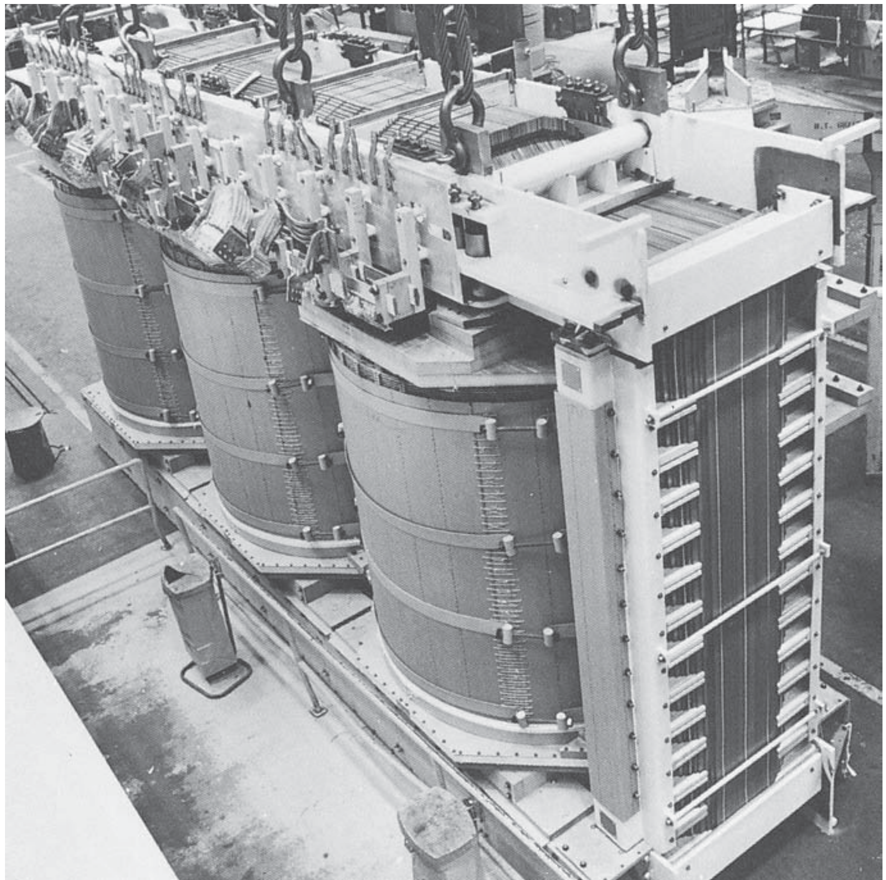


Core and coil assemblies of a three-phase 20.3 kVΔ/345 kVY step-up transformer. This oil-immersed transformer is rated 325 MVA self-cooled (OA)/542 MVA forced oil, forced air-cooled (FOA)/607MVA forced oil, forced air-cooled (FOA) (Courtesy of General Electric)

3

POWER TRANSFORMERS

The power transformer is a major power system component that permits economical power transmission with high efficiency and low series-voltage drops. Since electric power is proportional to the product of voltage and current, low current levels (and therefore low I^2R losses and low IZ voltage drops) can be maintained for given power levels via high voltages. Power transformers transform ac voltage and current to optimum levels for generation, transmission, distribution, and utilization of electric power.

The development in 1885 by William Stanley of a commercially practical transformer was what made ac power systems more attractive than dc power systems. The ac system with a transformer overcame voltage problems encountered in dc systems as load levels and transmission distances increased. Today's modern power transformers have nearly 100% efficiency, with ratings up to and beyond 1300 MVA.

In this chapter, we review basic transformer theory and develop equivalent circuits for practical transformers operating under sinusoidal-steady-state conditions. We look at models of single-phase two-winding, three-phase two-winding, and three-phase three-winding transformers, as well as auto-transformers and regulating transformers. Also, the per-unit system, which simplifies power system analysis by eliminating the ideal transformer winding in transformer equivalent circuits, is introduced in this chapter and used throughout the remainder of the text.

CASE STUDY

The following article describes how transmission transformers are managed in the Pennsylvania–New Jersey (PJM) Interconnection. PJM is a regional transmission organization (RTO) that operates approximately 19% of the transmission infrastructure of the U.S. Eastern Interconnection. As of 2007, there were 188 transmission transformers (500/230 kV) and 29 dedicated spare transformers in the PJM system. A Probabilistic Risk assessment (PRA) model is applied to PJM transformer asset management [8].

PJM Manages Aging Transformer Fleet: Risk-based tools enable regional transmission owner to optimize asset service life and manage spares.

BY DAVID EGAN AND KENNETH SEILER
PJM INTERCONNECTION

The PJM interconnection system has experienced both failures and degradation of older transmission transformers (Fig. 1). Steps required to mitigate potential system reliability issues, such as operation of out-of-merit generation, have led to higher operating costs of hundreds of millions of dollars for transmission system users over the last several years.

The PJM (Valley Forge, Pennsylvania, U.S.) system has 188 transmission transformers (500 kV/230 kV) in service and 29 dedicated spares. Figure 2 shows the age distribution of this transformer fleet. Note that 113 transformers are more than 30 years old and will reach or exceed their design life over the course of the next 10 years. To address increasing

("PJM Manages Aging Transformer Fleet" by David Egan and Kenneth Seiler, Transmission & Distribution World Magazine, *March 2007)*

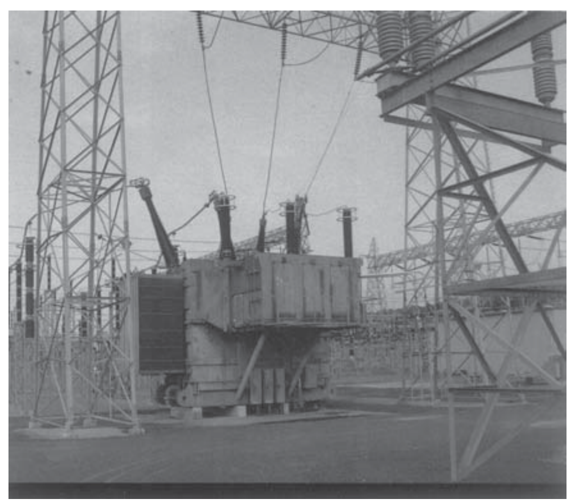


Figure 1
PJM is evaluating the risk of older transformers.
The Probabilistic Risk Assessment also considers the effectiveness of alternative spares strategies

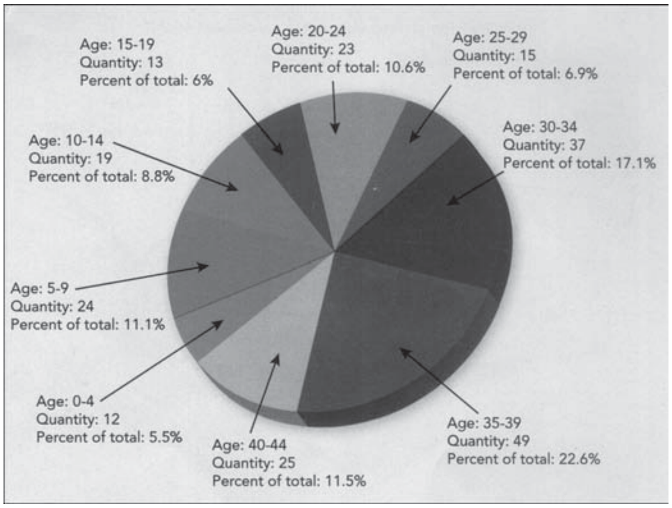


Figure 2
Age distribution of the PJM 500-kV/230-kV transformer fleet. Note that more than half of this population is over 30 years old

concerns regarding potential reliability impacts and the ability to replace failed transformer units in a timely fashion, PJM and its transmission-owning members are establishing a systematic, proactive transformer replacement program to mitigate negative impacts on PJM stakeholders, operations and ultimately the consumers. PJM now assesses the risk exposure from an aging 500-kV/230-kV transformer fleet through its Probabilistic Risk Assessment (PRA) model.

CONGESTION

Generally PJM's backbone high-voltage transmission system delivers lower-cost power from sources in the western side of the regional transmission organization (RTO) to serve load centers in the eastern side. Delivery of power in PJM includes transformation from 500-kV lines to 230-kV facilities for further delivery to and consumption by customers.

Congestion on the electric system can occur when a transmission transformer unit must be removed from service and the redirected electricity flow exceeds the capabilities of parallel transmission facilities. When congestion occurs, higher-cost generation on the restricted side of the constraint must operate to keep line flows under specified limits and to meet customer demand. The cost of congestion results from the expense of operating higher-cost generators. Congestion and its related costs exist on all electric power systems. However, in a RTO such as PJM, the cost of congestion is readily knowable and identified.

The failure impact of certain 500-kV/230-kV transformers on the PJM system can mean annual congestion costs of hundreds of millions of dollars if the failure cannot be addressed with a spare. Lead times for replacement transformer units at this voltage class can take up to 18 months, and each replacement unit cost is several million dollars. These costly transformer-loss consequences, coupled with the age distribution of the transformer population, have raised PJM's concern that the existing system spare quantities could be deficient and locations of existing spares suboptimal.

DEVELOPING PRA

PJM reviewed existing methods for determining transformer life expectancy, assessing failure impacts, mitigating transformer failures, ensuring spare-quantity adequacy and locating spares. Each of these methodologies has weaknesses when applied to an RTO scenario. In addition, no existing method identified the best locations for spare transformers on the system.

Transformer condition assessments are the primary means for predicting failures. Although technology advancements have improved condition-monitoring data, unless a transformer exhibits signs of imminent failure, predicting when a transformer will fail based on a condition assessment is still mostly guesswork. Traditional methods have quantified the impacts of transformer failure based on reliability criteria; they have not typically included economic considerations. Also, while annual failure rate analysis is used to determine the number of spares required, assuming a constant failure rate may be a poor assumption if a large portion of the transformer fleet is entering the wear-out stage of asset life.

Recognizing the vulnerabilities of existing methods, PJM proceeded to develop a risk-based approach to transformer asset management. The PJM PRA model couples the loss consequence of a transformer with its loss likelihood (Fig. 3). The product of these inputs, risk, is expressed in terms of annual risk-exposure dollars.

PRA requires a detailed understanding of failure consequences. PJM projects the dollar value of each transformer's failure consequence, including cost estimates for replacement, litigation, environmental impact and congestion. PJM's PRA also permits the assessment of various spare-unit and replacement policies based on sensitivity analysis of these four cost drivers.

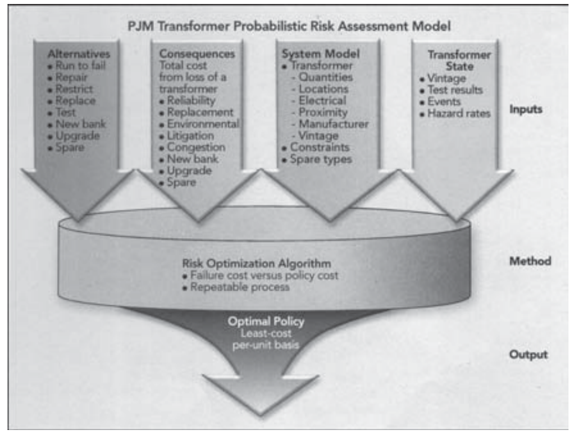


Figure 3
The PJM Probabilistic Risk Assessment model uses drivers to represent overall failure consequences: costs of replacement, litigation, environmental and congestion

PRA MODEL INPUTS

The PRA model depends on several inputs to determine the likelihood of asset failure. One key input is the number of existing fleet transformers. Individual utilities within PJM may not have enough transformers to develop statistically significant assessment results. However, PJM's region-wide perspective permits evaluation of the entire transformer population within its footprint.

Second, rather than applying the annual failure rate of the aggregate transformer population, each transformer's failure rate is determined as a function of its effective age. PJM developed its own method for determining this effective age-based failure rate, or hazard rate. Effective age combines condition data with age-based failure history. By way of analogy, consider a 50-year-old person who smokes and has high cholesterol and high blood pressure (condition data). This individual may have the same risk of death as a healthy 70-year-old non-smoker. Thus, while the individual's actual age is 50 years, his effective age could be as high as 70 years.

Third, the PRA model inputs also include transformers' interactions with each other in terms of the probabilities of cascading events and large-impact, low-likelihood events. For example, transformers are cooled with oil, which, if a transformer ruptures, can become a fuel source for fire. Such a fire can spread to neighboring units causing them to fail as well. PJM determined cascading event probability by reviewing industry events and consulting industry subject-matter experts. Further, the impacts of weather events also are considered. For example, a tornado could damage multiple transformer units at a substation. PJM uses National Oceanic and Atmospheric Administration statistical data for probabilities of such weather-related phenomena.

The remaining PRA model inputs include the possible risk-mitigation alternatives and transformer groupings. The possible risk-mitigation alternatives include running to failure, overhauling or retrofitting, restricting operations, replacing in-kind or with an upgraded unit, increasing test frequency to better assess condition, adding redundant transformers or purchasing a spare. The PRA model objective is to select the appropriate alternative commensurate with risk. To accomplish this objective, the PRA model also requires inputs of the cost and time to implement each alternative. The time to implement an alternative is important because failure consequences accumulate until restoration is completed.

Also, transformers must be grouped by spare applicability. Design parameters can limit the number of in-service transformers that can be served by a designated spare. Additionally, without executed sharing agreements in place between transmission owners, PJM cannot recognize transformer spare sharing beyond the owner's service territory.

THE QUESTION OF SPARES

PRA determines the amount of transformer-loss risk exposure to the PJM system and to PJM members. To calculate the total risk exposure from transformer loss, each transformer's risk is initially determined assuming no available spare. This initial total-system transformer-loss risk is a baseline for comparing potential mitigation approaches. For this baseline, with no spares available, US$553 million of annual risk exposure was identified.

A spare's value is equal to the cumulative risk reduction, across all facilities that can be served by a given spare. The existing system spares were shown to mitigate $396 million of the annual risk, leaving $157 million of annual exposure. The PRA showed that planned projects would further mitigate $65 million, leaving $92 million of exposed annual risk.

With the value of existing spares and planned reliability upgrade projects known, the PRA can then assess the value of additional spares in reducing this risk exposure. As long as the risk mitigated by an additional spare exceeds the payback value of a new transformer, purchasing a spare is justified. The PRA identified $75 million of justifiable risk mitigation from seven additional spares.

PRA also specifies the best spare type. If a spare can be cost justified, asset owners can use two types of spare transformers: used or new. As an in-service unit begins to show signs of failure, it can be replaced. Since the unit removed has not yet failed, it can be stored as an emergency spare. However, the downsides of this approach are the expense, work efforts and congestion associated with handling the spare twice. Also, the likelihood of a used spare unit's success is lower than that of a new unit because of its preexisting degradation.

PJM's PRA analysis revealed that it is more cost-effective to purchase a new unit as a spare. In this case, when a failure occurs, the spare transformer can be installed permanently to remedy the failure and a replacement spare purchased. This process allows expedient resolution of a failure and reduces handling.

Existing spares may not be located at optimal sites. PRA also reveals ideal locations for storing spares. A spare can be located on-site or at a remote location. An on-site spare provides the benefit of expedient installation. A remote spare requires added transportation and handling. Ideally, spares would be located at the highest risk sites. Remote spares serve lower risk sites. The PRA both identifies the best locations to position spares on the system to minimize risk and evaluates relocation of existing spares by providing the cost/benefit analysis of moving a spare to a higher risk site.

The PRA has shown that the type of spare (no spare, old spare or new spare) and a transformer's loss consequence strongly influence the most cost-effective retirement age. High-consequence transformers should be replaced at younger ages due to the risk they impose on the system as their effective age increases. PRA showed that using new spares maximizes a transformer's effective age for retirement.

STANDARDIZATION IMPACT

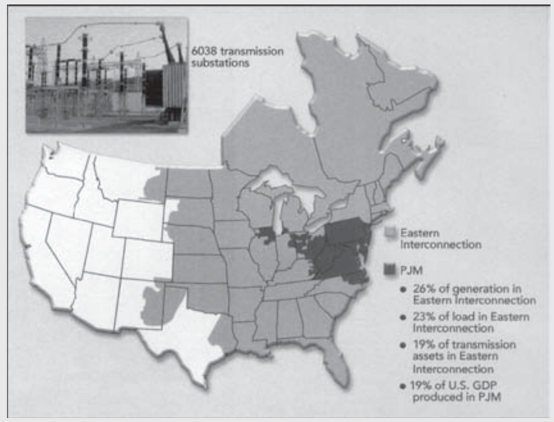
Approximately one-third of the number of current spares would be required if design standardization and sharing between asset owners were achieved. This allows a single spare to reduce the loss consequence for a larger number of in-service units. Increasing the number of transformers covered by a spare improves the spare's risk-mitigation value. Having more transformers covered by spares reduces the residual risk exposure that accumulates with having many spare subgroups.

PJM transmission asset owners have finalized a standardized 500-kV/230-kV transformer design to apply to future purchase decisions. For the benefits of standardization to be achieved, PJM asset owners

PJM BACKGROUND

Formally established on Sept. 16, 1927, the Pennsylvania-New Jersey Interconnection allowed Philadelphia Electric, Pennsylvania Power & Light, and Public Service Electric & Gas of New Jersey to share their electric loads and receive power from the huge new hydroelectric plant at Conowingo, Maryland, U.S. Throughout the years, neighboring utilities also connected into the system. Today, the interconnection, now called the PJM Interconnection, has far exceeded its original footprint.

PJM is the operator of the world's largest centrally dispatched grid, serving about 51 million people in 13 states and the District of Columbia. A regional transmission organization that operates 19% of the transmission infrastructure of the U.S. Eastern Interconnection on behalf of transmission system owners, PJM dispatches 164,634 MW of generating capacity over 56,000 miles (91,800 km) of transmission. Within PJM, 12 utilities individually own the 500-kV/230-kV transformer assets.



PJM system information breakdown and location

also are developing a spare-sharing agreement. Analysis showed that $50 million of current spare transformer requirements could be avoided by standardization and sharing.

The PRA model is a useful tool for managing PJM's aging 500-kV/230-kV transformer infrastructure. While creating the PRA model was challenging, system planners and asset owners have gained invaluable insights from both the development process and the model use. Knowing and understanding risk has better prepared PJM and its members to proactively and economically address their aging transformer fleet. PRA results have been incorporated into PJM's regional transmission-expansion planning process. PRA will be performed annually to ensure minimum transformer fleet risk exposure. PJM is also investigating the use of this risk quantification approach for other power-system assets.

Kenneth Seiler is manager of power system coordination at PJM Interconnection. He is responsible for the interconnection coordination of generation, substation and transmission projects, and outage planning. He has been actively involved in the PJM Planning Committee and the development of the PJM's aging infrastructure initiatives. Prior to working for PJM, he was with GPU Energy for nearly 15 years in the Electrical Equipment Construction and Maintenance and System Operation departments. Seiler earned his BSEE degree from Pennsylvania State University and MBA from Lebanon Valley College, **seilek@pjm.com**

David Egan is a senior engineer in PJM's Interconnection Planning department, where he has worked for three years. He earned his BSME degree from Binghamton University. Previously he worked at Oyster Creek Generating Station for 13 years. During this time, he worked as a thermal performance engineer and turbine-generator systems' manager, and coordinated implementation of the site's Maintenance Rule program, **egand@pjm.com**

3.1

THE IDEAL TRANSFORMER

Figure 3.1 shows a basic single-phase two-winding transformer, where the two windings are wrapped around a magnetic core [1, 2, 3]. It is assumed here that the transformer is operating under sinusoidal-steady-state excitation. Shown in the figure are the phasor voltages E_1 and E_2 across the windings, and the phasor currents I_1 entering winding 1, which has N_1 turns, and I_2 leaving winding 2, which has N_2 turns. A phasor flux Φ_c set up in the core and a magnetic field intensity phasor H_c are also shown. The core has a cross-sectional area denoted A_c, a mean length of the magnetic circuit l_c, and a magnetic permeability μ_c, assumed constant.

For an ideal transformer, the following are assumed:

1. The windings have zero resistance; therefore, the I^2R losses in the windings are zero.
2. The core permeability μ_c is infinite, which corresponds to zero core reluctance.
3. There is no leakage flux; that is, the entire flux Φ_c is confined to the core and links both windings.
4. There are no core losses.

A schematic representation of a two-winding transformer is shown in Figure 3.2. Ampere's and Faraday's laws can be used along with the preceding assumptions to derive the ideal transformer relationships. Ampere's law states that the tangential component of the magnetic field intensity vector

FIGURE 3.1

Basic single-phase two-winding transformer

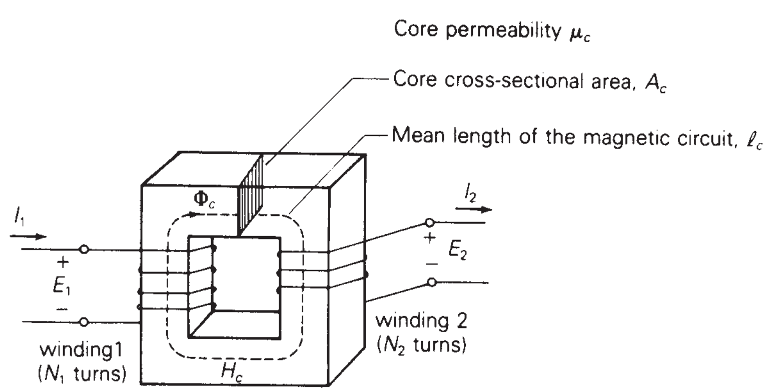
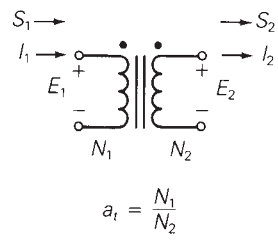


FIGURE 3.2 Schematic representation of a single-phase two-winding transformer



integrated along a closed path equals the net current enclosed by that path; that is,

$$\oint H_{\tan}\, dl = I_{\text{enclosed}} \tag{3.1.1}$$

If the core center line shown in Figure 3.1 is selected as the closed path, and if H_c is constant along the path as well as tangent to the path, then (3.1.1) becomes

$$H_c l_c = N_1 I_1 - N_2 I_2 \tag{3.1.2}$$

Note that the current I_1 is enclosed N_1 times and I_2 is enclosed N_2 times, one time for each turn of the coils. Also, using the right-hand rule*, current I_1 contributes to clockwise flux but current I_2 contributes to counter-clockwise flux. Thus, in (3.1.2) the net current enclosed is $N_1 I_1 - N_2 I_2$. For constant core permeability μ_c, the magnetic flux density B_c within the core, also constant, is

$$B_c = \mu_c H_c \quad \text{Wb/m}^2 \tag{3.1.3}$$

and the core flux Φ_c is

$$\Phi_c = B_c \text{A}_c \quad \text{Wb} \tag{3.1.4}$$

Using (3.1.3) and (3.1.4) in (3.1.2) yields

$$N_1 I_1 - N_2 I_2 = l_c B_c / \mu_c = \left(\frac{l_c}{\mu_c \text{A}_c}\right)\Phi_c \tag{3.1.5}$$

We define core reluctance R_c as

$$\text{R}_c = \frac{l_c}{\mu_c \text{A}_c} \tag{3.1.6}$$

Then (3.1.5) becomes

$$N_1 I_1 - N_2 I_2 = \text{R}_c \Phi_c \tag{3.1.7}$$

---

* The right-hand rule for a coil is as follows: Wrap the fingers of your right hand around the coil in the direction of the current. Your right thumb then points in the direction of the flux.

Equation (3.1.7) can be called "Ohm's law" for the magnetic circuit, wherein the net magnetomotive force mmf $= N_1 I_1 - N_2 I_2$ equals the product of the core reluctance R_c and the core flux Φ_c. Reluctance R_c, which impedes the establishment of flux in a magnetic circuit, is analogous to resistance in an electric circuit. For an ideal transformer, μ_c is assumed infinite, which, from (3.1.6), means that R_c is 0, and (3.1.7) becomes

$$N_1 I_1 = N_2 I_2 \tag{3.1.8}$$

In practice, power transformer windings and cores are contained within enclosures, and the winding directions are not visible. One way of conveying winding information is to place a dot at one end of each winding such that when current enters a winding at the dot, it produces an mmf acting in the *same* direction. This dot convention is shown in the schematic of Figure 3.2. The dots are conventionally called *polarity marks*.

Equation (3.1.8) is written for current I_1 *entering* its dotted terminal and current I_2 *leaving* its dotted terminal. As such, I_1 and I_2 are *in phase*, since $I_1 = (N_2/N_1)I_2$. If the direction chosen for I_2 were reversed, such that both currents entered their dotted terminals, then I_1 would be 180° *out of phase* with I_2.

Faraday's law states that the voltage $e(t)$ induced across an N-turn winding by a time-varying flux $\phi(t)$ linking the winding is

$$e(t) = N \frac{d\phi(t)}{dt} \tag{3.1.9}$$

Assuming a sinusoidal-steady-state flux with constant frequency ω, and representing $e(t)$ and $\phi(t)$ by their phasors E and Φ, (3.1.9) becomes

$$E = N(j\omega)\Phi \tag{3.1.10}$$

For an ideal transformer, the entire flux is assumed to be confined to the core, linking both windings. From Faraday's law, the induced voltages across the windings of Figure 3.1 are

$$E_1 = N_1(j\omega)\Phi_c \tag{3.1.11}$$

$$E_2 = N_2(j\omega)\Phi_c \tag{3.1.12}$$

Dividing (3.1.11) by (3.1.12) yields

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{3.1.13}$$

or

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \tag{3.1.14}$$

The dots shown in Figure 3.2 indicate that the voltages E_1 and E_2, both of which have their + polarities at the dotted terminals, are in phase. If the polarity chosen for one of the voltages in Figure 3.1 were reversed, then E_1 would be 180° out of phase with E_2.

The turns ratio a_t is defined as follows:

$$a_t = \frac{N_1}{N_2} \tag{3.1.15}$$

Using a_t in (3.1.8) and (3.1.14), the basic relations for an ideal single-phase two-winding transformer are

$$E_1 = \left(\frac{N_1}{N_2}\right) E_2 = a_t E_2 \tag{3.1.16}$$

$$I_1 = \left(\frac{N_2}{N_1}\right) I_2 = \frac{I_2}{a_t} \tag{3.1.17}$$

Two additional relations concerning complex power and impedance can be derived from (3.1.16) and (3.1.17) as follows. The complex power entering winding 1 in Figure 3.2 is

$$S_1 = E_1 I_1^* \tag{3.1.18}$$

Using (3.1.16) and (3.1.17),

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2 \tag{3.1.19}$$

As shown by (3.1.19), the complex power S_1 entering winding 1 equals the complex power S_2 leaving winding 2. That is, an ideal transformer has no real or reactive power loss.

If an impedance Z_2 is connected across winding 2 of the ideal transformer in Figure 3.2, then

$$Z_2 = \frac{E_2}{I_2} \tag{3.1.20}$$

This impedance, when measured from winding 1, is

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \tag{3.1.21}$$

Thus, the impedance Z_2 connected to winding 2 is referred to winding 1 by multiplying Z_2 by a_t^2, the square of the turns ratio.

EXAMPLE 3.1 Ideal, single-phase two-winding transformer

A single-phase two-winding transformer is rated 20 kVA, 480/120 V, 60 Hz. A source connected to the 480-V winding supplies an impedance load connected to the 120-V winding. The load absorbs 15 kVA at 0.8 p.f. lagging

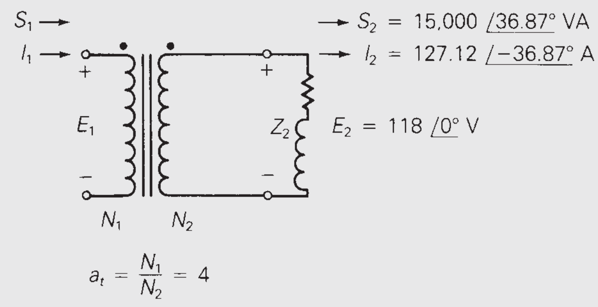


FIGURE 3.3

Circuit for Example 3.1

when the load voltage is 118 V. Assume that the transformer is ideal and calculate the following:

a. The voltage across the 480-V winding.

b. The load impedance.

c. The load impedance referred to the 480-V winding.

d. The real and reactive power supplied to the 480-V winding.

SOLUTION

a. The circuit is shown in Figure 3.3, where winding 1 denotes the 480-V winding and winding 2 denotes the 120-V winding. Selecting the load voltage E_2 as the reference,

$$E_2 = 118\underline{/0°} \quad \text{V}$$

The turns ratio is, from (3.1.13),

$$a_t = \frac{N_1}{N_2} = \frac{\mathrm{E}_{1\text{rated}}}{\mathrm{E}_{2\text{rated}}} = \frac{480}{120} = 4$$

and the voltage across winding 1 is

$$E_1 = a_t E_2 = 4(118\underline{/0°}) = 472\underline{/0°} \quad \text{V}$$

b. The complex power S_2 absorbed by the load is

$$S_2 = E_2 I_2^* = 118 I_2^* = 15{,}000\underline{/\cos^{-1}(0.8)} = 15{,}000\underline{/36.87°} \quad \text{VA}$$

Solving, the load current I_2 is

$$I_2 = 127.12\underline{/-36.87°} \quad \text{A}$$

The load impedance Z_2 is

$$Z_2 = \frac{E_2}{I_2} = \frac{118\underline{/0°}}{127.12\underline{/-36.87°}} = 0.9283\underline{/36.87°} \quad \Omega$$

c. From (3.1.21), the load impedance referred to the 480-V winding is

$$Z_2' = a_t^2 Z_2 = (4)^2(0.9283\underline{/36.87^\circ}) = 14.85\underline{/36.87^\circ} \quad \Omega$$

d. From (3.1.19)

$$S_1 = S_2 = 15{,}000\underline{/36.87^\circ} = 12{,}000 + j9000$$

Thus, the real and reactive powers supplied to the 480-V winding are

$$\mathrm{P}_1 = \mathrm{Re}\ S_1 = 12{,}000\ \mathrm{W} = 12\ \mathrm{kW}$$

$$\mathrm{Q}_1 = \mathrm{Im}\ S_1 = 9000\ \mathrm{var} = 9\ \mathrm{kvar}$$

■

Figure 3.4 shows a schematic of a conceptual single-phase, phase-shifting transformer. This transformer is not an idealization of an actual transformer since it is physically impossible to obtain a complex turns ratio. It will be used later in this chapter as a mathematical model for representing phase shift of three-phase transformers. As shown in Figure 3.4, the complex turns ratio a_t is defined for the phase-shifting transformer as

$$a_t = \frac{e^{j\phi}}{1} = e^{j\phi} \tag{3.1.22}$$

where ϕ is the phase-shift angle. The transformer relations are then

$$E_1 = a_t E_2 = e^{j\phi} E_2 \tag{3.1.23}$$

$$I_1 = \frac{I_2}{a_t^*} = e^{j\phi} I_2 \tag{3.1.24}$$

Note that the phase angle of E_1 leads the phase angle of E_2 by ϕ. Similarly, I_1 leads I_2 by the angle ϕ. However, the magnitudes are unchanged; that is, $|E_1| = |E_2|$ and $|I_1| = |I_2|$.

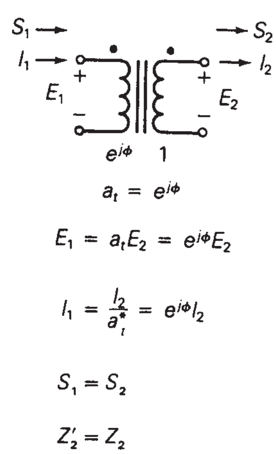


FIGURE 3.4 Schematic representation of a conceptual single-phase, phase-shifting transformer

From these two relations, the following two additional relations are derived:

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t^*}\right)^* = E_2 I_2^* = S_2 \tag{3.1.25}$$

$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{\dfrac{1}{a_t^*} I_2} = |a_t|^2 Z_2 = Z_2 \tag{3.1.26}$$

Thus, impedance is unchanged when it is referred from one side of an ideal phase-shifting transformer to the other. Also, the ideal phase-shifting transformer has no real or reactive power losses since $S_1 = S_2$.

Note that (3.1.23) and (3.1.24) for the phase-shifting transformer are the same as (3.1.16) and (3.1.17) for the ideal physical transformer except for the complex conjugate (*) in (3.1.24). The complex conjugate for the phase-shifting transformer is required to make $S_1 = S_2$ (complex power into winding 1 equals complex power out of winding 2), as shown in (3.1.25).

3.2 EQUIVALENT CIRCUITS FOR PRACTICAL TRANSFORMERS

Figure 3.5 shows an equivalent circuit for a practical single-phase two-winding transformer, which differs from the ideal transformer as follows:

1. The windings have resistance.
2. The core permeability μ_c is finite.
3. The magnetic flux is not entirely confined to the core.
4. There are real and reactive power losses in the core.

The resistance R_1 is included in series with winding 1 of the figure to account for I^2R losses in this winding. A reactance X_1, called the leakage

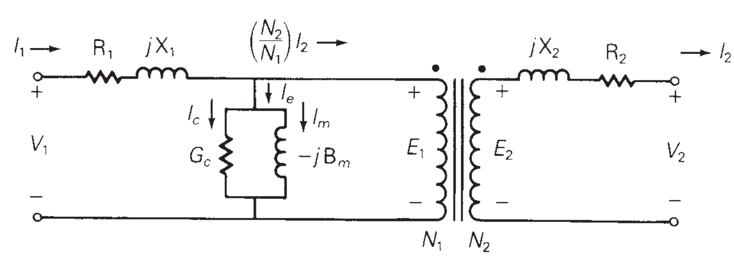


FIGURE 3.5 Equivalent circuit of a practical single-phase two-winding transformer

reactance of winding 1, is also included in series with winding 1 to account for the leakage flux of winding 1. This leakage flux is the component of the flux that links winding 1 but does not link winding 2; it causes a voltage drop $I_1(j\mathrm{X}_1)$, which is proportional to I_1 and leads I_1 by 90°. There is also a reactive power loss $\mathrm{I}_1^2\mathrm{X}_1$ associated with this leakage reactance. Similarly, there is a resistance R_2 and a leakage reactance X_2 in series with winding 2.

Equation (3.1.7) shows that for finite core permeability μ_c, the total mmf is not 0. Dividing (3.1.7) by N_1 and using (3.1.11), we get

$$I_1 - \left(\frac{N_2}{N_1}\right)I_2 = \frac{\mathrm{R}_c}{N_1}\Phi_c = \frac{\mathrm{R}_c}{N_1}\left(\frac{E_1}{j\omega N_1}\right) = -j\left(\frac{\mathrm{R}_c}{\omega N_1^2}\right)E_1 \tag{3.2.1}$$

Defining the term on the right-hand side of (3.2.1) to be I_m, called *magnetizing* current, it is evident that I_m lags E_1 by 90°, and can be represented by a shunt inductor with susceptance $\mathrm{B}_m = \left(\dfrac{\mathrm{R}_c}{\omega N_1^2}\right)$ mhos.* However, in reality there is an additional shunt branch, represented by a resistor with conductance G_c mhos, which carries a current I_c, called the *core loss* current. I_c is in phase with E_1. When the core loss current I_c is included, (3.2.1) becomes

$$I_1 - \left(\frac{N_2}{N_1}\right)I_2 = I_c + I_m = (\mathrm{G}_c - j\mathrm{B}_m)E_1 \tag{3.2.2}$$

The equivalent circuit of Figure 3.5, which includes the shunt branch with admittance $(\mathrm{G}_c - j\mathrm{B}_m)$ mhos, satisfies the KCL equation (3.2.2). Note that when winding 2 is open ($I_2 = 0$) and when a sinusoidal voltage V_1 is applied to winding 1, then (3.2.2) indicates that the current I_1 will have two components: the core loss current I_c and the magnetizing current I_m. Associated with I_c is a real power loss $\mathrm{I}_c^2/\mathrm{G}_c = \mathrm{E}_1^2\mathrm{G}_c$ W. This real power loss accounts for both hysteresis and eddy current losses within the core. Hysteresis loss occurs because a cyclic variation of flux within the core requires energy dissipated as heat. As such, hysteresis loss can be reduced by the use of special high grades of alloy steel as core material. Eddy current loss occurs because induced currents called eddy currents flow within the magnetic core perpendicular to the flux. As such, eddy current loss can be reduced by constructing the core with laminated sheets of alloy steel. Associated with I_m is a reactive power loss $\mathrm{I}_m^2/\mathrm{B}_m = \mathrm{E}_1^2\mathrm{B}_m$ var. This reactive power is required to magnetize the core. The phasor sum $(I_c + I_m)$ is called the *exciting* current I_e.

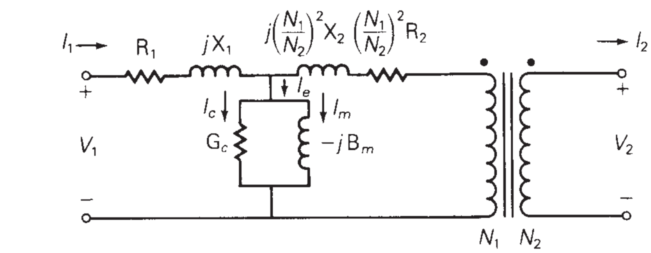
Figure 3.6 shows three alternative equivalent circuits for a practical single-phase two-winding transformer. In Figure 3.6(a), the resistance R_2 and leakage reactance X_2 of winding 2 are referred to winding 1 via (3.1.21).

---

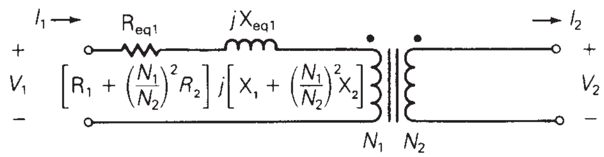
*The units of admittance, conductance, and susceptance, which in the SI system are siemens (with symbol S), are also called mhos (with symbol ℧) or ohms^{-1} (with symbol Ω^{-1}).

FIGURE 3.6

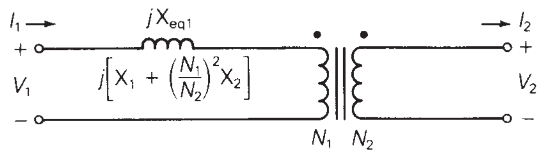
Equivalent circuits for a practical single-phase two-winding transformer



(a) R_2 and X_2 are referred to winding 1



(b) Neglecting exciting current



(c) Neglecting exciting current and I^2R winding loss

In Figure 3.6(b), the shunt branch is omitted, which corresponds to neglecting the exciting current. Since the exciting current is usually less than 5% of rated current, neglecting it in power system studies is often valid unless transformer efficiency or exciting current phenomena are of particular concern. For large power transformers rated more than 500 kVA, the winding resistances, which are small compared to the leakage reactances, can often be neglected, as shown in Figure 3.6(c).

Thus, a practical transformer operating in sinusoidal steady state is equivalent to an ideal transformer with external impedance and admittance branches, as shown in Figure 3.6. The external branches can be evaluated from short-circuit and open-circuit tests, as illustrated by the following example.

EXAMPLE 3.2 Transformer short-circuit and open-circuit tests

A single-phase two-winding transformer is rated 20 kVA, 480/120 volts, 60 Hz. During a short-circuit test, where rated current at rated frequency is applied to the 480-volt winding (denoted winding 1), with the 120-volt winding (winding 2) shorted, the following readings are obtained: $V_1 = 35$ volts, $P_1 = 300$ W. During an open-circuit test, where rated voltage is applied to winding 2, with winding 1 open, the following readings are obtained: $I_2 = 12$ A, $P_2 = 200$ W.

a. From the short-circuit test, determine the equivalent series impedance $Z_{eq1} = R_{eq1} + jX_{eq1}$ referred to winding 1. Neglect the shunt admittance.

b. From the open-circuit test, determine the shunt admittance $Y_m = G_c - jB_m$ referred to winding 1. Neglect the series impedance.

SOLUTION

a. The equivalent circuit for the short-circuit test is shown in Figure 3.7(a), where the shunt admittance branch is neglected. Rated current for winding 1 is

$$I_{1\text{rated}} = \frac{S_{\text{rated}}}{V_{1\text{rated}}} = \frac{20 \times 10^3}{480} = 41.667 \quad \text{A}$$

R_{eq1}, Z_{eq1}, and X_{eq1} are then determined as follows:

$$R_{eq1} = \frac{P_1}{I_{1\text{rated}}^2} = \frac{300}{(41.667)^2} = 0.1728 \quad \Omega$$

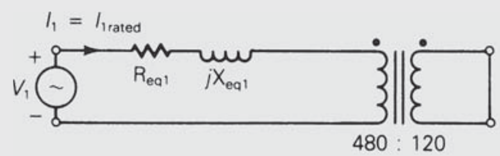
$$|Z_{eq1}| = \frac{V_1}{I_{1\text{rated}}} = \frac{35}{41.667} = 0.8400 \quad \Omega$$

$$X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = 0.8220 \quad \Omega$$

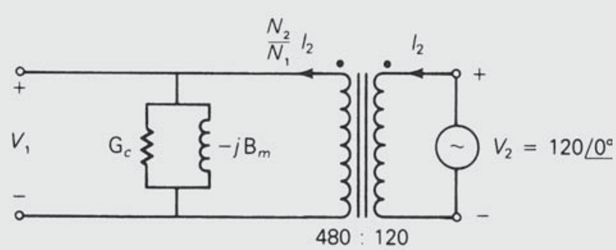
$$Z_{eq1} = R_{eq1} + jX_{eq1} = 0.1728 + j0.8220 = 0.8400\underline{/78.13^\circ} \quad \Omega$$

FIGURE 3.7

Circuits for Example 3.2



(a) Short-circuit test (neglecting shunt admittance)



(b) Open-circuit test (neglecting series impedance)

b. The equivalent circuit for the open-circuit test is shown in Figure 3.7(b), where the series impedance is neglected. From (3.1.16),

$$V_1 = E_1 = a_t E_2 = \frac{N_1}{N_2} V_{2\text{rated}} = \frac{480}{120}(120) = 480 \text{ volts}$$

G_c, Y_m, and B_m are then determined as follows:

$$G_c = \frac{P_2}{V_1^2} = \frac{200}{(480)^2} = 0.000868 \quad \text{S}$$

$$|Y_m| = \frac{\left(\frac{N_2}{N_1}\right) I_2}{V_1} = \frac{\left(\frac{120}{480}\right)(12)}{480} = 0.00625 \quad \text{S}$$

$$B_m = \sqrt{Y_m^2 - G_c^2} = \sqrt{(0.00625)^2 - (0.000868)^2} = 0.00619 \quad \text{S}$$

$$Y_m = G_c - jB_m = 0.000868 - j0.00619 = 0.00625\underline{/-82.02^\circ} \quad \text{S}$$

Note that the equivalent series impedance is usually evaluated at rated current from a short-circuit test, and the shunt admittance is evaluated at rated voltage from an open-circuit test. For small variations in transformer operation near rated conditions, the impedance and admittance values are often assumed constant. ■

The following are not represented by the equivalent circuit of Figure 3.5:

1. Saturation
2. Inrush current
3. Nonsinusoidal exciting current
4. Surge phenomena

They are briefly discussed in the following sections.

SATURATION

In deriving the equivalent circuit of the ideal and practical transformers, we have assumed constant core permeability μ_c and the linear relationship $B_c = \mu_c H_c$ of (3.1.3). However, the relationship between B and H for ferromagnetic materials used for transformer cores is nonlinear and multivalued. Figure 3.8 shows a set of B–H curves for a grain-oriented electrical steel typically used in transformers. As shown, each curve is multivalued, which is caused by hysteresis. For many engineering applications, the B–H curves can be adequately described by the dashed line drawn through the curves in Figure 3.8. Note that as H increases, the core becomes saturated; that is, the curves flatten out as B increases above 1 Wb/m^2. If the magnitude of the voltage applied to a transformer is too large, the core will saturate and a high

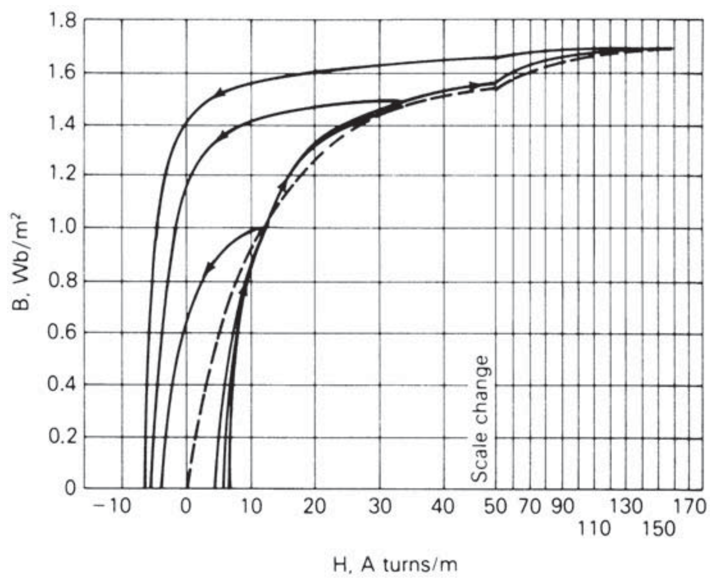


FIGURE 3.8

B–H curves for M-5 grain-oriented electrical steel 0.012 in. (0.305 mm) thick (Reprinted with permission of AK Steel Corporation)

magnetizing current will flow. In a well-designed transformer, the applied peak voltage causes the peak flux density in steady state to occur at the knee of the B–H curve, with a corresponding low value of magnetizing current.

INRUSH CURRENT

When a transformer is first energized, a transient current much larger than rated transformer current can flow for several cycles. This current, called *inrush current*, is nonsinusoidal and has a large dc component. To understand the cause of inrush, assume that before energization, the transformer core is magnetized with a residual flux density $B(0) = 1.5\ \mathrm{Wb/m^2}$ (near the knee of the dotted curve in Figure 3.8). If the transformer is then energized when the source voltage is positive and increasing, Faraday's law, (3.1.9), will cause the flux density $B(t)$ to increase further, since

$$B(t) = \frac{\phi(t)}{A} = \frac{1}{NA}\int_0^t e(t)\,dt + B(0)$$

As $B(t)$ moves into the saturation region of the B–H curve, large values of $H(t)$ will occur, and, from Ampere's law, (3.1.1), corresponding large values of current $i(t)$ will flow for several cycles until it has dissipated. Since normal inrush currents can be as large as abnormal short-circuit currents in transformers, transformer protection schemes must be able to distinguish between these two types of currents.

NONSINUSOIDAL EXCITING CURRENT

When a sinusoidal voltage is applied to one winding of a transformer with the other winding open, the flux $\phi(t)$ and flux density $B(t)$ will, from Faraday's law, (3.1.9), be very nearly sinusoidal in steady state. However, the magnetic field intensity $H(t)$ and the resulting exciting current will not be sinusoidal in steady state, due to the nonlinear B–H curve. If the exciting current is measured and analyzed by Fourier analysis techniques, one finds that it has a fundamental component and a set of odd harmonics. The principal harmonic is the third, whose rms value is typically about 40% of the total rms exciting current. However, the nonsinusoidal nature of exciting current is usually neglected unless harmonic effects are of direct concern, because the exciting current itself is usually less than 5% of rated current for power transformers.

SURGE PHENOMENA

When power transformers are subjected to transient overvoltages caused by lightning or switching surges, the capacitances of the transformer windings have important effects on transient response. Transformer winding capacitances and response to surges are discussed in Chapter 12.

3.3 THE PER-UNIT SYSTEM

Power-system quantities such as voltage, current, power, and impedance are often expressed in per-unit or percent of specified base values. For example, if a base voltage of 20 kV is specified, then the voltage 18 kV is $(18/20) = 0.9$ per unit or 90%. Calculations can then be made with per-unit quantities rather than with the actual quantities.

One advantage of the per-unit system is that by properly specifying base quantities, the transformer equivalent circuit can be simplified. The ideal transformer winding can be eliminated, such that voltages, currents, and external impedances and admittances expressed in per-unit do not change when they are referred from one side of a transformer to the other. This can be a significant advantage even in a power system of moderate size, where hundreds of transformers may be encountered. The per-unit system allows us to avoid the possibility of making serious calculation errors when referring quantities from one side of a transformer to the other. Another advantage of the per-unit system is that the per-unit impedances of electrical equipment of similar type usually lie within a narrow numerical range when the equipment ratings are used as base values. Because of this, per-unit impedance data can be checked rapidly for gross errors by someone familiar with per-unit quantities. In addition, manufacturers usually specify the impedances of machines and transformers in per-unit or percent of nameplate rating.

Per-unit quantities are calculated as follows:

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}} \tag{3.3.1}$$

where *actual quantity* is the value of the quantity in the actual units. The base value has the same units as the actual quantity, thus making the per-unit quantity dimensionless. Also, the base value is always a real number. Therefore, the angle of the per-unit quantity is the same as the angle of the actual quantity.

Two independent base values can be arbitrarily selected at one point in a power system. Usually the base voltage V_{baseLN} and base complex power $S_{base1\phi}$ are selected for either a single-phase circuit or for one phase of a three-phase circuit. Then, in order for electrical laws to be valid in the per-unit system, the following relations must be used for other base values:

$$P_{base1\phi} = Q_{base1\phi} = S_{base1\phi} \tag{3.3.2}$$

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}} \tag{3.3.3}$$

$$Z_{base} = R_{base} = X_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V^2_{baseLN}}{S_{base1\phi}} \tag{3.3.4}$$

$$Y_{base} = G_{base} = B_{base} = \frac{1}{Z_{base}} \tag{3.3.5}$$

In (3.3.2)–(3.3.5) the subscripts LN and 1ϕ denote "line-to-neutral" and "per-phase," respectively, for three-phase circuits. These equations are also valid for single-phase circuits, where subscripts can be omitted.

By convention, we adopt the following two rules for base quantities:

1. The value of $S_{base1\phi}$ is the same for the entire power system of concern.
2. The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.

With these two rules, a per-unit impedance remains unchanged when referred from one side of a transformer to the other.

EXAMPLE 3.3 Per-unit impedance: single-phase transformer

A single-phase two-winding transformer is rated 20 kVA, 480/120 volts, 60 Hz. The equivalent leakage impedance of the transformer referred to the 120-volt winding, denoted winding 2, is $Z_{eq2} = 0.0525\underline{/78.13^\circ}\ \Omega$. Using the transformer ratings as base values, determine the per-unit leakage impedance referred to winding 2 and referred to winding 1.

SOLUTION The values of S_{base}, V_{base1}, and V_{base2} are, from the transformer ratings,

$$S_{base} = 20 \text{ kVA}, \qquad V_{base1} = 480 \text{ volts}, \qquad V_{base2} = 120 \text{ volts}$$

Using (3.3.4), the base impedance on the 120-volt side of the transformer is

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{(120)^2}{20{,}000} = 0.72 \quad \Omega$$

Then, using (3.3.1), the per-unit leakage impedance referred to winding 2 is

$$Z_{eq2p.u.} = \frac{Z_{eq2}}{Z_{base2}} = \frac{0.0525\underline{/78.13^\circ}}{0.72} = 0.0729\underline{/78.13^\circ} \quad \text{per unit}$$

If Z_{eq2} is referred to winding 1,

$$Z_{eq1} = a_t^2 Z_{eq2} = \left(\frac{N_1}{N_2}\right)^2 Z_{eq2} = \left(\frac{480}{120}\right)^2 (0.0525\underline{/78.13^\circ})$$

$$= 0.84\underline{/78.13^\circ} \quad \Omega$$

The base impedance on the 480-volt side of the transformer is

$$Z_{base1} = \frac{V_{base1}^2}{S_{base}} = \frac{(480)^2}{20{,}000} = 11.52 \quad \Omega$$

and the per-unit leakage reactance referred to winding 1 is

$$Z_{eq1p.u.} = \frac{Z_{eq1}}{Z_{base1}} = \frac{0.84\underline{/78.13^\circ}}{11.52} = 0.0729\underline{/78.13^\circ} \text{ per unit} = Z_{eq2p.u.}$$

Thus, the *per-unit* leakage impedance remains unchanged when referred from winding 2 to winding 1. This has been achieved by specifying

$$\frac{V_{base1}}{V_{base2}} = \frac{V_{rated1}}{V_{rated2}} = \left(\frac{480}{120}\right) \qquad \blacksquare$$

Figure 3.9 shows three per-unit circuits of a single-phase two-winding transformer. The ideal transformer, shown in Figure 3.9(a), satisfies the per-unit relations $E_{1p.u.} = E_{2p.u.}$, and $I_{1p.u.} = I_{2p.u.}$, which can be derived as follows. First divide (3.1.16) by V_{base1}:

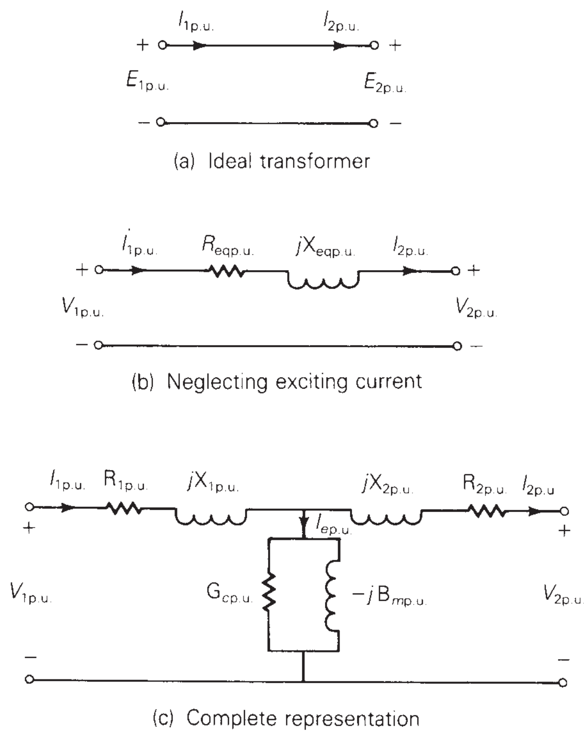
$$E_{1p.u.} = \frac{E_1}{V_{base1}} = \frac{N_1}{N_2} \times \frac{E_2}{V_{base1}} \tag{3.3.6}$$

Then, using $V_{base1}/V_{base2} = V_{rated1}/V_{rated2} = N_1/N_2$,

$$E_{1p.u.} = \frac{N_1}{N_2} \frac{E_2}{\left(\frac{N_1}{N_2}\right) V_{base2}} = \frac{E_2}{V_{base2}} = E_{2p.u.} \tag{3.3.7}$$

FIGURE 3.9

Per-unit equivalent circuits of a single-phase two-winding transformer



Similarly, divide (3.1.17) by $\mathrm{I}_{\text{base1}}$:

$$I_{1\text{p.u.}} = \frac{I_1}{\mathrm{I}_{\text{base1}}} = \frac{N_2}{N_1}\frac{I_2}{\mathrm{I}_{\text{base1}}} \tag{3.3.8}$$

Then, using $\mathrm{I}_{\text{base1}} = \mathrm{S}_{\text{base}}/\mathrm{V}_{\text{base1}} = \mathrm{S}_{\text{base}}/[(N_1/N_2)\mathrm{V}_{\text{base2}}] = (N_2/N_1)\mathrm{I}_{\text{base2}}$,

$$I_{1\text{p.u.}} = \frac{N_2}{N_1}\frac{I_2}{\left(\dfrac{N_2}{N_1}\right)\mathrm{I}_{\text{base2}}} = \frac{I_2}{\mathrm{I}_{\text{base2}}} = I_{2\text{p.u.}} \tag{3.3.9}$$

Thus, the ideal transformer winding in Figure 3.2 is eliminated from the per-unit circuit in Figure 3.9(a). The per-unit leakage impedance is included in Figure 3.9(b), and the per-unit shunt admittance branch is added in Figure 3.9(c) to obtain the complete representation.

When only one component, such as a transformer, is considered, the nameplate ratings of that component are usually selected as base values. When several components are involved, however, the system base values may be different from the nameplate ratings of any particular device. It is then necessary to convert the per-unit impedance of a device from its nameplate ratings to the system base values. To convert a per-unit impedance from "old" to "new" base values, use

$$Z_{\text{p.u.new}} = \frac{Z_{\text{actual}}}{Z_{\text{basenew}}} = \frac{Z_{\text{p.u.old}} Z_{\text{baseold}}}{Z_{\text{basenew}}} \tag{3.3.10}$$

or, from (3.3.4),

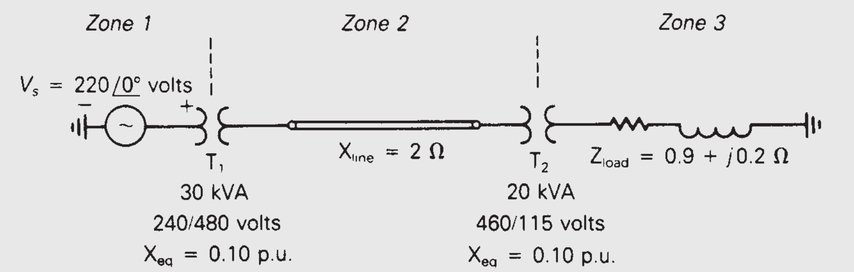
$$Z_{\text{p.u.new}} = Z_{\text{p.u.old}} \left(\frac{V_{\text{baseold}}}{V_{\text{basenew}}}\right)^2 \left(\frac{S_{\text{basenew}}}{S_{\text{baseold}}}\right) \tag{3.3.11}$$

EXAMPLE 3.4 Per-unit circuit: three-zone single-phase network

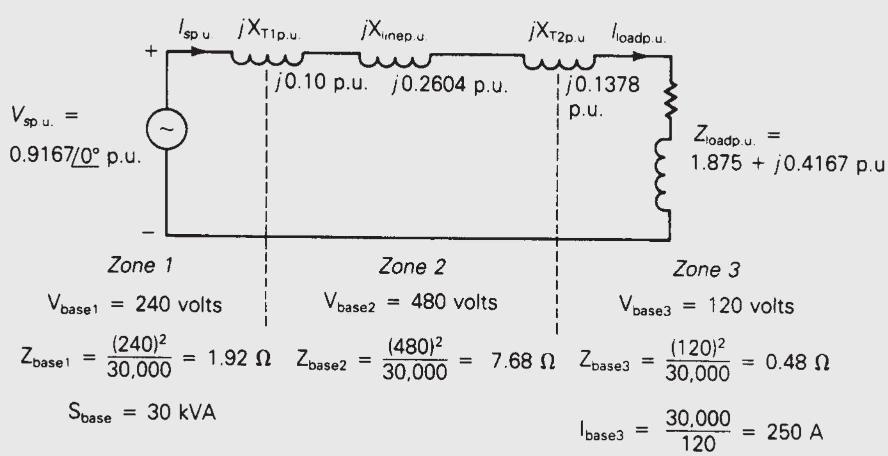
Three zones of a single-phase circuit are identified in Figure 3.10(a). The zones are connected by transformers T_1 and T_2, whose ratings are also shown. Using base values of 30 kVA and 240 volts in zone 1, draw the per-unit circuit and

FIGURE 3.10

Circuits for Example 3.4



(a) Single-phase circuit



(b) Per-unit circuit

determine the per-unit impedances and the per-unit source voltage. Then calculate the load current both in per-unit and in amperes. Transformer winding resistances and shunt admittance branches are neglected.

SOLUTION First the base values in each zone are determined. $S_{base} =$ 30 kVA is the same for the entire network. Also, $V_{base1} = 240$ volts, as specified for zone 1. When moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings. Thus,

$$V_{base2} = \left(\frac{480}{240}\right)(240) = 480 \quad \text{volts}$$

and

$$V_{base3} = \left(\frac{115}{460}\right)(480) = 120 \quad \text{volts}$$

The base impedances in zones 2 and 3 are

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30{,}000} = 7.68 \quad \Omega$$

and

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30{,}000} = 0.48 \quad \Omega$$

and the base current in zone 3 is

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30{,}000}{120} = 250 \quad \text{A}$$

Next, the per-unit circuit impedances are calculated using the system base values. Since $S_{base} = 30$ kVA is the same as the kVA rating of transformer T_1, and $V_{base1} = 240$ volts is the same as the voltage rating of the zone 1 side of transformer T_1, the per-unit leakage reactance of T_1 is the same as its nameplate value, $X_{T1p.u.} = 0.1$ per unit. However, the per-unit leakage reactance of transformer T_2 must be converted from its nameplate rating to the system base. Using (3.3.11) and $V_{base2} = 480$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

Alternatively, using $V_{base3} = 120$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

which gives the same result. The line, which is located in zone 2, has a per-unit reactance

$$X_{linep.u.} = \frac{X_{line}}{Z_{base2}} = \frac{2}{7.68} = 0.2604 \quad \text{per unit}$$

and the load, which is located in zone 3, has a per-unit impedance

$$Z_{\text{loadp.u.}} = \frac{Z_{\text{load}}}{Z_{\text{base3}}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \quad \text{per unit}$$

The per-unit circuit is shown in Figure 3.10(b), where the base values for each zone, per-unit impedances, and the per-unit source voltage are shown. The per-unit load current is then easily calculated from Figure 3.10(b) as follows:

$$\begin{aligned} I_{\text{loadp.u.}} = I_{\text{sp.u.}} &= \frac{V_{\text{sp.u.}}}{j(X_{\text{T1p.u.}} + X_{\text{linep.u.}} + X_{\text{T2p.u.}}) + Z_{\text{loadp.u.}}} \\ &= \frac{0.9167\angle 0^\circ}{j(0.10 + 0.2604 + 0.1378) + (1.875 + j0.4167)} \\ &= \frac{0.9167\angle 0^\circ}{1.875 + j0.9149} = \frac{0.9167\angle 0^\circ}{2.086\angle 26.01^\circ} \\ &= 0.4395\angle -26.01^\circ \quad \text{per unit} \end{aligned}$$

The actual load current is

$$I_{\text{load}} = (I_{\text{loadp.u.}})I_{\text{base3}} = (0.4395\angle -26.01^\circ)(250) = 109.9\angle -26.01^\circ \quad \text{A}$$

Note that the per-unit equivalent circuit of Figure 3.10(b) is relatively easy to analyze, since ideal transformer windings have been eliminated by proper selection of base values. ■

Balanced three-phase circuits can be solved in per-unit on a per-phase basis after converting Δ-load impedances to equivalent Y impedances. Base values can be selected either on a per-phase basis or on a three-phase basis. Equations (3.3.1)–(3.3.5) remain valid for three-phase circuits on a per-phase basis. Usually $S_{\text{base}3\phi}$ and V_{baseLL} are selected, where the subscripts 3ϕ and LL denote "three-phase" and "line-to-line," respectively. Then the following relations must be used for other base values:

$$S_{\text{base}1\phi} = \frac{S_{\text{base}3\phi}}{3} \tag{3.3.12}$$

$$V_{\text{baseLN}} = \frac{V_{\text{baseLL}}}{\sqrt{3}} \tag{3.3.13}$$

$$S_{\text{base}3\phi} = P_{\text{base}3\phi} = Q_{\text{base}3\phi} \tag{3.3.14}$$

$$I_{\text{base}} = \frac{S_{\text{base}1\phi}}{V_{\text{baseLN}}} = \frac{S_{\text{base}3\phi}}{\sqrt{3}V_{\text{baseLL}}} \tag{3.3.15}$$

$$Z_{\text{base}} = \frac{V_{\text{baseLN}}}{I_{\text{base}}} = \frac{V_{\text{baseLN}}^2}{S_{\text{base}1\phi}} = \frac{V_{\text{baseLL}}^2}{S_{\text{base}3\phi}} \tag{3.3.16}$$

$$R_{\text{base}} = X_{\text{base}} = Z_{\text{base}} = \frac{1}{Y_{\text{base}}} \tag{3.3.17}$$

EXAMPLE 3.5 Per-unit and actual currents in balanced three-phase networks

As in Example 2.5, a balanced-Y-connected voltage source with $E_{ab} = 480\underline{/0^\circ}$ volts is applied to a balanced-Δ load with $Z_\Delta = 30\underline{/40^\circ}\ \Omega$. The line impedance between the source and load is $Z_L = 1\underline{/85^\circ}\ \Omega$ for each phase. Calculate the per-unit and actual current in phase a of the line using $S_{base3\phi} = 10$ kVA and $V_{baseLL} = 480$ volts.

SOLUTION First, convert Z_Δ to an equivalent Z_Y; the equivalent line-to-neutral diagram is shown in Figure 2.17. The base impedance is, from (3.3.16),

$$Z_{base} = \frac{V^2_{baseLL}}{S_{base3\phi}} = \frac{(480)^2}{10{,}000} = 23.04 \quad \Omega$$

The per-unit line and load impedances are

$$Z_{Lp.u.} = \frac{Z_L}{Z_{base}} = \frac{1\underline{/85^\circ}}{23.04} = 0.04340\underline{/85^\circ} \quad \text{per unit}$$

and

$$Z_{Yp.u.} = \frac{Z_Y}{Z_{base}} = \frac{10\underline{/40^\circ}}{23.04} = 0.4340\underline{/40^\circ} \quad \text{per unit}$$

Also,

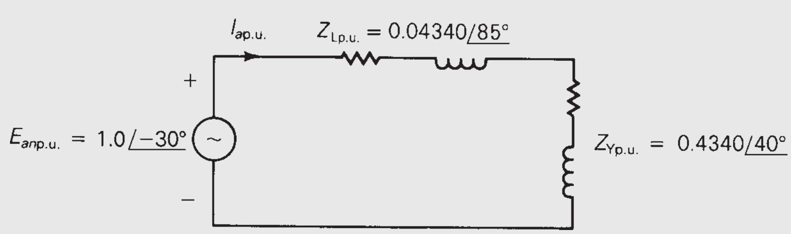
$$V_{baseLN} = \frac{V_{baseLL}}{\sqrt{3}} = \frac{480}{\sqrt{3}} = 277 \quad \text{volts}$$

and

$$E_{anp.u.} = \frac{E_{an}}{V_{baseLN}} = \frac{277\underline{/-30^\circ}}{277} = 1.0\underline{/-30^\circ} \quad \text{per unit}$$

The per-unit equivalent circuit is shown in Figure 3.11. The per-unit line current in phase a is then

FIGURE 3.11
Circuit for Example 3.5



$$I_{a\text{p.u.}} = \frac{E_{an\text{p.u.}}}{Z_{\text{Lp.u.}} + Z_{\text{Yp.u.}}} = \frac{1.0\underline{/-30^\circ}}{0.04340\underline{/85^\circ} + 0.4340\underline{/40^\circ}}$$

$$= \frac{1.0\underline{/-30^\circ}}{(0.00378 + j0.04323) + (0.3325 + j0.2790)}$$

$$= \frac{1.0\underline{/-30^\circ}}{0.3362 + j0.3222} = \frac{1.0\underline{/-30^\circ}}{0.4657\underline{/43.78^\circ}}$$

$$= 2.147\underline{/-73.78^\circ} \quad \text{per unit}$$

The base current is

$$\text{I}_{\text{base}} = \frac{\text{S}_{\text{base}3\phi}}{\sqrt{3}\text{V}_{\text{baseLL}}} = \frac{10{,}000}{\sqrt{3}(480)} = 12.03 \quad \text{A}$$

and the actual phase a line current is

$$I_a = (2.147\underline{/-73.78^\circ})(12.03) = 25.83\underline{/-73.78^\circ} \quad \text{A}$$

■

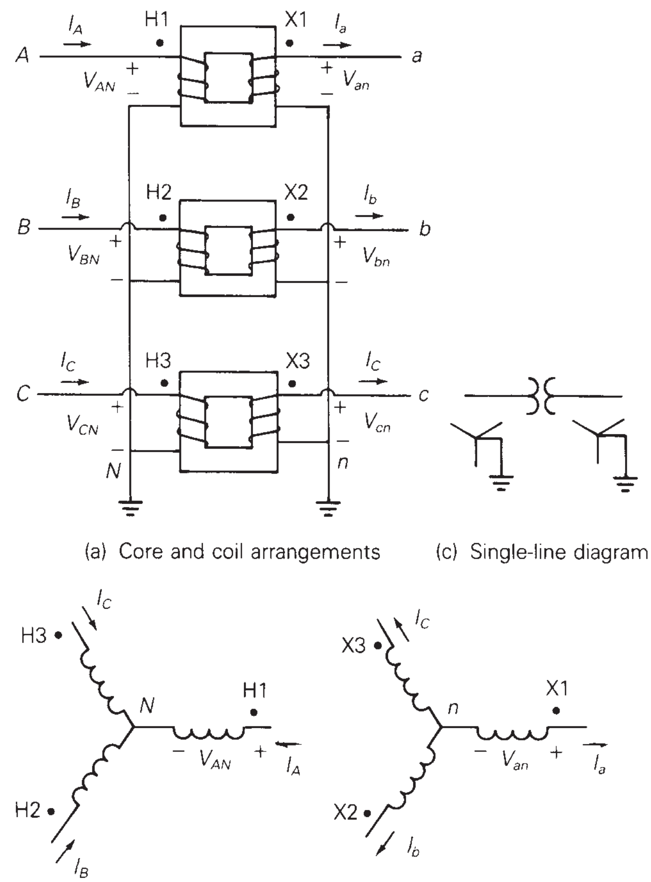
3.4 THREE-PHASE TRANSFORMER CONNECTIONS AND PHASE SHIFT

Three identical single-phase two-winding transformers may be connected to form a three-phase bank. Four ways to connect the windings are Y–Y, Y–Δ, Δ–Y, and Δ–Δ. For example, Figure 3.12 shows a three-phase Y–Y bank. Figure 3.12(a) shows the core and coil arrangements. The American standard for marking three-phase transformers substitutes H1, H2, and H3 on the high-voltage terminals and X1, X2, and X3 on the low-voltage terminals in place of the polarity dots. Also, in this text, we will use uppercase letters ABC to identify phases on the high-voltage side of the transformer and lowercase letters abc to identify phases on the low-voltage side of the transformer. In Figure 3.12(a) the transformer high-voltage terminals H1, H2, and H3 are connected to phases A, B, and C, and the low-voltage terminals X1, X2, and X3 are connected to phases a, b, and c, respectively.

Figure 3.12(b) shows a schematic representation of the three-phase Y–Y transformer. Windings on the same core are drawn in parallel, and the phasor relationship for balanced positive-sequence operation is shown. For example, high-voltage winding H1–N is on the same magnetic core as low-voltage winding X1–n in Figure 3.12(b). Also, V_{AN} is in phase with V_{an}. Figure 3.12(c) shows a single-line diagram of a Y–Y transformer. A single-line diagram shows one phase of a three-phase network with the neutral wire omitted and with components represented by symbols rather than equivalent circuits.

FIGURE 3.12

Three-phase two-winding Y–Y transformer bank



(a) Core and coil arrangements (c) Single-line diagram

(b) Schematic representation showing phasor relationship for positive sequence operation

The phases of a Y–Y or a Δ–Δ transformer can be labeled so there is no phase shift between corresponding quantities on the low- and high-voltage windings. However, for Y–Δ and Δ–Y transformers, there is always a phase shift. Figure 3.13 shows a Y–Δ transformer. The labeling of the windings and the schematic representation are in accordance with the American standard, which is as follows:

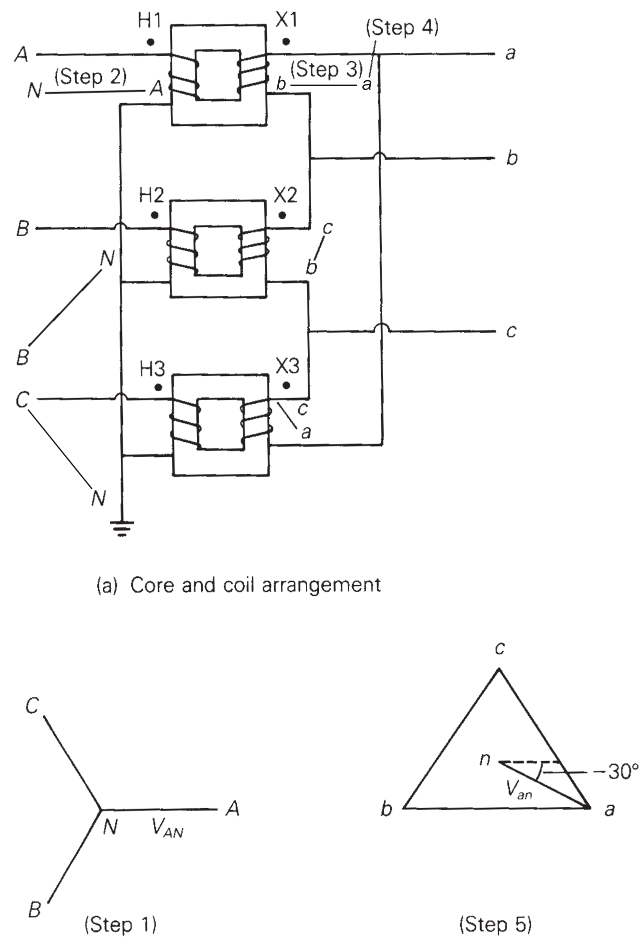
> In either a Y–Δ or Δ–Y transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by 30°.

As shown in Figure 3.13(b), V_{AN} leads V_{an} by 30°.

The positive-sequence phasor diagram shown in Figure 3.13(b) can be constructed via the following five steps, which are also indicated in Figure 3.13:

FIGURE 3.13

Three-phase two-winding Y–Δ transformer bank



(a) Core and coil arrangement

(b) Positive-sequence phasor diagram

STEP 1 Assume that balanced positive-sequence voltages are applied to the Y winding. Draw the positive-sequence phasor diagram for these voltages.

STEP 2 Move phasor *A–N* next to terminals *A–N* in Figure 3.13(a). Identify the ends of this line in the same manner as in the phasor diagram. Similarly, move phasors *B–N* and *C–N* next to terminals *B–N* and *C–N* in Figure 3.13(a).

STEP 3 For each single-phase transformer, the voltage across the low-voltage winding must be in phase with the voltage across the high-voltage winding, assuming an ideal transformer. Therefore, draw a line next to each low-voltage winding parallel to the corresponding line already drawn next to the high-voltage winding.

STEP 4 Label the ends of the lines drawn in Step 3 by inspecting the polarity marks. For example, phase A is connected to dotted terminal H1, and A appears on the *right* side of line A–N. Therefore, phase a, which is connected to dotted terminal X1, must be on the *right* side, and b on the left side of line a–b. Similarly, phase B is connected to dotted terminal H2, and B is *down* on line B–N. Therefore, phase b, connected to dotted terminal X2, must be *down* on line b–c. Similarly, c is *up* on line c–a.

STEP 5 Bring the three lines labeled in Step 4 together to complete the phasor diagram for the low-voltage Δ winding. Note that V_{AN} leads V_{an} by 30° in accordance with the American standard.

EXAMPLE 3.6 Phase shift in Δ–Y transformers

Assume that balanced negative-sequence voltages are applied to the high-voltage windings of the Y–Δ transformer shown in Figure 3.13. Determine the negative-sequence phase shift of this transformer.

SOLUTION The negative-sequence diagram, shown in Figure 3.14, is constructed from the following five steps, as outlined above:

STEP 1 Draw the phasor diagram of balanced negative-sequence voltages, which are applied to the Y winding.

STEP 2 Move the phasors A–N, B–N, and C–N next to the high-voltage Y windings.

STEP 3 For each single-phase transformer, draw a line next to the low-voltage winding that is parallel to the line drawn in Step 2 next to the high-voltage winding.

STEP 4 Label the lines drawn in Step 3. For example, phase B, which is connected to dotted terminal H2, is shown *up* on line B–N; therefore phase b, which is connected to dotted terminal X2, must be *up* on line b–c.

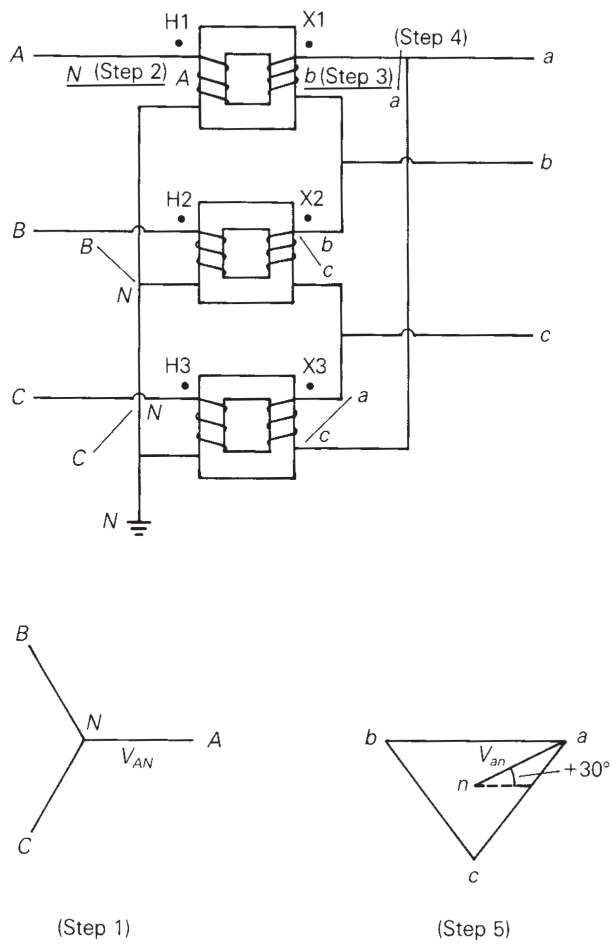
STEP 5 Bring the lines drawn in Step 4 together to form the negative-sequence phasor diagram for the low-voltage Δ winding.

As shown in Figure 3.14, the high-voltage phasors *lag* the low-voltage phasors by 30°. Thus the negative-sequence phase shift is the reverse of the positive-sequence phase shift. ■

The Δ–Y transformer is commonly used as a generator step-up transformer, where the Δ winding is connected to the generator terminals and the

FIGURE 3.14

Example 3.6—Construction of negative-sequence phasor diagram for Y–Δ transformer bank



Y winding is connected to a transmission line. One advantage of a high-voltage Y winding is that a neutral point N is provided for grounding on the high-voltage side. With a permanently grounded neutral, the insulation requirements for the high-voltage transformer windings are reduced. The high-voltage insulation can be graded or tapered from maximum insulation at terminals ABC to minimum insulation at grounded terminal N. One advantage of the Δ winding is that the undesirable third harmonic magnetizing current, caused by the nonlinear core B–H characteristic, remains trapped inside the Δ winding. Third harmonic currents are (triple-frequency) zero-sequence currents, which cannot enter or leave a Δ connection, but can flow within the Δ. The Y–Y transformer is seldom used because of difficulties with third harmonic exciting current.

The Δ–Δ transformer has the advantage that one phase can be removed for repair or maintenance while the remaining phases continue to operate as

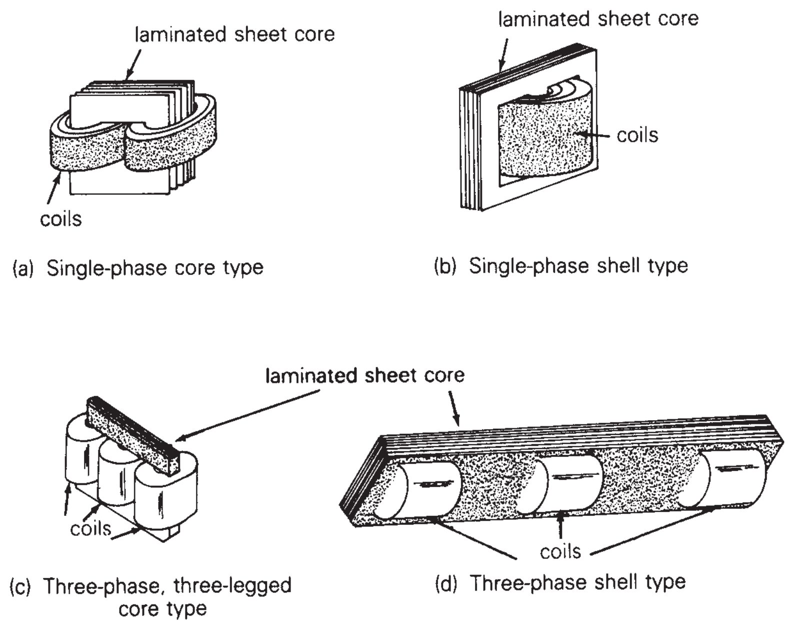


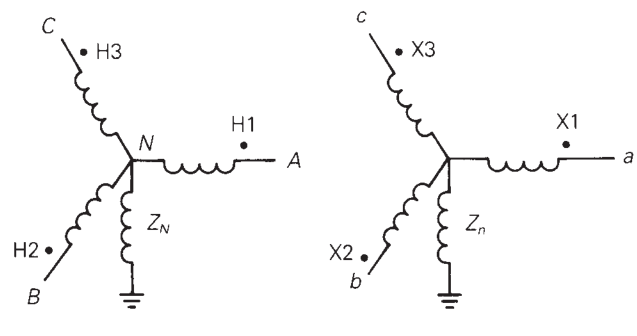
FIGURE 3.15
Transformer core configurations

a three-phase bank. This *open*-Δ connection permits balanced three-phase operation with the kVA rating reduced to 58% of the original bank (see Problem 3.36).

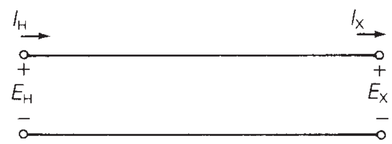
Instead of a bank of three single-phase transformers, all six windings may be placed on a common three-phase core to form a three-phase transformer, as shown in Figure 3.15. The three-phase core contains less iron than the three single-phase units; therefore it costs less, weighs less, requires less floor space, and has a slightly higher efficiency. However, a winding failure would require replacement of an entire three-phase transformer, compared to replacement of only one phase of a three-phase bank.

3.5 PER-UNIT EQUIVALENT CIRCUITS OF BALANCED THREE-PHASE TWO-WINDING TRANSFORMERS

Figure 3.16(a) is a schematic representation of an ideal Y–Y transformer grounded through neutral impedances Z_N and Z_n. Figure 3.16(b) shows the per-unit equivalent circuit of this ideal transformer for balanced three-phase operation. Throughout the remainder of this text, per-unit quantities will be used unless otherwise indicated. Also, the subscript "p.u.," used to indicate a per-unit quantity, will be omitted in most cases.



(a) Schematic representation



(b) Per-unit equivalent circuit for balanced three-phase operation

FIGURE 3.16 Ideal Y–Y transformer

By convention, we adopt the following two rules for selecting base quantities:

1. A common S_{base} is selected for both the H and X terminals.
2. The ratio of the voltage bases V_{baseH}/V_{baseX} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.

When balanced three-phase currents are applied to the transformer, the neutral currents are zero and there are no voltage drops across the neutral impedances. Therefore, the per-unit equivalent circuit of the ideal Y–Y transformer, Figure 3.16(b), is the same as the per-unit single-phase ideal transformer, Figure 3.9(a).

The per-unit equivalent circuit of a practical Y–Y transformer is shown in Figure 3.17(a). This network is obtained by adding external impedances to the equivalent circuit of the ideal transformer, as in Figure 3.9(c).

The per-unit equivalent circuit of the Y–Δ transformer, shown in Figure 3.17(b), includes a phase shift. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the Y–Δ transformer lead the corresponding quantities on the low-voltage side by 30°. The phase shift in the equivalent circuit of Figure 3.17(b) is represented by the phase-shifting transformer of Figure 3.4.

The per-unit equivalent circuit of the Δ–Δ transformer, shown in Figure 3.17(c), is the same as that of the Y–Y transformer. It is assumed that the windings are labeled so there is no phase shift. Also, the per-unit impedances do not depend on the winding connections, but the base voltages do.

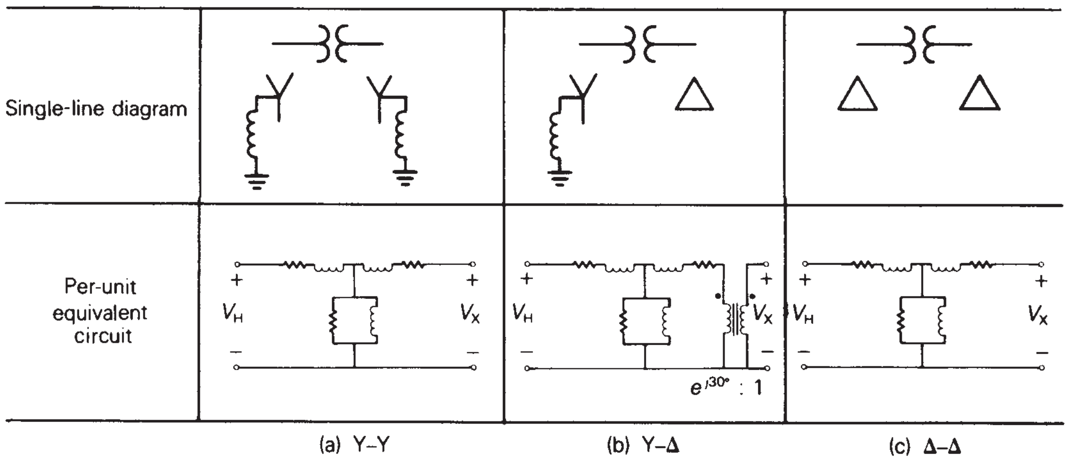


FIGURE 3.17 Per-unit equivalent circuits of practical Y–Y, Y–Δ, and Δ–Δ transformers for balanced three-phase operation

EXAMPLE 3.7 Voltage calculations: balanced Y–Y and Δ–Y transformers

Three single-phase two-winding transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance $X_{eq} = 0.10$ per unit, are connected to form a three-phase bank. Winding resistances and exciting current are neglected. The high-voltage windings are connected in Y. A three-phase load operating under balanced positive-sequence conditions on the high-voltage side absorbs 1000 MVA at 0.90 p.f. lagging, with $V_{AN} = 199.2\underline{/0°}$ kV. Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y, (b) in Δ.

SOLUTION The per-unit network is shown in Figure 3.18. Using the transformer bank ratings as base quantities, $S_{base3\phi} = 1200$ MVA, $V_{baseHLL} = 345$ kV, and $I_{baseH} = 1200/(345\sqrt{3}) = 2.008$ kA. The per-unit load voltage and load current are then

$$V_{AN} = 1.0\underline{/0°} \quad \text{per unit}$$

$$I_A = \frac{1000/(345\sqrt{3})}{2.008}\underline{/-\cos^{-1}0.9} = 0.8333\underline{/-25.84°} \quad \text{per unit}$$

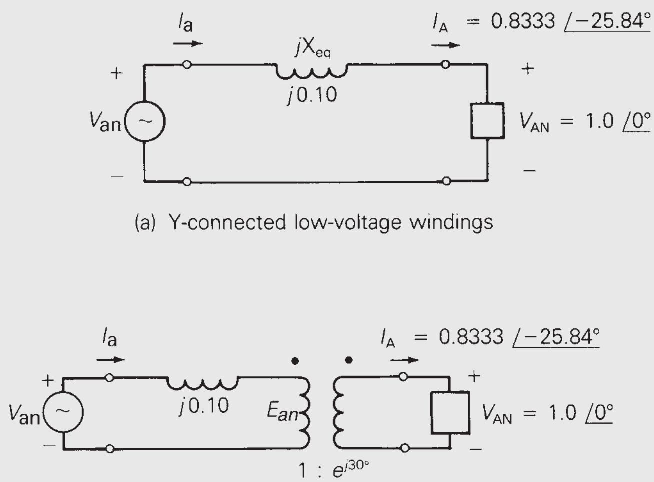
a. For the Y–Y transformer, Figure 3.18(a),

$$I_a = I_A = 0.8333\underline{/-25.84°} \quad \text{per unit}$$

$$\begin{aligned} V_{an} &= V_{AN} + (jX_{eq})I_A \\ &= 1.0\underline{/0°} + (j0.10)(0.8333\underline{/-25.84°}) \\ &= 1.0 + 0.08333\underline{/64.16°} = 1.0363 + j0.0750 = 1.039\underline{/4.139°} \\ &= 1.039\underline{/4.139°} \quad \text{per unit} \end{aligned}$$

FIGURE 3.18

Per-unit network for Example 3.7



(a) Y-connected low-voltage windings

(b) Δ-connected low-voltage windings

Further, since $V_{\text{baseXLN}} = 13.8$ kV for the low-voltage Y windings, $V_{an} = 1.039(13.8) = 14.34$ kV, and

$$V_{an} = 14.34\underline{/4.139^\circ} \quad \text{kV}$$

b. For the Δ–Y transformer, Figure 3.18(b),

$$E_{an} = e^{-j30^\circ} V_{AN} = 1.0\underline{/-30^\circ} \quad \text{per unit}$$

$$I_a = e^{-j30^\circ} I_A = 0.8333\underline{/-25.84^\circ - 30^\circ} = 0.8333\underline{/-55.84^\circ} \quad \text{per unit}$$

$$V_{an} = E_{an} + (j\mathrm{X}_{\text{eq}})I_a = 1.0\underline{/-30^\circ} + (j0.10)(0.8333\underline{/-55.84^\circ})$$

$$V_{an} = 1.039\underline{/-25.861^\circ} \quad \text{per unit}$$

Further, since $V_{\text{baseXLN}} = 13.8/\sqrt{3} = 7.967$ kV for the low-voltage Δ windings, $V_{an} = (1.039)(7.967) = 8.278$ kV, and

$$V_{an} = 8.278\underline{/-25.861^\circ} \quad \text{kV}$$

■

EXAMPLE 3.8 Per-unit voltage drop and per-unit fault current: balanced three-phase transformer

A 200-MVA, 345-kVΔ/34.5-kV Y substation transformer has an 8% leakage reactance. The transformer acts as a connecting link between 345-kV transmission and 34.5-kV distribution. Transformer winding resistances and exciting current are neglected. The high-voltage bus connected to the transformer is assumed to be an ideal 345-kV positive-sequence source with negligible source impedance. Using the transformer ratings as base values, determine:

a. The per-unit magnitudes of transformer voltage drop and voltage at the low-voltage terminals when rated transformer current at 0.8 p.f. lagging enters the high-voltage terminals

b. The per-unit magnitude of the fault current when a three-phase-to-ground bolted short circuit occurs at the low-voltage terminals

SOLUTION In both parts (a) and (b), only balanced positive-sequence current will flow, since there are no imbalances. Also, because we are interested only in voltage and current magnitudes, the Δ–Y transformer phase shift can be omitted.

a. As shown in Figure 3.19(a),

$$V_{\text{drop}} = \text{I}_{\text{rated}}\text{X}_{\text{eq}} = (1.0)(0.08) = 0.08 \quad \text{per unit}$$

and

$$\begin{aligned} V_{an} &= V_{\text{AN}} - (j\text{X}_{\text{eq}})I_{\text{rated}} \\ &= 1.0\underline{/0^\circ} - (j0.08)(1.0\underline{/-36.87^\circ}) \\ &= 1.0 - (j0.08)(0.8 - j0.6) = 0.952 - j0.064 \\ &= 0.954\underline{/-3.85^\circ} \quad \text{per unit} \end{aligned}$$

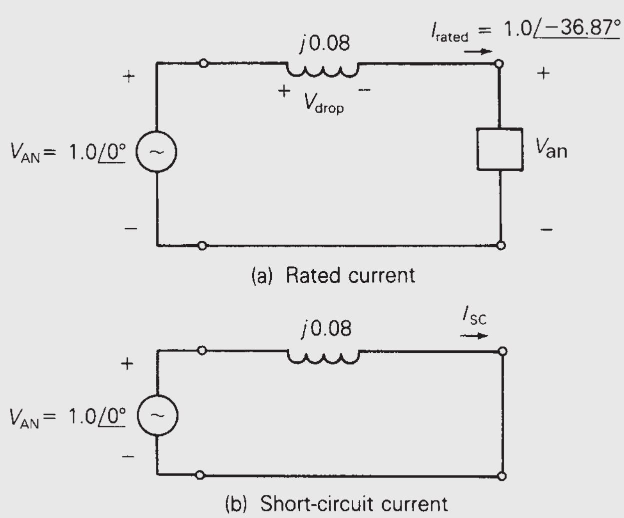
b. As shown in Figure 3.19(b),

$$\text{I}_{\text{SC}} = \frac{\text{V}_{\text{AN}}}{\text{X}_{\text{eq}}} = \frac{1.0}{0.08} = 12.5 \quad \text{per unit}$$

Under rated current conditions [part (a)], the 0.08 per-unit voltage drop across the transformer leakage reactance causes the voltage at the low-voltage terminals to be 0.954 per unit. Also, under three-phase short-circuit conditions

FIGURE 3.19

Circuits for Example 3.8



[part (b)], the fault current is 12.5 times the rated transformer current. This example illustrates a compromise in the design or specification of transformer leakage reactance. A low value is desired to minimize voltage drops, but a high value is desired to limit fault currents. Typical transformer leakage reactances are given in Table A.2 in the Appendix. ■

3.6 THREE-WINDING TRANSFORMERS

Figure 3.20(a) shows a basic single-phase three-winding transformer. The ideal transformer relations for a two-winding transformer, (3.1.8) and (3.1.14), can easily be extended to obtain corresponding relations for an ideal three-winding transformer. In actual units, these relations are

$$N_1 I_1 = N_2 I_2 + N_3 I_3 \tag{3.6.1}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3} \tag{3.6.2}$$

where I_1 enters the dotted terminal, I_2 and I_3 leave dotted terminals, and E_1, E_2, and E_3 have their + polarities at dotted terminals. In per-unit, (3.6.1) and (3.6.2) are

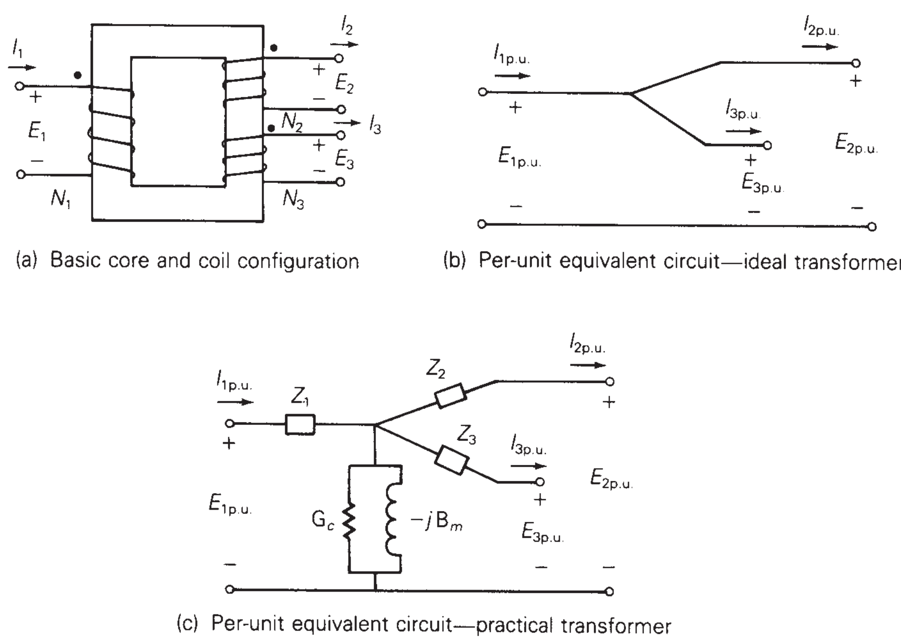


FIGURE 3.20 Single-phase three-winding transformer

$$I_{1\text{p.u.}} = I_{2\text{p.u.}} + I_{3\text{p.u.}} \tag{3.6.3}$$

$$E_{1\text{p.u.}} = E_{2\text{p.u.}} = E_{3\text{p.u.}} \tag{3.6.4}$$

where a common S_{base} is selected for all three windings, and voltage bases are selected in proportion to the rated voltages of the windings. These two perunit relations are satisfied by the per-unit equivalent circuit shown in Figure 3.20(b). Also, external series impedance and shunt admittance branches are included in the practical three-winding transformer circuit shown in Figure 3.20(c). The shunt admittance branch, a core loss resistor in parallel with a magnetizing inductor, can be evaluated from an open-circuit test. Also, when one winding is left open, the three-winding transformer behaves as a two-winding transformer, and standard short-circuit tests can be used to evaluate per-unit leakage impedances, which are defined as follows:

Z_{12} = per-unit leakage impedance measured from winding 1, with winding 2 shorted and winding 3 open

Z_{13} = per-unit leakage impedance measured from winding 1, with winding 3 shorted and winding 2 open

Z_{23} = per-unit leakage impedance measured from winding 2, with winding 3 shorted and winding 1 open

From Figure 3.20(c), with winding 2 shorted and winding 3 open, the leakage impedance measured from winding 1 is, neglecting the shunt admittance branch,

$$Z_{12} = Z_1 + Z_2 \tag{3.6.5}$$

Similarly,

$$Z_{13} = Z_1 + Z_3 \tag{3.6.6}$$

and

$$Z_{23} = Z_2 + Z_3 \tag{3.6.7}$$

Solving (3.6.5)–(3.6.7),

$$Z_1 = \tfrac{1}{2}(Z_{12} + Z_{13} - Z_{23}) \tag{3.6.8}$$

$$Z_2 = \tfrac{1}{2}(Z_{12} + Z_{23} - Z_{13}) \tag{3.6.9}$$

$$Z_3 = \tfrac{1}{2}(Z_{13} + Z_{23} - Z_{12}) \tag{3.6.10}$$

Equations (3.6.8)–(3.6.10) can be used to evaluate the per-unit series impedances Z_1, Z_2, and Z_3 of the three-winding transformer equivalent circuit from the per-unit leakage impedances Z_{12}, Z_{13}, and Z_{23}, which, in turn, are determined from short-circuit tests.

Note that each of the windings on a three-winding transformer may have a *different* kVA rating. If the leakage impedances from short-circuit tests are expressed in per-unit based on winding ratings, they must first be converted to per-unit on a common S_{base} *before* they are used in (3.6.8)–(3.6.10).

EXAMPLE 3.9 Three-winding single-phase transformer: per-unit impedances

The ratings of a single-phase three-winding transformer are

winding 1: 300 MVA, 13.8 kV

winding 2: 300 MVA, 199.2 kV

winding 3: 50 MVA, 19.92 kV

The leakage reactances, from short-circuit tests, are

$X_{12} = 0.10$ per unit on a 300-MVA, 13.8-kV base

$X_{13} = 0.16$ per unit on a 50-MVA, 13.8-kV base

$X_{23} = 0.14$ per unit on a 50-MVA, 199.2-kV base

Winding resistances and exciting current are neglected. Calculate the impedances of the per-unit equivalent circuit using a base of 300 MVA and 13.8 kV for terminal 1.

SOLUTION $S_{base} = 300$ MVA is the same for all three terminals. Also, the specified voltage base for terminal 1 is $V_{base1} = 13.8$ kV. The base voltages for terminals 2 and 3 are then $V_{base2} = 199.2$ kV and $V_{base3} = 19.92$ kV, which are the rated voltages of these windings. From the data given, $X_{12} = 0.10$ per unit was measured from terminal 1 using the same base values as those specified for the circuit. However, $X_{13} = 0.16$ and $X_{23} = 0.14$ per unit on a 50-MVA base are first converted to the 300-MVA circuit base.

$$X_{13} = (0.16)\left(\frac{300}{50}\right) = 0.96 \quad \text{per unit}$$

$$X_{23} = (0.14)\left(\frac{300}{50}\right) = 0.84 \quad \text{per unit}$$

Then, from (3.6.8)–(3.6.10),

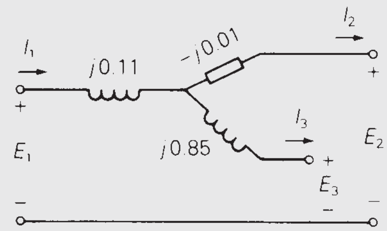
$$X_1 = \tfrac{1}{2}(0.10 + 0.96 - 0.84) = 0.11 \quad \text{per unit}$$

$$X_2 = \tfrac{1}{2}(0.10 + 0.84 - 0.96) = -0.01 \quad \text{per unit}$$

$$X_3 = \tfrac{1}{2}(0.84 + 0.96 - 0.10) = 0.85 \quad \text{per unit}$$

FIGURE 3.21

Circuit for Example 3.9



The per-unit equivalent circuit of this three-winding transformer is shown in Figure 3.21. Note that X_2 is negative. This illustrates the fact that X_1, X_2, and X_3 are *not* leakage reactances, but instead are equivalent reactances derived from the leakage reactances. Leakage reactances are always positive.

Note also that the node where the three equivalent circuit reactances are connected does not correspond to any physical location within the transformer. Rather, it is simply part of the equivalent circuit representation. ■

EXAMPLE 3.10 Three-winding three-phase transformer: balanced operation

Three transformers, each identical to that described in Example 3.9, are connected as a three-phase bank in order to feed power from a 900-MVA, 13.8-kV generator to a 345-kV transmission line and to a 34.5-kV distribution line. The transformer windings are connected as follows:

13.8-kV	windings (X):	Δ, to generator
199.2-kV	windings (H):	solidly grounded Y, to 345-kV line
19.92-kV	windings (M):	grounded Y through $Z_n = j0.10\ \Omega$, to 34.5-kV line

The positive-sequence voltages and currents of the high- and medium-voltage Y windings lead the corresponding quantities of the low-voltage Δ winding by 30°. Draw the per-unit network, using a three-phase base of 900 MVA and 13.8 kV for terminal X. Assume balanced positive-sequence operation.

SOLUTION The per-unit network is shown in Figure 3.22. $V_{baseX} = 13.8$ kV, which is the rated line-to-line voltage of terminal X. Since the M and H windings are Y-connected, $V_{baseM} = \sqrt{3}(19.92) = 34.5$ kV, and $V_{baseH} = \sqrt{3}(199.2) = 345$ kV, which are the rated line-to-line voltages of the M and H windings. Also, a phase-shifting transformer is included in the network. The neutral impedance is not included in the network, since there is no neutral current under balanced operation. ■

FIGURE 3.22

Per-unit network for Example 3.10

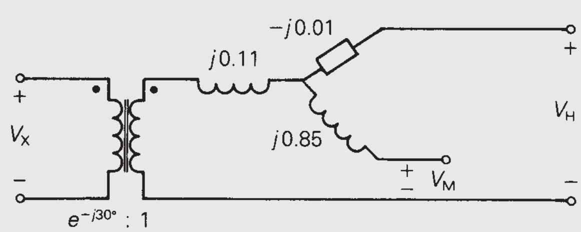
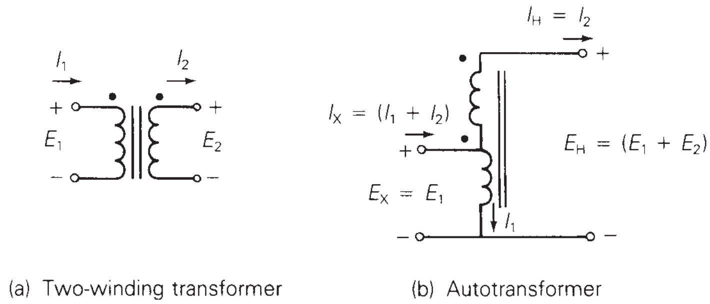


FIGURE 3.23

Ideal single-phase transformers



3.7

AUTOTRANSFORMERS

A single-phase two-winding transformer is shown in Figure 3.23(a) with two separate windings, which is the usual two-winding transformer; the same transformer is shown in Figure 3.23(b) with the two windings connected in series, which is called an *autotransformer*. For the usual transformer [Figure 3.23(a)] the two windings are coupled magnetically via the mutual core flux. For the autotransformer [Figure 3.23(b)] the windings are both electrically and magnetically coupled. The autotransformer has smaller per-unit leakage impedances than the usual transformer; this results in both smaller series-voltage drops (an advantage) and higher short-circuit currents (a disadvantage). The autotransformer also has lower per-unit losses (higher efficiency), lower exciting current, and lower cost if the turns ratio is not too large. The electrical connection of the windings, however, allows transient overvoltages to pass through the autotransformer more easily.

EXAMPLE 3.11 Autotransformer: single-phase

The single-phase two-winding 20-kVA, 480/120-volt transformer of Example 3.3 is connected as an autotransformer, as in Figure 3.23(b), where winding 1 is the 120-volt winding. For this autotransformer, determine (a) the voltage ratings E_X and E_H of the low- and high-voltage terminals, (b) the kVA rating, and (c) the per-unit leakage impedance.

SOLUTION

a. Since the 120-volt winding is connected to the low-voltage terminal, $E_X = 120$ volts. When $E_X = E_1 = 120$ volts is applied to the low-voltage terminal, $E_2 = 480$ volts is induced across the 480-volt winding, neglecting the voltage drop across the leakage impedance. Therefore, $E_H = E_1 + E_2 = 120 + 480 = 600$ volts.

b. As a normal two-winding transformer rated 20 kVA, the rated current of the 480-volt winding is $I_2 = I_H = 20{,}000/480 = 41.667$ A. As an autotransformer, the 480-volt winding can carry the same current. Therefore, the kVA rating $S_H = E_H I_H = (600)(41.667) = 25$ kVA. Note also that when $I_H = I_2 = 41.667$ A, a current $I_1 = 480/120(41.667) = 166.7$ A is induced in the 120-volt winding. Therefore, $I_X = I_1 + I_2 = 208.3$ A (neglecting exciting current) and $S_X = E_X I_X = (120)(208.3) = 25$ kVA, which is the same rating as calculated for the high-voltage terminal.

c. From Example 3.3, the leakage impedance is $0.0729\underline{/78.13^\circ}$ per unit as a normal, two-winding transformer. As an autotransformer, the leakage impedance *in ohms* is the same as for the normal transformer, since the core and windings are the same for both (only the external winding connections are different). However, the base impedances are different. For the high-voltage terminal, using (3.3.4),

$$Z_{\text{baseHold}} = \frac{(480)^2}{20{,}000} = 11.52 \quad \Omega \quad \text{as a normal transformer}$$

$$Z_{\text{baseHnew}} = \frac{(600)^2}{25{,}000} = 14.4 \quad \Omega \quad \text{as an autotransformer}$$

Therefore, using (3.3.10),

$$Z_{\text{p.u.new}} = (0.0729\underline{/78.13^\circ})\left(\frac{11.52}{14.4}\right) = 0.05832\underline{/78.13^\circ} \quad \text{per unit}$$

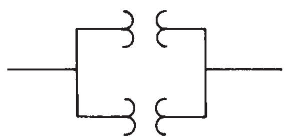
For this example, the rating is 25 kVA, 120/600 volts as an autotransformer versus 20 kVA, 120/480 volts as a normal transformer. The autotransformer has both a larger kVA rating and a larger voltage ratio for the same cost. Also, the per-unit leakage impedance of the autotransformer is smaller. However, the increased high-voltage rating as well as the electrical connection of the windings may require more insulation for both windings. ■

3.8 TRANSFORMERS WITH OFF-NOMINAL TURNS RATIOS

It has been shown that models of transformers that use per-unit quantities are simpler than those that use actual quantities. The ideal transformer winding is eliminated when the ratio of the selected voltage bases equals the ratio of the voltage ratings of the windings. In some cases, however, it is impossible to select voltage bases in this manner. For example, consider the two transformers connected in parallel in Figure 3.24. Transformer T_1 is rated 13.8/345 kV and T_2 is rated 13.2/345 kV. If we select $V_{\text{baseH}} = 345$ kV, then

FIGURE 3.24

Two transformers connected in parallel



transformer T_1 requires $V_{baseX} = 13.8$ kV and T_2 requires $V_{baseX} = 13.2$ kV. It is clearly impossible to select the appropriate voltage bases for both transformers.

To accommodate this situation, we will develop a per-unit model of a transformer whose voltage ratings are not in proportion to the selected base voltages. Such a transformer is said to have an "off-nominal turns ratio." Figure 3.25(a) shows a transformer with rated voltages V_{1rated} and V_{2rated}, which satisfy

$$V_{1rated} = a_t V_{2rated} \tag{3.8.1}$$

where a_t is assumed, in general, to be either real or complex. Suppose the selected voltage bases satisfy

$$V_{base1} = bV_{base2} \tag{3.8.2}$$

Defining $c = \dfrac{a_t}{b}$, (3.8.1) can be rewritten as

$$V_{1rated} = b\left(\frac{a_t}{b}\right)V_{2rated} = bcV_{2rated} \tag{3.8.3}$$

Equation (3.8.3) can be represented by two transformers in series, as shown in Figure 3.25(b). The first transformer has the same ratio of rated winding voltages as the ratio of the selected base voltages, b. Therefore, this transformer has a standard per-unit model, as shown in Figure 3.9 or 3.17. We will assume that the second transformer is ideal, and all real and reactive losses are associated with the first transformer. The resulting per-unit model is shown in Figure 3.25(c), where, for simplicity, the shunt-exciting branch is neglected. Note that if $a_t = b$, then the ideal transformer winding shown in this figure can be eliminated, since its turns ratio $c = (a_t/b) = 1$.

The per-unit model shown in Figure 3.25(c) is perfectly valid, but it is not suitable for some of the computer programs presented in later chapters because these programs do not accommodate ideal transformer windings. An alternative representation can be developed, however, by writing nodal equations for this figure as follows:

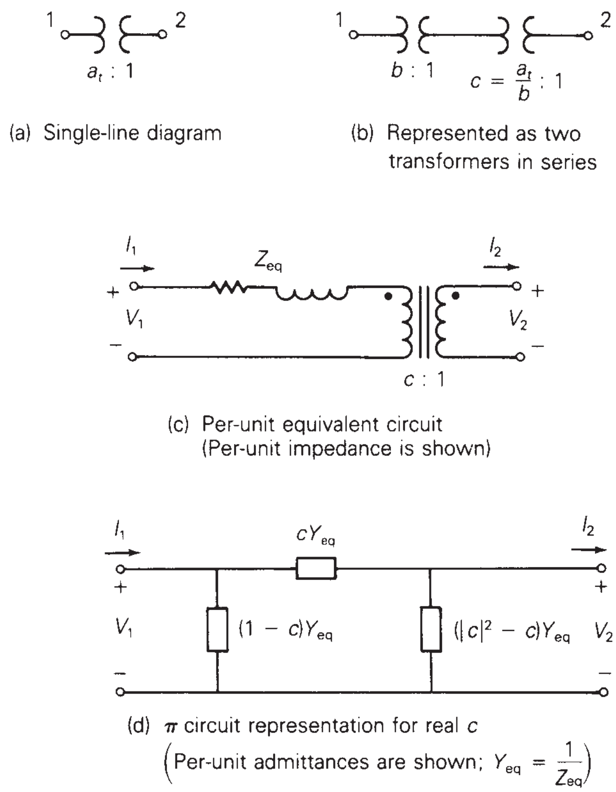


FIGURE 3.25 Transformer with off-nominal turns ratio

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{3.8.4}$$

where both I_1 and $-I_2$ are referenced *into* their nodes in accordance with the nodal equation method (Section 2.4). Recalling two-port network theory, the admittance parameters of (3.8.4) are, from Figure 3.23(c)

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} = \frac{1}{Z_{eq}} = Y_{eq} \tag{3.8.5}$$

$$Y_{22} = \frac{-I_2}{V_2}\bigg|_{V_1 = 0} = \frac{1}{Z_{eq}/|c|^2} = |c|^2 Y_{eq} \tag{3.8.6}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0} = \frac{-cV_2/Z_{eq}}{V_2} = -cY_{eq} \tag{3.8.7}$$

$$Y_{21} = \frac{-I_2}{V_1}\bigg|_{V_2 = 0} = \frac{-c^* I_1}{V_1} = -c^* Y_{eq} \tag{3.8.8}$$

Equations (3.8.4)–(3.8.8) with real or complex c are convenient for representing transformers with off-nominal turns ratios in the computer programs presented later. Note that when c is complex, Y_{12} is not equal to Y_{21}, and the preceding admittance parameters cannot be synthesized with a passive RLC circuit. However, the π network shown in Figure 3.25(d), which has the same admittance parameters as (3.8.4)–(3.8.8), can be synthesized for real c. Note also that when $c = 1$, the shunt branches in this figure become open circuits (zero per unit mhos), and the series branch becomes Y_{eq} per unit mhos (or Z_{eq} per unit ohms).

EXAMPLE 3.12 Tap-changing three-phase transformer: per-unit positive-sequence network

A three-phase generator step-up transformer is rated 1000 MVA, 13.8 kV Δ/345 kV Y with $Z_{eq} = j0.10$ per unit. The transformer high-voltage winding has $\pm$10% taps. The system base quantities are

$$S_{base3\phi} = 500 \quad \text{MVA}$$
$$V_{baseXLL} = 13.8 \quad \text{kV}$$
$$V_{baseHLL} = 345 \quad \text{kV}$$

Determine the per-unit equivalent circuit for the following tap settings:

a. Rated tap

b. -10% tap (providing a 10% voltage decrease for the high-voltage winding)

Assume balanced positive-sequence operation. Neglect transformer winding resistance, exciting current, and phase shift.

SOLUTION

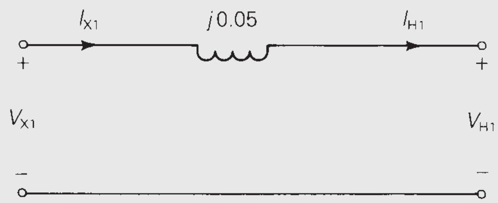
a. Using (3.8.1) and (3.8.2) with the low-voltage winding denoted winding 1,

$$a_t = \frac{13.8}{345} = 0.04 \qquad b = \frac{V_{baseXLL}}{V_{baseHLL}} = \frac{13.8}{345} = a_t \qquad c = 1$$

From (3.3.11)

$$Z_{p.u.new} = (j0.10)\left(\frac{500}{1000}\right) = j0.05 \quad \text{per unit}$$

The per-unit equivalent circuit, not including winding resistance, exciting current, and phase shift is:



(Per-unit impedance is shown)

b. Using (3.8.1) and (3.8.2),

$$a_t = \frac{13.8}{345(0.9)} = 0.04444 \qquad b = \frac{13.8}{345} = 0.04$$

$$c = \frac{a_t}{b} = \frac{0.04444}{0.04} = 1.1111$$

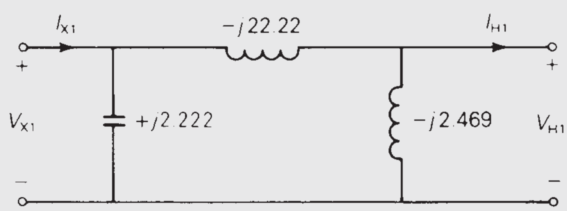
From Figure 3.23(d),

$$cY_{eq} = 1.1111\left(\frac{1}{j0.05}\right) = -j22.22 \quad \text{per unit}$$

$$(1-c)Y_{eq} = (-0.11111)(-j20) = +j2.222 \quad \text{per unit}$$

$$(|c|^2 - c)Y_{eq} = (1.2346 - 1.1)(-j20) = -j2.469 \quad \text{per unit}$$

The per-unit positive-sequence network is:



(Per-unit admittances are shown)

Open PowerWorld Simulator case Example 3.12 (see Figure 3.26) and select **Tools, Play** to see an animated view of this LTC transformer example. Initially the generator/step-up transformer feeds a 500 MW/100 Mvar load. As is typical in practice, the transformer's taps are adjusted in discrete steps, with each step changing the tap ratio by 0.625% (hence a 10% change requires 16 steps). Click on arrows next to the transformer's tap to manually adjust the tap by one step. Note that changing the tap directly changes the load voltage.

Because of the varying voltage drops caused by changing loads, LTCs are often operated to automatically regulate a bus voltage. This is particularly true when they are used as step-down transformers. To place the example transformer on automatic control, click on the "Manual" field. This toggles the transformer control mode to automatic. Now the transformer manually

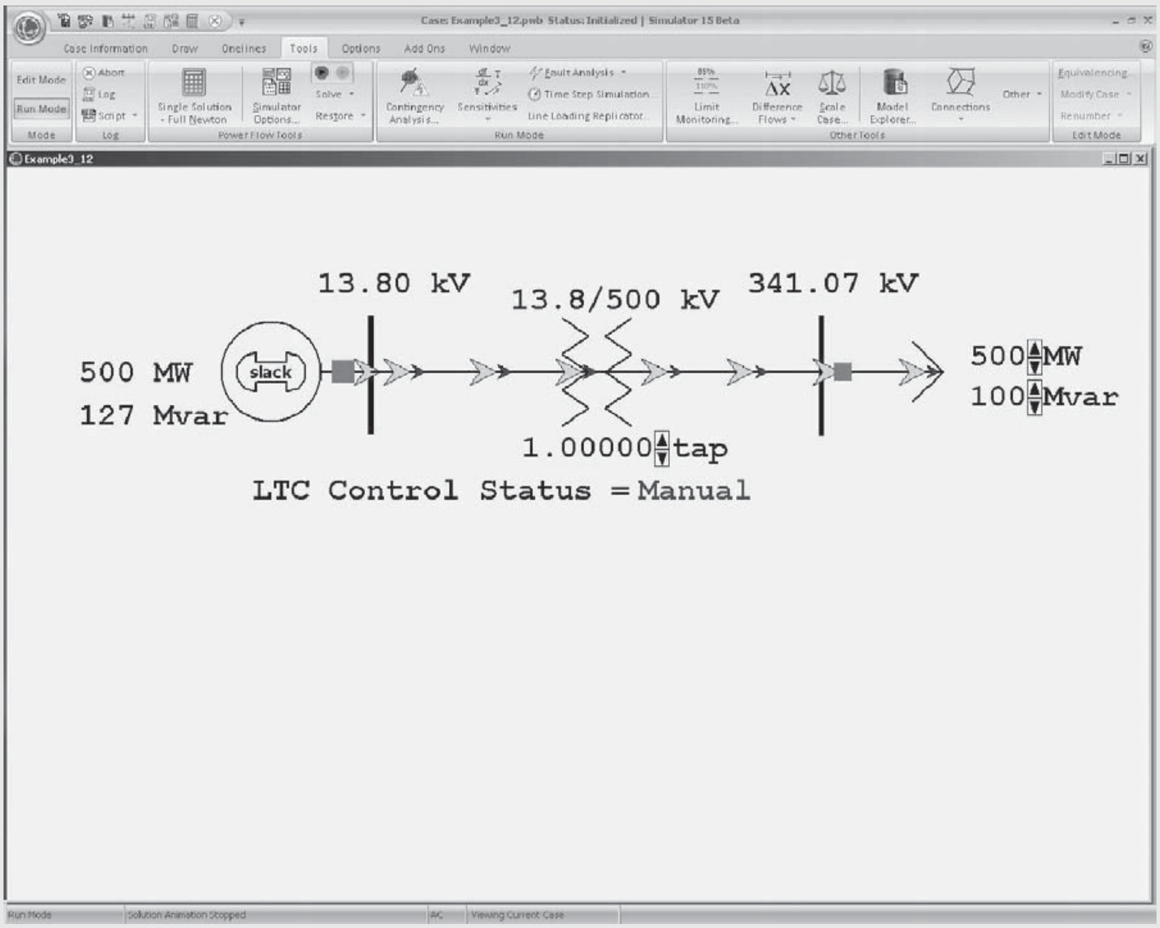


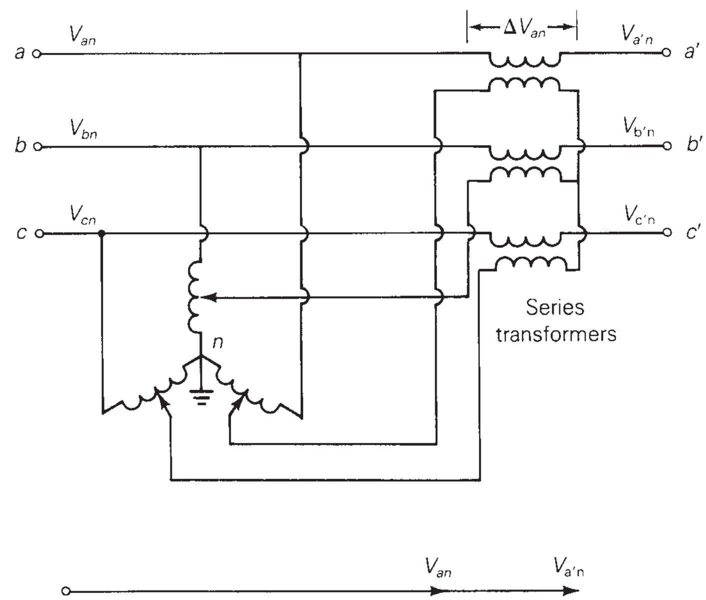
FIGURE 3.26 Screen for Example 3.12

changes its tap ratio to maintain the load voltage within a specified voltage range, between 0.995 and 1.005 per unit (343.3 to 346.7 kV) in this case. To see the LTC in automatic operation use the load arrows to vary the load, particularly the Mvar field, noting that the LTC changes to keep the load's voltage within the specified deadband. ■

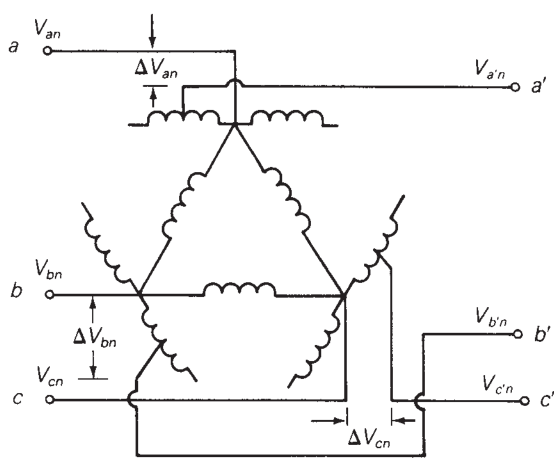
The three-phase regulating transformers shown in Figures 3.27 and 3.28 can be modeled as transformers with off-nominal turns ratios. For the voltage-magnitude-regulating transformer shown in Figure 3.27, adjustable voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn}, which have equal magnitudes ΔV and which are in phase with the phase voltages V_{an}, V_{bn}, and V_{cn}, are placed in the series link between buses a–a', b–b', and c–c'. Modeled as a transformer with an off-nominal turns ratio (see Figure 3.25), $c = (1 + \Delta \mathrm{V})$ for a voltage-magnitude increase toward bus abc, or $c = (1 + \Delta \mathrm{V})^{-1}$ for an increase toward bus $a'b'c'$.

FIGURE 3.27

An example of a voltage-magnitude-regulating transformer



For the phase-angle-regulating transformer in Figure 3.28, the series voltages ΔV_{an}, ΔV_{bn}, and ΔV_{cn} are $\pm 90^\circ$ out of phase with the phase voltages V_{an}, V_{bn}, and V_{cn}. The phasor diagram in Figure 3.28 indicates that each of the bus voltages $V_{a'n}$, $V_{b'n}$, and $V_{c'n}$ has a phase shift that is approximately proportional to the magnitude of the added series voltage. Modeled as a transformer with an off-nominal turns ratio (see Figure 3.25), $c \approx 1\underline{/\alpha}$ for a phase increase toward bus *abc* or $c \approx 1\underline{/-\alpha}$ for a phase increase toward bus $a'b'c'$.



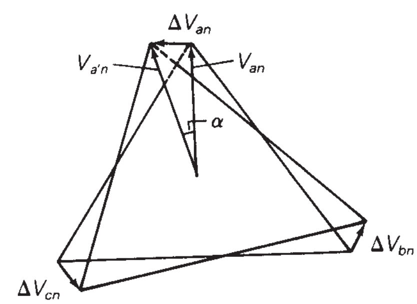


FIGURE 3.28 An example of a phase-angle-regulating transformer. Windings drawn in parallel are on the same core

EXAMPLE 3.13 Voltage-regulating and phase-shifting three-phase transformers

Two buses *abc* and $a'b'c'$ are connected by two parallel lines L1 and L2 with positive-sequence series reactances $X_{L1} = 0.25$ and $X_{L2} = 0.20$ per unit. A regulating transformer is placed in series with line L1 at bus $a'b'c'$. Determine the 2×2 bus admittance matrix when the regulating transformer (a) provides a 0.05 per-unit increase in voltage magnitude toward bus $a'b'c'$ and (b) advances the phase 3° toward bus $a'b'c'$. Assume that the regulating transformer is ideal. Also, the series resistance and shunt admittance of the lines are neglected.

SOLUTION The circuit is shown in Figure 3.29.

a. For the voltage-magnitude-regulating transformer, $c = (1 + \Delta V)^{-1} = (1.05)^{-1} = 0.9524$ per unit. From (3.7.5)–(3.7.8), the admittance parameters of the regulating transformer in series with line L1 are

$$Y_{11L1} = \frac{1}{j0.25} = -j4.0$$

$$Y_{22L1} = (0.9524)^2(-j4.0) = -j3.628$$

$$Y_{12L1} = Y_{21L1} = (-0.9524)(-j4.0) = j3.810$$

For line L2 alone,

$$Y_{11L2} = Y_{22L2} = \frac{1}{j0.20} = -j5.0$$

$$Y_{12L2} = Y_{21L2} = -(-j5.0) = j5.0$$

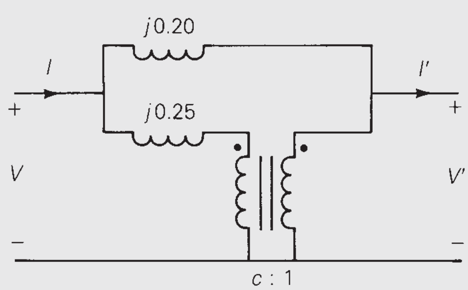
Combining the above admittances in parallel,

$$Y_{11} = Y_{11L1} + Y_{11L2} = -j4.0 - j5.0 = -j9.0$$

$$Y_{22} = Y_{22L1} + Y_{22L2} = -j3.628 - j5.0 = -j8.628$$

$$Y_{12} = Y_{21} = Y_{12L1} + Y_{12L2} = j3.810 + j5.0 = j8.810 \quad \text{per unit}$$

FIGURE 3.29 Positive-sequence circuit for Example 3.13



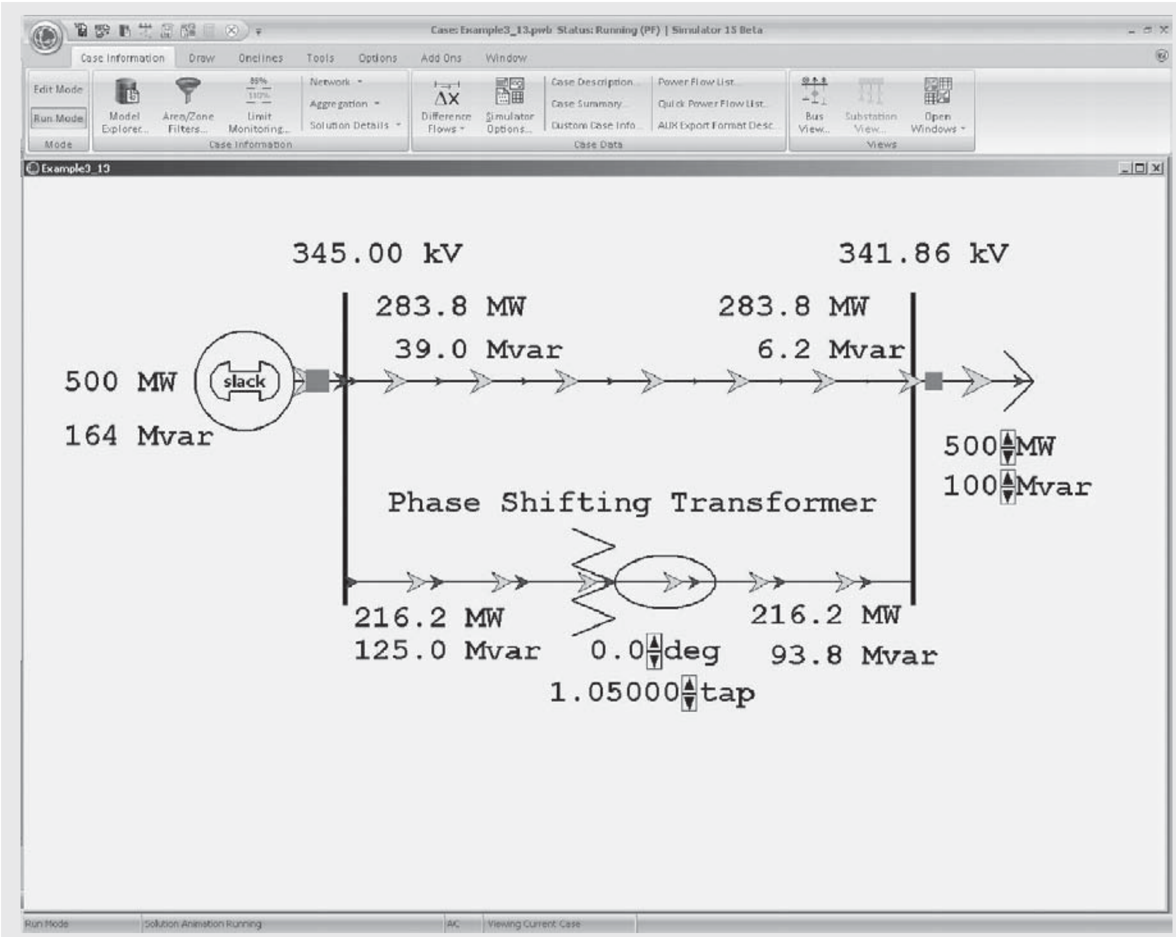


FIGURE 3.30 Screen for Example 3.13

b. For the phase-angle-regulating transformer, $c = 1\underline{/-\alpha} = 1\underline{/-3^\circ}$. Then, for this regulating transformer in series with line L1,

$$Y_{11L1} = \frac{1}{j0.25} = -j4.0$$

$$Y_{22L1} = |1.0\underline{/-3^\circ}|^2(-j4.0) = -j4.0$$

$$Y_{12L1} = -(1.0\underline{/-3^\circ})(-j4.0) = 4.0\underline{/87^\circ} = 0.2093 + j3.9945$$

$$Y_{21L1} = -(1.0\underline{/-3^\circ})^*(-j4.0) = 4.0\underline{/93^\circ} = -0.2093 + j3.9945$$

The admittance parameters for line L2 alone are given in part (a) above. Combining the admittances in parallel,

$$Y_{11} = Y_{22} = -j4.0 - j5.0 = -j9.0$$

$$Y_{12} = 0.2093 + j3.9945 + j5.0 = 0.2093 + j8.9945$$

$$Y_{21} = -0.2093 + j3.9945 + j5.0 = -0.2093 + j8.9945 \quad \text{per unit}$$

To see this example in PowerWorld Simulator open case Example 3.13 (see Figure 3.30). In this case, the transformer and a parallel transmission line are assumed to be supplying power from a 345-kV generator to a 345-kV load. Initially, the off-nominal turns ratio is set to the value in part (a) of the example (PowerWorld has the off-nominal turns ratio on the load side [right-hand] so its tap value of 1.05 = c^{-1}). To view the PowerWorld Simulator bus admittance matrix, select the **Case Information** ribbon, then **Solution Details, Ybus**. To see how the system flows vary with changes to the tap, select **Tools, Play,** and then click on the arrows next to the tap field to change the LTC tap in 0.625% steps. Next, to verify the results from part (b), change the tap field to 1.0 and the deg field to 3.0 degrees, and then again look at the bus admittance matrix. Click on the deg field arrow to vary the phase shift angle in one-degree steps. Notice that changing the phase angle primarily changes the real power flow, whereas changing the LTC tap changes the reactive power flow. In this example, the line flow fields show the absolute value of the real or reactive power flow; the direction of the flow is indicated with arrows. Traditional power flow programs usually indicate power flow direction using a convention that flow into a transmission line or transformer is assumed to be positive. You can display results in PowerWorld Simulator using this convention by first clicking on the **Onelines** ribbon and then selecting **Oneline Display Options.** Then on the Display Options tab uncheck the Use Absolute Values for MW/Mvar Line Flows'' fields. ■

Note that a voltage-magnitude-regulating transformer controls the *reactive* power flow in the series link in which it is installed, whereas a phase-angle-regulating transformer controls the *real* power flow (see Problem 3.59).

MULTIPLE CHOICE QUESTIONS

SECTION 3.1

3.1 The ''Ohm's law'' for the magnetic circuit states that the net magnetomotive force (mmf) equals the product of the core reluctance and the core flux.
(a) True (b) False

3.2 For an ideal transformer, the efficiency is
(a) 0% (b) 100% (c) 50%

3.3 For an ideal 2-winding transformer, the ampere-turns of the primary winding, N_1I_1, is equal to the ampere-turns of the secondary winding, N_2I_2.
(a) True (b) False

3.4 An ideal transformer has no real or reactive power loss.
(a) True (b) False

3.5 For an ideal 2-winding transformer, an impedance Z_2 connected across winding 2 (secondary) is referred to winding 1 (primary) by multiplying Z_2 by
(a) The turns ratio (N_1/N_2)
(b) The square of the turns ratio $(N_1/N_2)^2$
(c) The cubed turns ratio $(N_1/N_2)^3$

3.6 Consider Figure 3.4 of the text. For an ideal phase-shifting transformer, the impedance is unchanged when it is referred from one side to the other.
(a) True (b) False

SECTION 3.2

3.7 Consider Figure 3.5 of the text. Match the following, those on the left to those on the right.
(i) I_m (a) Exciting current
(ii) I_C (b) Magnetizing current
(iii) I_e (c) Core loss current

3.8 The units of admittance, conductance, and susceptance are siemens.
(a) True (b) False

3.9 Match the following:
(i) Hysteresis loss (a) Can be reduced by constructing the core with laminated sheets of alloy steel
(ii) Eddy current loss (b) Can be reduced by the use of special high grades of alloy steel as core material.

3.10 For large power transformers rated more than 500 kVA, the winding resistances, which are small compared with the leakage reactances, can often be neglected.
(a) True (b) False

3.11 For a short-circuit test on a 2-winding transformer, with one winding shorted, can you apply the rated voltage on the other winding?
(a) Yes (b) No

SECTION 3.3

3.12 The per-unit quantity is always dimensionless.
(a) True (b) False

3.13 Consider the adopted per-unit system for the transformers. Specify true or false for each of the following statements:
(a) For the entire power system of concern, the value of S_{base} is not the same.
(b) The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.
(c) Per-unit impedance remains unchanged when referred from one side of a transformer to the other.

3.14 The ideal transformer windings are eliminated from the per-unit equivalent circuit of a transformer.
(a) True (b) False

3.15 To convert a per-unit impedance from "old" to "new" base values, the equation to be used is

(a) $Z_{\text{p.u.new}} = Z_{\text{p.u.old}} \left(\frac{V_{\text{baseold}}}{V_{\text{basenew}}}\right)^2 \left(\frac{S_{\text{basenew}}}{S_{\text{baseold}}}\right)$

(b) $Z_{\text{p.u.new}} = Z_{\text{p.u.old}} \left(\frac{V_{\text{baseold}}}{V_{\text{basenew}}}\right)^2 \left(\frac{S_{\text{basenew}}}{S_{\text{baseold}}}\right)$

(c) $Z_{\text{p.u.new}} = Z_{\text{p.u.old}} \left(\frac{V_{\text{baseold}}}{V_{\text{basenew}}}\right)^2 \left(\frac{S_{\text{baseold}}}{S_{\text{basenew}}}\right)$

3.16 In developing per-unit circuits of systems such as the one shown in Figure 3.10 of the text, when moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings.
(a) True (b) False

3.17 Consider Figure 3.10 of the text. The per-unit leakage reactance of transformer T_1, given as 0.1 p.u., is based on the name plate ratings of transformer T_1.
(a) True (b) False

3.18 For balanced three-phase systems, Z_{base} is given by

$$Z_{\text{base}} = \frac{V^2_{\text{baseLL}}}{S_{\text{base3}}}$$

(a) True (b) False

SECTION 3.4

3.19 With the American Standard notation, in either Y–Δ OR Δ–Y transformer, positive-sequence quantities on the high-voltage side shall lead their corresponding quantities on the low-voltage side by 30°.
(a) True (b) False

3.20 In either Y–Δ or Δ–Y transformer, as per the American Standard notation, the negative-sequence phase shift is the reverse of the positive-sequence phase shift.
(a) True (b) False

3.21 In order to avoid difficulties with third-harmonic exciting current, which three-phase transformer connection is seldom used for step-up transformers between a generator and a transmission line in power systems.
(a) Y–Δ (b) Δ–Y (c) Y–Y

3.22 Does open –Δ connection permit balanced three-phase operation?
(a) Yes (b) No

3.23 With the open –Δ operation, the kVA rating compared to that of the original three-phase bank is
(a) 2/3 (b) 58% (c) 1

SECTION 3.5

3.24 It is stated that
(i) balanced three-phase circuits can be solved in per unit on a per-phase basis after converting Δ-load impedances to equivalent Y impedances.

(ii) Base values can be selected either on a per-phase basis or on a three-phase basis.
(a) Both statements are true.
(b) Neither is true.
(c) Only one of the above is true.

3.25 In developing per-unit equivalent circuits for three-phase transformers, under balanced three-phase operation,
(i) A common S_{base} is selected for both the H and X terminals.
(ii) The ratio of the voltage bases V_{baseH}/V_{baseX} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.
(a) Only one of the above is true.
(b) Neither is true.
(c) Both statements are true.

3.26 In per-unit equivalent circuits of practical three-phase transformers, under balanced three-phase operation, in which of the following connections would a phase-shifting transformer come up?
(a) Y–Y (b) Y–Δ (c) Δ–Δ

3.27 A low value of transformer leakage reactance is desired to minimize the voltage drop, but a high value is derived to limit the fault current, thereby leading to a compromise in the design specification.
(a) True (b) False

SECTION 3.6

3.28 Consider a single-phase three-winding transformer with the primary excited winding of N_1 turns carrying a current I_1 and two secondary windings of N_2 and N_3 turns, delivering currents of I_2 and I_3 respectively. For an ideal case, how are the ampere-turns balanced?
(a) $N_1I_1 = N_2I_2 - N_3I_3$ (b) $N_1I_1 = N_2I_2 + N_3I_3$
(c) $N_1I_1 = -(N_2I_2 - N_3I_3)$

3.29 For developing per-unit equivalent circuits of single-phase three-winding transformer, a common S_{base} is selected for all three windings, and voltage bases are selected in proportion to the rated voltage of the windings.
(a) True (b) False

3.30 Consider the equivalent circuit of Figure 3.20 (c) in the text. After neglecting the winding resistances and exciting current, could X_1, X_2, or X_3 become negative, even though the leakage reactance are always positive?
(a) Yes (b) No

SECTION 3.7

3.31 Consider an ideal single-phase 2-winding transformer of turns ratio N_1/N_2 = a. If it is converted to an autotransformer arrangement with a transformation ratio of $V_H/V_X = 1 + a$, (the autotransformer rating/two-winding transformer rating) would then be
(a) $1 + a$ (b) $1 + \frac{1}{a}$ (c) a

3.32 For the same output, the autotransformer (with not too large turns ratio) is smaller in size than a two-winding transformer and has high efficiency as well as superior voltage regulation.
(a) True (b) False

3.33 The direct electrical connection of the windings allows transient over voltages to pass through the autotransformer more easily, and that is an important disadvantage of the autotransformer.
(a) True (b) False

SECTION 3.8

3.34 Consider Figure 3.25 of the text for a transformer with off-nominal turns ratio.
(i) The per-unit equivalent circuit shown in Part (c) contains an ideal transformer which cannot be accommodated by some computer programs.
(a) True (b) False
(ii) In the π-circuit representation for real C in Part (d), the admittance parameters Y_{12} and Y_{21} would be unequal.
(a) True (b) False
(iii) For complex C, can the admittance, parameters the synthesized with a passive RLC circuit?
(a) Yes (b) No

PROBLEMS

SECTION 3.1

3.1 (a) An ideal single-phase two-winding transformer with turns ratio $a_t = N_1/N_2$ is connected with a series impedance Z_2 across winding 2. If one wants to replace Z_2, with a series impedance Z_1 across winding 1 and keep the terminal behavior of the two circuits to be identical, find Z_1 in terms of Z_2.
(b) Would the above result be true if instead of a series impedance there is a shunt impedance?
(c) Can one refer a ladder network on the secondary (2) side to the primary (1) side simply by multiplying every impendance by a_t^2?

3.2 An ideal transformer with $N_1 = 2000$ and $N_2 = 500$ is connected with an impedance Z_{2_2} across winding 2, called secondary. If $V_1 = 1000\ \underline{/0^\circ}$ V and $I_1 = 5\ \underline{/-30^\circ}$ A, determine V_2, I_2, Z_2, and the impedance Z_2', which is the value of Z_2 referred to the primary side of the transformer.

3.3 Consider an ideal transformer with $N_1 = 3000$ and $N_2 = 1000$ turns. Let winding 1 be connected to a source whose voltage is $e_1(t) = 100(1 - |t|)$ volts for $-1 \le t \le 1$ and $e_1(t) = 0$ for $|t| > 1$ second. A 2-farad capacitor is connected across winding 2. Sketch $e_1(t)$, $e_2(t)$, $i_1(t)$, and $i_2(t)$ versus time t.

3.4 A single-phase 100-kVA, 2400/240-volt, 60-Hz distribution transformer is used as a step-down transformer. The load, which is connected to the 240-volt secondary winding, absorbs 80 kVA at 0.8 power factor lagging and is at 230 volts. Assuming an ideal transformer, calculate the following: (a) primary voltage, (b) load impedance, (c) load impedance referred to the primary, and (d) the real and reactive power supplied to the primary winding.

3.5 Rework Problem 3.4 if the load connected to the 240-V secondary winding absorbs 110 kVA under short-term overload conditions at 0.85 power factor leading and at 230 volts.

3.6 For a conceptual single-phase, phase-shifting transformer, the primary voltage leads the secondary voltage by 30°. A load connected to the secondary winding absorbs 100 kVA at 0.9 power factor leading and at a voltage $E_2 = 277\underline{/0^\circ}$ volts. Determine (a) the primary voltage, (b) primary and secondary currents, (c) load impedance referred to the primary winding, and (d) complex power supplied to the primary winding.

3.7 Consider a source of voltage $v(t) = 10\sqrt{2}\sin(2t)$ V, with an internal resistance of 1800 Ω. A transformer that can be considered as ideal is used to couple a 50-Ω resistive load to the source. (a) Determine the transformer primary-to-secondary turns ratio required to ensure maximum power transfer by matching the load and source resistances. (b) Find the average power delivered to the load, assuming maximum power transfer.

3.8 For the circuit shown in Figure 3.31, determine $v_{out}(t)$.

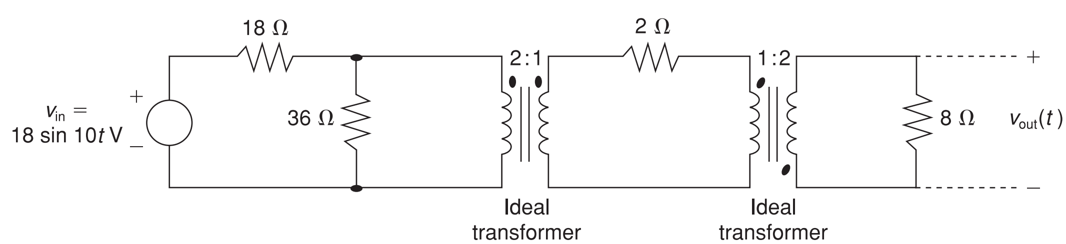


FIGURE 3.31 Problem 3.8

SECTION 3.2

3.9 A single-phase transformer has 2000 turns on the primary winding and 500 turns on the secondary. Winding resistances are $R_1 = 2$ Ω and $R_2 = 0.125$ Ω; leakage reactances are $X_1 = 8$ Ω and $X_2 = 0.5$ Ω. The resistance load on the secondary is 12 Ω.
(a) If the applied voltage at the terminals of the primary is 1000 V, determine V_2 at the load terminals of the transformer, neglecting magnetizing current.
(b) If the voltage regulation is defined as the difference between the voltage magnitude at the load terminals of the transformer at full load and at no load in percent of full-load voltage with input voltage held constant, compute the percent voltage regulation.

3.10 A single-phase step-down transformer is rated 15 MVA, 66 kV/11.5 kV. With the 11.5 kV winding short-circuited, rated current flows when the voltage applied to the primary is 5.5 kV. The power input is read as 100 kW. Determine R_{eq1} and X_{eq1} in ohms referred to the high-voltage winding.

3.11 For the transformer in Problem 3.10, the open-circuit test with 11.5 kV applied results in a power input of 65 kW and a current of 30 A. Compute the values for G_c and B_m

in siemens referred to the high-voltage winding. Compute the efficiency of the transformer for a load of 10 MW at 0.8 p.f. lagging at rated voltage.

3.12 The following data are obtained when open-circuit and short-circuit tests are performed on a single-phase, 50-kVA, 2400/240-volt, 60-Hz distribution transformer.

	VOLTAGE (volts)	CURRENT (amperes)	POWER (watts)
Measurements on low-voltage side with high-voltage winding open	240	4.85	173
Measurements on high-voltage side with low-voltage winding shorted	52.0	20.8	650

(a) Neglecting the series impedance, determine the exciting admittance referred to the high-voltage side. (b) Neglecting the exciting admittance, determine the equivalent series impedance referred to the high-voltage side. (c) Assuming equal series impedances for the primary and referred secondary, obtain an equivalent T-circuit referred to the high-voltage side.

3.13 A single-phase 50-kVA, 2400/240-volt, 60-Hz distribution transformer has a 1-ohm equivalent leakage reactance and a 5000-ohm magnetizing reactance referred to the high-voltage side. If rated voltage is applied to the high-voltage winding, calculate the open-circuit secondary voltage. Neglect I^2R and G_c^2V losses. Assume equal series leakage reactances for the primary and referred secondary.

3.14 A single-phase 50-kVA, 2400/240-volt, 60-Hz distribution transformer is used as a step-down transformer at the load end of a 2400-volt feeder whose series impedance is $(1.0 + j2.0)$ ohms. The equivalent series impedance of the transformer is $(1.0 + j2.5)$ ohms referred to the high-voltage (primary) side. The transformer is delivering rated load at 0.8 power factor lagging and at rated secondary voltage. Neglecting the transformer exciting current, determine (a) the voltage at the transformer primary terminals, (b) the voltage at the sending end of the feeder, and (c) the real and reactive power delivered to the sending end of the feeder.

3.15 Rework Problem 3.14 if the transformer is delivering rated load at rated secondary voltage and at (a) unity power factor, (b) 0.8 power factor leading. Compare the results with those of Problem 3.14.

3.16 A single-phase, 50-kVA, 2400/240-V, 60-Hz distribution transformer has the following parameters:

Resistance of the 2400-V winding: $R_1 = 0.75\ \Omega$

Resistance of the 240-V winding: $R_2 = 0.0075\ \Omega$

Leakage reactance of the 2400-V winding: $X_1 = 1.0\ \Omega$

Leakage reactance of the 240-V winding: $X_2 = 0.01\ \Omega$

Exciting admittance on the 240-V side $= 0.003 - j0.02$ S

(a) Draw the equivalent circuit referred to the high-voltage side of the transformer. (b) Draw the equivalent circuit referred to the low-voltage side of the transformer. Show the numerical values of impedances on the equivalent circuits.

3.17 The transformer of Problem 3.16 is supplying a rated load of 50 kVA at a rated secondary voltage of 240 V and at 0.8 power factor lagging. Neglecting the transformer exciting current, (a) Determine the input terminal voltage of the transformer on the high-voltage side. (b) Sketch the corresponding phasor diagram. (c) If the transformer is used as a step-down transformer at the load end of a feeder whose impedance is $0.5 + j2.0\ \Omega$, find the voltage V_S and the power factor at the sending end of the feeder.

SECTION 3.3

3.18 Using the transformer ratings as base quantities, work Problem 3.13 in per-unit.

3.19 Using the transformer ratings as base quantities, work Problem 3.14 in per-unit.

3.20 Using base values of 20 kVA and 115 volts in zone 3, rework Example 3.4.

3.21 Rework Example 3.5, using $S_{base3\phi} = 100$ kVA and $V_{baseLL} = 600$ volts.

3.22 A balanced Y-connected voltage source with $E_{ag} = 277\underline{/0^\circ}$ volts is applied to a balanced-Y load in parallel with a balanced-Δ load, where $Z_Y = 20 + j10$ and $Z_\Delta = 30 - j15$ ohms. The Y load is solidly grounded. Using base values of $S_{base1\phi} = 10$ kVA and $V_{baseLN} = 277$ volts, calculate the source current I_a in per-unit and in amperes.

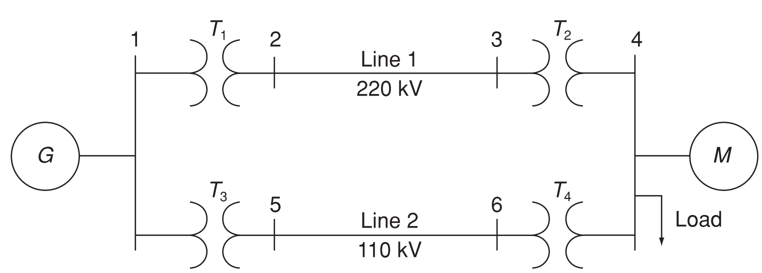
3.23 Figure 3.32 shows the one-line diagram of a three-phase power system. By selecting a common base of 100 MVA and 22 kV on the generator side, draw an impedance diagram showing all impedances including the load impedance in per-unit. The data are given as follows:

G:	90 MVA	22 kV	x = 0.18 per unit
$T1$:	50 MVA	22/220 kV	x = 0.10 per unit
$T2$:	40 MVA	220/11 kV	x = 0.06 per unit
$T3$:	40 MVA	22/110 kV	x = 0.064 per unit
$T4$:	40 MVA	110/11 kV	x = 0.08 per unit
M:	66.5 MVA	10.45 kV	x = 0.185 per unit

Lines 1 and 2 have series reactances of 48.4 and 65.43 Ω, respectively. At bus 4, the three-phase load absorbs 57 MVA at 10.45 kV and 0.6 power factor lagging.

FIGURE 3.32

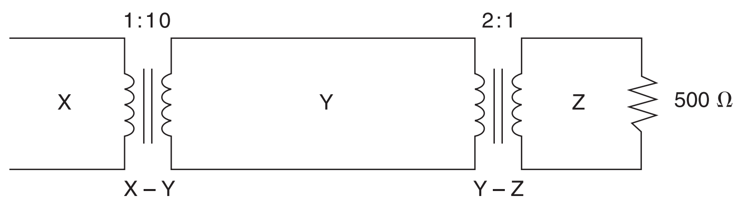
Problem 3.23



3.24 For Problem 3.18, the motor operates at full load, at 0.8 power factor leading, and at a terminal voltage of 10.45 kV. Determine (a) the voltage at bus 1, the generator bus, and (b) the generator and motor internal EMFs.

3.25 Consider a single-phase electric system shown in Figure 3.33.
Transformers are rated as follows:
X–Y 15 MVA, 13.8/138 kV, leakage reactance 10%
Y–Z 15 MVA, 138/69 kV, leakage reactance 8%
With the base in circuit Y chosen as 15 MVA, 138 kV, determine the per-unit impedance of the 500 Ω resistive load in circuit Z, referred to circuits Z, Y, and X. Neglecting magnetizing currents, transformer resistances, and line impedances, draw the impedance diagram in per unit.

FIGURE 3.33 Single-phase electric system for Problem 3.25

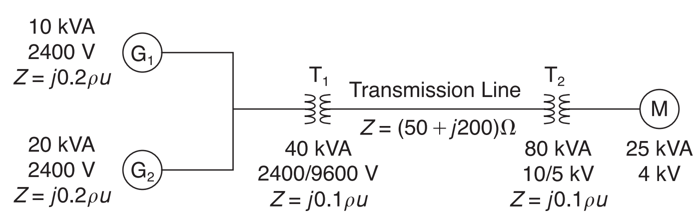


3.26 A bank of three single-phase transformers, each rated 30 MVA, 38.1/3.81 kV, are connected in Y–Δ with a balanced load of three 1-Ω, wye-connected resistors. Choosing a base of 90 MVA, 66 kV for the high-voltage side of the three-phase transformer, specify the base for the low-voltage side. Compute the per-unit resistance of the load on the base for the low-voltage side. Also, determine the load resistance in ohms referred to the high-voltage side and the per-unit value on the chosen base.

3.27 A three-phase transformer is rated 500 MVA, 220 Y/22 Δ kV. The wye-equivalent short-circuit impedance, considered equal to the leakage reactance, measured on the low-voltage side is 0.1 Ω. Compute the per-unit reactance of the transformer. In a system in which the base on the high-voltage side of the transformer is 100 MVA, 230 kV, what value of the per-unit reactance should be used to represent this transformer?

3.28 For the system shown in Figure 3.34, draw an impedance diagram in per unit, by choosing 100 kVA to be the base kVA and 2400 V as the base voltage for the generators.

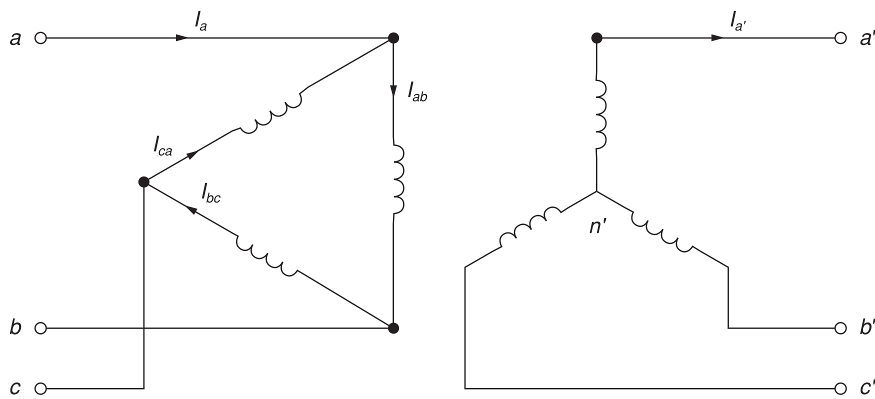
FIGURE 3.34 System for Problem 3.28



3.29 Consider three ideal single-phase transformers (with a voltage gain of η) put together as a delta-wye three-phase bank as shown in Figure 3.35. Assuming positive-sequence voltages for V_{an}, V_{bn}, and V_{cn}, find $V_{a'n'}$, $V_{b'n'}$, and $V_{c'n'}$ in terms of V_{an}, V_{bn}, and V_{cn}, respectively.
(a) Would such relationships hold for the line voltages as well?
(b) Looking into the current relationships, express I'_a, I'_b, and I'_c in terms of I_a, I_b, and I_c, respectively.
(c) Let S' and S be the per-phase complex power output and input, respectively. Find S' in terms of S.

FIGURE 3.35

Δ–Y connection for Problem 3.29



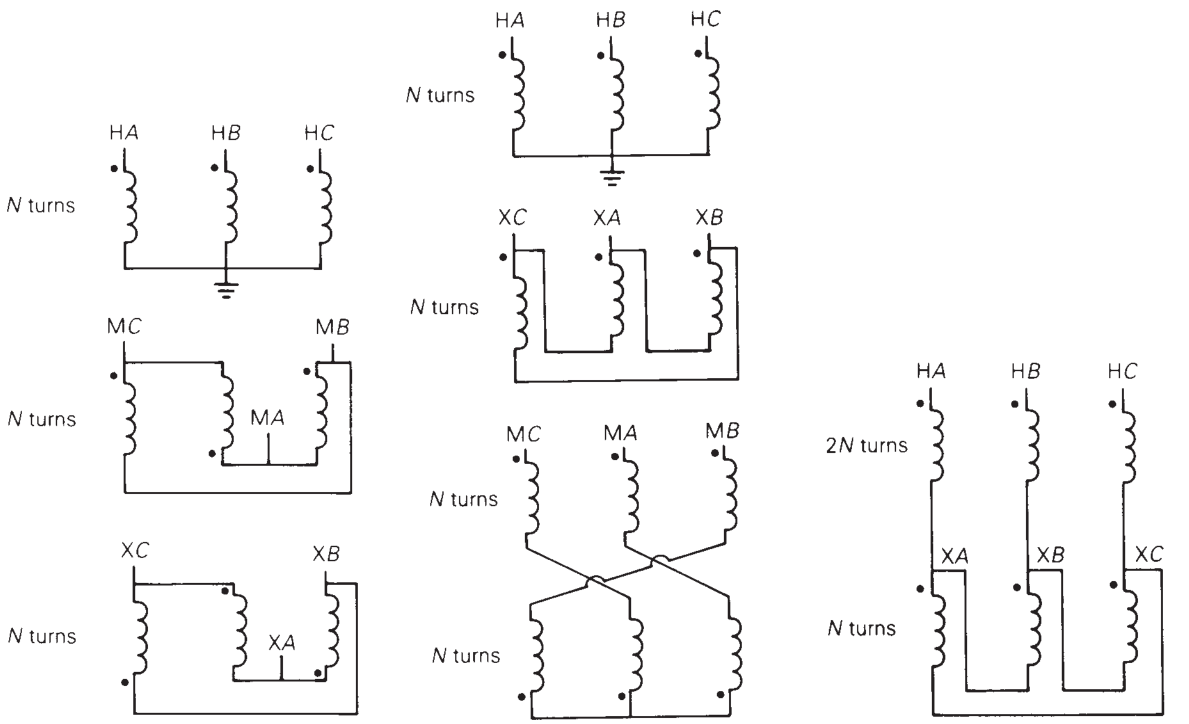
3.30 Reconsider Problem 3.29. If V_{an}, V_{bn}, and V_{cn} are a negative-sequence set, how would the voltage and current relationships change?
(a) If C_1 is the complex positive-sequence voltage gain in Problem 3.29, and C_2 is the negative sequence complex voltage gain, express the relationship between C_1 and C_2.

3.31 If positive-sequence voltages are assumed and the wye-delta connection is considered, again with ideal transformers as in Problem 3.29, find the complex voltage gain C_3.
(a) What would the gain be for a negative-sequence set?
(b) Comment on the complex power gain.
(c) When terminated in a symmetric wye-connected load, find the referred impedance Z'_L, the secondary impedance Z_L referred to primary (i.e., the per-phase driving-point impedance on the primary side), in terms of Z_L and the complex voltage gain C.

SECTION 3.4

3.32 Determine the positive- and negative-sequence phase shifts for the three-phase transformers shown in Figure 3.36.

3.33 Consider the three single-phase two-winding transformers shown in Figure 3.37. The high-voltage windings are connected in Y. (a) For the low-voltage side, connect the windings in Δ, place the polarity marks, and label the terminals a, b, and c in accordance with the American standard. (b) Relabel the terminals a', b', and c' such that V_{AN} is 90° out of phase with $V_{a'n}$ for positive sequence.

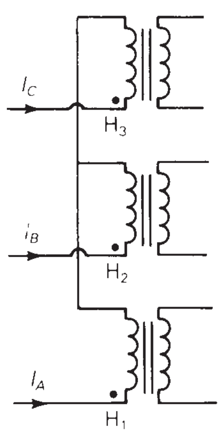


(a) Y–Δ–Δ transformer (b) Y–Δ–zig-zag transformer (c) Extended Δ autotransformer

FIGURE 3.36 Problems 3.32 and 3.52 (Coils drawn on the same vertical line are on the same core)

FIGURE 3.37

Problem 3.33



3.34 Three single-phase, two-winding transformers, each rated 450 MVA, 20 kV/288.7 kV, with leakage reactance $X_{eq} = 0.10$ per unit, are connected to form a three-phase bank. The high-voltage windings are connected in Y with a solidly grounded neutral. Draw the per-unit equivalent circuit if the low-voltage windings are connected (a) in Δ with American standard phase shift, (b) in Y with an open neutral. Use the transformer ratings as base quantities. Winding resistances and exciting current are neglected.

3.35 Consider a bank of three single-phase two-winding transformers whose high-voltage terminals are connected to a three-phase, 13.8-kV feeder. The low-voltage terminals are connected to a three-phase substation load rated 2.1 MVA and 2.3 kV. Determine the required voltage, current, and MVA ratings of both windings of each transformer, when the high-voltage/low-voltage windings are connected (a) Y–Δ, (b) Δ–Y, (c) Y–Y, and (d) Δ–Δ.

3.36 Three single-phase two-winding transformers, each rated 25 MVA, 34.5/13.8 kV, are connected to form a three-phase Δ–Δ bank. Balanced positive-sequence voltages are applied to the high-voltage terminals, and a balanced, resistive Y load connected to the low-voltage terminals absorbs 75 MW at 13.8 kV. If one of the single-phase transformers is removed (resulting in an open-Δ connection) and the balanced load is simultaneously reduced to 43.3 MW (57.7% of the original value), determine (a) the load voltages V_{an}, V_{bn}, and V_{cn}; (b) load currents I_a, I_b, and I_c; and (c) the MVA supplied by each of the remaining two transformers. Are balanced voltages still applied to the load? Is the open-Δ transformer overloaded?

3.37 Three single-phase two-winding transformers, each rated 25 MVA, 38.1/3.81 kV, are connected to form a three-phase Y–Δ bank with a balanced Y-connected resistive load of 0.6 Ω per phase on the low-voltage side. By choosing a base of 75 MVA (three phase) and 66 kV (line-to-line) for the high voltage side of the transformer bank, specify the base quantities for the low-voltage side. Determine the per-unit resistance of the load on the base for the low-voltage side. Then determine the load resistance R_L in ohms referred to the high-voltage side and the per-unit value of this load resistance on the chosen base.

3.38 Consider a three-phase generator rated 300 MVA, 23 kV, supplying a system load of 240 MVA and 0.9 power factor lagging at 230 kV through a 330 MVA, 23 Δ/230 Y-kV step-up transformer with a leakage reactance of 0.11 per unit. (a) Neglecting the exciting current and choosing base values at the load of 100 MVA and 230 kV, find the phasor currents I_A, I_B, and I_C supplied to the load in per unit. (b) By choosing the load terminal voltage V_A as reference, specify the proper base for the generator circuit and determine the generator voltage V as well as the phasor currents I_a, I_b, and I_c, from the generator. (*Note:* Take into account the phase shift of the transformer.) (c) Find the generator terminal voltage in kV and the real power supplied by the generator in MW. (d) By omitting the transformer phase shift altogether, check to see whether you get the same magnitude of generator terminal voltage and real power delivered by the generator.

SECTION 3.5

3.39 The leakage reactance of a three-phase, 300-MVA, 230 Y/23 Δ-kV transformer is 0.06 per unit based on its own ratings. The Y winding has a solidly grounded neutral. Draw the per-unit equivalent circuit. Neglect the exciting admittance and assume American standard phase shift.

3.40 Choosing system bases to be 240/24 kV and 100 MVA, redraw the per-unit equivalent circuit for Problem 3.39.

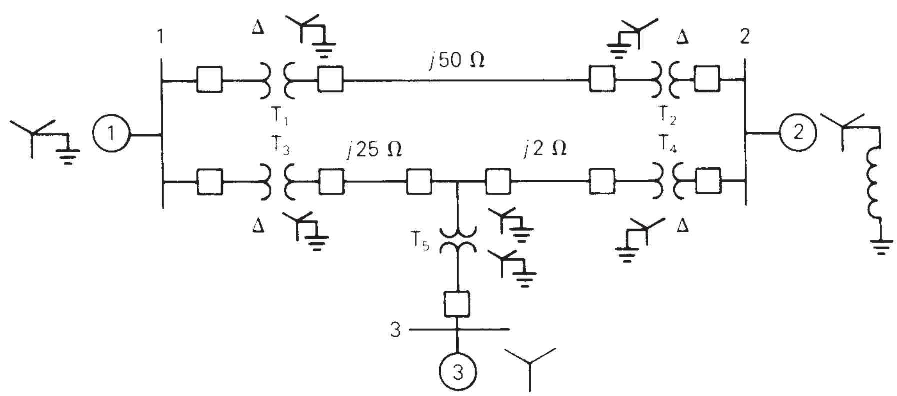
3.41 Consider the single-line diagram of the power system shown in Figure 3.38. Equipment ratings are:

Generator 1:	1000 MVA, 18 kV, $X'' = 0.2$ per unit
Generator 2:	1000 MVA, 18 kV, $X'' = 0.2$
Synchronous motor 3:	1500 MVA, 20 kV, $X'' = 0.2$
Three-phase Δ–Y transformers T_1, T_2, T_3, T_4:	1000 MVA, 500 kV Y/20 kV Δ, $X = 0.1$
Three-phase Y–Y transformer T_5:	1500 MVA, 500 kV Y/20 kV Y, $X = 0.1$

Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the equivalent reactance diagram. Use a base of 100 MVA and 500 kV for the 50-ohm line. Determine the per-unit reactances.

FIGURE 3.38

Problems 3.41 and 3.42



3.42 For the power system in Problem 3.41, the synchronous motor absorbs 1500 MW at 0.8 power factor leading with the bus 3 voltage at 18 kV. Determine the bus 1 and bus 2 voltages in kV. Assume that generators 1 and 2 deliver equal real powers and equal reactive powers. Also assume a balanced three-phase system with positive-sequence sources.

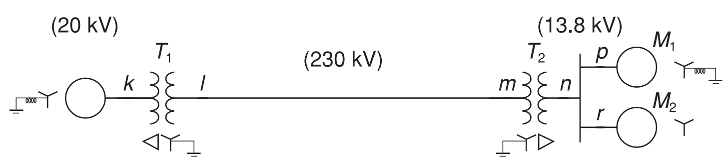
3.43 Three single-phase transformers, each rated 10 MVA, 66.4/12.5 kV, 60 Hz, with an equivalent series reactance of 0.1 per unit divided equally between primary and secondary, are connected in a three-phase bank. The high-voltage windings are Y connected and their terminals are directly connected to a 115-kV three-phase bus. The secondary terminals are all shorted together. Find the currents entering the high-voltage terminals and leaving the low-voltage terminals if the low-voltage windings are (a) Y connected, (b) Δ connected.

3.44 A 130-MVA, 13.2-kV three-phase generator, which has a positive-sequence reactance of 1.5 per unit on the generator base, is connected to a 135-MVA, 13.2 Δ/115 Y-kV step-up transformer with a series impedance of $(0.005 + j0.1)$ per unit on its own base. (a) Calculate the per-unit generator reactance on the transformer base. (b) The load at the transformer terminals is 15 MW at unity power factor and at 115 kV. Choosing the transformer high-side voltage as the reference phasor, draw a phasor diagram for this condition. (c) For the condition of part (b), find the transformer low-side voltage and the generator internal voltage behind its reactance. Also compute the generator output power and power factor.

3.45 Figure 3.39 shows a one-line diagram of a system in which the three-phase generator is rated 300 MVA, 20 kV with a subtransient reactance of 0.2 per unit and with its neutral grounded through a 0.4-Ω reactor. The transmission line is 64 km long with a series reactance of 0.5 Ω/km. The three-phase transformer T_1 is rated 350 MVA, 230/20 kV with a leakage reactance of 0.1 per unit. Transformer T_2 is composed of three single-phase transformers, each rated 100 MVA, 127/13.2 kV with a leakage reactance of 0.1 per unit. Two 13.2-kV motors M_1 and M_2 with a subtransient reactance of 0.2 per unit for each motor represent the load. M_1 has a rated input of 200 MVA with its neutral grounded through a 0.4-Ω current-limiting reactor. M_2 has a rated input of 100 MVA with its neutral not connected to ground. Neglect phase shifts associated with the transformers. Choose the generator rating as base in the generator circuit and draw the positive-sequence reactance diagram showing all reactances in per unit.

FIGURE 3.39

Problems 3.45 and 3.46

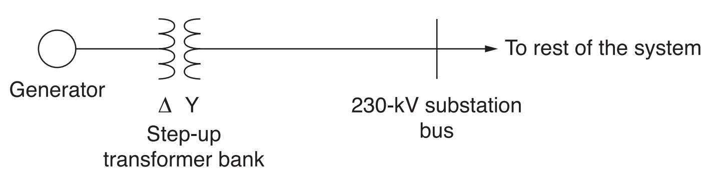


3.46 The motors M_1 and M_2 of Problem 3.45 have inputs of 120 and 60 MW, respectively, at 13.2 kV, and both operate at unity power factor. Determine the generator terminal voltage and voltage regulation of the line. Neglect transformer phase shifts.

3.47 Consider the one-line diagram shown in Figure 3.40. The three-phase transformer bank is made up of three identical single-phase transformers, each specified by $X_l = 0.24\ \Omega$ (on the low-voltage side), negligible resistance and magnetizing current, and turns ratio $\eta = N_2/N_1 = 10$. The transformer bank is delivering 100 MW at 0.8 p.f. lagging to a substation bus whose voltage is 230 kV.
(a) Determine the primary current magnitude, primary voltage (line-to-line) magnitude, and the three-phase complex power supplied by the generator. Choose the line-to-neutral voltage at the bus, $V_{a'n'}$, as the reference. Account for the phase shift, and assume positive-sequence operation.
(b) Find the phase shift between the primary and secondary voltages.

FIGURE 3.40

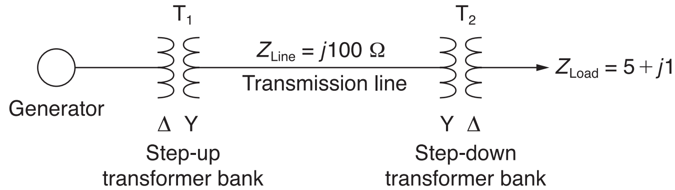
One-line diagram for Problem 3.47



3.48 With the same transformer banks as in Problem 3.47, Figure 3.41 shows the one-line diagram of a generator, a step-up transformer bank, a transmission line, a step-down

FIGURE 3.41

One-line diagram for Problem 3.48



transformer bank, and an impedance load. The generator terminal voltage is 15 kV (line-to-line).

(a) Draw the per-phase equivalent circuit, accounting for phase shifts for positive-sequence operation.

(b) By choosing the line-to-neutral generator terminal voltage as the reference, determine the magnitudes of the generator current, transmission-line current, load current, and line-to-line load voltage. Also, find the three-phase complex power delivered to the load.

3.49 Consider the single-line diagram of a power system shown in Figure 3.42 with equipment ratings given below:

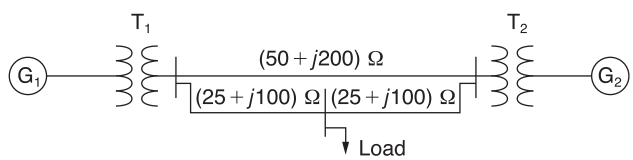
Generator G_1:	50 MVA, 13.2 kV, $x = 0.15$ pu
Generator G_2:	20 MVA, 13.8 kV, $x = 0.15$ pu
three-phase Δ–Y transformer T_1:	80 MVA, 13.2 Δ/165 Y kV, $X = 0.1$ pu
three-phase Y–Δ transformer T_2:	40 MVA, 165 Y/13.8 Δ kV, $X = 0.1$ pu
Load:	40 MVA, 0.8 p.f. lagging, operating at 150 kV

Choose a base of 100 MVA for the system and 132-kV base in the transmission-line circuit. Let the load be modeled as a parallel combination of resistance and inductance. Neglect transformer phase shifts.

Draw a per-phase equivalent circuit of the system showing all impedances in per unit.

FIGURE 3.42

One-line diagram for Problem 3.49



SECTION 3.6

3.50 A single-phase three-winding transformer has the following parameters: $Z_1 = Z_2 = Z_3 = 0 + j0.05$, $G_c = 0$, and $B_m = 0.2$ per unit. Three identical transformers, as described, are connected with their primaries in Y (solidly grounded neutral) and with their secondaries and tertiaries in Δ. Draw the per-unit sequence networks of this transformer bank.

3.51 The ratings of a three-phase three-winding transformer are:

Primary (1):	Y connected, 66 kV, 15 MVA
Secondary (2):	Y connected, 13.2 kV, 10 MVA
Tertiary (3):	Δ connected, 2.3 kV, 5 MVA

Neglecting winding resistances and exciting current, the per-unit leakage reactances are:

$$X_{12} = 0.08 \text{ on a 15-MVA, 66-kV base}$$

$$X_{13} = 0.10 \text{ on a 15-MVA, 66-kV base}$$

$$X_{23} = 0.09 \text{ on a 10-MVA, 13.2-kV base}$$

(a) Determine the per-unit reactances X_1, X_2, X_3 of the equivalent circuit on a 15-MVA, 66-kV base at the primary terminals. (b) Purely resistive loads of 7.5 MW at 13.2 kV and 5 MW at 2.3 kV are connected to the secondary and tertiary sides of the transformer, respectively. Draw the per-unit impedance diagram, showing the per-unit impedances on a 15-MVA, 66-kV base at the primary terminals.

3.52 Draw the per-unit equivalent circuit for the transformers shown in Figure 3.34. Include ideal phase-shifting transformers showing phase shifts determined in Problem 3.32. Assume that all windings have the same kVA rating and that the equivalent leakage reactance of any two windings with the third winding open is 0.10 per unit. Neglect the exciting admittance.

3.53 The ratings of a three-phase, three-winding transformer are:

Primary:	Y connected, 66 kV, 15 MVA
Secondary:	Y connected, 13.2 kV, 10 MVA
Tertiary:	Δ connected, 2.3 kV, 5 MVA

Neglecting resistances and exciting current, the leakage reactances are:

$$X_{PS} = 0.07 \text{ per unit on a 15-MVA, 66-kV base}$$

$$X_{PT} = 0.09 \text{ per unit on a 15-MVA, 66-kV base}$$

$$X_{ST} = 0.08 \text{ per unit on a 10-MVA, 13.2-kV base}$$

Determine the per-unit reactances of the per-phase equivalent circuit using a base of 15 MVA and 66 kV for the primary.

3.54 An infinite bus, which is a constant voltage source, is connected to the primary of the three-winding transformer of Problem 3.53. A 7.5-MVA, 13.2-kV synchronous motor with a subtransient reactance of 0.2 per unit is connected to the transformer secondary. A 5-MW, 2.3-kV three-phase resistive load is connected to the tertiary. Choosing a base of 66 kV and 15 MVA in the primary, draw the impedance diagram of the system showing per-unit impedances. Neglect transformer exciting current, phase shifts, and all resistances except the resistive load.

SECTION 3.7

3.55 A single-phase 10-kVA, 2300/230-volt, 60-Hz two-winding distribution transformer is connected as an autotransformer to step up the voltage from 2300 to 2530 volts. (a) Draw a schematic diagram of this arrangement, showing all voltages and currents when delivering full load at rated voltage. (b) Find the permissible kVA rating of the autotransformer if the winding currents and voltages are not to exceed the rated values as a two-winding transformer. How much of this kVA rating is transformed by magnetic induction? (c) The following data are obtained from tests carried out on the transformer when it is connected as a two-winding transformer:

Open-circuit test with the low-voltage terminals excited:
Applied voltage = 230 V, Input current = 0.45 A, Input power = 70 W.

Short-circuit test with the high-voltage terminals excited:
Applied voltage = 120 V, Input current = 4.5 A, Input power = 240 W.

Based on the data, compute the efficiency of the autotransformer corresponding to full load, rated voltage, and 0.8 power factor lagging. Comment on why the efficiency is higher as an autotransformer than as a two-winding transformer.

3.56 Three single-phase two-winding transformers, each rated 3 kVA, 220/110 volts, 60 Hz, with a 0.10 per-unit leakage reactance, are connected as a three-phase extended Δ autotransformer bank, as shown in Figure 3.31(c). The low-voltage Δ winding has a 110 volt rating. (a) Draw the positive-sequence phasor diagram and show that the high-voltage winding has a 479.5 volt rating. (b) A three-phase load connected to the low-voltage terminals absorbs 6 kW at 110 volts and at 0.8 power factor lagging. Draw the per-unit impedance diagram and calculate the voltage and current at the high-voltage terminals. Assume positive-sequence operation.

3.57 A two-winding single-phase transformer rated 60 kVA, 240/1200 V, 60 Hz, has an efficiency of 0.96 when operated at rated load, 0.8 power factor lagging. This transformer is to be utilized as a 1440/1200-V step-down autotransformer in a power distribution system. (a) Find the permissible kVA rating of the autotransformer if the winding currents and voltages are not to exceed the ratings as a two-winding transformer. Assume an ideal transformer. (b) Determine the efficiency of the autotransformer with the kVA loading of part (a) and 0.8 power factor leading.

3.58 A single-phase two-winding transformer rated 90 MVA, 80/120 kV is to be connected as an autotransformer rated 80/200 kV. Assume that the transformer is ideal. (a) Draw a schematic diagram of the ideal transformer connected as an autotransformer, showing the voltages, currents, and dot notation for polarity. (b) Determine the permissible kVA rating of the autotransformer if the winding currents and voltages are not to exceed the rated values as a two-winding transformer. How much of the kVA rating is transferred by magnetic induction?

SECTION 3.8

3.59 The two parallel lines in Example 3.13 supply a balanced load with a load current of $1.0\underline{/-30^\circ}$ per unit. Determine the real and reactive power supplied to the load bus from each parallel line with (a) no regulating transformer, (b) the voltage-magnitude-regulating transformer in Example 3.13(a), and (c) the phase-angle-regulating transformer in Example 3.13(b). Assume that the voltage at bus *abc* is adjusted so that the voltage at bus $a'b'c'$ remains constant at $1.0\underline{/0^\circ}$ per unit. Also assume positive sequence. Comment on the effects of the regulating transformers.

PW **3.60** PowerWorld Simulator case Problem 3.60 duplicates Example 3.13 except that a resistance term of 0.06 per unit has been added to the transformer and 0.05 per unit to the transmission line. Since the system is no longer lossless, a field showing the real power losses has also been added to the one-line. With the LTC tap fixed at 1.05, plot the real power losses as the phase shift angle is varied from -10 to $+10$ degrees. What value of phase shift minimizes the system losses?

PW **3.61** Repeat Problem 3.60, except keep the phase-shift angle fixed at 3.0 degrees, while varying the LTC tap between 0.9 and 1.1. What tap value minimizes the real power losses?

3.62 Rework Example 3.12 for a +10% tap, providing a 10% increase for the high-voltage winding.

3.63 A 23/230-kV step-up transformer feeds a three-phase transmission line, which in turn supplies a 150-MVA, 0.8 lagging power factor load through a step-down 230/23-kV transformer. The impedance of the line and transformers at 230 kV is $18 + j60\ \Omega$. Determine the tap setting for each transformer to maintain the voltage at the load at 23 kV.

3.64 The per-unit equivalent circuit of two transformers T_a and T_b connected in parallel, with the same nominal voltage ratio and the same reactance of 0.1 per unit on the same base, is shown in Figure 3.43. Transformer T_b has a voltage-magnitude step-up toward the load of 1.05 times that of T_b (that is, the tap on the secondary winding of T_a is set to 1.05). The load is represented by $0.8 + j0.6$ per unit at a voltage $V_2 = 1.0\underline{/0^\circ}$ per unit. Determine the complex power in per unit transmitted to the load through each transformer. Comment on how the transformers share the real and reactive powers.

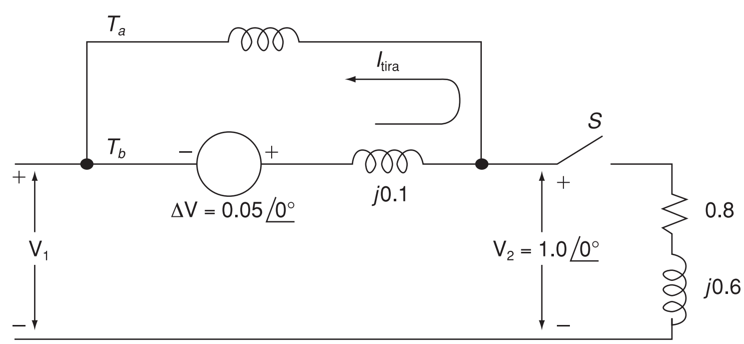


FIGURE 3.43
Problem 3.64

3.65 Reconsider Problem 3.64 with the change that now T_b includes both a transformer of the same turns ratio as T_a and a regulating transformer with a 3° phase shift. On the base of T_a, the impedance of the two components of T_b is $j0.1$ per unit. Determine the complex power in per unit transmitted to the load through each transformer. Comment on how the transformers share the real and reactive powers.

CASE STUDY QUESTIONS

A. What are the potential consequences of running a transmission transformer to failure with no available spare to replace it?

B. What are the benefits of sharing spare transmission transformers among utility companies?

C. Where should spare transmission be located?

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