Chapter 3

Power transmission and sizing

While the previous chapters have considered the analysis of a proposed motor-drive system and obtaining the application requirements, it must be recognised that the system comprises a large number of mechanical component. Each of these components, for example couplings, gearboxes and lead screws, will have their own inertias and frictional forces, which all need to be considered as part of the sizing process. This chapter considers power transmission components found in applications, and discusses their impact on overall system performance, and concludes with the process required to determine the detailed specifications of the motor and the drive.

The design parameters of the mechanical transmission system of the actuator must be identified at the earliest possible stage. However, it must be realised that the system will, in all probability, be subjected to detailed design changes as development proceeds. It should also be appreciated that the selection of a motor and its associated drive, together with their integration into a mechanical system, is by necessity an iterative process; any solution is a compromise. For this reason, this chapter can only give a broad outline of the procedures to be followed; the detail is determined by the engineer's insight into the problem, particularly for constraints of a non-engineering nature, such as a company's or a customer's policy, which may dictate that only a certain range of components or suppliers can be used.

In general, once the overall application, and the speed and torque (or in the case of a linear motor, speed and force) requirements of the total system have been clearly identified, various broad combinations of motors and drives can be reviewed. The principles governing the sizing of a motor drive are largely independent of the type of motor being considered. In brief, adequate sizing involves determining the motor's speed range, and determining the continuous and intermittent peak torque or force which are required to allow the overall system to perform to its specification. Once these factors have been determined, an iterative process using the manufacturer's specifications and data sheets will lead to as close an optimum solution as is possible.

3.1 Gearboxes

As discussed in Section 2.1.3 a conventional gear train is made up of two or more gears. There will be a change in the angular velocity and torque between an input and output shaft; the fundamental speed relationship is given by

$$n = \pm \frac{\omega_i}{\omega_o} = \pm \frac{N_o}{N_i} \tag{3.1}$$

where N_i and ω_i are the number of teeth on, and the angular velocity of, the input gear, and N_o and ω_o are the number of teeth on, and the angular velocity of, the output gear. In equation (3.1) a negative sign is used when two external gears are meshing, Figure 3.1(a), or a positive sign indicates that system where an internal gear is meshing with an internal gear, Figure 3.1(b).

In the case where an idler gear is included, the gear ratio can be calculated in an identical fashion, hence for an external gear train, Figures 3.1(c) and 3.1(d),

$$n = \frac{\omega_{in}}{\omega_{out}} = \left(-\frac{N_2}{N_1}\right)\left(-\frac{N_3}{N_2}\right) = \frac{N_3}{N_1} \tag{3.2}$$

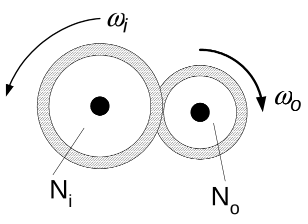
The direction of the output shaft is reversed for an internal gear train, Figure 3.1(d). In practice the actual gear train can consist of either a spur, or helical gear wheels. A spur gear (see Figure 3.2(a)) is normally employed within conventional gear trains, and has the advantage of producing minimal axial forces which reduce problems connected with motion of the gear bearings. Helical gears (see Figure 3.2(b)) are widely used in robotic systems since they give a higher contact ratio than spur gears for the same ratio; the penalty is axial gear load. The limiting factors in gear transmission are the stiffness of the gear teeth, which can be maximised by selecting the largest-diameter gear wheel which is practical for the application, and backlash or lost motion between individual gears. The net result of these problems is a loss in accuracy through the gear train, which can have an adverse affect on the overall accuracy of a controlled axis.

In many applications conventional gear trains can be replaced by complete gearboxes (in particular those of a planetary, harmonic, or cycloid design) to produce compact drives with high reduction ratios.

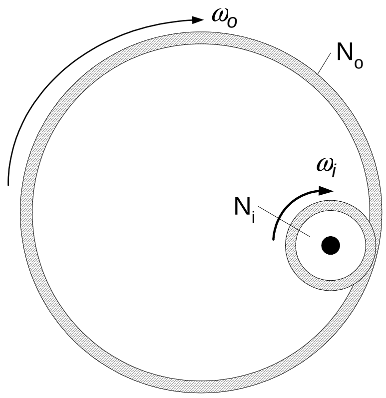
3.1.1 Planetary gearbox

A Planetary gearbox is co-axial and is particularly suitable for high torque, low speed applications. It is extremely price-competitive against other gear systems and offers high efficiency with minimum dimensions. For similar output torques the planetary gear system is the most compact gearbox on the market. The internal details of a planetary gearbox are shown in Figure 3.3; a typical planetary gear box consists of the following:

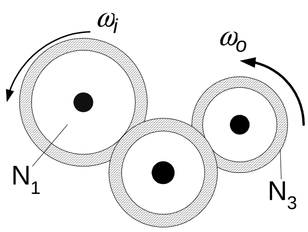
- A sun gear, which may or may not be fixed.



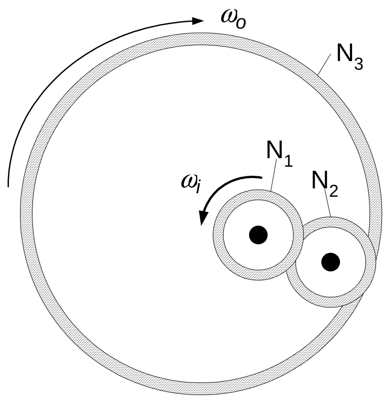
(a) External gears



(b) Internal gears



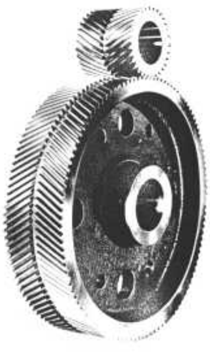
(c) External gear train



(d) Internal gear train

Figure 3.1. Examples of the dependency of direction and velocity of the output shaft on the type of gearing.

(a) Spur gears.



(b) Helical gears.

Figure 3.2. Conventional gears.

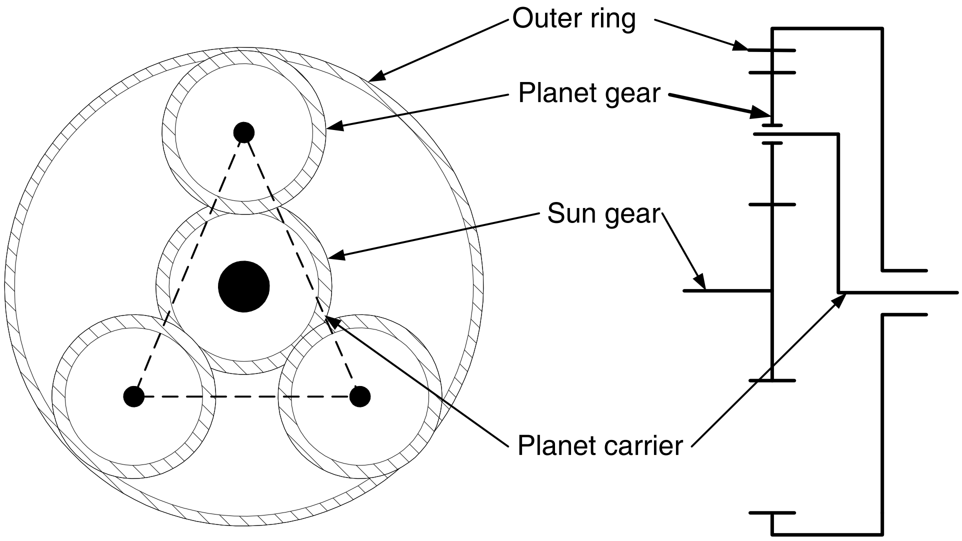


Figure 3.3. A planetary gearbox; the output from the gearbox is from the three planet gears via the planet carrier, while the sun gear is driven. In this case the outer ring is fixed, the input is via the sun, and the output via the planet carrier.

- A number of planetary gears.
- Planet gear carrier.
- An internal gear ring, which may not be used on all systems.

This design results in relatively low speeds between the individual gear wheels and this results in a highly efficient design. One particular advantage is that the gearbox has no bending moments generated by the transmitted torque; consequently, the stiffness is considerably higher than in comparable configuration. Also, they can be assembled coaxially with the motor, leading to a more compact overall design. The relationship for a planetary gearbox can be shown to be (Waldron and Kinzel, 1999)

$$\frac{\omega_{sun} - \omega_{carrier}}{\omega_{ring} - \omega_{carrier}} = -\frac{N_{ring}}{N_{sun}} \tag{3.3}$$

where ω_{sun}, $\omega_{carrier}$ and ω_{ring} are the angular speeds of the sun gear, planet carrier and ring with reference to ground. N_{ring} and N_{sun} are the number of teeth on the sun and ring respectively. Given any two angular velocities, the third can be calculated – normally the ring if fixed hence $\omega_{ring} = 0$. In addition it is important to define the direction of rotation; normally clockwise is positive, and counter-clockwise is negative.

Example 3.1

A planetary gearbox has 200 teeth on its ring, and 40 teeth on its sun gear. The input to the sun gear is 100 rev min^{-1} clockwise. Determine the output speed if the ring is fixed, or rotating at 5 rev min^{-1} either clockwise or counterclockwise.

Rearranging equation (3.3), gives

$$\omega_{carrier} = \frac{N_{sun}\omega_{sun} - N_{ring}\omega_{ring}}{N_{sun} - N_{ring}}$$

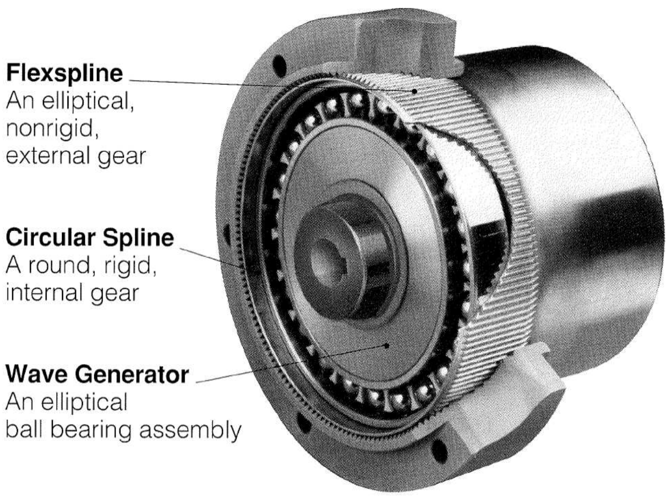
- When the ring is rotated at 5 rev min^{-1} clockwise, the output speed is 18.75 rev min^{-1} conterclockwise.
- When the ring is fixed, the output speed is 25 rev min^{-1} conterclockwise.
- When the ring is rotated at 5 rev min^{-1} counterclockwise, the output speed is 31.25 rev min^{-1} counterclockwise.

This simple example demonstrates that the output speed can be modified by changing the angular velocity of the ring, and that the direction of the ring adds or subtracts angular velocity to the output.

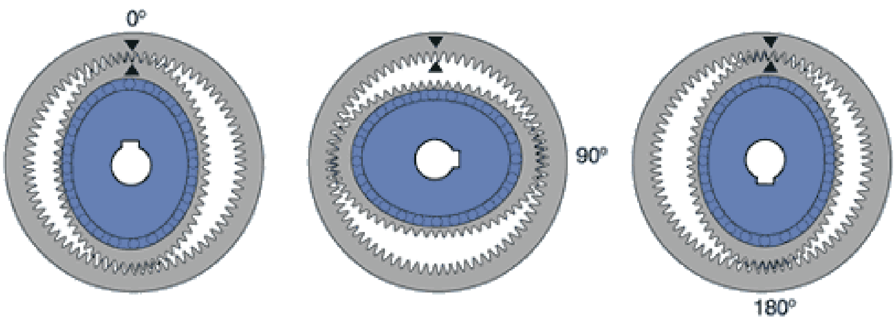
3.1.2 Harmonic gearbox

A harmonic gearbox will provide a very high gear ratio with minimal backlash within a compact unit. As shown in Figure 3.4(a), a harmonic drive is made up of three main parts, the circular spline, the wave generator, and the flexible flexspline. The design of these components depends on the type of gearbox, in this example the flexispline forms a cup. The operation of an harmonic gearbox can be appreciated by considering the circular spline to be fixed, with the teeth of the flexspline to engage on the circular spline. The key to the operation is the difference of two teeth (see Figure 3.4(b)) between the flexspline and the circular spline. The bearings on the elliptical-wave generator support the flexspline, while the wave generator causes it to flex. Only a small percentage of the flexispline's teeth are engaged at the ends of the oval shape assumed by the flexspline while it is rotating, so there is freedom for the flexspline to rotate by the equivalent of two teeth relative to the circular spline during rotation of the wave generator. Because of the large number of teeth which are in mesh at any one time, harmonic drives have a high torque capability; in addition the backlash is very small, being typically less that 30" of arc.

In practice, any two of the three components that make up the gearbox can be used as the input to, and the output from, the gearbox, giving the designer considerable flexibility. The robotic hand shown in Figure incorporates three harmonic gearboxes of a pancake design where the flexispline is a cylinder equal in width to the wave generator.



(a) Components of a harmonic gearbox



(b) Operation of a harmonic gear box, for each 360^o rotation of the wave generator the flexspline moves 2 teeth. The deflection of the flexspline has been exaggerated.

Figure 3.4. Construction and operation of an HDC harmonic gear box. Reproduced with permission from Harmonic Drive Technologies, Nabtesco Inc, Peabody, MA.

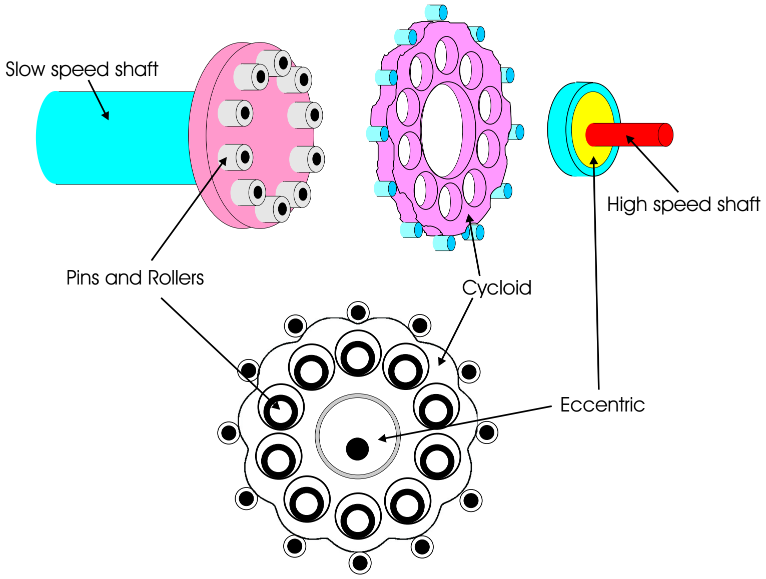


Figure 3.5. A schematic diagram of a cycloid speed reducer. The relationship between the eccentric, cylcoid and slow-speed output shaft is clearly visible. It should be noted that in the diagram only one cycloid disc is shown, commercial systems typically nave a number of discs, to improve power handelling.

3.1.3 Cycloid gearbox

The cycloid gearbox is of a co-axial design and offers high reduction ratios in a single stage, and is noted for its high stiffness and low backlash. The gearbox is suitable for heavy duty applications, since it has a very high shock load capability of up to 500%. Commercially cycloid gearboxes are available in a range of sizes with ratios between 6:1 and 120:1 and with a power transmission capability of up to approximately 100 kW. The gearbox design, which is both highly reliable and efficient, undertakes the speed conversion by using rolling actions, with the power being transmitted by cycloid discs driven by an eccentric bearing.

The significant features of this type of gearbox are shown in Figure 3.5. The gear box consists of four main components:

- A high speed shaft with an eccentric bearing.
- Cycloid disc(s).
- Ring gear housing with pins and rollers.
- Slow speed shaft with pins and rollers.

As the eccentric rotates, it rolls the cycloid disc around the inner circumference of the ring gear housing. The resultant action is similar to that of a disc

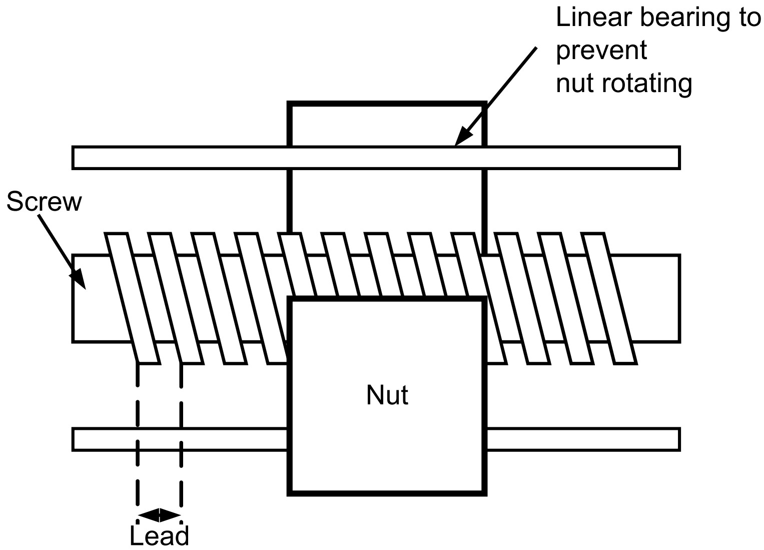


Figure 3.6. The construction of a lead screw. The screw illustrated is single start with an ACME thread.

rolling around the inside of a ring. As the cycloid disc travels clockwise around the gear ring, the disc turns counterclockwise on its axis. The teeth of the cycloid discs engage successively with the pins on the fixed gear ring, thus providing the reduction in angular velocity. The cycloid disc drives the low speed output shaft. The reduction ratio is determined by the number of 'teeth' on the cycloid disc, which has one less 'tooth' than there are rollers on the gear ring. The number of teeth on the cycloid disc equals the reduction ratio, as one revolution of the high speed shaft, causes the cycloid disc to move in the opposite direction by one 'tooth'.

3.2 Lead and ball screws

The general arrangement of a lead screw is shown in Figure 3.6. As the screw is rotated, the nut, which is constrained from rotating, moves along the thread. The linear speed of the load is determined by the rotational speed of the screw and the screw's lead. The distance moved by one turn of the lead screw is termed the lead: this should not be confused with the pitch, which is the distance between the threads. In the case of a single start thread, the lead is equal to the pitch; however the pitch is smaller than the lead on a multi-start thread. In a lead screw there is direct contact between the screw and the nut, and this leads to relatively high friction and hence an inefficient drive. For precision applications, ball screws are used due to their low friction and hence their good dynamic response. A ball screw is identical in principle to a lead screw, but the power is transmitted to the nut via ball bearings located in the thread on the nut (see Figure 3.7).

The relationship between the rotational and linear speed for both the lead and

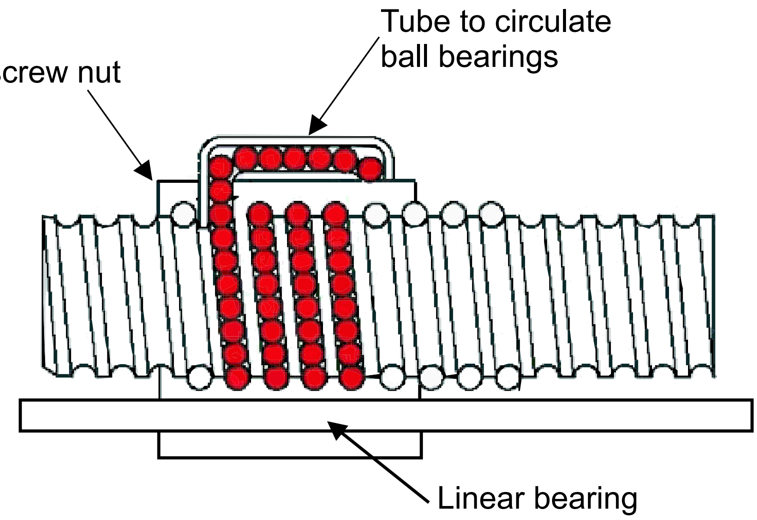


Figure 3.7. The cross section of a high performance ball screw, the circulating balls are clearly visible.

ball screw is given by:

$$N_L = \frac{V_L}{L} \tag{3.4}$$

where N_L is the rotational speed in rev min^{-1}, V$_L$ is the linear speed in m min^{-1} and L is the lead (in metres). The inertia of the complete system is the sum of the screw inertia J_s and the reflected inertia of the load J_L

$$I_{tot} = I_s + I_L \tag{3.5}$$

where

$$J_s = \frac{M_s r^2}{2} \tag{3.6}$$

$$J_L = M_L \left[\frac{L}{2\pi}\right]^2 \tag{3.7}$$

where M_L is the load's mass in kg, M_s is the screw's mass in kg and r is the radius of the lead screw (in metres). In addition, the static forces, both frictional and the forces required by the load, need to be converted to a torque at the lead screw's input. The torque caused by external forces, F_L, will result in a torque requirement of

$$T_L = \frac{LF_L}{2\pi} \tag{3.8}$$

and a possible torque resulting from slideway friction of

Table 3.1. Typical efficiencies for lead and ball screws

System type	Efficiency
Ball screw	0.95
Lead screw	0.90
Rolled-ball lead screw	0.80
ACME threaded lead screw	0.40

$$T_f = \frac{LM_L g \cos\theta \, \mu}{2\pi} \tag{3.9}$$

where θ is the inclination of the slideway. It has been assumed so far that the efficiency of the lead screw is one hundred per cent. In practice, losses will occur and the torques will need to be divided by the lead-screw efficiency, ϵ, see Table 3.1, hence

$$T_{required} = \frac{(T_f + T_L)}{\epsilon} \tag{3.10}$$

A number of linear digital actuators are based on stepper-motor technology, as discussed in Chapter 8, where the rotor has been modified to form the nut of the lead screw. Energisation of the windings will cause the lead screw to move a defined distance, which is typically in the range 0.025–0.1 mm depending on the step angle and the lead of the lead screw. For a motor with a step angle of θ radians, fitted to a lead screw of lead L, the incremental linear step, S, is given by

$$S = \frac{\theta L}{2\pi} \tag{3.11}$$

Example 3.2

Determine the speed and torque requirements for the following lead screw application:

- *The length (L_s) of a lead screw is 1 m, its radius (R_s) is 20 mm and is manufactured from steel (ρ = 7850 kg m^{-3}). The lead (L) is 6 mm rev^{-1}. The efficiency (ϵ) of the lead screw is 0.85.*

- *The total linear mass (M_L) to be moved is 150 kg. The coefficient of friction (μ)between the mass and its slipway is 0.5. A 50 N linear force (F_L) is being applied to the mass.*

- *The maximum speed of the load (V_L) has to be 6 m min^{-1} and the time (t) the system is required to reach this speed in 1 s.*

The mass of the lead screw and its inertia are calculated first:

$$M_s = \rho \pi R_s^2 L_s = 9.97 \text{ kg and } J_s = \frac{M_s R_s^2}{2} = 1.97 \times 10^{-3} \text{ kg m}^{-2}$$

The total inertia can be calculated by adding the reflected inertia from the load to the lead screw's inertia:

$$J_{tot} = J_s + M_L \left(\frac{L}{2\pi}\right) = 2.11 \text{ kg m}^{-2}$$

The torque required to drive the load against the external and frictional forces, allowing for the efficiency of the lead screw, is given by

$$T_{ext} = \frac{1}{\epsilon}\left(\frac{LF_L}{2\pi} + \frac{LM_L g\mu}{2\pi}\right) = 0.79 \text{ Nm}$$

The input speed required is given by

$$N_L = \frac{V_L}{L} = 1000 \text{ rev min}^{-1} = 104.7 \text{ rad s}^{-1}$$

and the input torque to accelerate the system is given by

$$T_{in} = \frac{N_L}{t} J_{tot} + T_{ext} = 1 \text{ Nm}$$

3.3 Belt drives

The use of a toothed belt or a chain drive is an effective method of power transmission between the motor and the load, while still retaining synchronism between the motor and the load (see Figure 3.8). The use of belts, manufactured in rubber or plastic, offers a potential cost saving over other methods of transmission. Typical applications that incorporate belt drives include printers, ticketing machines, robotics and scanners. In the selection of the a belt drive, careful consideration has to be given to ensuring that positional accuracy is not compromised by selection of an incorrect component. A belt drive can be used in one of two ways, either as a linear drive system (for example, positioning a printer head) or as speed changer.

In a linear drive application, the rotational input speed is given by

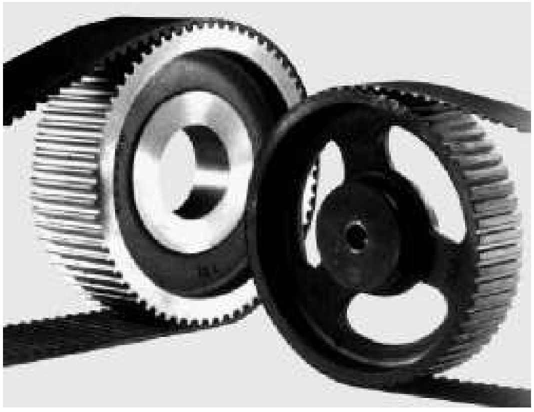


Figure 3.8. Synchronous belts and pulleys suitable for servo-drive applications.

$$N_i = \frac{V_L}{\pi D} \quad (3.12)$$

where D is the diameter of the driving pulley (in metres), and V_L is the required linear speed (in m s^{-1}). The inertia of the transmission system, J_{tot}, must include the contributions from all the rotational elements, including the idler pulleys, any rotating load, and the belt:

$$I_{tot} = I_p + I_L \quad (3.13)$$

where I_p is the sum of the inertias of all the rotating elements. The load and belt inertia is given by

$$I_L = \frac{MD^2}{4} \quad (3.14)$$

where the mass, M, is the sum of the linear load (if present) and the transmission-belt masses. An external linear force applied to the belt will result in a torque at the input drive shaft of

$$T_{in} = \frac{DF}{2} \quad (3.15)$$

In a linear application, the frictional force, F_f, must be carefully determined as it will result in an additional torque

$$T_f = \frac{DF_f}{2} \quad (3.16)$$

If a belt drive is used as a speed changer, the output speed is a ratio of the pulley diameters

$$n = \frac{D}{d} \quad (3.17)$$

and the input torque which is required to drive the load torque, T_i, is given by

$$T_i = \frac{T_{out}}{n} \tag{3.18}$$

The inertia seen at the input to the belt drive is the sum of the inertias of the pulleys, the belt, the idlers, and the load, taking into account the effects of the gearing ratio; that is

$$I_{tot} = I_{p1} + I_{belt} + \frac{I_L + I_{p2}}{n^2} \tag{3.19}$$

Where the inertia of the belt can be calculated from equation (3.14) and J_{p2} is the inertia of the driven pulley modified by the gear ratio. The drive torque which is required can then be computed; the losses can be taken into account by using equation (3.10).

The main selection criteria for a belt or chain is the distance, or pitch, between the belt's teeth (this must be identical to the value for the pulleys) and the drive characteristics. The belt pitch and the sizes of the pulleys will directly determine the number of teeth which are in mesh at a particular time, and hence the power that can be transmitted. The power that has to be transmitted can be determined by the input torque and speed. The greater the number of teeth in mesh, the greater is the power that can be transmitted; the number of teeth in mesh on the smaller pulley, which is the system's limiting value, and can be determined from

$$\text{Teeth in mesh} = \left[\pi - 2\,\sin^{-1}\frac{(D-d)}{C}\right] \times \frac{\text{Teeth on the small pulley}}{2\pi} \tag{3.20}$$

The selection of the correct belt requires detailed knowledge of the belt material, together with the load and drive characteristics. In the manufacturer's data sheets, belts and chains are normally classified by their power-transmission capabilities. In order to calculate the effect that the load and the drive have on the belt, use is made of an application factor, which is determined by the load and/or drive. Typical values of the application factors are given in Table 3.2, which are used to determine the belt's power rating, P_{belt}, using

$$P_{belt} = \text{Power requirements} \times \text{application factor} \tag{3.21}$$

Table 3.2. Typical application factors for belt drives

	Drive characteristic		
Load	Smooth running	Slight shocks	Moderate shocks
Smooth	1.0	1.1	1.3
Moderate shocks	1.4	1.5	1.7
Heavy shocks	1.8	1.9	2.0

Example 3.3

Determine the speed and torque requirements for the following belt drive:

- *A belt drive is required to position a 100 g load. The drive consists of two aluminium pullies ($\rho = 2770$ kgm^{-3}), 50 mm in diameter and 12 mm thick driving a belt weighting 20 g. The efficiency (ϵ) of the drive is 0.95.*
- *The maximum speed of the load (V_L) is 2 m min^{-1} and the acceleration time (t) is 0.1 s.*

Firstly calculate the moment of inertia of the pulley

$$M_p = \rho \pi R_p^2 t_p = 0.065 \text{ kg hence } I_p = \frac{M_p R_p^2}{2} = 2 \times 10^{-5} \text{ kg m}^2$$

The reflected inertia of the belt and load is given by

$$I_L = \frac{MD^2}{4} = 7.5 \times 10^{-5} \text{ kg m}^2$$

The total driven inertia can now be calculated

$$I_{tot} = 2J_p + I_L = 11.5 \times 10^{-5} \text{ kg m}^2$$

The required peak input speed is

$$N_i = \frac{V_L}{piD} = 763 \text{ rev min}^{-1}$$

and hence the the torque torque requirement can be determined

$$T_{in} = \frac{1}{\epsilon}\left(\frac{IN_i}{t}\right) = 0.098 \text{ Nm}$$

3.4 Bearings

In the case of a rotating shaft, the most widely used method of support is by using one or a number bearing. A considerable number of different types of bearing are commonly available. The system selected is a function of the loads and speeds experienced by the system; for very high speed application air or magnetic bearings are used instead of the conventional metal-on-metal, rolling contacts. When considering the dynamics of a system, the friction and inertia of individual bearings, though small, must need to be into account.

3.4.1 Conventional bearings

The bearing arrangement of a rotating component, e.g. a shaft, generally requires two bearings to support and locate the component radially and axially relative to the stationary part of the machine. Depending on the application, load, running accuracy and cost the following approaches can be considered:

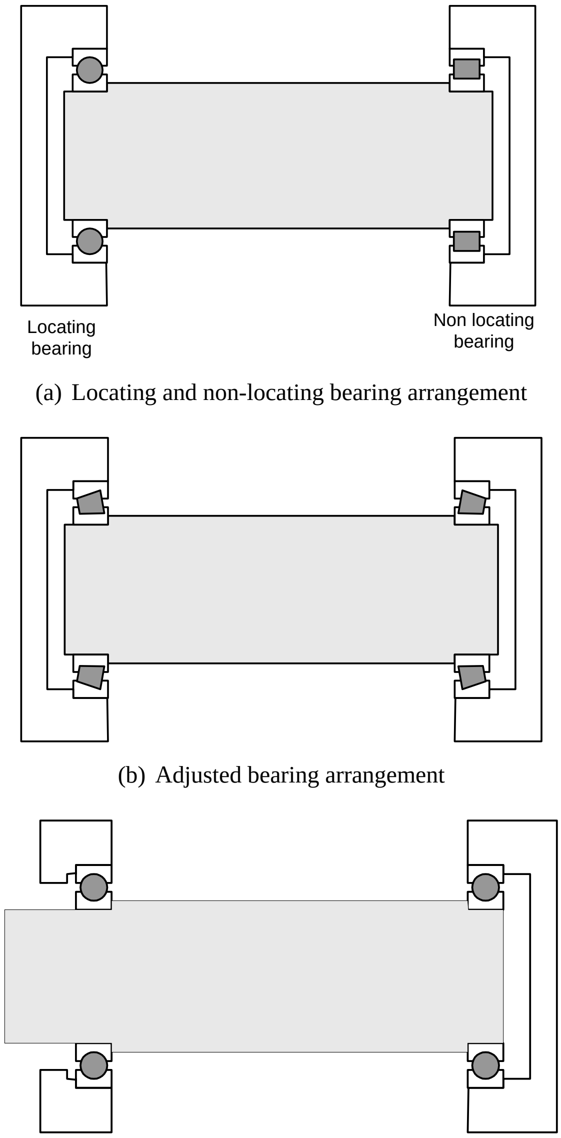
- Locating and non-locating bearing arrangements.
- Adjusted bearing arrangements.
- Floating bearing arrangements.

Locating and non-locating bearing arrangements

The locating bearing at one end of the shaft provides radial support and at the same time locates the shaft axially in both directions. It must, therefore, be fixed in position both on the shaft and in the housing. Suitable bearings are radial bearings which can accommodate combined loads, e.g. deep groove ball bearings. The second bearing then provides axial location in both directions but must be mounted with radial freedom (i.e. have a clearance fit) in its housing. The deep groove ball bearing and a cylindrical roller bearing, shown in Figure 3.9(a), illustrate this concept.

Adjusted bearing arrangements

In an adjusted bearing arrangements the shaft is axially located in one direction by the one bearing and in the opposite direction by the other bearing. This type of arrangement is referred to as *cross located* and is generally used on short shafts. Suitable bearings include all types of radial bearings that can accommodate axial loads in at least one direction, for example the taper roller bearings shown in Figure 3.9(b).



(a) Locating and non-locating bearing arrangement

(b) Adjusted bearing arrangement

(c) Floating bearing arrangements

Figure 3.9. Three approaches to supporting a rotating shaft.

Table 3.3. Typical coefficients of friction for roller bearings.

Bearing types	Coefficient of friction , μ_b
Deep grove	0.0015–0.003
Self-aligning	0.001–0.003
Needle	0.002
Cylindrical, thrust	0.004

Floating bearing arrangements

Floating bearing arrangements are also cross located and are suitable where demands regarding axial location are moderate or where other components on the shaft serve to locate it axially. Deep groove ball bearings will satisfy this arrangement, Figure 3.9(c).

Bearing friction

Friction within a bearing is made up of the rolling and sliding friction in the rolling contacts, in the contact areas between rolling elements and cage, as well as in the guiding surfaces for the rolling elements or the cage, the properties of the lubricant and the sliding friction of contact seals when applicable.

The friction in these bearing is either caused by the metal-to-metal contact of the balls or rollers on the bearing cage, or by the presence of lubrication within the bearing. The manufacturer will be able to supply complete data, but, as an indication, the friction torque, T_b, for a roller bearing can be determined using the following generally accepted relationship

$$T_b = 0.5 B_l d \mu_b \tag{3.22}$$

where d is the shaft diameter and B_l is the bearing load computed from the radial load, F_r and the axial load, F_a in the bearings, given by

$$B_l = \sqrt{F_r^2 + F_a^2} \tag{3.23}$$

The value of the coefficient of friction for the bearing, μ_b, will be supplied by the manufacturer; some typical values are given in Table 3.3.

The friction due to the lubrication depends on the amount of the lubricant, its viscosity, and on the speed of the shaft. At low speeds the friction is small, but it increases as the speed increases. If a high-viscosity grease is used rather than an oil, the lubrication friction will be higher and this can, in extreme cases, give rise to overheating problems. The contribution of the lubricant to the total bearing friction can be computed using standard equations.

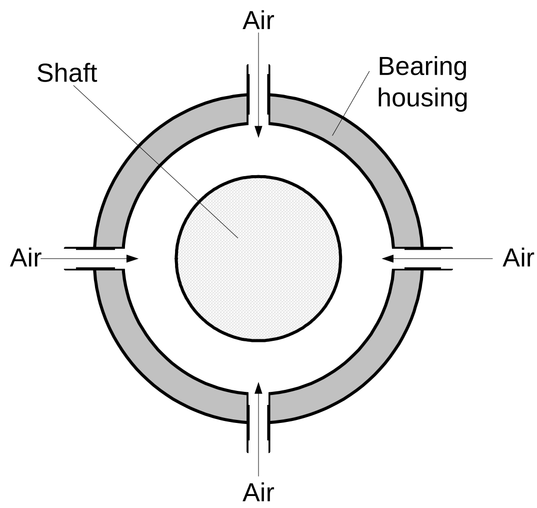


Figure 3.10. Cross section of an air bearing: the dimension of the airgap have been greatly exaggerated.

3.4.2 Air bearings

Air bearings can either be of an aerostatic or an aerodynamic design. In practice aerodynamic bearings are used in turbomachinery, where speeds of up to 36 000 rev min^{-1} in high temperature environments are typically found. In an aerostatic air bearing, Figure 3.10, the two bearing surfaces are separated by a thin film of pressurised air. The compressed air is supplied by a number of nozzles in the bearing housing. The distance between the bearing surfaces is about 5 to 30 μm. As the object is supported by a thin layer of air, the friction between the shaft and its housing can be considered to be virtually zero.

The use of an air bearing gives the system designer a number of advantages including:

- High rotational accuracy typically greater than 5×10^{-8} m is achievable and will remain constant over time as there is no wear due to the absence of contact between the rotating shaft and the housing.
- Low frictional drag, allow high rotational speeds; shaft speed of up to 200 000 rev min^{-1} with suitable bearings can be achieved.
- Unlimited life due to the absence of metal to metal contact, provided that the air supply is clean.
- High stiffness which is enhanced at speed due to a lift effect.

In machine tool applications,the lack of vibration and high rotational accuracy of an air bearing will allow surface finishes of up to 0.012 microns to be achieved.

Figure 3.11. A Radial magnetic bearing, manufactured by SKF Magnetic Bearings, Calgary, Canada.

3.4.3 Magnetic bearings

In a magnetic bearing the rotating shaft is supporting in a powerful magnetic field, and as with the air bearing gives a number of significant advantages:

- No contact, hence no wear, between the rotating and stationary parts. As particle generation due to wear is eliminated, magnetic bearings are suited to clean room applications.
- Operating through a wide temperature range, typically $-250°$C to 220°C: for this reason magnetic bearings are widely used in superconducting machines.
- A non-magnetic sheath between the stationary and rotating parts allows operation in corrosive environments.
- The bearing can be submerged in process fluid under pressure or operated in a vacuum without the need for seals.
- The frictional drag on the shaft is minimal, allowing exceptionally high speeds.

To maintain clearance, the shaft's position is under closed loop control by controlling the strength of the magnetic field, hence a magnetic bearing requires the following components:

- The bearing, consisting of a stator and rotor to apply electromagnetic forces to levitate the shaft.
- A five axis position measurement system.
- Controller and associated control algorithms to control the bearing's stator current to maintain the shaft at a pre-defined position.

The magnetic bearing stator has a similar construction to a brushless d.c. motor and consists of a stack of laminations wound to form a series of north and south poles. The current is supplied to each winding will produce an attractive force that levitates the shaft inside the bearing. The controller controls the current applied to the coils by monitoring the position signal from the positioning sensors in order to keep the shaft at the desired position through out the operating range of the machine. Usually there is 0.5 mm to 2 mm air gap between the rotor and stator depending on the application. A magnetic bearing is shown in Figure 3.11.

In addition to operation as a bearing, the magnetic field can be used to influence the motion of the shaft and therefore have the inherent capability to precisely control the position of the shaft to within microns and additionally to virtually eliminate vibrations.

3.5 Couplings

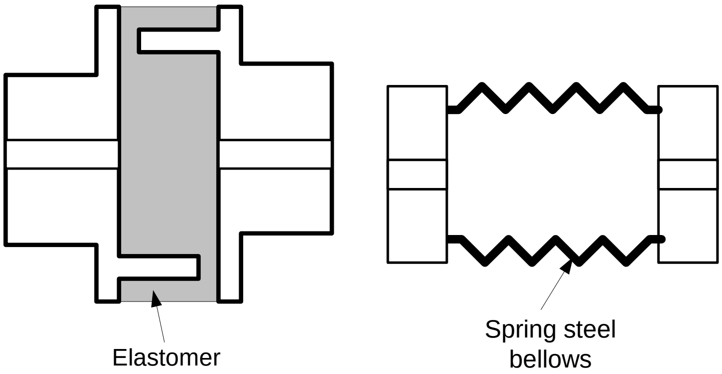
The purpose of a coupling is to connect two shafts, end-to-end, to transmit power. Depending on the application speed and power requirements a wide range of couplings are commercially available, and this section summarises the couplings commonly found in servo type applications.

A flexible coupling is capable of compensating for minor amounts of misalignment and random movement between the two shafts. Such compensation is vital because perfect alignment of two shafts is extremely difficult and rarely attained. The coupling will, to varying degrees, minimise the effect of misaligned shafts. If not properly compensated a minor shaft misalignment can result in unnecessary wear and premature replacement of other system components.

In certain cases, flexible couplings are selected for other functions. One significant application is to provide a break point between driving and driven shafts that will act as a *mechanical fuse* if a severe torque overload occurs. This assures that the coupling will fail before something more costly breaks elsewhere along the drive train. Another application is to use the coupling to dampen the torsional vibration that occurs naturally in the driving and/or driven system.

Currently there are a large number of flexible couplings due to the wide range of applications. However, in general flexible couplings fall into one of two broad categories, *elastomeric* or *metallic*. The key advantages and limitations of the designs are briefly summarised in Tables 3.4 and 3.5 to allow the user to select the match the correct coupling to the application.

Elastomeric couplings use a non-metallic element within the coupling, through which the power is transmitted, Figure 3.12(a). The element is manufactured from a compliant medium (for example rubber or plastic) and can be in compression or shear. Compression flexible couplings designs, include those based on jaw, pin and bushing, and doughnut designs while shear couplings include tyre and sleeve moulded elements.



(a) Flexible elastomer coupling. (b) Metallic bellows coupling.

Figure 3.12. Cross sections of commonly used couplings.

Table 3.4. Summary of the key characteristics of elastomeric couplings.

Advantages	Limitations
No lubrication required	Difficult to balance as an assembly
Good vibrational damping and shock absorption	Not torsionally stiff
Field replaceable elastomers elements	Larger than a metallic coupling of the same torque capacity
Capable of accommodating more misalignment than a metallic bellow coupling	Poor overload torque capacity

In practice there are two basic failure modes for elastomeric couplings. Firstly break down can be due to fatigue from cyclic loading when hysteresis that results in internal heat build up if the elastomer exceeds its design limits. This type of failure can occur from either misalignment or torque beyond its capacity. Secondly the compliant component can break down from environmental factors such as high ambient temperatures, ultraviolet light or chemical contamination. It should be noted that all elastomers have a limited shelf life and will in practice require replacement as part of maintenance programme, even if these failure conditions a not present.

Metallic couplings transmit the torque through designs where loose fitting parts are allowed to roll or slide against one another (for example in designs based on gear, grid, chain) or through the flexing/bending of a membrane (typically designed as a disc, diaphragm, beam, or bellows), Figure 3.12(b). Those with moving parts generally are less expensive, but need to be lubricated and maintained. Their pri-

Table 3.5. Summary of the key characteristics of metallic couplings.

Advantages	Limitations
Torsionally stiff	Fatigue or wear plays a major role in failure
High temperature capability	May need lubrication
Good chemical resistance possible	Complex assembly may be required
Low cost per unit torque transmitted	Require very careful alignment
High speed and large shaft size capability	Cannot damp vibration or absorb shock
Zero backlash	High electrical conductivity

mary cause of failure in a flexible metallic couplings is wear, so overloads generally shorten the couplings life through increased wear rather than sudden failure.

3.6 Shafts

A linear rotating shaft supported on bearings can be considered to be the simplest element in a drive system: their static and dynamic characteristics need to be considered. While it is relatively easy, in principle, to size a shaft, it can pose a number of challenges to the designer if the shaft is particularly long or difficult to support. In most systems the effects of transient behaviour can be neglected for the purpose of selecting the components of the mechanical drive train, as the electrical time constants are lower than the mechanical time constant, and therefore they can be considered independently. While such effects are not commonly found, they must be considered if a large-inertia load has to be driven by a relatively long shaft, where excitation generated either by the load (for example, by compressors) or by the drive's power electronics needs to be considered.

3.6.1 Static behaviour of shafts

In any shaft, torque is transmitted by the distribution of shear stress over its cross-section, where the following relationship, commonly termed the *Torsion Formula*, holds

$$\frac{T}{I_o} = \frac{G\theta}{L} = \frac{\tau}{r} \tag{3.24}$$

where T is the applied torque, I_o is the polar moment of area, G is the shear modulus of the material, θ is the angle of twist, L is the length of the shaft, τ is the shear stress and r the radius of the shaft.

In addition we can use the torsion equation to determine the stiffness of a circular shaft

$$K = \frac{T}{\theta} = \frac{G\pi r^4}{2L} \tag{3.25}$$

where the polar moment of area of a circular shaft is given by

$$I_o = \frac{\pi r^4}{2} \tag{3.26}$$

Example 3.4

Determine the diameter of a steel shaft required to transmit 3000 Nm, without exceed the shear stress of 50 MNm^{-1} or a twist of 0.1 rad m^{-1}. The shear modulus for steel is approximately 80 GNm^{-2}.

Using equation (3.24) and equation (3.26) it is possible to calculate the minimum radius for both the stress and twist conditions

$$r_{stress} = \frac{\tau_{max} I_o}{T} = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = 33.7mm$$

$$r_{twist} = \sqrt[4]{\frac{2TL}{G\theta\pi}} = 22.1mm$$

To satisfy both constraints the shaft should not have a radius of less than 33.7 mm.

3.6.2 Transient behaviour of shafts

In most systems the effects of transient behaviour can be neglected for the purpose of selecting the components of the mechanical drive train, because, in practice, the electrical time constants are normally smaller than the mechanical time constant. However, it is worth examining the effects of torque pulsations on a shaft within a system. These can be generated either by the load (such as a compressor) or by the drive's power electronics. While these problems are not commonly found, they must be considered if a large inertia load has to be driven by a relatively long shaft.

The effect can be understood by considering Figure 3.13; as the torque is transmitted to the load, the shaft will twist and carry the load. The twist at the motor end, θ_m will be greater than the twist at the load end, θ_L, because of the flexibility of the shaft; the transmitted torque will be proportional to this difference. If K is

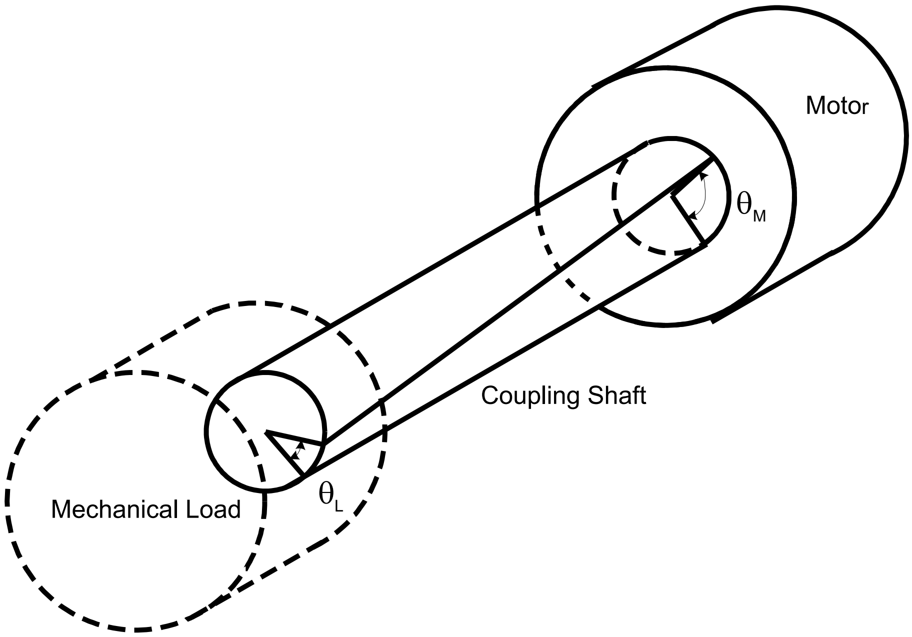


Figure 3.13. The effect of coupling a motor to a high-inertia load via a flexible shaft.

the shaft stiffness (Nm rad^{-1}), and B is the damping constant (Nm rad $^{-1}$s) then for the motor end of the shaft

$$T_m = I_m s^2 \theta_m + Bs(\theta_m - \theta_L) + K(\theta_m - \theta_L) \tag{3.27}$$

and at the load end the torque will turn the load in the same direction as the motor, hence

$$Bs(\theta_m - \theta_L) + K(\theta_m - \theta_L) = I_m s^2 \theta_m + T_L \tag{3.28}$$

where s is the differential operator, d/dt. If these equations are solved it can be shown that the undamped natural frequency of the system is given by

$$\omega_o = \sqrt{\frac{K}{I_m} + \frac{K}{I_L}} \tag{3.29}$$

$$\omega_o = \sqrt{1 - \zeta^2} \tag{3.30}$$

$$\zeta^2 = \sqrt{\frac{B}{2\sqrt{K}} \left(\frac{1}{I_m} + \frac{1}{I_L} \right)} \tag{3.31}$$

and the damped oscillation frequency is given by

$$\omega_n = \sqrt{1 + \frac{B^2}{4K}\left(\frac{1}{I_m} + \frac{1}{I_L}\right)} \tag{3.32}$$

In order to produce a stable system, the damped oscillation frequency must be significantly different to any torque pulsation frequencies produced by the system.

3.7 Linear drives

For many high performance linear applications, including robotic or similar high performance applications, the use of leadscrews, timing belts or rack and pinions driven by rotary motors, are not acceptable due to constrains imposed by backlash and limited acceleration. The use of a linear three phase brushless motor (Section 6.3) or the Piezoelectric motor (Section 9.3), provides a highly satisfactory solution to many motion control problems. If the required application requires only a small high-speed displacement, the voice coil (Section 9.1) can be considered.

The following advantages are apparent when a linear actuator is compared to conventional system based on a driving a belt or leadscrew:

- When compared to a belt and pulley system, a linear motor removes the problems associated with the compliance in the belt. The compliance will causes vibration when the load comes to rest, and this limits the speed and acceleration of a belt drive. It should be noted that a high performance belt drive can have a repeatability error in excess of 50 μm.

- As there are no moving parts are in contact, a linear motor has significant advantages over ball and leadscrew drives due to the removal of errors causes by wear on the nut and screw and e to friction, which is common if the drive has a high duty cycle. Even with the use of a high performance ballscrew the wear may become significant for certain application over time.

- As the length of a leadscrew or ballscrew is increased, so its maximum operating speed is limited, due to the flexibility of the shaft leading to vibration, particularly if a resonant frequency is hit – this is magnified as the length of the shaft increases. While the speed of the shaft can be decreased, by increasing the pitch, the system's resolution is compromised.

While the linear motor does provide a suitable solution for many applications, it is not inherently suitable for vertical operation, largely due to the problems associated with providing a fail-safe brake. In addition it is more difficult to seal against environmental problems compared with a rotary system, leading to restrictions when the environment is particularly hostile, for example when there is excessive abrasive dust or liquid present. Even with these issues, linear motors are widely used is many applications, including high speed robotics and other high performance positioning systems.

3.8 Review of motor-drive sizing

This chapter has so far discussed the power transmission elements of a drive system, while Chapter 2 has looked at issues related to the determination of a drive's requirements. This concluding section provides an overview of how this information is brought together, and the size of the motor and its associated drive are identified. The objective of the sizing procedure is to determine the required output speed and torque of the motor and hence to allow a required system to be selected. The process is normally started once the mechanical transmission system has been fully identified and quantified.

The main constraints that have to be considered during the sizing procedure when a conventional motor is being used can be summarised as follows:

- The peak torque required by the application must be less than both the peak stall torque of the motor and the motor's peak torque using the selected drive.
- The root-mean-square (r.m.s.) torque required by the application must be less than both the continuous torque rating of the motor and the continuous torque which can be delivered by the motor with the specified drive system.
- The maximum speed required by the application must be no greater than approximately eighty per cent of the maximum no-load speed of the motor drive combination; this allows for voltage fluctuations in the supply to the drive system.
- The motor's speed-torque characteristics must not be violated; in addition with a direct current (d.c.) brushed motor, the commutation characteristics of the motor must not be exceeded.

It should be noted that if a linear motor is used in an application the same set of constraints need to be considered, however force is considered to be the main driver as opposed to torque in the sizing process.

The operating regimes of the motor and its associated controller must be considered; two types of duty can be identified. The main determining factor is a comparison of the time spent accelerating and decelerating the load with the time spent at constant speed. In a continuous duty application the time spent accelerating and decelerating is not critical to the application, hence the maximum required torque (the external-load torque plus the drive-train's friction torque) needs to be provided on a continuous basis; the peak torque and the r.m.s. torque requirements are not significantly different to that of the continuous torque. The motor and the controller are therefore selected primarily by considering the maximum-speed and continuous-torque requirements.

An intermittent-duty application is defined as an application where the acceleration and deceleration of the load form a significant part of the motor's duty cycle. In this case the total system inertia, including the motor inertia, must be

Table 3.6. Typical d.c brushed motor motor data. All motors are rated for a maximum speed of 5000 rev min^{-1} at terminal voltage of 95 V.

Type	Continuous stall torque: Nm	Peak torque: Nm	Moment of inertia: kg m ^{2}	Voltage constant: V s rad $^{-1}$	Current constant: Nm A^{-1}
M1	0.5	2.0	1.7×10^{-4}	0.18	0.18
M2	0.7	4.0	2.8×10^{-4}	0.18	0.18
M3	1.2	8.0	6.0×10^{-4}	0.18	0.18

Table 3.7. Typical current data (in amps) supplied by manufacturers, for drives capable of driving d.c. brushed motor. All the drives are capable of supplied the 95 V required for the motors detailed in Table 3.6.

Type	Continuous current	Peak current
D1	5	10
D2	10	20
D3	14	20

considered when the acceleration torque is being determined. Thus, the acceleration torque plus the friction torque, and any continuous load torque present during acceleration, must be exceeded by the peak-torque capability of the motor-drive package. Additionally, the drive's continuous torque capability must exceed the required r.m.s. torque resulting from the worst-case positioning move.

The difference between these two application regimes can be illustrated by considering a lathe, shown in Figure 1.2. The spindle drive of a lathe can be considered to be a continuous-duty application since it runs at a constant speed under a constant load, while the axis drives are intermittent-duty applications because the acceleration and deceleration required to follow the tool path are critical selection factors.

The confirmation of suitable motor-drive combinations can be undertaken by the inspection of the supplier's motor-drive performance data, which provides information on the maximum no-load speed and on the continuous torque capability, together with the torque sensitivity of various motor frame sizes and windings. Tables 3.6 and 3.7 contain information extracted from typical manufacturer's data sheets relating to d.c brushed motors and drives, for a more detailed discussion see Chapter 5. In the sizing process it is normal to initially consider only a small number of the key electrical and mechanical parameters. If significant problems with motor and drive selection are experienced, a detailed discussion with the suppliers will normally resolve the problem.

As discussed above, two operating regimes can be identified: the following key features can be summarised as

- In a *continuous duty application* the acceleration and deceleration requirements are not considered critical; the motor and the controller can be satisfactorily selected by considering the maximum-speed and continuous-torque requirements.
- An *intermittent-duty application* is defined as an application where the acceleration and deceleration of the load form a significant part of the motor's duty cycle, and need to be considered during the sizing process.

3.8.1 Continuous duty

For continuous duty, where the acceleration performance is not of critical importance, the following approach can be used:

- Knowledge of the required speed range of the load, and an initial estimation of the gear ratios required, will permit the peak motor speed to be estimated. In order to prevent the motor from not reaching its required speed, due to fluctuations of the supply voltage, the maximum required speed should be increased by a factor of 1.2. It should be noted that this factor is satisfactory for most industrial applications, but it may be refined for special applications, for example, when the system has to operate from a restricted supply as is found in aircraft and offshore-oil platforms.
- Using the drive and the motor manufacturers' data sheet, it will be possible to locate a range of motors that meets the speed requirement when the drive operates at the specified supply voltage. If the speed range is not achievable, the gear ratio should be revised.
- From the motor's data, it will normally be possible to locate a motor-drive that meets the torque requirement; this will also allow the current rating of the drive to be determined. A check should then be undertaken to ensure that the selected system can accelerate the load to its required speed in an acceptable time.

Example 3.5

Determine the motor's speed and torque requirement for the system detailed below, and hence identify a suitable motor and associated drive:

- *The maximum load speed 300 rev min*$^{-1}$*, a non-optimal gearbox with a ratio of 10:1 has been selected. The gearbox's moment of inertia referred to its input shaft is* 3×10^{-4} *kgm*^{2}.

- *The load's moment of inertia has been determined to* 5×10^{-2} *kgm*^{2}
- *The maximum load torque is 8 Nm.*

Based on this information the minimum motor speed required can be determined including an allowance for voltage fluctuations

$$\text{Motor speed} = 300 \times \text{gear ratio} \times 1.2 = 3600 \text{ rev min}^{-1}$$

$$\text{Continuous torque} = \frac{8}{10} = 0.8 \text{ Nm}$$

Consideration of the motor data given in Table 3.6 indicates that motor M3 is capable of meeting the requirements. The required speed is below the peak motor speed of 5000 rev min $^{-1}$ $\pm$ 10%, and the required torque is below the motor's continuous torque rating. At the continuously torque demand the motor requires 4.5A, hence the most suitable drive from those detailed in Table 3.7, will be drive D1.

To ensure that the motor-drive combination is acceptable, the acceleration can be determined for the drives peak output of 10 A. At this current the torque generated by the motor is 1.8 Nm, well within the motor rating. Using equation (2.12), and noting that the gearbox's moment of inertia is added to that of the motor to give:

$$\alpha = \frac{T_{peak} - T_L/n}{n(I_d + I_L/n^2)} = \frac{1.8 - 8/10}{10(9 \times 10^{-4} + 5 \times 10^{-2}/10^2)} = 71.4 \text{ rad s}^{-1}$$

Hence the load will be accelerated to a peak speed of 300 rev min^{-1}, within 0.5 seconds, which is satisfactory. In practice the acceleration rate would be controlled, so that the system, it particular the gear teeth, would not experience significant shock loads.

3.8.2 Intermittent duty

When the acceleration performance is all important, the motor inertia must be considered, and the torque which is necessary to accelerate the total inertia must be determined early in the sizing process. A suitable algorithm is as follows:

- Using the application requirements and the required speed profile determine the required speeds and acceleration.
- Estimate the minimum motor torque for the application using equation 2.1.

- Select a motor-drive combination with a peak torque capability of at least 1.5 to 2 times the minimum motor-torque requirement to ensure a sufficient torque capability.

- Recalculate the acceleration torque required, this time including the inertia of the motor which has been selected.

- The peak torque of the motor-drive combination must exceed, by a safe margin of at least fifteen per cent, the sum of the estimated friction torque and the acceleration torque and any continuous torque loading which is present during acceleration. If this is not achievable, a different motor or gear ratio will be required.

- The motor's root-mean-square (r.m.s.) torque requirement can then be calculated as a weighted time average, using;

$$T_{rms} \leqslant T_{cm} + \sqrt{T_f^2 + dT_a^2} \tag{3.33}$$

  where T_{cm} is the continuous motor-torque requirement, T_f is the friction torque at the motor, T_a is the acceleration torque, and d is the duty cycle.

- The selected motor-drive combination is evaluated for maximum speed and continuous torque capabilities as in Section 3.8.1.

- If no motor of a given size can meet all the constraints, then a different, usually larger, frame must be considered, and the procedure must be repeated.

In practice, it is usual for one or two iterations to be undertaken in order find an acceptable motor-drive combination. The approximate r.m.s.-torque equation used above not only simplifies computation, but it also allows an easy examination of the effects of varying the acceleration/deceleration duty cycle. For example, the effects of changes in the dwell time on the value of r.m.s. torque can be immediately identified. Should no cost-effective motor-drive be identified, the effects of varying the speed-reduction ratio and inertias can easily be studied by trying alternative values and sizing the reconfigured system.

Sometimes, repeated selections of motors and drives will not yield a satisfactory result; in particular, no combination is able to simultaneously deliver the speed and the continuous torque which is required by the application, or to simultaneously deliver the peak torque and the r.m.s. torque required. In certain cases motor-drive combinations can be identified, but the size or cost of the equipment may appear to be too high for the application, and changes will again be required.

Example 3.6

Identify a suitable motor and its associated drive for the application detailed below:

- *The load is a rotary disc which has a moment of inertia of* 1.34 *kgm*^{2}*. The estimated frictional torque referred to the table's drive input is 5 Nm, and the external load torque is 8 Nm.*
- *The table is driven through a 20:1 gear box, which has a moment of inertia of* 3×10^{-4} *kgm*^{2} *referred to its input shaft.*
- *The table is required to index 90.0° (θ) in one second (t_m), and then dwell for a further two seconds. A polynomial speed profile is required.*

The selection process starts with the determination of the peak load speed and acceleration, using equation 2.28, the maximum speed occurs at $t = 0.5$ s and maximum acceleration occurs at $t = 0$ s.

$$\dot{\theta}(0.5) = \frac{6\theta t}{t_m^2} + \frac{6\theta t^2}{t_m^3} = 7.1 \text{ rad s}^{-1}$$
$$\ddot{\theta}(0) = \frac{6\theta}{t_m^2} = 9.4 \text{ rad s}^{-2}$$

The peak torque can now be calculated, at the input to the table. The torque is determined by the peak acceleration, and the load and friction torques, giving

$$T_{peak} = 8 + 5 + 1.34 \times 9.4 = 25.6 \text{ Nm}$$

This equates to 1.28 Nm peak torque from the motor. Using the motors defined in Table 3.6, it appears that motor M1 is a suitable candidate as it is capable of supplying over four times the required torque. If the motor's moment of inertia is now included in the calculation, the peak torque requirement is

$$T_{peak} = \frac{25.6}{20} + (3 \times 10^{-4} + 1.7 \times 10^{-4}) \times (9.4 \times 20) = 1.37 \text{ Nm}$$

which is well within the capabilities of the motor and drive D1, detailed in Table 3.7. The required peak current is 7 amps. The r.m.s. torque can now be calculated using equation (3.33):

$$T_{rms} \leqslant T_{cm} + \sqrt{T_f^2 + dT_a^2} = 0.67 \text{ Nm}$$

This figure is in excess of the continuous torque rating of the motor M1, and in certain applications could lead to the motor overheating. In addition, while the current is below the peak rating it it is greater than the continuous rating: in practice this could result in the drive cutting-out due to motor overheating. Thus a case can be made to change the motor and drive – in practice this decision would be made after careful consideration of the application.

If motor M2 is selected, and the above calculations are repeated, the motor's torque requirement becomes 1.4Nm, and the r.m.s torque becomes 0.69Nm. While marginal, M2 can be used along as the friction or load torque do not increase, if the drive is also changed to D2 there is no possibility of any overheating problems in the system.

As a final check the motor's peak speed is determined to be 1350 rev min^{-1}; this is well within the specification of the selected motor and drive.

This short example illustrates how a motor and drive can be selected, however the final decision needs a full understanding of the drive and its application. If the application only requires a few indexing moves, the selection of motor M1 could be justified however if a considerable number of indexes are required, motor M2 could be the better selection. This example has only considered the information given above; in practice the final decision will be influenced on the technical requirements of the complete process, and commercial requirements. While this example has been undertaken for a d.c. brushed motor, the same procedure is followed for any other type of drive – the only differences being the interpretation of the motor and drive specifications.

3.8.3 Inability to meet both the speed and the torque requirements

In the selection of motors, the limitations of both the motor and the drive forces a trade-off between the speed and the torque capabilities. Thus, it is usually advantageous to examine whether some alteration in the mechanical elements may improve the overall cost effectiveness of the application. Usually the speed-reduction ratios used in the application are the simplest mechanical parameter which can be investigated. If the speed required of the motor is high, but the torque seems manageable, a reduction in the gear ratio may solve the problem. If the torque required seems high but additional speed is obtainable, then the gear ratio should be increased. The goal is to use the smallest motor-drive combination that exceeds both the speed and torque requirement by a minimum of ten to twenty per cent. Sometimes the simple changing of a gear or pulley size may enable a suitable system to be selected.

A further problem may be the inability to select a drive that meets both the peak- and the continuous- torque requirements. This is particularly common in intermittent-motion applications. Often, the peak torque is achievable but the drive is unable to supply the continuous current required by the motor. As has been shown earlier, while optimum power transfer occurs when the motor's rotor and the reflected load inertia are equal this may not give the optimum performance for an intermittent drive, hence the gear ratios in the system need to be modified and the sizing process repeated.

Example 3.7

Consider Example 3.6 above, and consider the impact of performance due to a change in the reduction ratio.

In Example 3.6 the speed and torque requirements, using motor M1, were calculated to be 1.35 Nm and 1350 rev min^{-1}. The speed requirement is well within the motor specifications. If the gear ratio was changed to 40:1, the motor's peak speed requirement increases to 2700 rev min^{-1} and the r.m.s. torque drops to 0.35 Nm, and a peak torque of 0.818 Nm. These figures are well within the specification of motor M1 and drive D1.

This example illustrates a different approach to resolving the sizing problem encountered earlier. The change in gear ratio can easily be achieved at the design state, and in all possibility be cheaper that going to a larger motor and drive system.

3.8.4 Linear motor sizing

So far in this section we have considered the sizing of conventional rotary motors. We will now consider the sizing of a linear motor. Due to the simplicity of a linear drive, the process is straightforward compared to combining a leadscrew, ballscrew or belt drive with a conventional motor. As with all other sizing exercises, the initial process is to identify the key parameters, before undertaking the detailed sizing process. A suitable algorithm is as follows:

- Using the application requirements and the required speed profile determine the required speed and acceleration.
- Estimate the minimum motor force for the application using equation 2.2.
- Select a motor-drive combination with a peak force capability of at least 1.5 to 2 times the minimum force requirement to ensure a sufficient capability.

- Recalculate the acceleration force required, this time including the mass of the moving part of the selected motor.
- The peak force of the motor-drive combination must exceed, by a safe margin of at least fifteen per cent, the sum of the estimated friction force and the acceleration force and any continuous force which is present during acceleration. If this is not achievable, a different motor will be required.
- The motor's root-mean-square (r.m.s.) torque requirement can then be calculated as a weighted time average, in addition this will allow the motor's temperature to be estimated.

Example 3.8

Determine the size of the linear motor, and drive required to move a mass of $M_L = 40\ kg$, a distance of $d = 750$ mm in time of $t_m = 400$ ms.

- *The system has a dwell time of $t_d = 300$ ms, before the cycle repeats.*
- *Assume that the speed profile is triangular, and equal times are spent accelerating, decelerating and at constant speed.*
- *Assume the frictional force, $F_f = 3N$.*
- *The motors's parameters are: force constant is $K_F = 40NA^{-1}$, back emf constant $K_{emf} = 50\ Vm^{-1}s$, winding resistance, $Rt_w = 2\ \Omega$ and thermal resistance of the coil assembly, $Rt_{c-a} = 0.15\ °CW^{-1}$.*

The acceleration and peak speed can be determined using the process determined in Section 2.4, hence

$$\dot{x} = \frac{3d}{2t_m} = 2.4 \text{ m s}^{-1}$$

and

$$\ddot{x} = \frac{3\,\dot{x}}{t_m} = 28.8 \text{ m s}^{-2}$$

The acceleration force required is given by

$$F_a = M_L\ddot{x} + F_f = 1155 \text{ N}$$

This now allows the calculation of the root mean square force requirement

$$F_{rms} = \sqrt{\frac{2t_m F_a^2 + t_m F_f^2}{t_m + t_d}} = 635.5 \text{ N}$$

The drives current and voltage requirements can therefore be calculated

$$V_{drive} = \dot{x} K_{emf} + I_{peak} R_w = 178 \text{ V}$$

$$I_{peak} = \frac{F_a}{K_F} = 28.8 \text{ A}$$

$$I_{continuous} = \frac{F_{rms}}{K_F} = 15.9 \text{ A}$$

As linear motors are normally restricted to temperature rises of less that $100°$C, the temperature rise over ambient needs to be calculated

$$T_{rise} = I_{continuous}^2 R_w Rt_{c-a} = 76°\text{C}$$

3.9 Summary

This chapter has reviewed the characteristics of the main mechanical power transmission components commonly used in the construction of a drive system, together with their impact on the selection of the overall drive package. The chapter concluded by discussing the approach to sizing drives. One of the key points to be noted is that the motor-drive package must be able to supply torques and speed which ensure that the required motion profile can be followed. To assist with the determination of the required values, a sizing procedure was presented. It should be remembered over-sizing a drive is as under-sizing.