



Practical Design to Eurocode 2

The webinar will start at 12.30



Course Outline

Lecture	Date	Speaker	Title
1	21 Sep	Charles Goodchild	Introduction, Background and Codes
2	28 Sep	Charles Goodchild	EC2 Background, Materials, Cover and effective spans
3	5 Oct	Paul Gregory	Bending and Shear in Beams
4	12 Oct	Charles Goodchild	Analysis
5	19 Oct	Paul Gregory	Slabs and Flat Slabs
6	26 Oct	Charles Goodchild	Deflection and Crack Control
7	2 Nov	Paul Gregory	Detailing
8	9 Nov	Jenny Burridge	Columns
9	16 Nov	Jenny Burridge	Fire
10	23 Nov	Jenny Burridge	Foundations



Columns


Lecture 8
9th November 2016



Model Answers

Lecture 7 Exercise:
Lap length for column longitudinal bars

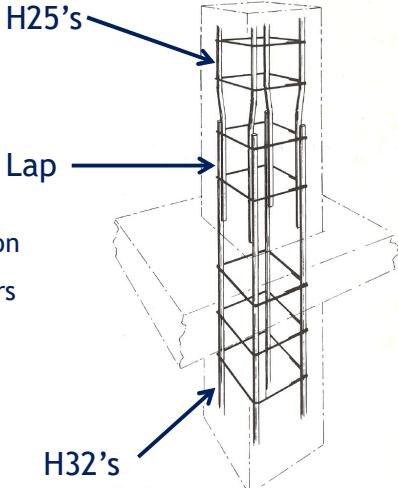
Column lap length exercise




Design information

- C40/50 concrete
- 400 mm square column
- 45mm nominal cover to main bars
- Longitudinal bars are in compression
- Maximum ultimate stress in the bars is 390 MPa

Exercise:
Calculate the minimum lap length using EC2 equation 8.10:

$$l_0 = \alpha_1 \alpha_2 \alpha_3 \alpha_5 \alpha_6 l_{b,rqd} \geq l_{0,min}$$


Column lap length exercise



Procedure

- Determine the ultimate bond stress, f_{bd} EC2 Equ. 8.2
- Determine the basic anchorage length, $l_{b,req}$ EC2 Equ. 8.3
- Determine the design anchorage length, l_{bd} EC2 Equ. 8.4
- Determine the lap length, $l_0 = \text{anchorage length} \times \alpha_6$

Solution - Column lap length



Determine the ultimate bond stress, f_{bd}

$$f_{bd} = 2.25 \eta_1 \eta_2 f_{ctd} \quad \text{EC2 Equ. 8.2}$$

$$\eta_1 = 1.0 \text{ 'Good' bond conditions}$$

$$\eta_2 = 1.0 \text{ bar size } \leq 32$$

$$f_{ctd} = \alpha_{ct} f_{ctk,0,05} / \gamma_c \quad \text{EC2 cl 3.1.6(2), Equ 3.16}$$

$$\alpha_{ct} = 1.0 \quad \gamma_c = 1.5$$

$$f_{ctk,0,05} = 0.7 \times 0.3 f_{ck}^{2/3} \quad \text{EC2 Table 3.1}$$

$$= 0.21 \times 40^{2/3}$$

$$= 2.456 \text{ MPa}$$

$$f_{ctd} = \alpha_{ct} f_{ctk,0,05} / \gamma_c = 2.456 / 1.5 = 1.637$$

$$f_{bd} = 2.25 \times 1.637 = 3.684 \text{ MPa}$$

Solution - Column lap length



Determine the basic anchorage length, $l_{b,req}$

$$l_{b,req} = (\emptyset/4) (\sigma_{sd} / f_{bd}) \quad \text{EC2 Equ 8.3}$$

$$\text{Max ultimate stress in the bar, } \sigma_{sd} = 390 \text{ MPa.}$$

$$l_{b,req} = (\emptyset/4) (390 / 3.684)$$

$$= 26.47 \emptyset$$

For concrete class C40/50

Solution - Column lap length



Determine the design anchorage length, l_{bd}

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,req} \geq l_{b,min} \quad \text{Equ. 8.4}$$

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 (26.47\emptyset) \quad \text{For concrete class C40/50}$$

For bars in compression $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1.0$

$$\text{Hence } l_{bd} = 26.47\emptyset$$

Solution - Column lap length



Determine the lap length, $l_0 = \text{anchorage length} \times \alpha_6$

All the bars are being lapped at the same section, $\alpha_6 = 1.5$

A lap length is based on the smallest bar in the lap, 25mm


Hence,

$$l_0 = l_{bd} \times \alpha_6$$

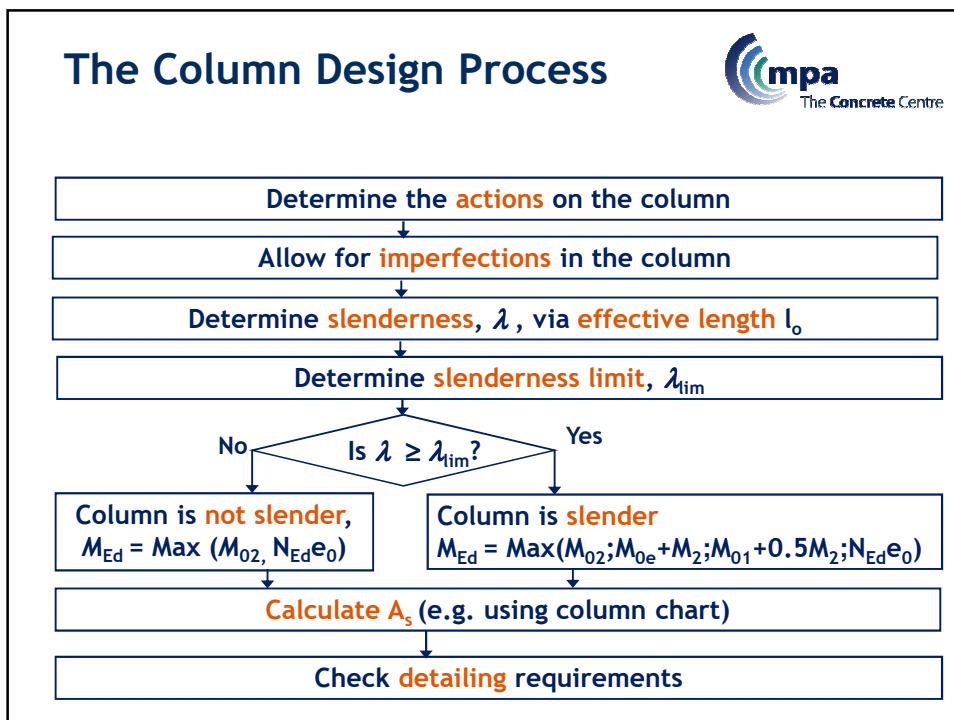
$$l_0 = 26.47 \emptyset \times 1.5$$

$$l_0 = 39.71 \emptyset = 39.71 \times 25$$

$$l_0 = 993 \text{ mm}$$



Columns




Actions

Actions on the columns are determined using one of the analysis methods we looked at for flexural design.

From the analysis obtain the following data:

- Ultimate axial load, N_{Ed}
- Ultimate moment at the top of the column, M_{top}
- Ultimate moment at the bottom of the column, M_{bottom}

Allow for imperfections . . .



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graph TD
    A[Actions] --> B[Imperfections]
    B --> C[Slenderness, λ]
    C --> D[Effective length, l0]
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    E --> F{Is λ ≥ λlim?}
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    I --> J[Detailing]
            
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Geometric Imperfections:

Cl. 5.2

5.5

Deviations in cross-section dimensions are normally taken into account in the material factors and should not be included in structural analysis

Imperfections need not be considered for SLS

But out-of-plumb needs to be considered and is represented by an inclination, θ_i

$$\theta_i = \theta_0 \alpha_h \alpha_m$$

where $\theta_0 = 1/200$


$$\alpha_h = 2/\sqrt{l}; \quad 2/3 \leq \alpha_h \leq 1$$

$$\alpha_m = \sqrt{0.5(1+1/m)}$$

where l = the length or height (m)
(see 5.2(6))
 m = no. of vert. members

For isolated columns in braced systems, α_m and α_h may be taken as 1.0

i.e. $\theta_i = \theta_0 = 1/200$



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Geometric Imperfections

Cl. 5.2 (7) & (9)

5.6.2.1



For isolated members at ULS, the effect of imperfections may be taken into account in two ways:

a) as an eccentricity, $e_i = \theta_1 l_0 / 2$

So for isolated columns in a braced system,
 $e_i = l_0 / 400$ may be used.

b) as a transverse force, H_i

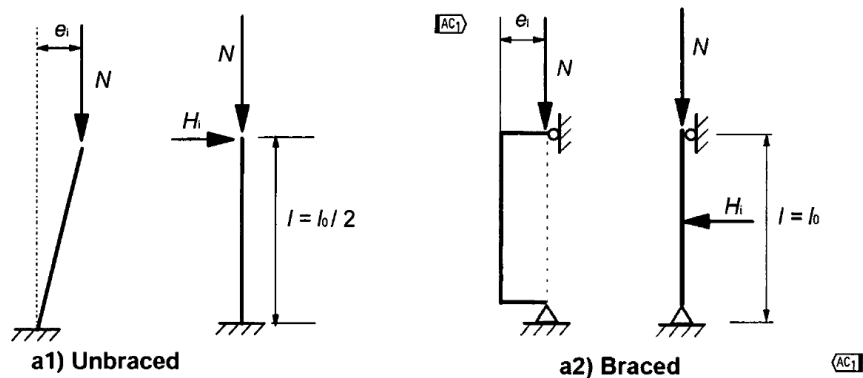
$H_i = \theta_1 N$ for un-braced members

$H_i = 2\theta_1 N$ for braced members = $N / 100$

Examples of Isolated Members

Figure 5.1a

5.5.2



Geometric Imperfections:

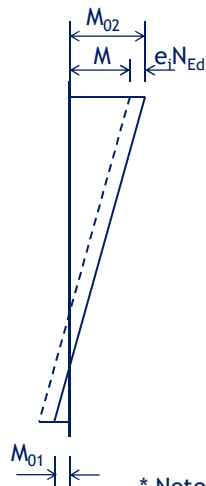


Cl 6.1(4)

Subject to a minimum eccentricity:

$$e_0 = h/30 \text{ but } \geq 20 \text{ mm}$$

Design Moments - Stocky Columns



$$M_{01} = [\text{Min}\{|M_{\text{top}}|, |M_{\text{bottom}}|\} - e_i N_{\text{Ed}}] *$$

$$M_{02} = [\text{Max}\{|M_{\text{top}}|, |M_{\text{bottom}}|\} + e_i N_{\text{Ed}}] *$$

$$e_i = l_o/400$$

N_{Ed} = design load in the column

For a stocky column,

Design moment

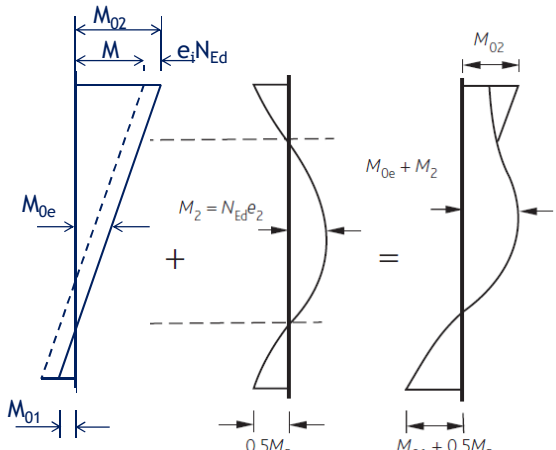
$$M_{\text{Ed}} = \text{Max}\{M_{02}, e_0 N_{\text{Ed}}\}$$

$$e_0 = \text{Max}\{h/30, 20\text{mm}\}$$

* Note: M_{01} and M_{02} return to their original signs after this calculation has been carried out

Moments in Slender Columns

Cl. 5.8.8.2 Fig 5.10



Design Moment,

$$M_{Ed} = \text{Max} \{ M_{02}; M_{0e} + M_2; M_{01} + 0.5M_2; N_{Ed}e_0 \}$$

a) First order moments for 'stocky' columns

b) Additional second order moments for 'slender' columns

c) Total moment diagram for 'slender' columns

Is the Column Slender?

Cl. 5.8.2, 5.8.3.1 5.6.1

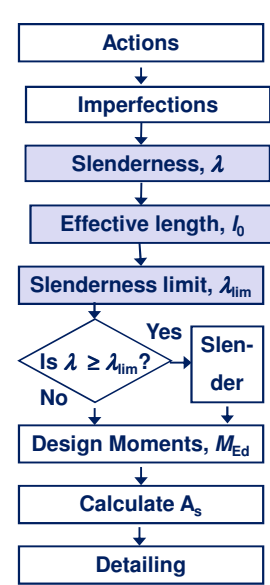
2nd order effects

Second order effects may be ignored if they are less than 10% of the corresponding first order effects


Second order effects may be ignored if the slenderness, λ is less than λ_{lim} where

$$\lambda_{lim} = 20 A B C / \sqrt{n}$$

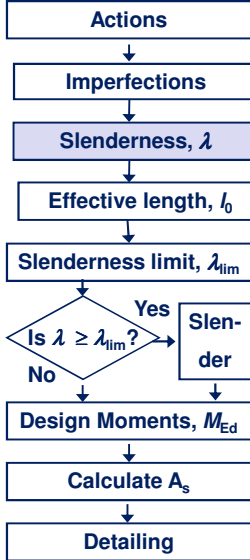
With biaxial bending the slenderness should be checked separately for each direction and only need be considered in the directions where λ_{lim} is exceeded



Slenderness



Slenderness, $\lambda = l_0 / i$
 l_0 = effective length = $F \cdot l$
 (l = actual length)
 i = radius of gyration = $\sqrt{I/A}$
 hence
 for a rectangular section $\lambda = 3.46 l_0 / h$
 for a circular section $\lambda = 4 l_0 / h$



Effective length


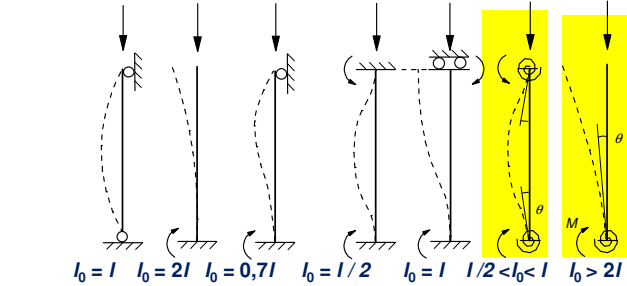


Figure 5.7, 5.8.3.2

Figure 5.6, 5.6.1.2

Effective length, $l_0 = F l$



$l_0 = l$ $l_0 = 0.5l$ $l_0 = 0.7l$ $l_0 = 1.5l$ $l_0 = l$ $l_0 > 2l$

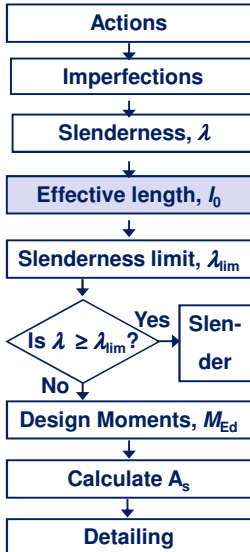
Braced members:

$$F = 0.5 \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0.45 + k_2}\right)}$$

Unbraced members:


$$F = \max \left\{ \sqrt{1 + 10 \cdot \frac{k_1 \cdot k_2}{k_1 + k_2}} ; \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right) \right\}$$

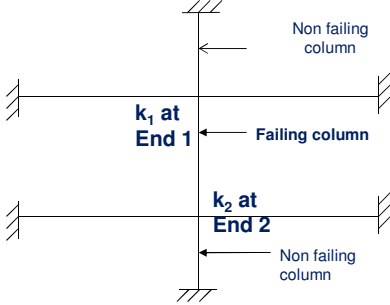
where $k = (\theta / M) \cdot (EI / l)$



Effective length & k factors

PD 6687 Cl.2.10





From PD 6687

The contribution of 'non failing' columns to the joint stiffness may be ignored

For beams θ/M may be taken as $1/2EI$ (allowing for cracking in the beams)

$$k_1 = [EI / l]_{col} / [\sum 2EI / l]_{beams1}$$

$$k_2 = [EI / l]_{col} / [\sum 2EI / l]_{beams2}$$

Assuming that the beams are symmetrical about the column and their sizes are the same in the two storeys shown, then:

$$k_1 = k_2 = [EI / l]_{col} / [2 \times 2EI / l]_{beams} \geq 0.1$$

Effective length & Slenderness

Cl. 5.8.3.1

5.6.1.4

Slenderness $\lambda = l_0 / i$

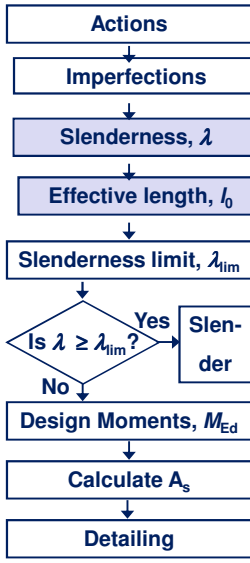
where

$$l_0 = F l$$

k = relative stiffness

$$k = \frac{EI_c}{\sum 2EI_b} \geq 0.1$$


$i = \sqrt{I/A}$
for a rectangular section
for a circular section



Effective length factor, F, for braced columns

k2 \ k1	0.10	0.20	0.30	0.40	0.50	0.70	1.00	2.00	5.00	9.00	Pinned
0.10	0.59	0.62	0.64	0.66	0.67	0.69	0.71	0.73	0.75	0.76	0.77
0.20	0.62	0.65	0.68	0.69	0.71	0.73	0.74	0.77	0.79	0.80	0.81
0.30	0.64	0.68	0.70	0.72	0.73	0.75	0.77	0.80	0.82	0.83	0.84
0.40	0.66	0.69	0.72	0.74	0.75	0.77	0.79	0.82	0.84	0.85	0.86
0.50	0.67	0.71	0.73	0.75	0.76	0.78	0.80	0.83	0.86	0.86	0.87
0.70	0.69	0.73	0.75	0.77	0.78	0.80	0.82	0.85	0.88	0.89	0.90
1.00	0.71	0.74	0.77	0.79	0.80	0.82	0.84	0.88	0.90	0.91	0.92
2.00	0.73	0.77	0.80	0.82	0.83	0.85	0.88	0.91	0.93	0.94	0.95
5.00	0.75	0.79	0.82	0.84	0.86	0.88	0.90	0.93	0.96	0.97	0.98
9.00	0.76	0.80	0.83	0.85	0.86	0.89	0.91	0.94	0.97	0.98	0.99
Pinned	0.77	0.81	0.84	0.86	0.87	0.90	0.92	0.95	0.98	0.99	1.00

Slenderness limit



Allowable Slenderness

$$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$$

where:

$A = 1 / (1 + 0.2 \phi_{ef})$
 ϕ_{ef} is the effective creep ratio;
 (if ϕ_{ef} is not known, $A = 0.7$ may be used)

$B = \sqrt{1 + 2\omega}$ $\omega = A_s f_{yd} / (A_c f_{cd})$
 (if ω is not known, $B = 1.1$ may be used)

$C = 1.7 - r_m$
 $r_m = M_{01} / M_{02}$
 M_{01}, M_{02} are first order end moments,
 $|M_{02}| \geq |M_{01}|$
 (if r_m is not known, $C = 0.7$ may be used)

$n = N_{Ed} / (A_c f_{cd})$

Actions

↓

First order moments

↓

Slenderness, λ

↓

Effective length, l_0

↓

Slenderness limit, λ_{lim}

↓

Is $\lambda \geq \lambda_{lim}$?

Yes → Slender

No ↓

Design Moments, M_{Ed}


↓

Calculate A_s

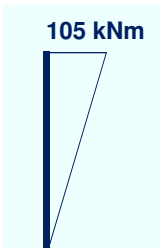
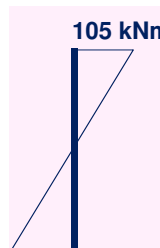
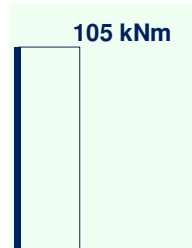
↓

Detailing

Factor C




$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$

 <p style="text-align: center;">105 kNm</p> <p style="text-align: center;">$r_m = M_{01} / M_{02}$ $= 0 / 105$ $= 0$ $C = 1.7 - 0$ $= 1.7$</p>	 <p style="text-align: center;">105 kNm</p> <p style="text-align: center;">-105 kNm</p> <p style="text-align: center;">$r_m = M_{01} / M_{02}$ $= 105 / -105$ $= -1$ $C = 1.7 + 1$ $= 2.7$</p>	 <p style="text-align: center;">105 kNm</p> <p style="text-align: center;">105 kNm</p> <p style="text-align: center;">$r_m = M_{01} / M_{02}$ $= 105 / 105$ $= 1$ $C = 1.7 - 1$ $= 0.7$</p>
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Is $\lambda \geq \lambda_{lim}$?

Is $l_0/i = \lambda \geq \lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$?

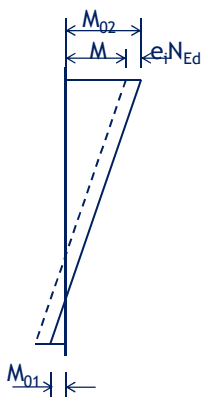


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graph TD
    A[Actions] --> B[Imperfections]
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    D --> E[Slenderness limit, λₗᵢₘ]
    E --> F{Is λ ≥ λₗᵢₘ?}
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    H --> I[Calculate Aₛ]
    I --> J[Detailing]
            
```

No, $\lambda < \lambda_{lim}$

Design Moments - Stocky Columns
(aka 1st order moments plus effects of imperfections) *As before !*



$M_{01} = \text{Min}\{|M_{top}|, |M_{bottom}|\} - e_i N_{Ed}$
 $M_{02} = \text{Max}\{|M_{top}|, |M_{bottom}|\} + e_i N_{Ed}$
 $e_i = l_0 / 400$
 N_{Ed} = design load in the column
 For a stocky column,
 Design moment
 $M_{Ed} = \text{Max}\{M_{02}, e_0 N_{Ed}\}$
 $e_0 = \text{Max}\{h/30, 20\text{mm}\}$



```

graph TD
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```


Yes, $\lambda \geq \lambda_{lim}$ 2nd order effects

CI 5.8.5, cl 5.8.8 5.6.2.1

The methods of analysis include a general method, for 2nd order effects based on non-linear second order analysis and the following two simplified methods:

- Method based on nominal stiffness
- Method based on nominal curvature

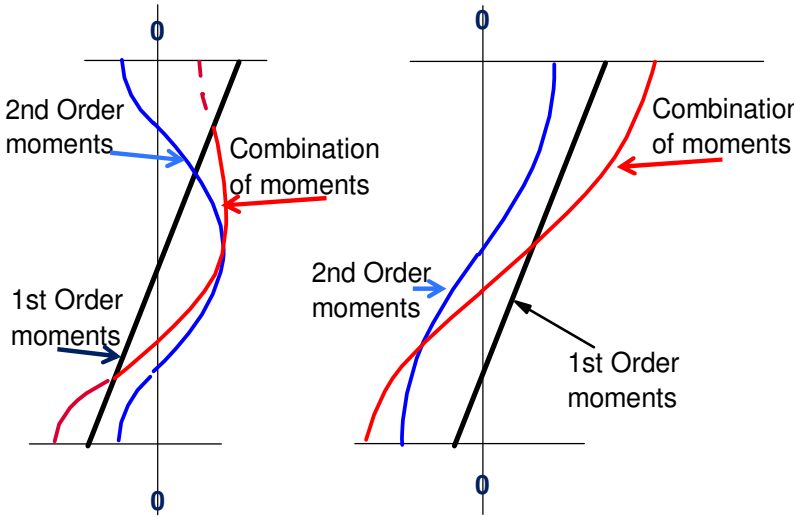
This method is primarily suitable for isolated members with constant normal force and defined effective length. The method gives a nominal second order moment based on a deflection, which in turn is based on the effective length and an estimated maximum curvature. (preferred in UK)




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```

Moments in Slender Columns



Typical braced column Typical unbraced column



Nominal Curvature Method

Cl. 5.8.8.2

5.6.2.2



$$M_{Ed} = M_{0Ed} + M_2$$

M_{0Ed} is the 1st order moment including the effect of imperfections
 M_2 is the nominal 2nd order moment.

Differing 1st order end moments M_{01} and M_{02} may be replaced by an equivalent 1st order end moment M_{0e} :

$$M_{0e} = (0.6 M_{02} + 0.4 M_{01}) \geq 0.4 M_{02}$$

HOWEVER, this is only the mid-height moment the two end moments should be considered too. PD 6687 advises for braced structures:

$$M_{Ed} = \text{Max} \{M_{02}, M_{0e} + M_2; M_{01} + 0.5M_2\} \geq e_0 N_{Ed}$$

where $M_{02} = \text{Max}\{|M_{top}|; |M_{bot}|\} + e_i N_{Ed}$

$$M_{01} = \text{Min}\{|M_{top}|; |M_{bot}|\} + e_i N_{Ed}$$

M_{top} & M_{bot} are frame analysis 1st order end moments

Effectively:
$$M_{Ed} = \text{Max} \{M_{02}, M_{0e} + M_2; M_{01} + 0.5M_2; e_0 N_{Ed}\}$$

Second order moment

Cl. 5.8.8

5.6.2.2



$$M_2 = N_{Ed} e_2$$

$$e_2 = (1/r) l_0^2 / c$$

$$1/r = K_r K_\phi / r_0$$

$$K_r = (n_u - n) / (n_u - n_{bal}) \leq 1 \quad (\text{or see Column charts})$$

$$n = N_{Ed} / (A_c f_{cd})$$

$$n_u = 1 + \omega$$

$$\omega = A_s f_{yd} / (A_c f_{cd})$$

$$n_{bal} = 0.4$$

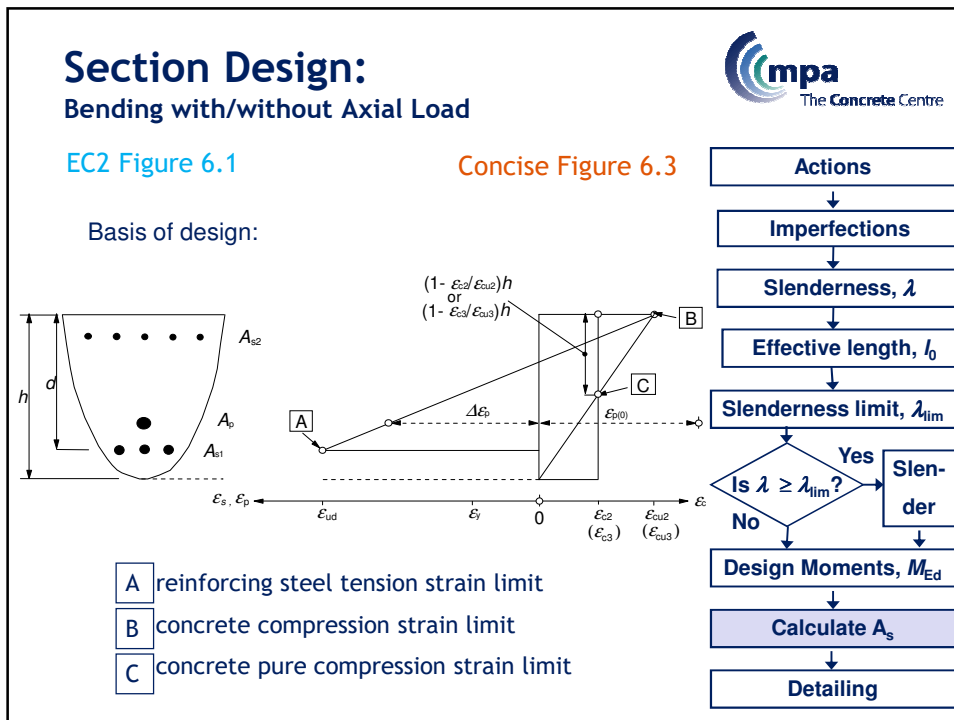
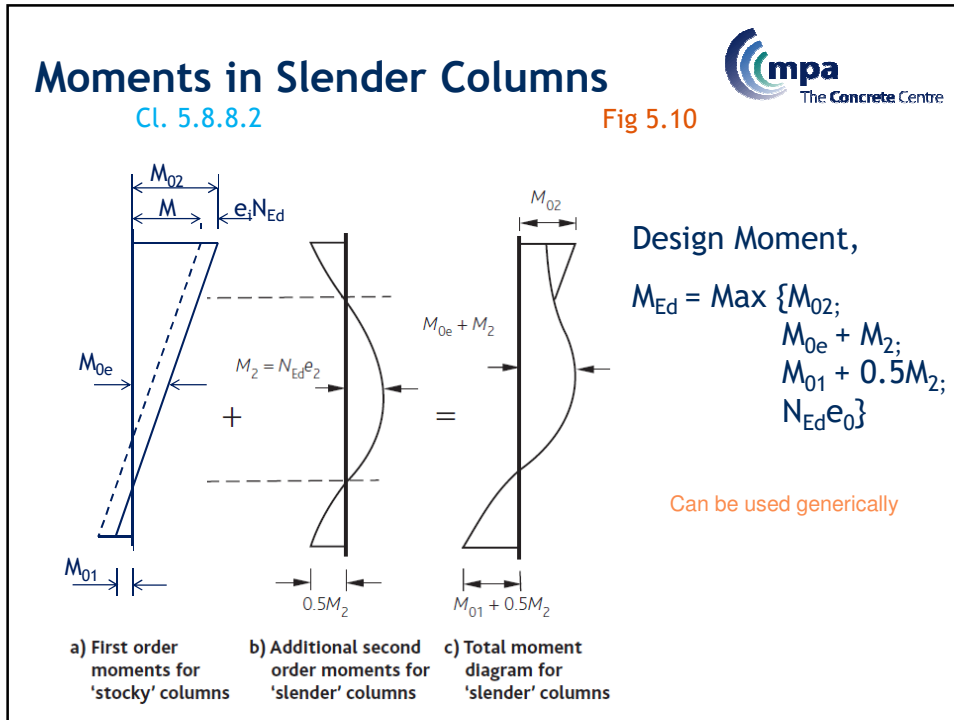
$$K_\phi = 1 + \beta \phi_{ef} \geq 1$$

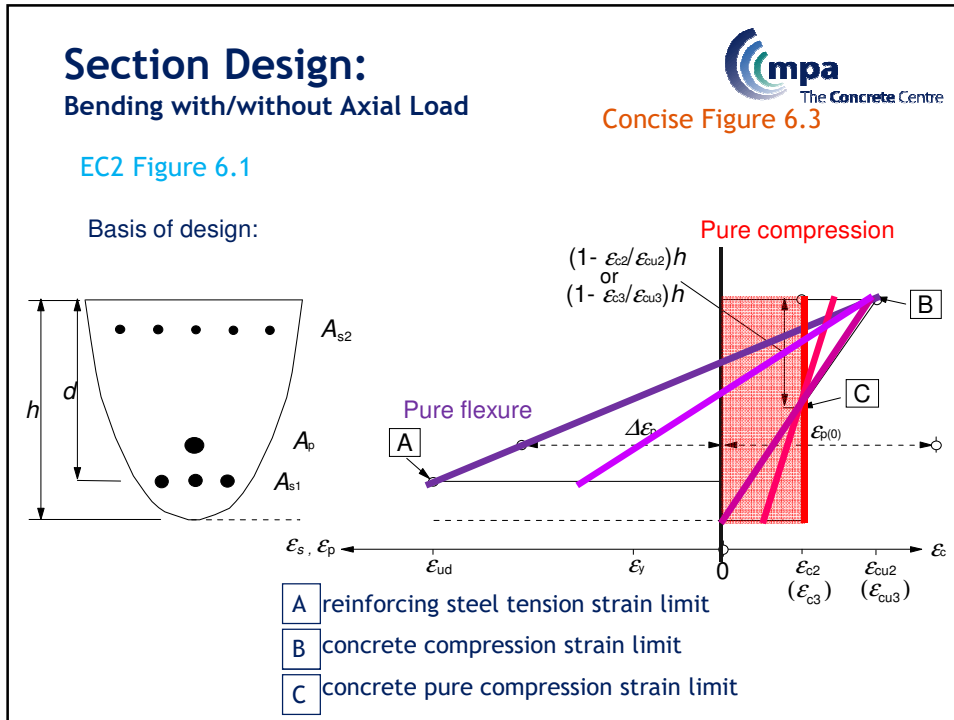
$$\beta = 0.35 + f_{ck} / 200 - \lambda / 150$$

$$1/r_0 = \varepsilon_{yd} / (0.45d)$$


$$\varepsilon_{yd} = f_{yd} / E_s$$

$$c = 10 \quad (\text{for a constant cross section})$$





Section Design:
Bending with/without Axial Load


Concise 15.9.2, 15.9.3

Design

Either: iterate such that $A_{sN} = A_{sM}$

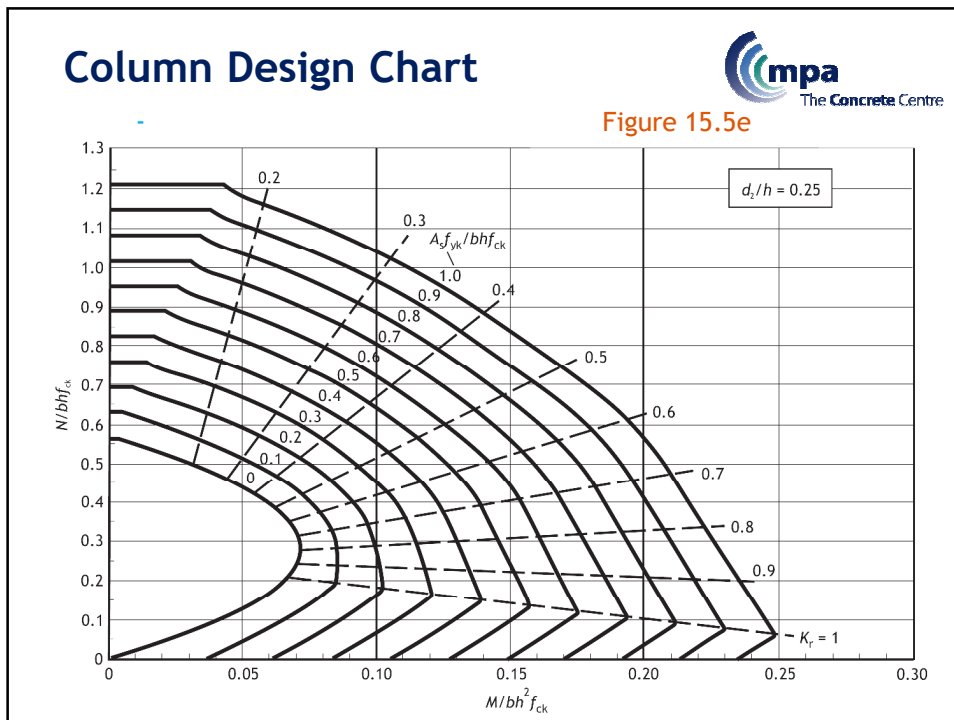
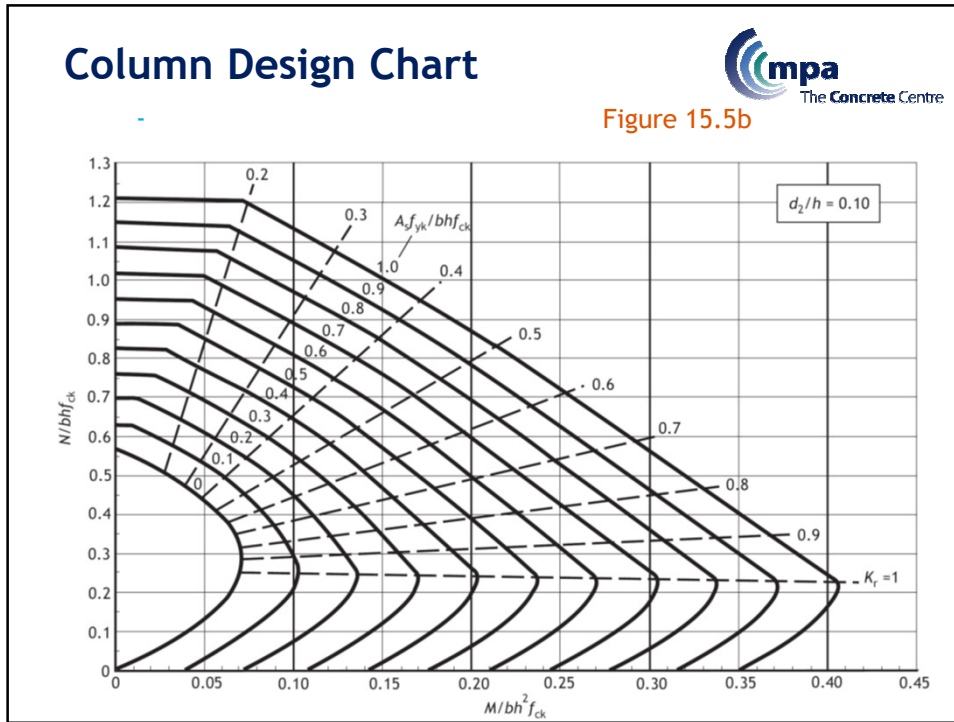
- For axial load

$$A_{sN}/2 = (N_{Ed} - \alpha_{cc} h f_{ck} b d_c / \gamma_c) / (\sigma_{sc} - \sigma_{st})$$
- For moment

$$A_{sM}/2 = [M_{Ed} - \alpha_{cc} h f_{ck} b d_c (h/2 - d_c/2) / \gamma_c] / [(h/2 - d_2) (\sigma_{sc} + \sigma_{st})]$$

Or: Calculate d_2/h , $N_{Ed}/b h f_{ck}$ and $M_{Ed}/b h^2 f_{ck}$
And use column charts

to find $A_s f_{yk} / b h f_{ck}$ and thus A_s



Biaxial Bending

Cl. 5.8.9

5.6.3



Having done the analysis and design one way, you have to do it in the other direction and check biaxial bending. Often it will be non-critical by inspection but one should check

$$\left(\frac{M_{Edz}}{M_{Rdz}} \right)^a + \left(\frac{M_{Edy}}{M_{Rdy}} \right)^a \leq 1,0$$

Note: imperfections need only be taken in the one, more critical direction so either M_{Edz} or M_{Edy} might be reduced in this check

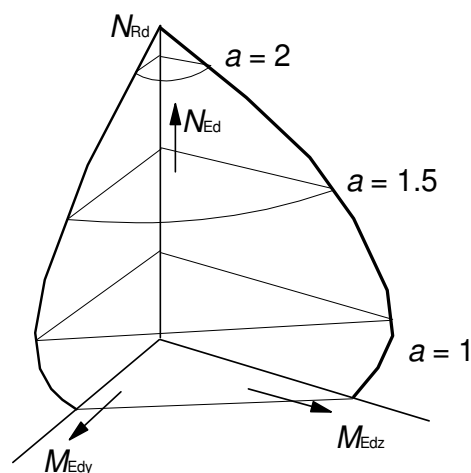
For rectangular cross-sections

N_{Ed}/N_{Rd}	0.1	0.7	1.0
a	1.0	1.5	2.0

where $N_{Rd} = A_c f_{cd} + A_s f_{yd}$

For circular cross-sections $a = 2.0$


Biaxial bending for a rectangular column



Details/Detailing

EC2 (9.5.2)

- $h \leq 4b$ (otherwise a wall)
- $\phi_{min} \geq 12$
- $A_{s,min} = 0,10N_{Ed}/f_{yd}$ but $\geq 0,002 A_c$
- $A_{s,max} = 0.04 A_c$ (0,08A_c at laps)
- Minimum number of bars in a circular column is 4.
- Where direction of longitudinal bars changes more than 1:12 the spacing of transverse reinforcement should be calculated.

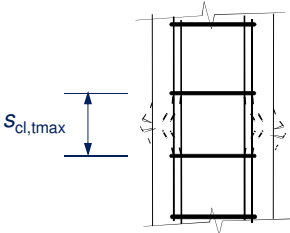
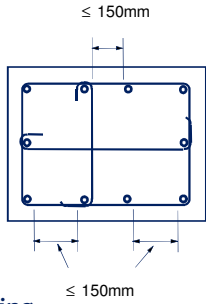


```


graph TD
    A[Actions] --> B[Imperfections]
    B --> C[Slenderness, λ]
    C --> D[Effective length, l0]
    D --> E[Slenderness limit, λlim]
    E --> F{Is λ ≥ λlim?}
    F -- Yes --> G[Slender]
    F -- No --> H[Design Moments, MEd]
    G --> H
    H --> I[Calculate As]
    I --> J[Detailing]
            
```

Links

EC2 (9.5.3)

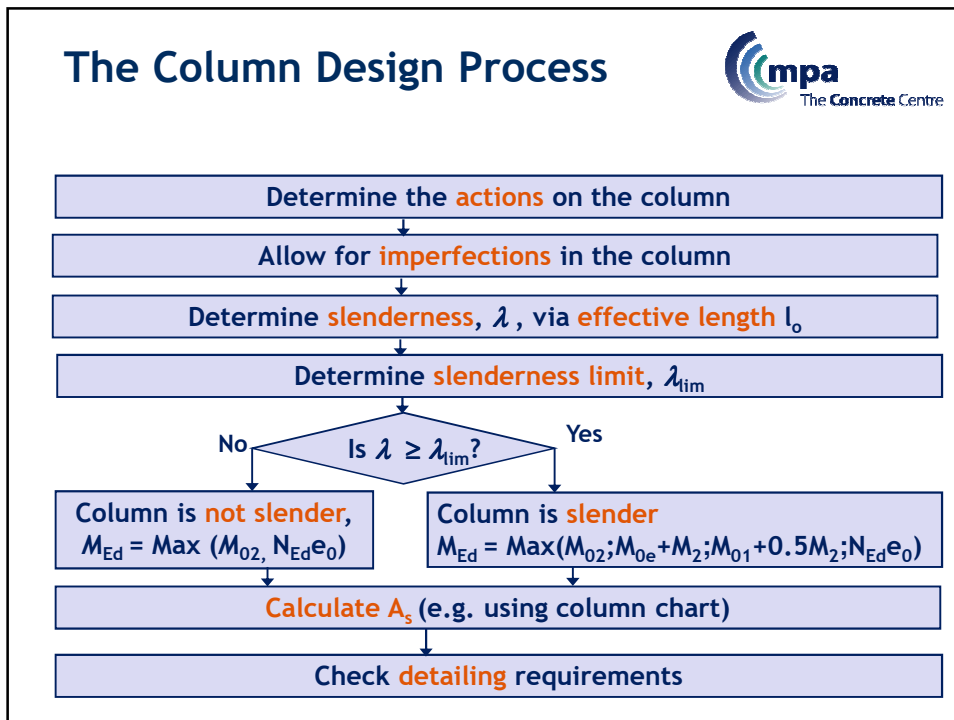



No compression bar > 150 mm from a restraining bar




Link diam
 $= \max(6, \phi_{max}/4)$
 (For HSC columns see NA)

Link spacing
 $s_{cl,tmax} = \min \{20 \phi_{min}; b; 400\text{mm}\}$
but $s_{cl,tmax}$ should be reduced by a factor 0.6:
 – in sections within h above or below a beam or slab
 – near lapped joints where $\phi > 14$.
 $s_{cl,tmax} = \min \{12 \phi_{min}; 0.6b; 240\text{mm}\}$
 A min of 3 links are required in lap length

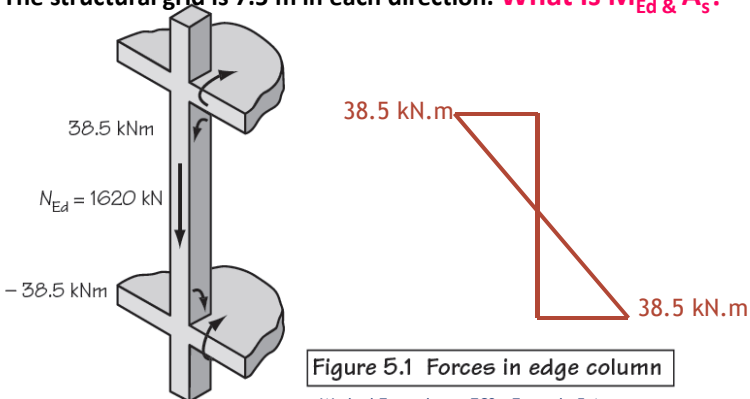


Worked Example



A 300 mm square column on the edge of a flat slab structure supports an axial load of 1620 kN and first order moments of 38.5 kNm top and -38.5 kNm bottom in one direction only[†]. The concrete is grade C30/37, $f_{ck} = 30$ MPa and cover, $c_{nom} = 25$ mm. The 250 mm thick flat slabs are at 4000 mm vertical centres.


The structural grid is 7.5 m in each direction. **What is M_{Ed} & A_s ?**



The diagram shows a 3D view of a square column on the edge of a slab. The column is subjected to an axial load $N_{Ed} = 1620$ kN and moments of 38.5 kNm at the top and -38.5 kNm at the bottom. To the right, a bending moment diagram shows a linear variation from 38.5 kNm at the top to -38.5 kNm at the bottom.

Figure 5.1 Forces in edge column
Worked Examples to EC2 - Example 5.1

Solution - effective length & slenderness (M_2)

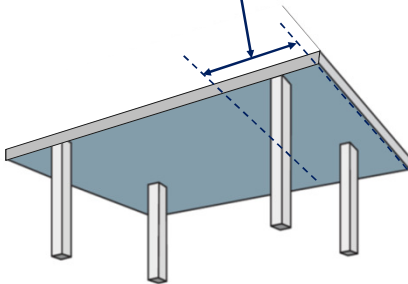


Using PD 6687 method

$$k = \frac{\frac{EI_c}{L_c}}{\sum \frac{2EI_b}{L_b}} = \frac{\frac{300^4/12}{3750}}{\frac{2 \times 3750 \times 250^3/12}{7500}} = 0.14$$

From table (Table 4 of How to Columns)

k2	k1										
	0.10	0.20	0.30	0.40	0.50	0.70	1.00	2.00	5.00	9.00	Pinned
0.10	0.59	0.62	0.64	0.66	0.67	0.69	0.71	0.73	0.75	0.76	0.77
0.20	0.62	0.65	0.68	0.69	0.71	0.73	0.74	0.77	0.79	0.80	0.81
0.30	0.64	0.68	0.70	0.72	0.73	0.75	0.77	0.80	0.82	0.83	0.84
0.40	0.66	0.69	0.72	0.74	0.75	0.77	0.79	0.82	0.84	0.85	0.86
0.50	0.67	0.71	0.73	0.75	0.76	0.78	0.80	0.83	0.86	0.86	0.87
0.70	0.69	0.73	0.75	0.77	0.78	0.80	0.82	0.85	0.88	0.89	0.90
1.00	0.71	0.74	0.77	0.79	0.80	0.82	0.84	0.88	0.90	0.91	0.92
2.00	0.73	0.77	0.80	0.82	0.83	0.85	0.88	0.91	0.93	0.94	0.95
5.00	0.75	0.79	0.82	0.84	0.86	0.88	0.90	0.93	0.96	0.97	0.98
9.00	0.76	0.80	0.83	0.85	0.86	0.89	0.91	0.94	0.97	0.98	0.99
Pinned	0.77	0.81	0.84	0.86	0.87	0.90	0.92	0.95	0.98	0.99	1.00




NB. 3750 \equiv half bay width for flat slab

$F = 0.61$. So $l_0 = 0.61 \times 3.750 = 2.29\text{m}$

Calculate slenderness:
 $\lambda = 3.46 l_0/h$
 $= 3.46 \times 2.29 / 0.3 = 26.4$

Solution - slenderness limit (M_2)



Limiting slenderness:
 $\lambda_{lim} = 20 ABC / \sqrt{n}$
 where

$A = 0.7$ (default)

$B = 1.1$ (default)

$C = 1.7 - r_m$
 $= 1.7 - M_{01}/M_{02}$
 $= 1.7 - (-38.5 + 9.3) / (38.5 + 9.3)$
 $= 1.7 - -29.2 / 47.8 = 2.31$

$n = N_{Ed} / A_c f_{cd}$
 $= 1620 \times 10^3 / (300^2 \times 0.85 \times 30 / 1.5)$
 $= 1.06$

$\lambda_{lim} = 20 ABC / \sqrt{n} = 20 \times 0.7 \times 1.1 \times 2.31 / 1.06^{0.5} = 34.5$

So
 $\lambda_{lim} = 34.5$ i.e. > 26.4 \therefore column is not slender.
 And $M_2 = 0$ kNm.

$e_i = l_0 / 400$
 $= 2290 / 400$
 $= 5.7$ mm

$N_{Ed} = 1620$ kN

$e_i N_{Ed} = 1620 \times 0.0057$
 $= 9.3$ kNm

Solution - design moments



$$M_{Ed} = \max[M_{02}; M_{0Ed} + M_2; M_{01} + 0.5M_2; N_{Ed}e_0]$$

where:

$$M_{02} = M + e_i N_{Ed}$$

where

$$M = 38.5 \text{ kNm}$$

$$e_i = l_0 / 400 = 2290 / 400 = 5.7 \text{ mm}$$

$$N_{Ed} = 1620 \text{ kN}$$

$$M_{02} = 38.5 + 1620 \times 0.0057$$

$$= 38.5 + 9.3$$

$$= 47.8 \text{ kNm}$$

$$M_{01} = -38.5 + 9.3 = -29.2 \text{ kNm}$$

$$M_{0Ed} = (0.6M_{02} + 0.4M_{01}) \geq 0.4M_{02}$$

$$= 0.6 \times 47.8 + 0.4 \times (-29.2) \geq 0.4 \times 47.8$$

$$= 17.0 \leq 19.1$$

$$= 19.1 \text{ kNm}$$

$$M_2 = 0 \text{ kNm}$$

Solution - design moments



Minimum moment

$$e_0 N_{Ed}$$

$$e_0 = \max[h/30; 20] = \max[300/30; 20] = 20 \text{ mm}$$

$$N_{Ed} = 1620 \text{ kN}$$

$$e_0 N_{Ed} = 0.02 \times 1620$$

$$= 32.4 \text{ kNm}$$

Solution - design moments



$$M_{Ed} = \max[M_{02}; M_{0Ed} + M_2; M_{01} + 0.5M_2; N_{Ed}e_0]$$

where:

$$\begin{aligned} M_{02} &= 47.8 \text{ kNm} \\ M_{01} &= -29.2 \text{ kNm} \\ M_{0Ed} &= 19.1 \text{ kNm} \\ M_2 &= 0 \text{ kNm} \\ e_0 N_{Ed} &= 32.4 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{Ed} &= \max[47.8; 19.1 + 0; -29.2 + 0; 32.4] \\ &= \underline{47.8 \text{ kNm}} \end{aligned}$$

Solution - determine A_s



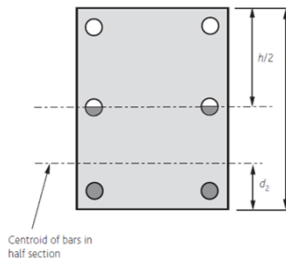
Using design charts:

Worked Examples to EC2

Require d_2/h to determine which chart(s) to use:

$$\begin{aligned} d_2 &= c_{nom} + \text{link} + \phi / 2 = 25 + 8 + \text{say } 32/2 = 49 \text{ mm} \\ d_2/h &= 49 / 300 \\ &= 0.163 \end{aligned}$$

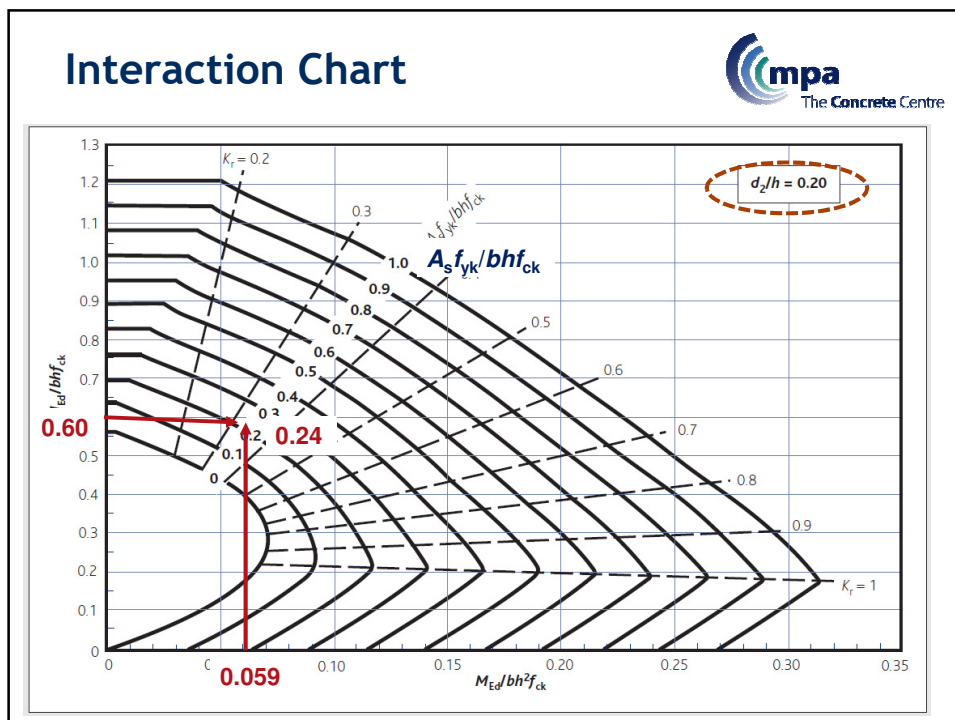
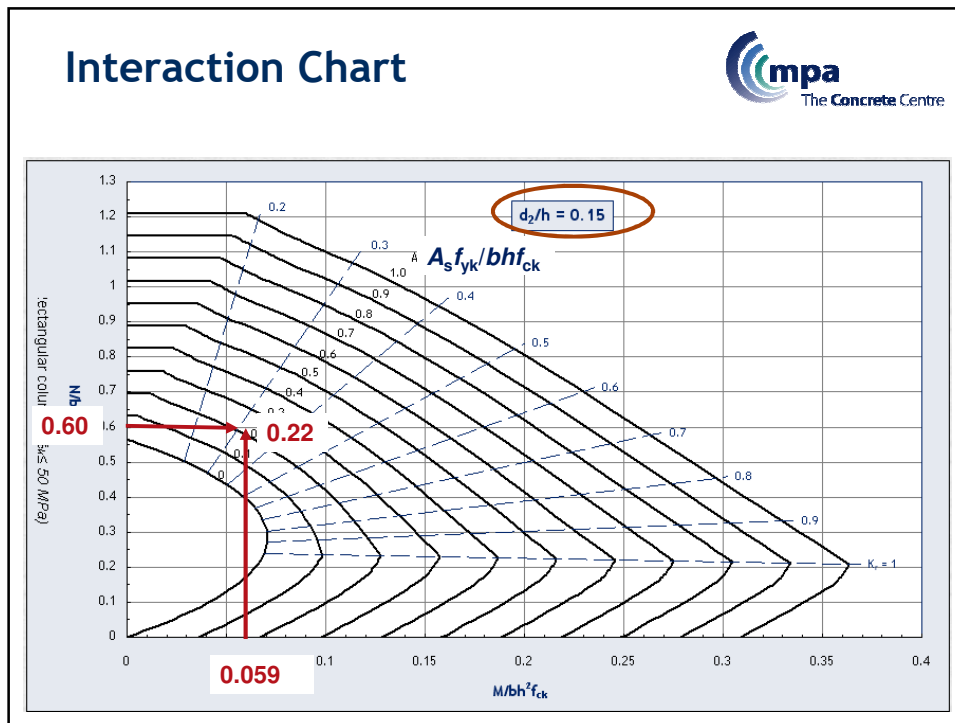
NB.
Charts are for symmetrically reinforced. They actually work on centroid of the reinforcement in half the section. So when no. of bars > 4 beware. See Concise 15.9.3



∴ Assuming 4 bars and interpolating between $d_2/h = 0.15$ (Fig 15.5c) and 0.20 (Fig 15.5d) for:-

$$N_{Ed} / bhf_{ck} = 1620 \times 10^3 / (300^2 \times 30) = 0.60$$


$$M_{Ed} / bh^2f_{ck} = 47.8 \times 10^6 / (300^3 \times 30) = 0.059$$



Solution - determine A_s

$A_s f_{yk} / bh f_{ck} = 0.225$ by interpolation
 $A_s = 0.225 \times 300^2 \times 30 / 500 = 1215 \text{ mm}^2$

Try 4 no. H20 (1260 mm²)



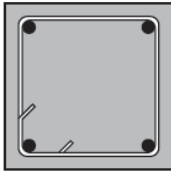
Links

Diam min. $\geq \phi / 4$
 $\geq 20 / 4 = 5 \text{ mm}$

Max. spacing = 0.6×300
 $= 180 \text{ mm}$

So use H8 links @ 175c/c

Design summary




4H20
 H8 links @175
 25 mm cover
 $f_{ck} = 30 \text{ MPa}$

Design summary: edge column

Often the analysis and design would have to be undertaken for the other axis and where necessary checked for biaxial bending: in this case neither is critical

Workshop Problem - Your turn

The suspended slabs (including the ground floor slab) are 300 mm thick flat slabs at 4500 mm vertical centres. Between ground and 5th floors the columns at C2 are 500 mm square; above 5th floor they are 405 mm circular. Assume an internal environment, 1-hour fire resistance and $f_{ck} = 50 \text{ MPa}$.



Assume the following:

- Axial load: 7146kN
- Top Moment: 95.7kN
- Bottom Moment: -95.7kN
- Nominal cover: 35mm
- Bay width is 6.0 m
- Elastic modulus is the same for column and slab

Design the column C2 between 1st and 2nd floors for bending about axis parallel to line 2.

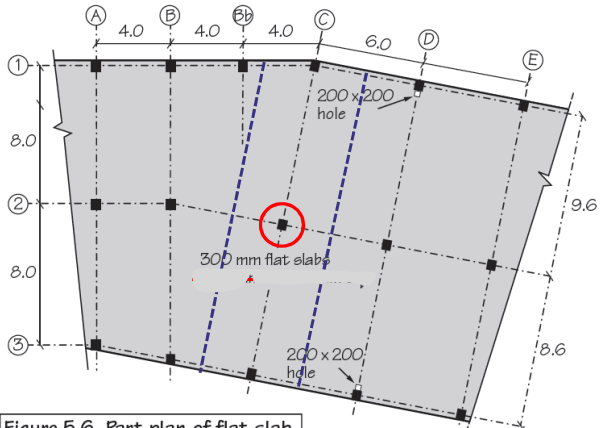


Figure 5.6 Part plan of flat slab

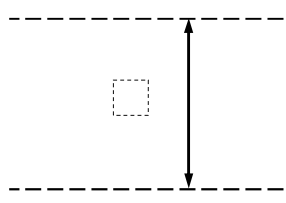
Your solution

Use PD 6687 method
 Column clear span is 4500 - 300 = 4200 mm and $k_1 = k_2$

$$k = \frac{\frac{EI_c}{L_c}}{\sum \frac{2EI_b}{L_b}} =$$

From Table (Table 4 of How to...Columns)

k2	k1										
	0.10	0.20	0.30	0.40	0.50	0.70	1.00	2.00	5.00	9.00	Pinned
0.10	0.59	0.62	0.64	0.66	0.67	0.69	0.71	0.73	0.75	0.76	0.77
0.20	0.62	0.65	0.68	0.69	0.71	0.73	0.74	0.77	0.79	0.80	0.81
0.30	0.64	0.68	0.70	0.72	0.73	0.75	0.77	0.80	0.82	0.83	0.84
0.40	0.66	0.69	0.72	0.74	0.75	0.77	0.79	0.82	0.84	0.85	0.86
0.50	0.67	0.71	0.73	0.75	0.76	0.78	0.80	0.83	0.86	0.86	0.87
0.70	0.69	0.73	0.75	0.77	0.78	0.80	0.82	0.85	0.88	0.89	0.90
1.00	0.71	0.74	0.77	0.79	0.80	0.82	0.84	0.88	0.90	0.91	0.92
2.00	0.73	0.77	0.80	0.82	0.83	0.85	0.88	0.91	0.93	0.94	0.95
5.00	0.75	0.79	0.82	0.84	0.86	0.88	0.90	0.93	0.96	0.97	0.98
9.00	0.76	0.80	0.83	0.85	0.86	0.89	0.91	0.94	0.97	0.98	0.99
Pinned	0.77	0.81	0.84	0.86	0.87	0.90	0.92	0.95	0.98	0.99	1.00



Take 'beam' width as, say, half the bay width

F =
 $\therefore l_o =$

Check slenderness:
 $\lambda = 3.46 l_o/h$
 =

Your solution

$$M_{01} = \text{Min}\{|M_{\text{top}}|, |M_{\text{bottom}}|\} + e_i N_{Ed}$$

$$M_{02} = \text{Max}\{|M_{\text{top}}|, |M_{\text{bottom}}|\} + e_i N_{Ed}$$

$$e_i N_{Ed} : \quad e_i = l_o/400 = \quad e_i N_{Ed} = \quad \text{kNm}$$


$$N_{Ed} =$$

$$\therefore M_{02} = \quad M_{01} =$$

$$M_{0Ed} = (0.6M_{02} + 0.4M_{01}) \geq 0.4M_{02}$$

$$=$$

Minimum moment ($e_0 N_{Ed}$)
 $e_0 = \text{Max}\{h/30; 20\text{mm}\}$



Your solution

Check whether the column is stocky or slender using λ_{lim} method:

$$A = 0.7 \text{ (use default value)}$$

$$B = 1.1 \text{ (use default value)}$$

$$C = 1.7 - r_m = 1.7 - M_{01}/M_{02} =$$

$$n = N_{\text{Ed}}/A_c f_{\text{cd}} =$$

$$(f_{\text{ck}} = 50 \text{ MPa}, f_{\text{cd}} = \alpha_{\text{cc}} \cdot f_{\text{ck}}/\gamma_m)$$

$$\lambda_{\text{lim}} = 20 ABC / \Gamma n$$

=

$$\lambda =$$

The column is/is not slender

$$M_2 =$$

Your solution

$$M_{\text{Ed}} = \max[M_{02}; M_{0\text{Ed}} + M_2; M_{01} + 0.5M_2; e_0 N_{\text{Ed}}]$$

$$M_{02} =$$

$$M_{0\text{Ed}} + M_2 =$$

$$M_{01} + 0.5M_2 =$$

$$e_0 N_{\text{Ed}} =$$

$$\therefore M_{\text{Ed}} =$$

Your Solution - determine A_s 

$$d_2 = c_{\text{nom}} + \text{link} + \phi/2 =$$

$$d_2/h =$$

$$M_{\text{Ed}}/(bh^2f_{\text{ck}}) =$$

$$N_{\text{Ed}}/(bhf_{\text{ck}}) =$$

Look up charts are in "Concise" pages 97-99

Your Solution - determine A_s 

$$A_s f_{yk} / bh f_{ck} =$$

$$A_s = \quad \text{mm}^2 \quad \text{Try H } (\quad \text{mm}^2)$$

Links

$$\text{Diameter} = \max \{6, \phi/4\} = \quad \text{mm}$$

$$s_{\text{cl,tmax}} = \min \{12 \phi_{\text{min}}; 0.6b; 240\text{mm}\} = \quad \text{mm}$$

$$\text{Try H } @ \quad \text{c/c}$$



End of Lecture 8