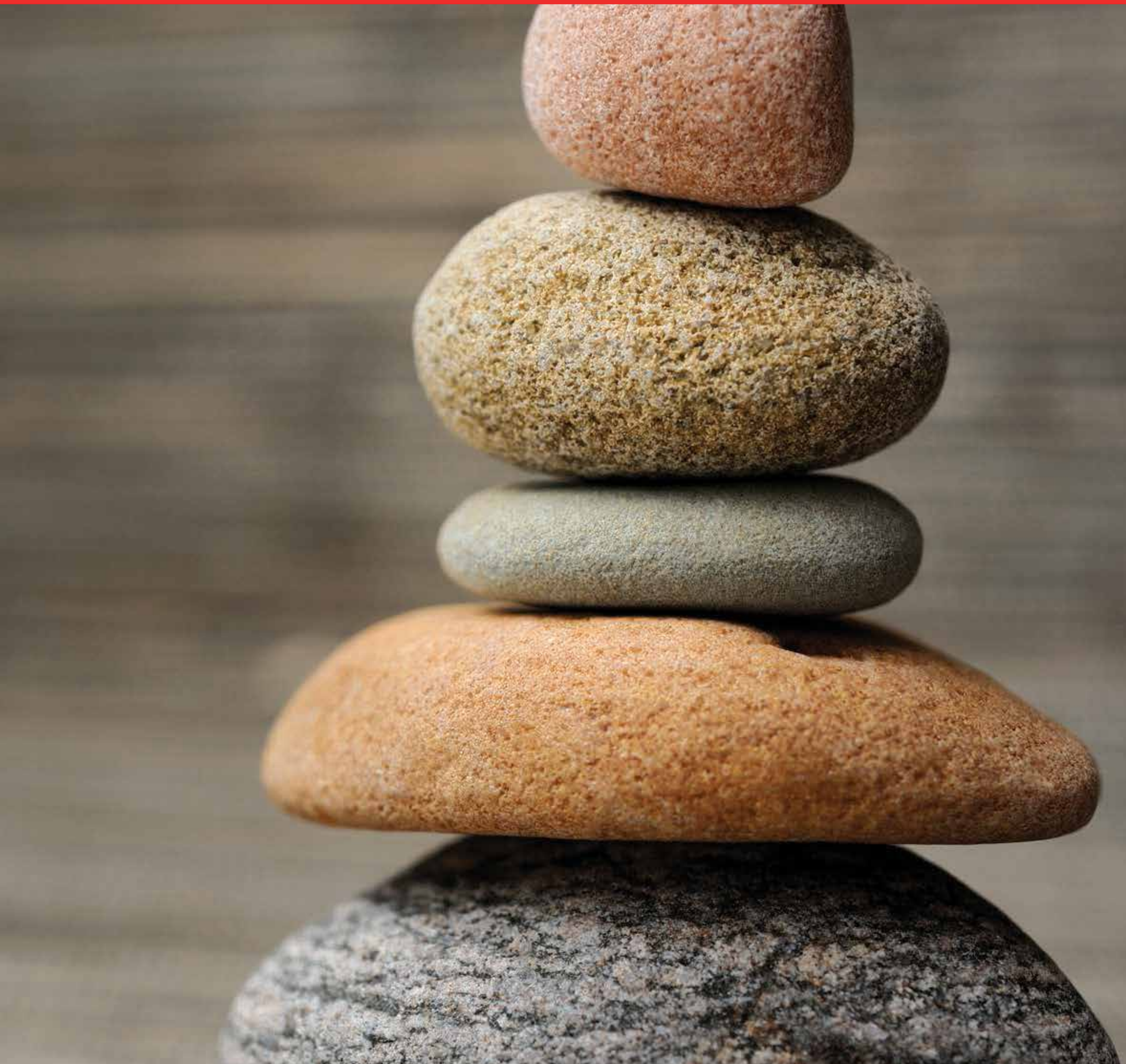
PRACTICAL MATHEMATICS & COMPUTING (FA2)

CERTIFICATE IN FINANCIAL ACCOUNTING



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IN MALAWI

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PREFACE

INTRODUCTION

The Institute noted a number of difficulties faced by students when preparing for the Institute's examinations. One of the difficulties has been the unavailability of study manuals specifically written for the Institute's examinations. In the past students have relied on text books which were not tailor-made for the Institute's examinations and the Malawian environment.

AIM OF THE MANUALS

The manual has been developed in order to provide resources that will help the Institute's students attain the needed skills. It is therefore recommended that each student should have their own copy.

HOW TO USE THE MANUAL

Students are being advised to read chapter by chapter since subsequent work often builds on topics covered earlier.

Students should also attempt questions at the end of the chapter to test their understanding. The manual will also be supported with a number of resources which students should keep checking on the ICAM website.

FA2: PRACTICAL MATHEMATICS & COMPUTING

COURSE AIM

To enable the candidate to understand basic mathematical principles and techniques, and use them in business operations including space, volume, sales, interest rates, investments, data summarizationand uncertainty.

OBJECTIVES

On completion of this module, the candidate will be able to:

i. apply arithmetic and algebraic principles in business operations
ii. apply graphical techniques to solve business problems
iii. compute basic summary statistics
iv. interpret basic summary statistics
v. formulate equations and inequalities from word problems
vi. solve equations and inequalities
vii. demonstrate the use of techniques for presenting data including charts and graphs
viii. calculate simple probabilities
ix. apply basic computing skills in business

METHOD OF ASSESSMENT

The Practical Mathematics and Computing module will be assessed using a traditional 3 hour paper-based examination. The examination paper will consist of two sections; section A and section B. Section A will be compulsory and it will carry 60 marks. Section B will have 3 questions each carrying 20 marks. Candidates will be required to answer any 2 questions.

SPECIFICATION GRID

This grid shows the relative weightings of topics within this subject and should guide the relative study time spent on each. Over time the marks available in the assessment will equate to the weightings below, while slight variations may occur in individual assessments to enable suitably rigorous questions to be set.

Syllabus Area	Weighting %
Basic Mathematics	15
Financial Mathematics	20
Basic Statistics	15
Probability	10
Computing	40
Total	100

Learning outcomes

A. Basic Mathematics

The candidate will be able to demonstrate the use of basic mathematics in business, and solve equations.

In the assessment, the candidate may be required to:

1. Sets and numbers

i. define key terms: set, element, subset, complement and disjoint sets

ii. find intersections and unions of sets

iii. describe a set using either list notation or set-builder notation

iv. demonstrate the relationships between sets using Venn Diagrams

v. apply sets in real life problems

vi. differentiate between natural numbers, integers, rational numbers, irrational numbers

a. and real numbers.

2. Fractions, decimals and reciprocals

i. differentiate between proper and improper fractions

ii. carry out operations involving fractions

iii. solve problems requiring the use of fractions

3. **Percentage**

    i. solve problems involving percentages

    ii. convert fractions to percentages, and vice versa

    iii. solve word problems involving fractions and percentages

4. **Approximation, error and tolerance**

    i. round off a number to a given number of decimal places and significant figures

    ii. find the absolute error and maximum error of a given measurement

    iii. explain sources of error

    iv. explain how rounding can affect the results of a calculation

5. **Perimeter, area and volume**

    i. find the perimeters and areas of regular plane shapes

    ii. calculate surface areas of regular objects

    iii. calculate volumes of regular objects

    iv. calculate perimeters and areas of regular plane shapes

6. **Financial mathematics**

The candidate will be able to solve problems and apply financial mathematical techniques involving interest, profit, basic taxes and premiums.

In the assessment, the candidate may be required to:

i. define interest

ii. calculate simple and compound interest

iii. explain the difference between simple and compound interest

iv. calculate profit or loss

v. calculate profit mark up and margin

vi. distinguish trade discount from cash discount

vii. calculate discount

viii. convert money in other currencies to Malawi Kwacha and vice versa

ix. calculate PAYE given a person's taxable income

x. calculate VAT on various goods and services

xi. prepare electricity and water bills

xii. calculate insurance premiums

xiii. calculate instalments when buying in instalments/hire purchase

7. Basic statistics

The candidate will be able to demonstrate the use of various techniques for summarising date including tabulation, graphing and basic summary measures.

In the assessment, the candidate may be required to:

i. explain the need for statistics in business environment

ii. explain the difference between data and information

iii. identify the properties of good information

iv. tabulate data including frequency distributions

v. prepare graphs, charts and diagrams

vi. calculate summary measures for ungrouped data including arithmetic mean, mode, median, range, variance and standard deviation

8. Introduction to probability

The candidate will be able to calculate basic probabilities and demonstrate the use of probability where uncertainty exists in business

In the assessment, the candidate may be required to:

i. describe the role of probability in decision making

ii. define probability, experiment, sample space, experiment, outcome and event

iii. describe the different types of events including complementary, mutually exclusive and independent events

iv. calculate simple probabilities

v. use the addition and multiplication rules of probability

COMPUTING

9. Introduction to Information Systems

Candidates should be able to explain the role of information systems in an organization.

In the exam candidates may be required to:

i. Describe essential components and role of an information systems in a business

ii. Describe various components of office automation

iii. Explain uses of Computer Networks, Emails and Internet

10. Data Processing

Candidates should be able to explain the following: data processing operations, data correction techniques, organizational levels and their information needs, role of computers in data processing and qualities of good information.

In the exam candidates may be required to:

i. Define data processing

ii. Describe Data processing operations

iii. Describe the role of computers in data processing

iv. Describe factors that make information valuable

v. Describe qualities of good information

11. Computer Hardware

Candidates should be able to explain the various parts of computer hardware.

In the exam candidates may be required to:

i. Define computer hardware

ii. Describe basic components of a computer hardware

iii. Describe functions and parts of computer system unit

iv. Explain internal data representation

v. Describe different types of computers

vi. Explain embedded systems

vii. Explain computer generations

viii. Describe computer classification

12. Computer Software

Candidates should be able to explain the uses and classifications of various software products on the market

In the exam candidates may be required to:

i. Define computer software

ii. Describe classification of software

iii. Explain uses of computer software packages

iv. Describe generations of computer languages

13. Data Communication

Candidates should be able to explain how organizations can benefit by using the following technologies :- Internet, intranet and extranets.

In the exam candidates may be required to:

i. Describe components and use of internets, extranets and intranets

ii. Define E - Business

iii. Explain uses of Computer Networks

13. Data Security and Controls

Candidates should be able to explain risks to information systems and various safeguards.

In the exam candidates may be required to:

i. Describe data security

ii. Explain the need for security and controls

iii. Explain logical and physical controls

iv. Explain the spread of and safeguards against computer viruses.

v. Explain systematic development of a Disaster Recovery Plan

REFERENCES

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Swokowski, E. & Cole, J. A. (2009). *Fundamentals of college algebra* (Custom ed.). London: Cengage Learning

Swokowski, E. & Cole, J. A. (2005). *Fundamentals of college algebra* (11thed.). London: Brooks/Cole

Loudon & Loudon: Management Information Systems : Managing the digital Firm

Turban: Information technology for management: Transforming organizations in the digital economy

Bocij P.: Business information Systems

C.S. French: Computer Science

Internet Resources

http://www.answers.com/Analog computers

http://en.wikipedia.org/wiki/Embedded_system

http://www.wordiq.com

http://www.computermuseum.li

http://www.columbia.edu/acis/history/generations.html

Table of Contents

CHAPTER 1
SETS AND NUMBERS

LEARNING OBJECTIVES

By the end of this chapter the student should be able to:

i) Define key terms: set, element, subset, complement and disjoint sets
ii) Find the intersection and union of sets..
iii) Describe a set using either list notation or set-builder notation.
iv) Demonstrate the relationships between sets using Venn Diagrams.
v) Differentiate between Natural numbers, Integers, Rational Numbers, Irrational numbers and Real Numbers.

1.0 Introduction

This chapter deals with sets and numbers. Here we will see that items that have common characteristics can be put into groups which we call sets. If we have many sets we can then consider the relationships between different sets. The notion of a set is a powerful mathematical tool that is used to solve real-world problems in science, business, and many other fields as this chapter will demonstrate. This chapter will also introduce you to different number systems. This is important because numbers belong to different systems and it is important that we know which numbers can be added, subtracted, multiplied and divided without being taken out of the number system under consideration.

1.1 Sets

Definition

A set is well-defined collection of objects. For example, the employees of a company in the marketing department form a well-defined set. The objects that make up a set are called its **elements** or **members.**

1.1.1 Terminology and Notation

We use upper case letters to denote sets and elements of a set are enclosed between curly brackets, $\{\ldots\}$. If the set of vowels is Vthen $V = \{a, e, i, o, u\}$. In this case we know that a is an element of V and we will write this as $a \in V$. The notation $k \notin V$denotes that $k$is not an element of the set V.

Empty Set

We allow as a set the empty or null set, denoted by $\emptyset$ which is the set that contains no elements.

Subset

If we have a set $K = \{1,2,3,4\}$ then we can form another set with elements drawn from K. For example, $M = \{1,2,4\}$(elements taken from K in no particular order) has all of its elements taken from K. In other words, every element of $M$is also an element of K. In this case where every element of a set, say M, is also an element of K we call M a subset of K. This is written as $M \subseteq K$. Note that a set is a subset of itself!

Cardinality of a set

If a set A has k elements then we say that the cardinality of A, written $|A|$or$n(A)$, is k. So the cardinality of the set $A = \{2,3,4,5,6,7\}$is 6.

Equal Sets

Two sets A and B are said to be equal, written A= B, if they have the same number of elements and every element of A is an element of B and every element of B is an element of A. Equivalently, $A = B$ if and only if $A \subseteq B$and $B \subseteq A$.
Example: The two sets $P = \{a, b, c, d\}$ and $Q = \{d, b, a, c\}$ are equal.

1.1.2 Set Operations

Here we look at how sets can be combined to form new sets.

Universal Set

If we are considering a number of sets and we put all the elements of the sets under discussion in one set then we form a set called the universal set.

Union

Let $X$and Y be non-empty sets then union of X and Y denoted $X \cup Y$ consists of those elements that belong either to X or Y or both. For example if X= $\{1,2,3,4\}$ and $Y = \{1,4,6,8\}$then $X \cup Y = \{1,2,3,4,6,8\}$. You will note that $4 \in X$ and $4 \in Y$ and that 4 is appearing only once in $X \cup Y$ because **WE DO NOT REPEAT ELEMENTS IN A SET**. It is also worth noting that the way we order elements in a set does not matter. This means we could have written $X \cup Y$as $X \cup Y = \{1,2,4,3,8,6\}$.

Intersection

Let $X$and Y be non-empty sets then the intersection between X and Y denoted $X \cap Y$ consists of those elements that are common to both sets $X$and Y. For example if X= $\{1,2,3,4\}$ and $Y = \{1,4,6,8\}$ then $X \cap Y = \{4\}$ because this is the only element that is found in both sets. If two sets do not have any elements in common then the sets are said to be **disjoint.**

Example 1

Let $\mathrm{A} = \{1,2,3,4\}$, $\mathrm{B} = \{3,4,5,6,7\}$ and $\mathrm{C} = \{2,3,8,9\}$.
Then
$\mathrm{A} \cup \mathrm{B} = \{1,2,3,4,5,6,7\}$, $\mathrm{A} \cup \mathrm{C} = \{1,2,3,4,8,9\}$, $\mathrm{B} \cup \mathrm{C} = \{2,3,4,5,6,7,8,9\}$,
$\mathrm{A} \cap \mathrm{B} = \{3,4\}$, $\mathrm{A} \cap \mathrm{C} = \{2,3\}$ and $\mathrm{B} \cap \mathrm{C} = \{3\}$.

Complement

Let $\boldsymbol{U}$ be the universal setand X be a non-empty set, then the complement of X in $\boldsymbol{U}$ denoted X' is the set of all elements in $\boldsymbol{U}$ not in X. For example, let $\boldsymbol{U} = \{2,4,6,8,10\}$ and $X = \{4,8\}$ then $X' = \{2,6,10\}$.

1.1.3 Set Description

In this section we look at some of the of ways of describing a set.

List Notation

A set can be described by listing down its elements. For example, the set of counting numbers can be given as $\{1,2,3, \ldots\}$. In this example the three dots at the end, called an ellipsis, tell us that the list of numbers continues.

Predicate Notation/Set-Builder Notation

A set can be described by specifying a condition for membership in the set. For example, the set of vowels can be described in this way as

$$V = \{\mathrm{x} : \mathrm{x \text{ is a vowel}}\}.$$

In general, when we use the predicate notation to describe a set, we will write $A = \{\mathrm{x} : \mathrm{P(x)}\}$ this is read as ***'x such that P(x)'***. In this notation we look for elements x having the property described by P. How can we describe the set

$$D = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$$

using set-builder notation?

Example 2

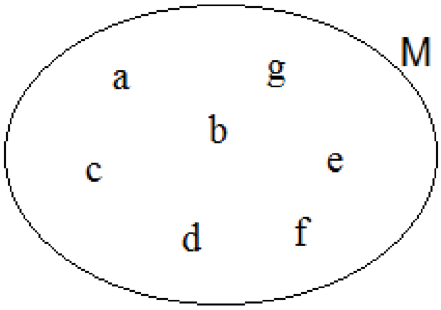
Use predicate notation to describe the set $C = \{\text{Blantyre, Lilongwe, Mzuzu, Zomba}\}$.
Solution

We note that this set contains all the cities of Malawi hence we can write:

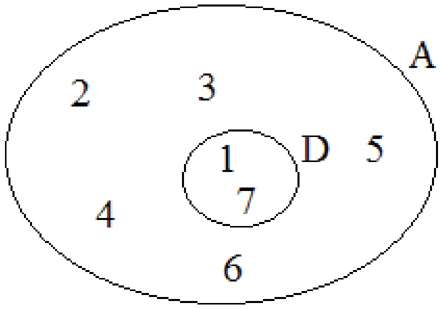
$$C = \{\mathrm{c} : \text{c is a city in Malawi}\}.$$

1.1.4 Venn Diagrams

A *Venn diagram* is a simple pictorial representation of a set. For example, if $R = \{a, b, c, d, e, f, g\}$ then we could represent this information in the form of a Venn diagram as:



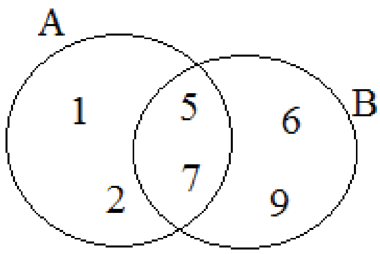
Venn diagrams are useful for demonstrating general relationships between sets. For example, if a firm maintains a fleet of 7 cars, we might write $A = \{1,2,3,4,5,6,7\}$ (each car being numbered for convenience). If also it was important to identify those cars of the fleet that were being used by the directors, we might have $D = \{1,7\}$. i.e. Cars *1* and *7* are director's cars. This situation is represented in the Venn diagram overleaf.



This diagram nicely demonstrates the fact that D is a subset of A.

Example 3

Discuss the Venn diagram below with respect to intersection and union.



Solution

It is easy to see that $A = \{1,2,5,7\}$ and that $B = \{5,6,7,9\}$. Also from the diagram $A \cap B = \{5,7\}$and $A \cup B = \{1,2,5,6,7,9\}$.

Example 4

Let M, P and C be the sets of students taking Business Mathematics, Practical Maths and Costing respectively at a college. Assume $|M| = 300$, $|P| = 350$, $|C| = 450$, $|M \cap P| = 100$, $|M \cap C| = 150$, $|P \cap C| = 75$, $|M \cap P \cap C| = 10$. How many students are taking exactly one of those courses?

Solution

From the information given and using the Venn diagram below:
We want to find $a + c + g$. Now, $e = |M \cap P \cap C| = 10$, so since $|M \cap P| = 100$, $b = 100 - 10 = 90$. In the same way, $d = 150 - 10 = 140$, $f = 75 - 10 = 65$.

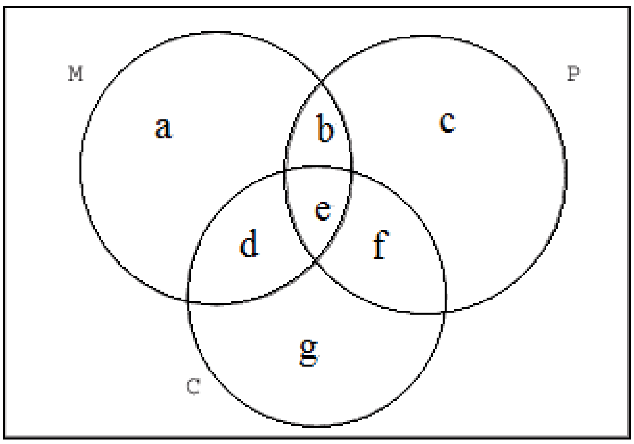
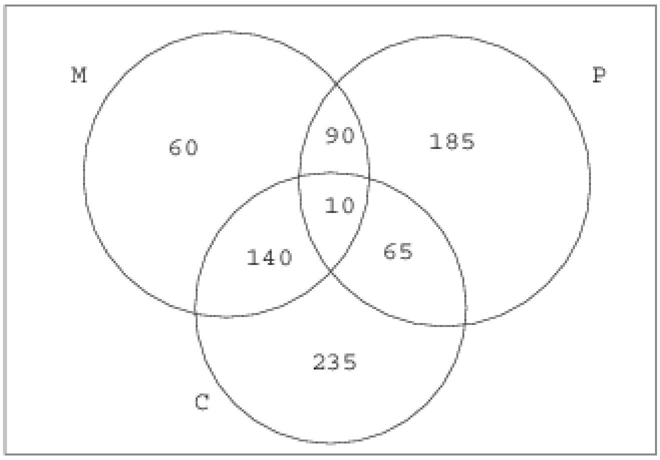


Figure 1

We also do the same for the rest of the letters and we obtain the Venn diagram below.



Hence $a + c + g = 60 + 185 + 235 = 480$ is the number of students taking exactly one of those courses.

1.2 Number Systems

Certain sets of numbers are so frequently referred to that they are given special symbolic names.

1.2.1 Natural Numbers

This is the set of counting numbers 1,2,3,…. It is usually denoted as ***N*.**
So ***N***={1, 2, 3 . . .}. Elements of this set can be multiplied and added with the result being another natural number. However, if we consider 1-2 the result is -1 which is not a natural number. We will say that a set of numbers is closed under a given operation if combining two elements under the operation does not take us out of the given set of numbers. We can therefore say that ***N***is closed under addition and multiplication but not addition and division.

1.2.2 Integers

This is the set ***Z*** = { . . . ,−2,−1, 0, 1, 2, . . .} containing natural numbers, zero and negative whole numbers. ***Z*** is closed (i.e. well defined) under addition, subtraction and multiplication but not division. The set of natural numbers is a subset of the set of integers.

1.2.3 Rational Numbers

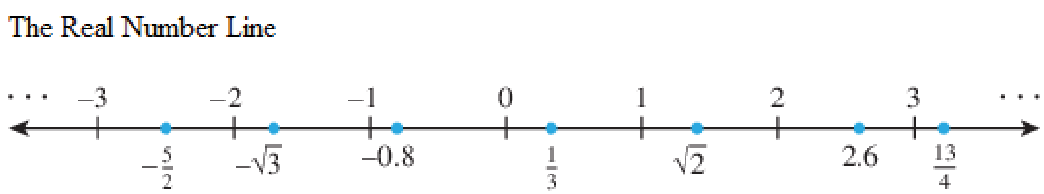
The setof rational numbersdenoted by $\boldsymbol{Q}$ containsall numbers that can be written as quotient of two integers. We can write $\boldsymbol{Q} = \{\frac{\boldsymbol{a}}{\boldsymbol{b}} : a, b \in \boldsymbol{Z} \; and \; b \neq 0\}$**.** It is worth noting that the set of integers is a subset of the rationals. In our definition we have said that an element belonging to $\boldsymbol{Q}$ can be written as a quotient of two integers and as such π is not a rational number. It is not correct to say that π can be written as $\frac{22}{7}$ because the value of π is not equal to $\frac{22}{7}$, however we are allowed to take $\frac{22}{7}$ as an approximation to π. The set $\boldsymbol{Q}$ is closed under all the four basic operations of addition, subtraction, multiplication and division.

1.2.4 Irrational Numbers

These are numbers that cannot be written as a fraction of two integers. We may denote irrational numbers as $\boldsymbol{I}$. They are nonrepeating, nonterminating decimals. π, e and $\sqrt{2}$ are irrational numbers.e has an approximate value of 2.718.

1.2.5 Real Numbers

This set is denoted by $\boldsymbol{R}$ and is the union of the rational and irrational numbers. That is, $\boldsymbol{R} = \boldsymbol{Q} \cup \boldsymbol{I}$. Observe that$\boldsymbol{N} \subseteq \boldsymbol{Z} \subseteq \boldsymbol{Q} \subseteq \boldsymbol{R}$.
The set of real numbers is usually pictured as the set of all points on a line, as shown overleaf. The number 0 corresponds to a middle point, called the *origin*. A unit of distance is marked off, and each point to the right of the origin corresponds to a positive real number found by computing its distance from the origin. Each point to the left of the origin corresponds to a negative real number, which is found by computing its distance from the origin and putting a minus sign in front of the resulting number. The set of real numbers can be divided into three parts: the set of positive real numbers, the set of negative real numbers, and the number 0. *Note that 0 is neither positive nor negative.*



Example 5

Show that each number is a rational number by finding two integers whose ratio equals the given number.

a) $-\frac{4}{7}$
b) 8
c) $0.\overline{6}$

Solution

a) $-\frac{4}{7}$ is a rational number because it can be expressed as the ratio of the integers -4 and 7.

b) 8 is a rational number because it can be expressed as the ratio of the integers 8 and 1$(8 = \frac{8}{1})$.

c) $0.\overline{6}$ represents the repeating decimal 0.6666666… and can be expressed as the ratio of 2 and 3 $\left(0.\overline{6} = \frac{2}{3}\right)$.

Example 6

Check the set(s) to which each number belongs. The numbers may belong to more than one set.

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6						
$\sqrt{23}$						
$-\frac{2}{7}$						
3						
2.35						

Solution

	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-6			✓	✓		✓
$\sqrt{23}$					✓	✓
$-\frac{2}{7}$				✓		✓
3	✓	✓	✓	✓		✓
2.35				✓		✓

1.3 Number Bases

In everyday life we use decimal notation to express integers. For example, 123 is used to denote

$$1 \times 10^2 + 3 \times 10^1 \times 10^0.$$

However, it is often convenient to use bases other than 10. Any integer greater than 1 may be used as the base when expressing integers. However, the common number bases used other than 10 are binary (with 2 as the base), octal (base 8) and hexadecimal (base 16). Computers usually use binary notation when carrying out arithmetic and octal or hexadecimal notation when expressing characters, such as letters or digits.

1.3.1 Place Values

Each numeration system, or base, uses different digits and has different place values depending on the base being used. Place values are powers of a given base. The ten-based decimal system, which seems most natural to us, is constructed in powers of ten.

Place Value	1000_{10}	100_{10}	10_{10}	1_{10}
Exponent	10^3	10^2	10^1	10^0
Decimal Value	1000	100	10	1

Base nine is constructed in powers of nine.

Place Value	1000_9	100_9	10_9	1_9
Exponent	9^3	9^2	9^1	9^0
Decimal Value	729	81	9	1

In the ten-based decimal system, we have the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. There is no single digit for *ten*. The quantity *ten* is represented by writing the numeral 10, which means, "1 ten and 0 ones." These two individual digits represent one number. Base nine only has the digits 0, 1, 2, 3, 4, 5, 6, 7, and 8. The single digit 9 does not exist in base nine. *Nine* in base nine would be, "1 nine and 0 ones," or 10_9, just as *ten* is, "1 ten and 0 ones," or 10, in base ten. Letters are used to serve as extra digits in bases greater than ten.

1.3.2 Reading Numerals

It is important to be able to read and verbalize numbers in other number bases. Any number without a subscript is assumed to be in base ten. For example, a number in base ten can be properly written as 34_{10}, but 34 is assumed to be in base ten. Numbers in other bases are not verbalized the same way as base ten. For example: 29_7 should be vocalized as, "two nine base seven." It should not be read as, "twenty nine," because this would imply base ten.

1.3.3 The Difference Between Bases

Here we look at the first fifteen numbers of the different systems side by side so we can visualize the difference between them.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

1.3.4 Converting from Other Bases to Base Ten

By using multiplication and the place values of the original base, one can convert from any number base to base ten.

Example 7

Convert 52031_6 to base ten.

Solution

The first step is to find out what quantity each place value represents. Since the number is in base six, it can be represented this way

5	2	0	3
6^3	6^2	6^1	6^0

Then, multiply each number by its place value using expanded notation.

$(5 \times 6^3) + (2 \times 6^2) + (0 \times 6^1) + (3 \times 6^0) = (5 \times 216) + (2 \times 36) + (0 \times 6) + (3 \times 1) = 1155.$

Therefore, 52031_6 is the same as 1,155 in base ten.

Example 8

Convert $2AE0B_{16}$ to base ten.

Solution

As we did above, we will multiply each number by its corresponding place value and add the resulting products.

2	A	E	0	B
16^4	16^3	16^2	16^1	16^0

Then, multiply each number by its place value using expanded notation.

$$(2 \times 16^4) + (A \times 16^3) + (E \times 16^2) + (0 \times 16^1) + (B \times 16^0)$$
$$= (2 \times 16^4) + (10 \times 16^3) + (14 \times 16^2) + (0 \times 16^1) + (11 \times 16^0) = 175{,}627.$$

Note that we have used the fact that $A = 10, B = 11$and $E = 14$.

1.3.5 Converting from Base Ten to any other Base

When changing from base ten to another number base, there are two different methods. The first method involves dividing by the place values of the desired base. The second method involves using a division method which keeps track of remainders and progressively dividing by the desired base.

Example 9

Convert 1155 to base six.

Solution

Method 1

The place values of base are 6^0 ,6^1, 6^2, 6^3, 6^4, 6^5 and so on. Now, to convert, divide the original number by the largest place value that is still smaller than the original number, which in this case is $6^3 = 216$ (We cannot take $6^4 = 1296$which is greater than 1155.) Next, divide the remainder from the previous step by the next smaller place value. Repeat this step for each of the remaining place values. Let us do that.

- $1155 \div 216 = 5$with a remainder of 75.
- We now divide the remainder by the next smaller place value. We see that $6^2 = 36$ is smaller than 75 so we will use 36. Now ,$75 \div 36 = 2$ with a remainder of 3.
- We take the remainder 3 and find a suitable place value that is smaller than 3. Obviously, $6^0 = 1$ is the only place value we can use. Let us divide 3 by this place value.We obtain $3 \div 1 = 3$ with no remainder.
- Let us now put all this information together using a table. We get:

5	2	0	3
216	36	6	1
6^3	6^2	6^1	6^0

- above each place value.

Method 2

In this method, we divide the number we want to convert by the desired base, and keep track of remainders in a separate column. Proceed by dividing the non-remainder part of the dividend by the desired base until the result is less than one.

Example 10

Convert 407 to base sixteen.

Solution

$$\mathbf{407 \div 16 = 25} \xrightarrow{\textit{Remainder}} \mathbf{7}$$

$$\mathbf{25 \div 16 = 1} \xrightarrow{\textit{Remainder}} \mathbf{9}$$

$$\mathbf{1 \div 16 = 0} \xrightarrow{\textit{Remainder}} \mathbf{1}$$

We can use a table to do these calculations as follows;

		Remainder
16	**407**	**7**
16	**25**	**9**
16	**1**	**1**
	0	

To get the answer, write the numerals in the remainder column from bottom to top. Therefore, $407 = 197_{16}$. Notice that while converting, only the non-remainder part of the dividend is used in the next step; remainders are recorded in a separate column as whole numbers and not as decimals. As a result, the final equation equals 0 with a remainder of 1 and not $\frac{1}{16}$. Since the final dividend is 0 with a remainder, division is no longer necessary.

1.3.6 Converting from One Non-Decimal Number to another Non-decimal Number

To convert from one non-decimal base to another, convert first to base ten, then to the desired base. For example, to convert 21_8 to base sixteen and binary, convert from base eight to base ten first, and then convert the base ten number to base sixteen and binary.

$$21_8 = 17_{10} = 11_{16} = 10001_2.$$

1.3.7 Conversion between binary, octal, and hexadecimal expansions

Even though we have said that it is better to convert to base ten first when converting between non-decimal bases, conversion between binary and octal and between binary and hexadecimal expansions is extremely easy because each octal digit corresponds to a block of three binary digits and each hexadecimal digit corresponds to a block of four binary digits.

Example 11

Find

a) the octal and hexadecimal expansions of $(11\ 1110\ 1011\ 1100)_2$.
b) the binary expansions of $(765)_8$ and $(A8D)_{16}$.

Solution

a) To convert $(11\ 1110\ 1011\ 1100)_2$ into octal notation we group the binary digits into blocks of three, adding initial zeros at the start of the leftmost block if necessary. These blocks, from left to right, are 011, 111, 010, 111, and 100, corresponding to 3, 7, 2, 7, and 4, respectively. Consequently, $(11\ 1110\ 1011\ 1100)_2 = (37274)_8$.

To convert $(11\ 1110\ 1011\ 1100)_2$ into hexadecimal notation we group the binary digits into blocks of four, adding initial zeros at the start of the leftmost block if necessary. These blocks, from left to right, are 0011, 1110, 1011, and 1100, corresponding to the hexadecimal digits 3, E, B, and C, respectively. Consequently, $(11\ 1110\ 1011\ 1100)_2 = (3EBC)_{16}$.

b) To convert $(765)_8$ into binary notation, we replace each octal digit by a block of three binary digits. These blocks are 111, 110, and 101. Hence, $(765)_8 = (1\ 1111\ 0101)_2$.

To convert $(A8D)_{16}$ into binary notation, we replace each hexadecimal digit by a block of four binary digits. These blocks are 1010, 1000, and 1101. Hence, $(A8D)_{16} = (1010\ 1000\ 1101)_2$.

1.3.8 Addition and Subtraction of Numbers in bases other than ten.

If we are asked to find the value of the sum of 364 and 197 using pencil-and-paper we would proceed as shown below;

$$\begin{array}{r} 11 \\ 364 \\ +197 \\ \hline 561 \\ \hline \end{array}$$

Remember that addition is done by columns and that if the result of an addition is more than nine then the number is divided by ten. We then record the remainder and carry the quotient. In the example above, $7 + 4 = 11$ and $11 \div 10 = 1$ remainder 1 so we record 1carry 1. This 1 is then added to the sum of the numbers in the next column. You will see that we have two 1's above 364 as in the first column we carried one and also in the second column we carried one. Subtraction also works the same way the difference being that when, in any given column, the number in the upper row is less than the number in the lower row we have to borrow (take) 1 from the next column and add to the number in the current column. In base ten, whenever we take a one from the next column what we add to the current number is ten. The next example shows this;

$$\begin{array}{r} 25 \\ 364 \\ -197 \\ \hline 167 \\ \hline \end{array}$$

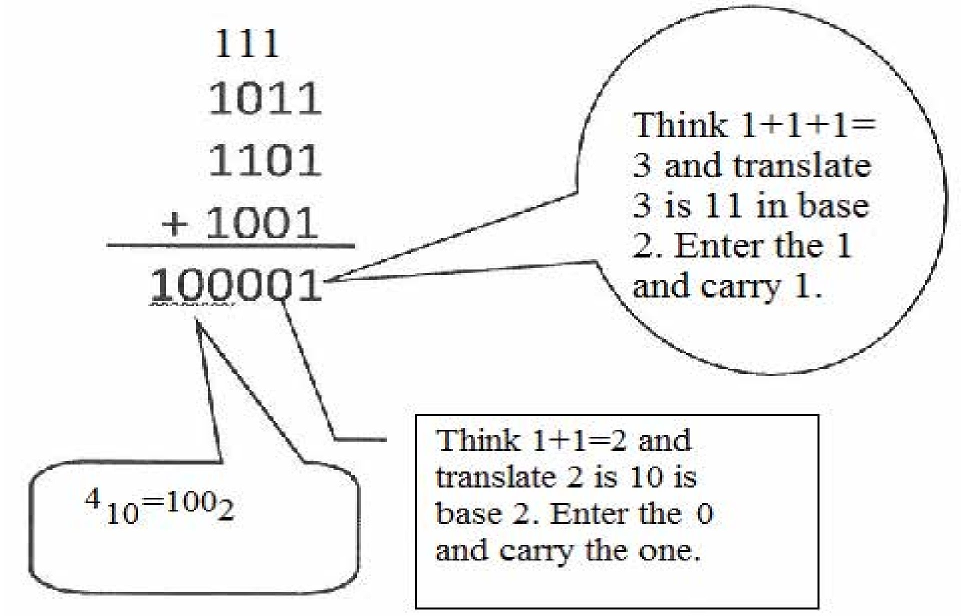
For numbers in other bases addition and subtraction can be done in the same fashion. What we have to bear in mind is the fact that the base we are working in determines the dividing factor. For example if we are working in base eight then sums will be reduced by dividing by eight. Also when subtracting numbers that are in base eight whenever we take a 1 from the next column what we will add to the current number will be eight (10_8). Lets do a couple of examples.

Example 12

Find the sum of 1011_2, $1101_2$and 1001_2.

Solution

Note that we could convert all these numbers to base ten, find their sum and convert back to base. But what are interested in here is to add these numbers in base two.



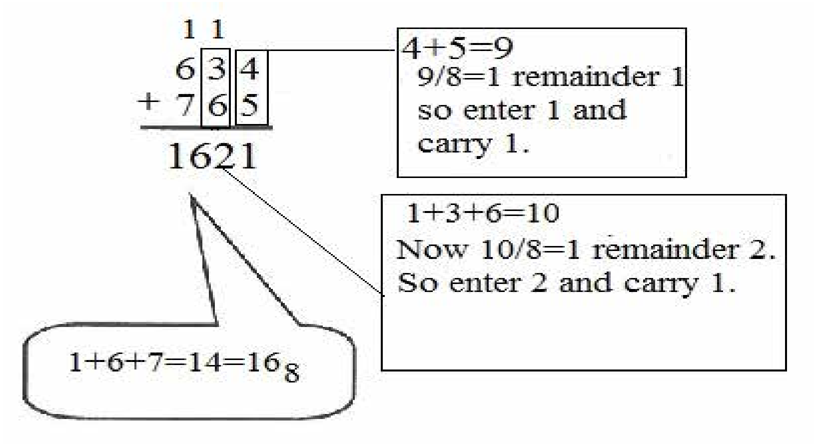
Therefore $1011_2 + 1101_2 + 1001_2 = 100001_2$.

Example 13

Find the sum of 634_8 and 765_8.

Solution

These numbers are in base eight so we will be dividing any number greater than 7 by 8.



We can see that $634_8 + 765_8 = 1621_8$.

Example 14

Find the difference between $101_2$and 100_2.

Solution

This one is straight forward.

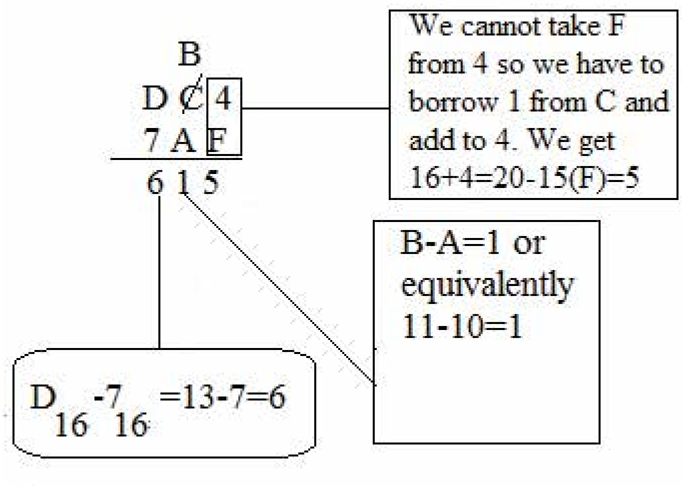
$$\begin{array}{r} 101 \\ \underline{100} \\ 001 \end{array}$$

Example 15

Find the difference between $DC4_{16}$ and $7AF_{16}$.

Solution

Because we are doing subtraction whenever we borrow 1 from the next column that 1 will be equivalent to sixteen.



CHAPTER SUMMARY

In this chapter we have looked at the following:

- Meanings of: Set, empty set, subset, cardinality of a set, equal sets and universal set.
- Set operations: Union, Intersection and Complement
- Set description: List Notation and Set-builder notation.
- Venn diagrams.
- Number Systems: Natural numbers, Integers, Rational numbers, Irrational numbers and Real numbers.
- Number Bases: Conversion between different bases, Addition and Subtraction of numbers in a base other than ten

END OF CHAPTER EXERCISES

1. Which of these sets are equal: $\{x, y, z\}$, $\{z, y, z, x\}$, $\{y, x, y, z\}$, $\{y, z, x, y\}$?

2. Let **U** = {1,2, …, 9} be the universal set, and let $A = \{1, 2, 3, 4, 5\}$, $C = \{5,6,7,8,9\}$, $E = \{2, 4, 6, 8\}$, $B = \{4, 5, 6, 7\}$, $D = \{1, 3, 5, 7, 9\}$, $F = \{1, 5, 9\}$.

   Find the following:

   a) $A \cup B$ and $A \cap B$

   b) $A \cup C$ and $A \cap C$

c) $D \cup F$ and $D \cap F$.

3. Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

4. Let $A = \{0, 2, 4, 6, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{0, 3, 6, 9\}$. Find the following:

   a) $A \cup B \cup C$

   b) $A \cap B \cap C$.

5. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

6. In a particular accounts office, employees Kondwani, Duncan, William and Bande have MSCE, with Kondwani and Bande also having a degree. Kondwani, Melvin, William, Tina, Momo and King are associate members of Institute of Chartered Accountants in Malawi(ICAM) with Tina, and Momo having MSCE. Identifying set A as those employee with MSCE, set *C* as those employees who are ICAM and set *D* as graduates:

   a) Specify the elements of sets *A, C* and *D.*

   b) Draw a Venn diagram representing sets *A, C* and *D*, together with their known elements.

   c) What special relationship exists between sets *A* and D?

   d) Specify the elements of the following sets and for each set, state in words what information is being conveyed.

      i. $A \cap C$
      ii. $D \cup C$
      iii. $D \cap C$

   e) What would be a suitable universal set for this situation?

7. Check the sets to which each number belongs.

	Natural Numbers	**Integers**	**Rational**	**Irrational Numbers**	**Real Numbers**
5					
$-\sqrt{9}$					
-1.7					

$\frac{1}{2}$					
$0.\overline{2}$					
$\sqrt{7}$					
$\frac{0}{4}$					

8. Convert the following numbers to base 10:

   a) 304_8

   b) FFD_{16}

   c) 1011_2

   d) 4133_6

9. Convert the following base 10 numbers to the indicated base:

   a) 851 to base 8

   b) 1389 to hexadecimal

   c) 322 to binary

   d) 677 to base 9

10. Evaluate

    a) $223_7 + 561_7$

    b) $AB78_{16} + DF8C_{16}$

    c) $10111_2 + 10101_2$

11. Perform the following:

    a) $467_8 - 365_8$

    b) $3211_6 - 455_6$

    c) $101011_2 - 111_2$

    d) $CAA8_{16} - B9D_{16}$

CHAPTER 2

FRACTIONS, DECIMALS AND PERCENTAGES

LEARNING OBJECTIVES

By the end of this chapter, the student should be able to:

i. Differentiate between proper and improper fractions.
ii. Carry out operations involving fractions.
iii. Solve applied problems requiring the use of fractions.
iv. Solve problems involving percentages.
v. Convert fractions to percentage.
vi. Convert percentages to fractions.
vii. Solve word problems involving fractions and percentages.

2.0 Introduction

This chapter seeks to introduce fractions, decimals and percentages. Firstly, we will define fractions and look at different forms of fractions. We will also look at how to add, subtract, multiply and divide fractions. Secondly, we will consider decimals and the interplay between decimals and fractions. Basic arithmetic operations will also performed on decimals with emphasis placed on multiplication and division. Lastly, we will see how percentages are related to the notions of fractions and decimals. It is worth noting that collectively fractions, decimals and percentages are important tools in understanding and manipulating business transactions where calculations are involved.

2.1 Fractions

A fractionrepresents a part of a whole or,more generally, any number of equal parts. It describes how many parts of a certain size there are, forexample, one-half, eight-fifths, three-quarters. We could also define a fraction as a ratio of two integers a and b written as $\frac{a}{b}$ or ${}^{a}\!/_{b}$ where $b \neq 0$.

2.1.1 Forms of Fractions

a) Simple, common, or vulgar fractions

A common, vulgar, or simplefraction (for example $\frac{26}{3}$) consists of an integer numeratorand a non-zero integer denominator. Simple fractions can be positive or negative, proper, or improper (see below).

b) Proper and Improper Fractions

Proper fractions are those expressed with the numerator smaller than the denominator.

Examples of proper fractions include ¾, ½, $^{21}/_{40}$, $^{98}/_{99}$.

Improper fractions on the other hand are those whose numerators are larger than the denominators. Thus $^{7}/_{2}$, $^{30}/_{20}$ are improper fractions.

c) Mixed numbers

A mixed number is a sum of a non-zero integer and a proper fraction. This sum is implied without the use of any visible operator such as "+". $2\frac{1}{4}$ is a mixed number which is the sum of 2 and $\frac{1}{4}$, we can therefore write $2 + \frac{1}{4} = 2\frac{1}{4}$.

There is a close connection between mixed numbers and improper fractions. In fact a mixed number can be changed to an improper fraction. To do this, we multiply the whole number and the denominator. Then add the numerator. Write the sum over the denominator.

Example 1

Change $2\frac{1}{4}$ and $3\frac{2}{5}$ to an improper fraction.

Solution

$2\frac{1}{4} = \frac{(2\times4)+1}{4} = \frac{9}{4}$ and $3\frac{2}{5} = \frac{(3\times5)+2}{5} = \frac{17}{5}$.

An improper fraction can be changed to either a whole number or a mixed number. This is done by:

STEP 1: Dividing the numerator by the denominator.

STEP 2: The quotient (without the remainder) becomes the whole number part of the mixed number. The remainder becomes the numerator of the fractional.

STEP 3: The new denominator is the same as the denominator of the improper fraction.

Example 2

Change $\frac{23}{4}$ and $\frac{21}{7}$ to a mixed number.

Solution

a. $\frac{23}{4} = 5$ with remainder 3. As such $\frac{23}{4} = 5\frac{3}{4}$.

b. $\frac{21}{7} = 3$ with remainder 0. So $\frac{21}{7} = 3$.

d) Reciprocals and the "invisible denominator"

The reciprocal of a fraction is another fraction with the numerator anddenominator exchanged. The reciprocal of $\frac{13}{17}$, for instance, is $\frac{17}{13}$. The product of a fraction and its reciprocal is 1. Any integer can be written as a fraction with the numberone as denominator. For example, 15 can be written as $\frac{15}{1}$, where 1 issometimes referred to as the invisible denominator. Therefore, every fractionor integer except for zero has a reciprocal. The reciprocal of 17 is $\frac{1}{17}$.

2.1.2 Operations on Fractions

a) Simplifying fractions to lowest terms

To simplify the work involved when working with fractions, they are usually reduced to the lowest terms. This is an operation where the numerator and the denominator are divided by their highest common factor. This is the largest number that divides both without leaving a remainder.

Example 3

Reduce $\frac{20}{30}$ to its lowest term.

Solution The HCF of 20 and 30

$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$

$HCF = 2 \times 5 = 10$. Taking numbers common to bothfactorisations.

Dividing 20 by 10 = 2 and 30 by 10 = 3

$$\therefore \quad \frac{20}{30} = \frac{2}{3}$$

The same answer could be obtained by successive cancellations (divisions) of the numerator and denominator by highest common factors of the numerators and denominators.

Thus $\frac{20}{30} = \frac{10}{15} = \frac{2}{3}$..

That is first divide top and bottom by 2 then the resulting fraction by 5.

b) Equivalent fractions

Multiplying the numerator and denominator of a fraction by the same(non-zero) number results in a fraction that is equivalent to the originalfraction. For example, $\frac{1}{2} = \frac{1\times 2}{2\times 2} = \frac{2}{4}$ means that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Dividing the numerator and denominator of a fraction by the same non-zeronumber will also yield an equivalent fraction. This is called *reducing* or *simplifying* the fraction. A simple fraction in which the numerator anddenominator are coprime (that is, the only positive integer that goes intoboth the numerator and denominator evenly is 1) is said to be irreducible, inlowest terms, or in simplest terms. For example, $\frac{18}{27}$ is not in lowest terms because both 18 and 27 can be exactly divided by 27. From the work we did in the previous section we can say that $\frac{20}{30}$ and $\frac{2}{3}$ are equivalent fractions.

c) Comparing Fractions

In order to compare fractions we need to find a common denominator (the LCM of the denominators) and express all the fractions to be compared in terms of the common denominator. The fraction with the largest numerator is the largest. Thus to compare $\frac{a}{b}$ and $\frac{b}{d}$, these are converted to $\frac{ad}{bd}$ and $\frac{bc}{bd}$. Then bd is the common denominator and the numerators ad andbc can be compared.

Example 4

Which is larger between $\frac{3}{4}$ and $\frac{4}{5}$?

Solution

Common denominator is $4 \times 5 = 20$.

$$^{3}/_{4} = {}^{15}/_{20} \text{ and } {}^{4}/_{5} = {}^{16}/_{20}$$

$$\therefore {}^{4}/_{5} \text{ is larger.}$$

d) Addition and Subtraction

To add or subtract fractions one must find a common denominator which is normally the lowest common multiple of the denominators of the respective fractions. The numerators are multiplied by relevant factors obtained by dividing the original numerators into the common denominator. Essentially, we get the fractions to have the same denominators then we add or subtract the numerators as required.

Example 5

Add $\frac{1}{2}$ and $\frac{1}{3}$ (common denominator is LCM of 2 of 3 which is 6)

Solution

$$\frac{1}{2}+\frac{1}{3}=\frac{1\times 3+1\times 2}{6}=\frac{5}{6}$$

Example 6

Find the sum of $2\frac{1}{3}$, $1\frac{7}{9}$ and $3\frac{4}{7}$.

Solution

We note that we have mixed numbers in the given problem.
To add or subtract mixed numbers:

STEP 1: Add or subtract the fractions. It is easier when the denominators are equal.

STEP 2: Add or subtract the whole numbers.

STEP 3: Simplify

We will use the steps given above to work this problem.

$$2\frac{1}{3}+1\frac{7}{9}+3\frac{4}{7} = 6\frac{1\times 21+7\times 7+4\times 9}{63}$$, 63 is the LCM of 3, 7 and 9.

$$= 6\frac{21+49+36}{63}$$

$$= 6\frac{106}{63}=(6+1)\frac{43}{63}=7\frac{43}{63}$$

Example 7

Evaluate $^{7}/_{9}$ - $^{3}/_{5}$

Solution

Here the Common denominator is 45

$$\frac{7\times5-3\times9}{45}=\frac{35-27}{45}=\frac{8}{27}$$

Example 8

Evaluate 2 ½ - $1^{1}/_{4}$

Solution

2 ½ - $1^{1}/_{4}$ = 1½ - ¼
= 2 $^{2}/_{4}$ – ¼
= $1^{1}/_{4}$

e) **Multiplication and Division**

Multiplication

When multiplying proper or improper fractions the answer can be obtained by a direct multiplication of the numerators and denominators (respectively). To multiply mixed numbers, convert the fraction to an improper fraction then multiply.

Example 9

Multiply $\frac{3}{4}\times\frac{1}{5}$

Solution:

$$\frac{3}{4}\times\frac{1}{5}=\frac{3\times1}{4\times5}=\frac{3}{20}$$

Example 10

Find the product of $\frac{1}{4},\frac{3}{7}$ and $\frac{2}{3}$.

Solution

$$\frac{1}{4}\times\frac{3}{7}\times\ \frac{2}{3}=\frac{1\times3\times2}{4\times7\times3}=\frac{6}{84}=\frac{1}{14}$$

Example 11 Multiply $1^3/_4$ and $2^1/_5$

Solution

We have to convert the mixed numbers into improper fractions and then multiply. Let us do that.

$1^3/_4 = {}^7/_4$ and $2^1/_5 = {}^{11}/_5$

So we have $1^3/_4 \times 2^1/_5 = {}^7/_4 \times {}^{11}/_5 = {}^{77}/_{20} = 3\frac{17}{20}$.

Division

When dividing fractions the divisor must be inverted and then multiplied into the dividend. In other words the inverted divisor becomes the multiplier.

If mixed numbers are involved like in the multiplication process, they must be converted to improper fractions as first step.

Example 12

Evaluate $\frac{3}{8} \div \frac{3}{5}$

Solution

$$\frac{3}{8} \div \frac{3}{5} = \frac{3}{8} \times \frac{5}{3} = \frac{15}{24} = \frac{5}{8}$$

Example 13

Divide $1^4/_5$ by $2\ ^3/_4$

Solution

We must first convert the mixed numbers to improper fractions then we will carry out the division.

$$1\frac{4}{5} = \frac{9}{5} \text{ and } 2\frac{3}{4} = \frac{11}{4}$$

So $1\frac{4}{5} \div 2\frac{3}{4} = \frac{9}{5} \div \frac{11}{4} = \frac{9}{5} \times \frac{4}{11} = \frac{36}{55}$

2.2 Decimals

Decimals have a wide application in business, particularly in currencies and metric measurements where 10 or powers of 10 make the basis of counting.

A decimal is a fraction expressed as a fractional part of 10 or power of 10. Thus 0.2, 0.04, 0.005 are decimal fractions. If expressed as proper fractions, they would be 2/10, 4/100, 5/1000 respectively. A decimal is written with a dot which separates the whole number (to the left) and the fraction part (to the right). For example, in 5.643, 5 is the whole number and .643 is the fraction part.

Conventionally a decimal fraction like 0.4 is said to have 1 decimal place, 0.03 has 2 decimal places 0.4214 has 4 decimal places and so on.

2.2.1 Addition and Subtraction of Decimals

To add and subtract decimals the number must be written with the decimal point running down (aligned) a column before the addition or subtraction operation is carried out. In other words corresponding decimal places must be in the same columns.

Example 14

Add 3.2, 41.003, 20.0341

Solution:

$$\begin{array}{l} 3.2 \\ 41.003 \\ 20.0341 \\ \boxed{64.2371} \end{array}$$

Example 15

Find the sum of 111.1196, 3212.2115, 6406.3728, 518.9337 and 27.0188

Solution:

$$\begin{array}{r} 111.1196 \\ 3212.2115 \\ 6406.3728 \\ 518.9337 \\ +\ 27.0181 \\ \boxed{10275.6554} \end{array}$$

Example 16

Subtract 1.93 from 2.20

Solution

$$\begin{array}{r} 2.20 \\ -\underline{1.93} \end{array}$$

0.27

Example 17

Evaluate 149.297 – 122.229

Solution

$$\begin{array}{r} 149.297 \\ -\underline{122.229} \\ \underline{27.068} \end{array}$$

Notice that after some practice it is not necessary to write the decimal fraction in a column, addition or subtraction can be done as figures are presented as long as corresponding decimal places are added or subtracted. Further if a calculator is used figures can simply be entered.

2.2.2 Multiplication and Division of Decimals

To multiply decimals one may proceed as though they were ordinary numbers. When the product has been obtained the decimal point can be placed by counting the total number of decimal places of the original numbers multiplied. This number becomes the number of decimal places in the product.

Example 18

Multiply 3.43 by 10.003

Solution:

$$\begin{array}{r} 343 \\ \times\underline{10003} \\ 1029 \\ \underline{343} \\ \underline{3431029} \end{array}$$

There are two decimal places in 3.43 and three in 10.003. Therefore the total number of decimal places is 5. The answer has 5 decimal places hence 34.31029

When carrying out division the divisor must always be converted into a whole number. This is done by multiplying it by an appropriate power of 10. The dividend should also be multiplied by the same factor of ten.

Example 19

Divide 4.4526 by 1.23

Solution

Multiply 1.23 by 100
The divisor becomes 123

∴ the dividend is 4.4526 x 100 = 445.26

$$
\begin{array}{r|r}
 & 3.62 \\
\hline
123 & 445.26 \\
 & -369 \\
\hline
 & 762 \\
 & -738 \\
\hline
 & 246 \\
 & -246 \\
\hline
 & 0
\end{array}
$$

2.2.3 Converting vulgar fractions to decimals

A common fraction can be converted to decimal by dividing the numerator by the denominator.

Example 20

Convert $\frac{3}{8}$ to a decimal.

Solution

We perform the division 3 ÷ 8. The result is 0.375.

$$
\begin{array}{rrrrrrr}
 & & .3 & 7 & 5 & & \\
\hline
8 &)3 & .0 & 0 & 0 & 0 & 0 \\
- & 2 & 4 & & & & \\
\hline
 & & 6 & 0 & & & \\
 & - & 5 & 6 & & & \\
\hline
 & & & 4 & 0 & & \\
 & & - & 4 & 0 & & \\
\hline
 & & & & 0 & &
\end{array}
$$

Example 21

Convert $\frac{71}{80}$ to a decimal.

Solution

We perform the division $71 \div 80$. The result is 0.8875.

$$
\begin{array}{r|l}
 & 0.8875 \\
\cline{2-2}
80 & 71 \\
 & 710 \\
 & \underline{-640} \\
 & 700 \\
 & \underline{-640} \\
 & 600 \\
 & \underline{-560} \\
 & 400 \\
 & \underline{-400} \\
 & 0
\end{array}
$$

$$\therefore\ 71 \div 80 = \frac{71}{80} = 0.8875$$

2.2.4 Converting decimals to vulgar fractions

Here we will look at how two types of decimals, terminating and non-terminating and can be converted to fractions.

Terminating Decimals

A decimal is terminating if it has a last digit. It is quite easy to turn them into fractions. Create a fraction whose numerator is the decimal number (without the decimal point) and denominator is a power of 10 where the power is the number of decimal places in the decimal number.

Example 22

Convert 0.45 to a reduced fraction.

Solution

The numerator of our fraction is 45. The number of decimal places is two so the denominator will be 10^2. Therefore:

$$0.45 = \frac{45}{10^2} = \frac{45}{100} = \frac{9}{20}.$$

Example 23

Convert 23.044 to a reduced fraction.

Solution

The number of decimal places is three so we will divide by 10^3. Let us do that

$$23.044 = \frac{23044}{10^3} = \frac{23044}{1000} = \frac{5761}{250} = 23\frac{11}{250}..$$

Alternatively, $23.044 = 23 + 0.044 = 23 + \frac{44}{1000} = 23 + \frac{11}{250} = 23\frac{11}{250}$

Non-terminating Decimals

A decimal is non-terminating if it has infinitely many digits. If there is a repeating block, we denote it by abar drawn over the repeating digit. For example, the number $2.\overline{35}$denotes $2.35353535 \ldots$. Turning these decimals into fractions is an application of linear equations. We will go through the method with the aid of an example.

Example 24

Convert the repeating decimal $7.\overline{4}$ to a fraction.

Solution

Step 1. We call our number x and write it without the bar notation.

$$x = 7.444444 \ldots$$

Step 2. We multiply both sides of this equation by 10.

$$10x = 74.44444 \ldots$$

Step 3. We write these equations together, starting with the second one.

$$\begin{aligned} 10x &= 74.44444 \ldots \\ x &= 7.44444 \ldots \end{aligned}$$

Step 4. We subtract the second equation from the first one.

$$67 = 9x$$

Step 5. We solve the equation for x.

$$67 = 9x\,, x = \frac{67}{9}.$$

Thus the answer is $\frac{67}{9}$.

.

Example 25

Convert the repeating decimal $0.\overline{405}$ to a fraction.

Solution

Step 1. We call our number x and write it without the bar notation

$$x = 0.405405405405405 \ldots$$

Steps 2 and 3. This decimal has a three-digit long repeating block. To obtain proper alignment of the digits,we will move the decimal point by three digits, i.e. we will multiply by 1000. We multiply both sides of thisequation by 1000. We write these equations together, starting with the second one.

$$1000x = 405.405405405405 \ldots$$
$$x = 0.405405405405 \ldots$$

Step 3. We subtract the second equation from the first one.

$$999x = 405$$

Step 4. We solve the equation for x.

$$999x = 405, \text{then} x = \frac{405}{999} = \frac{15}{37}.$$

2.3 Percentages

The idea of percentages is very important in business because it is another way of representing fractions and decimals. Thus a fraction or a decimal can be converted into a percentage. A percentage also gives a better way of comparing quantities which have fractional parts. That is, to compare numbers having decimals, one may simply convert the numbers as percentages and then compare the resulting percentages.

The term 'percent' means out of hundred or per hundred. Therefore 5 percent means 5 per hundred, 5 out of hundred or $^5/_{100}$. Similarly 67 percent means 67 out of hundred.

From the discussion above one can see that a percentage is a fraction whose denominator is 100 except that the latter is not written. It is instead replaced by the symbol %. Thus 5 percent is written as 5%.

Example 26

Express in percentage form the statement: In a school 3 boys in every 4 own bicycles.

Solution

¾ of the total number of boys own bicycles.

∴ In every hundred boys (¾ of 100) boys own bicycles

∴ In every hundred boys 75 own bicycle or 75% own bicycles

Example 27

A man spends 10% of his income on rent and 55% on household expenses. What percentage remained?

Solution

The man spends a total of 10% + 55% = 65%.

∴ He has 100 – 65 = 35% remaining.

Example 28

65% of bus conductors are men, what percentage of bus conductors are women

Solution In every 100 conductors there are 65 men

∴ 100 – 65 = 35 are women

∴ 35% of the bus conductors are women.

Example 29

What percentage of K 8140 is K 2035?

Solution

The required percentage is $\frac{2035}{8140} \times 100 = 25\%$.

Example 30

A pair of shoes costing K 1480 is sold at a profit of 20%; find the selling price.

Solution

The actual profit is $\frac{20}{100}$ ×K 1480 =K 296.

Hence the selling price is K 1480 +K296 =K1776.

2.3.1 Converting fractions to percentages

To convert fraction to percentage one simply multiplies the fraction by a hundred and append a % symbol.

Example 31 :

Express the following as percentages

a. $\frac{1}{5}$

b. $\frac{3}{20}$

Solution

a. $^{1}/_{5} \times 100 = 20\%$

b. $^{3}/_{20} \times 100 = {}^{300}/_{20} = 15\%$.

2.3.2 Converting decimals to percentages

Decimals are converted to percentages by, again, multiplying by 100%.

Example 32

Write the following as percentages $0.75, 0.33, 0.125$.

Solution

a. $0.75 \times 100 = 75,$

$\therefore\ 0.75 = 75\%$

b. $0.33 \times 100 = 33,$

$\therefore 0.33 = 33\%$

c. $0.125 \times 100 = 12.5,$

$\therefore 0.125 = 12.5\%$

2.3.3 Converting percentages to fractions or decimals

This conversion is done by writing the percentage with its denominator. The former gives a common fraction while the latter results with a decimal.

Example 33

Convert the following percentages to decimals.

a. 75% b. 115%

Solution

First we re-write the percent as a fraction. The we perform the division indicated by the fraction.

a. $75\% = \frac{75}{100} = 0.75$

b. $115\% = \frac{115}{100} = 1.15$

Example 34

Convert 55% to a reduced fraction.

Solution

We rewrite the percent as a fraction and simplify. $55\% = \frac{55}{100} = \frac{11}{20}$.

Example 35

Convert 37.5% to a reduced fraction.

Solution

We rewrite the percent as a fraction and simplify.

$$37.5\% = \frac{37.5}{100} = \frac{37.5\times 10}{100\times 10} = \frac{375}{1000} = \frac{3}{8}.$$

CHAPTER SUMMARY

In this chapter we have looked at the following:

- The difference between proper and improper fractions.
- Addition, subtraction, multiplication and division of fractions.
- Addition, subtraction, multiplication and division of decimals.
- Conversion between fractions, decimals and percentages.

END OF CHAPTER EXERCISES

1. Subtract

   a. $^3/_8$ from $\frac{3}{4}$

   b. $^2/_9$ from $^1/_{11}$

2. Add

   a. $^2/_7$ and $\frac{1}{4}$

   b. $4\,^3/_2$ and $2\,^1/_3$

3. Evaluate the following

   a. $^5/_6 - ^1/_3 \times \frac{1}{2}$

   b. $24\frac{1}{4} - ^9/_{11} - 13\frac{1}{2}$

   c. $7^1/_{20}$ of $(9^1/_2 - 5\,^1/_{14}) - (2^1/_7$ of $- ^1/_{14})$

4. Which of the following fractions are equivalent?

$$\frac{3}{4}, \frac{1}{3}, \frac{6}{8}, \frac{2}{6}, \frac{9}{5}, \frac{75}{100}, \frac{10}{30}, \frac{15}{20}$$

5. Convert the following numbers to decimals

$$\frac{1927}{11}, \frac{12}{14}$$

6. Add or subtract. Write each sum or difference in simplest form.

a. $3\frac{1}{6} + 5\frac{1}{6}$

b. $9\frac{4}{5} - 2\frac{3}{10}$

c. $13\frac{1}{8} - 1\frac{7}{10}$

7. Express 7 minutes and 12 seconds as a fraction of 1 hour and write the answer as a decimal.

8. Express in percentage the following statements:

   a. 1 day in every 5 days is wet

   b. A tax is at the rate of 10 in every kwacha

   c. One out of every 20 goats is male.

9. Express the following as percentages:

   a. $\frac{11}{25}$

   b. $\frac{13}{14}$

   c. 0.39

10. A certain product weighs 10 kilograms 13 grams. What is its cost to the nearest tambala if the retail selling price is K8.53 per kilo?

11. Four men own $^1/_6$, $^1/_5$, ¼ and $^2/_{15}$ of the capital of a company. How much of the capital do they own between them?

12. After spending 25% of his money, Ryan is still left with K 2070. How much money did he have at first?

13. An article is sold for K 800; find the cost price

    a. if it is sold at a profit of 20%,

    b. ifisis sold at a loss of 5%.

14. The owner of a business dies and the business has to be wound up. In his will, he leaves ½ of his money to his wife, $^1/_3$ to be shared by his three sons and the rest for his daughter. If the daughter receives K300,000 determine;

    a. the total amount of the will,

b. the amount his wife receives.

15. Sales of the Gona Corporation were 7500 units in 1985. Find 1986 sales if:

    a. 1986 sales are 130% of 1985 sales

    b. 1986 sales are 130% more than 1985 sales?

    c. 1986 sales are 30% less than 1985 sales.

16. Given the following are sales figures of a company, find:

    a. the rate of increase from 1985 to 1986

    b. the rate of decrease from 1986 to 1987

1985	K40,500.00
1986	K43,740.00
1987	K41,115.00

CHAPTER 3

APPROXIMATION, ERROR AND TOLERANCE

LEARNING OBJECTIVES

By the end of this chapter the student should be able to:

i) Differentiate between rounding a number using decimal places and significant figures.
ii) Round off a number to a given decimal places.
iii) Round a number to a given number of significant figures.
iv) Find the absolute error and maximum error of a given measurement.
v) Discuss sources of error.
vi) Discuss how rounding can affect the results of a calculation.

3.0 Introduction

In practice a lot of measurements and results of computations cannot be exact. Cases of measurements which cannot be exact include distance between places, population of a town or country, crop yields and the length of cloth manufactured in a period. As for computations usually those resulting in very large figures or involving very small fractions cannot be exact. In these cases approximations are used and they are normally sufficient.

3.1 Approximations

An approximate figure is one which is sufficiently close to the true value for the purpose it is intended. For example the population of Malawi can be quoted as 7.5 million. This, it may be known, is not exact. But for practical purposes it is useful. Another example is that of room temperature: the exact one may be 25.9213 centigrade but a figure of 26 will be acceptable. It is not very far from the true value and the applications for it will not be sensitive to the minor difference.

Approximations are arrived at by expressing a figure (measurement, result of computation):

(a) to the nearest selected unit or denomination,

(a) to a specified number of decimal places.

Example 1

A bought 2 pieces of cloth each 1m 1 cm long, B bought 1 piece 3m 70cm long. How much cloth to the nearest meter did A and B buy?

Solution

(1m 10cm) x 2 + 3m 70cm = 2m 20cm x 3m 70cm
= 5m 90cm $\simeq$ 6m

The symbol '$\simeq$' is approximately equal to.

3.1.1 Significant figures

When counting or computation has been carried out the result can be expressed with a number of digits depending on how many digits have meaning. The rest of the digits in the figure are set to zero.

Example 2

Round the following numbers correct to 3 significant figures:
(a) 430719
(b) 43.0717
(c) 430119
(d) 0.0824

Solution

(a) 430719 becomes 431000
(b) 43.071 becomes 43.1 here zeros are left out because they won't mean anything.
(c) 430191 becomes 430000
(d) 0.0824 becomes 0.0824. Zeroes placed at the beginning of a decimal number are not significant.

Thus in rounding or correcting to a number of significant figures, only the specified number of digits is kept starting from left to right. The rest are set to zero.

3.1.2 Rounding or correcting to a number of decimal places

This is similar to the idea of significant figures only that the emphasis is on the fraction part of the number or result.

Example 3

Write the following correct to 3 decimal places.

(a) 1.3492
(b) 2.01973
(c) 0.04722

Solution

(a)	1.349	➜	1.349
(b)	2.01973	➜	2.020
(c)	0.04722	➜	0.047

Example 4

Find and correct to one decimal place the sum of 186.34725, 8.91567, 100.5723, 47.83191

Solution

186.34725 + 8.91567 + 47.83191
= 343.66713

To 1 decimal place we have 343.7

3.1.4 Contracted addition and subtraction

When adding or subtracting figures whose result is required to a certain number of significant figures the work involved can be reduced by using the contracted method. Here the individuals figures are rounded to same level of significant figures usually about 2 more than the level (number) required for the answer.

Example 5

Find the sum of the following figures 171775, 181392, 96184, 247449 correct to three significant figures.

Solution

Normal method	Contracted method
171775	171780
181392	181390
+96184	+96180
449351	449350

Answer = 449000 Answer = 449000

Since the answer is required to 3 decimal places, it is easier to round the individual numbers to 5 significant figures. This has greater advantage when the numbers involved are large.

Example 6

Find the sum of 14.62894, 0.07968, 3.97815 and 0.02948. Correct to two decimal places.

Solution	14.62894	14.6290
	0.07968	0.0797
	3.97815	3.9782
	0.02948	0.0295
	18.71625	18.7164
Answer=	18.72	Answer = 18.72

Example 7

Find the difference between 18.17397 and 13.45752 correct to 2 decimal places.

Solution

Notice that it is necessary to carry the numbers to 4 decimal places only during the calculations.

18.17397
13.45752
4.71645

Answer = 4.72

3.1.5 Multiplication

Again multiplication requires the use of more significant figures than required in the answer. Further in dealing with decimals it may be necessary to adjust both the multiplier and multiplicand, so that the multiplicand is in "semi-standard form". For example 53.732 x 26.771 can be written as 537.32 x 2.6771. Before carrying out the actual multiplication a rough estimate must be obtain. This helps in ascertaining the answer.

Example 8

Find the product of 53.732 and 26.771

Solution

Check 53.732 x 26.771

= 537.32 x 2.6771 (= 500 x 3 = 1500)

Working:

```
      537.32
      2.6771
    ________
107464
 322392
  376124
   376124
     53732
   _______
1438.459372
```

Answer 1438.46

3.1.6 Division

Where whole numbers are concerned, division is simple. It can proceed in the usual way. However where decimal places are involved care is needed.

Example 9

Divide 5972.46 by 46.5.

Solution

Note that 5972.46 ÷ 46.5 can be worked out as 59724.6÷ 465 where the divisor is now a whole number.

```
          128.44
465 ⌈ 59724.6
      -465
       1322
       -930
        3924
       -3720
         2046
        -1860
          1860
         -1860
             0
```

Answer = 128.44

It should be noted that with the advent of a calculator, calculations of this kind are easy to handle.

3.2 Error

3.2.1 Absolute error

Absolute error is the numerical difference between a measurement and its true value.

Example 10

Suppose the true length of a metal rode is 4.5m and a welder measures it to be 5m. What is the absolute error.

Solution

The absolute error is $5 - 4.5\text{m} \quad = \quad 0.5\text{m}$

Example 11

The length of the collar of a size 15 shirt is 36.5 cm. Suppose a tailor makes one with a length of 37.5 cm. What is the absolute error?

Solution

$37.5 \text{ cm} - 36.5 \text{ cm} \quad = 1 \text{ cm}$

$\therefore \qquad$ absolute error = 1 cm

The absolute error, a measure of how far the measurement is from the true or expected value or measurement, is given by subtracting the true figures from the measured one.

If the result is negative the minus is ignored.

3.2.2 Maximum absolute error

In practice one might be interested to know the maximum value an absolute error some measurements can reach. An example will help clear this point.

Example 12

Consider a packet of sugar whose weight is quoted as 0.5 kg, correct to one decimal place. What is its maximum absolute error?

Solution

One can see that the true weight will beanywhere between 0.45 and 0. 54 (from the rounding up).

The maximum the error can be 0.5 – 0.45 = 0.05 kg.

or 0.5 - 0.54 = 0.04. So the maximum error is 0.05.

Example 13

What is the maximum absolute error in a length of 8.3 cm corrected to one decimal place?

Solution

The true length can be said to lie between 8.25 cm and 8.34 cm.

$\therefore$maximum absolute error = 8.3 – 8.25 cm
= 0.05 cm

or 8.34-8.3= 0.04 cm.

Here the maximum error is 0.05.

Example 14

Find the maximum absolute error, when the weight of an article is given as 0.434 kg corrected to the nearest gram after weighing?

Solution

The true weight can be said to lie between 0.4344Kg and 0.4335kg the maximum absolute error = 0.0005 (i.e; 0.4335 – 0.434).

3.2.3 Relative Error

Absolute error is the numerical difference of the measurement (usually rounded) and the true value. The absolute error however does not tell the seriousness of the error. For example one may not quickly judge the relative seriousness of 1 gm error in the measurement of a 90kg bag of maize on one hand and the same error in the measurement of the weight of 10 gram bottle of medicine. In practice in the former the error is not important but in the latter it is serious. This judgement is best done when the error is expressed as a ratio of the measurement in question. Using the same example, in the case of maize the error is $^{1}/_{90,000}$, a very small fraction while in the other it is $^{1}/_{10}$ which is quite significant.

Expressing the error as a ratio of the measurement gives the **relative error.**

Example 15

The length of a line is 7.2 cm, correct to the nearest mm. Find:

(a) the maximum absolute error

(b) the relative error and error percent to 1 significant figure

Solution

(a) The true length is between 7.15 and 7.24 cm

$\therefore$ maximum absolute error = 0.05

(b) The relative error = $\frac{0.05}{7.2} \approx 0.007$

3.2.4 Error Percentage

When the absolute error is expressed as a percentage of the true or expected value, it is known as error percentage. It is the same idea as relative error, multiplied by 100.

Example 15

The problem in example 14 has an error percentage of

$$0.007 \times 100 = 0.7\%$$

Example 16

A bottle of fanta is supposed to contain 300 ml of the drink. If the actual fill is 302 ml. What is the error percentage?

Solution

302 – 300 = 2

Error percentage = $\frac{2}{300} \times 100 = 0.67\%$

In calculating the error percentage or relative error one uses the absolute error or the maximum absolute error. The error percentage and relative error give an indication of how serious the error is.

3.2.5 Sources of Error

Errors can result from a variety of areas the commonest of which are physical measurement, calculations, and rounding.

Errors in measurement

Consider a Mathematics student who wants to measure the time taken by a man to walk 1km. The student will time the man's walk over the 1 km with a watch. Certainly the time taken will not be exact because of reading error and faults in the watch. However, an acceptable approximation say of 12 minutes may be given. Notice this may be only to the nearest minute. If a finely calibrated instrument is used to measure the time, the result will be more accurate.

Calculations and Rounding up

Another source of error is the calculations themselves. Calculations become a source of error in themselves if there is a mistake in the process of making them, otherwise the practical error which is the subject matter here results from rounding up.

Example 17

A district of 414.972 hectares has a population of 1,659,878. Find the number of people per hectare correct to the nearest whole number.

Solution

It is sufficient to approximate 414.972 hectares to 415 hectares and 1,659,878 people to 1,660,000.

$\therefore$ the number people per hectares will be

$$\frac{1{,}660{,}000}{415} = 4{,}000 \text{ people}$$

The actual result would be:

$$= \frac{1659878}{414.972}$$

$$= \underline{\underline{3999.9}}76 \text{ (to 7 sig. figures)}$$

3.3 Tolerance

Tolerance is a concept used when describing how much error should be allowed. The idea stems from the fact that exact figures are impossible, not practical, too expensive to arrive at or they may not be necessary.

Examples of a situation where the above is true is in population counts. If one is measuring the population of a city an exact figure is both impractical and meaningless because of the dynamic nature of persons. In such a case a number rounded and specified to be accurate within certain limits is often given. Other examples would be manufacture and bottling of drinks, bagging of cloth and measurement of distance.

Tolerance is, as it can be judged from the above, an idea that uses absolute relative and percentage error.

3.3.1 Maximum Absolute Error and Tolerance

Example 18

What is the maximum absolute error of a measurement given as 30.2cm to 1 decimal place. Calculate the tolerance and tolerance limits.

Solution
Maximum absolute error:

True figure lies between 30.15 and 30.24

∴ Maximum absolute error

= 30.2 – 30.15
= $\underline{\underline{0.05}}$

Tolerance = 0.05

Tolerance limits are:

30.2 – 0.05 = 30.15
and 30.2 + 0.05 = 30.25

Example 19

The average value of an asset is given to be 25,000 to 2 significant figures. What is the tolerance and calculate the limits with which the average is accurate.

Solution

True value ranges between K24,500 and K25,400

∴ Maximum absolute error = K500
Tolerance = K500

The average is accurate within the limits:

25,000 – 500 = K24,500

and 25,000 + 500 = K25,500

3.3.3 Tolerance specification

For examples 18 and 19 tolerance is the same as maximum absolute error. The idea which should be held here is, however, that tolerance gives an indication of the limits of accuracy to any given figure. Conventionally it is expressed as a figure plus or minus the error.

Example 20

What are the tolerance specifications for the problem in example 4.4.1?

Solution

(30.2 ± 0.05) cm
(read as 30.2 plus or minus 0.05)

For example 19 tolerance specification:
$= (25{,}000 \pm 500)$ kwacha

4.4.4 Practical aspect of Tolerance

Tolerance can be set deliberately. A good example is in the filling of bottles with liquid products. Cutting of rods and pipes, cutting glass for windows, grooves and spaces for fittings etc. In these examples dimensions can be specified but one will realize that the measurements will not be exact in practice.

Example 21

In cutting glass for windows a carpenter tells the glass cutter that the required size is 45cm. In addition the carpenter instructs him that he should be within 2mm of the length required. Express the measurement as tolerance. Specify lower and upper limits for cutting the glass.

Solution

(a) Tolerance = 45cm ± 2mm
Or 45cm ± 0.2cm

(b) Lower limit = 45cm – 0.2
= 44.8cm

Upper limit = 45cm + 0.2
= 45.2cm

Example 22

A manufacturer of a soft drink would like the fill of a bottle to be not lower than 295 mls and no more than 305 mls. Specify the tolerance.

Solution

295 represents lower limit
305 represents upper limit

$\therefore$The average fill (safe fill) = $\frac{295+305}{2}$

= 300

Tolerance specifications:

Fill should be (300 ± 5) mls

Example 23

The error percent allowed on the length of the bookshelf in the Chief Accountants Office is 2%. If the length is 2.5m, what is the tolerance specification?

Solution

Absolute error$=0.02 \times 2.5 = 0.05$
Tolerance limits = $2.5\text{m} \pm 0.05\text{m}$

(note tolerance specifications are tolerance limits).

Example 24

An expenditure item on the budget for 1999 for Chagonaplc amounts to K57,540. The management accountant thinks that this budgeted figure be followed within a variance which is not more than 5% of the figure.

Express the idea in terms of tolerance.

Solution

The variance allowed can be taken as the error to be allowed.

Error $= \frac{57540 \times 5}{100} =$ K2877.00

Tolerance specifications K57,540$\pm$K2877.00

Example 25

For the budget item in example 24: Suppose one of the following figures turned out to be the actual expenditures. Which one would be acceptable and which one would necessitate remedial action?

(a)	K57,540
(b)	K54,662
(c)	K57,563
(d)	K60,417
(e)	K60,467

Solution

The management would accept any of the following if they turned out:

(a) 57,540 (c) 57,563 and (d) 60,417

because they are in the tolerance limits.

The following would not be accepted and therefore necessitate action: (b) 54,662 and (e) 60,467.

CHAPTER SUMMARY

In this chapter we have looked at the following:

- Rounding off a number to a given number of decimal places.
- Rounding off a number to a given number of significant figures.
- Finding the absolute error and maximum error of a given measurement.
- Sources of error.
- Maximum absolute error and error tolerance

END OF CHAPTER EXERCISES

1. Correct the following to the nearest thousand:

(a)	89247
(b)	721
(c)	2496
(d)	2575.3

2. Evaluate and correct to 2 significant figures:

(a) 81.17 / 32.54

(b) 4.273 / 681.3

(c) 596.7 / 0.08134

(d) $\frac{32.736 \times 2.4683}{4.2973}$

3. The area of Chikopa Township is 1458.3 square miles. If its population at a census was found to be 472022, what was the average per square mile to the nearest whole number?

4. Find to 1 significant figure the maximum absolute error, the relative error and error percent.

(a) A length of 4.5cm, correct to the nearest mm.
(b) A weight of 0.328Kg, correct to the nearest gm.
(c) A length of 2480, correct to 3 figures.

5. What is the relative error and error percent?

(a) Calculate length 5.32, measurement 5.4cm.

(b) Calculate volume 86.62cc, by measurement 86.5cc.

(c) If π is taken as $3^1/_7$ instead of 3.14159.

(d) 1m is taken as 39 in instead of 39.3707.

(e) A line is measured as 7.5cm, if the maximum error percent is 4%, between limits (to 2 figures), does the true length lie?

6. Given two numbers 12 and 14 as rounded to the nearest whole number. Find:

(a) Lower limit of their sum.
(b) Upper limit of their sum.
(c) Tolerance specifications.

7. If the calculated length of an object is 5.32cm and the length of the same object by measurement is 5.4cm. What is the relative error?

CHAPTER 4

MULTIPLES, POWERS AND INDICES

LEARNING OBJECTIVES

By the end of this chapter, the student should be able to:

i. Define a multiple of a number
ii. Multiply numbers with the same bases by adding the powers
iii. divide numbers with the same bases by subtracting the powers
iv. define logarithm
v. apply the multiplication, division and the power rules of logarithm
vi. present figures in standard form

4.0 Introduction

It will be noted from the examples and explanations in this chapter that multiples and powers are important aspects of mathematics in that the former helps in calculations which involve cumulation of smaller figures and the latter provides a short form of expressing figures which are multiplied several times by themselves. This chapter will first of all explore the idea of multiples and then look at powers of number, also called indices.

4.1 Multiples

A multiple of a number is the number which is formed by multiplying that number by smaller or a larger integer. 20 is a multiple of 10 because it is formed by multiplying 10 by 2. Another way of defining a multiple is to consider division. Another way of defining a multiple is to consider division. In this connection of a number is that which can be divided by that number without leaving a remainder.

4.2 Powers and Indices

As mentioned in the introduction powers are a way of simplifying the writing of long expressions.

Example 1

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

Two to power 4 is the same as 16. Notice if 16 has to be written in terms of its prime factors it is:

2 x 2 x 2 x 2

which can be shortened to 2^4

Example 2

2^3 = third power of 2 or 2 to power 3

2^5 = fifth power of 2 or 2 to power 5

10^6 = the sixth power of 10 or 10 to power 6

Powers are not an end by themselves apart from shortening the writing of some factors, they simply calculations as will be seen in the following examples.

Example 3 (Multiplication)

(i) $2^2 \times 2^3 = 2^5$

(ii) $3^4 \times 3^5 = 3^9$

In general we can say $x^m \times x^n = x^{m+n}$

In other words in multiplying powers of a number are simply adds the powers.

Example 4 (Division)

(i) $2^4 / 2^2 = 2^{4-2} = 2^2$

(ii) $5^7 / 5^3 = 5^{7-3} = 5^4$

Thus when dividing powers of a number we simply subtracts the powers or indices.

Therefore

$$\boxed{x^m \div x^n = x^{m-n}}$$

Example 5 (Powers of indices)

(i) $(2^4)^2 = 2^{4x2} = 2^8$

(ii) $(7^3)^2 = 7^{3x2} = 7^6$

In other words, if a power of a number is raised to another power, one multiplies the original power with the new indices.

Thus

$$\boxed{(X^m)^n = X^{m \times n}}$$

4.3 Roots

Example 6:

(i) What is $16^{½}$?

Solution

Consider the following:

$$16^{½} \text{ x } 16^{½} = 16^1 = 16$$

∴ multiplying $16^{½}$ by itself gives 16 by definition $16^{½}$ is the square root of 16.

∴ $16^{½} = \sqrt{16} = 4$

(ii) What is $27^{\frac{1}{3}}$?

Notice: $27^{\frac{1}{3}} \times 27^{\frac{1}{3}} \times 27^{\frac{1}{3}}$

$= 27^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 27$

∴ $27^{\frac{1}{3}}$ can be multiplied by itself three times to make 27.

∴ $27^{\frac{1}{3}}$ is the cube root of 27

$27^{\frac{1}{3}}$ is the $\sqrt[3]{27}$, read as the cube root of 27.

$$\text{Generally} \quad X^{\frac{1}{m}} = \sqrt[m]{X}$$

Example 7:

Evaluate: 7^{-2}

Notice : $7^3 \text{ x } 7^{-2} = 7^{3-2} = 7^1$

But : $7^{3-2} \Rightarrow 7^3 / 7^2$

Or : $\frac{7^3}{7^2}$ or $7^3 \times \frac{1}{7^2}$

$\therefore$ $7^{-2} = \frac{1}{7^2}$ 7^{-2} = $\frac{1}{7^2}$

$$\text{Generally:} \quad x^{-m} = \frac{1}{x^m}$$

4.4 Logarithms

Logarithms are the "opposite" of exponentials, just as subtraction is the opposite of addition and division is the opposite of multiplication. Logarithms "undo" exponentials. Technically speaking, logs are the inverses exponentials

The **logarithm** of a number is the exponent to which another fixed value, the base, must be raised to produce that number. For example, the logarithm of 1000 to base 10 is 3, because 1000 is 10 to the power 3: $1000 = 10 \times 10 \times 10 = 10^3$. More generally, if $x = b^y$, then y is the logarithm of x to base b, and is written $y = \log_b(x)$, so $\log_{10}(1000) = 3$.

$$2^3 = 2 \times 2 \times 2 = 8$$

It follows that the logarithm of 8 with respect to base 2 is 3, so $\log_2 8 = 3$.

4.4.1 Laws of logarithm

a) Multiplication

The logarithm of a product is the sumof the logarithms of the factors:

$$\log_b (xy) = \log_b x + \log_b y \qquad \text{(Law 1)}$$

Example 8:

a. $\log_{10}(10x) = \log_{10} 10 + \log_{10} x = 1 + \log_{10} x$

b. $\log_3 243 = \log_3 (9 \times 27) = \log_3 9 + \log_3 27 = \log_3 3^2 + \log_3 3^3 = 2 + 3 = 5$

c. $\log 20 = \log(2 \times 10) = \log 2 + \log 10 = 0.3010 + 1 = 1.3010$

b) Division

The logarithm of the quotient is the difference between the numerator and the denominator.

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y \qquad \text{(Law 2)}$$

Example 9

$$\log_2 \left(\frac{16}{12}\right) = \log_2 16 - \log_2 12 = \log_2 2^4 - \log_2 (4 \times 3) = \log_2 2^4 - \log_2 4 - \log_2 3$$
$$= \log_2 2^4 - \log_2 2^2 - \log_2 3$$
$$= 4 - 2 - \log_2 3 = 2 - \log_2 3$$

c) Power

The logarithm of the *p*-th power of a number is *p* times the logarithm of the number itself; these identities with examples:

$$\log_b\left(x^p\right) = p\log_b x \qquad \text{(Law 3)}$$

Example 10

$\log_2 64 = \log_2 2^6 = 6\log_2 2 = 6\times 1 = 6$

$\log_{10} 10^2 = 2\log_{10} 10 = 2\times 1 = 2$.

d) Roots

The logarithm of a *p*-th root is the logarithm of the number divided by *p*.

$$\log_b \sqrt[p]{x} = \log_b x^{\frac{1}{p}} = \frac{1}{p}\log_b x \qquad \text{(Law 4)}$$

$$\log_{10}\sqrt[3]{10000} = \log_{10} 10000^{\frac{1}{3}} = \frac{1}{3}\log_{10} 10^4 = 4\times\frac{1}{3}\log_{10} 10 = \frac{4}{3}$$

Example 11

Simplify:

i. $\log_{10}5 + \log_{10}20 = \log_{10}(5\text{x}20) = \log_{10}100 = 2$

ii. $\log_{10} 3000 - \log_{10}3 = \log_{10}(3000/3) = \log_{10}1000 = 3$

iii. $\log_{10}(6^2 + 8^2)^3 = 3\log_{10}(6^2 + 8^2) = 3\log_{10}(36 + 64) = 3\log_{10}100 = 3\text{x}2 = 6$

4.5 Standard form

Standard form is a convenient way of writing very large or very small numbers. It is used on a scientific calculator when a number is too large or too small to be displayed on the screen.

> In *standard form*, numbers are written as
>
> $a \times 10^n$
>
> where $1 \leq a < 10$ and n is an integer.

Example 12

4000 in standard form is 4.0×10^3
10^3 because the decimal has been move 3 times to the left while $0.004 = 4.0 \times 10^{-3}$ the decimal has moved 3 times to the right.

To write 81 900 000 000 in standard form move the decimal to the left 13 times this will give

8.19×10^{13}.

Also $0.000000\ 012 = 1.2 \times 10^{-8}$ since the decimal has been moved 8 times to the right.

Using Calculators

Your calculator will have a key EE or EXP for entering numbers in standard form.
For example, for 3.2×10^7,

press 3.2 EXP 7

which will appear on your display like this:

3.2 ^{07}

Some calculators also display the ' × 10 ' part of the number, but not all do. You need to find out what your calculator displays. Remember, you must always write the ' × 10 ' part when you are asked to give an answer in standard form

CHAPTER SUMMARY

In this chapter we have learnt that:

i. A multiple of a number is one that can be divided by that number without leaving a remainder

vii. We can multiply numbers with the same bases by adding the powers

viii. We can divide numbers with the same bases by subtracting the powers

ix. Logarithm is the opposite of exponential

x. There are three rules of logarithm i.e. multiplication, division and power

ii. Figures can be presented in standard form a x 10^n where $1 \leq a \leq 10$ and n is an integer.

END OF CHAPTER EXERCISES

1. Evaluate the following:

(a) $3^3 \times 3^2$

(b) $\frac{4^5}{4^3}$

(c) 9^{-2}

2. Write the following in as short a way as possible:

(a) $x^3 \times y^{-3} \times a \times b^2$

(b) $\dfrac{ac^2d^2}{a^2dc^3}$

3. Evaluate:

(a) $16^{½}$

(b) $36^{½}$

(c) $27^{-1/3}$

(d) $25^{½} \times 4^{-½}$

4. Simplify the following (assume all the logs are to base 10):

a) log 60 +log 2 – log 12

b) 3log 3 + log 3000 - 4log 3

c) log ($10^{1/2}$ x 100^2)

d) log a^2 –log a^5 + 3log a

5. Write the following numbers in standard form:

(a) 5720

(b) 7.4

(c) 473 000

(d) 6 000 000

(e) 0.09

(f) 0.000621

(g) 0.0000097

(h) 0.00000000000021

6. Convert each of the following numbers from standard form to the normal decimal notation:

(a) 3×10^4

(b) 3.6×10^4

(c) 8.2×10^3

(d) 3.1×10^2

(e) 1.6×10^4

(f) 1.72×10^5

(g) 6.67×10^{-1}

(h) 3.86×10^{-5}

(i) 9.27×10^{-7}

CHAPTER 5

PERIMETER, AREA AND VOLUME

LEARNING OBJECTIVES

By the end of this chapter the student should be able to:

i. Find the perimeters of various regular plane shapes
ii. Calculate areas of various regular plane shapes
iii. Calculate surface areas of regular objects
iv. Calculate volumes of regular objects

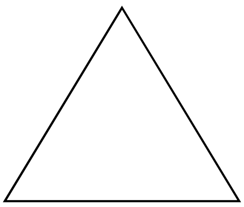
5.0 Introduction

Calculations of perimeter, area and volume may seem irrelevant to commerce. They however, occupy as much an important place in the business world as other business calculations. It should be born in mind here that the basis of commerce are physical commodities which occupy space. Examples of physical objects that are of interest to the business world include walls, mats, containers, etc. For these, calculations of length, area and volume are very necessary for knowledge and costing. A good example where the calculations are useful is in trying to arrive at the cost of putting curtains in a hotel. One will need to know the total amount (area) of curtain cloth required to cover the hotel.

5.1 Perimeters and areas of common plane figures

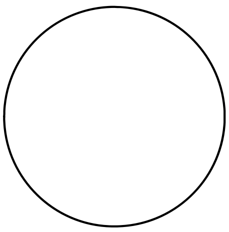
A plane figure is a two-dimensional shape which is bounded by lines called sides.

Some examples of plane figures:

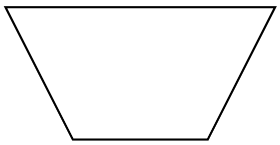


a

b



c



d

Plane figures bounded by lines are called polygons.

Perimeter of a plane figure is the total length of the sides of the figure.

Area of a plane figure is the amount of surface covered by the figure. Area is measured in square units.

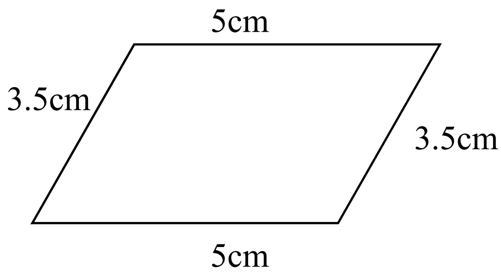
In practice, not all objects occur in perfect regular shapes. Often one meets irregular shapes and approximations have to be made. The following however, are the basic regular shapes and formulas we can use to calculate their perimeters and areas

5.1.1 Quadrilaterals

A quadrilateral is a figure with four sides. There are different types of quadrilaterals among them:

(a) Parallelogram

A Parallelogram is a quadrilateral with opposite sides parallel. Because the opposite sides are parallel then they are also equal and the diagonals bisect each other. The figure below shows the shape of a parallelogram.



For a parallelogram with a base side *b* and another side *s*

$$\begin{aligned} Perimeter &= b + s + b + s \\ &= 2(b + s) \end{aligned}$$

Example 1

For the parallelogram above, if we let $b = 5cm$ and $s = 3.5cm$, then

$$Perimeter = 2(5cm + 3.5cm)$$
$$= 17cm$$

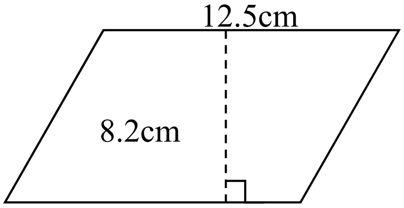
$$Area\ of\ a\ parallelogram = base \times perpendicular\ height$$
$$= bh\text{w}$$

where h is the perpendicular height.

Example 2

Find the area of a parallelogram with a base of 12.5 cm and height 8.2 cm.

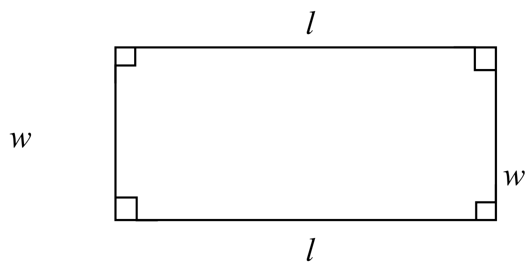
Solution



$$Area = b \times h$$
$$= 12.5cm \times 8.2cm$$
$$= 102.5cm^2$$

(b) Rectangle

A rectangle is parallelogram with sides meeting at right angles. The opposite sides are equal and parallel. The longer side of a rectangle is called the *length* and the shorter *width (breadth)*. The figure below illustrates a rectangle of length l and width w.

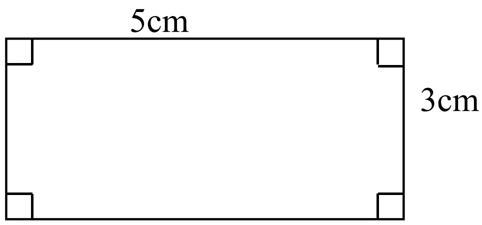


For a rectangle with length l and width w,

$$Perimeter = 2(l + w)$$
$$Area = l \times w$$

Example 3

Find the perimeter and area of the rectangle below

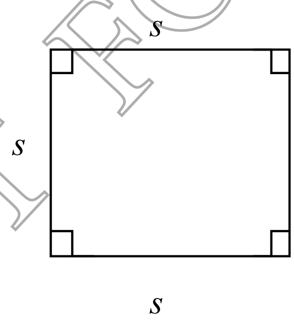


Solution

$$Perimeter = 2(l + w)$$
$$= 2(5cm + 3cm)$$
$$= 16cm$$
$$Area = l \times w$$
$$= 5cm \times 3cm$$
$$= 15cm^2$$

(c) Square

A square is a parallelogram with all four sides equal and meeting at right angles as shown below

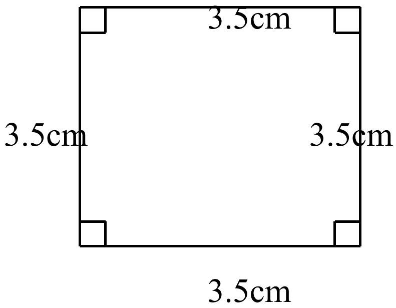


For a square of side *s*,

$$Perimeter = 4s$$
$$Area = s^2$$

Example 4

Find the perimeter and area of the square below



Solution

$$Perimeter = 4s$$
$$= 4 \times 3.5cm$$
$$= 14cm$$

$$Area = s^2$$
$$= (3.5cm)^2$$
$$= 12.25cm^2$$

(d) **Rhombus**

A rhombus is a Parallelogram with all sides equal but not meeting at right angles. Basically a rhombus is distorted square.

Just like a square, a rhombus of base side *s* has

$$Perimeter = 4s$$

$$Area = s \times perpendicular\ height$$

Example 5

Given that the figure below is a rhombus, find its area and perimeter

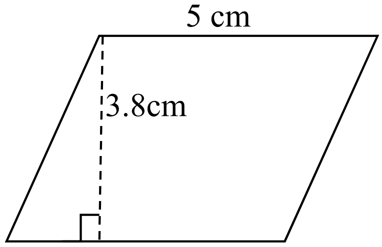
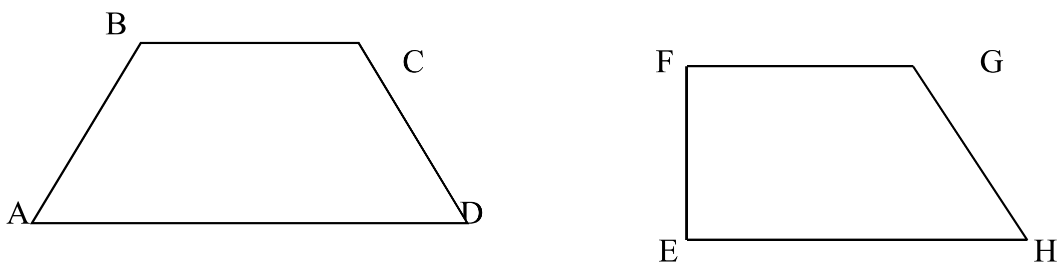


Figure 8

$$Perimeter = 4s$$
$$= 4 \times 5cm$$
$$= 20\ cm$$
$$Area = s \times h = 5cm \times 3.8cm$$
$$= 19cm^2$$

(e) Trapezium

A Trapezium is a quadrilateral with one pair of opposite sides parallel as shown in the following figures AD is parallel to BC in trapezium (a) and EH is parallel to FG in trapezium (b).



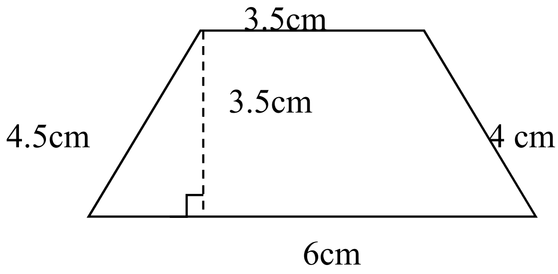
(a) (b)

$$Perimeter = sum\ of\ all\ sides$$

$$Area = \frac{sum\ of\ parallel\ sides \times height}{2}$$

Example 6

Find the perimeter and area of the trapezium shown below



Solution

$$Perimeter = sum\ of\ all\ sides$$
$$= 6cm + 4.5cm + 3.5cm + 4cm$$
$$= 18cm$$

$$Area = \frac{sum\ of\ parallel\ sides \times height}{2}$$

$$= \frac{(6cm + 3.5cm) \times 3.5cm}{2}$$
$$= 16.625cm^2$$

5.1.2 Circle

Circumference and area of a circle

The perimeter of a circle is called a circumference and it is given by

$circumference = 2\pi r,$

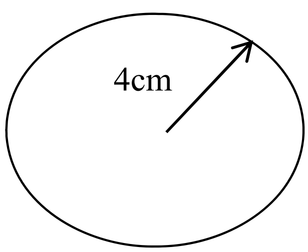
Where r is the radius of the circle.

For a circle with radius r,

$$area = \pi r^2$$

Example 7

What is the circumference and area of the following circle with radius $r = 4cm$?



Solution

$$\begin{aligned} Perimeter &= 2\pi r \\ &= 2\pi \times 4cm \\ &= 8\pi cm \\ &= 25.13cm \end{aligned}$$

$$\begin{aligned} Area &= \pi r^2 \\ &= \pi \times (4cm)^2 \\ &= 16\pi cm^2 \\ &= 50.27cm^2 \end{aligned}$$

5.1.3 Triangle

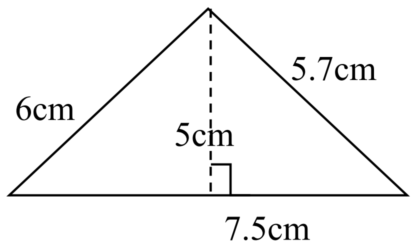
A triangle is a three sided figure.

$$Perimeter\ of\ a\ triangle = sum\ of\ all\ sides$$

$Area\ of\ a\ triangle = \frac{1}{2} \times b \times h$,
where b is the base side and h is the perpendicular height.

Example 8

Find the area and perimeter of the triangle shown below



Solution

$$\begin{aligned} Area &= \frac{1}{2} b \times h \\ &= \frac{1}{2} \times 7.5cm \times 5cm \\ &= 18.75cm^2 \end{aligned}$$

Example 9

A triangle has area of $26.46cm^2$ and a height of $4.2\ cm$. What is the length of its base?

Solution

$$Area = \frac{b \times h}{2}$$

$$26.46cm^2 = \frac{b \times 4.2cm}{2}$$

$$b = \frac{26.46cm^2 \times 2}{4.2cm} = 12.6cm$$

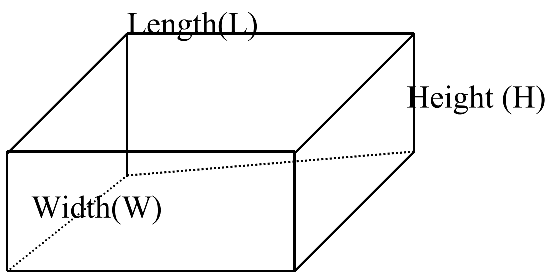
5.2 Volume

Volume refers to the space in three dimensions occupied by an object. In practice the volume of an object refers to its capacity. Thus in calculating the volumes of a room, tank cylinder, we are calculating their capacities. Below are some objects and formulas for calculating their volumes.

Note:

- The standard units for measuring volume are cubic metres (m^3) and litres(l) are for capacity
- $1000cm^3 = 1l$

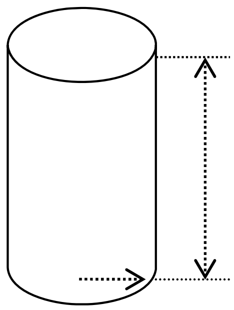
5.2.1 Cuboid



The volume of a cuboid of length L, width W and height H is given by

$$\boxed{Volume = L \times W \times H}$$

5.2.2 Cylinder

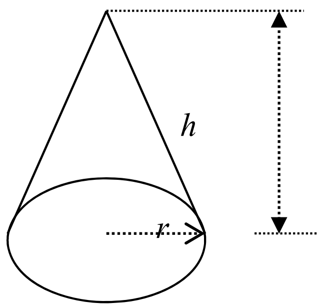


$$Volume = \pi r^2 h$$

$$Curved\ surface\ area = 2\pi rh$$

where r is the radius of the circular base and is the perpendicular height of the cylinder

5.2.3 Cone

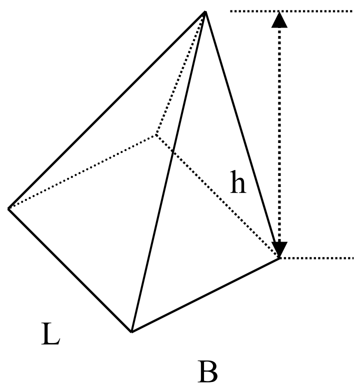


$$Volume = \frac{base\ area \times height}{3}$$

$$= \frac{\pi r^2 h}{3}$$

$$Curved\ Surface\ Area = \pi rs$$

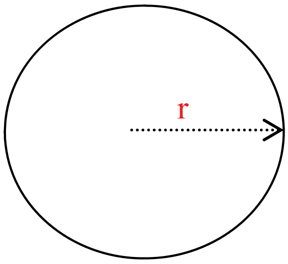
5.2.4 Pyramid



$$Volume = \frac{Base\ Area \times Height}{3}$$

$$= \frac{L \times B \times H}{3}$$

5.2.5 Sphere



$$Volume = \frac{4\pi r^3}{3}$$
$$Curved\ Surface\ Area = 4\pi r^2$$

Example 10

Find the volume of a cuboid of length *12cm,* width *8cm,* and height *6cm*

Solution

$$\begin{aligned} Volume\ of\ a\ cuboid &= L \times B \times H \\ &= 12cm \times 8cm \times 6cm \\ &= 576cm^3 \end{aligned}$$

Example 11

A cylinder and a cone have the same circular base with a radius of $8cm$ and the same height of $12cm$.

a) Find the volume and curved surface area of the cylinder.

b) Find the volume of the cone.

Solution

a) $Volume\ of\ a\ cylinder = \pi r^2 h$

$$= \pi \times (8cm)^2 \times 12cm = 768\pi cm^3$$

Curve $Surface\ Area = 2\pi rh$

$$\begin{aligned} &= 2 \times \pi \times 8cm \times 12cm \\ &= 192\pi\ cm^2 \end{aligned}$$

b) $Volume\ of\ a\ cone = \frac{\pi r^2 h}{3}$

$$= \frac{1}{3}\pi \times (8cm)^2 \times 12cm = 256\pi cm^3$$

Example 12

The volume of a sphere is $280cm^3$. What is its radius?

Solution

$$Volume\ of\ a\ sphere = \frac{4\pi r^3}{3}$$
$$280cm^3 = \frac{4\pi r^3}{3}$$

$$r = \sqrt[3]{\frac{280cm^3 \times 3}{4\pi}}$$
$$= 4.058cm$$

CHAPTER SUMMARY

In this chapter we have looked at:

- Definitions of perimeter, area and volume
- Calculation of perimeter for quadrilaterals
- Calculation of area for quadrilaterals and triangles
- Circumference and area of a circle
- Volumes and surface areas of various objects

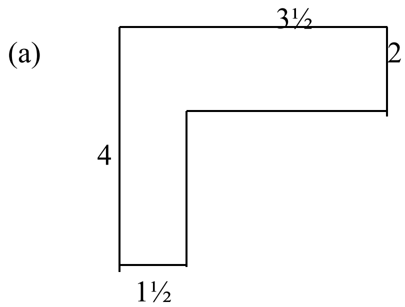
END OF CHAPTER EXERCISES

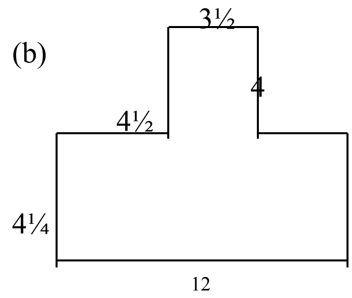
1. Find the perimeters and areas of the following rectangles:

   (a) 6.4cm by 3.8cm
   (b) 3.2m by 5dm (in m)

2. A rectangular sheet of cardboard, 25cm by 16cm weighs 18.3gm, an oval is cut out of it and the latter found to weigh 10.2gm. Find the area of the oval to the nearest sq.cm.

3. Find the area of the following figures in which all the corners are at right angles. The lengths of all sides are in cm.





4. What is the area for each of the following figures?

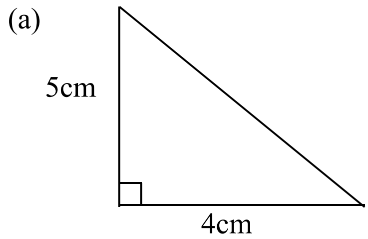
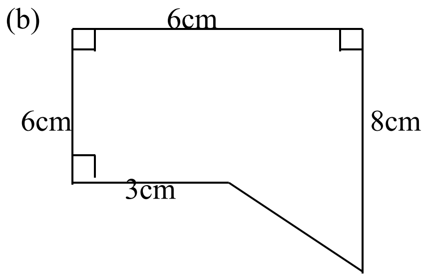


Figure 19



5. A man walks 4km from A to B where B is west of A. Then he walks on towards the south to C for 3km. Calculate the total distance the man will have walked if he decided to go back to A waking straight from C to A.

6. Find the area of a circle with radius:

(a) 7m
(b) 10cm

7. Find the radii of circles with the following area

(a) 22 cm^2
(b) 56 cm^2

8. The circumference of a circle is 25.1cm. Find its area.

9. Find the area of semi-circle of diameter 1.02m.

10. Find the total area of a surface of a solid cylinder with base-radius 3 cm and height 4cm.

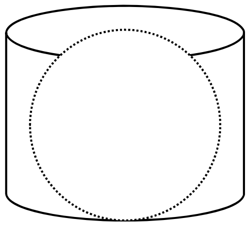
11. Find the area of the curved surfaces of the cylinders with the following measurements:

(a) 7cm radius and 5cm height
(b) 10 cm height and 3 cm radius.

12. A swimming pool is 50m long and 16m wide. The depth of water at the shallow end is 1m, increasing uniformly to a depth of 3m at the deep end.

(a) Calculate the volume of water in the pool.
(b) Express the volume of water (in cubic meters) in the pool as capacity in litres.
(c) If the pool is to be painted, find the cost of painting the entire side walls of the pool if painting costs K12 per square meter.

13. The following figure shows a sphere or radius r fitting exactly into a cylinder. The means the sphere touches the cylinder at the top, bottom and sides.



If the volume of the cylinder is 269.5cc, find

(a) theheight, and

(b) the volume of the sphere.

14. Find the volume of a cone with the same circular base and height as the cylinder and show that the volumes of the cone, sphere, and cylinder are in the ratio 1:4:3.

CHAPTER 6

BASIC FINANCIAL MATHEMATICS

LEARNING OBJECTIVES

By the end of this chapter,the student should be able to:

i. Define interest
ii. Describe Simple and Compound Interest
iii. Calculate Simple and Compound Interest
iv. Explain the difference between Simple and Compound Interest
v. Calculate profit or loss of an investment
vi. Calculate profit mark up and margin of an investment
vii. Distinguish trade discount from cash discount
viii. Calculate discount
ix. Convert money in other currencies to Malawi Kwacha and vice versa
x. Calculate PAYE given a person's taxable income
xi. Calculate Value Added Tax on various goods and services
xii. Prepare electricity and water bills
xiii. Define Insurance and premium
xiv. Calculate premiums for various insurance policies
xv. Distinguish instalment buying from hire purchase
xvi. Determine periodical instalments when buying in instalments/hire purchase

6.0 Introduction

When a person borrows money from an organization such as a bank, building society or from another individual, a charge called *interest* is made on that money for its use. Here interest is viewed as the cost of borrowing money.Similarly, if a person invests money by lending to another person or depositing in a bank, that person receives a sum of money over and above what is lent or deposited. This sum is *interest*. In this perspective interest is the reward received for lending money or depositing it. In the real sense, in case of lending money, the interest is a charge made by the lender for the risk taken. Therefore, the higher the risk the higher the interest that will be charged.

While interest refers to sum charged or received over a basic amount lent or deposited, the basic amount is termed the **principal**. The interest is charged on a specific amount paid and refers to a period that money is held, normally one year. Hence the term rate of interest or interest per annum. Calculations for shorter or longer periods can be made basing on this rate.

There are two types of interest:

(a) Simple interest and

(b) Compound interest.

6.1 Simple Interest

Simple interest is the interest charged on a fixed principal. For example a simple interest on K100 for 2 years at 10% per annum is 2 x 100 x 0.10 = K20.00

The underlying concept is that each year a charge of 10% (0.1) is made on the fixed principal K100. Or simply K10 is charged on K100 per annum, for 2 years, therefore, the charge is K20.

6.1.1 Formula for Simple Interest

Simple interest on a principal P is over T years at an interest rate of R% is given by:

$$\boxed{I = \frac{PTR}{100}}$$

Example 1

Calculate the simple interest on a loan of K600,000 at the rate of 11% for 4 years.

Solution

Simple Interest:

$$I = \frac{PTR}{100}$$

$$P = 600{,}000; R = 11; T = 4$$

$$\text{Hence } I = \frac{K600{,}000 \times 4 \times 11}{100}$$

$$= 264{,}000$$

Example 2

Calculate the length of time necessary for an investment of K100,000 to reach K125,000 when invested at 5% per annum.

Solution

The question is asking for the period T.
Since

$$I = \frac{PTR}{100}$$

Then

$$T = \frac{I \times 100}{P \times R}$$

If K100,000 accrues to K125,000 then

$$I = K125{,}000 - K100{,}000$$
$$= 25{,}000$$

$$\therefore \quad T = \frac{K25{,}000 \times 100}{K100{,}000 \times 5}$$

$$= 5\ years$$

Example 3

Find the simple interest on a loan of K800,000 at the rate of 8% per annum for 6 months.

Solution

$$I = \frac{PTR}{100}$$

$$P = K800{,}000; R = 8; T = 0.5yrs\ (6\ months)$$

$$I = \frac{K800{,}000 \times 0.5 \times 8}{100} = K32{,}000$$

6.2 Compound Interest

While simple interest is the interest on a fixed principal, for Compound Interest the initial principal changes as time passes as interest of the previous year gets added on the principal in a particular year and the interest is charged on the resulting sum. The basic concept is that each year's principal (in which interest is charged) is the current principal plus previous year's interest. Compound Interest therefore keeps growing as long as no amounts are withdrawn.

Example 4

Calculate the compound interest on K100 for 3 years at 5% per annum.

Solution

Interest for 1 year	=	5% of 100 = 5
At the end of year 1 amount	=	100 + 5
	=	K105
Interest in year 2	=	5% of 105
	=	K5.25
At the end of year 2 amount	=	105 + 5.25
	=	K110.25
Interest in year 3	=	5% of 110.25
	=	K5.51
Compound Interest	=	5 + 5.25 + 5.51
	=	K15.76

6.2.1 Calculating Compound Interest

You will notice that in the business world loans are contracted for periods in much longer than 2 or 3 years. In this case the method outlined above would be cumbersome.

If a principal is invested under compound interest at a rate of r% per annum over n compounding periods, then the accrued amount A, is given by

$$\boxed{A = P\left(1 + \frac{r}{100}\right)^n}$$

The compound interest would therefore be obtained by subtracting the principal from the accrued amount

i.e.

$$I = A - P$$

where I is the compound interest.

Example 5

What is the compound interest on K2,000 at 5% for 2 years calculated half yearly.

Solution

Compound Interest:

$$I = A - P$$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Since compound interest is calculated half-annually, number of compounding periods,

$$n = 4;\ halfannul\ rate = \frac{5}{2}\%$$

$$A = K2000\left(1 + \frac{2.5}{100}\right)^4$$

$$= K2000(1.025)^4$$

$$= K2207.63$$

$$\therefore I = K2207.63 - K2000$$
$$= K207.63$$

6.3 Applications

6.3.1 Discounts

A discount is a reduction in the price of a commodity. It is often expressed as a percentage of the original price. If for example, a customer is given a 5% discount on a K100 articlehe will K100 – K5 or K95.

Discount is a general term used for reduction in price as described above. The word *Trade Discount* is used to refer to discounts given where one buys on wholesale basis to sell on retail. For example a manufacturer will give a trade discount to retailer who buys goods in bulk for retail.

On the other hand, *Cash Discount* refers to reduction in prices being sold on retail.

Example 6

Gonani Wholesalers allow Mr.Chikoko, a retailer a trade discount of 30% on the goods worth K750,000 he is purchasing. Calculate the amount of money that Mr.Chikoko will pay.

Solution

Discount is $30\%\ of\ K750{,}000 = \frac{30 \times K750{,}000}{100}$

$$= K225{,}000$$

$$Cash\ Price = K750{,}000 - K225{,}000$$
$$= K525{,}000$$

Alternatively,

$$Cash\ price = (100\% - 30\%) \times K750{,}000$$
$$= K525{,}000$$

Example 7

Mr.Gulu orders goods worth K1,000,000 from a ABC Limited. For this amount ABC Limited allows a discount of 25% and a further 5% discount if the goods are paid for in cash on collection or delivery. Calculate how much Mr Gulu will pay if

(a) he does not settle the invoice immediately

(b) if he pays on collection of goods.

Solution

(a) $Discount = 25\%\ of\ K1{,}000{,}000$

$$= \frac{25}{100} \times K1{,}000{,}000$$

$$= K250{,}000$$

$$Net\ Price = K1{,}000{,}000 - K250{,}000$$
$$= K750{,}000$$

$$Cash\ Discount = 5\%\ of\ K750{,}000$$
$$= \frac{5}{100} \times K750{,}000$$
$$= K37{,}500$$

$$Cash\ price = K750{,}000 - K37{,}500$$
$$= K712{,}500$$

* Notice that the second discount is on the K750,000 and not on the original amount. The standard is to calculate the trade discount first to find the net price. Then the cash discount is calculated on the net price. It is wrong to add the discounts together as 25% + 5%.

6.3.2 Mark up, Margin, Profit and Loss

In as much as discounts benefit the customer and are meant to encourage buying, the seller is also concerned with the price at which he sells the goods because this determines how much he earns from the business.

a) Mark up

In setting a price the seller will start from the cost of the article and add expenses incurred and a certain amount he would like to earn which is called **net profit.** These expenses (e.g; transport and labour) + net profit are called **mark up** and they are expressed as a percentage of the cost price.

Therefore,

$$\boxed{\text{Mark up} = \text{Selling price} - \text{Cost price}}$$

If put as a percentage:

$$\boxed{Mark\ up = \frac{Selling\ Price - Cost\ Price}{Cost\ Price} \times 100\%}$$

Example 8

A company applies a uniform mark up rate of 35% on cost of all products. Find the retail price of:

(a) a toaster costing K1,800 and

(b) an iron costing K7,950.

Solution

a) Retail price of toaster:

$$Cost\ Price = K1{,}800$$
$$Mark\ up: 35\%\ of\ K1{,}800 = K630$$
$$Retail\ Price = K2{,}430$$

b) Retail price of iron:

$$Cost\ Price = K7{,}950$$
$$Mark\ up: 35\%\ of\ K1{,}800 = K2782.50$$

$$Retail\ Price \quad = K10{,}732.50$$

Example 9

A football is sold on retail at K9,500 after 25% mark up. What is the maximum price at which the retailer would be willing to purchase the football for his shop?

Solution

If we let C be the cost price then,

$$K9{,}500 = C + 25\%\ of\ C$$

$$\rightarrow K9{,}500 = 1.25C$$

$$C = \frac{K9{,}500}{1.25}$$

$$= K7{,}600$$

The maximum price at which the retailer would be willing to acquire the ball is K7,600.

b) Margin

Although calculation of mark up is a standard practice in business, there are also many, particularly retailers who prefer to relate profit to sales. In this case expenses plus net profit are expressed as a component or percentage of total sales rather than purchases. This is known as margin.

Thus

$$\boxed{Margin = \frac{Selling\ Price - Cost\ Price}{Selling\ Price} \times 100\%}$$

Example 10

A basket of fish was purchased at K20,000 and sold at a mark up of 30%. Calculate the margin obtained.

Solution

$$Cost\ price = K20{,}000$$

$$Mark\ up = \frac{K20{,}000 \times 30}{100} = K6{,}000$$

$$Selling\ price = K20{,}000 + K6{,}000 = K26{,}000$$

$$Margin = \frac{K6000}{26{,}000} \times 100\% = 23.08\%$$

c) Profit and Loss

As suggested under Mark Up and Margin, the difference between the cost price and selling price is *profit or loss*. In strict terms this difference is called *Gross Profit*. However, to sell any article there are expenses for labour, transport etc, which must be incurred. If these expenses are deducted from the Gross Profit the result is *Net Profit*.

Hence

$$Gross\ Profit = Selling\ Price - Cost\ Price$$

$$Net\ Profit = Gross\ Profit - Expenses$$

Example 11

A company sales an item whose cost price was K180,000 at K260,000. Calculate the gross profit and net profit if the company incurred expenses of K70,000 on the item.

Solution

Gross Profit = *K260,000 - K180,000 = K80,000*

Net Profit = *K 80,000 - K 70,000 = K10,000*

Example 12

A firm's sales for a year were K80,000. The cost of goods was K50,000 and overhead (expenses) were K10,000. Find:

(a) Gross profit
(b) Net profit
(c) Percentage mark up and margin

Solution

(a) Gross Profit = Sales – Cost of goods
= K80,000 – K50,000
= K30,000

(b) Net Profit = K30,000 – K10,000
= K20,000

(c)

$$Mark\ \ Up = \frac{Sales - Cost\ of\ goods}{Cost\ of\ goods} \times 100\%$$

$$= \frac{Gross\ Profit}{Cost\ of\ goods} \times 100\%$$

$$= \frac{K30{,}000}{K50{,}000} \times 100\%$$

$$= 60\%$$

$$Margin = \frac{Gross\ Profit}{Sales}$$
$$= \frac{K30{,}000}{K80{,}000} \times 100\%$$

$$= 37.5\%$$

Note:

Mark Up and Margins sometimes have the term profit attached tothem and therefore one can talk of profit mark up or profit margin.

Example 13

A company producing wheelbarrows had the following results for its financial year ending 31st December 1998. Purchases of materials: K425,761; Turnover of K392,798. Salaries K27,000; Rent K4,500; Lightening K1,000; Telephone and Postage K800; Transport K1,900; Stationery K700; Opening Stock K192,552 and closing stock K72,540. Calculate the

(a) gross and
(b) net profit if any.

Solution

In order to calculate the gross and net profits it will be simpler to create simple and brief Trading and Profit and Loss Accounts.

TRADING ACCOUNT AT 31 DECEMBER

Sales			K392,798
Opening Stock			K192,552
Purchases	(K425,761)		
Closing stock	(K72,540)		
		(K305,749)	
Gross Profit			**K87,049**
Expenses			
Rent	(K4,500)		
Salaries	(K27,000)		
Lighting	(K4,500)		
Transport	(K1,900)		
Stationery	(K700)		
		(K38,600)	**Net Profit**
K48,449			

6.3.4 Instalment buying and Hire Purchase

a) Instalment buying

Buying by instalments is common where one cannot afford the price of an article. In such cases the person is allowed to use the items and agrees to pay specified smaller amounts over a number of weeks, months or years. Usually a down payment called deposit is made and the balance is paid later. The scheme is sometimes described as 'buy now and pay later'.

Example 14

Chisu has agreed to buy a suit priced at K25,000 (if bought by cash) for K5000 down payment and the balance in twelve monthly instalments of K1,875 each. Calculate the final price of the suit and the cost of buying by instalments (finance charge).

Solution

Cost of Suit = *5000 + (12 x K1,875)*
= *K27,500*

Finance charge = *K27,500 – K25,000*
= *K2500*

Example 15

Jane is going to buy typewriter originally priced at K15,000 plus 8% add on charge. Assuming no down payment, calculate her monthly instalment if she is given a period of 20 months over which to make the payments.

Solution

$$Add\ on\ Charge = \frac{8}{100} \times K15{,}000 = K1{,}200$$

$$Total\ cost = K15{,}000 + K1{,}200$$

$$= K16{,}200$$

$$Monthly\ Instalment = \frac{K16{,}200}{20}$$

$$= K810$$

b) Hire Purchase

Hire purchase is when a person acquires an item by paying in smaller amounts, but does not have full ownership until payment is finished.

Example 16

A man is buying a TV set for about K300,000. He is asked to pay a deposit of 30% and is charged a fixed interest of 10% per annum on the balance which is to be paid in 2 years. Calculate the deposit, the interest, and his monthly instalments.

Solution

Deposit = 30% of K300,000

= $\frac{30}{100} \times K300{,}000$

= K90,000

Balance = K300,000 – K90,000

= K210,000

Interest = $\frac{10}{100} \times K210{,}000 \times 2$

= K42,000

Total Balance = K252,000

Monthly Instalment $= \frac{K252,000}{24}$
$= K10,500$

6.3.5 Loans

Sometimes a person may buy an article, he can otherwise not afford, by obtaining a loan from a financing institution or employers. The lending organization often operates differently from the retail or producer in that compound interest on the loan for the agreed period is charged.

6.3.6 Mortgage

A concept similar to hire purchase is the one used for purchasing houses or buildings. Although individuals can buy a house or building and settle the bill immediately, the prices of buildings are so high that they require a loan from a financing organisation usually a building society or a bank. An individual normally contracts a loan from the organization to pay for the house and then pays back the loan at an interest over a period. The period can be up to 20 years and the interest is calculated on the balance. Such loans have the building as security which is called mortgage.

The process is the same as that for hire purchase with the following exceptions:

- The loan is paid over a very long period (It is a long term loan).
- The loan has the house as security.

6.3.7 Insurance

Insurance can be defined as pooling of risks. To understand this, consider the following:

> That the chances of a shop catching fire are small but should if it catches fire, the owner suffers heavy losses often not recoverable. The aim of insurance is to provide protection or cover against such losses. The persons with interest in their property contribute small amounts annually to the insurance company. The company in turn pays for the losses up to an agreed or determined value in case of disaster or accident to the insured.

The small amounts paid to the insurer (Insurance company) are called *premiums* and are calculated based on the value of the asset or assets being insured. The payment by the insurer (to the insured) for the loss is called *compensation*.

This manual will consider as examples, motor vehicle and life insurance.

Example 17

For a full cover car insurance premium is calculated as 1.5% of the car value. Calculate the premium per annum if the value of the car if K5,000,000.

Solution

Premium $= 1.5\%$ of K5,000,000

$$= \frac{1.5}{100} \times K5{,}000{,}000$$

$$= K75{,}000 \text{per annum.}$$

Example 18

For a car purchased now and whose owner has no licence the premium for full cover insurance is 2% of the car value. In making an accident and the car can be repaired the driver will bear the first K60,000 of the repair cost. The value of the car is valued at K3,000,000.

a) Calculate the annual premium

b) If the car makes an accident and it is written off, how much will the owner be given?

c) If he has an accident but the car can be repaired and the garage gives a quotation (cost of repairs to be made) of K350,000. How much will the insurance company pay?

Solution

a) Premium = 2% of K3,000,000
= 3,000,000 x 0.02
= K60,000

b) The company will pay the market value of the car usually it will be the K3,000,000.

c) The first K60,000 will be paid by the driver

∴ Company will pay K350,000 – K60,000
= K290,000

Example 20

A man takes a life insurance policy for K1,500,000 which matures after 10 years. Assuming the man is 22 years old when takes the insurance. His annual premiums are K25,000 or K2083.00 per month. If he dies at the age of 36, how much more does the Insurance Company pay to his estate.

Solution

At the age of 36, the policy would have stayed for 36 – 22 = 14 years which is more than the maturity period (10 years) hence the policy would have matured and the insurance company will pay K1,500,000 which was the sum assured.

6.3.8 Electricity Charges

Electricity charges are based on the Kilowatt-hour. In other words one Kilowatt-hour is equivalent to one unit. Since electricity is used for various purposes in domestic, commercial and others, there are various *tariffs* available. An electricity tariff is a category of charges on each unit of electricity used plus a fixed charge. Tariffs depend on the types (and therefore power required) of appliances expected to be used in various places where electricity is consumed.

In Malawi examples of tariffs are Domestic and Industrial. These two can also be subdivided into further divisions. For example for domestic tariffs are dividedinto high density tariffs and low density tariffs.

Example 21

Suppose Mr Gamundas' electricity meter reading is 5605 at 29/01/2013 and the previous one taken on 29/12/2013 is 5238.

a) How much will he pay for the month of April if he must pay a fixed charge of K500 and his tariff is that one unit (a kilowatt) costs K10.

b) Make up a bill for Mr Gamunda.

Solution

a) Notice that electricity charges are done for a month that is they pertain to electricity used in one month.

New reading:	*5,605*
Old reading:	*5,238*
Consumption:	*367*

Charge:	*367 ×K10*	=	*K3670*
	Fixed charge	=	*500*
			K4170

c) A typical bill would be as follows:

RELIABLE ELECTRICITY COMPANY P. O. BOX 3044 CHILUNDA				
J. J. GAMUNDA, ESQ P. O. Box 3102 CHILUNDA				
ACCOUNT NO.		0024 CII	Date of reading 29/01/2014	
Previous Reading	Present	Tariff	Units	Amount K
5238	5605	Domestic	367	3670
			Fixed charge	500
			Total due	4170
Date due 15/05/2008				

Example 22

An industrial consumer Chanthu Ltd pays a basic charge of K5000 and then K40 per unit for the first 5,000 units and K30 for units in excess of 5,000 units. His previous meter reading was 420434 and the present taken on 27/01/2014 is 425714. Chathu did not however settle his previous bill of K2,470. Calculate his present charge and show how much he should pay on the bill.

Solution

New meter reading:	425,714	
Old meter reading:	420,434	
Current consumption:	5,280	
Charge first for 5000 units:	5,000 x K40	= K200,000
Next 280 units:	280 x K30	= K8,400
Fixed charge:		K5000
Current Bill:		K213, 400
Balance brought forward:		K2,470
		K215,870

BILL

RELIABLE ELECTRICITY COMPANY
P. O. BOX 3044
CHILUNDA

CHATHU LIMITED
P. O. Box 5200
CHALUNDA

ACCOUNT NUMBER 0032 127 DATE 03/02/2014

				AMOUNT (K)
Balance Brought forward				2,470
Previous Reading	Current Reading	Tariff	Units	
420434	425714	Industrial	5280	208,400
			Fixed charge	5000
			Current	213,400
			Total due	215,870

6.3.9 Water Charges

Water is usually charged per cubic meter used. Like electricity there are various tariffs available depending on types of consumers. Typical water charges and bills are shown in the examples that follow.

Example 23

Mr Jamus's water meter gave the following reading on 31st January 2014: 5741. On 31st December 2001 the meter reading was 5703. Mr Jamu lives in an area with the following water tariff: K300 for the first 10 cubic meters; K400 for consumption in excess of 10 cubic meters. A minimum charge of K200 is made on all bills. Prepare Mr.Jamus' water bill.

Solution

New reading:	5741
Old reading:	5703
Consumption:	38

Charges		
First 10 cubic meters:	10×K300 =	K3,000
Next 28 cubic meters:	28×K400 =	K11,200
Fixed charge:		K200
Bill for the month:		K14,400

BILL

SOUTHERN WATER
BOX 421
KADOMA

Mr. G. JAMU
P. O. BOX 222
KADOMA

ACCOUNT NUMBER JB/13/4518

WATER ACCOUNT FOR THE MONTH OF January :
DATE DUE :

DATE	DESCRIPTION		AMOUNT (K)
1st February 2002	New meter reading	5741	
	Old meter reading	5703	
	Current consumption	380	14,200.00
	Fixed Charge		200.00
	Amount due		K14,400.00

Example 24

Mr.Kalambo is in the same area as Mr Jamu above. His meter reading on 15th April, 2001 is 0054080 and it was 005359 on 14th March, 2013. Mr Kalambos' latest previous bill amounted to K2063. Due to lack of funds Mr.Kalambo paid K1000 only of this amount on 1st April, 2001. Prepare Mr.Kalambo's bill.

Solution

New reading 005408

Old reading	005359	
Consumption:	49	

Charges

First 10 cubic meters:	10×K300 =	K3,000
Next 28 cubic meters:	39×K400 =	K15,600
Fixed charge:		K200
Bill for the month:		K18,800

BILL

SOUTHERN WATER
BOX 421
KADOMA

Mr. G. JAMU
P. O. BOX 222
KADOMA

ACCOUNT NUMBER JB/13/4518

WATER ACCOUNT FOR THE MONTH OF APRIL :
DATE DUE :

DATE	DESCRIPTION	AMOUNT(K)
	Balance brought forward	2063.00
1st April 2001	Receipts	-1000.00
15th April 2001	New meter reading5741	
	Old meter reading 5703	
	Current consumption 380	18,600.00
	Fixed Charge	200.00
	Amount due	K19,863.00

6.3.10 Foreign Exchange

In Malawi the currency used is the Kwacha (K) which can be subdivided into 100 tambala. Other countries have their own currencies too. For example, the currency used in Zambia is the Zambian Kwacha (K) and it can be divided into 100 ngwee (n). Zimbabweans use the dollar (Z$) which is made up of 100 cents.

The following table gives currencies of various countries:

COUNTRY	UNIT OF CURRENCY	SUB UNITS
Malawi	Kwacha (K)	100 tambala (t)
Zambia	Kwacha (K)	100 ngwee (n)
Zimbabwe	Dollar ($)	100 cents (c)
Tanzania	Shilling (s)	
Kenya	Shilling (s)	
Uganda	Shilling (s)	
Mozambique	Meticais (MZN)	
Botswana	Pula	
Swaziland	Lilangeni	
Lesotho	Molito	
South Africa	Rand	100 cents
Britain	Pound (£)	100 pence
USA	Dollar ($)	100 cents
West Germany	Deutschmark (DM)	100 preunings
Japan	Yen (¥)	
France	Franc (F)	100 centimes
Canada	Dollar ($)	100 cents
Holland	Guinder (g)	100 cents
Europe	Euro (€)	

When a Malawian wishes to buy goods from South Africa he must find or buy South African Rands. The rate at which he is going to buy the rands is called the Exchange Rate.

For example, if R1 = K37 and he wants R1000 he must give up
$37 \times K1000 = \quad K37{,}000$

Exchange rate is therefore the rate at which one county's currency is exchanged for another. It is used also to convert one currency into the equivalent of another.

The exchange rate depends on the demand and supply of the currencies and the level of economic activities in the countries involved. The higher the level of economic activity in one country, the more the number of people wanting to buy goods from it and sothe higher the value of its currency. That is people will require relatively more of their currency in order to acquire currency of that country.

The following table gives exchange rates between the Malawi Kwacha and a selected currencies as at 8th November 2013.

MALAWI KWACHA PER UNIT OF FOREIGN CURRENCY

CURRENCY		MALAWI KWACHA
United Kingdom	£ Sterling	618.28
United States	Dollar	384.17
Zambian	Kwacha	70
Kenyan	Shilling	4.50
Tanzanian	Shilling	0.24
RSA	Rand	37.58
Europe	Euro	515.13
Japanese	Yen	4.16

Example 25

Given K100,000. How much is this amount worth in the UK, Japan, Tanzania, and Europe. Use the table given.

Solution

In UK:

K618.28 = £1

Therefore

$$K100{,}000 = \frac{K100{,}000}{K618.28} \times £1$$

$$= £171.74$$

In Japan:

K4.16 = ¥1

$$K100{,}000 = \frac{K100{,}000}{K4.16} \times ¥1$$

$$= ¥2{,}403.46$$

In Tanzania:

K0.24= S1

Hence

$$K100{,}000 = \frac{K100{,}000}{K0.24} \times S1$$
$$= S416{,}666.67$$

In Europe:

K515.13 = €1
Hence

$$K100{,}000 = \frac{K100{,}000}{K515.13} \times €1$$
$$= €194.13$$

Example 26

a) Convert £150 into Malawi Kwacha
b) How much is £200 to RSA rand if £1 = R16.44

Solution

a) $K618.28 = £1$

Then
$$£150 = K618.28 \times 150 = K92{,}742.00$$

b) $K16.44 = £1$

Then
$$£200 = R16.44 \times 200 = R3{,}288.00$$

7.3.11 Income Tax and Value Added Tax

a) Income Tax

Income Tax is a tax that is based on the amount a person earns in tax year that begins on 1st July and ends on 30th June the following year.

Normally, income tax is deducted from employed people's monthly pay before they receive it by their employer who then pays the tax to Government through the Malawi Revenue Authority. This method of paying income tax is called *Pay As You Earn (PAYE).*

Amounts of each person's income that are not taxed are called *tax allowances*

Example 27

0% on the first K15,000
15% on the next K5,000 and
30% on the remaining taxable income

Calculate the monthly PAYE which is payable by Mr.Phiri.

Solution

PAYE

On the first K15,000: $\frac{0}{100} \times K15{,}000 = K0.00$

On the next K5000: $\frac{15}{100} \times K5{,}000 = K750.00$

On the remaining K31,500: $\frac{30}{100} \times K31{,}500 = K9450.00$

Total PAYE payable = K10,200

b) Value-added Tax (VAT)

Apart from income tax, Government also raises money by taxes on goods and services. These are called indirect taxes as they are included in the prices of commodities. Value-added tax (VAT) is the main indirect tax and it is charged on most goods we buy such as electricity, cooking oil, furniture, e.t.c. However, they also certain commodities that are zero-rated i.e. no VAT is charged. Usually, these are basic necessities such as maize flour, bread and school books

In Malawi for some time now the standard rate for VAT has been 16.5%.

Note that sometimes prices are quoted exclusive of VAT. The prices you will pay will include VAT and be 16.5% more than the quoted price.

Example 28

A pair of new car tyres costs K88,700 exclusive of VAT. How much will be the total bill for the tyres?

Solution

$$VAT = 16.5\%\ of\ K88{,}700$$

$$= \frac{16.5}{100} \times K88{,}700$$

$$= K14{,}635.50$$

$$Total\ Bill = K88{,}700 + K14635.50$$

$$= K103{,}335.50$$

CHAPTER SUMMARY

In this chapter we have looked at

- Description of Simple and Compound interest and how they calculated
- Calculation of Profit and Loss of an investment
- Calculation and interpretation Mark up and margin
- Payment options: Instalment and hire purchase buying
- Insurance: premiums and compensation
- Loans and mortgages
- Foreign exchange
- Calculation and preparation of water and utility bills
- Definitions and calculations of income tax (PAYE) and Value added tax

END OF CHAPTER EXERCISES

1. Find the interest earned if a man deposits K110,000 in an account for 3 years at 5% simple interest.

2. What would be the interest earned in problem 1 if the interest is compound.

3. The population of Kapeti grows at 2.3% per annum. Estimate the population for 2015 if it were at 4.7 million in 2012.

4. How much should Mugawa invest now if he has to earn K1,500 at the end of 2 years if the interest is:

   (a) at 8% simple interest?
   (b) at 8% at compound interest?

5. A man investing K1,000,000 got K1,500,000 in total after 5 years. What was the rate of interest if simple interest was used.

6. Calculate the interest earned on K2,000 invested for 3 years at a compound interest of 8% earned quarterly.

7. A wholesaler offers a 10% discount on orders of more than 50 pens originally at K20 each. How much will a retailer pay if he buys:

   a) 60 pens?
   b) 45 pens?

8. Mr.Mozondeka has would like to hire the following articles from Chipayika Furniture Ltd.

   - 2 Arm Chairs, K75 each
   - 6 Dining Chairs, K40 each
   - 8 stools at K6 each
   - 1 Dining Table at K300 each
   - 8 Rocking Chairs at K55 each

   a) Copy and complete the following invoice

INVOICE				
Number : 9/2407 Mr. B. Mazondeka 17 Kalikosi MBAMBA			Chipayika Furniture 72 Kapeti Street Box 14 KANDE	
TERMS 5% CASH DISCOUNT IF PAID IN 1 MONTH				
SERIAL TOTAL NUMBER	DESCRIPTION	QUANTITY	UNIT	PRICE
			K T	K T
1	Arm Chair	2	75 00	140 00
2				
3				
4				
			Total	
			Less 20% Trade Discount	
			Payable	

   b) How much will Mazondeka pay if he settles the invoice on sight?

9. A retailer sells his goods to the value of K30,000 per week and makes a 25% profit on his sales. If his total running expenses for the year amount to K78,000.

Calculate:

a) His net profit for the year
b) Assuming all goods sold in the year were purchased in that year his:

    i) margin
    ii) markup

on the goods themselves.

10. In order to buy a house a man borrows K15,000,000 from the Building Society. If the interest rate is 11% per annum simple interest, calculate his monthly repayments to the Building Society if he will repay the mortgages over 25 years.
How much interest will he pay?.(Mortgage calculations in real life are more complicated than this).

11. Mr.Gondwe buys an electric fan priced at K15,000. He pays 20% deposit and asked to pay 12 instalments of K1,200 each. Calculate:

a) How much the fun will cost
b) Interest paid by Mr.Gondwe
c) Interest rate (at simple interest).

12. Given that the following PAYE tax bands: 0% on the first K15,000; 15% on the next K3,000 and30% on the remaining taxable income are applicable to all people on employment in the country. Calculate the monthly PAYE payable in each of the following cases

a) Mr. Gama earns an annual salary of K4,500,000.00
b) James earns a monthly salary of K126,667.00
c) Mr Kabichi earns a monthly salary of K14,500.00

13. The bill of a meal at one the restaurants in Blantyre was K6,390.00, VAT inclusive. How much was the VAT, if the rate for VAT is 16.5%?

14. All prices at LK Building Products are shown exclusive of VAT. Mrs Phiri buys eight boxes of tiles priced at K9,900 per box, a large packet of tile cement at K2,390 and some ready mixed tile grout at K1,380.
If VAT is at 16.5%, calculate Mrs Phiri's total Bill including VAT.

CHAPTER 7

RATIOS AND PROPORTIONS

LEARNING OBJECTIVES

By the end of this chapter, the student should be able to:

i. Define the terms ratio and proportion
ii. Check proportionality of the problem
iii. Solve simple proportions
iv. Differentiate between simple and compound proportions

7.0 Introduction

It is normal to make comparisons betwee tow different things. The comparions may take different forms depending on an aspect of interest. These could be in terms of various attributes such as colour, weight, height and so on. However, what we are interested in is a comparison between quantities or relationship between values. The relationship between values are studied under ratios and proportions.

7.1 Ratios

A "ratio" is just a comparison between two different things. For instance, someone can look at a group of people, count noses, and refer to the "ratio of men to women" in the group. Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20.

Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15".

Expressing the ratio of men to women as "15 to 20" is expressing the ratio in words. There are two other notations for this "15 to 20" ratio:

odds notation: 15 **:** 20

fractional notation: $^{15}/_{20}$

You should be able to recognize all three notations; you will probably be expected to know them.

Given a pair of numbers, you should be able to write down the ratios.

Example 1

There are 16 ducks and 9 geese in a certain park. Express the ratio of ducks to geese in all three formats.

Solution

$16:9$, $\frac{16}{9}$, 16 to 9

Example 2:

Consider the above park. Express the ratio of geese to ducks in all three formats.

Solution

$9:16$, $\frac{9}{16}$, 9 to 16

The numbers were the same in each of the above exercises, but the *order* in which they were listed differed, varying according to the order in which the elements of the ratio were expressed. In ratios, order is very important.

Let's return to the 15 men and 20 women in our original group. I had expressed the ratio as a fraction, namely, $^{15}/_{20}$. This fraction reduces to $^{3}/_{4}$. This means that you can also express the ratio of men to women as $^{3}/_{4}$, 3 **:** 4, or "3 to 4".

This points out something important about ratios: the numbers used in the ratio might not be the *absolute* measured values. The ratio "15 to 20" refers to the *absolute* numbers of men and women, respectively, in the group of thirty-five people. The simplified or reduced ratio "3 to 4" tells you only that, for every three men, there are four women. The simplified ratio also tells you that, in any representative set of seven people (3 + 4 = 7) from this group, three will be men. In other words, the men comprise $^{3}/_{7}$ of the people in the group. These relationships and reasoning are what you use to solve many word problems.

Example 3

In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

Solution

The ratio, "7 to 5" (or 7 : 5 or $^{7}/_{5}$), tells me that, of every 7 + 5 = 12 students, five failed. That is, $^{5}/_{12}$ of the class flunked. Then ($^{5}/_{12}$)(36) = 15 students failed.

Example 4

In the park mentioned above, the ratio of ducks to geese is 16 to 9. How many of the 300 birds are geese?

Solution

The ratio tells me that, of every 16 + 9 = 25 birds, 9 are geese. That is, $^{9}/_{25}$ of the birds are geese. Then there are ($^{9}/_{25}$)(300) = 108 geese.

Generally, ratio problems will just be a matter of stating ratios or simplifying them.

Example 5

Express the ratio in simplest form: K10 to K45.

Solution

This exercise wants me to write the ratio as a *reduced* fraction:

$$^{10}/_{45} = \mathbf{^{2}/_{9}}.$$

This reduced fraction is the ratio's expression in simplest fractional form. Note that the units (the "kwacha" signs) "canceled" on the fraction, since the units, "K", were the same on both values. When both values in a ratio have the same unit, there should generally be no unit on the reduced form.

Example 6

Express the ratio in simplest form: 240 Km to 8 Km

Solution

This is done as follows:

$$\frac{240\,km}{8\,km} = \frac{30}{1} = 30:1$$

.Example 7

a) Chakadza Industries has 60 employees of whose 10% are machine operators and 50% are packers. What is the ratio of:

i) machine operators to packers
ii) machine operators to employees
iii) packers to machine operators

b) A company has a total number of 400 sales workers and 1200 production workers

i) What fraction of the staff are in sales?
ii) What is the ratio of production to sales workers?
iii) Find the ratio of K14.70 to K31.85
iv) Divide K1,500 in the ratio of 3 : 2

Solution

a)

i) 10 : 50 = 1 : 5
ii) 10 : 60 = 1 : 6
iii) 50 : 10 = 5 : 1

b)

i. Total work force = 400 + 1200 = 1,600
Sales/Total work force= $^{400}/_{1600}$ = ¼

ii. Ratio of Production : Sales = $^{1200}/_{400}$ = 3 : 1

iii. $\frac{1470}{3181} = \frac{6}{13}$

iv. The amount K1,560 is to be divided into 5 parts (3 + 2) which is to be shared in 3 parts to 2 parts or proportions $^3/_5$ and $^2/_5$ using the proportions

$^3/_5$ of K1,560 = K936
$^2/_5$ of K1,560 = K624

Exercise 1

1) The ratio of the ages of two people is 5 : 9. If the older person is 54 years old. How old is the younger one?

2) A length of timber is cut into two sections in the ratio 3 : 2. The larger section is later cut in the ratio 5 : 4. What is the ratio of the smallest section to the largest.

3) Two business partners shared their annual profits in the ratio of 3 : 2 up to K2,700 and equally for any profit in excess of this amount. If in a certain year the profits amount toK5,900, how much did each receive?

7.2 Proportions

A proportion is two ratios that have been set equal to each other; a proportion is an equation that can be solved. When I say that a proportion is two ratios that are equal to each other, I mean this in the sense of two fractions being equal to each other. For instance, $^5/_{10}$ equals $^1/_2$. Solving a proportion means that you are missing one part of one of the fractions, and you need to solve for that missing value. For instance, suppose you were given the following equation:

$$\frac{x}{10} = \frac{1}{2}$$

You already know, by just looking at this equation and comparing the two fractions, that x must be 5, but suppose you hadn't noticed this. You can solve the equation by multiplying through on both sides by 10 to clear the denominators:

$$\frac{x}{10} = \frac{1}{2}$$

$$10 \times \frac{x}{10} = 10 \times \frac{1}{2}$$

$$x = 5$$

Verifying what we already knew, we get that $x = 5$.

Proportions would not be of much use if you only used them for reducing fractions. A more typical use would be something like the following example.

Example 8

Consider those ducks and geese we counted back at the park. Their ratio was 16 ducks to 9 geese. Suppose that there are 192 ducks. How many geese are there?

Solution

We will let "G" stand for the unknown number of geese. Then we will clearly label the orientation of my ratios, and set up my proportional equation:

$$\frac{ducks}{geese} : \frac{16}{9} = \frac{192}{G}$$

We will do "cross multiplication" get rid of the fractions as follows:

$$\frac{16}{9}=\frac{192}{G},$$

Then $16 \times G = 192 \times 9$ i.e. $16G = 1728$.

So $G = \frac{1728}{16} = 108$

Then there are 108 geese.

Note that "Cross-multiplying" is standard language, in that it is very commonly used, but it is not technically a mathematical term. You might not see it in your book, but you will almost certainly hear it in your class or study group.

Notice how, in the equation at the beginning of the solution above, we wrote out the ratio in words:

$$\frac{ducks}{geese}$$

This is *not* standard notation, but it can be very useful for setting up your proportion. Clearly labeling what values are represented by the numerators and denominators will help you keep track of what each number stands for. In other words, it will help you set up your proportion correctly. If you do not set up the ratios consistently (if, in the above example, you mix up where the "ducks" and the "geese" go in the various fractions), you will get an incorrect answer. Clarity can be very important.

7.3 Checking proportionality

There is some terminology related to proportions that you may need to know. In the proportion:

$$\frac{a}{b}=\frac{c}{d}$$

the values in the "b" and "c" positions are called the "means" of the proportion, while the values in the "a" and "d" positions are called the "extremes" of the proportion. A basic defining property of a proportion is that the product of the means is equal to the product of the extremes. In other words, given the proportional statement:

$$\frac{a}{b}=\frac{c}{d}$$

you can conclude that $ad = bc$. (This is, in effect, the cross-multiplication) This relationship is occasionally turned into a homework problem. Let us look at the following example.

Example 9

Check if $\frac{24}{140}$ proportional to $\frac{30}{176}$.

Solution

For these ratios to be proportional (that is, for them to be a true proportion when they are set equal to each other), We have to be able to show that the product of the means is equal to the product of the extremes. In other words, the product of 140 and 30 and the product of 24 and 176, must be then the ratios are proportional:

$$140 \times 30 = 4200$$
$$24 \times 176 = 4224$$

While these values are close, they are not equal, so we know the original fractions cannot be proportional to each other. So the answer is that **they are not proportional.**

The other technical exercise based on terminology is the finding of the "mean proportional" between two numbers. Mean proportionals are a special class of proportions, where the means of the proportion are equal to each other. An example would be:

$$\frac{1}{2} = \frac{2}{4}$$

because the means are both "2", while the extremes are 1 and 4. This tells you that 2 is the "mean proportional" between 1 and 4. You may be given two values and be asked to find the mean proportional between them.

Example 10

Find the mean proportional of 3 and 12.

Solution

Let "x" be the number that you are looking for. Since x will also be both of the means, You will set up the proportion with 3 and 12 as the extremes, and x as both means:

$$\frac{3}{x} = \frac{x}{12}$$

Now solve for x:

$$\frac{3}{x}=\frac{x}{12}$$

$$3 \times 12 = x^2$$

$$36 = x^2$$

$$\pm 6 = x$$

Since you are looking for the mean proportional of 3 and 12, you would figure that you would need to take the positive answer, so that the mean proportional would be just the 6. However, considering the fractions, either value would work:

$$\frac{3}{-6}=\frac{-6}{12}$$

$$\frac{3}{6}=\frac{6}{12}$$

So, actually, there are *two* mean proportionals: **–6 and 6**

Example 11

Find the mean proportional of $^3/_2$ and $^3/_8$.

We set up the proportion using fractions within fractions, and proceed normally:

$$\frac{3/2}{x}=\frac{x}{3/8}$$

Then $x^2 = \frac{3}{2}\times\frac{3}{8}$ or $x^2 = \frac{9}{16}$

So $x = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$

So the two mean proportionals are $\frac{3}{4}$ and $-\frac{3}{4}$.

You may be interested only with the positive i.e "$\frac{3}{4}$ ".

We have seen that solving proportions is simply a matter of stating the ratios as fractions, setting the two fractions equal to each other, cross-multiplying, and solving the resulting equation. We look at some more examples.

Example 12

Find the unknown value in the proportion: $2 : x = 3 : 9$.

$$2 : x = 3 : 9$$

Solution

First, convert the colon-based odds-notation ratios to fractional form:

$$\frac{2}{.\,x} = \frac{3}{9}$$

Then solve the proportion:

$$\frac{2}{x} = \frac{3}{9}$$

$$9(2) = x(3)$$

$$18 = 3x$$

$$6 = x$$

Example 13

Find the unknown value in the proportion: $(2x + 1) : 2 = (x + 2) : 5$

$$(2x + 1) : 2 = (x + 2) : 5$$

Solution

First, convert the colon-based odds-notation ratios to fractional form:

$$\frac{2x+1}{2} = \frac{x+2}{5}$$

Then solve the proportion:

$$\frac{2x+1}{2} = \frac{x+2}{5}$$

$$5(2x+1)=2(x+2)$$

$$10x+5=2x+4$$

$$8x=-1 \text{ or } x=-\frac{1}{8}$$

Once you have solved a few problems involving proportions, you will likely then move into word problems where you will first have to invent the proportion, extracting it from the word problem, before solving it.

Example 14

If twelve inches correspond to 30.48 centimeters, how many centimeters are there in thirty inches?

Solution

We will set up your ratios with "inches" on top, and will use "*c*" to stand for the number of centimeters.

$$\frac{inches}{centimetres} : \frac{12}{30.8} = \frac{30}{c}$$

$$\frac{12}{30.8} = \frac{30}{c}$$

$$12c = (30)(30.48)$$
$$12c = 914.4$$
$$c = 76.2$$

Thirty inches correspond to 76.2 cm.

Example 15

The tax on a property with an assessed value of K70 000 is K1 100. What is the assessed value of a property if the tax is K1 400?

Solution

Set up ratios with the assessed valuation on top, and use "*v*" to stand for the value that you need to find. Then:

$$\frac{value}{tax}:\frac{70000}{1100}=\frac{v}{1400}$$

$$\frac{70000}{1100}=\frac{v}{1400}$$

$$98\ 000\ 000 = 1\ 100v$$
$$89\ 090.909\ 090\ 9... = v$$

Since the solution is a Kwacha-and-tambala value, you need to round the final answer to two decimal places:

The assessed value is K89 090.91.

Example 16

One piece of pipe 21 meters long is to be cut into two pieces, with the lengths of the pieces being in a 2 : 5 ratio. What are the lengths of the pieces?

Solution

Label the length of the short piece as "x". Then the long piece, being the total piece less what was cut off for the short piece, must have a length of $21 - x$.

(short piece) : (long piece): $2 : 5 = x : (21 - x)$

$$\frac{2}{5}=\frac{x}{21-x}$$

$$2(21 - x) = 5x$$

$$42 - 2x = 5x$$

$$42 = 7x$$

$$6 = x$$

Then the length of the longer piece is given by:

$$21 - x = 21 - 6 = 15$$

The two pieces have lengths of 6 meters and 15 meters.

Example 17

You are installing rain gutters across the back of your house. The directions say that the gutters should decline $^1/_4$ inch for every four feet. The gutters will be spanning thirty-seven feet. How much lower than the starting point (that is, the high end) should the low end be?

Solution

Rain gutters have to be slightly sloped so the rainwater will drain toward and then down the downspout. As I go from the high end of the guttering to the low end, for every four-foot length that I go sideways, the gutters should decline [be lower by] one-quarter inch. So how much must the guttering decline over the thirty-seven foot span? I'll set up the proportion.

$$\frac{declination\ (in)}{length\ (ft)} : \frac{1/4}{4} = \frac{d}{37}$$

$$\frac{1/4}{4} = \frac{d}{37}$$

$$1/4 \times 37 = 4d$$

$$9.25 = 4d$$

$$2.3125 = d$$

The lower end should be $2\,^5/_{16}$ inches lower than the high end.

Example 18

Biologists need to know roughly how many fish live in a certain lake, but they don't want to stress or otherwise harm the fish by draining or drag netting the lake. Instead, they let down small nets in a few different spots around the lake, catching, tagging, and releasing 96 fish. A week later, after the tagged fish have had a chance to mix thoroughly with the general population, the biologists come back and let down their nets again. They catch 72 fish, of which 4 are tagged. Assuming that the catch is representative, how many fish live in the lake?

Solution

The idea is that, after allowing the tagged fish to circulate, they are evenly mixed in with the total population. When the researchers catch some fish later, the ratio of tagged fish in the sample is representative of the ratio of the 96 fish that they tagged with the total population.

Use "f" to stand for the total number of fish in the lake, and set up my ratios with the numbers of "tagged" fish on top. Then set up and solve the proportion:

$$\frac{tagged}{total}:\frac{4}{72}=\frac{96}{f}$$

$$\frac{4}{72}=\frac{96}{f}$$

$$f\times 4 = 72 \times 96$$

$$4f = 6912$$

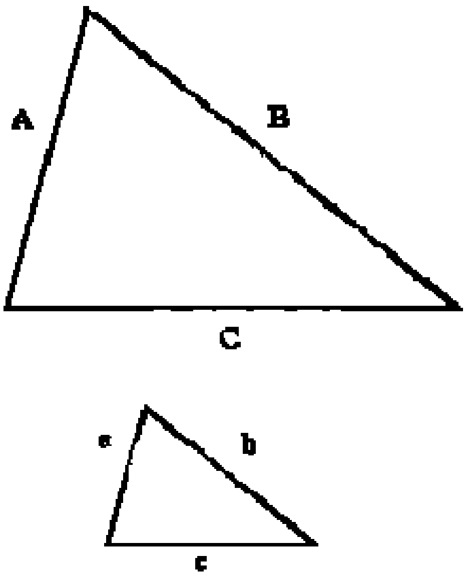
$$f = 1728$$

There are about 1728 fish in the lake.

Another category of proportion problem is that of "similar figures". "Similar" is a geometric term, referring to geometric shapes that are the same, except that one is larger than the other. Think of what happens when you use the "enlarge" or "reduce" setting on a copier, or when you get an eight-by-ten enlargement of a picture you really like, and you will have the right idea. If you have used a graphics program, think "aspect ratio".

In the context of ratios and proportions, the point is that the corresponding sides of similar figures are proportional.

For instance, look at the similar triangles at the right:



The "corresponding sides" are the pairs of sides that "match", except for the enlargement / reduction aspect of their relative sizes. So A corresponds to a, B corresponds to b, and C corresponds to c.

Since these triangles are similar, the pairs of corresponding sides are proportional. That is, $A:a = B:b = C:c$. This proportionality of corresponding sides can be used to find the length of a side of a figure.

Example 19

In the displayed triangles, the lengths of the sides are given by $A = 48\,mm$, $B = 81\,mm$, $C = 68\,mm$ and $a = 21$ mm. Find the lengths of sides b and c, rounded to the nearest whole number.

Solution

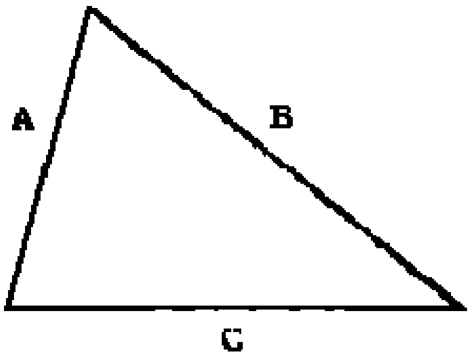
Set up the proportions, using ratios in the form (big triangle length) / (little triangle length), and then solve the proportions. Since the length of only side a for the little triangle, the reference ratio will be A**:**a.

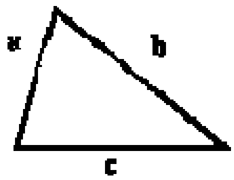
First, find the length of b.

$$\frac{A}{a} = \frac{B}{b}$$

$$\frac{48}{21} = \frac{81}{b}$$

$$b \times 48 = 21 \times 81$$
$$48b = 1701$$
$$b = 35.4375$$





Now find the length of c.

$$\frac{A}{a} = \frac{C}{c}$$

$$\frac{48}{21} = \frac{68}{c}$$

$c \times 48 = 21 \times 68$

$48c = 1428$

$c = 29.75$

$b = 35$ mm and $c = 30$ mm.

Example 20

A picture measuring 3.5" high by 5" wide is to be enlarged so that the width is now 9 inches. How tall will the picture be?

Solution

In other words, the photo lab will be maintaining the aspect ratio; the rectangles representing the outer edges of the pictures will be similar figures. So I set up my proportion and solve:

$$\frac{height}{width} : \frac{3.5}{5} = \frac{h}{9}$$

$$\frac{3.5}{5} = \frac{h}{9}$$

$9 \times 3.5 = 5 \times h$
$31.5 = 5h$
$6.3 = h$

The picture will be 6.3 inches high.

Example 21

The instructions for mixing a certain type of concrete call for 1 part cement, 2 parts sand, and 3 parts gravel. (The amount of water to add will vary, of course, with the wetness of the sand used.) You have four cubic feet of sand. How much cement and gravel should you mix with this sand?

Solution

Since the sand is measured in cubic feet and the "recipe" is given in terms of "parts", I will let "one cubic foot" be "one part

The ratio of cement to sand is 1 **:** 2, and there are four cubic feet of sand. Let "c" stand for the amount of cement that you need, then set up and solve the proportion.

$$\frac{cement}{sand} : \frac{1}{2} = \frac{c}{4}$$

$$\frac{1}{2} = \frac{c}{4}$$

$$2c = 4 \text{ i.e. } c = 2$$

Now solve for the amount of gravel to add.

The ratio of sand to gravel is 2 **:** 3, and we have four cubic feet of sand. We will define "g" to stand for the amount of gravel that we need, and we set up and solve the proportion:

$$\frac{sand}{gravel} : \frac{2}{3} = \frac{4}{g}$$

$$\frac{2}{3} = \frac{4}{g}$$

$$2g = 12 \text{, so } g = 6$$

You need two cubic feet of cement and six cubic feet of gravel.

A proportion can also be defined as the relationship between a number of unconnected things or parts.

Example 22

If a student is able to complete 3 problems in 15 minutes, how long will it take him to complete 9 problems? Assuming he works at the same speed?

Solution

3 problems take 15 minutes
1 problem will take $^{15}/_{3}$ = 5 minutes
Then 9 problems will take 5 x 9 = <u>45 minutes</u>

Proportions can be classified according to the variations (relationships) between the parts being considered. These are direct variations and unverse variations.

7.4 Direct and Inverse Variation

a) Direct Variations

Direct variation is when the components of the proportions vary (decrease and increase) together.

Example 23

Suppose a train is running at a constant speed of 48 k.p.h. How far will it travel in 10 minutes?

Solution

In 60 min it travels 48 Km
In 1 min it will travel $^{48}/_{60}$

In 1 min :$^{48}/_{60}$ x 10 = 8 Km

It should be noted in the example that at the constant speed, if time is reduced the distance traveled is also reduced. This is the direct relationship or variation.

a) Inverse Variations

Inverse variation is when the components of the proportions vary inversely i.e. one component increase while the other decreases.

Example 24

Suppose a man makes a journey of 120 km in 12 hours if he travels at a speed of 10 k.p.h. How long will it take him if he travels at 20 k.p.h.?

Solution

At 10 k.p.h it takes $^{120}/_{10}$ = 12 hours

At 20 k.p.h it takes $^{120}/_{20}$ = 6 hours

It should be noted here that as the speed increases the time taken to make the journey decreases. This is inverse relationship or variation.

7.5 Compound Proportions

Compound is the proportion that involves two or more quantities. In compound proportion, the ratio between different quantities is absolutely different and may not be realted in any way. We will illustrate this by looking at some examples.

Example 25

If 12 men earn K81 in 10 days, how long will 14 men earn in 8 days, if the daily wage is the same for each man?

Solution

If the money earned by 1 man in 1 day is referred to as 1 man-day.

(12 x 10 =120) man days equal K81

Then (14 x 8 = 112) man days = $\frac{14 \times 8}{12 \times 10} \times K81 = K75.60$.

Example 26

A laboratory technician has two solutions, A and B. Solution A has 50% sulphuric acid concentration while solution B has 75% sulphuric acid concentration. If the technician decide to mix the solutions, how many litres of the two solutions does the technician need to make 100 litres of the mixture with 60% sulphuric acid concentration?

Solution

This is a compound proportion problem. Recall that we are looking to find the number of litres of solution A and B that should up to 100 litres and contain 60% of the acid.

Let x the number of litres of solution A.
Thus the number of litres of solution B is $100 - x$.
Let us draw the following table that will simplify the relationship between the different variables.

Solution	% of acid	No. of litres	Concentration of Acid
A	50	x	$0.5x$
B	75	$100 - x$	$0.75(100 - x)$
Mixture	60	100	$0.6(100)$

This implies that: $0.5x + 0.75(100 - x) = 0.6(100)$

$$0.5x + 75 - 0.75x = 60$$

$$-0.25x = -15$$

$$\text{So} \quad x = \frac{-15}{-0.25} = 60$$

Hence A: $x = 60$ and B: $100 - 60 = 40$

The technician needs 60 litres of solution A and 40 litres of solution B.

Exercise 2

1. 20 tablets of soap cost K18; find the cost of 25 tablets.

2. 12 men can cultivate a field in 10 days, how long will 15 men take?

3. A hotel charges at the rate of K65 per day; find the charges for 10 days.

4. If 40 men are able to complete a building in 24 days. How long would it take 48 men to complete a similar construction?

7.6 Proportional Parts

The concept of proportional parts is closely related to the idea of ratios: An entire article length or measure is divided into parts and each part's share is established as a ratio to the other (or proportion). Sometimes the items compared may not come from the same thing but by the nature of their similarities they can be expressed as ratios in order to show their relative sizes.

Example 27

A rod of one meter is broken into 3 pieces whose length are in the ratio 4 : 3 : 3. Find the length of each piece.

Solution

4 + 3 + 3 = 10

$^{100cm}/_{10} = 10$ cm

one piece is 4 x 10 cm = 40 cm long
the second piece: 3 x 10 cm = 30 cm long
the last piece : 3 x 10 = 30 cm long

Proportional parts are more useful when the relationship between components of two or more sets of ratios is to be established.

Example 28

If a : b = 4 : 7, b : c = 5 : 3, find the ratio a: b : c

Solution

a : b = 4 : 7, b : c = 5 : 3

Find the LCM of the numbers for b which is common

i.e; LCM of 7, 5, = 35

So a and c will change.

$a : b = ? : 35 \rightarrow 20 : 35$
$b : c = 35 : ? \rightarrow 35 : 21$

Then $a : b : c = 20 : 35 : 21$

CHAPTER SUMMARY

In this chapter you have learned:

i. The definition of ratio and proportion
ii. Calculating actual numbers from the given ratios
iii. Checking proportionalities
iv. Solving problems involving simple and compound proportions

END OF CHAPTER EXERCISES

1. Divide 5 tins into three parts in the ratio $\frac{1}{2} : \frac{1}{3} : 1\frac{1}{4}$

2. A, B, C provide K250, K520, K750 respectively to buy a business, and their shares of the profits are proportional to the capital they provide. If the profits are K245, what does each receive?

3. Find the three smallest whole numbers proportional to a, b, c in:

i)	$a : b = 5 : 6$		$b : c = 9 : 4$
ii)	$a : b = 12 : 5$		$a : c = 8 : 3$

4. Profits amounting to K2415 are divided between A,B, C so that for every K4 A gets K5 and for every K9 B gets C gets K16. Find the shares.

5. A puts K900 into a business for 1 year and B puts into it K400 for 9 months. The profits shared between A and B are K180; how much should each receive?

6. A train takes 50 minutes for a journey if it runs 48 Km per hour. At what rate must it run to reduce the time by 10 minutes?

7. The total number of students in a College was 2736, the ratio of male to female being 14 : 5. How many more males were there than females?

8. If 6 people spend a total of K18,000 on travel in 24 weeks, what would be the expenditure at the same rate of 10 people over 30 weeks?

9. A manuscript when set up in type takes 192 pages of 28 lines with an average of 10 words per line. How many pages will be required if this is changed to 35 lines per page and an average of 12 words per line.

10. The cost of 4 articles each weighing 5 Kg is K9.20. What will the retailer be charged for 6 articles of the same quality each weighing 4.5 kg?

CHAPTER 8

ALGEBRAIC EXPRESSIONS

LEARNING OBJECTIVES

By the end of this chapter, the student should be able to

i. write out an algebraic expression
ii. identify polynomials
iii. classify polynomials
iv. expand polynomials
v. factorize algebraic expressions
vi. reduce a rational expression to the lowest terms

8.0 Introduction

Mathematical relationships between variables are sometimes given in form of algebraic expressions and/or equations. An algebraic expression or equation can involve one or more variables. The main concept of in algebraic expressions and equations is that one variable depends on or is influenced by other variables. Sometimes these algebraic expressions/equations are synonymous with the concept of a function. Thus for y influenced by or depending on x can be given by an expression that relates y and x. In this chapter, we discus linear and quadratic expressions/equations.

8.1 Important terminology

Variable

A **variable** is a symbol that takes on different values (i.e. its value may change within the scope of a given problem or set of operations). A variable is usually in form of a letter in a mathematical expression. In $y = 3x^2 + 5$, x and y are variables because these can assume any values.

Equation
An **equation** is an expression that show how variables are related. For instance, $y = x^2 + 4$ is an equation relating the variables x and y.

Constant
A **constant** is a value that remains unchanged in an expression/equation. In the equation $y = 3x^2 + 5$, 5 is a constant.

Term
The word "term" means a number or the product of a number and one or more of the variables involved. For example, the expression $2x^3 + 3xy$ has two terms: $2x^3$ and $3xy$.

Exponent
In mathematics an **exponent** is the power to which a variable, constant or term is raised. For example in $y = 3x^2 + 5$, 2 is the exponent for x and 3 is an exponent for $4xy$ in $(4xy)^3$.

8.2 Polynomials

8.2.1 Definition

A polynomial is basically an expression consisting of one or more terms and the exponents are non-negative integers (0, 1, 2, 3, 4.....). The concept of polynomials is the same as that of a function. The difference is that the term polynomial draws attention to the fact of the nature of the exponents on the variables of an expression.

For instance, the expressions $5x^3$, $3x^2 - 4x + 4$ and $6x$ are polynomials while the expressions $x^3 + 2x^{1/2} + 1$ and $4x^6 + 15x^{-8} + 1$ are not polynomials because the first one has non-integer exponent and the second has a negative exponent.

We first look at polynomials in one variable. A polynomial in one variable are algebraic expressions of the form ax^n where n is a zero or any positive integer and a is any real number. The degree of a polynomial in one variable is the highest exponent in the polynomial. Some examples of polynomials and their degrees are given below:

$3x^2 - 4x + 4$	Polynomial of degree 2
$11x^{13} + 10x^{10} - x^7 - 4$	Polynomial of degree 13
$5x^{28}$	Polynomial of degree 28
$6x + 13$	Polynomial of degree 1
20	Polynomial of degree 0

Note that a polynomial does not have to have all powers of x as in the first example. Furthermore, a polynomial can consist of just a single term as the third and last example. Another way of writing the last example is $20x^0$. It is now clear in this form that the exponent on the x is zero hence polynomial of degree 0.

Next we look at polynomials in two variables. Polynomials in two variables are algebraic expressions of the form $ax^n y^m$. The degree of each term in a polynomial in two variables is the sum of the exponents in each term and the degree of the polynomial is largest sum. We give examples of polynomials in two variables below:

$3xy^2 - 4x + 4$	Polynomial of degree 3

$$10x^6 + 5x^2y^3 + 2y \qquad \text{Polynomial of degree 6}$$
$$xy - 4x^5y^8 - x^3y^3 + 12 \qquad \text{Polynomial of degree 13}$$

Note that for polynomial in two variables, not all terms need to have x and y in them.

For purposes of this chapter, our interest is addition, subtraction, multiplication, division, factorization and simplification of the polynomials. These operations are important in solving both linear and quadratic equations later.

8.2.2 Addition and subtraction of polynomials

To add or subtract polynomials it is the like terms that are added or subtracted.

Example 1

Given $y = 3x^3 + x^2 - 5$ and $z = 2x^3 - 5x - 2$, find:

i) $y + z$
ii) $y - z$

Solution

i) $y + z = (3x^3 + x^2 + x - 5) + (2x^3 - 5x - 2)$
$= (3x^3 + 2x^3) + (x^2) + (x - 5x) + (-5 - 2)$
$= 5x^3 + x^2 - 4x - 7$

ii) $y - z = (3x^3 + x^2 + x - 5) - (2x^3 - 5x - 2)$
$= 3x^3 + x^2 + x - 5 - 2x^3 + 5x + 2$
$= (3x^3 - 2x^3) + (x^2) + (x + 5x) + (-5 + 2)$
$= x^3 + x^2 + 6x - 3$

8.2.3 Multiplication of polynomials

To multiply two polynomials, each term in one expression multiplies every term in the other expression. This process is called expansion.

Example 2

Simplify each of the following:

i) $2x^2(x^2 - 5x + 3)$
ii) $(2x + 5)(x - 7)$
iii) $(3x^2y)(4x^3y^3)$
iv) $(2x + 3)(x^2 - x + 1)$

v) $5(x+2)^2$

Solution

i) $2x^2(x^2-5x+3)$

To simplify this one, multiply each term of the polynomial in the brackets by $2x^2$

$$2x^2(x^2-5x+3)=2x^4-10x^3+6x^2$$

ii) $(2x+5)(x-7)$

To simply this one, multiply every term of the second polynomial by every term of the first polynomial, and then add or subtract the like terms

$$\begin{aligned}(2x+5)(x-7)&=2x^2-14x+5x-35\\&=2x^2-9x-35\end{aligned}$$

iii) $(3x^2y)(4x^3y^3)$

To simply this one, we multiply the coefficients and then multiply similar variables.

$$\begin{aligned}(3x^2y)(4x^3y^3)&=(3\times4)(x^2\times x^3)(y\times y^3)\\&=12x^{2+3}y^{1+3}\\&=12x^5y^4\end{aligned}$$

iv) $(2x+3)(x^2-x+1)$

Just before, this one too will be simplified by multiplying every term of the second polynomial by every term of the first polynomial, and then subtracting or adding the like terms.

$$\begin{aligned}(2x+3)(x^2-x+1)&=2x^3-2x^2+2x+3x^2-3x+3\\&=2x^3+x^2-x+3\end{aligned}$$

v) $5(x+2)^2$

We included this expression specifically to remind ourselves that we need to deal with the exponent first before multiplying the coefficient

$$\begin{aligned}5(x+2)^2&=5(x+2)(x+2)=5(x^2+2x+2x+4)\\&=5(x^2+4x+4)=5x^2+20x+20\end{aligned}$$

The last example reminds us of the following special products:

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)(x-y) = x^2 - y^2$$

However, we must ensure that we do not make the following common **mistakes** that students often make when they learn how to multiply polynomials: $(x+y)^2 = x^2 + y^2$ and $(x-y)^2 = x^2 - y^2$

8.2.4 Factorization of polynomials

Factorization is the process of breakdown a polynomial into smaller components whose product is the polynomial itself. Some polynomials can be factorized while others cannot. In this section, we familiarize ourselves with common techniques for factorizing polynomials.

Greatest common factor

The first technique for factorizing polynomials is 'factoring out' the **greatest common factor**. When factorizing in general this will also be the first step that we should try as it will often simplify the problem. To use this method all that we do is look at all the terms and determine if there is a factor that is common to all the terms. If there is, we will factor it out of the polynomial. For instance, given the expression $ax + ay$, we notice that an 'a'is common to both terms so it can be factored out as follows: $ax + ay = a(x+y)$

Example 3

Factor out the greatest common factor from each of the following:

i) $8x^4 - 4x^3 + 10x^2$
ii) $x^3y^2 + 3x^4y + 5x^5y^3$
iii) $3x^6 - 9x^2 + 3x$
iv) $9x^2(2x+7) - 12x(2x+7)$

Solution

i) $8x^4 - 4x^3 + 10x^2$

We notice that we can factor out 2 and x^2 from each term. So let's factor out $2x^2$

$$8x^4 - 4x^3 + 10x^2 = 2x^2(4x^2 - 2x + 5)$$

ii) $x^3y^2 + 3x^4y + 5x^5y^3$

We have both x's and y's in the polynomial. However, this does not affect the factoring process. Since each term has x^3 and y, we can factor out x^3y from the polynomial to get

$$x^3y^2 + 3x^4y + 5x^5y^3 = x^3y(y + 3x + 5x^2y^2)$$

iii) $3x^6 - 9x^2 + 3x$

Here we facto out $3x$ from the polynomial and this gives us

$$3x^6 - 9x^2 + 3x = 3x(x^5 - 3x + 1)$$

Notice the '+1' where $3x$ was, originally. This is so because the $3x$ is the term we factored out. Most common mistake with these types of factorization problems is to leave out the '1' and hence getting $3x^6 - 9x^2 + 3x = 3x(x^5 - 3x)$ which is wrong.

iv) $9x^2(2x + 7) - 12x(2x + 7)$

Although this looks different from the first problems, the factoring process is the same. Each term has $3x$ and $(2x + 7)$. So we can factor out $3x$ and $(2x + 7)$ to get

$$9x^2(2x + 7) - 12x(2x + 7) = 3x\{3x(2x + 7) - 4(2x + 7)\} = 3x(2x + 7)(3x - 4)$$

Note: We can always check the factorization by multiplying out the components (factors) to get the original polynomial.

Factorization by grouping

This is a technique that isn't used all that often, but when it can be used it can be somehow useful. We illustrate the technique with an example.

Example 4

Factorize the following by grouping

i) $3x^2 - 2x + 12x - 8$
ii) $x^5 + 5 - 2x^4 - 2$
iii) $x^5 - 3x^3 - 2x^2 + 6$

Solution

i) $3x^2 - 2x + 12x - 8$

To factorize this polynomial, we group the first two terms and the last two terms as follows

$$3x^2 - 2x + 12x - 8 = (3x^2 - 2x) + (12x - 8)$$

Now, we can factor out x from the first group and 4 from the second group to get

$$3x^2 - 2x + 12x - 8 = x(3x - 2) + 4(3x - 2)$$

Finally, we can now factor out a common factor of $(3x - 2)$ to obtain

$$3x^2 - 2x + 12x - 8 = (3x - 2)(x + 4)$$

ii) $x^5 + 5 - 2x^4 - 2$

In this case we will proceed as in example i). However, notice that both of the last two terms are negative, so we will factor out the '–' as well when grouping the terms. This is shown below

$$x^5 + 5 - 2x^4 - 2 = (x^5 + x) - (2x^4 + 2)$$

We then factor out x from the first group and 2 from the second group to get

$$x^5 + 5 - 2x^4 - 2 = x(x^4 + 1) - 2(x^4 + 1)$$

Finally, we factor out $(x^4 + 1)$ from both terms to get

$$x^5 + 5 - 2x^4 - 2 = (x^4 + 1)(x - 2)$$

iii) $x^5 - 3x^3 - 2x^2 + 6$

As in example ii) above, the third term has a '–'. However, the fourth term has a '+'. We will factor out the '–' when grouping the last two terms and then change the '+' on the fourth term to '–' as shown below

$$x^5 - 3x^3 - 2x^2 + 6 = (x^5 - 3x^3) - (2x^2 - 6)$$

Now we proceed as in the first two example

$$x^5 - 3x^3 - 2x^2 + 6 = x^3(x^2 - 3) - 2(x^2 - 6) = (x^2 - 3)(x^3 - 2))$$

Note: Factorization by grouping is easy but it does not always work. The only way to check if it works is by trying it out. Remember that if the third term is negative, we will often have to factor the '–' out of the third and fourth terms when grouping them.

Factorization of quadratic polynomial: The 'ac' technique

A 'quadratic' is another term for second degree polynomial. So we know that the highest exponent in a quadratic polynomial will be a 2. In these problems we will be attempting to factor quadratic polynomials into two first degree polynomials.

There are various techniques for factorizing quadratic polynomials. However, the easiest is the 'ac' technique. The technique is also very useful in determining whether or not a particular quadratic expression can be factorized. In order to use the 'ac' technique, the quadratic expression must be in the form $ax^2 + bx + c$ where a b and c are really number, possibly integers.

Let's consider the following example.

Example 5

Factorize each of the following

i) $x^2 + 6x + 9$
ii) $x^2 + 2x - 15$
iii) $x^2 - 10x + 24$
iv) $x^2 - 2x - 8$
v) $3x^2 + 2x - 8$
vi) $5x^2 - 17x + 6$

Solution

i) $x^2 + 6x + 9$

Relating $x^2 + 6x + 9$ to $ax^2 + bx + c$, shows that $a = 1, b = 6$ and $c = 9$. Multiplying a and$c$ gives 9. We therefore proceed to find factors of 9 (i.e. $a \times c$) whose sum is 6 (i.e. b). The factors of 9 that add up to 6 are 3 and 3. We can therefore express $6x$ as $6x = 3x + 3x$. The original quadratic polynomial can then be expressed as

$$x^2 + 6x + 9 = x^2 + 3x + 3x + 9$$

From this point we proceed as in the previous section, factorization by grouping:

$$x^2 + 6x + 9 = (x^2 + 3x) + (3x + 9) = x(x + 3) + 3(x + 3) = (x + 3)(x + 3)$$
$$\therefore x^2 + 6x + 9 = (x + 3)^2$$

ii) $x^2 + 2x - 15$

In this case $a = 1, b = 2$ and $c = -15$. Multiplying a and c gives –15. We therefore proceed to find factors of –15 whose sum is 2. The factors of 15 are 1 and 15, and 3 and 5. Since the value of b is positive, one or both of the factors must be positive. In fact the larger factor must have the same sign as b. So we must have 15 or 5. Furthermore, to obtain –15 the other factor must be negative. So we must have –1 or –3. The possible combinations are –1 and 15, or –3 and 5. The factors of –15 that add up to 2 are therefore –3 and 5 (i.e. $-3+5=2$ while $-1+15=14 \neq 2$) We can therefore express $2x$ as $2x = -3x + 5x$. The original quadratic polynomial can then be expressed as

$$x^2 + 2x - 15 = x^2 - 3x + 5x - 15$$

Grouping and factorization gives

$$x^2 + 2x - 15 = x^2 - 3x + 5x - 15 = (x^2 - 3x) + (5x - 15) = x(x-3) + 5(x-3)$$
$$\therefore x^2 + 2x - 15 = (x-3)(x+5)$$

iii) $x^2 - 10x + 24$

In this case $a = 1, b = -10$ and $c = 24$. Multiplying a and c gives 24. We therefore proceed to find factors of 24 whose sum is –10. The factors of 24 are 1 and 24, 2 and 12, 3 and 8, and 4 and 6. Since the value of b is negative, one or both of the factors must be negative. In fact the larger factor must have be negative. So we must have –24, –12, –8, or –6. Furthermore, to obtain [+] 24, the other factor must also be negative. So we must have –1, –2, –3 or –4. The possible combinations are –1 and –24, –2 and –12, –3 and –8, or –4 and –6. The factors of 24 that add up to -10 are therefore –4 and –6 (i.e. $-4-6=-10$). We can therefore express $-10x$ as $-10x = -4x - 6x$. The original quadratic polynomial can then be expressed as

$$x^2 - 10x + 24 = x^2 - 4x - 6x + 24$$

Grouping and factorizing gives

$$x^2 - 10x + 24 = x^2 - 4x - 6x + 24 = (x^2 - 4x) - (6x - 24) = x(x-4) - 6(x-4)$$
$$\therefore x^2 - 10x + 24 = (x-4)(x-6)$$

iv) $x^2 - 2x - 8$

As before, multiplying a and c gives –8 and the factors of –8 that add up to –2 are 2 and –4. The quadratic expression becomes $x^2 - 2x - 8 = x^2 + 2x - 4x - 8$

Grouping and factorizing gives

$$x^2 - 2x - 8 = x^2 + 2x - 4x - 8 = (x^2 + 2x) - (4x + 8) = x(x+2) - 4(x+2)$$

$\therefore x^2 - 2x - 8 = (x+2)(x-4)$

v) $3x^2 + 2x - 8$

In this case $a = 3, b = 2$ and $c = -8$. Multiplying a and c gives –24. We therefore need factors of –24 whose sum is 2. The factors of 24 are 1 and 24, 2 and 12, 3 and 8, and 4 and 6. Since the value of b is positive, one or both of the factors must be positive. In fact the larger factor must have be positive. So we must have 24, 12, 8, or 6. Furthermore, to obtain –24, the other factor must be negative. So we must have –1, –2, –3 or –4. The possible combinations are –1 and 24, –2 and 12, –3 and 8, or –4 and 6. The factors of 24 that add up to 2 are therefore –4 and 6 (i.e. $-4+6=2$). We can therefore express $2x$ as $2x = -4x + 6x$. The original quadratic polynomial can then be expressed as

$$3x^2 + 2x - 8 = 3x^2 - 4x + 6x - 8$$

Grouping and factorizing gives

$$3x^2 + 2x - 8 = 3x^2 - 4x + 6x - 8 = (3x^2 - 4x) + (6x - 8) = x(3x-4) + 2(3x-4)$$
$$\therefore 3x^2 + 2x - 8 = (3x-4)(x+2)$$

vi) $5x^2 - 17x + 6$

In this case $a = 5, b = -17$ and $c = 6$. Multiplying a and c gives 30. We therefore need factors of 30 whose sum is –15. The factors of 30 are 1 and 30, 2 and 15, 3 and 10, and 5 and 6. Since the value of b is negative, one or both of the factors must be negative. In fact, the larger factor must have be negative. So we must have –30, –15, –10, or –6. Furthermore, to obtain [+] 30, the other factor must also be negative. So we must have –1, –2, –3 or –5. The possible combinations are –1 and –30, –2 and –15, –3 and –10, or –5 and –6. The factors of 30 that add up to –17 are therefore –2 and –15 (i.e. $-2-15=-17$). We can therefore express $-17x$ as $-17x = -2x - 15x$. The original quadratic polynomial can then be expressed as

$$5x^2 - 17x + 6 = 5x^2 - 2x - 15x + 6$$

Grouping and factorizing gives

$$5x^2 - 17x + 6 = 5x^2 - 2x - 15x + 6 = (5x^2 - 2x) - (15x - 6) = x(5x-2) - 3(5x-2)$$
$$\therefore 5x^2 - 17x + 6 = (5x-2)(x-3)$$

Special forms

There some special forms of quadratic polynomials that can be factorized easily without necessarily following the 'ac' technique. Here are some of the forms:

i) $a^2 + 2ab + b^2 = (a^2 + b^2)$ e.g. $x^2 + 6x + 9 = x^2 + 2 \times 3 \times x + 3^2 = (x+3)^2$

ii) $a^2 - 2ab + b^2 = (a^2 - b^2)$ e.g. $x^2 - 20x + 100 = x^2 - 2 \times 10 \times x + 10^2 = (x-10)^2$

iii) $a^2 - b^2 = (a+b)(a-b)$ e.g. $25x^2 - 9 = (5x)^2 - 3^3 = (5x+3)(5x-3)$

We leave it to you to confirm these using the 'ac' technique.

8.3 Rational expressions

We now look at rational expressions. A **rational expression** is a fraction in which the numerator and/or the denominator are polynomials. Here are some examples of rational expressions.

$$\frac{22}{2x-5}, \quad \frac{x+3}{x^2+5x-2}, \quad \frac{a^5+17a+1}{a^2-7}, \quad \frac{3x^2-2x-10}{1}$$

The last one may look a little strange since it is more commonly written without the denominator 1. It is therefore important to note that possibly all polynomials can be thought of as rational expression,

When dealing with numbers we know that division by zero is not allowed. Well the same is true for rational expressions. So, when dealing with rational expressions we will always assume that whatever x is, the denominator won't be zero. So given any expression, avoid the value/s of x that would lead into dividing by zero. Our focus in this section is reducing rational expressions to their lowest form.

A rational expression is in its lowest terms if all common factors from the numerator and denominator have been canceled. We already know how to do this with number fractions so let us look at examples involving algebraic expressions using factorization and cancelation of common terms.

Example 6

Reduce the following rational expressions to their lowest terms.

i) $\dfrac{12x^5}{4x^3}$

ii) $\dfrac{12x^3+8x^2}{4x^2}$

iii) $\dfrac{x^2+7x+10}{x+2}$

Solution

i) $\dfrac{12x^5}{4x^3}$

We factor out $4x^3$ and cancel out the common terms as below

$$\frac{12x^5}{4x^3} = \frac{4x^3(3x^2)}{4x^3} = \frac{\cancel{4x^3}(3x^2)}{\cancel{4x^3}} = 3x^2$$

ii) $\dfrac{12x^3 + 8x^2}{4x^2} = \dfrac{4x^2(3x+2)}{4x^2} = \dfrac{\cancel{4x^2}(3x+2)}{\cancel{4x^2}} = 3x+2$

iii) $\dfrac{x^2 + 7x + 10}{5x^2 + 10x} = \dfrac{(x+5)(x+2)}{5x(x+2)} = \dfrac{x+5}{5x}$

CHAPTER SUMMARY

In order to evaluate an algebraic expression, sometimes one needs to simplify it. The following are the basic steps to simplify an algebraic expression:

- A polynomial is an expression consisting of one or more terms with non-negative integral exponents including zero
- To add polynomials add like terms by adding their coefficients
- Remove parentheses by multiplying factors (a process called expansion). To expand polynomials, multiply every term in one polynomial by every term in the other polynomial. Finish the expansion process by adding or subtracting like terms.
- Use rules of exponents to combine exponents when removing parentheses in expansion
- Factorization of polynomial can be carried out factoring out greatest common factor, using the grouping method or the 'ac' technique for quadratic polynomials
- Rational expression can be reduced to the lowest terms by factorizing then and then cancelling all common factors from the numerator and denominator.

END OF CHAPTER EXERCISES

1. Perform the following operations

a) $(x^2 - 5x - 14) + (x^3 - 3x - 10)$

b) $(x^3 + 9)(3 - 7)$

c) $(m^2 + 5m + 6) - (6m^2 - 5m + 4)$

2. Perform the indicated operations and reduce the answers to the lowest term

a) $\dfrac{x^2 - 5x - 14}{x^2 - 3x + 2} \times \dfrac{x^2 - 4}{x^2 - 14x + 49}$

b) $\dfrac{m^2 - 9}{m^2 + 5m + 6} \div \dfrac{3 - m}{m + 2}$

c) $\dfrac{y^2 + 5y + 4}{\dfrac{y^2 - 1}{y + 5}}$

3. Factorize the following

a) $x^2 - x - 12$

b) $x^2 + 14x + 40$

c) $y^2 + 12y + 36$

d) $4m^2 - 1$

e) $3x^2 - 2x - 8$

f) $10z^2 + 19z + 6$

g) $5x^2 - 2x$

CHAPTER 9

EQUATIONS AND GRAPHS

LEARNING OBJECTIVES

By the end of this chapter the student should be able to

i. Distinguish linear between linear and quadratic equations
ii. Plot graphs of linear and quadratic equations
iii. Find slope and intercepts of linear equations
iv. Solve linear and quadratic equations using algebraic and graphical methods

9.0 Introduction

Many real life problems are solved quickly if they are expressed in algebraic terms, like equations. An equation is an algebraic statement which specifies equality between two expressions. Basically an equation consists of two algebraic expressions and an equal sign between them. Graphs of equations are an important tool in solving and portraying the behavior of algebraic expressions as well as presenting statistical data.
This chapter outlines linear, non-linear and statistical graphs. The chapter focuses on drawing the graphs and interpreting them.

9.1 Linear Equations

Linear equations are equations which take the general form $y = mx + c$,
where:

x and *y* are variables; *m* and *c* are fixed. *m*is called the *slope* of the equation and *c* the *y-intercept* (the value of *y* when *x= 0*)

Non-linear equations will have variables whose powers are different from 1 (and 0)

Example 1

The following are some examples of linear equations

i. $y = x$
ii. $3y + 7x = 4$
iii. $4y = 0.5x + 3$

By expressing any linear equation in the general form, one can easily deduce the slope and y-intercept.

Example 2

Find the slope and y-intercept of the equation $5y - 9x = 45$, in example 5 above

Solution

First we convert the equation into the form $y = mx + c$ by making y subject of the equation. Thus

$y = 1.8x + 9$. Relating this equation to the general form we obtain

$slope, m = 1.8$ and $y - intercept, c = 9$

Exercise: Find the slope and y-intercept of each of the linear equations in example 1.

9.1.1 Linear Graphs

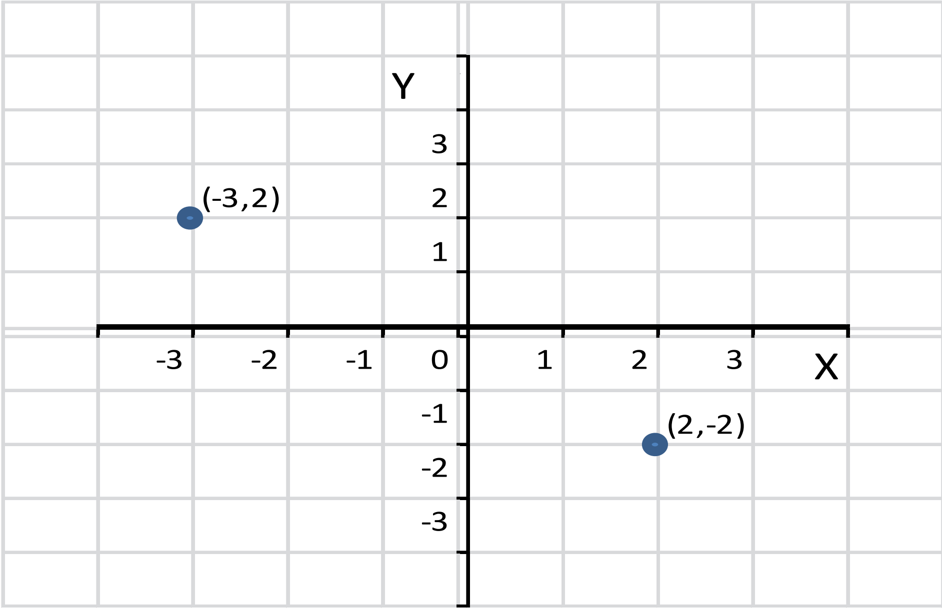
Linear graphs are graphs drawn to represent linear equations. A linear equation can have one or more variables. In this text we will limit ourselves to linear equations of up to two variables.

Generally linear graphs are straight lines. We will discover at the end that as a result of being a straight line, all points of a linear graph lie on the line and therefore only two coordinate pointsa are sufficient to draw the graph.

Axes

These are the two lines that provide the framework within which a graph is drawn. One axis is horizontal and usually (not always) referred to as the x-axis while the other is vertical, the y-axis.

The axes are shown in the following diagram as the two dark lines:



The axes are calibrated according to the ranges of data being graphed.

Coordinates and ordered pairs of data

A coordinate is a set of two values indicating an x and a y value on the graph. The set (-3, 2) and (2, 2) shown on the framework above are examples of coordinates

Coordinates comprise two values; the first one is an x-value and the second a y-value. This pair is referred to as an ordered pair of values.

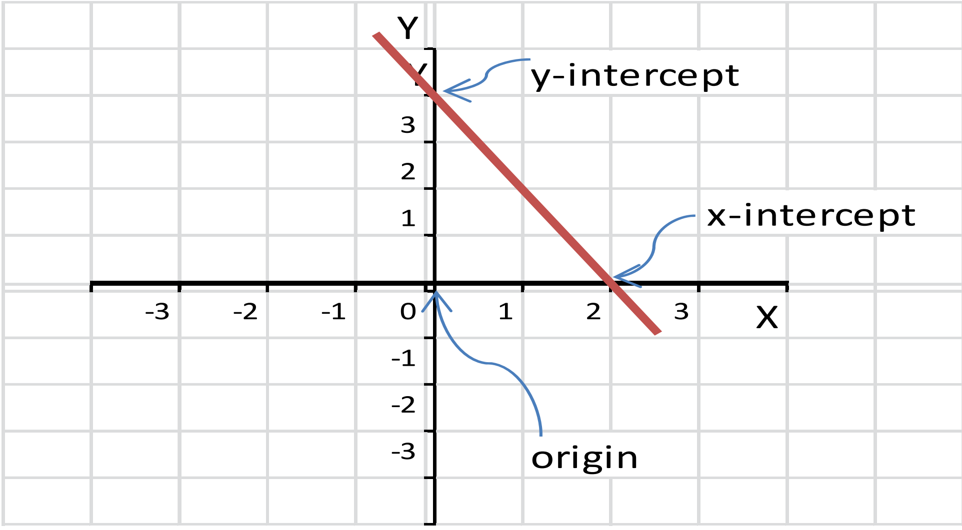
Origin

The word *origin* is used to describe the point on the graph where the two axes cross each other. At this point $x = 0$ and $y = 0$. The coordinate is (0,0)

Intercept

Generally the term means crossing. In the context of graphs, these are points where two graphs cross. More often than not, the concept is used where the graph cuts (crosses) the x – axis or the y – axis.

The term x intercept is used to mean the point where a graph crosses the x axis and y intercept is the point where the graph cuts the y axis. These are illustrated below.

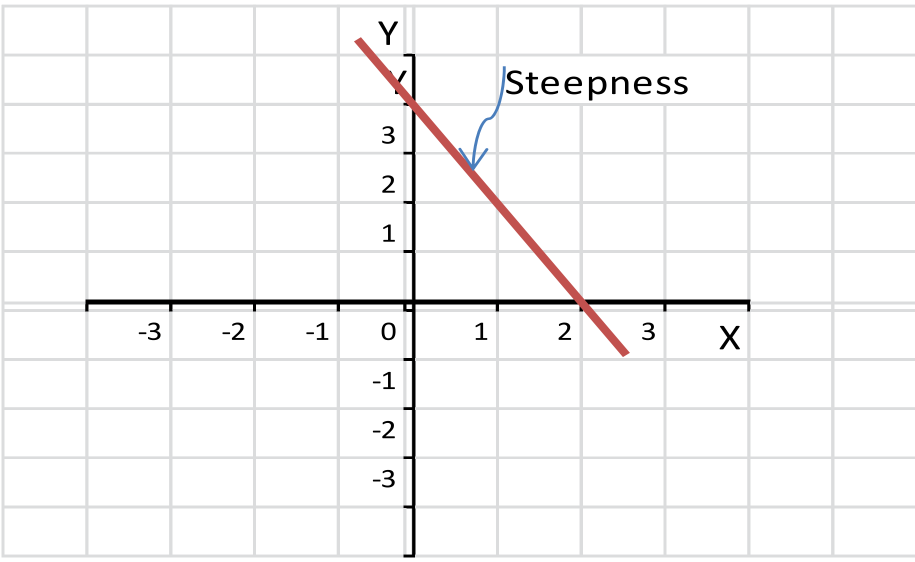


Note from the graph above that:

a) At the y-intercept $x = 0$ and
b) At the x-intercept $y = 0$

Slope

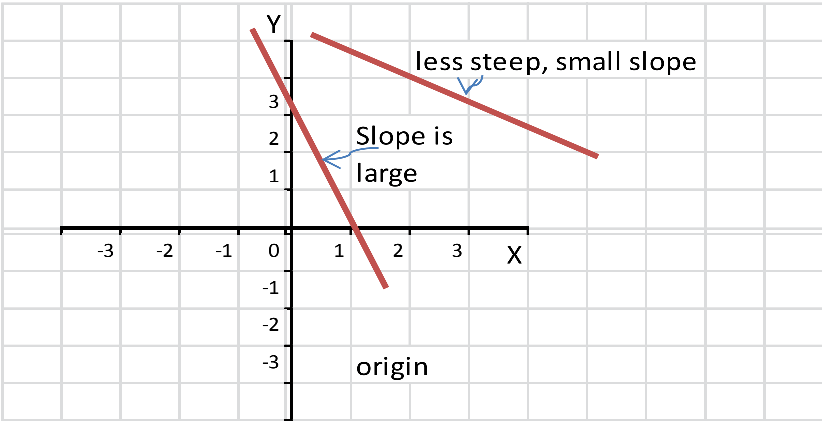
The slope of a graph is the measure of the steepness (gradient) of the graph. The concept is developed further below.



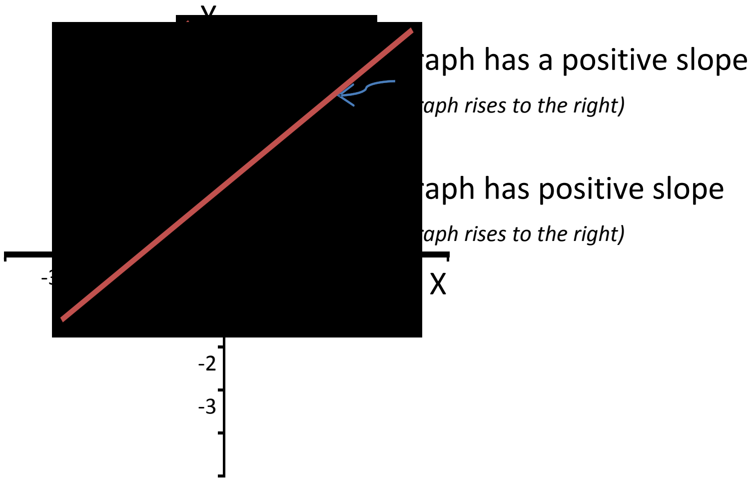
In general the slope is the change in the vertical axis resulting from the change a unit change in the horizontal axis.

Notes on slope:

- The bigger the slope, the steeper the graph.



- The slope of graph can either be positive or negative



9.1.2 Plotting a linear graph

The first step in plotting a graph is to construct a table of values. For a linear graph this step involves generating at least two coordinate points. For a given linear equation and for any values of an independent variable x calculate corresponding values for the independent variable y.

The second step is to plot the points on a pair of axes and join them with a line. Given a linear equation all the points will fall on a straight line.

Example 3

Plot a graph of $y = 4x + 3$

Solution

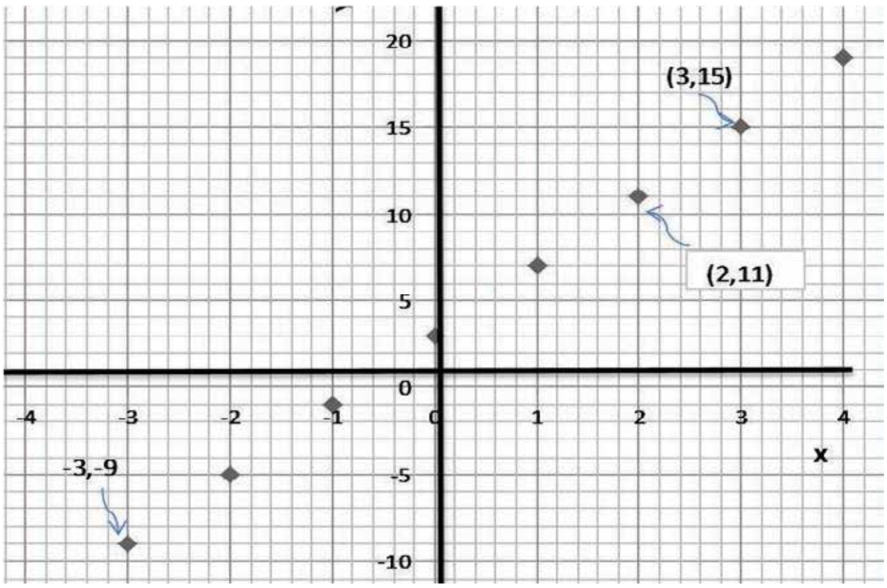
Generating coordinates: Picking the following set of x values of

-4, -3, -2, -1, 0, 1, 2, 3, and 4

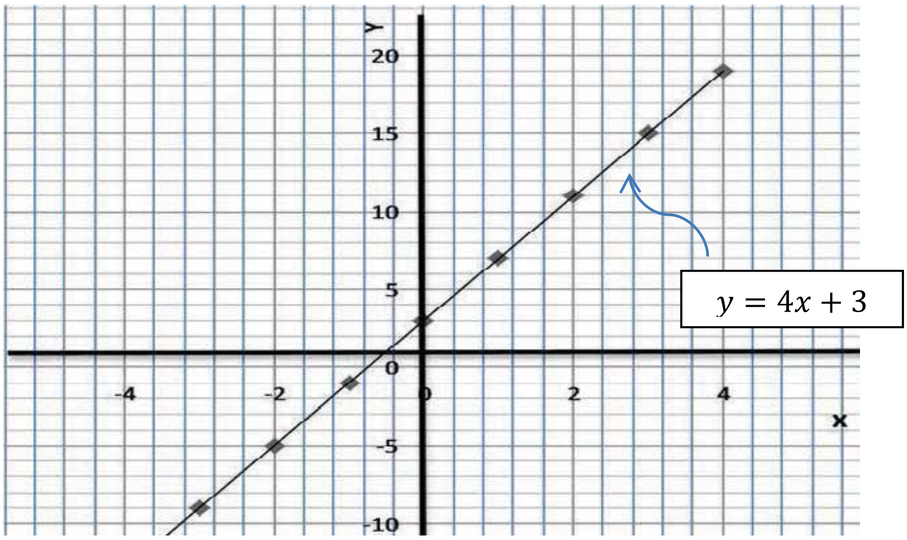
The coordinates are

x	-4	-3	-2	-1	0	1	2	3	4
y (=4x + 3)	-13	-9	-5	-1	3	7	11	15	19

Plotting these on a pair of axes we have the following points in the framework.



When these points are joined, they follow a straight line as shown below



Since a linear graph is a straight line, only two coordinates would have been used to draw the graph.

Example 4

Draw a graph of $4y + 8x = 21$

Solution

The equation has to be rearranged so that it is in the form $y = a + bx$

Thus$4y + 8x = 21$ gives $4y = 21 - 8x4$

$$y = 5.25 - 2x$$

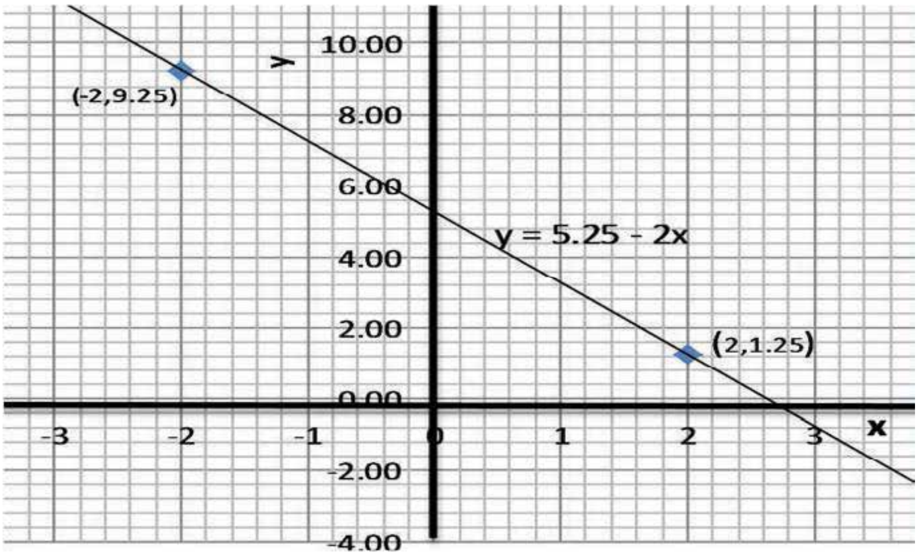
To generate any two coordinates we select any two values of x

Let these be x = -2 and x = 2

When x = -2, y= 5.25 – (-4) = 9.25 the first coordinate is (-2, 9.25)

When x = 2 y = 5.25 - 4 = 1.25 the other coordinate is (2,1.25)

The graph:



Note the following (a) two points plotted on the graph
(b) the graph has a negative slope

Using intercepts

A graph can be drawn by picking the intercepts as the coordinates. That is using points where $x = 0$ and where $y = 0$.

The technique is the same as that of using any two coordinates except that the intercepts are targeted for simplicity as one value in the coordinate is zero.

Example 5

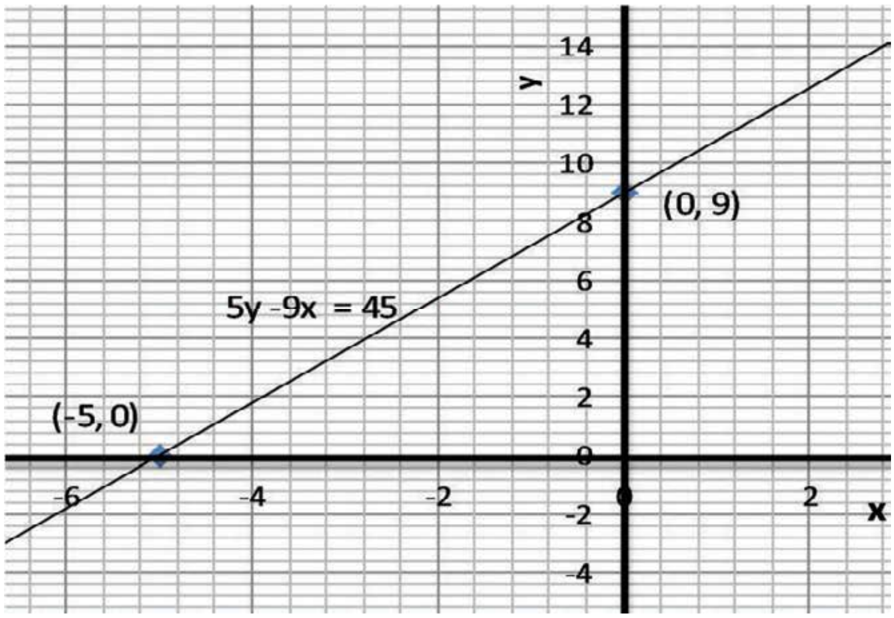
Draw a graph of $5y - 9x = 45$

Solution

The intercepts are $y = 0;\ x = -5$ by substitution and
$x = 0;\ y = 9$ again by substitution

The two coordinates are (-5, 0) and (0, 9)

The graph



9.1.3 Slope of a linear graph

As stated earlier, slope is the change in the vertical (*y*) axis resulting from a unit change in the horizontal (*x)* axis.

In other words,

$$Slope = \frac{change\ in\ y}{change\ in\ x}$$

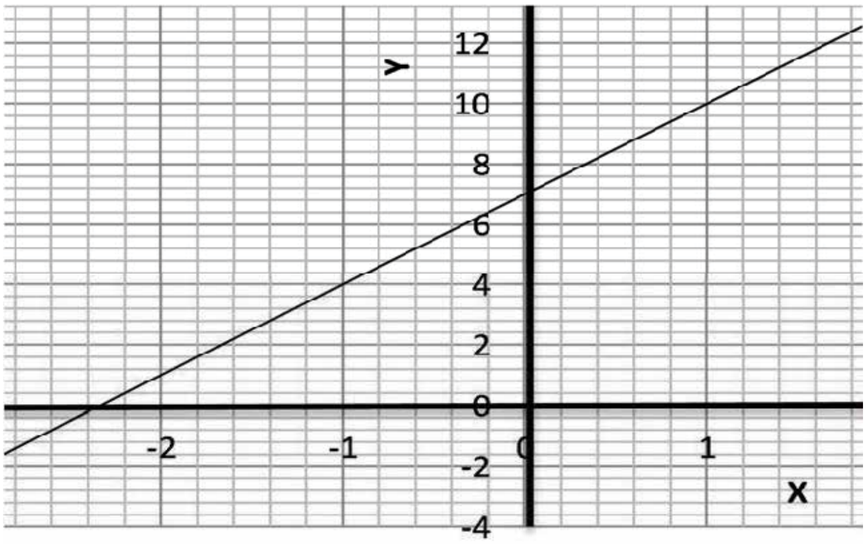
We usually denote slope by *m.*

From the above formula, if (x_1, y_1) and (x_2, y_2) are any two points lying on a graph of a linear equation then the slope of the graph is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

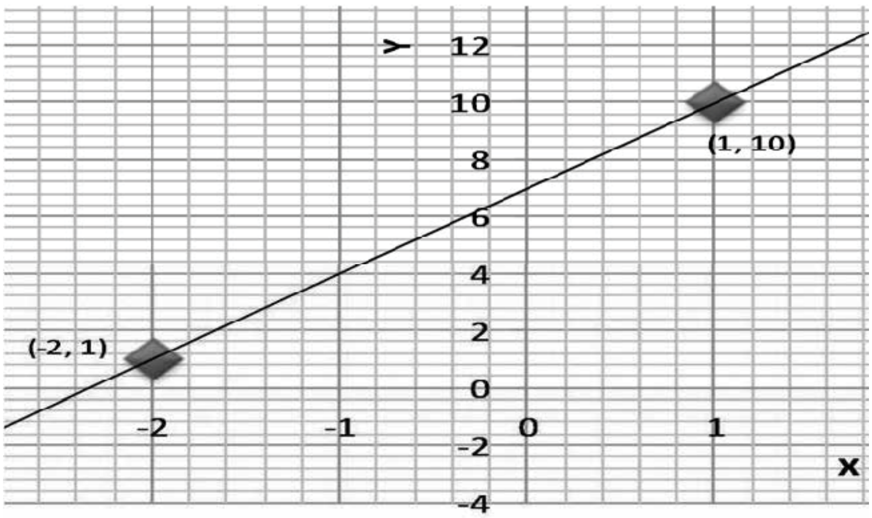
Example 6

Find the slope of the following graph



Solution

Notice that the points $(-2,1)$ and $(1,10)$are lying on the graph as shown below



$$Slope = \frac{y_2 - y_1}{x_2 - x_1}$$

Letting $(x_1, y_1) = (-2,1)$ and $(x_2, y_2) = (1,10)$ then

Slope of the graph:

$$m = \frac{10-1}{1-(-2)} = \frac{9}{3} = 3$$

Example 7

Find the slope of the equation $5y - 9x = 45$ example 3 above

Solution

Refer to the graph drawn above in example 3 and let
$(x_1, y_1) = (-2, 9.25)$and$(x_2, y_2) = (2, 1.25)$ as drawn on the graph

$$Slope = \frac{1.25 - 9.25}{2 - -2}$$
$$= -2$$

Hence $Slope\ of\ the\ equation = -2$

Slope of horizontal and vertical lines.

Notice that the slopes dealt with above indicate recognizable inclinations of the line. They are not perfectly horizontal (parallel to the x-axis) nor vertical (parallel to the y-axis).

The values of the slopes are defined numbers (negative or positive) different from zero. Slopes for strictly horizontal or vertical lines are different.

A horizontal line has slope =0

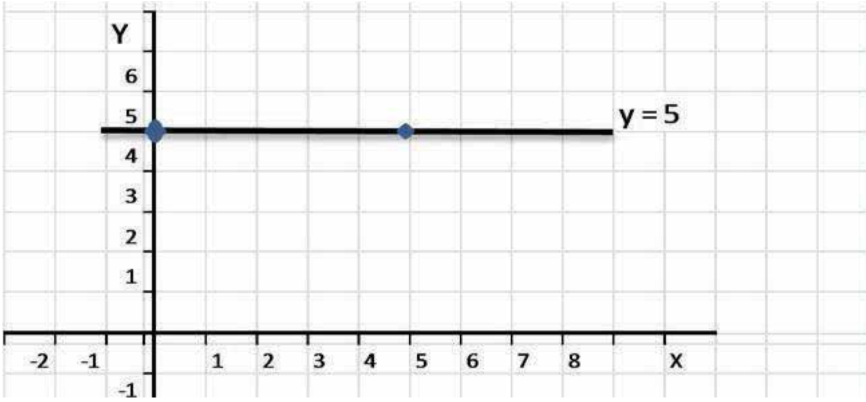
Example 8

Draw the graphs of $y = 5$ and $y = 6$ and calculate the slopes in each case

Solution

The graph of $y = 5$

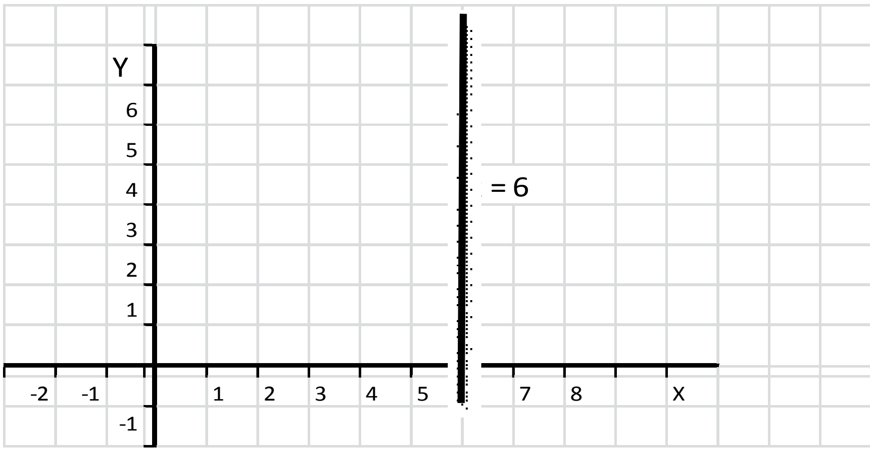
y is constant at x=5. The points (0,5) and 4,5) lie on this graph and plotting these we have the following graph



Picking any two points (1,5) and (7,5) the slope is

$$\frac{5-5}{7-1}$$
$$=\frac{0}{6}$$
$$=0$$

The graph of x = 6



The graph of x = 6 is a vertical line through x =6

Picking any two point (6,1) and (6,5) the slope is supposed to be calculated as

$$m=\frac{5-1}{6-6}$$

$$=\frac{4}{0}$$

which is undefined.

In general the slope of any horizontal graph (a graph for the equation $y = c$,where$c$ is a constant) is **zero** (no rise) and the slope of a vertical line (a graph for the equation $x = k$, where k is a constant) **is undefined**.

9.1.4 Simultaneous Linear Equations

Solving simultaneous equations involves finding unique values for the variables which satisfy all the given equations. If an equation has two unknown variables it cannot be solved on its own. Two variables require two equations to be solved at the same time (simultaneously).

A system of two equations in two variables is called a 2×2 system of equations.

Solving 2×2 simultaneous equations

Methods for solving simultaneous equations

a) Graphical technique
b) Algebraic technique

a) The Graphical technique

For purposes of a level addressed by this text the graphical method is suitable for systems of equations with two variables and two equations. To determine the solution:

i) Draw the linear equations on the same axes
ii) Note the values of x and y where the graphs intersect. These values are the solution.

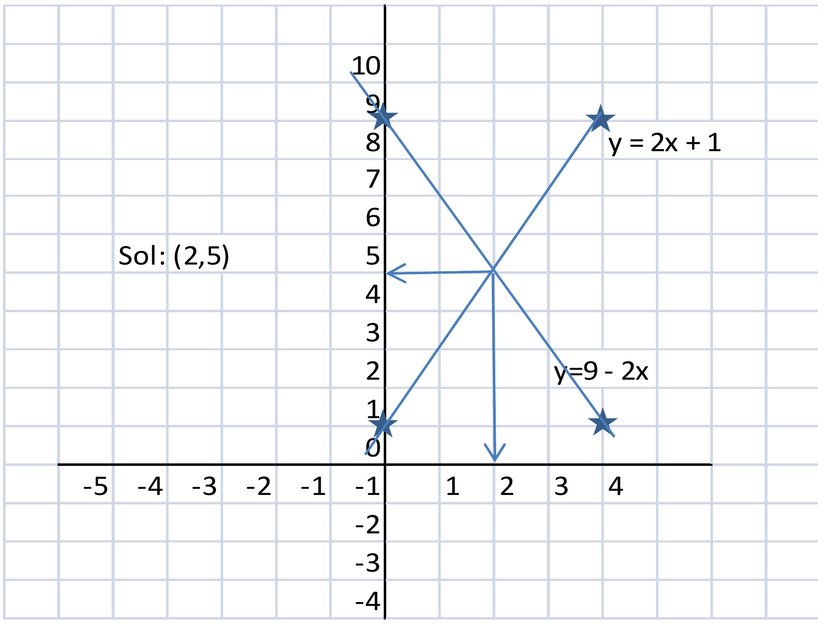
Example 9

Solve the following system of equations using the graphical technique:$y = 2x + 1$ and $y = 9 - 2x$

Solution

Two points on the graph of $y = 2x + 1$ are $(0, 1)$ and $(4,9)$
For $y = 9 - 2x$ we have $(0,9)$ and $(4, 1)$

Plotting these points and joining them by straight lines we obtain:



From the graph x = 2 and y = 5 (point of intersection)

b) Using the algebraic Method (elimination):

This method involves arranging the two equations involved into the form

$ax + by = c$…..i
$rx + sy = t$….ii

and then eliminating one of the two variables so that we remain with an equation with a single variable which can be solved using simple manipulations.

Example 10

Solve the equations $4x + 3y = 24$ and $2x + 3y = 18$ silmuteneously.

Solution

$$4x + 3y = 24$$
$$2x + 3y = 18$$

Subtracting the second from the first equation we get

$$4x + 3y = 2$$
$$\underline{2x + 3y = 18}$$
$$2x = 6$$

Hence

$$=> x = 3$$

We substitute the value of x back into the one of the original equation to solve for y.

$$4(3) + 3y = 24$$
$$3y = 24 - 12$$
$$y = \frac{24}{3}$$
$$= 4$$

Another way is by making one the variables in one of the two equations subject and substituting the resulting expression into the other equation.

Example 11

Solve the equations $2x + 3y = 11$ and $x - y = -2$

Solution

Making y subject of the second equation we obtain $y = x + 2$
Substituting $x + 2$ for y in the first equation:

$$2x + 3(x + 2) = 11$$
$$2x + 3x + 6 = 11$$
$$5x = 11 - 6$$
$$x = 1$$
$$y = x + 2$$
$$= 1 + 2$$
$$= 3$$

The solution to the simultaneous equations is $x = 1; y = 3$

Example 12

The price of the particular product is set at 12 kwacha per item for ten (10) items and falls to K8.50 per item when 30 are ordered.

Derive the demand function. Assuming it is linear

Working

q	p
10	K12
30	K8.50

These can be used to form two equations of the form $p = a + bq$

$12 \ = a + 10b$
$8.5 = a + 30b$

$12 = a + 10b$ -------------- (i)
$8.5 = a + 30b$ ------------- (ii)
$a = 12 - 10b$ -------------- (iii)
(iii) into (ii)
$8.5 = 12 - 10b + 30b$
$-3.5 = 20b$
$-3.5 = b$
20
$-0.175 = b$ into (iii) $a = 12 - 10 x (-0.175)$
$= 13.75$

The demand function: $\underline{p = 13.\ 75 - 0.175q}$

In general,

i. the slope of a(horizontal) graph for an equation a $y = c$, (where y is the vertical axis, and c a constant) is **zero** (no rise)
ii. The slope of a vertical graph for an equation $x = k$, where x is the horizontal axis and k is a constant) is **undefined.**

9.2 Quadratic Equations

Quadratic equations are sometimes known as second order equations. The main feature is that the independent variable has power equal to 2.

Generally a quadratic equation will take the general form:

$$y = ax^2 + bx + c,$$

where a, b,c are constants and $a \neq 0$

Example 13

The following are some examples of quadratic equations:

i) $y = x^2 - 5x + 6$
ii) $y = x^2 + 9x + 8$
iii) $y=x^2$

9.2.1 Quadratic Graphs

To plot a graph of a quadratic equation, we follow these steps

i. Construct a table of values.
ii. Plot the pairs of coordinates from the table of values on a Cartesian plane
iii. Join the points with a smooth curve

Example 14

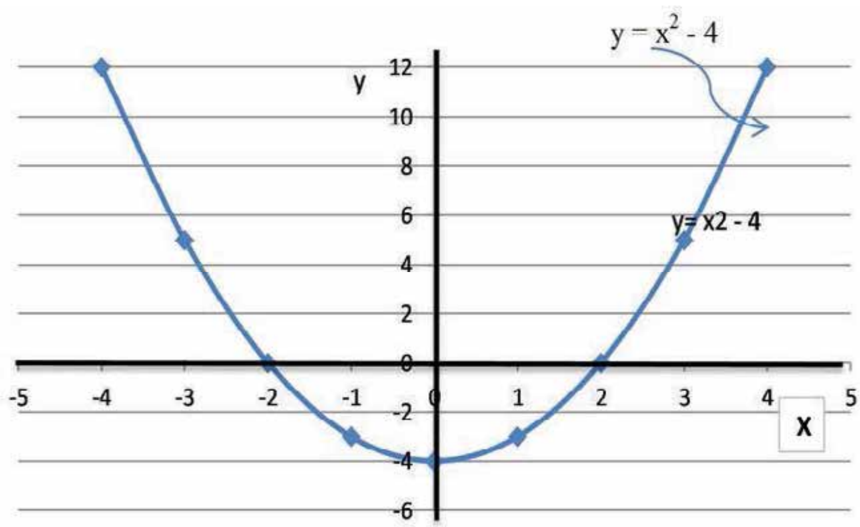
Plot the graph of $y = x^2 - 4$

Solution

For values of *x* from -4 to 4 we calculate corresponding *y* values and enter them in the table below:

x	-4	-3	-2	-1	0	1	2	3	4
y (=x^2 - 4)	12	5	0	-3	-4	-3	0	5	12

Plotting the coordinates and joining the points we have the following graph.



Example 15

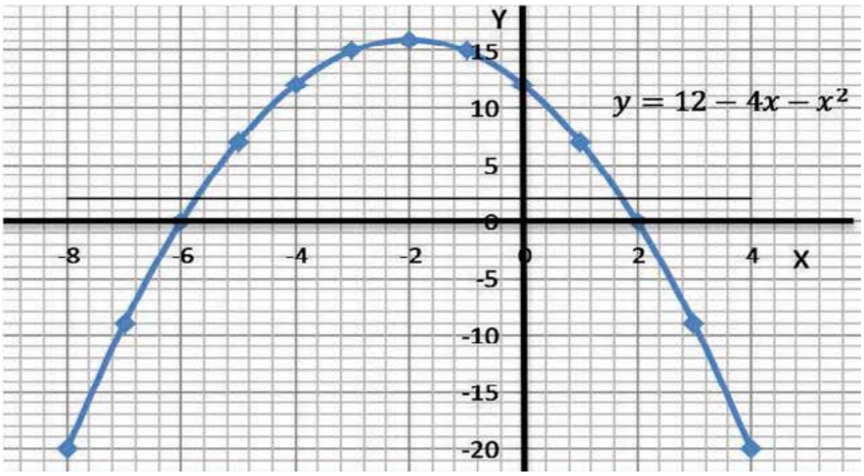
Draw a graph $y = 12 - 4x - x^2$

Solution

Table of values:

x	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	-20	-9	0	7	12	15	16	15	12	7	0	-9	-20

The graph



9.2.2 Solving quadratic equations

Solving a quadratic equation involves finding the values of *x* which satisfy the equation

$$ax^2 + bx + c = 0$$

We can solve this type of an equation

(a) factorization
(b) formula
(c) graph

a) Solving quadratic equation by factorization

This method is based on the fact that if we have two numbers p and q such that $p.q = 0$, then either $p = 0$ or $q = 0$. Similarly, if the quadratic expression of the equation $ax^2 + bx + c = 0$ can be written as a product of its linear factors, either one or both of those linear factors is zero.

Example 16

Solve the quadratic equation $x^2 - 5x + 6 = 0$

Solution

$$(x - 2)(x - 3) = 0$$

=> Either

$$(x - 2) = 0$$
$$=> \; x = 2$$
$$x - 3 = 0$$
$$=> x = 3$$

Example 17

Solve $8x^2 - 2 = 0$

Solution

$$8x^2 - 2 = 0 => 4x^2x - 1 = 0$$
$$=> (2x - 1)(2x + 1) = 0$$
$$=> Either\ (2x - 1) = 0$$
$$=> 2x = 1$$
$$=> x = \frac{1}{2}$$

$$or\ \ 2x + 1 = 0$$
$$=> x = -\frac{1}{2}$$

b) Quadratic formula

Sometimes a quadratic equation may not have simple factors. In that case, the best option is to use a quadratic formula to find the solution to the given quadratic equation.

Given a quadratic equation of the form $ax^2 + bx + c = 0$, the solution is given by

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 18

$$x^2 - 5x + 6 = 0$$

Solution

Relating the equation $x^2 - 5x + 6 = 0$ to the general form we get

$$a = 1, \qquad b = -5 \; and \; c = 6$$

Substituting these values into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We have

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$
$$= \frac{5 \pm \sqrt{1}}{2}$$

Either

$$x = 3$$

Or

$$x = 2$$

c) Using a graph

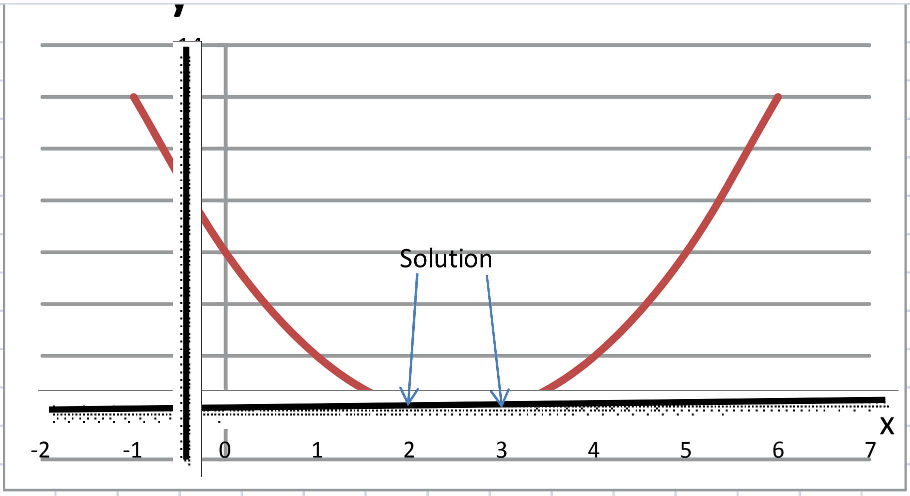
To solve a quadratic equation $ax^2 + bx + c = 0$, plot the graph of $y = ax^2 + bx + c$.

The solution to the quadratic equation are the x-coordinates of the points at which the graph intercepts the x-axis.

Example 19

Find the solution to the quadratic equation using a graphical method.

Solution



The graph cuts the x axis where x = 2 and 3

Therefore x = 2 or 3

CHAPTER OF SUMMARY

In this chapter we have looked at

- Linear and quadratic graphs
- Plotting graphs for linear and quadratic equations
- find the slope of a linear graph and the intercepts
- plot quadratic graphs

END OF CHAPTER EXERCISES

1. In each of the following, identify the slope and the *y*- intercept

   a) $y = 3.5x - 5$

   b) $2y + 3x = 5$

2. Draw a fully labeled graph of $y = 4x + 3$

3. A graph passes though (0,5) and (7,0), find

   a) The slop

   b) The y intercept

c) The value of y when $x = 3$.

4. Determine the y- intercept of the equation $3y + 4x = 12$.

a) Plot the graphs of $y = 14x^2 - 2$ and $y = 12 - 3x$ on the same axes.

b) Find the values of x and y where the two graphs meet.

5. Draw a graph of $y = 8x - x^2$, for values of x from –2 to 10. Show the data table and label the graph properly.

6. Using factorization, solve the quadratic equation $y = x^2 - 4$.

7. Solve the equation: $4x^2 + 3x - 2.5 = 0$ using the quadratic formula.

8. Find the roots of the quadratic equation $x^2 - 5x - 14 = 0$ using

a) Factorization
b) Quadratic formula
c) Graphical method.

CHAPTER 10

INTRODUCTION TO STATISTICS

LEARNING OBJECTIVES

By the end of this chapter,the student should be able to:

i. explain the need for statistics in business environment
ii. understand the various techniques for presenting data
iii. select appropriate data presentation techniques for specific types of data
iv. construct diagrammatic displays of data
v. calculate summary measures/statistics
vi. interpret summary measures

10.0 Introduction

Statistics usually causes anxiety among students because they only associate it with complicated mathematical procedures and formulae. Statistics involves data collection, presentation, analysis and interpretation for effective decision making in business. It is therefore important that you study statistics. We will start our study of statistics by looking at what statistics is, its application in business and basic/common statistical terminologies. After we have introduced statistics, we shall discuss some common techniques for presenting data.

Before looking at the different techniques for presenting data, it is necessary to consider the purpose for which the data was collected. For instant, the data you collected might have been wanted for your company's annual report. A straightforward list of all data values could be presented but, particularly if there were a lot of items, this would not be very helpful and even very boring. Data presentation therefore simplifies large amounts of data, shows key facts and patterns, and display data in an interesting and easily understandable way.

Sometimes it is not enough just to summarise and present data diagrammatically. The last section of this chapter will be dedicated to a discussion of basic numerical analysis of a distribution of data values. These include measures of central location and dispersion collectively referred to as summary statistics.

Definition:

There are at least three definitions of the term statistics,

First statistics can be defined as

a) as a set of figures calculated or tabulated from a collection of data

b) tools and techniques used in the data collection and analysis. This is also referred to as statistical methods

c) further analysis and conclusions drawn. This is sometimes called inferential statistics

d) the science of data. It involves collecting, classifying, summarizing, organizing, analysing, and interpreting numerical information.

Whatever the definition data and its analysis is key. Statistics has an important role in commerce in that it helps in decision making.

10.1 Statistical Applications in Business

Statistics means "numerical descriptions" to most people. Monthly unemployment figures, the failure rate of a new business, and the proportion of female executives in a particular industry all represent statistical descriptions of large sets of data collected on some phenomenon. Often the data are selected from some larger set of data whose characteristics we wish to estimate. We call this selection process *sampling.*

For example, you might collect the ages of a sample of customers at a video store to estimate the average age of *all* customers of the store. Then you could use your estimate to target the store's advertisements to the appropriate age group.

Notice that statistics involves two different processes:

- describing sets of data and
- drawing conclusions (making estimates, decisions, predictions, etc.) about the sets of data based on sampling.

So, the applications of statistics can be divided into two broad areas: *descriptive statistics* and *inferential statistics.*

Definition

Descriptive statistics utilizes numerical and graphical methods to look for patterns in a data set, to summarize the information revealed in a data set, and to present the information in a convenient form.

Definition

Inferential statistics utilizes sample data to make estimates, decisions, predictions, or other generalizations about a population.

10.2 Fundamental Elements of Statistics

Definition

A **population** is a set of units (usually people, objects, transactions, or events) that we are interested in studying.

For example, populations may include:

a) *all* PAEC registered candidates,

b) *all* registered voters in Zomba,

c) *everyone* who has purchased a particular brand of cellular telephone,

d) *all* the cars produced last year by a particular assembly line,

e) the *entire* stock of spare parts at Toyota Malawi's maintenance facility,

f) *all* sales made at the drive-through window of a Noninoni restaurant during a given year, and

g) the set of *all* accidents occurring on a particular stretch of MasaukoChipembere highway during a holiday period.

Notice that the first three population examples (a-c) are sets (groups) of people, the next two (d-e) are sets of objects, the next (f) is a set of transactions, and the last (g) is a set of events. Also notice that each set includes all the units in the population of interest.

In studying a population, we focus on one or more characteristics or properties of the units in the population. We call such characteristics *variables*. For example, we may be interested in the variables age, gender, income, and/or the number of years of education of the people currently unemployed in the United States.

Definition

A **variable** is a characteristic or property of an individual population unit.

The name "variable" is derived from the fact that any particular characteristic may vary among the units in a population.

In studying a particular variable it is helpful to be able to obtain a numerical representation for it. Often, however, numerical representations are not readily available, so the process of measurement plays an important supporting role in statistical studies. **Measurement** is the process we use to assign a value or label of a variables to individual population units. We might, for instance, measure the preference for a food product by asking a consumer to rate the product's taste on a scale from 1 to 10. Or we might measure workforce age by simply asking each worker how old she is.

When we measure a variable for every unit of a population, the result is called a **census** of the population. Typically, however, the populations of interest in most applications are much larger, involving perhaps many thousands or even an infinite number of units.

A reasonable alternative would be to select and study a *subset* (or portion) of the units in the population.

Definition

A **sample** is a subset or a portion of the units of a population.

For example, from the population of PAEC candidates, a sample could be all candidates aged between 18 years and 23 years.

Another example: suppose a company is being audited for invoice errors. Instead of examining all 15,472 invoices produced by the company during a given year, an auditor may select and examine a sample of just 100 invoices. If he is interested in the variable "invoice error status," he would record (measure) the status (error or no error) of each sampled invoice.

Definition

A **statistical inference** is an estimate or prediction or some other generalization about a population based on information from a sample.

That is, we use the information contained in the sample to learn about the larger population." Thus, from the sample of 100 invoices, the auditor may estimate the total number of invoices containing errors in the population of 15,472 invoices.

The auditor's inference about the quality of the firm's invoices can be used in deciding whether to modify the firm's billing operations.

Example 1

A large paint retailer has had numerous complaints from customers about under filled paint cans. As a result, the retailer has begun inspecting incoming shipments of paint from suppliers. Shipments with under fill prob1ems will be returned to the supplier. A recent shipment contained 2,440 gallon-size cans. The retailer sampled 50 cans and weighed each on a scale capable of measuring weight to four decimal places. Properly filled cans weigh 10 pounds.

a) Describe the population.

b) Describe the variable of interest.

c) Describe the sample.

d) Describe the inference.

Solution

a) The population is the set of units of interest to the retailer, which is the shipment of 2,440 cans of paint.

b) The weight of the paint cans is the variable the retailer wishes to evaluate.

c) The sample is a subset of the population. In this case, it is the 50 cans of paint selected by the retailer.

d) The inference of interest involves the *generalization* of the information contained

in the weights of the sample of paint cans to the population of paint cans. In particular, the retailer wants to learn about the extent of the under fill problem (if any) in the population. This might be accomplished by finding the average weight of the cans in the sample and using it to estimate the average weight of the cans in the population.

10.3 Data and Data Sources

10.3.1 Data and Information

Before looking at the sources of data it will be worthwhile defining the term data. A data refers to an item figure which does not make sense on its own. For example on a pay roll days worked is a data item. Only when days worked are combined with says rate of pay can one begin to make some sense out of it. A collection of related which has been processed:

10.3.2 Types of Data

Data can be classified into

a) Quantitative or qualitative data

b) Discrete or continuous

c) Primary or secondary

a) Quantitative data relates to quantifiable items or entities.

Examples of quantitative data

- 243 motor vehicles passing through a sport on the road as recorded by a transport survey enumerator
- Production value of MK4,231,040 in the company books
- Poverty levels of 15% as registered by researchers
- Rainfall of 27mm observed by a met station.

b) Qualitative data are descriptions and opinions. Examples include

- Reasons given for high divorce rates,
- Why is the accounting profession dominated by men or
- Descriptions of buying behaviour of customers.

c) Discrete data relates to countable data. An examples may be number of cars owned by a company

d) Continuous data is characterized by the presence of fractions. An example is a length of 2.35m

Both discrete and continuous data are quantitative.

Primary data is data obtained from an original source for specific purpose or objective. Secondary data is data already collected and published. The use to which a current user puts it may not be the original one.

10.3.3 Sources of Data

Sources of data can be grouped into

a) Primary and Secondary

b) Internal and External

- Primary source is a source that provides original data. Examples are:

i) administrative records
ii) sales and production records
iii) surveys
iv) censuses

- Secondary sources are those that provide data is published data or data collected for a purpose which is different from the one at hand. Examples include

  a) News papers

  b) Magazines

  c) The internet (excluding surveys on the internet)

Internal and External sources

Internal sources are sources within the organization and they include the organizations records. External sources are sources of information from outside the organization. They can be primary or secondary.

10.4 Data presentation

When data has been collected and analysed it must be presented in such a way that the characteristics of whatever it is expressing should be clear to the user. Common data presentation techniques include:

a) Tabulation

b) Graphs

c) Charts (pie chart, bar chart, histogram)

10.4.1 Tabulation

Tabulation involves putting data in tables. A table is an array of numbers arranged in rows and columns. Tabulation condenses a large mass of data and brings out the distinct pattern in a data in an attractive form. It enables comparison to be made easily among classes of data and takes up less space than data presented in narrative form.

Example 2

Consider aales (in K millions) for XYZ Chain Shops in four locations in the country. This is summarized in table form below.

LOCATION OF SHOP	Men Clothing	Womens clothing	Children Clothing	Hardware items	Food items	Total
Blatyre	25	35	34	13	12	119
Zomba	11	14	9	22	7	63
Lilongwe	12	27	29	26	14	108
Mzuzu	10	10	9	11	10	50
ToTal	58	86	81	72	43	340

Important principles of a table

- The table should have an overall title
- Each column and row should have a heading
- Total column and rows should be included
- The table should not contain too many variables (sub headings)

10.4.2 Frequency Distribution

A **frequency** is the number of observations or items or data values that belong to each category or class of data. For qualitative or discrete quantitative data, a frequency is simply a record of how many of each type were present. Consequently, a **Frequency Distribution** can be said to be a grouping of data into mutually exclusive classes showing the number of observations or items in each class.

The main aim of a frequency distribution is to summarise data in a logical manner that enables an overall perspective of data to be obtained quickly. A frequency distribution can be represented in form of a table, a graph or a formula. In this module, however, we shall only look at frequency distributions that take the forms of a table and/or graph.

Frequency distributions can be classified into simple and grouped distributions. While simple distributions involve qualitative and discrete quantitative data, grouped distributions involve continuous quantitative data.

The following table represents a frequency distribution showing the frequency with which some ports of exit were used by departing visitors in 2009.

Port of exit	Number of visitors
Chileka	76,308
Chiponde	52,836
Chitipa/Chisenga	6,816

Dedza	10,916
Kaporo/Songwe	90,350
Lilongwe (KIA)	174,652
Mchinji/Chimaliro	82,304
Muloza	41,203
Mwanza	141,956
Nayuchi	14,759
Nsanje/Marka	17,322
Other	45,609
Total	755,031

Source: 2009 Tourism Report - NSO

From the table above, it is easy to see that visitors departing Malawi frequently used Kamuzu International Airport (KIA) as their port of exit in 2009. A total of 174,652 visitors exited Malawi through Lilongwe (KIA). The simple frequency distribution above can be transformed into a complex distribution by among other things including the reasons for visiting and the gender of the visitor as shown in the following table.

Port of exit	**Reason for visit**							
	Holiday/vacation		**Work/business**		**Visit friends/relatives**		**Conference**	
	Female	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**
Chileka	7226	8353	9378	42638	3587	5022	52	52
Chiponde	10250	28391	2511	8968	1537	1179	0	0
Chitipa/Chisenga	512	769	256	4254	51	974	0	0
Dedza	4049	5688	154	307	462	256	0	0
Kaporo/Songwe	7533	17065	3075	35772	8712	18142	0	51
Lilongwe (KIA)	17117	28084	17527	89837	6560	15221	102	204
Mchinji/Chimaliro	5842	11018	5074	41716	7790	10864	0	0
Muloza	8251	17527	1845	9327	1537	2716	0	0
Mwanza	16758	23523	8866	62010	12351	18448	0	0
Nayuchi	2716	6047	1230	4561	0	205	0	0
Nsanje/Marka	564	2716	1947	9532	1281	1230	0	52
Other	2716	12813	3382	16963	3588	5943	204	0
Total	83534	161994	55245	325884	47456	80200	358	359

Source: 2009 Tourism Report – NSO

Let us now construct our own frequency distribution.

Example 3

A BMS class at an Accountancy College has 42 students. Of these, 19 have a Certificate in Financial Accounting (CIFA) background. There are 4 male students with CIFA background while 15 female students have the CIFA background. In total there are 19 female and 23 male students. Present the data on a table.

Solution

Number of students in a BMS class

	CIFA background	Other	Total
Male	4	15	19
Female	15	8	23
Total	19	23	42

Constructing a Frequency Distribution for Quantitative Data

We can construct a frequency distribution from an array or table of numbers (data vales) using the following steps as a guide:

Step 1:Decide on the number of classes, k

We are supposed to use just enough classes or groups of data values so that we reveal the shape of the distribution. The number of classes depends on the number of observations (n) in the data set. Traditionally, the number of classes must vary from 5 to 20 i.e. $5 \le k \le 20$

A quick hint to decide on the number of classes is the "2 raised to the k rule". The rule states that we must selected the smallest k number of classes such that $2^k > n$

Step 2: Determine the class width or interval

The class interval or width must, in general, be the same for all the classes. The class width is expressed as:

$$i \geq \frac{x_{\max} - x_{\min}}{k}$$

where i is the class interval/width, $x_{\max}$ is the largest or highest data value, $x_{\max}$ is the smallest or lowest data value in the raw data and k is the number of classes.

Step 3: Determine the individual class boundaries

Set the class boundaries such that each data value can be put into only one category. This means we must avoid overlapping class boundaries. For example, the classes 10 – 20 and 15 – 25 should not be used in the same distribution because they are overlapping.

In order to get the classes right, you need to set up the correct lower boundary of the first or lowest class. The lower boundary for the first class can be the lowest data value ($x_{\min}$) in the data set or any number slightly lower than the lowest value. For instance, if the lowest data value is 21 then the lower boundary of the first class can be 21 or any number slightly lower that 21 e.g. 20. The rest of the class boundaries are then determined based on this lower boundary.

Note that as you decide on the classes they will be adjustments.Ensure that figures used for class boundaries are numbers which are easy to work with e.g.numbers divisible by 2, 5 or 10.

Step 4: Tally the data values

Use tallies to allocate counts of data values for each class.

Step 5: Count the number of items

Count the tally marks in each class to obtain the ***class frequencies***. The frequency distribution/table can then be rewritten so that it is presented without the tally marks.

Example 4

A fisherman using a line and rod recorded his catch per day for 50 days and his records are as following.

23	27	24	7	28	15	25	29	15	46
5	9	12	10	17	22	23	17	16	32
11	12	18	20	38	13	27	38	18	22
22	20	13	14	26	14	19	19	40	31
33	17	34	21	23	26	18	14	21	27

Construct a frequency distribution for these data.

Solution

Step 1: Number of classes

Since there are $n = 50$ data values, we need to have k classes such that $2^k \geq 50$. For $k = 5$, $2^5 = 32$ which is less than 50. Consequently, 5 classes are not enough for this data set. If we set $k = 6$, then $2^6 = 64$, which is greater than 50. So we recommend $k = 6$ classes for the data set.

Step 2: Class width/interval

Since the lowest catch/day is 5 and the highest catch/day is 46, and that we need 6 classes, the class width should be at least $i = \frac{46-5}{6} = 6.8$. Usually, such a class width would be rounded up to some convenient number such as the next integer, a multiple of 5, 10 or 100. In our case, we round 6.8 to 7.

Step 3: Individual class boundaries

Since the lowest catch/day is 5, the lowest boundary for the first class can be 5 or any number slightly smaller than 5 such as 4 and 3. We are of the view that '5' is much easier to work with, so we set the lower boundary of the first class at 5. Consequently, here are the 6 classes for this data set:
5 – 12, 12 – 19, 19 – 26, 26 – 33, 33 – 40, 40 – 47

Step 4: Tallying

Daily catch	Tallies
5 – 12	~~////~~
12 – 19	~~////~~~~////~~~~////~~ /
19 – 26	~~////~~~~////~~ ////
26 – 33	~~////~~ ////
33 – 40	////
40 – 47	//

Note: Each class includes the lower boundary but not the upper boundary. For example, the data value 26 is in the class 26 – 33 and not in the class 19 – 26.

Step 5: Counting the number of items or tallies to obtain the class frequencies

Daily catch	Frequency
5 – 12	5
12 – 19	16
19 – 26	14
26 – 33	9

33 – 40	4
40 – 47	2

General Forms of Frequency Distributions

The following examples show typical forms of frequency distributions. Each example tries to depict a special feature that one may find in a frequency distribution.

i) A simple frequency distribution shows singles values and their frequencies (counts), and it is useful when summarizing simple discrete data over a limited range.

x	**f**
10	2
14	7
17	8
21	6
25	3
30	5

ii) The following frequency distribution shows the number of orders that a company received per week over a 40-weeks period. Notice that the classes are continuous and of equal width

Number of order	**Number of weeks (f)**
5 but less than 10	1
10 but less than 15	5
15 but less than 20	7
20 but less than 25	10
25 but less than 30	7
30 but less than 35	4
35 but less than 40	3

iii) Here the classes are not continuous. There are gaps for example between 14 and 15, between 19 and 20, etc. These are known as class limits and not boundaries. This is typical for a frequency distribution where data is discrete.

Age (years)	Number of students
10 – 14	50
15 – 19	80
20 – 24	45
25 – 29	12
30 – 34	10

We can transform the class limits to boundaries by simply averaging the adjacent limits. The class boundary between the classes 10 – 14 and 15 – 19 would be the average of 14 and 15, i.e. $\frac{14+15}{2} = 14.5$, and the boundary between the classes 15 – 19 and 20 – 24 would be the average of 19 and 20 i.e. $\frac{19+20}{2} = 20.5$.

We can therefore represent the distribution above using class boundaries as shown below:

Age (years)	Number of students
9.5 – 14.5	50
14.5 – 19.5	80
19.5 – 24.5	45
24.5 – 29.5	12
29.5 – 34.5	10

Note: The classes are now continuous

iv) The classes in the following distribution do not have equal width. In this case, the width will depend on the requirements of the individual presenting the data.

Days	f
0 – 10	8
10 – 15	12
15 – 20	10
20 – 22	16
22 – 25	11

v) This distribution has two open ended classes: under 100 and 250 and over. Open ended classes are used when data at the ends of the distribution are too widely and sparsely spread.

Income (in K'000)	Number of employees
Under 100	116

100 and under 120	89
120 and under 150	184
150 and under 170	142
170 and under 200	159
200 and under 250	124
250 and over	124

10.4.3 Class Intervals/width and Midpoints

In this module, the terms 'class interval/width' and 'class midpoints' are used frequently. You need, therefore, to have a clear understand of the two. The **class midpoint or class mark** is halfway the between the lower and upper boundaries (or limits) of the same class. It is computed by adding the lower and upper boundaries and dividing the results by 2. Refer to the fisherman's example, the midpoint for first class (5 – 12) with 5 as the lower boundary and 12 as its upper boundary is 8.5 found by $(5+12)\div 2$. The rest of the midpoints are:

Daily catch	F	Midpoint, x	Midpoint found by
5 – 12	5	8.5	$(5+12)\div 2$
12 – 19	16	15.5	$(12+19)\div 2$
19 – 26	14	22.5	$(19+26)\div 2$
26 – 33	9	29.5	$(26+33)\div 2$
33 – 40	4	36.5	$(33+40)\div 2$
40 – 47	2	43.5	$(40+47)\div 2$

To determine the class interval/width, subtract the lower class boundary from the upper class boundary. We can also determine the class interval/width by finding the difference between consecutive midpoints. The class interval for the fisherman's example was determined as 7, which we find by subtracting the lower boundary of the first class, 5, from the upper boundary of the same class, 12 i.e. 12 – 5 = 5. Alternatively, since the midpoint of the first class is 8.5 and the midpoint of the second class is 15.5, then the class width is 15.5 – 8.5 = 7.

10.4.4 Relative and Cumulative Frequencies

It may be desirable to convert frequencies to relative and/or cumulative frequencies. A **relative frequency** is the ratio of the class frequency to the total number of observations. It shows the fraction (or percentage) of the total number of data values in each class.

Mathematically, a relative frequency of each class/data value is defined as:

$$\frac{Frequency\ of\ a\ class\ or\ data\ value}{Sum\ of\ the\ frequencies} = \frac{f}{\sum f}$$

In our fisherman example, we may want to show the proportion of days on which specific intervals of fish catch were recorded as shown below:

Daily catch	Frequency	Relative frequency	Found by
5 – 12	5	0.10	$5 \div 50$
12 – 19	16	0.32	$16 \div 50$
19 – 26	14	0.28	$14 \div 50$
26 – 33	9	0.18	$9 \div 50$
33 – 40	4	0.08	$4 \div 50$
40 – 47	2	0.04	$2 \div 50$
Total	50	1	

Note: We can convert the relative frequencies to percentages simply by multiplying them by 100 (%).

Definition: Less-than and more-than cumulative frequencies

A **less-than cumulative frequency** is the number of data values/observations that are less than a specific data value fall or below the upper boundary of a specific class in case of grouped frequency distribution. The less-than cumulative frequency is the default cumulative frequency. However, **a more-than cumulative frequency** is the number of data values/observations that are equal to or greater than a specific data value or the lower boundary of specific class in case of grouped frequency distribution.
Referring to our fisherman example, we may want to show the number of days on which he recorded less than 19 fishes. In this case we would simply add 5 and 16 to give us 21 days. And for less than 26 fishes, we add 5, 16 and 14 to get 35 days. Furthermore, if we wanted the number of days on which he recorded either equal to or more that 26 fishes, we would simply add 9, 4 and 2 to get 15 days. The cumulative frequencies for the remaining classes are shown in the table below:

Less-than cumulative frequency distribution

Daily catch	Frequency F	Less-than cumulative, F	Found by
5 – 12	5	5	5
12 – 19	16	21	5+16

19 – 26	14	35	5+16+14
26 – 33	9	44	5+16+14+9
33 – 40	4	48	5+16+14+9+4
40 – 47	2	50	5+16+14+9+4+2

More-than cumulative frequency distribution

Daily catch	Frequency F	Less-than cumulative, F	Found by
5 – 12	5	50	2+4+9+14+16+5
12 – 19	16	45	2+4+9+14+16
19 – 26	14	29	2+4+9+14
26 – 33	9	15	2+4+9
33 – 40	4	6	2+4
40 – 47	2	2	2

Advantages of Tabular Presentation of Data

- Enhanced level of accuracy and precision
- Data can be presented in more than one dimension (more than one variable). For example in Table 10.2, we have the following variables presented: Port of exit (name), reason for visit and gender of the visitors.

Disadvantage of a Table

- Tables lack visual impression of the distribution
- The figures may be cumbersome (especially in large tables)

10.5 Graphical and Diagrammatic Presentation of Frequency Distribution

Managers and other busy individuals often need a quick picture of the trend in variables of interest such as sales, expenditure, cost, revenue, and profit. These trends are usually easy to depict using charts and graphs. A graph can be defined as a chart or diagram that shows the relationship of sets of related data. A graph is normally drawn with two lines (called the axes) intersecting at right angles. One is vertical and it represents one set of the data to a chosen scale and the other is horizontal again representing the other set of dat. The axes are referred to (usually) as the Y and X axes respectively.

10.5.1 Uses of Graphs

There are three main uses of graph.

a) Graphs are used to represent comparative (in relation to period) data on production, sales, revenue, taxes and returns among others.

b) Graphs can be used to convert data from one measure such as weight to another for instance value.

c) Graphs can be used or show of calculate the rate of change of one set of data relative to the other set.

10.5.2 Pictograms

A pictogram is a technique where pictures or symbols are used to represent data. For example, if 500 houses were built in Ndirande and 250 houses in Zolozolo, then using a scale of 1 picture of a house to 50 houses, we can represent the information as follows:

Ndirande:
Zolozolo:

Key:  = 50 houses

Example 5

A survey was carried out to find the number of school buses operating in the major towns in Malawi. The results are presented in the distribution below. Present the distribution using a pictogram.

Town	Number of school buses
Blantyre	200
Lilongwe	150
Zomba	50
Mzuzu	20

Solution:

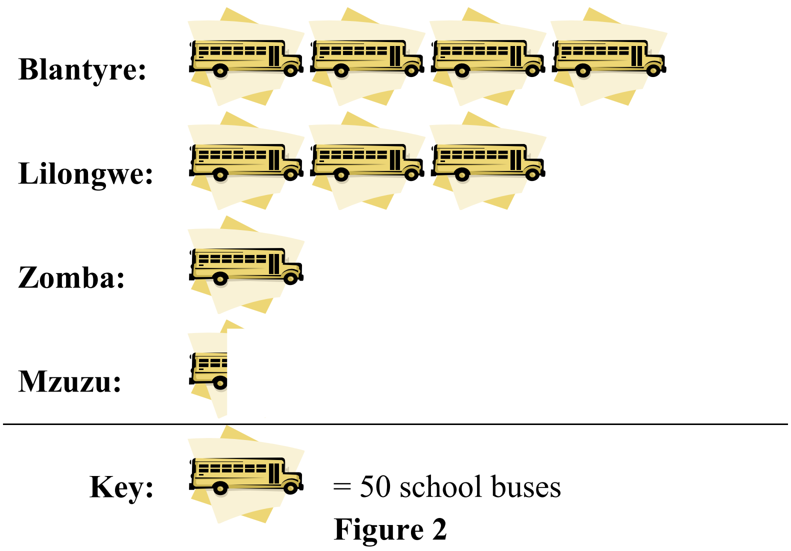


Figure 2

Advantage of a Pictogram

- A pictogram provides a quick visual impression of the data (magnitude)

Disadvantages of a Pictogram

- It lacks precision specially interpreting fractions of the pictures/symbols used. For example, the immediate problem that comes out is how to express the 20 buses in Mzuzu in the above problem.
- The technique has limited dimensions in which data can be presented.

10.5.3 Bar Chart

A bar chart or bar graph is a chart with non-joining rectangular bars. Data is represented by the bars and the lengths or heights are proportional to the values or frequencies that they represent. The bars can be plotted vertically or horizontally.

There are three common forms of bar charts namely:

a) The simple bar chart
b) The multiple or compound bar chart
c) The component or stacked bar chart

How to Construct a Bar Chart

- Place the data categories on the x-axis (no scaling is required)

- Place the frequencies/counts/percentages on y-axis and must be scaled accordingly. The height or length of each bar will show the frequency/percentage/count for each category.
- The bars must be equal equally spaced and of equal width.

a) The Simple Bar Chart

A simple bar chart presents data only in two dimensions.

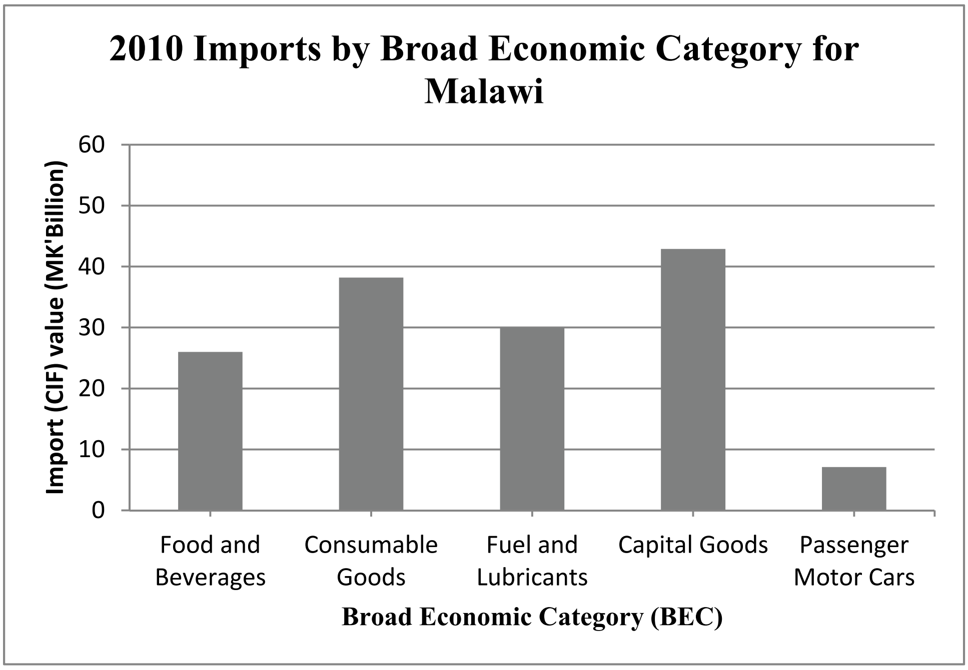
We typify the construction of a simple bar chart using the information in the table below on the levels of imports into Malawi in 2010.

Malawi's imports by Broad Economic Category (BEC) in 2010

BEC description	**CIF Value (MK'Billion)**
Food and beverages	26.0
Consumable goods	38.2
Fuel and lubricants	30.0
Capital goods	42.9
Passenger motor cars	7.1
Parts and accessories	8.2
Total	**152.4**

Source: Annual Statistics Trade Report 2010

To construct the bar chart, we shall place BEC item descriptions on the x-axis and the CIF values in the y-axis. The bar graph is presented below.



With this chart it is easy to see which BEC registered the highest CIF value in 2010. We can also see that CIF value for consumable goods was more than four times the CIF value for passenger cars in 2010.

b) Multiple or Compound Bar Chart

A multiple bar chart is similar to a simple bar chart only that it does show more than one aspect of the data/variable. In a multiple bar chart, each bar represents a specific of the major category of a variable.

Example 6

The following data shows the annual external trade values for Malawi from 2005 to 2011.

Annual external trade values for Malawi

Year	Trade values (MK'Billion)	
	Exports	**Imports**
2005	59.6	138.8
2006	93.8	174.6
2007	122.2	194.6
2008	123.6	285.8
2009	168.0	298.2

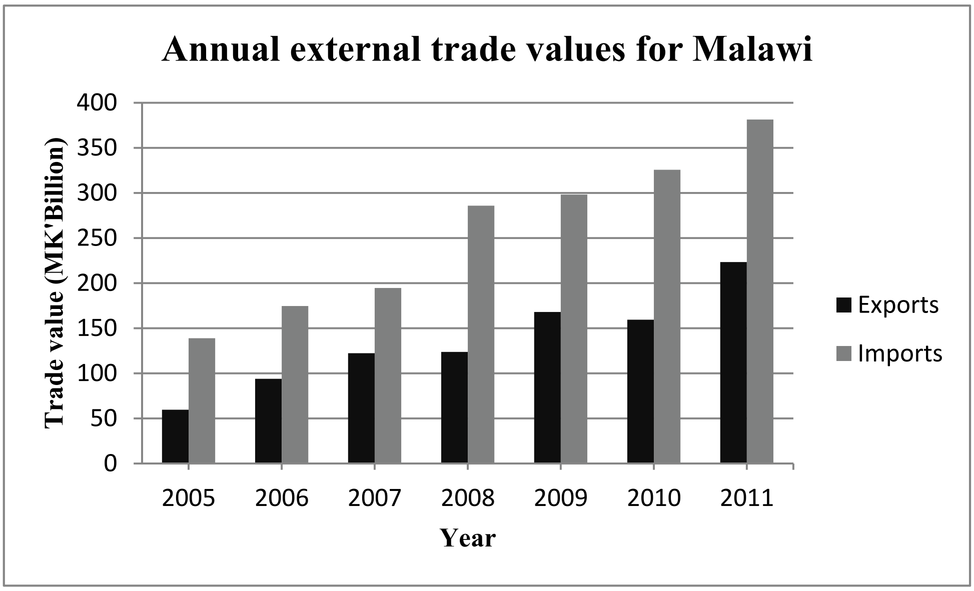
2010	159.4	325.6
2011	223.3	381.3

Source: NSO-July/December 2012 Trade Brief

Present the trade values on a multiple or compound bar chart.

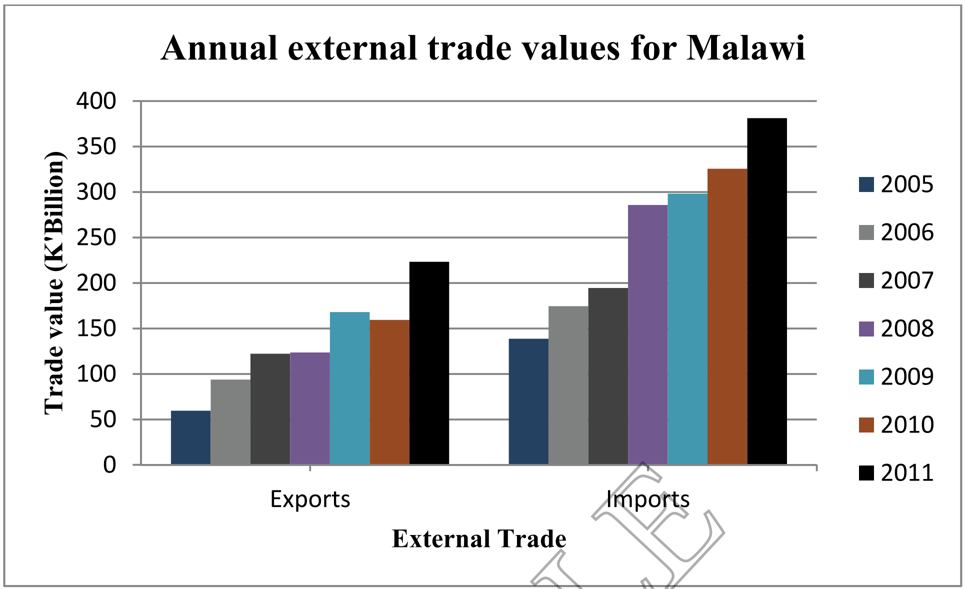
Solution

We proceed by constructing the bar chart clustered by year (i.e. years appear along the x-axis) and the charts is presented below.



One can tell from the chart that Malawi had been importing more than it had been export over the period 2005-2011.

It is also possible to construct a multiple bar chart clustered by trade type. In this case we shall put the trade in the x-axis. The bar chart would look like the one below:



We can tell from the chart that while exports had generally increased from 2005 to 2011, imports into Malawi had drastically increased over the same period.

c) Component Bar Chart

A component or stacked bar chart is more like the simple bar chart except that each bar, while representing the total (frequency/count/value), it is split up into the specific components of the variable category which are stacked on top of one another in the same order. For instance, if the bars are representing the number of students in a class, the bars would be split up show the number of male and female students in the class.

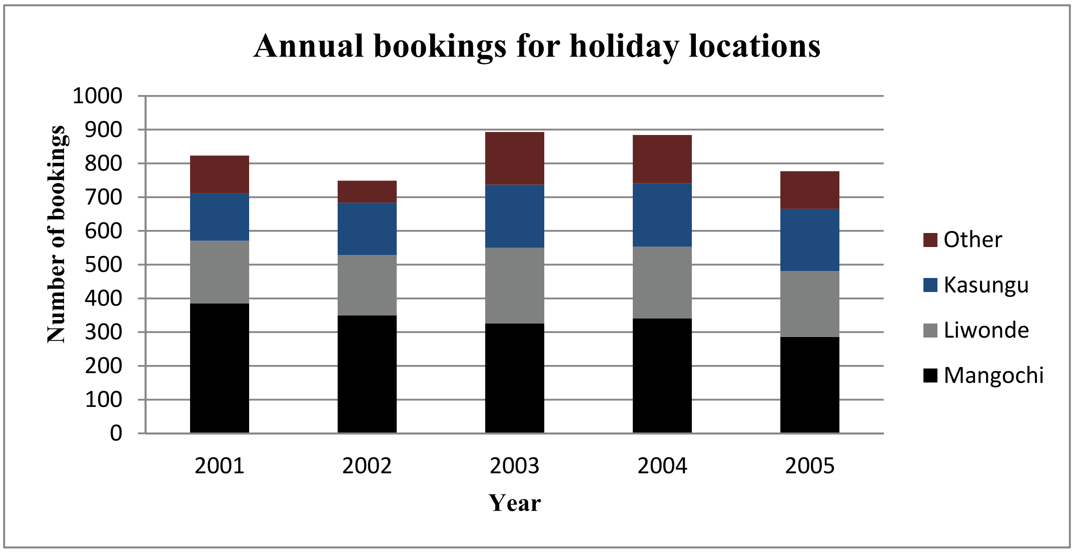
The following table shows holiday locations booked through a travel agent in Malawi.

Number of bookings for holiday locations

Holiday location	Annual bookings				
	2001	**2002**	**2003**	**2004**	**2005**
Mangochi	385	350	326	341	286
Liwonde	186	178	224	212	195
Kasungu	140	156	187	188	184
Other	112	65	156	143	112
Total	**823**	**749**	**893**	**884**	**777**

We shall now proceed to construct a component bar chart for the data clustered by year and leave for your practice to construct a component bar chart clustered by holiday location.

The first step in constructing component bar chart is to find the totals. Luckily, the totals for each year (since we are clustering by year) are already provided in the table above. We place the years in the x-axis and number of bookings in the y-axis, scaled according. The component bar chart is presented below.



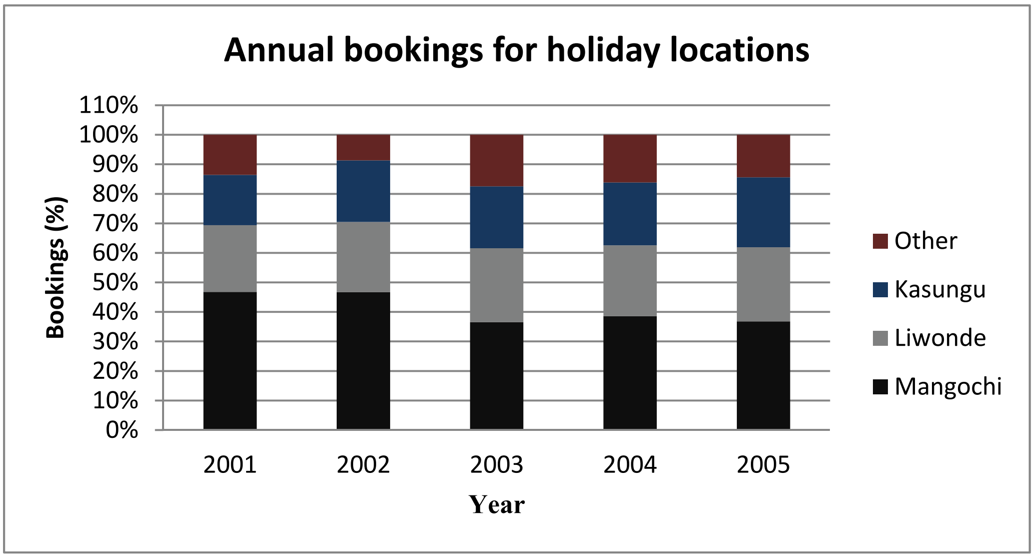
Note that for each year the four segments of the bar are stacked together to give the total number of bookings for each particular year. For 2001, the bars for Other (112), Kasungu (140) and Liwonde (186) are stacked onto the Mangochi bar (385) resulting into a combined bar for 2001 whose height/length is 823.

Quite often component bar charts are presented in terms of percentages. Unlike the absolute component bar chart we just constructed which requires the use of raw data frequencies, to construct a percentage bar chart we need to express the composition for each year as percentage of the total for that year. We explain the process of constructing a percentage component bar chart using the information on number of holiday bookings from 2001 to 2005 seen earlier.

The first step in constructing a percentage component chart is to express the components as a percentage of the total (for each year since we are clustering by year). For example, the Mangochi bookings are $385 \div 823 \times 100 = 46.7 \approx 47\%$ of 2001 total. In 2005, Kasungu bookings are $184 \div 777 \times 100 = 23.7 \approx 24\%$ of the total. The rest of the percentages are given in the following table.

Location	2001		2002		2003		2004		2005	
	Raw	%	Raw	%	Raw	%	Raw	%	Raw	%
Mangochi	385	47	350	47	326	37	341	39	286	37
Liwonde	186	23	178	24	224	25	212	24	195	25
Kasungu	140	17	156	21	187	21	188	21	184	24
Other	112	14	65	9	156	17	143	16	112	14
Total	823	100	749	100	893	100	884	100	777	100

We then construct a component bar chart just as before but this time around we will have percentages in the y-axis instead of raw number of bookings.



The chart shows that where asMangochi had above 40% in 2001 and 2002 of the total number of bookings, its number of bookings dropped in percentage terms while that of Kasungu and Liwonde gained prominence in the subsequent years.

10.5.4 Pie Chart

A pie chart is normally useful for illustrating nominal level variables or data. In a pie chart data is presented on sectors of a circle. The size of a particular sector is proportional to the magnitude of a data item or the frequency of a data class. The proportionality is determined by assigning an appropriate angle at the centre of the circle.

How to construct a Pie Chart

Step 1: Find angles

Convert the data frequencies/values to angular measurements. This is done by dividing each frequency/value by their sum and then multiplying by 360°

Step 2: Draw a circle (pie)

Draw a circle of reasonable radius and divide it into segments/sectors proportional to the frequencies (or counts) as measured by the angles. It is expected that the sum of the sectors or angular measurements will be 360° (or 100%)

Step 3: Shade the sectors or segments different and these must be explained in a “legend” or “key”

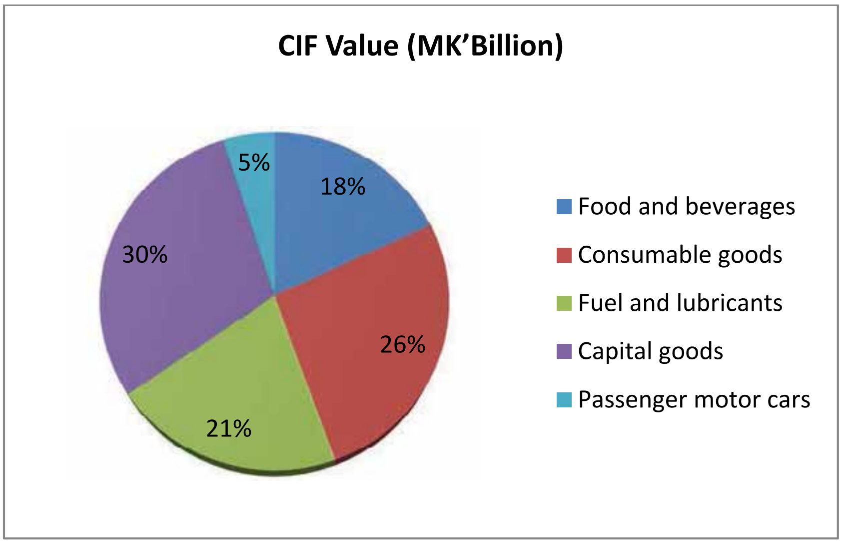
We reproduce the 2010 Malawi imports data from Table 13.3. Our intention is to construct a pie chart for the data.

BEC description	**CIF Value (MK’Billion)**
Food and beverages	26.0
Consumable goods	38.2
Fuel and lubricants	30.0
Capital goods	42.9
Passenger motor cars	7.1
Parts and accessories	8.2
Total	**152.4**

The first step is to convert the CIF values to angles.

BEC description	**Value**	**Angle at the centre**
Food and beverages	26.0	$\frac{26}{152.4} \times 360^\circ \approx 61.4^\circ$
Consumable goods	38.2	$\frac{38.2}{152.4} \times 360^\circ \approx 90.2$
Fuel and lubricants	30.0	$\frac{30}{152.4} \times 360^\circ \approx 70.9^\circ$
Capital goods	42.9	$\frac{42.9}{152.4} \times 360^\circ \approx 101.3^\circ$
Passenger motor cars	7.1	$\frac{7.1}{152.4} \times 360^\circ \approx 16.8^\circ$
Parts and accessories	8.2	$\frac{8.2}{152.4} \times 360^\circ \approx 19.4^\circ$
Total	**152.4**	$\mathbf{360^\circ}$

The second step involves using a compass to draw a circle and a protractor to measure and assign angles at the centre of the circle. And finally the in third step you shade/colour the pie chart sectors. The final pie chart is as follows:



Because the area of the pie represents the relative share of each import item, we can easily compare them. From the pie chart, it is clear that capital goods accounted for the largest proportion of imports for Malawi in 2010. 'Passenger car' category was the least at 5% of the five broad economic categories in terms of their CIF values.

Advantages:

- A pie chart can give a visual impression of the comparative sizes of the components
- A pie chart is relatively easy to understand
- Much as it is meant to provide a comparative picture of the components of the data, a pie chart can have actual data and/or percentages embedded in the diagram to show magnitudes

Disadvantages:

- Lack of accuracy in general if figures are not embedded
- Limited dimensions in which data can be presented since data can only be presented in one dimension

10.5.5 Histogram

A histogram is one of the most common ways of presenting grouped frequency distributions pictorially. A histogram is very similar to a bar chart. The difference is that while the bar chart has spaces between bars, the bars in a histogram are drawn adjacent to each other.

Example 7

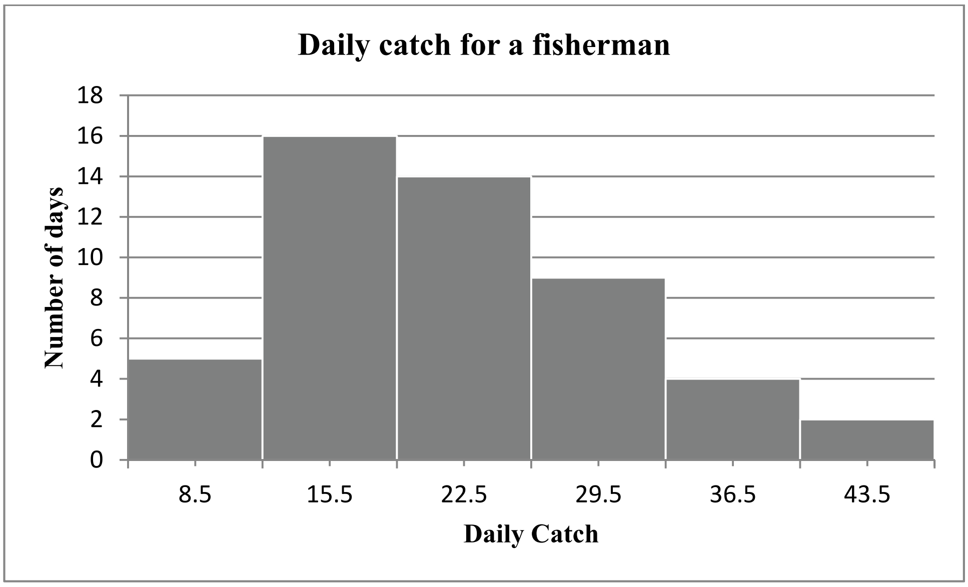
We illustrate the construction of a histogram by recalling the frequency distribution in our fisherman example. Below is the frequency distribution.

Daily catch	Number of days
5 – 12	5
12 – 19	16
19 – 26	14
26 – 33	9
33 – 40	4
40 – 47	2
Total	50

Construct a histogram for the distribution

Solution

Since all the class intervals are then same, therefore the frequencies (number of days) will be represented by the heights of the bars. So we proceed by placing the daily catch values along the x-axis scaled to ensure that it fits the values from 5 to 47 and the frequencies are scaled along the y-axis.



Points to note on the drawing of a histogram

- Each bar represents just one class, the bar width corresponds to the class width i.e. each bar extends from the lower boundary to the upper boundary of the class.
- The bars are joined together (i.e. the values on the x-axis should be continuous)
- If class width vary, then the areas of the bars (width xheight) corresponds to the class frequencies. In this case the height of bar, referred to as the frequency density, is given by.

$$\boxed{\frac{frequency}{Width} = \frac{f}{i}}$$

Example 8

An accountant of a progressive tea estate has gathered data on wages paid to tea pickers in the month and has presented the data on a frequency distribution as follows

Wage (MK'000)	Number of pickers
10 – 15	20
15 – 18	12
18 – 25	35
25 – 30	27
30 – 40	8
40 and over	5

Construct a histogram for the distribution.

Solution

Note that the last class is open-ended and that the class widths are not equal. The open ended class is dealt with by assigning it the most common width or the width of the preceding class. In our case we assign it the class width of 5 and the class becomes 40 45.

The presence of unequal class width requires that we use frequency densities for the heights of the bars. The frequency densities are calculated in the table below.

Wage (MK'000)	f	Frequency density, $\frac{f}{i}$
10 – 15	20	$20 \div 5 = 4$
15 – 18	12	$12 \div 3 = 4$
18 – 25	35	$35 \div 7 = 5$
25 – 30	27	$27 \div 5 = 5.4$
30 – 40	8	$8 \div 10 = 0.8$
40 – 45	5	$5 \div 5 = 1$

Here is the histogram for the distribution

Example 9

Below is age frequency distribution for students from Tayamba Pvt. Secondary School. Represent the distribution on a histogram.

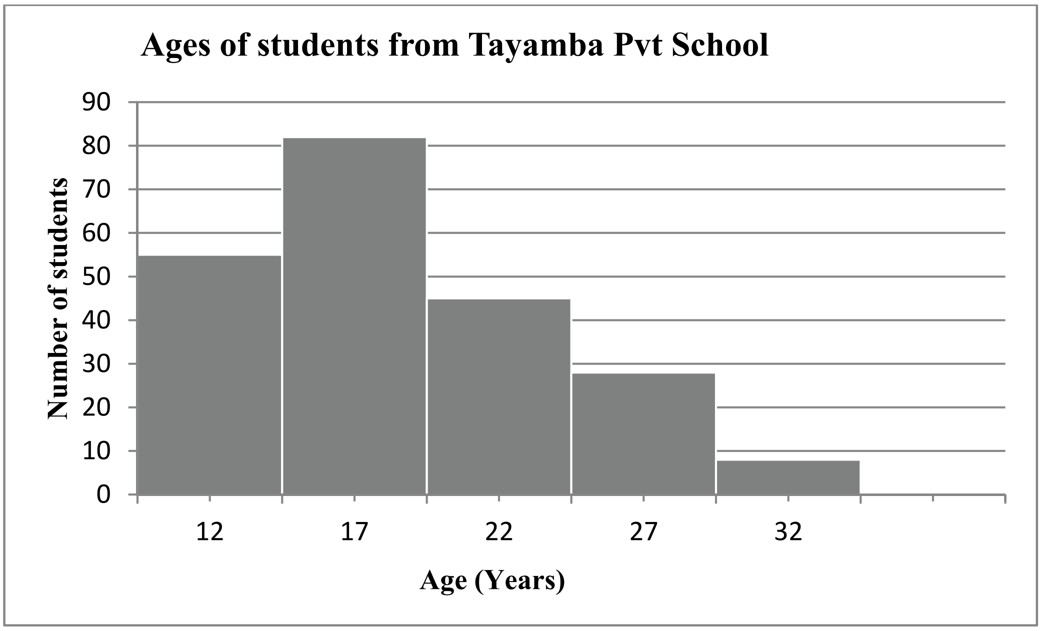
Age (years)	Number of students
10 – 14	55
15 – 19	82
20 – 24	45
25 – 29	28
30 – 34	8
Total	218

Solution

This is case data classes are not continuous. We need therefore to close the gaps between the age groups by converting the class limits into boundaries (see section ____) as shown below

Age (years)	Boundaries	Number of students
10 – 14	**9.5 – 14.5**	55
15 – 19	**14.5 – 19.5**	82
20 – 24	**19.5 – 24.5**	45
25 – 29	**24.5 – 29.5**	28
30 – 34	**29.5 – 34.5**	8
Total		218

The histogram is presented below.



Although a histogram provides a strong visual appeal, one cannot read the exact data values since data values are grouped.

10.5.6 Frequency Polygon

A frequency polygon is similar to a histogram. It consists of line segments connecting the points formed by the intersections of the class marks (midpoints) and the class frequencies i.e. connecting the top centres of the bars in a histogram.

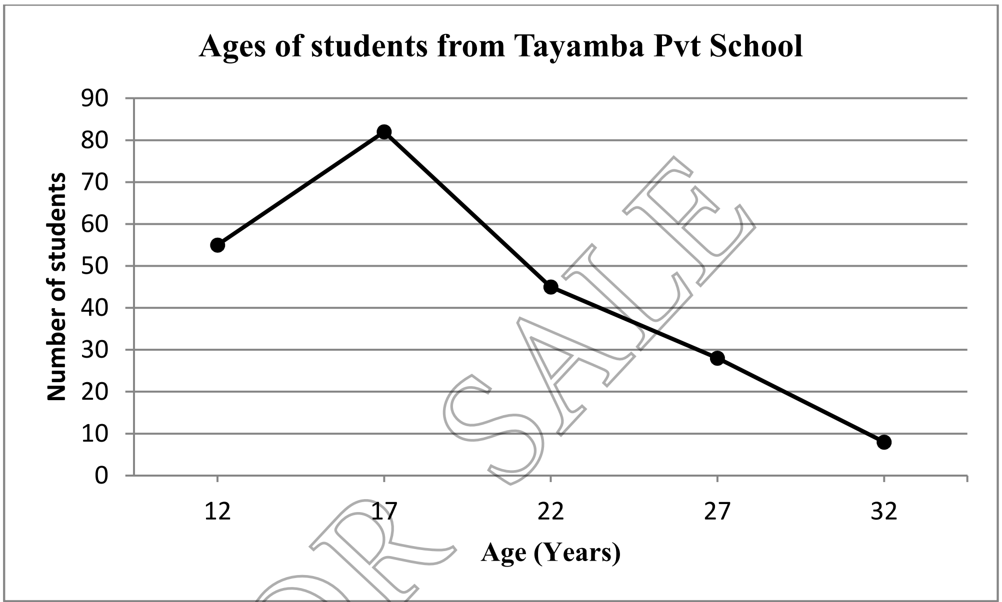
To construct a frequency polygon, we scale the class midpoints along the X-axis and the frequencies along the Y-axis. We illustrate the construction of the frequency polygon using the data from the previous example. The data is reproduced below.

Age (years)	Number of students
10 – 14	55
15 – 19	82
20 – 24	45
25 – 29	28
30 – 34	8
Total	218

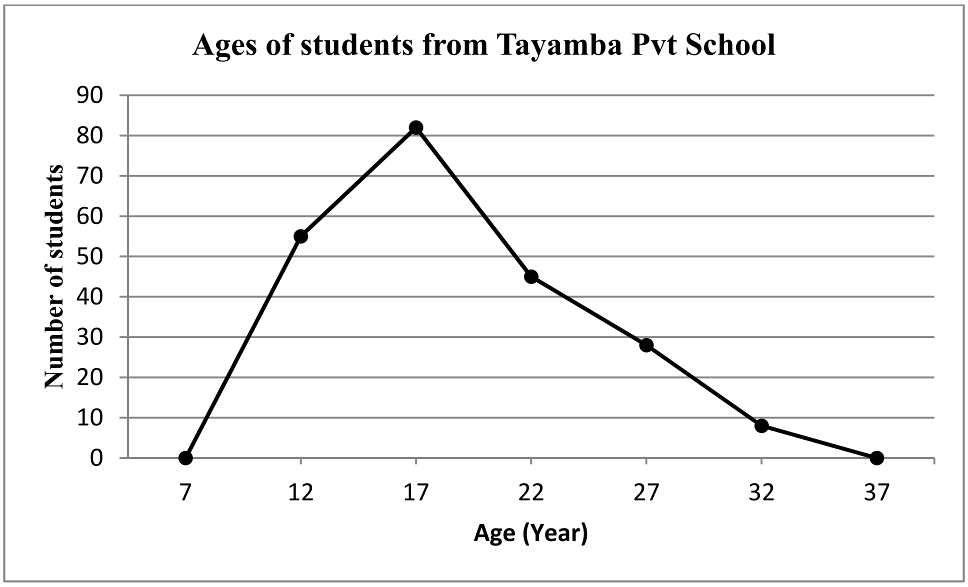
To construct the frequency polygon we first need to find the class midpoint. The midpoints are given in the following table.

Age (years)	**Midpoints**	Number of students
10 – 14	**12**	55
15 – 19	**17**	82
20 – 24	**22**	45
25 – 29	**27**	28
30 – 34	**32**	8
Total		218

Next we plot the midpoints and their corresponding frequencies on the Cartesian (X-Y) plane. The points to be plotted have the following coordinates (12,55), (17,82), (22,45), (27,28) and (32,8). The points are then connected in order as shown below



Oops! This does look like a polygon. In order to complete the frequency polygon, the ends must be connected to the x-axis at zero frequencies. To do this, add two imaginary classes, one at either end, and connected the ends of the line graph to the X-axis at their midpoints. In our case the classes are 5 – 9 (midpoint is 7) and 35 – 39 (midpoint is 37). The ends of the line graph will then be connect to the points (7,0) and (37,0) to complete our frequency polygon as shown below.



Both the histogram and frequency polygon allow us to get a quick picture of the main characteristics of the data i.e. highs, lows, points of concentration etc. However, the frequency polygon has an advantage over the histogram in the sense that it allows for direct comparison of two or more distribution.

10.5.7 Ogive

An ogive is a cumulative frequency graph. There are two forms of an ogive and these are: The "less than" o give and the "more than" ogive. We illustrate the steps followed in constructing an ogive using this example.

Example 10

We shall once again use the data on the daily catches of fish. The distribution is below:

Daily catch	Number of days
5 – 12	5
12 – 19	16
19 – 26	14
26 – 33	9
33 – 40	4
40 – 47	2
Total	50

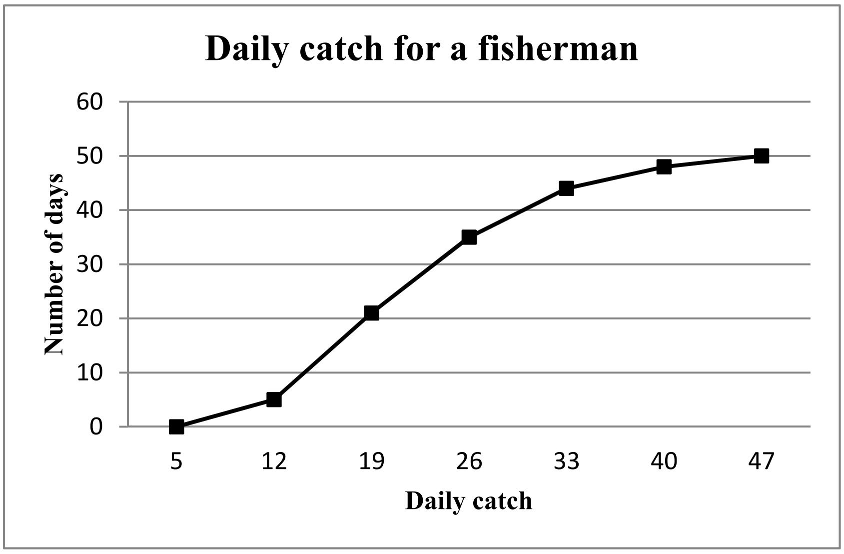
Construct less-than and more-than ogives.

Solution:

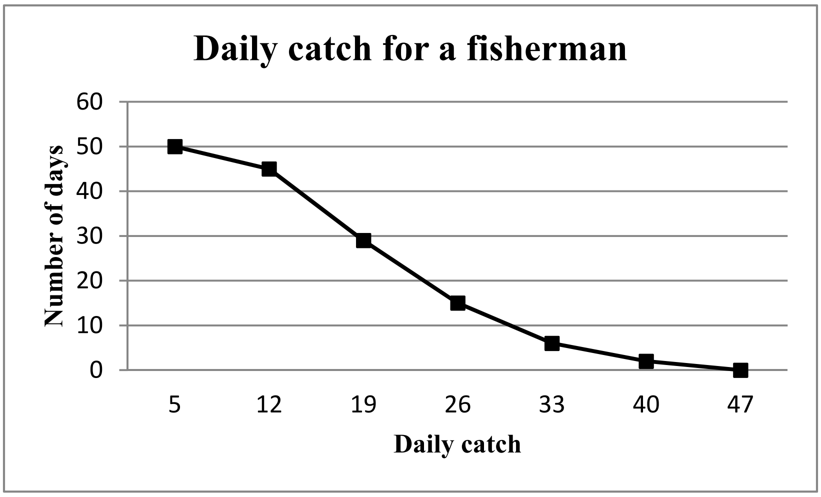
As the description of an ogive suggests, an ogive requires cumulative frequencies. We present the cumulative frequencies in the table below.

Daily catch	f	Less-than cum freq F	More-than cum freq, F
5 – 12	5	5	50
12 – 19	16	21	45
19 – 26	14	35	29
26 – 33	9	44	15
33 – 40	4	48	6
40 – 47	2	50	2
Total	50		

The next step is to plot the points. For a less-than ogive, we plot the class upper boundaries and the corresponding cumulative frequencies. In our case the first plot is at (12,5) and the next is (19,21). The rest of the points follow in this order (26,35), (33,44), (40,48) and (47,50). Once plotted the points are connected to produce a less-than ogive. To complete the less-than ogive, the lower suspended end is connected to the X-axis at the lower boundary of the first class. A completed less-than ogive is given below.



For a more-than ogive, we plot the class lower boundaries and the corresponding more-than cumulative frequencies. In our case the first plot would be at (5, 50) and the next is (12,45). The rest of the points follow in this order (19,29), (33,6), and (40,2). Once plotted the points are connected to produce a more-than ogive. To complete the more-than ogive, the lower suspended end is connected to the X-axis at the upper boundary of the last class. A completed more-than ogive is given below.



10.6 Summary Measures

Data analysis is both a vast and complex area and it therefore involves many and complicated statistical tools. However, basically it is concerned with making judgement and conclusions, from the data and thus necessitates calculation of some statistics that help describe what the data portrays. This manual considers two categories of the statistics: of measures of central tendency and measures of dispersion.Common measures of central location are the mean, mode and median.These measures indicate some impression of the magnitude of all the items in the data set.Measures of dispersion give some idea of the degree of variability of a variable. The main measures of variation to be considered in this chapter are the range, standard deviation, and variance.

10.6.1 The Mode

The mode of a distribution of figures is the figure that occurs most frequently or the figure with the highest frequency of occurrence

Example 11

i) Find the mode of the following data 7,9,6,4,6,5,8,10.

ii) Find the mode of the following data 5,4,7,8,7,9,9,10,12.

Solution:

i) Mode is 6

ii) The data has actually 2 modes 7 and 9. This kind of data is referred to as bimodal.

The mode is easy to find and it can be determined by inspection. However as an, average, it has the problem of sometimes not being unique. Further it cannot be used in further analysis of data.

The mode can also be determined from a simple frequency distribution. Consider the following example.

Example 12

Consider the simple frequency distribution given below and find the mode.

x	f
0	2
1	3
2	7
3	**8**
4	6
5	4

Solution

The mode is 3 (item with the highest frequency of **8**)

10.6.2 The Median

This is the middle most value in a ranked or ordered distribution. If the number of data items n is odd then the median is the $\frac{(n+1)}{2}$ thvalue or item If on the other hand n is even the median is the average or mean of the two most central values namely the $\frac{n}{2}$ thand the $\left(\frac{n}{2}+1\right)$th items.

Example 13

Calculate the median for each of the following sets of data.

i) 3, 7,8,2,4,5, 7
ii) 10,12,9,11,17,14,16, 13

Solution:

i) Rearranging the data:

2,3,4,5,7,7,8

Since n = 7 (odd),

$\therefore$the median is the $\frac{(n+1)}{2} = \frac{7+1}{2} = 4$ -th data value = 5

ii) Rearranging the data values:

9,10,11,12,13,14,16,17

Since n = 8 (even)

$\therefore$the median is the mean of the $\frac{n}{2}$ thand the $\left(\frac{n}{2}+1\right)$th items

$= \text{mean of the } \frac{8}{2} = 4$ -th data value and $\frac{8}{2}+1 = 5$ -th data value

$= \text{mean of } 4^{th} \text{ and } 5^{th} \text{ value} = \frac{12+13}{2} = 12.5$

Median for a simple frequency distribution

We exemplify how to find the median for a frequency distribution using the following example:

Example 14

Consider the following simple frequency distribution

X	0	1	2	3	4	5	6
F	2	4	6	10	7	4	1

Find the median.

Solution

Step 1: We need to obtain the less-than cumulative frequencies (F) for the data values. These are given below:

X	0	1	2	3	4	5	6
f	2	4	6	10	7	4	1
F	**2**	**6**	**12**	**22**	**29**	**33**	**34**

Step 2: We need to obtain the median position:

Median is the $\frac{n+1}{2} = \frac{1+\sum f}{2}$ th data value

$\therefore$ Median position $\frac{1+34}{2} = 17.5$ i.e. median is the 17.5^{th} data value

Step 3: We determine the median by simply getting the data value whose cumulative frequency is either equal to or first exceeds the median position (in this case 17.5). Since no cumulative frequency is equal to 17.5, we opt for the first cumulative frequency to exceed 17.5. The cumulative frequency is 22

x	0	1	2	3	4	5	6
f	2	4	6	10	7	4	1
F	**2**	**6**	**12**	**22**	**29**	**33**	**34**

The median is therefore 3 (whose cumulative frequency is 22)

Let's look at another example to depict a situation in **Example 13** (ii) where the median was obtained by averaging two adjacent data values but this time from a frequency distribution.

Example 15

A newspaper vendor records the sales volume (number of newspaper) he makes on a daily basis. A random sample of 50 days was taken resulting into the following frequency distribution.

Sales volume	Number of days
37	5
38	5
40	10
43	8
45	7
47	6
48	5
51	4

Find the median sales volume.

Solution

The steps to be followed are as in **Example 14**

Step 1: We obtain the less-than cumulative frequencies (F)

Sales volume	37	38	40	43	45	47	48	51
Frequency, f	3	5	12	5	8	6	5	6
F	**3**	**8**	**20**	**25**	**33**	**39**	**44**	**50**

Step 2: We obtain the median position:

Median is the $\dfrac{n+1}{2} = \dfrac{1+\sum f}{2}$ th data value

$\therefore$ Median position $\dfrac{1+50}{2} = 25.5$ i.e. median is the 25.5$^{\text{th}}$ data value

Step 3: We determine the median by simply getting the data value whose cumulative frequency is either equal to or first exceeds the median position (in this case 25.5). Since no cumulative frequency is equal to 25.5, we should have opted for the first cumulative frequency to exceed 25.5 i.e. **33**. However, we observe that we have a cumulative frequency of 25 which is the integral part of the median position, **25**.5. For this reason we determine the median by averaging the data values whose cumulative frequencies are respectively **25** and **33**.

Sales volume	37	38	40	43	45	47	48	51
Frequency, f	3	5	12	5	8	6	5	6
F	**3**	**8**	**20**	**25**	**33**	**39**	**44**	**50**

The median is therefore the average of 43 and 45

$\therefore$ Median is $\dfrac{43+45}{2} = 44$ newspapers

Whereas the median is simple to compute and that it is not influenced by extreme values, it possess the problem of data rearrangement. Like the mode it cannot be used in further analysis.

10.6.3 The Mean

Probably the most used measure of central tendency. The mean is found by dividing the sum of all data items (values) by the number of the data items.

The formula for the arithmetic mean:

Given the items $x_1, x_2, x_3, ..., x_n$, then their mean denoted $\bar{x}$ is given by the formula

$$\boxed{\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{\sum x}{n}}$$

where $\sum x$ denotes the sum of all data items (values) and n the number of the data items (values).

Example 16

Calculate the arithmetic mean of the following data:

5,10,7,6,3,7,9, 6, 4,8

Solution

$n = 10$ and $\sum x = 5+10+7+6+3+7+9+6+4+8 = 65$

$$\therefore \bar{x} = \frac{\sum x}{n} = \frac{65}{10} = 6.5$$

Example 17

The following are prices of a certain size of avocado pears (in Kwacha) at a market on 15 days:

100, 110, 140, 150, 110, 150, 130, 130, 160, 140, 120, 130, 120, 130, 130

What is the daily average price?

Solution

Here, $n = 15, \ \sum x = 1950$

$$\therefore \text{Mean, } \bar{x} = \frac{\sum x}{n} = \frac{1950}{15} = 130$$

The daily average price is K130.00

Example 18

The following tables shows the number of cars that passed through a checkpoint on the main road of a certain town.

Month	Number of cars	
	Registered	Unregistered
January	223	31
February	154	13
March	283	14
April	196	36
May	252	22
June	275	20
July	274	23
August	263	23
September	173	13
October	318	22
November	106	16
December	122	18

Calculate the following for the cars passing through the checkpoint:

a) monthly average of the number of registered cars
b) monthly average of the number of unregistered cars
c) monthly average of the number of all cars

Solution

a) Average number of registered cars:

$$n = 12, \ \sum x = 2639$$

$$\bar{x} = \frac{\sum x}{n} = \frac{2639}{12} = 219.92 \approx 220 \text{ registered cars}$$

b) Average number of unregistered cars:

$$n = 12, \ \sum x = 251$$

$$\bar{x} = \frac{\sum x}{n} = \frac{251}{12} = 20.92 \approx 21 \text{ unregistered cars}$$

c) Average number of all cars

$$n = 12, \ \sum x = 2639 + 251 = 2890$$

$$\bar{x} = \frac{2890}{12} = 240.83 \approx 241 \text{ cars}$$

NOTE: Two averages can be combined and a new average obtain without going through the long computations again as we show below

$$\bar{x} = 220 + 21 = 241 \text{ cars}$$

Mean of a simple frequency distribution

A frequency distribution shows how often or how many time each data values appears in a data set. A simple way to find the mean of simple distribution is therefore to multiply each data values/item by the number of times it appears, sum up the products and then divide by sum of the frequencies. In general this is written as follows:

$$\bar{x} = \frac{\sum fx \leftarrow \textit{Sum of the products of } x - \textit{values } \& \textit{ their frequencies}}{\sum f \leftarrow \textit{Sum of the frequencies}}$$

Example 19

Suppose that a restaurant sells pizzas of different sizes and prices. The following shows the number of pizzas sold over a certain period.

Price (K’000)	Number of pizzas sold
2.5	10
3.0	16
3.4	12
3.8	8
4.0	6
4.5	8

Find the average price, to nearest K100, of the sample of pizzas.

Solution

The mean price, $\bar{x} = \dfrac{\sum fx}{\sum f}$

We proceed by constructing a table of sums

Price	f	fx
2.5	10	25.0
3.0	16	48.0
3.4	12	40.8
3.8	8	30.4
4.0	6	24.0
4.5	8	36.0
Totals, $\sum_s$	**60**	**204.2**

$$\therefore \bar{x} = \frac{\sum fx}{\sum f} = \frac{204.2}{60} = 3.40333\,(\text{K'000})$$

The mean price of pizza was K3,400.00

10.6.4 Measures of Dispersion

While averages (mode, median and mean) give describe the centre of the data or measure of a typical value, measures of dispersion describe how spread out the data values are. A small value of a measure of dispersion indicates that the values are clustered closely. On the contrary, a large value of a measure of dispersion shows that data values are widely spread out.

10.6.5 The Range

The simplest measure of dispersion is the range. The range is the difference between the largest and the smallest data values in a data set or distribution i.e

.

$$\text{Range } = x_{\max} - x_{\min}$$

Example 20

Calculate the range for the following sales figures.

K100, K90, K120, K140, K110

Solution

Range =K140 – K90=K50

The range is simple to calculate but notice that it ignores all values and considers the two end values only. This makes the range very susceptible to extreme values.

10.6.6 The Variance and Standard Deviation

As pointed out earlier, the main disadvantage of the range is that it does not take into consideration all the data items. The variance and standard deviation use all items in a data set or distribution. They indicate how far data items deviate from the mean as they measure the average amount by which data items in a data set/distribution vary from the mean.

The variance is mean of the squared deviations from the mean and is denoted S^2 or σ^2. The symbols S and σ (sigma) can be used interchangeably in this course. The variance can only assume none negative values and will be zero if and only if all data items are equal.

The standard deviation is the square root of the variance, hence is denoted S or σ.

Formulae for variance and standard deviation:
To find the variance and standard deviation of the data items $x_1, x_2, x_3, \ldots, x_n$ we have to follow these steps:
Step 1: Calculate the mean, $\bar{x}$
Step 2: Find the squares of the deviations from the mean, $(x - \bar{x})^2$
Step 3: Sum up the squares of the deviations from the mean, $\sum (x - \bar{x})^2$,
Step 4: Calculate the mean by dividing the sum by the number of the items to obtain the variance, $\sigma^2 = \dfrac{\sum (x - \bar{x})^2}{n}$
Step 5: Find the square root of the variance to obtain the standard deviation.

$$\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

Example 21

Given that the following are prices of a pineapple on various market days.

K150, K100, K90, K130, K140, K170, K110, K110, K180, K120.

Calculate the standard deviation of the prices to the nearest Kwacha:

Solution

$$\text{Mean, } \bar{x} = \frac{\sum x}{n} = \frac{1300}{10} = 130$$

We will now construct a table to ease our calculation of deviations and their sum

Price, x	Deviation $x-\bar{x}$	Deviation squared $(x-x)^2$
150	20	400
100	-30	900
90	-40	1,600
130	0	0
140	10	100
170	40	1,600
110	-20	400
110	-20	400
180	50	2,500
120	-10	100
Totals, $\sum_s$	**0**	**8,000**

$$\therefore \sigma^2 = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{8000}{10}} = \sqrt{800} = 28.28427 \text{ (K'000)}$$

The standard deviation is K28,284.00 (nearest kwacha)

NB: If we were interested in the variance, it should have been $\frac{8000}{10} = 800(\text{'}000)$ i.e. $K^2 800{,}000.00$. Notice that the unit of the variance is always squared.

Alternative Formula for the Standard Deviation

We now show you how to use the alternative formula for the standard deviation. To find the variance and standard deviation of the data items $x_1, x_2, x_3, ..., x_n$ we can follow these steps:

Step 1: Sum all the items, $\sum x$

Step 2: Sum the squares of all the items, $\sum x^2$

Step 3: Divide each of the sums from steps 1 and 2 by the number of the items, $\frac{\sum x^2}{n}$ and $\frac{\sum x}{n}$

Step 4: Subtract the square of $\frac{\sum x}{n}$ from $\frac{\sum x^2}{n}$ to obtain the variance,

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

Step 5: Find the square root of the variance to obtain the standard deviation

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

Example 22

Use the alternative formula to find the standard deviation of the prices given in Example 21.

Solution

To use the alternative formula, we need to find $\sum x$ and $\sum x^2$. So we shall construct a table of sums just as we did in example 21.

	Price, x	x-Squared, x^2
	150	22,500
	100	10,000
	90	8,100
	130	16,900
	140	19,600
	170	28,900
	110	12,100
	110	12,100
	180	32,400
	120	14,400
Totals, $\sum_s$	**1,300**	**177,000**

$$\therefore \sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{177{,}000}{10} - \left(\frac{1{,}300}{10}\right)^2} = \sqrt{800} = 28.28427 \text{ (K'000)}$$

The standard deviation is K28,284.00 (nearest kwacha)

We now modify the alternative formula so that it can be used to calculate standard deviation for a frequency distribution. The modified formula is

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$, where f is the frequency and x data value/item

Example 23

Calculate the standard deviation of the data given in example 19.

Solution

Since the standard deviation is given by $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$, we therefore need $\sum f$, $\sum fx$ and $\sum fx^2$ in order to calculate the standard deviation.

Price	f	fx	fx^2
2.5	10	25.0	62.50
3.0	16	48.0	144.00
3.4	12	40.8	138.72
3.8	8	30.4	115.52
4.0	6	24.0	96.00
4.5	8	36.0	162.00
Totals, $\sum_s$	**60**	**204.2**	**718.74**

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{718.74}{60} - \left(\frac{204.2}{60}\right)^2} = \sqrt{0.39632} = 0.62954 \text{ (K'000)}$$

The standard deviation of the prices of pizza is K629.54

CHAPTER SUMMARY

In this chapter we have looked at

- Frequency tables as simple ways of presenting information. However, charts often express the meaning of data more clearly than a table. We have looked at three main forms of bar charts, namely
    - simple,
    - component (stacked) and
    - multiple(compound).
    - The pie chart, where comparative composition of the whole is displayed.

- Graphical representation of frequency distributions using a histogram, frequency polygon, or an ogive (cumulative frequency graph).
- The arithmetic average, the best known type of measure of central location and widely used in business.
- The mode, an average which means the most frequently appearing value.
- the median, the value of the middle item of an ordered data set.
- Three measures of dispersion namely, The range, standard deviation and variance are It is emphasised that you should be able to both calculate measures of central location and dispersion, and interpret their values.

END OF CHAPTER EXERCISES

1. a) Find the median of the following examination results: 29, 72, 36 54, 73, 76, 49, 51, 81, 62, 65.

b) What is the mode of the data set in part a) above?

2. A list of units produced by a machine during a three week period is as follows: 17, 23, 26, 23, 18, 19, 23, 21, 20, 23, 16, 19, 17, 23, 22.

Calculate:
(a) mean
(b) median
(c) mode

3. Over a period of 10 days a production department produces the following output:

Day	Output (units)
1	120
2	145
3	130
4	167
5	123
6	134
7	152
8	132
9	157
10	162

(a) What is the mean daily production output?
(b) Calculate the range within which output fell.
(c) What is the standard deviation of the production output?

4. If the mean of 5 numbers is 24.8 and 4 of the numbers are 25, 30, 20 and 27. What is the fifth number?

5. The following are sales for XYZ Ltd for the years 2010 to2012. Present them on a bar chart:

	Year		
Item	**2010**	**2011**	**2012**
Food Item	500	800	1000
Drinks	600	400	200
Clothing	400	400	800
TOTAL	1500	1600	2000

6. Students at a certain school were surveyed to find out the mode of transport they used when going to school. The results were:

Walking: 9, Bicycle: 10, Car: 6 and Bus: 15.

Construct

i. a pie chart of radius 4cm to present this information
ii. a simple bar chart of the same data
iii. Comment on both the charts

7. A company decided to research the price of laptops on the market. An analysis of advertisements in the press and specialised IT bulletins produced the following information:

Price (MK'000)	Number of laptops
200 – 250	5
250 – 300	12
300 – 400	15
400 – 420	9
420 – 450	6
450 – 680	3

Draw a histogram of the prices.

8. The following set of data represents the age distribution of a company's workforce of 170 employees.

Age (Year)	Frequency
18 but under 30	56
30 but under 40	44
40 but under 50	35
50 but under 60	27
60 but under 70	7
70 and over	1

a) Draw a histogram of the data
b) Draw an ogive
c) How are the two charts (the histogram and ogive) different

9. About 60% of small and medium sized businesses are family owned. An international survey asked chief executive officers (CEOs) of family owned businesses how they became CEO. Responses were that the CEO inherited the business; the CEO built the business, or the CEO was hired by the family owned

firm. A sample of 26 CEOs of family owned business provided the following data on how each one of them became CEO.

Built	Built	Built	Built
Inherited	Built	Inherited	Built
Inherited	Built	Built	Hired
Built	Hired	Hired	Built
Inherited	Inherited	Inherited	Hired
Built	Built	Built	
Built	Inherited	Inherited	

a) Construct a frequency distribution table of the above data
b) Draw a bar chart for the data
c) What could be the main reason a person becomes CEO of a family owned business?

10. A food processor makes chambo fillet from chambo supplied from the lake. The bigger the mass the better (larger and juicer) the fillet and therefore the higher the price. It is known that fish of weight greater than 1 Kg make good fillets which fetch better prices. In order to forecast his revenue he decides to weigh the fish coming in. He asks his buyer to take a sample of 100 fish from different consignments in accordance to a predetermined scientific technique. The results are as follows:

Weights of 100 fish in grams:

393	**1327**	**776**	**1383**	**1179**	**913**	**1958**	**651**	**1307**	**1909**
426	**592**	**1691**	**412**	**1233**	**1490**	**1010**	**1481**	**1400**	**1280**
722	**1612**	**628**	**1483**	**718**	**1313**	**642**	**1478**	**630**	**1454**
818	**881**	***1227***	**929**	**1341**	**865**	**982**	**1488**	**983**	**833**
1145	**1444**	**634**	**1010**	**1727**	**1076**	**1137**	**1439**	**1149**	**1074**
1296	**1321**	**1690**	**1364**	**1916**	**1206**	**1309**	**1756**	**1536**	**1366**
1439	**810**	**1478**	**1532**	**784**	**1182**	**1573**	**429**	**1519**	**1303**
1639	**1689**	**1083**	**1670**	**1617**	**1610**	**1144**	**1370**	**1762**	**1748**
1840	**3890**	**806**	**1733**	**1917**	**1435**	**1845**	**1380**	**1631**	**214**
2027	**1212**	**1332**	**1482**	**1442**	**1553**	**1416**	**1170**	**1763**	**1129**

a) Suggest suitable classes of a frequency distribution for the weights of fish

b) Construct the frequency distribution with one open ended class and state a reason for your choice
c) Construct a less than ogive of the data
d) Construct a less than cumulative diagram of the weights

CHAPTER 11

INTRODUCTION TO PROBABILITY

LEARNING OBJECTIVES

By the end of this chapter the student, should be able to:

i. Define probability
ii. Describe the different types of events
iii. Calculate probability of simple events
iv. Define complementary, mutually exclusive, and independent events
v. Use the addition rule to calculate probability of mutually exclusive events
vi. Use the multiplication rule to calculate probability of independent events.

11.0 Introduction

Probability has often times been taken as a complex, and scientific concept. Whereas it could be true considering the mathematics behind some of probability problems, it is surprising to note that the concept has a very important place in business and everyday life.

It is not uncommon to hear people utter statements "the chances of success in this type of business are high"; He had fifty percent chances of surviving, after his road accident" or "her odds of being elected the women's chief co-ordinator were one to five." If carefully examined the above statements share one factor: the outcomes or results of the situations are uncertain. This factor is common in a number of business dealings. Some decisions have to be made even in the face of uncertainty; use is therefore, made of probability which is a concept that seeks to quantify the chances or likelihood that an event will occur.

11.1 Definition and Notation

Probability is therefore, a measure of the chance or likelihood of something occurring A single distinct result which might occur is called an *outcome* and a set containing all possible outcomes is called a *sample space* or *outcome set.* An *event* is a subset of the sample space.

We denote the size of the outcome set (total number of possible outcomes) by N and number of possible ways in which an event E can occur by $n(E)$

If E is any event, we write $\Pr(E)$ to mean the probability that the event E will occur.

There are two types of probabilities

a. Theoretical probability

Theoretical probability is calculated based on logical thought. If E is an event, then the probability that E will occur if an experiment is performed is given by:

$$\Pr(E) = \frac{Number\ of\ times\ that\ E\ can\ occur}{Total\ number\ of\ possible\ outcomes} = \frac{n(E)}{N}$$

Note: Under theoretical probability, we assume that the experiment was not performed before and make prediction on how likely a possible event is to occur if the experiment is performed.

Example 1

An unbiased die labelled 1 to 6 is thrown, what is the probability of obtaining a 4?

Solution

$$\Pr(4) = \frac{n(4)}{N}$$

Since the die has 6 sides and only one is labelled 4 then

$$N = 6$$
$$n(4) = 1$$

Hence

$$\Pr(4) = \frac{1}{6}$$

b. Empirical/Experimental Probability

This is the probability that is calculated based on results of an experiment that has already been calculated a number of times. If an experimental has been performed a relatively large number of times and a frequency distribution is given, the empirical probability of an event *E* when the experiment is performed one more time is given by:

$$\Pr(E) = \frac{Number\ of\ times\ that\ E\ occurred\ in\ previous\ experiment}{Number\ of\ times\ the\ experiment\ was\ performed} = \frac{f(E)}{\sum f}$$

Example 2

Data on education status of a child was collected from a sample of 36 children of ages 10-15 from Kadyalunda village. The following table summarises the findings:

Education Status	In school	Dropped out	Never attended School
Number of children	13	16	7

What is the probability that a child chosen at random from the village will be

a) in school
b) a drop out?

Solution

a. Let S be the event that a child is in school

$$\Pr(S) = \frac{f(S)}{\sum f}$$

$$= \frac{13}{36}$$

b. Let D be the event that a child is a drop out

$$\Pr(D) = \frac{f(D)}{\sum f} = \frac{16}{36} = \frac{4}{9}$$

Note:

- Empirical probabilities are just relative frequencies
- Empirical probability is used when the theoretical probability cannot be calculated i.e. when the outcome set is not known or when outcomes are not equally likely.

11.2 Types of Events

a) Complementary event

If E is some event, the compliment of E, denoted $\overline{E}$, is the event that 'E will not occur'.

- e.g. If E is the event that 'it will rain today', then $\bar{E}$ is the event that 'it will not rain today'

b) Mutually Exclusive Events

Two events are said to be mutually exclusive if they can never occur at the same time

- e.g. If M is the event of being a Muslim and C the event of being a Christian, then M and C are mutually exclusive events as one can never be a Muslim and a Christian at the same time.
- While if G is the event of being a graduate and C a chartered accountant then G and C are **NOT** mutually exclusive events as it is possible for a person to be both a graduate and a chartered accountant.

c) Independent Events

Two events are said to be independent if the occurrence (or not occurrence) of one event does not affect the occurrence (or not occurrence) of the other event. Independenteventsmay or may not occur together

- e.g. *P,* the event of John passing his CIFA examinations and *W* the event of John's favourite team winning a league can be thought of being independent.

11.3 Properties of probabilities

Suppose S is a sample space and E any event in S then

i. The probability of E is no less than 0 and no more than 1

i.e. $0 \leq Pr(E) \leq 1$

If E is bound to occur then $\Pr(E) = 1$, and E is called a *certainty*
If E can never occur then $\Pr(E) = 0$, and E is called an impossibility.

ii. The sum of probabilities for all possible outcomes in S is 1

i.e$\sum_{all E} \Pr(E) = 1$

iii. $\Pr(\bar{E}) = 1 - \Pr(E)$

11.4 Mutually Exclusive Events and the Addition Rule

If A and B are mutually exclusive events then the probability that one of them will occur is given by

$$\boxed{\Pr(A\ or\ B) = \Pr(A) + \Pr(B)}$$

However, if A and B are not mutually exclusive the following formula applies:

$$P(A\,or\,B) = P(A) + P(B) - P(A \cap B)$$

Example 3

In a business undertaking, the probability of making a loss is 0.4, that of breaking even (neither losing nor making profits) is 0.4 and for making a profit it is 0.2. Calculate the probability for making a profit or breaking even.

Solution

Since its one business one will make a loss, break even or make a profit. These events are mutually exclusive as no two of them can happen at the same time.

Let A be the event making profit and B the event of breaking even. From the given information,

$\Pr(A) = 0.2, \ \Pr(B) = 0.4$

Then $\quad \Pr(A\ or\ B) = \Pr(A) + \Pr(B) = 0.2 + 0.4 \quad = 0.6$

Example 4

A fair (unbiased) die is tossed once. What is the probability an even number or a multiple of 3 turns up?

Solution

Let A be the event an even number turns up and B be the event a multiple of 3 turns up.

Then $A = \{2,4,6\}$ and $B = \{3,6\}$. Now $A \cap B = \{6\}$
So Pr(an even number or a multiple of 3 turns up)

$$\begin{aligned}
&= P(A\,or\,B) \\
&= P(A) + P(B) - P(A \cap B) \\
&= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}
\end{aligned}$$

11.5 Independent events and the multiplication rule

If A and B are independent events, then the probability that both events will occur is given by

$$\boxed{P(A\, and\ B) = P(A) \times P(B)}$$

However, if A and B are not independent the following formula is applied:

$$\boxed{P(A\, and\ B) = P(A) \times P(B \mid A)}$$

Note that $P(B \mid A)$, read as 'probability B given A', is the probability that B will occur on condition that A has already occurred.

Example 5

Afirm bids for two contracts A and B. If the firm's chances of winning contracts A and B are 24% and 30% respectively, find the probability that the firm will be awarded both contracts given that the contracts are awarded independently.

Solution

Let A be the event that the firm will be awarded contract A and B the event that it will be awarded contract B

$$\Pr(A) = 0.25$$
$$P(B) = 0.30$$

Since the contracts are awarded independently, then A and B are independent events and

$$\Pr(both\ A\ and\ B) = \Pr(A) \times \Pr(B) = 0.25 \times 0.30 = 0.075$$

Example 6

You randomly select two cards from a standard 52-card deck. What is the probability that the first card is not a face card (a king, queen, or jack) and the second card is a face card if:

(a) you replace the first card before selecting the second, and

(b) you do not replace the first card?

Solution

Let A be the event the first card drawn is not a face card and B be the event the second card is a face card

a) If you replace the first card before selecting the second card, then A and B are independent events. So, the probability is:

$$P(A\,and\;B) = P(A)\times P(B) = \frac{40}{52}\times\frac{12}{52} = \frac{30}{169} = 0.178 \text{ correct to 3 decimal places.}$$

b) If you do not replace the first card before selecting the second card, then A and B are dependent events. So, the probability is:

$$P(A\,and\;B) = P(A)\times P(B|A) = \frac{40}{52}\times\frac{12}{51} = \frac{120}{663} = 0.181 \text{ correct to 3 decimal places.}$$

Example 7

You and two friends go to a restaurant and order a sandwich. The menu has 10 types of sandwiches and each of you is equally likely to order any type. What is the probability that each of you orders a different type?

Solution

Let event A be that you order a sandwich, event B be that one friend orders a different type, and event C be that your other friend orders a third type. These events are dependent. So, the probability that each of you orders a different type is:

$$P(A\,and\;B\,and\,C) = P(A)\times P(B|A)\times P(C|A\,and\,B)$$

$$= \frac{10}{10}\times\frac{9}{10}\times\frac{8}{10} = \frac{18}{25} = 0.72$$

11.6 Tree diagrams

When two or more events take place it is often simpler to find the probabilities by means of diagrams that show all possible events and their probabilities. Examples of such diagrams are the tree diagrams

Example 8

A box contains 5 black pens and 7 red pens. A pen is selected at random, its colour noted, and it is then returned to the box. The box is shaken and a second pen selected. Draw tree diagram to show all possible outcomes of the selection and use it to find the probability of selecting

a) one of each type of pen

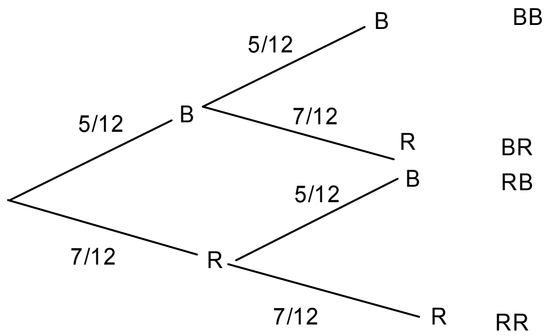
b) two red pens

Solution

In a tree diagram each possible outcome is represented by a tree branch with the event label written at the end of the branch and its probability along the branch.

If we let B be the event of selecting a black pen and R the event select a red pen then, $\Pr(B) = \frac{5}{12}$ and $\Pr(R) = \frac{7}{12}$

Using a tree diagram, the possible outcomes of the two selections and their probabilities would be:



Notes:

- The Probability of an event represented by consecutive branches of the tree is the product of the probabilities along those branches
- The probability of any event which can occur in two or more ways is the sum of the probabilities at the end of the relevant branches

a) The event of picking one of each type of pen can occur in two ways:

Red first then Black (RB) or
Black first then Red (BR)

Hence the probability of picking each type is the sum of probabilities at the end of the respective branches.

i.e. P(selecting each type of pen)

$$= P(RB\,or\,BR)$$
$$= P(RB) + P(BR)$$
$$= \frac{7}{12}\times\frac{5}{12}+\frac{5}{12}\times\frac{7}{12}$$

$$= \frac{35}{144} + \frac{35}{144} = \frac{70}{144} = \frac{35}{72}$$

b) Probability of pick two red pens would happen in one way (R,R)

$$\Pr(R, R) = \frac{49}{144}$$

CHAPTER SUMMARY

In this chapter we have looked at:

- The definition of probability
- Types of probabilities
- Calculation of simple probabilities
- Types of events
- The addition rule and its use in calculating probability for mutually exclusive events
- The multiplication rule and its use in calculating probability for independent events
- Calculation of probabilities using tree diagrams

END OF CHAPTER EXERCISES

1. A fair die is rolled once. What is the probability of:
   a) Obtaining an even score
   b) Obtaining a 6
   c) Obtaining a 3 or a 4

2. A coin is tossed twice. What is the probability of obtaining two heads in a row?

3. A box contains 3 white balls and 7 black balls. A boy blind folded picks a ball, its colour noted and returned to the box. The bag is shaken and the blindfolded boy randomly picks another ball.

   a) Draw a tree diagram showing all possible selections and their corresponding probabilities
   b) Use the tree diagram in (a) above to find the probability that
      i) Of the two balls selected, one is black and one is white
      ii) The first ball selected was black and the second ball white
      iii) Both balls selected are white

4. Jane has a box containing 15 chocolates which look identical. 10 have soft centres and the rest have hard centres. She picks a chocolate at random and eats it.She picks a second chocolate at random and eats it.

   a) Draw a probability tree diagram to show the outcomes when the two chocolates are eaten
   b) Calculate the probability that she eats two chocolates of different types.

5. A firm is independently working on two separate jobs. There is a probability of 0.25 that either of the jobs will be finished on time.

   Find the probability that

   a) Both jobs will be finished on time
   b) Neither of the jobs will be finished on time
   c) Just one of the jobs will be finished on time
   d) At least one of the jobs will be finished on time

6. The following data relate to the number of sales made by a company over a number of weeks

Weekly sales	Number of weeks
Up to 10	2
10 to 19	12
20 to 29	22
30 to 39	10
40 and above	4

   Based on these sales, calculate the probability that next week the firm will make the following number of sales

   a) At least 20
   b) No more than 39
   c) Between 20 and 29 inclusive

7. The Ministry of Health estimates that 5.9% of Malawians have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it.

   If the medical lab gives the test to a randomly selected person, what is the probability that the diagnosis is correct?

8. The owner of a one-man lawn mowing business owns three old and unreliable riding mowers. As long as one of the three is working he can stay productive. From past experience, one of the mowers is unusable 12 percent of the time, one 6 percent of the time, and one 20 percent of the time.

Find the probability that all three mowers are unusable on a given day.

9. At a particular company 64% of the employees are forty years old or over. Of those employees, 83% are enrolled in the company's retirement plan. Only 61% of the employees under forty years old are enrolled in the plan.

   Make a probability tree diagram and use it to find the probability that a randomly selected employee is enrolled in the company's retirement plan.

COMPUTING

CHAPTER 12

THE COMPUTER AND ITS IMPORTANCE

12.0 Introduction

In modern business world, where efficiency in all operations has become the order of the day , data processing and computing are becoming a necessity. The purpose of computers, it must be said on the onset is to help process business data into useful information that can be used in decision making or planning of operations. To appreciate the idea, one needs to know what is meant by data and information and then consider advantages of a computer.

12.1 Data and Information

a) Data

Data refers to the raw material for data processing which does not give meaning on its own i.e. unprocessed facts, figures or events which are not put in a form that people can understand and use.

Examples of data are: hours worked, Price list and cloths made per day in clothing factory, Hours worked on their on their own do not tell user much, neither does cloth made. However when hours worked are multiplied byh wage rate per hour, it gives the wages earned which useful information for decision making. *Price list* does not provide any information in terms of what and how much has been sold

b) Information

Information is the data that has been processed in so as to provide meaning to the person who receives it (i.e. facts that have been interpreted & understood by users). Data is said to be information when it is converted into a more useful form. Examples of information may include a bank statement, a sales forecast, a cash flow, a telephone directory etc

Qualities of good information

The main characteristics of good information can be summarized in the mnemonic **ACCURATE**

i) Accuracy

Information needs to be accurate enough for its purpose. Incorrect information leads to making wrong and poor decisions
Figures should add up correctly and the degree of rounding should be appropriate. Accuracy of information can be improved with use of computerized systems

ii) **C**omplete

Information should include everything that it needed for someone to do his/her job properly and void making wrong decisions. Information should include for example internal and external data, financial and non-financial data as long as it is relevant to the purpose.

iii) **C**ost-Effective

Information is cost-effective is the benefits derived from the information exceed the cost of acquiring it. Information should also lead to a decision to take action which results in cost reduction, eliminating losses, increasing sales, better utilization of resources, prevention of fraud etc.

iv) **U**ser-targeted

Every individual must be given the information he/she needs to carry out certain tasks. The needs of the users must be taken into consideration when preparing information. For example senior managers need summaries, junior managers need detailed information.

iv) **R**elevant

Information must be relevant to the purpose for which the user wants it. Information that is not needed for a decision must not be included no matter how interesting it might be. Provision of irrelevant information just wastes time and money
For instance if sales information for Region B is requested, sales figures for Region A should be omitted, it is irrelevant

v) **A**uthoritative (Reliability)

Information must be reliable and trusted by the managers/ user who are expected to use it. It must be collected from reliable sources using reliable collection methods. Information from the Internet for instance may not always be unreliable

vi) **T**imely

Information should be readily available when it is needed. Time is very crucial. If information is not available until after a decision is made will be of little or no value.

v) Easy-to-use

Information should be clearly presented to ease understandability by the user. Use of computerized systems can increase presentation by use of tables or graphs and charts. It should be easy for the user to take key points at a glaceand to find the details if required.

12.2 Advantages of using a computers

Because of the speed and accuracy which are realized through use of computers there are direct benefits that businesses may set out of using computers. These include the following:

a) To automate repetitive processes

- Automate basic functions like calculations, which can be carried out faster and more cheaply.

b) As a provider of information

- Provides a regular flow of information via information systems to individuals who require it.

c) As a means of communication

- Data communications has revolutionalised the way companies operate and the way that they are organized
- Communication is done via e-mails, cell phones, faxes

d) As a means of communication

- Information can be shared via computer networks and the Internet

e) As an Integrator

- IT can act as a focal point of communications activities, which help link together different departments, and activities within an organization.

12.3 Data Processing

These are systems that designed to perform and record routine and repetitive transactions on a daily basis.

Data processing can be done manually or by use of computers. Figure 3.1 below shows the activities comprising the data processing cycle which are:

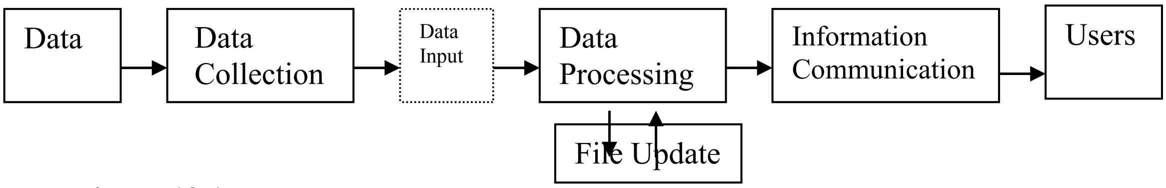


Figure 12.1

Data collection

All data needed for data processing is collected and made available and there must be a system or procedure to ensure data is collected and made available. Data can be collected through surveys, research or Informal gathering or individual collection

Sources of Data & Information

Data & information can be sourced from both inside and outside the organization therefore information systems need to be designed to help in obtaining the relevant data.

Internal sources

Internal sources may include:

- Accounting records like Sales Ledger, Purchases Ledger, Cash book
- Payroll system
- Production department, Sales & marketing department
- Members of staff – information may be obtained informally in the course of the day-to-day business or through meetings, interviews, questionnaires and discussions.

External Sources

- Government
- Newspapers, Libraries, Information bureaus, Legal experts, TV reports, Internet

Datacan be gathered using formal methods of collection which can be entrusted to particular individuals.

Notice that these stages are common in many types of data processing. However where the volume of data is small some stages may not be clearly identified.

12.4 The Computer

Figure 12.2: Personal Computer

Definition

According to R G Anderson in his book 'Data Processing' a computer is defined as " a machine which accepts data from input devices, performs arithmetic and logical operations in accordance with a predetermined program and finally transfers the processed data to an output device".

The History of Computers

<u>First Generation Computers (1940 - 1955)</u>

During the years **1943 to 1946**, Dr. John W. Mauchly and J. Presper Eckert, Jr. completed the ENIAC (Electronic Numerical Integrator and Computer), the first large-scale electronic digital computer. It weighed 30 tons, contained 18,000 vacuum tubes, and occupied a 30' x 50' space.

Characteristics

- Built using thousands of **vacuum tubes (electronic valves)** which controlled internal operations and were huge
- Used **Magnetic drum** as primary internal-storage medium with very little storage capacity
- **Limited main-storage** capacity
- Used a **lot of electricity**
- Produced great deal of heat and subject to frequent burnout
- Required extensiveand special air-conditioning
- Bulky; occupied a lot of floor space
- Slow with Slow input/output, punched-card-oriented: Operators performed input and output operations through the use of punched cards.
- **Low level symbolic- programming language**: The computer used machine language which was cumbersome and accomplished through long strings of numbers made up of Zeroes and Ones.
- **Applications**: payroll processing and record keeping though still oriented toward scientific applications that business data processing.

The type of computers in the first generation were mainframes

Examples: IBM 650, UNIVAC I

Second Generation Computers (1955 – 1964)

In **1958**, computers built with transistors marked the beginning of the second generation of computer hardware.

Characteristics

- Used **transistors**for internal operations: tiny solid state transistors replaced vacuum tubes in computers. Transistor were much smaller than vacuum tube
- Used **Magnetic core** as primary internal-storage medium: Electric currents pass through wires which magnetize the core to represent on and off states.
- Increased main-storage capacity: The internal or main storage was supplemented by use of magnetic tapes for external storage. These tapes substituted for punched cards.
- **Faster input/output**: Devices could be connected directly to the computer and considered "on-line".
- **Magnetic disks** were also developed that stored information on circular tracks that looked like phonograph records.
- **Used high-level programming languages** (COBOL,FORTRAN, Pascal)
- **Increased speed and reliability**: Modular-hardware was developed through the design of electronic circuits.

- **Less bulky;** computers were made smaller and faster.
- Less expensive
- The heat problem was then minimized
- Consumed less energy
- Faster and more reliable data processing
- **Batch-oriented applications** like billing, payroll processing, updating and inventory files: Batch processing allowed for collection of data over a period time and then one processed in one computer run.

The type of computers in the second generation were mainframes

Examples: LEO Mark III, ATLAS, IBM 7000, IBM 1401

Third Generation Computers (1964-1971)

In **1969**, Dr. Ted Hoff of Intel Corporation developed a microprocessor, or microprogrammable computer chip, the Intel 4004.

This was the start of commercial computers

Characteristics

- Used **integrated circuits (ICs):***ICs* replaced the transistors of the second-generation machines.
- **Magnetic core** and **solid-state** main storage: hence greater storage capacity.
- More **flexibility with input/output** and**disk-oriented:**
- Extensive use of **high-level programming languages**: The software industry evolved during this time. Many users found that it was more cost effective to buy pre-programmed packages than to write the programs themselves. The programs from the second generation had to be rewritten since many of the programs were based on second generation architecture.
- Smaller size, more powerful and better performance and reliability: Advances in solid-state technology allowed for the design and building of smaller and faster computers.
- **Remote processing** and **time-sharing** ability through communication: Computers were then able to perform several operations at the same time. Remote terminals were developed to communicate with a central computer over a specific geographic location. Time sharing environments were established.
- Availability of **operating-system software** to control Input/Output and do tasks handled by human operators:
- **Applications** such **as airline reservation systems, market forecasting**, credit card billing: The applications also included inventory, control, and scheduling labor and materials. Multitasking was also accomplished.

The type of computers in the third generation were mainframes and minicomputers

Examples: IBM 360 series, NCR 395, ICL 1900 series, Burroughs B6500

Fourth Generation Computers (1971 – Present computers)

Computers in use today

Characteristics

- Uses **a general-purpose microprocessor chip** with control unit and ALU on a single chip classified **as Large Scale Integration (LSI)**
- The development of 8 bit microprocessor computers
- **Increased storage capacity** and speed.
- **Big memory and storage capacity**
- **Modular design** with complex integrated circuits classified as Large Scale Integration **(LSI)**
- Compatibility between equipment
- **Special application programs**
- **Versatility** of input/ output devices

a. In **1976**, Steve Wozniak and Steve Jobs built the first Apple computer.

b. In **1980**, IBM offered Microsoft Corporation's founder, Bill Gates, the opportunity to develop the operating system for the soon-to-be announced IBM personal computer. With the development of MS-DOS, Microsoft achieved tremendous growth and success.

c. The IBM PC was introduced in **1981**, signaling IBM's entrance into the personal computer marketplace. The IBM PC quickly garnered the largest share of personal computer market and became the personal computer of choice in business.

d. In **1984**, IBM introduced a personal computer, called the PC AT, that used the Intel 80286 microprocessor. Apple introduced the Macintosh computer, which incorporated a unique graphical interface, making it easy to learn.

e. In **1990**, Microsoft released Windows 3.0, a substantially enhanced version of its Windows graphical user interface first introduced in 1985. The software allowed users to run multiple applications on a personal computer and more easily move data from one application to another. It was an instant success and by 1990, more than 54 million computers were using this software in the United States.

f. In **1993**, several companies introduced computer systems using the Pentium microprocessor from Intel. The Pentium chip is the successor to the Intel 486 microprocessor.

g. In **1995**, Intel began shipment of Pentium Pro microprocessor, the successor to its widely used Pentium chip. Microsoft released Windows 95. In the US, 2 out of 3 employees have access to a PC. One out of every 3 homes has a PC. More than 50 million PCs were sold worldwide in 1995, over 250 million are in use.

h. In June of **1998**, Microsoft released Windows 98 which was considered a minor upgrade to Microsoft Windows 95.

i. In **2000**, Microsoft released Windows 2000 which was considered by some to be the best version to date.

j. **Today**, Microsoft Windows XP is widely used. It is available in a number of different editions. .

12.5 Types of Computers

Computer History and Developments

Computers were developed along two separate engineering paths, producing two distinct types of computer, **Digitaland Analog computers**

Analog Computers
An analog computer was designed to operate on continuously varying data where numbers were presented by electrical voltage. They accept inputs which vary with respect to time. Analog computers work by translating data from constantly changing physical conditions into corresponding mechanical or electrical quantities..

Characteristics of analog computers:

- Operates using continuous variables**.** Meaning it uses numbers that change not in steps, but change in a smooth continuous manner.
- Performs operations in a truly parallel manner; meaning that it can perform many calculations all at the same time.
- They do not have the ability to store data in large quantities,
- They do not have the comprehensive logical programming facilities hence they can not perform comparisons.
- Analog computers are extraordinarily fast.
- Limited precision - the precision of analog computers is not good; they are limited to three, or at most, four digits of precision.
- The cost of hardware required to provide high degree of accuracy is often prohibitive and therefore they do not offer high degree of accuracy and display.

Analog computers were well known in the 1940s but they are now virtually extinct i.e. they largely have been made obsolete for general-purpose mathematical computations. These computers are now kept in museums for historical purposes.

However the analog mechanism is still used in many devices. For example, some automobile speedometers are mechanical analog computers that measure the rotations per minute of the drive shaft. Measuring instruments with pointers on a circular disk like thermometers and voltmeters are also good examples.

Examples of analog computers:

- The slide rule. This was the first analog computer to be developed

- Car Speedmeter

Digital Computers

Digital computers are computers that were designed to operate using discrete data (non continuous data). The computers require the variables of the problem to be expressed in terms of discrete numbers (digits) like 0,1,2,3,4…… in steps and proceeds in these discrete steps from one state to the next.

Characteristics of digital computers:
- They operate using discrete data**.**
- They have the ability to store data in large quantities,
- They have comprehensive logical programming facilities hence can compare results with other data.
- Lower speed than analog computers
- They have higher resolution hence better display
- Digital computers have almost unlimited precision, but quite slow compared to analog computers.

Examples of digital computers are desktop computers, laptop computers etc.

Hybrid computers

These are computers that have the combined features of digital and analog computers. It is designed to handle both analog and digital data. The digital component normally serves as the controller and provides logical operations, while the analog component normally serves as a solver of differential equations.

Example of a hybrid computer is an ATM.

Common Types of Computers Classifactions

Computers are categorized by both size and processing capabilities

i) Supercomputers

A supercomputer is a computer which performs at a rate of speed which is far above that of other computers. Given the constantly changing world of computing, it should come as no surprise to learn that most supercomputers bear their superlative titles for a few years, at best. Computer programmers are fond of saying that today's supercomputer will become tomorrow's computer; the computer you are reading this article on is probably more powerful than most historic supercomputers.

The term “supercomputer” was coined in 1929 by the New York World, referring to tabulators manufactured by IBM. To modern computer users, these tabulators would probably appear awkward, slow, and cumbersome to use, but at the time, they represented the cutting edge of technology. This continues to be true of supercomputers today, which harness immense processing power so that they are incredibly fast, sophisticated, and powerful.

The primary use for supercomputers is in scientific computing, which requires high-powered computers to perform complex calculations.

In many cases, a supercomputer is custom-assembled, utilizing elements from a range of computer manufacturers and tailored for its intended use. Most supercomputers run on a Linux or Unix operating system, as these operating systems are extremely flexible, stable, and efficient. Supercomputers typically have multiple processors and a variety of other technological tricks to ensure that they run smoothly.

Supercomputers are the most powerful & sophisticated machines designed to perform complex calculations at very high speed. They are able to process very large amounts of data. Supercomputers are used for applications such as meteorological applications (weather patterns) or astronomical applications. They are expensive and therefore not meant for commercial use. Manufacturers of supercomputers include ***Cray and Fujitsu***

ii) Mainframe computers / Enterprise servers

Mainframes are the largest (huge) and next most powerful computer. They are designed to meet the computing needs of a large organization with large data processing requirements. They have very extensive storage facilities & large memories.

A mainframe is normally used by having a number computer terminals linked to it by cables. Manufacturers include ***IBM and HP***

iii) **Minicomputers**

These are computers whose size & processing capabilities lie somewhere between those of a mainframe and a Personal Computer (PC). They have more processing power than a PC but less than a mainframe. Though somewhat smaller, they are intended to meet the needs of a small company by serving up to a hundred terminals. Manufacturers include **ICL, IBM and DELL**

iv) **Personal computers (PCs) / *Microcomputer (Desktop computer)***

A PC is a general-purpose, single user computer powered by a **Microprocessor chip.** PCs are the most common type of computers found in organisations. They are small in size and have small storage capacity. They are used for small to medium sized businesses. Most often they are linked together in **Networks** or to enable sharing of information between users.

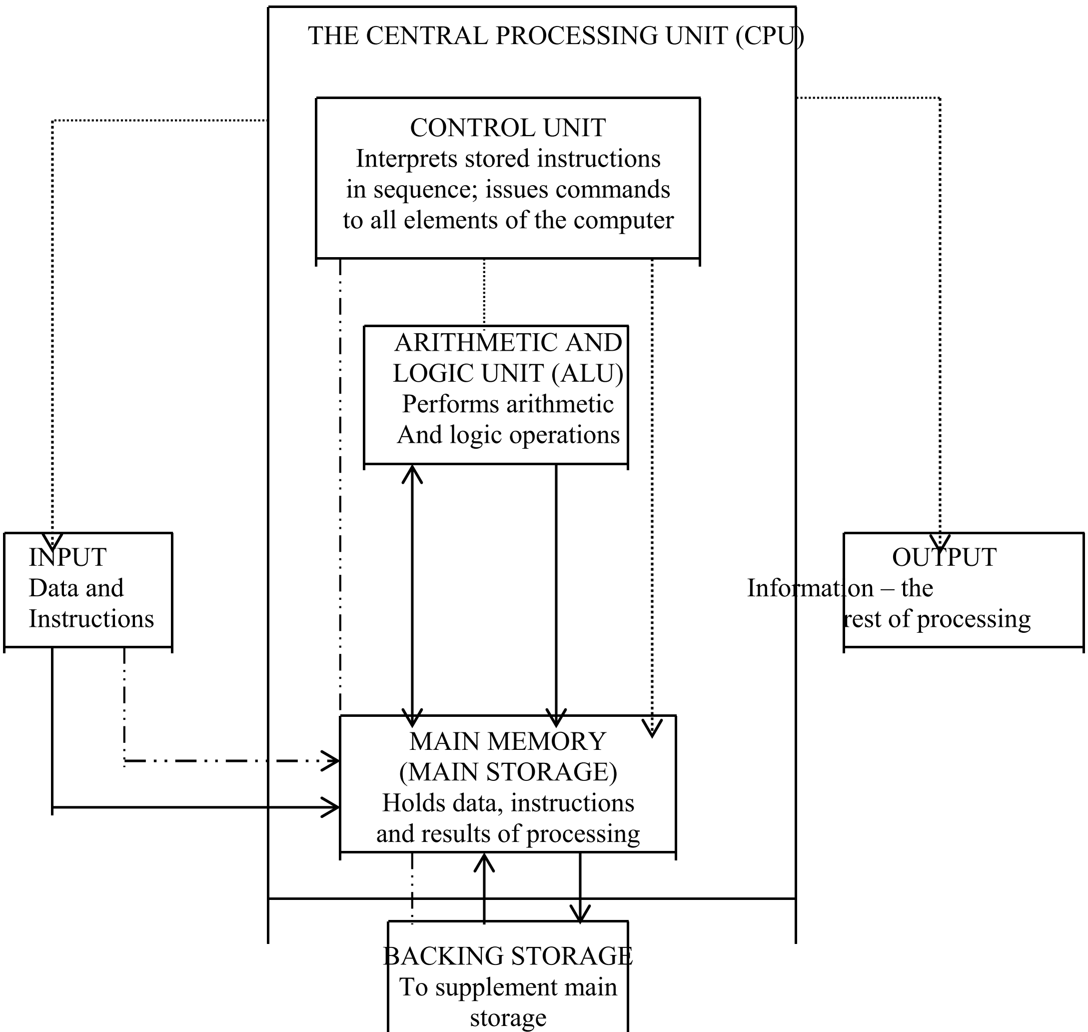
v) **Portables**

These include Laptops/briefcase computers and Palmtops/handheld computer. They usually have similar processing capabilities to PC and are usually powered by a battery. However their main drawbacks are keyboard ergonomics (keys are small and too close together) and limited battery power

12.6 Elements of Computer Systems

The Computer Hardware is basically composed of four major components:

1) Input Devices
2) Storage Devices
3) The Processor (The Central Processing Unit)
4) Output Devices



Commands

Data flow

Instruction flow

A BRIEF DESCRIPTION IF EACH ELEMENT

INPUT

Input devices are computer hardware peripherals that feed data and instructions into the computer or other computational devices for display, storage, processing, or outputting or transmission. They convert the instructions and data into digital signals that can be processed by a computer. Examples of input devices are mouse, bar-code reader, magnetic-stripe reader, keyboard, modem, scanner, graphic tablet and stylus.

The following are examples of input devices:

KEYBOARD

Figure 12.3 : Keyboard

Keyboard is an input device partly modelled after the typewriter keyboard which uses an arrangement of buttons or keys to act as mechanical levers or electronic switches. A keyboard typically has characters engraved or printed on the keys and each press of a key typically corresponds to a single written symbol. However, pressing some symbols require pressing and holding several keys to produce letters, numbers or signs (characters), other keys or simultaneous key presses can produce actions or computer commands.
A computer keyboard distinguishes each physical key from every other and reports all key presses to the controlling software.

A keyboard is also used to give command to the operating system of a computer eg in Windows, Ctrl+Alt+Del combination bring up a task window or shuts down the machine.

MOUSE

Figure 12.4 : mouse

A mouse is a device used to point, select, drag and open programs and files. A standard mouse has two or three buttons which can be pressed (clicked) to send specific signals to the processor.

Types

i) **Wheeled mouse**. This mouse has a removable rubber ball protruding from a small hole in its base.

ii) **Optical mouse.** This mouse has a small light-emitting diode (LED) that bounces light off the surface when the mouse is moved. The sensors convert the movement into coordinates that the computer understands. This mouse is replacing the wheeled mouse.

LIGHT PEN

Figure 12.5 light pen

Is an input device that utilizes a light-sensitive detector to selectobjects on a display screen. A light pen is similar to a mouse, except that with a light pen you can move the pointer and select objects on the display screen by directly pointing to the objects with the pen

BAR CODE

a machine-readable code in the form of numbers and a pattern of parallel lines of varying widths, printed on and identifying a product.

Barcodes can be read by obarcode readers, or scanned from an image by special software.

Figure 12.6 Bar code

Uses

Barcodes have slowly become an essentialpart of modern civilization. Their use is widespread, and the technology behind barcodes is constantly improving. Some modern applications of barcodes include:

- Almost every item purchased from a grocery store, department store, and mass merchandiser has a barcode on it. Eg from Shoprite, Sana, Cash and Carry. This greatly helps in keeping track of a large number of items in a store and also reduces instances of shoplifting involving price tag swapping. .
- Barcodes are widely used in shop floor control applications software where employees can scan work orders and enter the time spent on a job.
- Retail chain membership cards (issued mostly by grocery stores and specialty "big box" retail stores such as sporting equipment, office supply, or pet stores) use bar codes to uniquely identify a consumer. Retailers benefit by being able to offer customized marketing and greater understanding of individual consumer shopping patterns.
- The tracking of item movement, including rental cars, airline luggage, mail, express mail and parcels.

Entertainment event tickets can have barcodes that need to be validated before allowing the holder to enter sports arenas, cinemas, theatres, fairgrounds, transportation etc. This can allow the proprietor to identify duplicate or fraudulent tickets more easily

Storage Devices

The storage devices are used for permanent or long-term storage of data. The data and programs are stored on backing storage devices before they can be loaded into the memory for processing by the CPU.

These storage devices are also referred to as:-

- Backing Storage
- Secondary Storage Area
- Auxiliary storage

Types of storage devices

a) Magnetic Disks

Magnetic disks offer direct access to data in which the data is stored and accessed randomly. The disks are coated with magnetic materials. Each surface of the disk is made up of concentric cycles called **tracks** and these are divided into **sectors.** A group of sectors with space enough to store an independent block of data that can be accessed independently are called **clusters.**

Types of Magnetic Disks

i) Hard Disk

Data is stored by magnetising the surface of flat, circular plates called **platters.** Disks constantly rotate at very high speed. A read/write head floats on a cushion of air a fraction of a millimetre above the surface of the disc. The drive is inside a sealed unit because even a speck of dust could cause the heads to crash.

Figure 12.7: Hard disk

Programs and data are held on the disc in blocks formed by **tracks** and **sectors**. These are created when the hard disc is first **formatted** and this must take place before the disc can be used. Disc are usually supplied pre-formatted.

For a drive to read data from a disc, the read/write head must move in or out to align with the correct track (*the time to do this is called the seek time*). It must wait then until the correct sector rotates round until it underneath the read/write head.

A hard disk is a storage device fixed in the computer system itself. The hard disk capacity varies widely. At the time writing, modern computers can have up to 500 Gb on average. Larger computer systems may have removable disk packs.

Typical uses:

The hard disc is usually the **main backing storage media for a typical computer** or server. It is used to store:

The operating system (*e.g. Microsoft Windows*)

Applications software (*e.g. word-processor, database, spreadsheet, etc.*)

Files such as documents, music, video etc.

A typical home/school microcomputer would have a disc capacity of over 100 gigabytes.

Advantages:

Very fast access to data. Data can be read directly from any part of the hard disc (**random access**). The access speed is about **1000 KB per second**.

Disadvantages:

It can however be a real disaster when they eventually fail because few users have the data on their computer hard drive backed up.

ii) *Floppy disks*

These are small and removable disks, they provide a cost-effective means of electronic storage for small amounts of information. The size of a floppy disk is 3.5" disk can hold up to 1.44 Mb of data.

iii) **A zip disk**

It is a type of removable disk, with much larger capacity (100 Mb) that stores data in a compressed form. A Zip disk is suitable for back-up, storage or for moving files between computers.

b) **USB Flash disk or Memory Stick**

Flash disks offer direct access to data in which the data is stored and accessed randomly.
It is a storage module made of flash memory chips i.e. contents can be flashed off to provide room for new entry. Flash memory is a non-volatile computer memory that can be electrically erased and reprogrammed.

Flash disk capacity ranges from 128MB to infinite. Writing and reading speed is much faster than for magnetic disks. It can allow data to be written and wiped over 1 million times and can keep data for a long period of time.

Flash disks are mainly used for storage and transferring data between computers and other digital devices. The flash is inserted into one of the USB ports.

c) **Optical Disks**
Optical disks offer direct access to data in which the data is stored and accessed randomly. An optical disc is an electronic data storage medium that can be written to and read from using a low-powered laser beam. **Laser** stands for "light amplification by stimulated emission of radiation." On this disc, data such as music, text, or graphic images is digitally encoded.

CD-ROM - means **C**ompact **D**isc - **R**ead **O**nly **M**emory. This means you can only read from the disc, not write or store data onto it.

Figure 12.8 DC

They are also known as **optical discs** because the data is read by a laser beam reflecting or not reflecting from the disc surface.

A CD-ROM only starts spinning when requested and it has to spin up to the correct speed each time it is accessed.

Typical uses:

Most software programs are now sold on CD-Rom.

Advantages:

CD-ROM's hold large quantities of data (*650 MB*).

They are relatively tough as long as the surface does not get too scratched.

Disadvantages:

Since it can hold large quantities of data then when damaged the will be much loss.

Types

i) **CD-ROM (Compact Disc – Read Only Memory)**
The discs on which data is recorded once and there after can only be read. A CD-ROM can store 650 megabytes of data. It is mainly used to store commercial software products or music.

ii) **CD-R (Compact Disc- Recordable)**
CD recorders are now available for general business use with blank CDs (CD-R). CD-R is a Write Once, Read Many times (WORM) and the writing can be done in sessions. They are used mainly for music, software programs and data transfers.

iii) **CD-RW** (*C*ompact **D**isc **Re**writable) or CD-Erasable
These are rewritable disks (CD-RW) i.e. it can be written and erased as many times as possible. The capacity is between 650Mb to 700Mb. However they are not reliable for long term storage. They are used mainly for music, software programs and data transfers.

iv) **DVD (Digital Versatile Disc) or (Digital Video disc)**
The CD format has started to be superseded by DVD. DVDs are used for multimedia files with video graphics and sound – requiring greater disk capacity. The minimum storage capacity is 5Gb. They are mainly used for movies, software and data archives. They can also be found as DVD-ROM, DVD-R, DVD-RW (holds data that can be rewritten multiple times or HD-DVD (High density DVD).

v) VCD (Versatile Compact Disc) or Video Compact Disc
It is a CD that contains moving pictures and sound. Its capacity is between 650MB to 700 MB. They can be played on almost all standalone DVD players and computers with DVD-ROM or CD-ROM drive. DVDs are better than VCDs in terms of quality and sound.

vi) Mini DVD-R
It is a recordable media ideal for a broad range of mobile storage applications including multimedia content like video, still images, business presentations, internet downloads, data transfer etc.

Magnetic media - Magnetic Tape:

Just like the tape in a tape-recorder, the data is written to or read from the tape as it passes the magnetic heads.

Typical uses:

Magnetic tapes are often used to make a copy of hard discs for back-up reasons.

Advantages:
Magnetic tape is **relatively cheap** and tape cassettes can **store very large quantities of data** (*typically 26 GB*).

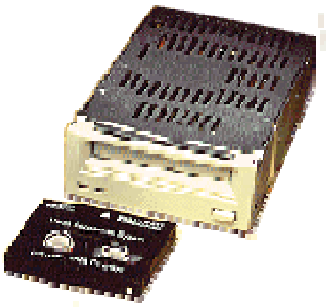


Figure 12.9 Magnetic tape

Disadvantages:

Accessing data is very slow and you cannot go directly to an item of data on the tape as you can with a disc. It is necessary to start at the beginning of the tape and search for the data as the tape goes past the heads (**serial access**).

Tapes do not offer direct access to data but sequential. Like an audio or videocassette, data has to be recorded along the length of a computer tape and it is more difficult time consuming to access on a tape than data on disk (i.e. direct access is not possible with tape). Reading and writing are separate operations. Tape cartridges have a much larger capacity than DVDs and they are still widely used as a backing storage medium. Fast tapes which can be used to create a back-up file very quickly are known as Tape streamers.

Each type of disk has a drive in the main system unit. A drive is the mechanism that reads from or writes to storage media. You need to have a floppy disk drive in order to read data from or write data to a floppy disk. And the disk drive for the floppy disk is drive A or drive B, while that of a hard disk is drive C: and that of a CD-ROM (Compact Disc Read-Only memory) is drive D. Tapes are slotted in tape units

PROCESSING

With particular instructions, arithmetic operating and other data manipulation are carried out in the computer. Specifically, these are carried out in the Arithmetic and logic unit.

OUTPUT

The output devices convert data from the computer into forms which humans can understand and use. The devices allow you to see the processed data or information.

These devices convert data/information which is in machine readable form to human readable form. The most common methods of the computer output are printers and screen displays. It is also possible to output on to microfilm or microfiche and onto transparences for overhead projection. Many computers also provide output through speakers..

12.7 The Choice of the Output Media

Choosing suitable output medium depends on a number of factors as outlined below:

FACTOR	COMMENT
Hardcopy	Is hardcopy of the output required? i.e. is the printed version of the output needed? If so, what quality must the output be?
Volume	The volume of the information produced may affect the choice of output. For example a VDU screen can hold a certain amount of data but it becomes more difficult to read when information goes “off-screen” and can only be read a page at a time.
Speed	The speed at which output is required may be crucial e.g. to print a large volume of information, a high speed printer might be most suitable to finish the work more quickly. If a single enquiry is required, it may be quicker to make notes from a VDU display.
Suitability for further use	The suitability of the output medium to the purpose for which he output is needed. Output onto a magnetic disk would be appropriate if the data is for further processing. Large volumes of reference data for human users to hold in a library might be held in microfilm or microfiche and so output in these forms would be appropriate.

Cost	Some output devices would not be worth having because their advantage would not justify their cost and so another output medium should be chosen as "second best".

COMPUTER MONITOR (SCREEN)

Figure 12.10

A monitor is a piece of electrical equipment which displays images generated by devices such as a computer. The monitor comprises the display device, circuit and enclosure. The display device in the modern monitors is typically a Liquid Crystal Display while older monitors used Cathode Ray Tube (CRT)

Screen Size

The size of a display is usually given as the distance between two opposite screen corners.

PRINTERS

These are devices that print text or illustrations on paper. In terms of technology , printers fall into the following categories:

- daisy wheel printer

- dot matrix printer
- ink jet Printer
- laser Printer
- LCD and LED Printer
- Line printer
- Thermal printer

The printers are also classified by the following characteristics:

- Quality of Type: the output produced by printers is said to be either letter quality (as good as a type writer) near letter quality print, or draft quality. Only daisy wheel, ink jet and laser printers produce letter quality output.
- Speed: Measured in characters per second (cps) or page per minute (ppm), the speed of the printers vary widely, Daisy Wheel to be the slowest printing about 30cps.
- Impact or Non-Impact: Impact printers include all printers that work by striking an ink ribbon. Daisy Wheel, dot matrix and line printers are impact printers. Non-impact printers include laser printers and ink jet printers. Impact printers are much noisier than non-impact printers.
- Graphic: Some printers (daisy-wheel and line printers) can print only text. Other printers can print both text and graphic.
- Font: Some printers, notably dot matrix printers are limited to one or a few fonts. In contrast, laser and ink jet printers are capable of printing an almost unlimited variety of fonts.

A basic classification of printer is:

- Character Printers: these print one character at a time and examples include daisy wheel and dot matrix printers.

  Daisy wheel printers are slow and noisy but produce high quality prints. Companies are unlikely to buy new daisy wheel printers today because other types of printers are more versatile.

Dot Matrix Printers are widely used in accounting departments. Their main problem is low resolution of the printed characters which is unsuitable for many forms of printed output. They are also very slow.

- Line Printers these printer print a couple of lines per minute and the printing of lines per minute is done is single operation. These printers are ideal for bulk printing

LCD Printer

It is similar to a laser printer. Instead of using a laser to create an image on the drum, however it shines a light through a liquid crystal panel. Individual pixels in the panel either let the light pass or block the light thereby creating an image composed of dots on the drum. The liquid crystal shutter printers print quality equivalent to that of laser printers.

Daisy Wheel Printer

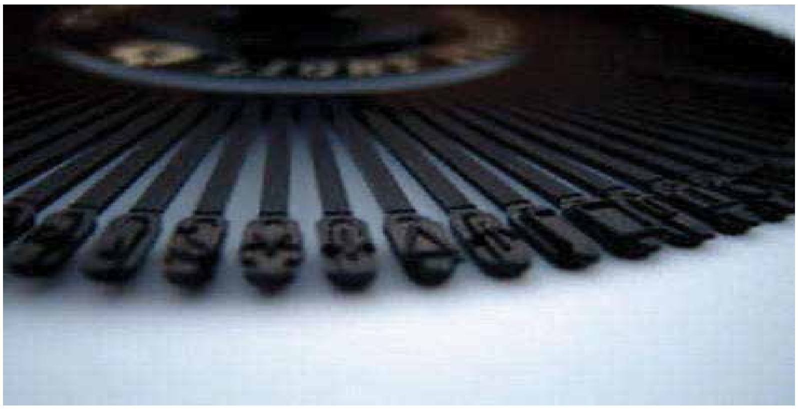


Figure 12.11 Daisy wheel printer

A type of a printer that produces letter-quality output. The daisy wheel printer works on the same principle as the ball head typewriter. The daisy wheel is a disk made up of plastic or metal on which characters stand out in relief along the outer edge. To print a character, the printer rotates the disk until the desired character is facing the paper. Then a hammer strikes the disk forcing the character to hit the paper. You can change the daisy wheel to print different fonts. Daisy wheel printers cannot print graphics and they are generally noisy and slow, printing from 10 to 75 characters/second. As a price of laser and ink jet printers has declined and the quality of dot matrix printers has improved, daisy wheel printers are becoming obsolete.

Thermal Printer

Figure 12.11 Thermal printer

A printer that uses heat to transfer an impression onto paper. There are two types of the thermal printers.

Thermal Wax Transfer: A printer that adheres a wax-based ink onto paper. A thermal print-head melts wax-based ink from the transfer ribbon onto paper. When cool, the wax is permanent. This type of thermal printer uses an equivalent panel of ink for each page to be printed, no matter if a full page or only one line of the print is transferred.

Monochrome Printers have a blank page for each page to be printed while coloured panels for each page. Unlike thermal dye transfer printers, these printers print images as dots which means images must be dithered first. As a result images are not quite photorealistic although they are very good. Thermo Dye Printers are fast and do not require special paper.

Direct thermal: a printer that prints image by burning dots on to coated papers when the paper passes over a line of heating elements. Early fax machines used direct thermal printing.

(*dithering – creating the illusion of new colours and shades by varying the pattern of dots e.g. newspaper photographs are dithered*)

Line Printer

High speed printer capable of printing an entire line at one time. A fast line printer can print 300 lines per minute. These printers cannot print graphics, print quality is low and they are noisy.

Dot Matrix

Figure 12.12 Dot matrix printer

A type of printer that produces characters and illustrations by striking pins against an ink ribbon to print closely spaced dots in appropriate shape. Dot Matrix printers are relatively cheap and do not produce high quality output. However, they can print to multi-page forms (i.e. carbon copies) something laser and ink jet printers cannot do.

Dot Matrix Printers vary in two important characteristics:

- Speed: Given in character per second (cps), the speed can vary from 50 to over 500. Most dot matrix printers offer different speeds depending on the quality of print desired.
- Print Quality: Determined by the sum of pins (the mechanism that print the dot). It can vary from 9 to 24. The best matrix printers (24 pins) can produce near-letter quality type output,

Dot matrix printers are noisy

Laser Printer

Figure 12.13 laser printer

This is a type of printer that utilizes a laser beam to produce an image on a drum. The light of the laser alters the electrical charge on the drum wherever it hits. The drum is then rolled through a reservoir of tonner which is picked up by the charged portions of the drum. Finally the tonner is transferred on to the paper through a combination of heat and pressure. This is also the way a copy machine works. Because an entire page is transmitted to a drum before the tonner is applied laser printers are sometimes called page printers.

Laser printers produce very high quality print and are capable of printing almost unlimited variety of fonts. Laser printers are much quieter than dot matrix or daisy wheel printers. They are also relatively fast. Laser printers are controlled through page description languages (PDLs).

Ink-Jet Printer

Figure 12.14: ink-jet printer

A type of printer that works by spraying ionized ink at a sheet of paper. Magnetized plates in the ink's path direct the ink out the paper in the desired shapes. Ink-jet printers are capable of producing high quality print. Ink-jet printers are slower than laser printers. Ink-jet printers require special type of ink. Because ink-jet printers require smaller mechanical parts than laser printers, they are especially popular as portable printers.

How to care for your computer?

Computer

• Always unplug your computer from the mains when not in use..

• Install your computer in a cool and clean environment.

• Eating and drinking near your computers should be forbidden.

Monitor

• Clean your monitor with a soft clean cloth . Always turn off the monitor prior to cleaning and do not use any detergents or chemicals.

Keyboard

• You may use a soft clean cloth dampened with cleaning alcohol to clean your keyboard. Try to refrain from eating food while using your computer, since small pieces of food could become lodged in the keyboard.

CHAPTER 13

DATA REPRESENTATION

13.1 Description of Main Storage

The primary storage which is also known as **main memory** is resident on the computer itself. Memory is internal storage area in the computer that is used to temporally store data and programs during computer operations.

The Memory

Computer Memory are the internal storage areas in a computer that are used to temporarily or permanently store data or instructions to be executed.

Types of Memory

a) Random Access Memory (RAM).

Random Access Memory is also referred to as Primary Storage consists of computer chips, which are capable of holding data and programs whilst the computer is operating. The Primary Storage is used for **four purposes:**

- To hold programs and data while the computer is operating. The CPU acts on program instructions that are held in the memory
- To hold input data - the data waiting to be processed
- To hold data currently being processed - provides a working storage space when data is being processed
- To hold output data - provides a temporary storage space for finished results of the processing operations until they can be released to output devices

The reason for holding programs in memory is to speed up the processing. The processing capacity of a computer is dictated by the capacity of its memory among other factors.

RAM is the memory chip which is directly available to the processor. It is used to hold data while the computer is operating. The size of the RAM is extremely important, it determines the power of the computer.
The RAM chip is slotted on the Motherboard

Characteristics of RAM

- Data can be written or read

- It is volatile i.e. data and programs held get lost when the machine is switched off. As such there is need to save the work on permanent storage devices before the computer is switched off.
- It is temporary
- It is expandable i.e. the size of RAM can be increased
- It holds data on which the CPU is working on (i.e. the Memory directly available to the processor)
- It is the memory that is used by computer application programs
- The size of RAM determines the speed of the computer.

b) Read Only Memory (ROM)

ROM is a memory chip into which fixed data is written permanently at the time of manufacture.

Data can only be read but can not be written to or deleted from it i.e. the data on this chip is unchangeable or irremovable. It contains a computer start-up program called the "Bootstrap" and the BIOS (Basic Input/Output system) which a ROM chip containing programs needed to control the keyboard, monitor and disk drives. This type of memory chip is used to store system programs that test the system and load the Operating System when the machine is switched on.

Characteristics of ROM

- Data can only be read
- It is Non-volatile. It contains non-volatile and non-alterable programs which are hard-wired and are used to boot the computer. The contents will not be lost when power is switched off.
- It is permanent
- It is supplied only by the computer manufacturer. The programs and data are installed during the process of computer manufacture
- It is non-expandable i.e. can not be upgraded
- Not used by application programs

13.2 Representation of Data

Data is represented on the machine electronically by storage cells which are either charged or discharged. Another way of saying the same thing is that the cells which may be viewed as electronic switches are either on (charged) or off (discharged). In short the storage is based on the two state idea of the cells, on or off. If an on can be taken to mean 1 and off to mean 0 then use can be made of the binary system which has 0 and 1 as the only digits.

How memory works:

Programs and data files are stored as binary numbers. Binary Systems is made up of just 0's or 1's as its digits unlike decimal numbering system which (0-9) as its digits.

To **store** the **0**'s and **1**'s while the computer is running you need a memory chip. This is made up of millions of tiny electrical switches called transistors. They can store a **0** or a **1**by the 'switch' being either **open** or **closed**. This **0** or **1** is the simplest unit of memory and is called a '**bit**' (***B****inary Dig****it***).

Bits are arranged in units of eight to make a **byte**. One byte can therefore store eight **0**'s or **1**'s in **256** different combinations. (00101011 *and* 01110110 *would be just 2 possible combinations for example*).

One byte is a very small amount of memory and it is more usual to refer to **kilobytes** (**KB**), **megabytes** (**MB**) and **gigabytes** (**GB**).

1kB (kilobytes) = 1024 bytes (*approximately 1 thousand bytes*)
1MB (megabytes) = 1024KB (*approximately 1 million bytes*)
1GB(gigabyte) = 1024MB (*approximately 1 thousand million bytes*)
1KB of memory could store roughly one full A4 page of text.
600 MB (on a CD-ROM) could store roughly the text contents of a 10 volume encyclopaedia.

Encoding Data:

Memory chips can **only** store **binary numbers** so other data such as sounds, images or text has to be **encoded** into binary (*digitised*).

If you want to store a **character** from the keyboard, the computer gives it a number code made up of eight bits (*1 byte*). These text codes are the same internationally and are called the **ASCII code** (***A****merican* ***S****tandard* ***C****ode for* ***I****nformation* ***I****nterchange*).

The code for the letter '**a**' is **01100001** (*see below*). **One byte** of memory is therefore used to store the letter '**a**' (*in code*) on a memory chip.

Remember - since computers can only store binary numbers, all computer data has to be in this digital format. . Images, sounds, video etc. all have to be **digitised** before they can be processed by a computer.

How the letter 'a' is stored in 1 byte of computer memory:

	1 BYTE (*8 bits of memory*)							
Contents of each **bit** =>	**0**	**1**	**1**	**0**	**0**	**0**	**0**	**1**

13.3 Binary System

The binary system has two binary digits 0 or 1 only. Each position is a power of 2. The examples below will help to make the concept clear.

Example Express the following numbers using the power of 2:

a)	2	b)	4	c)	8
d)	16	e)	32	f)	64

Solution :

$2 = 2^1$
$4 = 2^2$
$8 = 2^3$
$16 = 2^4$
$32 = 2^5$
$64 = 2^6$

The actual presentation is done by arranging the powers of two from right to left.

Example 2^6, 2^5, 2^4, 2^3, 2^2, 2^1, 2^0

The value of any number is made by a combination of the powers of two. If there is a 1 in that position it means there is a power of 2 and if there is 0 there is no power of 2 meaning the value in that position is zero. The sum of the values of the powers of 2 gives the binary or decimal number equivalent in value.

Example Express the following numbers in binary:
0, 1, 2, 4, 8

Solution (Using the positions)

2^3	2^2	2^1	2^0
4th	3rd	2nd	1st position

1 = 1 *(ie 2^0)*

2 = 10

Note first position (2^0) has 0: Value = 0 = 0
Second position (2^1) has 1: value = 2^1 = 1
Total = 2

4 = 100

1st position (2^0) has 0 :value ----> 0 = 0

		2^{nd} position (2^1) has 0: value ---→ 0	=	*0*
		3^{rd} position (2^2) has 1: value	=	*4*
				4

8	=	1000		
		1^{st} position (2^0) has 0: value	=	*0*
		2^{nd} position (2^1) has 0: value	=	*0*
		3^{rd} position (2^2) has 0: value	=	*0*
		4^{th} position (2^3) has 1: value	=	*8*
		Total	=	*8*
5	=	101		
		1^{st} position (2^0) has 1 value	=	*1*
		2^{nd} position (2^1) has 0 value	=	*0*
		3^{rd} position (2^2) has 1 value	=	*4*
		Total	=	*5*
7	=	111		
10	=	1010		
20	=	10100		

What comes out here is that any decimal number can be represented by a combination of the digits 0 and 1 in their appropriate positions.

It can be extended here that on the computer the alphabetic characters, and other are given numerical values and therefore are represented by binary digits as well.

END OF CHAPTER EXERCISES

1. What is a computer?

2. Distinguish a mini computer from a macro computer.

3. Write short notes on any 2 output devices and any 2 input devices.

4. Describe briefly the binary system of numbers. Write 26,30, 7, 10, and 14 in binary form.

5. A manager would like to buy a computer for your department. What points would advise him to consider before he make the purchase.

6. Describe the functions of the control unit of the CPU of the computer.

CHAPTER 14

SOFTWARE

LEARNIING OBJECTIVES

By the end of this chapter, the student should be able to:

- Distinguish between systems software and application software
- Outline the functions of the operating System
- Give advice on the choice of the operating system
- Identify parts of the Spreadsheet
- Use formula on the spreadsheet
- Outline uses of the spreadsheet
- Specify uses of the wordprocessor

14.1 Introduction

This chapter discusses the different types of computer software. Software is essential for the operation of the computer.

Definition

Software is defined as an organized collections of computer data and instructions, often broken into two major categories: system software that provides the basic non-task-specific functions of the computer, and application software which is used by users to accomplish specific tasks.

A computer program is a series of statements or instructions to the computer. Computers function by obeying these series of instructions (the program). The program instructions are held in the main memory and passed one by one into the control unit, where they are decoded so that the control unit can set up the circuits appropriately.

14.2 Types of Software

There are basically two types of software:

a) Systems Software
b) Application Software

14.3 Systems Software

System software (or **systems software**) is computer software designed to operate and control the computer hardware and to provide a platform for running application software
Systems software is usually provided by the manufacturer of the computer.

Operating Systems

These are programs that aid in the basic operations of the machine and control the way in which the computer is used. Examples are Windows 95, MSDOS, windows 2000, Windows NT and Windows XP

Software Utilities

Utility software is system software designed to help analyze, configure, optimize or maintain a computer.

Utility software usually focuses on how the computer infrastructure (including the computer hardware, operating system, application software and data storage) operates.

Examples of utility software:

- Copying
- Sorting
- Deleting
- Tracking

14.4 Operating Systems

Definition

Operating system is a suite of programs which take control over the operation of the computer to the extent of being able to allow a number of programs to run without human intervention. The more advanced operating systems allow a number of application programs to run concurrently and in sequence without the intervention of the user.

The purpose of the operating system is to control the way the software uses the hardware. The purpose of this control is to make the computer operate in the way intended by the user and in a systematic, reliable and efficient manner.

FUNCTIONS OF OPERATING SYSTEM

Operating system has the following functions:

i. The scheduling and loading of programs: in order to provide a continuous job processing sequence or to provide appropriate response to events.
ii. Control over hardware recourses: e.g. Control over the selection and operation of devices used for input, output or storage.
iii. Handling errors when they occur and using corrective routines where possible.
iv. Passing on control from one program to another under a system of priority when more one application program occupies main storage
v. Protecting hardware, software and data from improper use.
vi. Communication with computer user or operator by means of terminals or consoles through the use of monitor commands and responses. The operator or the user may also be able to communicate with the operating system by means of command language.
vii. Furnishing a complete record of what has happened during operation. Some of the details of this log may be stored for accounting purposes.

Choice of operating system

The application for which a computer is needed largely determine the choice of hardware and the accompanying software. The operating system supplier will need to consider the following factors:

i. The hardware provision and basic design of the computer.

ii. The applications intended for the computer

iii. The method of communication with the computer e.g. many or few peripherals.

iv. The method of operating the computer.

14.5 Capabilities of Operating Systems

Multiprogramming

Multiple programs can share computer resources at any time through concurrent use of the CPU.

Multiprocessing

This links two or more processors (CPUs) to work parallel in a single computer system.

The operating system can assign multiple CPUs to execute different instructions from the same program or from different programs simultaneously, dividing the work between CPUs.

Virtual storage

It handles programs more efficiently by breaking down the program into tiny sections that are read into Memory only when needed. The rest of each program is stored on disk unit until it is required

This allows very large number of programs to be executed concurrently by a single machine.

14.6 Application Software

Applications software is the name given to software that aids users to perform specific tasks on the computer.

Applications software can be obtained in a variety of ways and pricing structures. Most software is bought but some software can be obtained free. Bought software is usually copyrighted and protected by law from illegal copying (*software piracy*).

When software is acquired it needs to be installed in your computer before you can use it. Most software comes on CD-ROMs.

Examples of application software include:

- Word processors e.g. Microsoft word

- Spreadsheets e.g. Microsoft excel
- Presentation software e.g. Microsoft PowerPoint, open office impress
- Database management software e.g. Microsoft access
- Web browser e.g. Mozilla Firefox, Microsoft Internet Explorer.
- Computer games

Off-the-shelf packages

Off-the-shelf packages are ready made software products. Their use shortens the system development process, as they are readily available for use. More specifically they offer many **advantages including:**

- They are readily available
- They are well documented (i.e, among other things, the installation and user manuals are clear, concise and complete).
- They are of high quality (sincethey are produced by experts)
- They are more reliable (many people and organisations will have used them before you)
- They are relatively cheap (the cost of production is shared among many users)
- They are regularly updated and upgraded (updating refers to correcting errors in the software package and upgrading refers to releasing new versions of the software package. Normally new versions are sold at a discount to bona fide customers of earlier versions)
- They can be customized. (To customize software means to adapt it to suit the needs of a particular organisation or individual).

Limitations / Disadvantages of using an off-the-shelf package

Off-the-shelf packages have some problems associated with their use, including the following:

- As earlier on pointed out, the user requirements would need to be modified. This implies that the system '*will **not meet** all the user requirements*'
- User dependence on supplier on maintenance.
- May not provide a competitive advantage since your competitors could be using the same package.

Specific purpose software packages

With a specific purpose package, such as a payroll system or an accounting package, the user has very little control over the process since the package is written to perform a prescribed set of tasks. The user will typically just enter data via the keyboard or the mouse, probably selecting from a set of menus, and the software would subsequently perform a lot of related processes.

With a payroll software package, for example, the user enters the employee details and the number of hours worked, etc. The package then calculates the tax and other deductions and the net wage, and prints the employee's pay slip.

General-purpose software packages

General purpose programs are those which perform particular types of common information processing activity. A spreadsheet package such as Ms Excel is designed to help users perform calculation-oriented tasks. One user of Microsoft Excel will use it for a *budget control system* while another will use it for *stock control purposes* and yet another user can use it for a *payroll system*

Specialist Application Software (Bespoke)

These are programs with associated documentation designed specifically to carry out particular task e.g. solving sets of mathematical equations, controlling company's inventory.

Application Packages

These are suites of programs with associated documentation used for a particular type of problem. Most of the packages are designed to be used for a variety of applications.

14.7 Spreadsheet

A spreadsheet is thus a table used to store various types of data. The data is arranged in rows and columns to make it easier to store, organize and analyse the information.

A spreadsheet application is a computer program such as excel, lotus 123ce or google . It has a number of built in features and tools such as functions, formulas, charts and data analysis tools that make it easier to work with large amounts of data.

THE MAIN SCREEN PARTS (Excel)

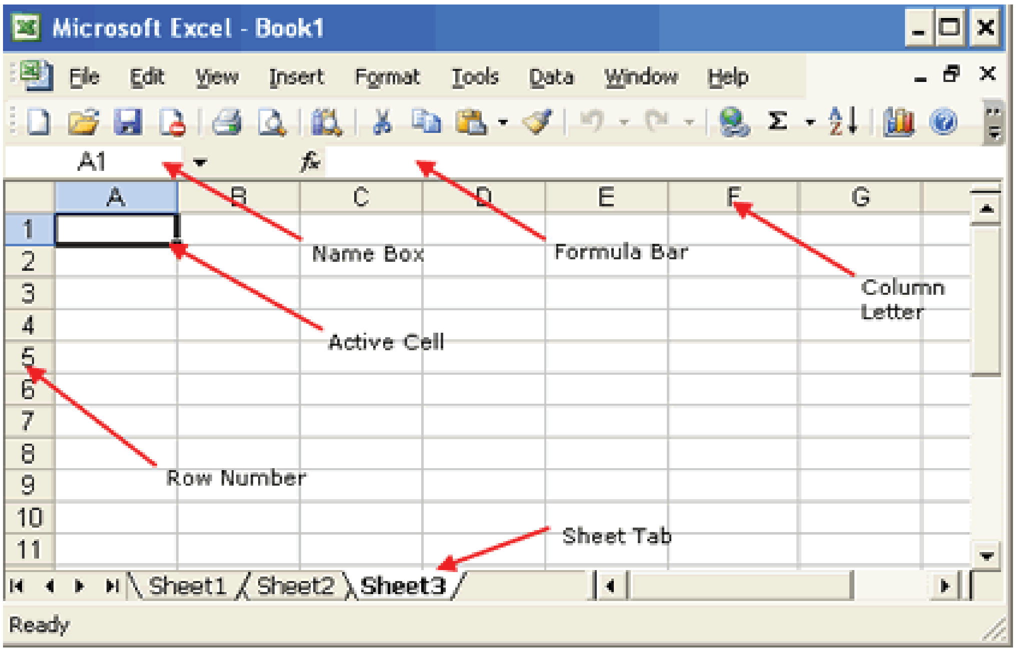


Figure 14.1 : worksheeet

Active Cell

The cell with a black outline. Data is always entered into the active cell.

Column Letter

Columns run vertically on a worksheet and each one is identified by the column header.

Formula Bar

Located above the worksheet, this area displays the contents of the active cell. It can also be used for entering or editing data and formulas.

Name Box

Located next to the formula bar, the name box displays the cell reference or the name of the active cell. The cell is named after the column letter and row number e.g. column B row 2, the cell is B2 (cell reference is B2).

Row Number

Rows are horizontal in a spreadsheet and are identified by the row header.

Sheet Tab

Switching between worksheets in a spreadsheet file is done by clicking on the sheet tab at the bottom of the screen.

Office Button

Clicking on the office button displays a dropdown menu containing a number of options such as open, save and print. The options in the office button menu are very similar to those found under the file menu in other versions of the spreadsheet.

Ribbon

The ribbon is the strip of buttons and icons located above the work area. The ribbon replaces the menus and the toolbars found in other versions of excel.

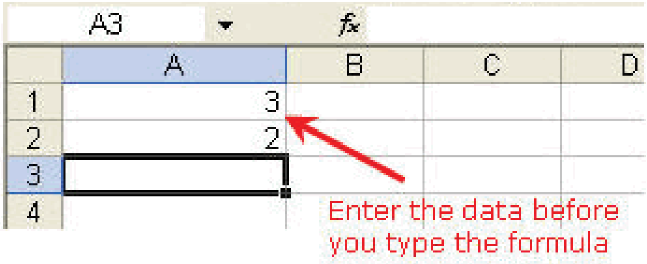
Quick Access Toolbar

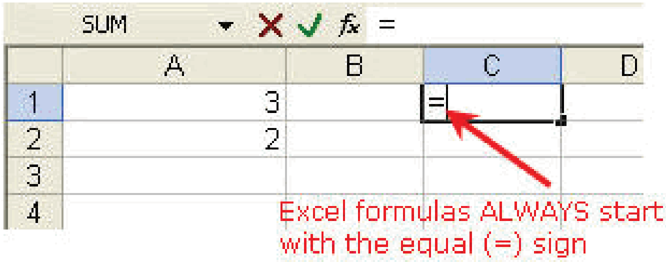
This customizable tool bar allows you to add frequently used commands. Click on the down arrow at the end of the toolbar to display the toolbars options.

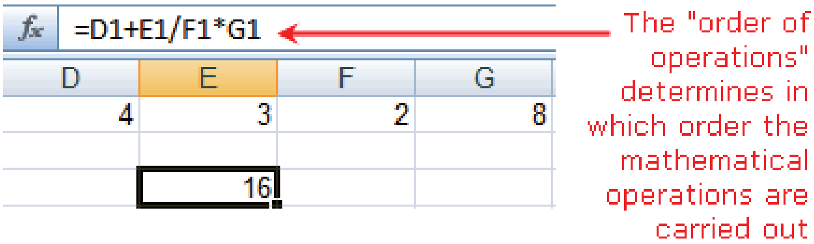
WRITING THE FORMULA

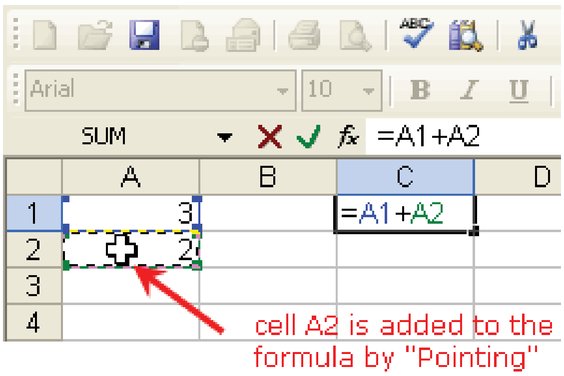
Writing excel formulas is different from the way it is done in a mathematics class. Excel formulas start with the equal sign (=) rather than ending with it.

The equal sign goes into the cell where you want the formula answer to appear. The equal sign informs Excel that what follows is part of a formula and not just a name or number. For example, Excel formula will work like this: = 3+2 rather than 3+2=.


A3
fx
A
B
C
D
3
2
Enter the data before
you type the formula


SUM
fx =
A
B
C
D
3
2
=
Excel formulas ALWAYS start
with the equal (=) sign


fx =D1+E1/F1*G1
D
E
F
G
4
3
2
8
16
The "order of
operations"
determines in
which order the
mathematical
operations are
carried out


Arial
10
SUM
fx =A1+A2
A
B
C
D
3
2
=A1+A2
cell A2 is added to the
formula by "Pointing"

E4 =D4*(B4*1.5)

	A	B	C	D	E
1	Overtime Calculations				
2	Name	Rate/Hr	Hours	Overtime Hours	Overtime Pay
3					
4	Jones B.	$16.75	42.0	2.0	$50.25
5	Smith M.	$17.75	43.0	3.0	$79.88
6	Turner T.	$15.75	36.5	0.0	$0.00

USES OF SPREADSHEET

i. Spreadsheets act as a calculator by automatically doing calculations.

ii. Spreadsheets are used for tracking personal investments, budgeting, invoices, inventory tracking, statistical analysis, numerical modeling, address book, telephone books, printing labels, etc.

iii. Spreadsheets are used in almost every profession to calculate, graph, analyse and store information.

iv. Spreadsheets are used for what-if calculations. Change one number in a spreadsheet and all the calculations in a large spreadsheet will recalculate, will change automatically (when cell reference are used in the formulas).

PRACTICAL

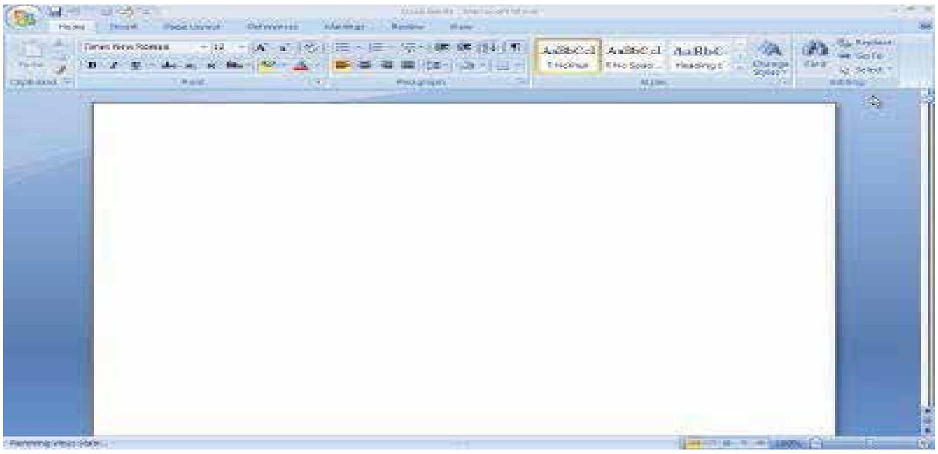
Open a spreadsheet.

If on the desktop, click start then l Programs then Microsoft Office then Excel

If you are in Excel click File then New

In either case a spreadsheet appears on your screen.

14.8 Word Processor



Blank Screen for word processor

Figure:14.2

Word processor is a general program for word processing.

Word processing is using a computer to create, edit, and print a document.

A word processor enables you to create a document, store it electronically on a disk, display it on a screen, modify it by entering commands and characters from the key board and print it on a printer.

Advantages of using Word processor

i. You can make changes to without retyping the entire document.
ii. If you want to delete a paragraph, you simply remove it without leaving a trace.
iii. It is easy to insert a word, sentence or paragraph in the middle of a document.
iv. It is easy to move text from one place to another within a document or between documents
v. You can easily send the document to a printer to get a hard copy.

Basic features of Word processor

Insert text: Allows you to insert text anywhere in the document

Delete text: Allows you to erase character, word, lines or pages as easily as you as you can across them out on paper.

Cut and paste: Allows you to move (cut) a section from one place in a document and insert (paste) it somewhere else.

Copy: Allows you to duplicate a section of text.

Page size and margins: Allows you to define various page sizes and margins and the word processor will automatically readjust the text so that it fits the set margins.

Search and replace: Allows you to direct the word processor to search for a particular word or phrase. You can also direct the word processor to replace one or group of characters with another everywhere that the first group appears.

Word wrap: The word processor automatically moves to the next line when you have filled the line with text and it will readjust text if you change the margins.

Print: Allows you to send a document to a printer to get a hard copy.

Word processor that supports only these features are called "text editor".

Features of full featured word processor

File management:

Many word processors contain file management capabilities that allows you to create, delete, move and search for files

Font Specification: Allows you to change font within a document e.g you can specify bold, italics and underlying. You can also change font size and typeface.

Footnotes and Cross references : Allows you to automate the numbering and placements of foot notes and enables you to easily cross-reference other sections of the document.

Graphics: Allows you to embed illustrations and graphs into a document.

Headers, footers and page numbering: Allows you to specify customized headers and footers that the word processor will put at the top and bottom of every page. Word processor keeps track of page numbers so that the correct number appears on each page.

Layout: Allows you to specify different margins within a single document and specify various methods for indenting paragraphs

Macros: A macro is a character or word that represents a series of key strokes. The key strokes can represent text or commands. The ability to define macro allows you to save yourself a lot of time by replacing common combination of key strokes.

Merges: Allows you to merge text from one file into another file. This is particularly useful for generating many files that have the same format but different data.

Spell checker: Ability that allows you to check the spelling of words. It will highlight any word it does not recognize.

Tables of contents and indexes: Allows you to automatically create a table of content and index based on special codes that you insert in the document.

Thesaurus: A built-in thesaurus that allows you to check for synonyms without leaving the word processor.

Windows: Allows you to edit two or more documents at the same time. Each document appears in a separate window. This is particularly valuable when working on a large project that consists of several different files.

WYSIWYG(What You See Is What You Get): With WYSIWYG, a document appears on the display screen exactly as it will look when printed.

What are File Extensions? - The extra three letters located after the period following your file name are file extensions. They tell both the operating system and the user what type of file it is. The operating system is usually configured to know what default program to use to view or edit each type of file. The following are just a few examples:

- Text Documents: . .txt
- Audio Files: . . .mp3
- Video Files : .mpg .
- Graphic Picture .jpg .gif .tif
- Hypertext Files: .htm .cgi .php
- Other Data Files: . pdf .x
- Configuration Files: .ini .inf .set
- Compressed File Sets: .zip .rar .tar

How are files saved ?

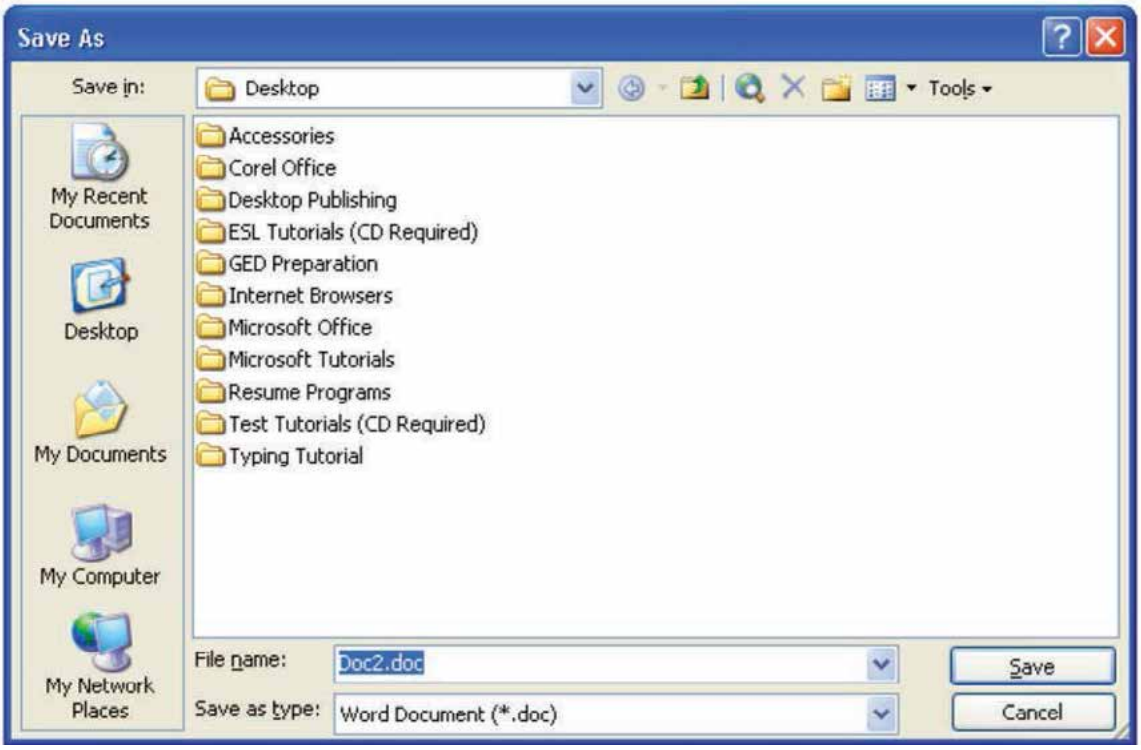


Figure:14.3 Saving window

• To save a file, click the **Save** button on the Standard toolbar or go to the **Menu Bar** and from File, choose **Save As.** Figure 16.5 will then appear.
• Type the name for the file to be saved. It will replace the default name in the “File Name” box. It will automatically be assigned a 3-letter extension.
• Click the **Folder Name** and then choose the folder where you wish to store the file.
(A:) as the Folder
Name. Now, from the drop down list, locate Drive C and then the users folder. Save your file to that folder.

• The File Type will default to the application’s type. If you want to save to a different format, choose that format in the **File Type** box.
• Click the **Save** button.

How do I open a program and create a file?

• Go to Start button and locate the Microsoft Word program. Click on that program.
• Type one sentence such as “My name is ________ .”
• After you have typed that sentence, now practice saving that document.

How to close document?
• To close a file, from the **Menu Bar** click on **File**, then close.
• Or click on upper right, black “x”.
• To close the entire program, click on the upper right, red “X.”

How do I open my document?

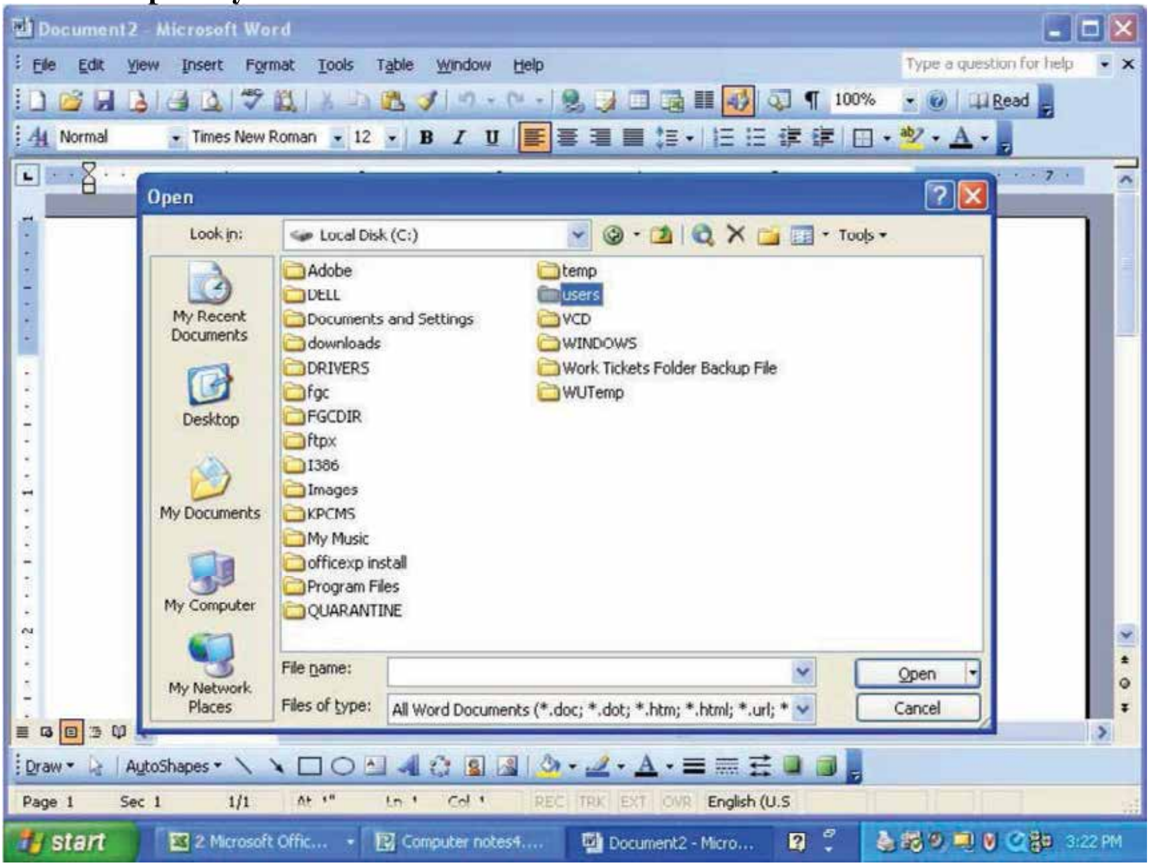


Figure:14.4 file open window

To open an existing file, choose **Open** icon from the Standard toolbar. Choose the Folder Name where the file is located and then click on the file when it appears in the Folder Contents area (see Figure 16.6). Then click on the **Open** button.

• If you know a file is on the disk in that folder, and it isn't listed in the Folder Contents area, try changing the FILE TYPE to ALL FILES. If it is in that folder, the file name will appear.

• To start a new, blank document, click on the new document icon on the Standard toolbar

• To start a new document using a template, from the **Menu Bar**, choose **New** from the **File** menu. The different tabs contain different templates and wizards that can be used to create a new document.Now, close that saved document.

What is Window's Explorer?

• Window's Explorer is a file management tool similar to **My Computer**. It is a program that allows you to view the contents of the computer, the hierarchy of folders, and the contents of each folder.
• It also allows you to organize your files and folders by copying, moving, deleting and renaming them.

• To access Window's Explorer, right-click on your **Start** icon and select **Explore**. (see Figure 16.8)

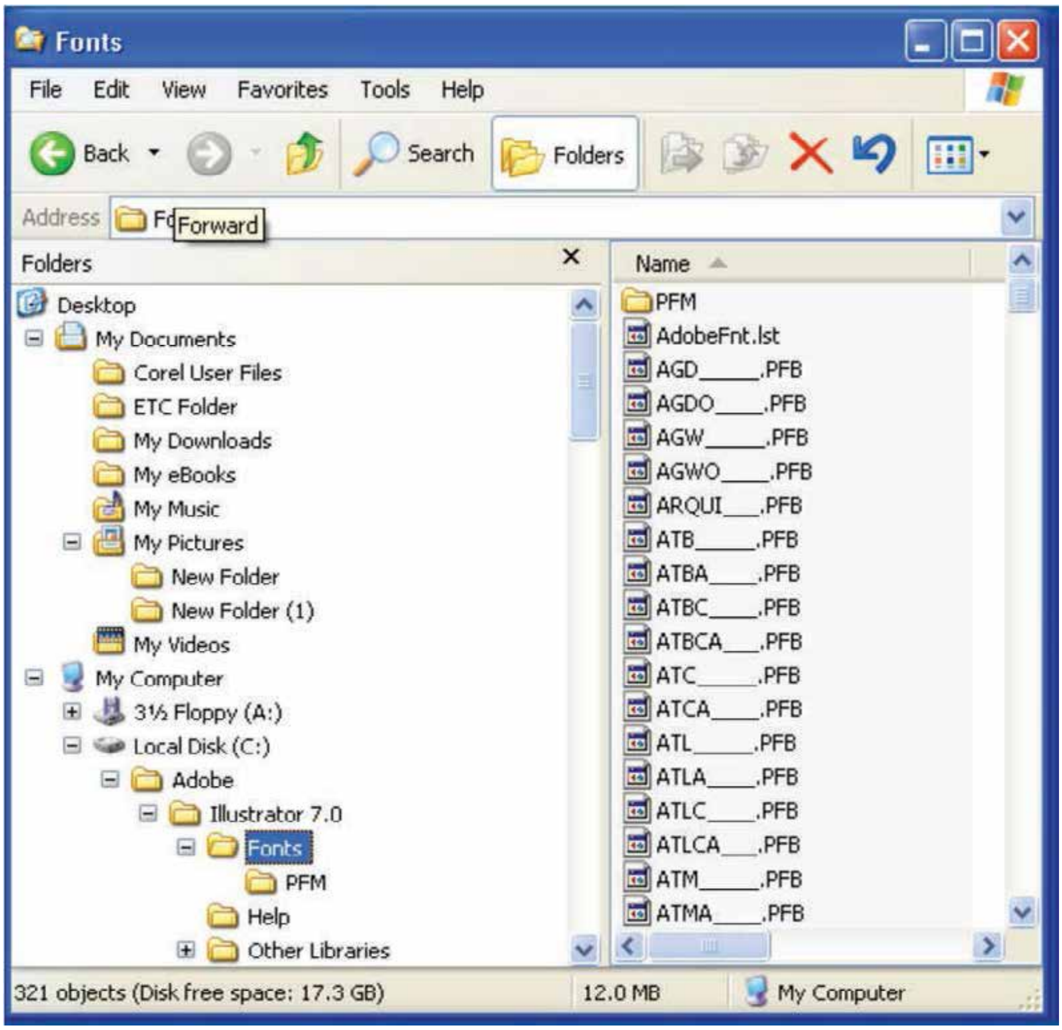


Figure 14.5 windows explorer

How do I select files?

- Click on the file in the right pane to select it. Locate your new file.

- Practice opening and closing your new file from Windows Explorer. Change your file and now "Save As" a new file. Open that file from Windows Explorer. Now close that new file.

- Remember, to select multiple files, press the **CTRL** key then click each file. If the files are all next to each other, you can select the first file, press the **Shift** key, and then select the last file in the list.

How to I create and rename a folder?

- To create a new folder, first select the source folder by clicking on its icon in the left pane. The right pane will show the contents of the current folder.
- Right-click in the right pane, click on New, then click on Folder. The new folder is created in the right pane and is waiting for you to type in a name for it. Alternatively, you can select the File menu, then New, then Create New Folder.

- To rename a file or folder, simply right-click the file or folder and choose Rename from the shortcut menu. The current name will be highlighted and anything you type will replace the existing text. Press the **Enter** key when you are done.

- Drag means you point to an item, hold down the primary mouse button, move the item to the desired location on the screen, and then release the primary mouse button. Drag your folder on the desktop.

- You can also move any open window to another location on the desktop by pointing to the title bar of the window and dragging the window.

How to copy a file or folder?

- To copy a file means that you wish to make a duplicate of an existing file. The original file remains.
-
- After selecting the files to be copied, right-drag them to the destination folder in the left pane. Choose "Copy Here" from the shortcut menu.

- Alternatively, after selecting the files, you can choose Copy from the Edit menu, then click the destination folder in the left pane, and then choose Paste from the Edit menu.

- Another method is to select the files, right-click, choose Copy from the shortcut menu right-click the destination folder in the left pane, and then choose Paste from the shortcut menu.

How to file or folder?

- To move a file means that you wish to move the file from one location to the other. The file is no longer in the original location.

- After selecting the files to be moved, right-drag them to the destination folder in the left pane. Choose **Move Here** from the shortcut menu.

- Alternatively, after selecting the files, you can choose Cut from the Edit menu, then click the destination folder in the left pane, and then choose Paste from the Edit menu.

- Remember, if you select **Move Here**, your original file or folder will be removed from the source folder and moved to the destination drive or folder.

- Let's move your new file. Go to the **C: Drive**, the **Users** folder and locate your new file.
- Right-click and drag that file to your new folder. Now you will be asked if you want to copy or move this file. Select move. Open your file from that folder. Now, close your file.

14.9 Database

A database can be defined as an organized collection of structured data, stored with a minimum of duplication of data items so as to provide a consistent and controlled pool of data.

A data bases is constructed in a step manner and created file at a time. Files can be merged, deleted or new ones added as time goes by until the data base is created to provide for the information needs of an organization examples include the following.

1. Records of customer accounts of their purchases
2. Records of spare parts shaving location, qualities at hand and prices
3. Student, grades, names, addresses, background qualifications.

A data base nowadays can be constructed using some application packages and for this course for DBASE III will be used.

Exercise
Create a data base of the following data using Dbase III.

Create a small data base for the following Water Accounts.

AccountName Number	Address	Consumption Cubic metres

2021	Chagunda R	Box 451, Blantyre	592
2221	Chimera M	Box 610, Lunzu	241
2304	Mzoma R	Box 5304, Limbe	671
2921	Kamchacha Z	Box 2323, Blantyre	578
1045	Kanyumba D	Box 214, Kande	28
1045	Kanjedza R	Box 217, Kande	30
1145	Gawaza L	Box 221, Kande	45
1001	Chembezi D	Box 24, Mwanza	601
2194	Chimenya G	Box 717, Blantyre	795
2147	Chafuwa Z	Box 382, Blantyre	687
1421	Kalanza B	Box 324, Kande	56

Accounts that start with 1 are domestic and tariff are 7 tambala per cubic meter with a fixed charge of K1.

After accounts are industrial whose tariff carry a fixed charge of K10, water is charged as follows:

8 tambala per cu. meter for 0 to 250 cu. meter
9 tambala per cu. meter consumption of more than 250 cu. meters.

14.10 SAGE Accounting Package

Introduction

Sage supports the needs and challenges of more than 3.1 million small and mid-sized business customers.

Customer Management

Sage offers a complete range of customer management software solutions to support the needs of small and midsized businesses and organizations.. challenges and situation.

Financial Management

Sage offers a complete range of financial management software solutions, including everything from accounting—to asset management—to full ERP systems. financial solutions are finely honed and elegantly simple so you can manage, access, and control every detail of your business with ease.

Healthcare

Sage's family of healthcare software positions you to gain the business insight you need to form a full view of your medical practice. Find out how Electronic Health Records and practice management solutions can help your office run more efficiently, reduce costs and improve quality of care.

Inventory Management

Sage offers a broad range of solutions for manufacturers and distributors that have challenges of inventory management..

Nonprofit

Sage offers a range of solutions that are well-suited to the needs and missions of nonprofit and government organizations. Sage nonprofit solutions are created to work the way you work, recognizing the unique challenges of different types of nonprofit and government organizations.

Operational Management

Sage offers a complete range of operational management software solutions to support the needs of small and midsized businesses and organizations. The software solutions support proactive planning and scheduling, ongoing reporting, quality control, regulatory compliance, process, and more.

Payment Solutions

Sage offers a complete range of merchant services and payment solutions to support the needs of small and midsized businesses and organizations. Our solutions are focused on credit card and debit card acceptance and check conversion. We also offer gift card, loyalty programs, and more.

People Management

Sage offers a complete range of people management solutions that offer flexible, end-to-end HR management and payroll solutions to match your growing needs and technology platform. Solutions include recruitment solutions, benefits administration capabilities, employee self-service, and much more.

Supplier & Partner Management

Sage offers a range of solutions that enable businesses and organizations to proactively manage suppliers and partners. Our offerings include EDI solutions that allow you to effectively manage procurement and purchasing processes, vendors and subcontractors.

14.11 Programming Language

Evolution of programming languages

Application software is primarily concerned with accomplishing the tasks of end users.

Programming languages and software tools can be used to develop application software.

Programming language also called computer language is used by programmers to pass instructions and communicate with the computer, computer language is used when programmers are writing programs on the computer.

Programming languages are categorized into low level and high level languages.

Low level or machine level languages

A low level language is a computer language that is in machine code or assembly code, a low level language is usually in pure binary form, these languages are mainly hardware oriented.
Low level languages are closely related to a computer's internal codes. It consists chiefly of a set of letters (mnemonics) which are translated by a program called assembler into machine code

- **First generation programming languages 1GL**

1GL is machine language or the level of instructions and data that the processor is given to work on (which in conventional computers is a string of 0s and 1s). Here a program is usually written in a symbolic language which is then translated into binary by the computers operating system.
Programming using machine language was a very slow and labour intensive process.

- **Second generation programming languages 2GL**

This is assembler sometimes called assembly language
An assembler is a special program written in machine language used to translate – convert symbolic language instructions (source program) into machine language instructions (object program). The resulting program can only be executed when the assembly process is completed.
The assembler also detects syntax errors in the source program so that these can be corrected before the program is tested.

High level languages

These languages were developed in order to ease the work of programmers by making the programming language more procedure oriented. Statements of a high level source program are closer to natural English or other natural languages.

High level languages are also problem oriented, they are developed to solve different types of application problems.

A high level source program is translated i.e. converted into object program by means of a *compiler* or *interpreter.*

- **Third generation programming languages 3GLs**

These are problem oriented languages, these are languages designed to handle a particular class of problem. Translation is done using compilers or interpreters, third generation languages are structured programming languages. E.g.

- COBOL (Common Business Oriented Language)
- BASIC (Beginners All Purpose Symbolic Instruction Code)
- FORTRAN(Formula Translation)
- Pascal, named after a French mathematician
- PL/1 (Programming Language 1)

  - **Fourth generation programming languages 4GLs**

This generation is higher than 3GLs
It has programming languages that can be employed directly by end users or less skilled programmers to develop computer applications more rapidly than conventional programming languages.

These are non- procedural, they need one only to specify what has to be accomplished rather than provide details about how to carry out the task.

4GLs are slow in processing the job at hand, they need powerful processors. 4GLs are usually programmed interactively- meaning that programming errors are detected at an early stage. An example of 4GLs is SQL for databases.

Programming as such is beyond the scope of the manual. However, it pays to know that a program is written using computer languages and there are different computer languages. The term "Computer Programming Language" is used rather just a language.

CHAPTER 15
NETWORKS

LEARNINIG OBJECTIVES

By the end of this chapter students should be able to:

- Define a network
- Differentiate between WAN and LAN
- Identify the different types of LAN topology
- Outline advantages and disadvantages of using network

15.1 Introduction

A computer network consists of two or more computers that are linked in order to share resources (e.g. printers and CD ROMs), exchange files or allow electronic communication. The computers in a network may be linked through cables, telephone lines, radio waves, satellite or infrared light beams.

15.2 Types of Networks

Two major types of networks are:

(a) Local Area Network (LAN)

(b) Wide Area Network (WAN)

LAN

A computer network that spans a relatively small area. Most LANs are confined to a single building or group of buildings.

LAN configurations consists of:

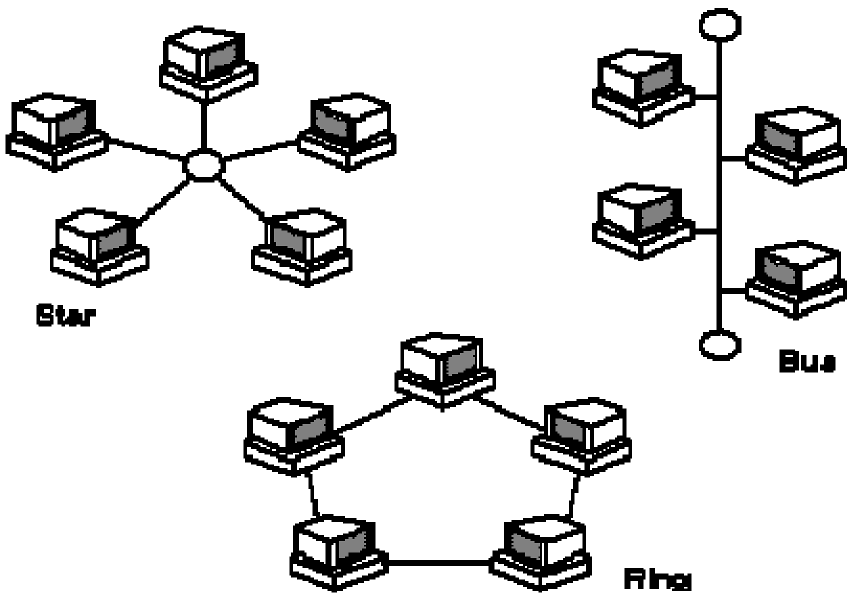
A file server: stores all the software that controls the network as well as the software that can be shared by the computers.

A work station: computers connected to the server. These are less powerful than the file server.

Cables: used to connect the computer interface card in each computer.

LAN Structure (Topology)

There are four principal topologies used in LAN:



Bus topology – all devices are connected to a central cable called the bus or backbone. Bus networks are relatively cheap and easy to install, suitable for shorter distances.

Ring topology – all devices are connected to one another in a shape of a closed loop so that each device is connected directly to two other devices one on either side of it. Ring topologies are relatively expensive and difficult to install but they offer high bandwidth and can span larger distances.

Star Topologies – all devices are connected to central hub. Star networks are relatively easy to install and manage but bottlenecks can occur because all data must pass through the hub.

Tree topology – a tree topology combines characteristics of linear bus and star topologies it consists of star configured work stations connected to a linear bus backbone cable.

15.3 Advantages of Networks

Speed – sharing and transferring files within networks are very rapid. Thus saving time while maintaining integrity of the file.

Cost – individually licensed copies of many software programs can be costly. Network versions are available at considerable version.

Shared programs on a network allows for easier upgrading of the program on one single file server instead of upgrading individual work stations.

Security – sensitive files and programs on a network are password protected or designated as “copy inhibit” so that you do not have to worry about illegal copying of programs.

Centralised Software Management – Software can be loaded on one computer (file server) eliminating the need to spend time and energy installing updates and tracking files on independent computers throughout the building.

Resource sharing – resources such as printers for machines and modems can be shared.

Electronic mail – e-mail aids in personal and professional communication. E-mail on a LAN can enable staff to communicate within the building having not to leave their desks.

Flexible access – people access their files from any computer in the organization (building).

Workgroup computing – workgroup software e.g. Microsoft Back Office allows many users to work on a document or project concurrently.

15.4 Disadvantages of Networks

- server faults stops applications being available.
- network faults can cause loss of data
- network faults could lead to loss of resources
- system is open to hackers
- users work dependent on network
- could become insufficient
- resources could be located too far from the users
- decisions tend to become centralized

Wide Area Network (WAN)

WAN connects computers over a wide geographical area. The largest WAN is an Internet.

CHAPTER 16
INTERNET

LEARNING OBJECTIVES

By the end of this chapter, the students should be able to:

- Define the term Internet
- List the uses of internet
- Explain advantages and disadvantages of internet
- Discuss security issues of internet

16.1 Computer Networks

Computers can be connected to other computers in what is referred to as a network. A Computer network is made up of a number of connected computers each having their own processor.

A network allows computers to connect or communicate with each other to share resources e.g files, printers, software, therefore a network allow computing resources to be used more efficiently between groups of users.

The computers on a network may be linked through cables, telephone lines, radio waves, satellites, or infrared light beams.

A ***Networked computer*** is a computer that is connected to other computers whilst***Stand alone*** computer is a computer that is not connected to other computers.

Advantages of being on a network.

-
- Expensive resources e.g. printers can be shared
- Files stored on a single hard disk can be shared
- Incompatible hardware can be linked and share things between them.
- Data is less likely to be lost because of such things as backup
- Memos or messages can be passed from one machine to another providing a fast, low cost messaging system.
- Greater power, because each additional machine will bring in additional processing power.
- If one machine breaks down the other will continue providing services.
- Can be used for mailing

Benefits of Client/Server Computing)
A Client/server LAN consists of requesting microcomouters, called clients and supplying devices that provide a service called servers.

Processing power is spread over several computers (Greater resilience). If the server or one PC breaks down, other locations can carry on the processing.

(i) Shared data & programs **-** Programs and data are centrally held on the server and shared by all PCs – no duplication of data on individual printers.
(ii) Shared work-load. Each PC in the network can the same work i.e operators can share the work
(iii) Shared peripherals. Expensive resources like printers can be shared.
(iv) Incompatible hardware such as PCs and Macs can be linked by means of the network and files passed between them
(v) Information stored on remote computers can be accessed
(vi) Data is less likely to be lost or accidentally erased because formal house keeping procedures like regular backups are regularly instituted
(vii) Improved communications. Memos and other massages can be passed from one machine to another. e-mails can be used to send messages
(viii) Cheaper – by use of shared resources and can use diskless terminals
(ix) Scalability – computing power can be added as necessary and the network can be extended as the organisation grows.

Two main types of networks are local area network (LAN) and wide area network (WAN).

Local Area Networks

This is a computer network covering a small physical area like a home ,office or small group of buildings such as school or an airport, a LAN is located in a single building or on a single site.
LANs are mostly used to connect personal computers and workstations in a company offices and factories to share resources.
LANs are restricted in size, they may use a transmission technology consisting of a cable to which all the machines are attached.
LANs often use microcomputers (PCs) as the ones that keep the data messages circulating.

16.2 Current Uses of the Internet

- Dissemination of information
- Product/service development through instantaneous test marketing.
- Transaction processing e-commerce.
- Relationship enhancement between various groups of stakeholders.

- Recruitment and job search.
- Entertainment – music, games, humour.

16.3 Growth of Internet

i. Many households are establishing multiple internet access portals e.g. digital TV

ii. Changes in the telecommunication market are likely to mean that internet connection will become cheaper

iii. Digital television and WAP enable mobile phone permit internet to be accessed without using PC

iv. For many internet interface is PDA (Personal Digital Assistance)

v. Internet kiosks are becoming increasing in shopping centres

vi. The degree to which a customer can be persuaded to believe that using the internet will deliver some value-added interms of speed, simplicity and price.

16.4 Business to business (B2B) is widely used because:

- Internet shifts power from seller to buyer by switching costs – buyers want one stop shopping with information they believe and advice they can trust. Internet reduces transaction cost thus stimulates economic activity
- The speed, range and accessibility of information on the internet and the low cost of capturing and distributing it create new commercial possibilities

- The internet provides opportunities to organize for and automate tasks which would previously have required more costly interaction with the organisation (low touch or zero touch approaches).
- FAQ (Frequently Asked Questions) carefully structured set of answers can deal with many customer interactions
- Status checking – what is my bank balance, where is my order.
- Keyword search – the ability to search provides web users with opportunity to find information in large and complex websites.

- Wizards (interview style interface) and intelligent algorithms. These can help diagonosis.
- E-mail and systems to route and track unbound e-mail.
- Bulletin boards enable customers to interact with each other.
- Call back buttons enable customers to speak to someone in order to deal with and resolve a problem.
- Transaction processing – the tracking of orders and payment online.

16.5 Problems with the Internet

- The internet is not owned by any one body and there are no clear guidelines on how it should develop
- The quality of much of the information on the internet leaves much to be desired
- Downloading is time consuming
- Faster technology are expensive e.g. ISDN (Integrated Service Digital Network) and ADSL (Asymetric Digital Subscriber Live)
- Staff spend too much time browsing through non-work-related sites
- Security is a big issue

16.6 Internet Security Issues

- Corruption e.g. viruses can spread through the network
- Employees can get access to unauthorised parts of the system and can deliberately damage
- Hackers can access your systems via internet
- Employees may download inaccurate information or imperfect or virus-ridden software from external network
- Communication link may breakdown
- Information transmitted may be intercepted

- Hacking: involves attempting to gain unauthorized access to a computer system usually through telecommunication.

- Virus: is a software which infects programs and data and possibly damages them and which replicates itself. Viruses need opportunity spread. Therefore, they are usually placed in a software which is most likely to be copied e.g.
    - Free software (from the internet)
    - Pirated software (cheaper than original)
    - Games software
    - E-mail attachment

Examples of type of Virues

- Melissa – corrupts Microsoft Office documents
- Love Bug – attacks the operating system
- File viruses
- Boot sector virus
- Overwriting virus
- Worm
- Macro-virus

Safety measures

- Encryption
- Authentication
- Firewalls
- Dial back system

16.7 Electronic Commerce

E-commerce means conducting business electronically via a communication link.

Electronic Data Interchange (EDI)

- Reduces delays caused by post
- Avoids re-key data and save time and reduces errors
- Reduces administration costs

- Improves customer service

Problems

- Different file structures
- Compatibility of computers
- Different time schedules
- Difficult to manage a large number of trading partners

E-COMMERCE AND THE WEB

Distribution: internet can be used to get certain products directly into people's homes.

E-marketing: businesses use internet to provide information about their own products and services.

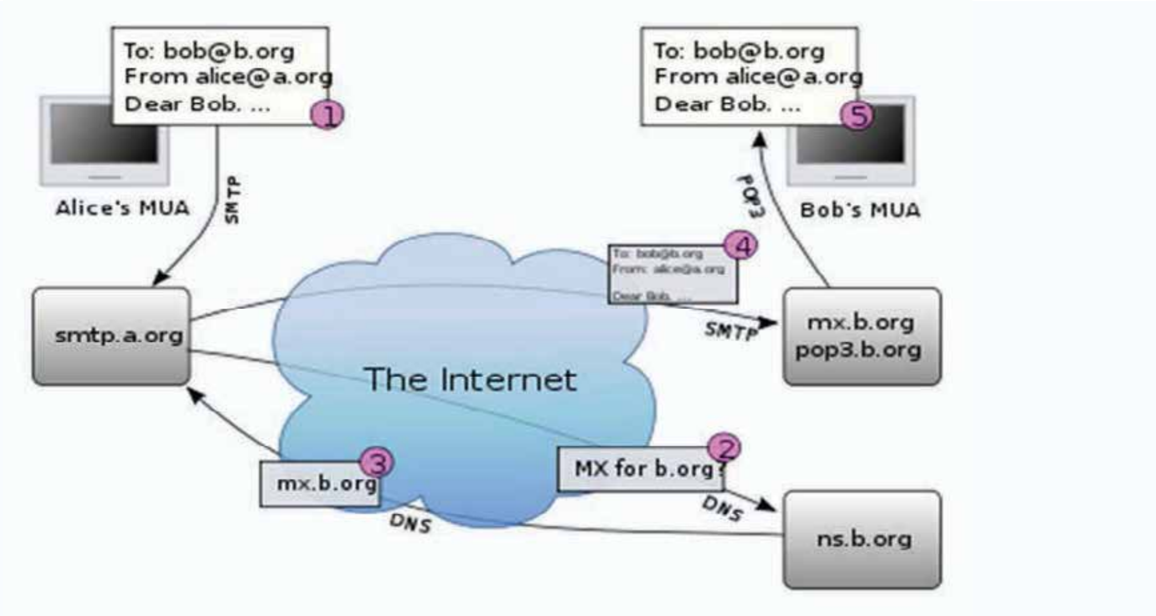
16.8 Electroni Mail (E-mail)

Electronic mail, most commonly abbreviated **email** or **e-mail**, is a method of exchanging digital messages. E-mail systems are based on a store-and-forward model in which e-mail server computer systems accept, forward, deliver and store messages on behalf of users, who only need to connect to the e-mail infrastructure, typically an e-mail server, with a network-enabled device for the duration of message submission or retrieval. Originally, e-mail was always transmitted directly from one user's device to another's; nowadays this is rarely the case.

An electronic mail message consists of two components, the message *header*, and the message *body*, which is the email's content. The message header contains control information, including, minimally, an originator's email address and one or more recipient addresses. Usually additional information is added, such as a subject header field.

Operation overview

The diagram to the right shows a typical sequence of events that takes place when Alice composes a message using her mail user agent (MUA). She enters the e-mail address of her receipient, and hits the "send" button.



1. Her MUA formats the message in e-mail format and uses the Simple Mail Transfer Protocol (SMTP) to send the message to the local mail transfer agent (MTA), in this case smtp.a.org, run by Alice's Internet Service Provider (ISP).

2. The MTA looks at the destination address provided in the SMTP protocol (not from the message header), in this case bob@b.org. An Internet e-mail address is a string of the form localpart@exampledomain. The part before the @ sign is the *local part* of the address, often the username of the recipient, and the part after the @ sign is a domain name or a fully qualified domain name. The MTA resolves a domain name to determine the fully qualified domain name of the mail exchange server in the Domain Name System (DNS).

3. The DNS server for the b.org domain, ns.b.org, responds with any MX records listing the mail exchange servers for that domain, in this case mx.b.org, a server run by Bob's ISP.
4. smtp.a.org sends the message to mx.b.org using SMTP, which delivers it to the mailbox of the user bob.
5. Bob presses the "get mail" button in his MUA, which picks up the message using the Post Office Protocol (POP3).

That sequence of events applies to the majority of e-mail users. However, there are many alternative possibilities and complications to the e-mail system:

Internet e-mail messages consist of two major sections:

- *Header* — Structured into fields such as summary, sender, receiver, and other information about the e-mail.
- *Body* — The message itself as unstructured text; sometimes containing a signature block at the end. This is exactly the same as the body of a regular letter.

16.9 Uses of e-mail

In society

There are numerous ways in which people have changed the way they communicate in the last 50 years; e-mail is certainly one of them. Traditionally, social interaction in the local community was the basis for communication – face to face. Yet, today face-to-face meetings are no longer the primary way to communicate as one can use a landline telephone, mobile phones, fax services, or any number of the computer mediated communications such as e-mail.

Research has shown that people actively use e-mail to maintain core social networks, particularly when others live at a distance. However, contradictory to previous research, the results suggest that increases in Internet usage are associated with decreases in other modes of communication, with proficiency of Internet and e-mail use serving as a mediating factor in this relationship. With the introduction of chat messengers and video conference, there are more ways to communicate.

Flaming

Flaming occurs when a person sends a message with angry or antagonistic content. Flaming is assumed to be more common today because of the ease and impersonality of e-mail communications: confrontations in person or via telephone require direct interaction, where social norms encourage civility, whereas typing a message to another person is an indirect interaction, so civility may be forgotten] Flaming is generally looked down upon by Internet communities as it is considered rude and non-productive.

E-mail bankruptcy

Also known as "e-mail fatigue", e-mail bankruptcy is when a user ignores a large number of e-mail messages after falling behind in reading and answering them. The reason for falling behind is often due to information overload and a general sense there is so much information that it is not possible to read it all.

In business

E-mail was widely accepted by the business community as the first broad electronic communication medium and was the first ‘e-revolution’ in business communication. E-mail is very simple to understand and like postal mail, e-mail solves two basic problems of communication: logistics and synchronization

LAN based email is also an emerging form of usage for business. It not only allows the business user to download mail when *offline*, it also provides the small business user to have multiple users e-mail ID's with just *one e-mail connection*.

Problems

- *The problem of logistics*

Much of the business world relies upon communications between people who are not physically in the same building, area or even country; setting up and attending an in-person meeting, telephone call, or conference call can be inconvenient, time-consuming, and costly. E-mail provides a way to exchange information between two or more people with no set-up costs and that is generally far less expensive than physical meetings or phone calls.

- *The problem of synchronisation*

With real time communication by meetings or phone calls, participants have to work on the same schedule, and each participant must spend the same amount of time in the meeting or call. E-mail allows asynchrony: each participant may control their schedule independently.

16.10 Disadvantages of E-mail

Most business workers today spend from one to two hours of their working day on e-mail: reading, ordering, sorting, 're-contextualizing' fragmented information, and writing e-mail. The use of e-mail is increasing due to increasing levels of globalisation—labour division and outsourcing amongst other things. E-mail can lead to some well-known problems:

- *Loss of Context*: which means that the context is lost forever; there is no way to get the text back. Information in context (as in a newspaper) is much easier and faster to understand than unedited and sometimes unrelated fragments of information. Communicating in context can only be achieved when both parties have a full understanding of the context and issue in question.
- *Information overload*: E-mail is a push technology—the sender controls who receives the information. Convenient availability of mailing lists and use of "copy all" can lead to people receiving unwanted or irrelevant information of no use to them.
- *Inconsistency*: E-mail can duplicate information. This can be a problem when a large team is working on documents and information while not in constant contact with the other members of their team.

Despite these disadvantages, e-mail has become the most widely used medium of communication within the business world.

Privacy concerns about e-mail

E-mail privacy, without some security precautions, can be compromised because:

- e-mail messages are generally not encrypted
- e-mail messages have to go through intermediate computers before reaching their destination, meaning it is relatively easy for others to intercept and read messages
- many Internet Service Providers (ISP) store copies of e-mail messages on their mail servers before they are delivered. The backups of these can remain for up to several months on their server, despite deletion from the mailbox.
- the "Received:"-fields and other information in the e-mail can often identify the sender, preventing anonymous communication.

CHAPTER 17

PROTECTING DATA

The risks to data:

The data stored in a computer can often be far more valuable than the actual computer equipment. Losing such data could put a company out of business.

Examples of valuable data are:

a company's financial records;
customers' details;
records of company stocks and shares;
data collected from experiments or research.

Data can be damaged or destroyed in the following ways:

- mistakes by users such as deleting files
- hackers gaining access to systems and changing or deleting data
- computer fraud where data is changed to benefit individuals
- theft of computer hardware such as laptops with data on the hard disk(s)
- infection of systems and data by computer viruses.
- deliberate and malicious damage by users of the system, possible in revenge for being made redundant for example.
- disasters such as fire, floods, earthquakes etc. destroying equipment
- breakdown of hardware, particularly disk drives

Why computer data is easier to misuse than paper-based data:

Action	Computer data	Paper-based data
Gaining access to the data	Anonymous remote accessing using the Internet (hacking) can be done if someone has the skills with a low risk of being caught.	Someone would have to physically break into where the printed data is stored.
Making a copy of data and removing it	Files can easily copied onto removable storage media (*for example, a USB flash drive which can easily be hidden and removed*). Files could also be removed by being sent as an email attachment.	Paper would have to photocopied or photographed. Large amounts of photocopied pages could be bulky to remove easily without detection.
Altering data without being detected	It is hard to spot altered data, even for an expert looking closely at the file properties.	It is usually easier to spot where printed data has been altered.
Searching for valuable data	It is easy and fast to search and sort computer data using software.	It is very time consuming to search and sort pages of printed data.

Reducing the risks of data loss:
Making a backup of data:
Keeping hardware secure:

Protect the computer itself by using locks on doors and windows and using security bolts to fix the computer permanently to the desktop.
Laptops are often fitted with locking points so a cable can be used to secure them to a desk etc.
Floppy disks are easily physically damaged and must also be kept away from magnetic fields and dust.

Illegal access to computers:

Hacking involves unauthorised access to computer systems, usually by finding out or breaking passwords. Once into a computer system a hacker can do an enormous amount of damage. Stand-alone computers are less of a risk as the damage is limited to just that computer. Computers which form part of a network or those with external links, such as attached modems, are at greater risk as they can be accessed from other computers.

There are **three levels of hacking**, all of which are punishable by fines or imprisonment under the **Computer Misuse Act of 1990 (UK)**.

Accessing information without permission - viewing information (*i.e. looking through someone's personal email*) or even attempting to access it is illegal.

Accessing information and using it for illegal purposes - the information that has been accessed is used by the hacker or passed to someone else who uses it for illegal purposes (*i.e. looking up details of a rival companies research projects or using information for blackmail*)

Accessing information and altering it for fraudulent purposes - the hacker alters the information they have accessed in some way (*i.e. altering financial details of customers in a bank or introducing a virus*)

Ways to restrict access:

Using user Ids (*user identification*) and passwords

Blocking external access - external hackers can be blocked by disconnecting modems from the telephone line when not being used. Systems with permanent phone connections need special software called a 'firewall' to try and prevent unauthorised access over networks and the Internet.

Physically making a computer or network difficult to access - by keeping individual computers in secure areas and blocking access to removable media so data cannot easily be copied.

Encryption - this involves transforming the data so that it is unreadable without a digital key. Encrypted data is therefore meaningless if it is accessed or intercepted. It is particularly important to encrypt data transferred over wireless networks because the data is so easy to intercept.

Computer viruses:

There are **three** things to remember when explaining what a virus:

It is a **program**: a set of instructions which can be introduced into a computer via a floppy disk, email attachment or the Internet.

It makes **copies** of itself: the program contains instructions that make it attach copies of itself to system files or programs. It can therefore spread to other programs on your hard disk and onto floppy disks or email itself to all the contacts in your email address book.

It causes **damage**: the effects can be devastating and cost millions of pounds to fix. They can alter programs or files and completely disable a computer's operating system.

Because a virus alters files it is illegal under the **Computer Misuse Act of 1990** to deliberately infect a computer system with a virus and the offence is punishable by fines or imprisonment in the same way as other hacking offences.

Protecting against computer viruses:

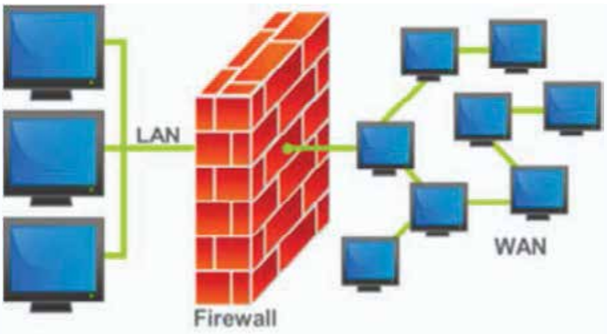
Anti-virus software is usually required to detect and then destroy them but it is important to have **regular updates** to deal with new viruses. The anti-virus software scans the files on a computer looking for any viruses which it then removes and alerts the user.

To reduce the risk:

Never use a floppy disk of memory stick given to you from an untrustworthy source or pass floppy disks around between your friends.

- Do not start up a computer with a floppy disk left in the drive.
- Set the computer BIOS so the boot sequence does not start with the floppy disk drive.
- Keep the write-protection hole covered on floppy disks that are used to store original "clean" programs - so they can be reinstalled with confidence.
- Never open an email **attachment** that comes from someone unknown or is not clearly explained in the email message that it is attached to. Even then **BE SUSPICIOUS** - virus program writers are always thinking of new ways to get computer users to open attachments and run the virus!
- Install a virus protection program and keep it up-to-date!
- Be careful of the websites you visit and the hyperlinks you click on. Many websites are designed to trick visitors into downloading viruses, trojans or spyware by offering free downloads. If you are unsure of a hyperlink, always hover over it with the mouse before clicking so you can see (*in the bottom-left of the browser screen*) where it will take you to.

Firewall (computing)



A **firewall** is a part of a computer system or network that is designed to block unauthorized access while permitting authorized communications. It is a device or set of devices which is configured to permit or deny computer based application upon a set of rules and other criteria.

Firewalls can be implemented in either hardware or software, or a combination of both. Firewalls are frequently used to prevent unauthorized Internet users from accessing private networks connected to the Internet, especially **intranets**. All messages entering or leaving the intranet pass through the firewall, which examines each message and blocks those that do not meet the specified security criteria.

There are several types of firewall techniques:

1. Packet filter: Packet filtering inspects each packet passing through the network and accepts or rejects it based on user-defined rules. Although difficult to configure, it is fairly effective and mostly transparent to its users. It is susceptible to IP spoofing.
2. **Application gateway**: Applies security mechanisms to specific applications, such as **FTP** and Telnet servers. This is very effective, but can impose a performance degradation.
3. **Circuit-level gateway**: Applies security mechanisms when a TCP or UDPconnection is established. Once the connection has been made, packets can flow between the hosts without further checking.
4. Proxy server: Intercepts all messages entering and leaving the network. The proxy server effectively hides the true network addresses.

Function

A **firewall** is a dedicated appliance, or software running on a computer, which inspects network traffic passing through it, and denies or permits passage based on a set of rules.

It is normally placed between a protected network and an unprotected network and acts like a gate to protect assets to ensure that nothing private goes out and nothing malicious comes in.

A firewall's basic task is to regulate some of the flow of traffic between computer networks of different trust levels. Typical examples are the Internet which is a zone with no trust and an internal network which is a zone of higher trust. A zone with an intermediate trust level, situated between the Internet and a trusted internal network, is often referred to as a "perimeter network" or Demilitarized zone (DMZ).

A firewall's function within a network is similar to physical firewalls with fire doors in building construction. In the former case, it is used to prevent network intrusion to the private network. In the latter case, it is intended to contain and delay structural fire from spreading to adjacent structures.

CHAPTER 18

COMPUTER FILES

18.1 Introduction

It should be mentioned that basic work on computerized data is organized in files. To understand a file it is good to look at its components.

18.2 Components of a File

a) **Field** : A field is a group of characters that make up a data item eg. name, number, etc. A field is a basic item that a computer can access in a file. A field can have one or more characters.

b) **Record** : A record is a group of related fields. This is the basic aggregation of data items that would make sense (information) an example of a record would be as follows:

Name	W.K. Chidyaonga
Account No.	020447
Balance	500.00

c) **File** : A group of related records form a file. For example records about several customers to a supply will form a file. A computer file can be likened to physical paper file or office file. In it one finds several documents but act their because of some common characteristic they share.

Computer files are stored on the computer itself or on backing storage when being created each file is given a name and, it is this name which is used to retrieve or access it.

18.3 Types of Files

a) **Transaction Files**

These are files that are accessed frequently and the data is inserted, deleted or updated fairly frequently. They contain records for day to day transactions.

b) Master Files

This contains important back up information records and they are accessed less frequently. To update master files transaction file data is used.

c) **Reference Files**

These contain reference information only like catalogues and other materials like standard prices, measurements and descriptions.

END OF CHAPTER EXERCISES

1. Distinguish giving examples between the terms "output devices" and "output media".

2. What considerations would you make in selecting a media?

3. List the types of computer files and briefly describe each one.

4 The quality of products at various stages of a production run necessarily suffers a certain amount of variability. The dimensions of a standard part of a machine, for example, will not be precisely the same from one unit to another, nor will otherwise identical products precisely resemble each of other in surface finish. Bags of fertilizer, sugar and so forth having the same advertised net weight may not have precisely the same weights. Inspection methods of some kind are therefore necessary in order to ensure that products meet minimum quality requirements before delivery to the consumer. Traditionally, inspection has usually been of the finished product or component, the product being passed or rejected, often on the basis of a sample inspection. This method has the disadvantage of determining the quality of the product only when the manufacturing process is complete, and is responsible for a great deal of avoidable waste. Modern practice is to aim at controlling quality, i.e; not to look for defective productive products or components after they have been made, but to anticipate tendencies in the process that might cause them to become defective. This is achieved by maintaining a continuous inspection, at regular stages during the manufacturing process, by examining samples of a product or its components and charting the results in a particular way. The chart gives visual indication enabling corrective action to be taken as soon as the productive process begins to go wrong.

Using the passage practice the following special features:

Operation

Deleting character,	Deleting words
Inserting character;	Inserting words or section
Copying; Moving;	General movement
Bold print	
Column selling	
Saving of file	
Retrieving of the file	

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PRACTICAL MATHEMATICS & COMPUTING (FA2)
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