

Practice Coach PLUS 

Coached Instruction Supplement

Mathematics **7**

Practice Coach PLUS, Coached Instruction Supplement, Mathematics, Grade 7
678NASP

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Table of Contents

Domain 1	The Number System	
Lesson 1	Relate Fractions, Decimals, and Percents	4
Lesson 2	Solve Problems with Percents	8
Lesson 3	Terminating and Repeating Decimals	13
Lesson 4	Divide Fractions	15
Lesson 5	Add and Subtract Integers	19
Lesson 6	Multiply and Divide Integers	25
Lesson 7	Add and Subtract Rational Numbers	29
Lesson 8	Multiply and Divide Rational Numbers	34
Domain 2	Ratios and Proportional Relationships	
Lesson 9	Ratios and Rates	40
Lesson 10	Proportions	43
Lesson 11	Proportional Relationships	46
Lesson 12	Represent Proportional Relationships	49
Domain 3	Expressions and Equations	
Lesson 13	Write Algebraic Expressions	54
Lesson 14	Simplify and Evaluate Algebraic Expressions	57
Lesson 15	Add and Subtract Algebraic Expressions	62
Lesson 16	Write Algebraic Equations	65
Lesson 17	Solve Equations	68
Lesson 18	Use Algebra to Solve Word Problems	72
Lesson 19	Inequalities	76
Domain 4	Geometry	
Lesson 20	Scale Drawings	82
Lesson 21	Construct Geometric Shapes	87
Lesson 22	Cross Sections of Three-Dimensional Figures	91
Lesson 23	Circles	94
Lesson 24	Angles	98
Lesson 25	Area	103
Lesson 26	Surface Area	107
Lesson 27	Volume	110

Domain 5 Statistics and Probability

Lesson 28	Probability	112
Lesson 29	Compound Events	116
Lesson 30	Samples	120
Lesson 31	Measures of Central Tendency	125
Lesson 32	Measures of Variation	131
Lesson 33	Mean Absolute Deviation	137
Lesson 34	Make Predictions Using Data	143
Lesson 35	Compare Data Sets	146
Answer Key		152

Relate Fractions, Decimals, and Percents



Getting the Idea

Rational numbers are numbers that can be expressed in the form $\frac{a}{b}$, where a and b are **integers** and $b \neq 0$. **Fractions, decimals, and percents** are rational numbers that can be used to show parts of a whole. Percent means *per hundred*. For example, 70% of a number means $\frac{70}{100}$ times the quantity. The symbol for percent is %.

You can convert rational numbers to different forms. To convert a percent to a fraction, write the percent as the **numerator** over a **denominator** of 100. Then write the fraction in **simplest form** using the **greatest common factor (GCF)**.

Example 1

Write 72% as a fraction in simplest form.

Strategy Write the percent as a fraction with a denominator of 100. Simplify.

Step 1

Remove the percent sign. Write the percent as the numerator and 100 as the denominator.

$$72\% \rightarrow \frac{72}{100}$$

Step 2

Simplify the fraction using the GCF.

The GCF of 72 and 100 is 4.

Divide the numerator and denominator by 4.

$$\frac{72}{100} = \frac{72 \div 4}{100 \div 4} = \frac{18}{25}$$

Solution $72\% = \frac{18}{25}$

Example 2

What is 84% written as a decimal?

Strategy Remove the percent sign and move the decimal point two places to the left.

$$84\% \rightarrow 84. \rightarrow 0.84$$

Solution $84\% = 0.84$

Example 3

What is $\frac{2}{5}$ written as a decimal?

Strategy Write an equivalent fraction with a denominator of 10.

Step 1

Find a fraction equivalent to $\frac{2}{5}$ that has a denominator of 10.

Since $5 \times 2 = 10$, multiply the numerator and denominator by 2.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

Step 2

Write the decimal equivalent of $\frac{4}{10}$.

$\frac{4}{10}$ is read “four tenths.”

$$\frac{4}{10} = 0.4$$

Solution $\frac{2}{5} = 0.4$

Example 4

Write 0.65 as a fraction in simplest form.

Strategy Write the digits after the decimal point as the numerator.
The denominator is the place value of the last digit. Simplify.

Step 1

Write the digits 65 as the numerator of the fraction.

The denominator is 100 because the last digit, 5, is in the hundredths place.

$$0.65 = \frac{65}{100}$$

Step 2

Simplify using the GCF.

The GCF of 65 and 100 is 5.

$$\frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$$

Solution $0.65 = \frac{13}{20}$

To convert a decimal to a percent, multiply the decimal by 100 and insert a percent sign. Multiplying a decimal by 100 is the same as moving the decimal point 2 places to the right.

Example 5

What is 0.875 written as a percent?

Strategy **Multiply the decimal by 100.**

Move the decimal point two places to the right.

$$0.875 \times 100 = 87.5$$

Insert a percent sign.

$$87.5\%$$

Solution **$0.875 = 87.5\%$**

Example 6

Write $\frac{16}{25}$ as a percent.

Strategy **Write an equivalent fraction.**

Step 1

Percent means per hundred, so write an equivalent fraction with a denominator of 100.

$25 \times 4 = 100$, so multiply the numerator and denominator by 4.

$$\frac{16}{25} = \frac{16 \times 4}{25 \times 4} = \frac{64}{100}$$

Step 2

Insert a percent sign next to the numerator.

$$\frac{64}{100} \rightarrow 64\%$$

Solution **$\frac{16}{25} = 64\%$**

If the denominator is not a factor of 100, convert the fraction to a decimal.

Then convert the decimal to a percent. Some decimals, such as $\frac{1}{3}$, are repeating decimals.

To write a repeating decimal as a percent, write the percent and the part that repeats as a fraction.

Example 7

What is $\frac{2}{3}$ written as a percent?

Strategy Convert the fraction to a decimal. Then convert the decimal to a percent.

Step 1

Divide the numerator by the denominator.

$$\frac{2}{3} = 2 \div 3 = 0.\overline{6}$$

Step 2

Multiply the decimal by 100.

$$0.\overline{6} \times 100 = 66.\overline{6}$$

Insert a percent sign.

$$66.\overline{6}\% \text{ or } 66\frac{2}{3}\%$$

Solution $\frac{2}{3} = 66\frac{2}{3}\%$



Coached Example

Maria received 55% of the vote in a student council election. What decimal and fraction, written in simplest form, are equivalent to the percentage of the vote Maria received?

To convert a percent to a decimal, remove the percent sign and move the decimal point 2 places to the _____.

The decimal _____ is equivalent to 55%.

To convert a percent to a fraction, write the percent as the numerator over a denominator of _____.

What is the GCF of the numerator and denominator? _____

Divide the numerator and denominator by _____.

Simplify. _____

55% written as a decimal is _____. 55% written as a fraction is _____.

Solve Problems with Percents



Getting the Idea

Percents are used for many things, such as the sale price of an item, the sales tax you pay on an item, and the interest earned on a bank deposit. Sometimes it is necessary to change a percent to a fraction or a decimal to solve a percent problem.

Example 1

What is 8% of 214?

Strategy **Change the percent to a decimal. Then multiply.**

Step 1

Change the percent to a decimal.

Remove the percent symbol and divide by 100.

$$8\% \longrightarrow \overset{\text{0}}{\underset{\text{0}}{8}} \longrightarrow 0.08$$

Step 2

Multiply the decimal by 214.

$$0.08 \times 214 = 17.12$$

Solution **8% of 214 is 17.12.**

A **discount** is the amount of money that is taken off the original price of an item.

The **sale price** is the cost of the item after the discount has been applied.

Example 2

Debbie sees a pair of jeans that was originally priced at \$36. The jeans are on sale for 25% off. What is the sale price of the jeans?

Strategy **Change the percent to a fraction. Then multiply to find the discount.**

Step 1

Write an expression to represent the discount.

Find 25% of \$36.

Step 2

Rename 25% as a fraction in simplest form.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

Step 3

Find $\frac{1}{4}$ of \$36, the amount of the discount.

$$\frac{1}{4} \times \$36 = \frac{1}{\cancel{4}^1} \times \frac{\overset{9}{\cancel{36}}}{1} = \frac{9}{1} = \$9$$

So, the amount of the discount is \$9.

Step 4

Subtract the discount from the original price.

$$\$36 - \$9 = \$27$$

Solution **The sale price of the jeans is \$27.**

Sales tax is a tax based on the cost of an item. You can find sales tax in the same way you find the percent of a number.

Example 3

A DVD costs \$19.80. The sales tax is 7%. What is the cost of the DVD, with tax included?

Strategy

Find the amount of sales tax. Then add the tax to the cost of the DVD.

Step 1

Change 7% to a decimal.

$$7\% = 0.07$$

Step 2

Multiply the decimal by the cost of the DVD.

$$0.07 \times \$19.80 = \$1.386$$

Round \$1.386 to \$1.39.

So, the tax is \$1.39.

Step 3

Add the sales tax to the cost of the DVD.

$$\$19.80 + \$1.39 = \$21.19$$

Solution **The cost of the DVD, with tax included, is \$21.19.**

Interest is the amount of money the bank pays you for its use of the money in a savings account. Interest could also be the amount of money you pay a lender for the use of borrowed money.

The amount of money in your savings account or the amount you borrow is the **principal**. One type of interest is **simple interest**. To calculate simple interest, use the formula $I = prt$, where I represents simple interest, p represents the principal, r represents the rate of interest, and t represents time, in years.

Example 4

Jen deposited \$750 in her savings account. She had the money in the account for 18 months without making any deposits or withdrawals. The account earns 2% annual simple interest. How much money did Jen earn in simple interest?

Strategy Use the formula $I = prt$.

Step 1

Convert the time to years and the rate to a decimal.

$$18 \text{ months} = 1\frac{1}{2} \text{ years} = 1.5 \text{ years}$$

$$2\% \rightarrow 0.02$$

Step 2

Substitute the known values into the formula.

$$I = prt$$

$$I = \$750 \times 0.02 \times 1.5$$

Step 3

Multiply.

$$I = \$750 \times (0.02 \times 1.5)$$

$$= \$750 \times 0.03$$

$$= \$22.50$$

Solution Jen earned **\$22.50** in simple interest.

To find the percent increase or the percent decrease, write the difference in amounts as the numerator of a fraction with the original amount as the denominator. For prices, a percent of increase is a markup and a percent of decrease is a markdown. Then change the fraction to a percent.

Example 5

A computer game that originally cost \$40 is on sale for \$28. What is the percent decrease in the price of the game?

Strategy Write a fraction to represent the decrease.

Step 1

Write and simplify a fraction.

$$\frac{\text{original price} - \text{sale price}}{\text{original price}} = \frac{\$40 - \$28}{\$40} = \frac{12}{40} = \frac{3}{10}$$

Step 2

Convert the fraction to a percent.

$$\frac{3}{10} = \frac{30}{100} = 30\%$$

Solution The percent decrease in the price of the game is 30%.

Example 6

A basketball team won 15 games during the 2008–2009 season. In the 2009–2010 season, the team won 21 games. What was the percent of increase in games won?

Strategy Write a fraction to represent the increase.

Step 1

Write and simplify a fraction.

$$\frac{\text{games won in 2009–2010} - \text{games won in 2008–2009}}{\text{games won in 2008–2009}} = \frac{21 - 15}{15} = \frac{6}{15} = \frac{2}{5}$$

Step 2

Convert the fraction to a percent.

$$\frac{2}{5} = 0.4 = 40\%$$

Solution The percent of increase in games won is 40%.

A **percent error** measures how far off an estimate is to the actual value. The percent error is the **absolute value** of the difference between an estimate and the actual value, divided by the actual value. Multiply by 100 to express the error as a percent.

Example 7

Johnny used mental math to estimate that 11×5 is approximately 50.

What is his percent error?

Strategy Find the actual value. Then calculate the percent error.

Step 1 Find the actual value of 11×5 .

$$11 \times 5 = 55$$

Step 2 Find the difference between the estimate and the actual value.

$$55 - 50 = 5$$

Step 3 Divide the difference by the actual value. Round to the nearest hundredth.

$$\frac{5}{55} \approx 0.09$$

Step 4 Convert the decimal to a percent.

$$0.09 \rightarrow 9\%$$

Solution Johnny's percent error is about 9%.



Coached Example

Angela and Sadie had dinner at a restaurant. Their bill was \$24.50. They left a 15% tip for their server. How much money did Angela and Sadie spend for their bill and tip?

To solve this problem, add the amount of the _____ and the amount of the _____.

The bill was \$_____.

To find the amount of the tip, first change _____ to a decimal.

_____ % \rightarrow _____

Then multiply the decimal by the amount of the _____.

_____ \times \$_____ = \$_____

Round the amount of the tip to the nearest cent. \$_____

Add the amount of the _____ and the amount of the _____.

\$_____ + \$_____ = \$_____

Angela and Sadie spent \$_____ for their bill and tip.

Terminating and Repeating Decimals



Getting the Idea

Terminating and repeating decimals are also rational numbers. A **terminating decimal** ends. A **repeating decimal** does not end. Instead, it repeats a digit or pattern of digits over and over.

To determine whether a fraction can be expressed as a terminating or repeating decimal, convert the fraction to a decimal using long division. The **quotient** will terminate or repeat.

Example 1

Can the fraction $\frac{1}{3}$ be expressed as a terminating or repeating decimal?

Strategy Divide the numerator by the denominator. Analyze the quotient.

Step 1

Divide the numerator by the denominator.

$$\begin{array}{r}
 0.333 \dots \\
 3 \overline{)1.000} \\
 \underline{-9} \\
 10 \\
 \underline{-9} \\
 10 \\
 \underline{-9} \\
 1
 \end{array}$$

Step 2

Does the decimal end?

No, the decimal does not end, so it is not terminating.

Step 3

Does one digit or a pattern of digits in the decimal repeat?

Yes, the digit 3 repeats.

So, the decimal is repeating.

Solution The fraction $\frac{1}{3}$ can be expressed as a repeating decimal.

To indicate the numbers that repeat in a repeating decimal, draw a bar over the repeating digit or digits. The repeating decimal $0.333 \dots$ can be written as $0.\overline{3}$.

Example 2

Can the fraction $\frac{7}{8}$ be expressed as a terminating or repeating decimal?

Strategy Divide the numerator by the denominator. Analyze the quotient.

Step 1

Divide the numerator by the denominator.

$$\begin{array}{r} 0.875 \\ 8 \overline{)7.000} \\ \underline{-64} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Step 2

Does the decimal end?

Yes, the decimal ends.

So, the decimal is terminating.

Solution The fraction $\frac{7}{8}$ can be expressed as a terminating decimal.



Coached Example

Can the fraction $\frac{3}{8}$ be expressed as a terminating or repeating decimal?

Divide the _____ by the _____.

$$8 \overline{)3}$$

Does the decimal end? _____

The fraction $\frac{7}{8}$ can be expressed as a _____ decimal.

Divide Fractions



Getting the Idea

To divide fractions, first find the **reciprocal** of the **divisor**. Then multiply the **dividend** by the reciprocal of the divisor. Reciprocals are two numbers whose **product** is 1.

You can find the reciprocal of a fraction or whole number by switching the numerator and the denominator. For example, $\frac{3}{8}$ and $\frac{8}{3}$ are reciprocals because $\frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$.

Example 1

Divide.

$$\frac{3}{5} \div \frac{2}{3} = \square$$

Strategy **Multiply the dividend by the reciprocal of the divisor.**

Step 1

Rewrite as a multiplication problem, using the reciprocal of the divisor.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2}$$

Step 2

Multiply.

$$\frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

$\frac{9}{10}$ is in simplest form.

Solution $\frac{3}{5} \div \frac{2}{3} = \frac{9}{10}$

A **complex fraction** is a fraction in which the numerator and/or denominator contains a fraction. Recall that a fraction represents a quotient. The quotient is the numerator divided by the denominator (where the denominator is not equal to 0). For example, $\frac{3}{4} = 3 \div 4$.

The division expression in Example 1 can be written as a complex fraction: $\frac{\frac{3}{5}}{\frac{2}{3}}$.

The numerator of the complex fraction is $\frac{3}{5}$ and the denominator is $\frac{2}{3}$.

The complex fraction and the division expression are equivalent: $\frac{\frac{3}{5}}{\frac{2}{3}} = \frac{3}{5} \div \frac{2}{3}$.

Simplifying a complex fraction is the same as dividing its numerator by its denominator.

Example 2

Simplify.

$$\frac{\frac{1}{4}}{\frac{1}{12}} = \square$$

Strategy Multiply the numerator by the reciprocal of the denominator.

Step 1

Rewrite as a multiplication problem using the reciprocal of the denominator.

The reciprocal of $\frac{1}{12}$ is $\frac{12}{1}$.

$$\frac{1}{4} \div \frac{1}{12} = \frac{1}{4} \times \frac{12}{1}$$

Step 2

Multiply.

$$\frac{1}{4} \times \frac{12}{1} = \frac{12}{4}$$

Step 3

Write the answer in simplest form.

$$\frac{12 \div 4}{4 \div 4} = \frac{3}{1} = 3$$

Solution

$$\frac{\frac{1}{4}}{\frac{1}{12}} = 3$$

Example 3

Simplify.

$$\frac{\frac{5}{8}}{\frac{2}{3}} = \square$$

Strategy Multiply the numerator by the reciprocal of the denominator.

Step 1

Rewrite as a multiplication problem using the reciprocal of the denominator.

$$\frac{5}{8} \div \frac{2}{3} = \frac{5}{8} \times \frac{3}{2}$$

Step 2

Multiply.

$$\frac{5}{8} \times \frac{3}{2} = \frac{15}{16}$$

Solution

$$\frac{\frac{5}{8}}{\frac{2}{3}} = \frac{15}{16}$$

To divide **mixed numbers**, first rewrite the mixed numbers as **improper fractions**. Then follow the rules for dividing fractions.

Example 4

Jamie divided $5\frac{1}{4}$ pounds of apples into baskets that hold $1\frac{3}{4}$ pounds each.

How many baskets did she use?

Strategy Rewrite the mixed numbers as improper fractions. Then divide.

Step 1

Write an expression to represent the problem.

$$\text{Find } 5\frac{1}{4} \div 1\frac{3}{4}.$$

Step 2

Rewrite the mixed numbers as improper fractions.

$$\begin{aligned} 5\frac{1}{4} &\rightarrow \frac{(5 \times 4) + 1}{4} = \frac{21}{4} \\ 1\frac{3}{4} &\rightarrow \frac{(1 \times 4) + 3}{4} = \frac{7}{4} \\ 5\frac{1}{4} \div 1\frac{3}{4} &= \frac{21}{4} \div \frac{7}{4} \end{aligned}$$

Step 3

Rewrite as a multiplication problem using the reciprocal of the divisor.

$$\begin{aligned} \text{The reciprocal of } \frac{7}{4} \text{ is } \frac{4}{7}. \\ \frac{21}{4} \div \frac{7}{4} &= \frac{21}{4} \times \frac{4}{7} \end{aligned}$$

Step 4

Simplify the factors and multiply.

$$\begin{array}{c} 3 \\ \cancel{21}^1 \times \cancel{4}^1 \\ 1 \end{array} = \frac{3 \times 1}{1 \times 1} = \frac{3}{1}$$

Step 5

Simplify.

$$\frac{3}{1} = 3$$

Solution Jamie used 3 baskets.

Any whole number can be expressed as a fraction. For example, $4 = \frac{4}{1}$. So, the reciprocal of a whole number divisor is a unit fraction. For example, the reciprocal of 4 is $\frac{1}{4}$.

Example 5

Divide.

$$6\frac{5}{8} \div 3 = \boxed{}$$

Strategy Rewrite the whole number as a fraction. Then find the reciprocal.

Step 1

Rewrite $6\frac{5}{8}$ as an improper fraction. Write the reciprocal of 3.

$$6\frac{5}{8} \rightarrow \frac{(6 \times 8) + 5}{8} = \frac{53}{8}$$

The reciprocal of 3 is $\frac{1}{3}$.

Step 2

Rewrite as a multiplication problem and solve.

$$6\frac{5}{8} \div 3 = \frac{53}{8} \times \frac{1}{3}$$

Step 3

Multiply.

$$\frac{53}{8} \times \frac{1}{3} = \frac{53 \times 1}{8 \times 3} = \frac{53}{24}$$

Step 4

Simplify the product.

$$\frac{53}{24} = 2\frac{5}{24}$$

Solution

$$6\frac{5}{8} \div 3 = 2\frac{5}{24}$$



Coached Example

Mr. Camara cuts a 15-foot wooden board into pieces that are each $1\frac{2}{3}$ feet long. How many pieces of wood does he have?

Let w represent the number of pieces of wood.

Write a number sentence to represent this problem. _____

Rewrite 15 as an improper fraction. _____

Rewrite $1\frac{2}{3}$ as an improper fraction. _____

Rewrite the number sentence using improper fractions. _____

To divide fractions, multiply the dividend by the _____ of the divisor.

The reciprocal of the divisor is _____.

Rewrite as a multiplication problem using the reciprocal of the divisor.

$$\text{_____} \div \text{_____} = \text{_____} \times \text{_____}$$

Multiply.

Simplify the product. _____

Mr. Camara has _____ pieces of wood.

Add and Subtract Integers



Getting the Idea

You can use a number line to add integers. Start at the point that represents the first integer. To add a positive integer, move to the right. To add a negative integer, move to the left.

Example 1

Find the sum of 3 and its additive inverse.

Strategy Use a number line.

Step 1

Write an addition expression for the sum.

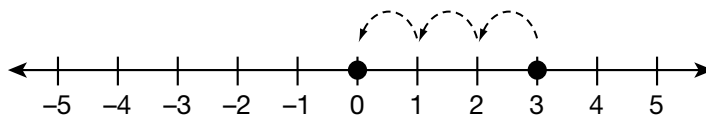
The additive inverse of 3 is -3 .

Find $3 + (-3)$.

Step 2

Use a number line to add.

Start at 3. Since you are adding a negative integer, move 3 units to the left.



The sum is 0.

Solution The sum of 3 and its additive inverse is 0.

$3 + (-3) = 0$ is an example of the **existence of the additive inverse property**. It states that the sum of a number and its **additive inverse** is 0.

In Example 1, the sum of $3 + (-3)$ is 0, located a distance of 3 units to the left of 3.

So, $(-3) + 3$ will also have the sum of 0 because it is located 3 units to the right of -3 .

Let a and b represent two integers. To find the sum of $a + b$ on a number line, start at a and move a distance of $|b|$. Move to the right of a if b is positive and to the left of a if b is negative. The sign of the sum depends upon the direction and the number of units moved from a .

Example 2

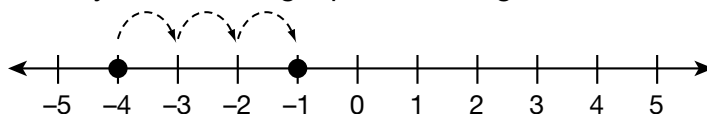
Find the sum.

$$-4 + 3 = \square$$

Strategy Use a number line to add the two integers.

Start at -4 .

Since you are adding a positive integer, move 3 units to the right.



The sum is -1 .

Solution $-4 + 3 = -1$

You can use the following rules to add integers.

Rules for Adding Two Integers

- When integers have the same sign, add the absolute values and use the sign of the addends in the sum.
- When integers have different signs, find the difference of their absolute values. Then use the sign of the addend with the greater absolute value in the sum.

Example 3

Add.

$$-11 + (-8) = \square$$

Strategy **Apply the rules for adding two integers.**

The integers have the same sign, so add the absolute values. The sum will be negative.

$$|-11| = 11$$

$$|-8| = 8$$

$$11 + 8 = 19$$

Solution $-11 + (-8) = -19$

You can also use the properties of addition to add integers.

Example 4

Add.

$$24 + (-10) = \square$$

Strategy **Use the properties of addition.****Step 1**

Rewrite 24 as a sum with an addend of 10.

$$24 = (14 + 10)$$

Step 2

Rewrite the problem using the new form of 24.

$$24 + (-10) = (14 + 10) + (-10)$$

Step 3

Use the associative property of addition.

$$(14 + 10) + (-10) = 14 + (10 + (-10))$$

$$= 14 + 0$$

← The sum of a number and its additive inverse is 0.

$$= 14$$

Solution $24 + (-10) = 14$

A number line can also be used to subtract integers. To subtract a positive integer, move to the left. To subtract a negative integer, move to the right.

Example 5

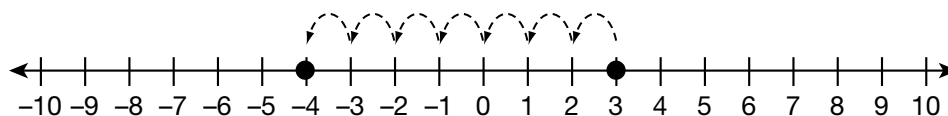
Find the difference.

$$3 - 7 = \square$$

Strategy Use a number line to subtract two integers.

Start at 3.

Since you are subtracting a positive integer, move 7 units to the left.



Solution $3 - 7 = -4$

Use these rules to subtract integers.

Rules for Subtracting Two Integers

- Write the additive inverse (opposite) of the number to be subtracted (the **subtrahend**).
- Change the minus sign to a plus sign.
- Apply the rules for adding two integers.

Example 6

Subtract.

$$-5 - 4 = \square$$

Strategy Add the opposite of the subtrahend.

Step 1

Find the opposite of the number to be subtracted.

The subtrahend is 4.

The opposite of 4 is -4 .

Step 2

Add the opposite of the subtrahend to the minuend.

$$-5 - 4 = -5 + (-4)$$

Both integers being added have a negative sign. The sum will be negative.

Step 3

Add the absolute values of the integers.

$$|-5| = 5 \text{ and } |-4| = 4$$

$$5 + 4 = 9$$

Solution $-5 - 4 = -9$

Example 7

Subtract.

$$2 - (-8) = \square$$

Strategy **Add the opposite of the subtrahend.****Step 1**

Find the opposite of the number to be subtracted.

The subtrahend is -8 .The opposite of -8 is 8 .**Step 2**

Add the opposite of the subtrahend to the minuend.

$$2 - (-8) = 2 + 8$$

Both integers being added are positive.

Step 3

Add the integers.

$$2 + 8 = 10$$

Since both integers are positive, the sum will also be positive.

Solution **$2 - (-8) = 10$**

The properties of addition and subtraction can be used to show that $a - (b + c) = a - b - c$ if a , b , and c are integers.

$$a - (b + c) = a + -(b + c)$$

Add the opposite.

$$= a + (-b) + (-c)$$

Rewrite the sum using the distributive property.

$$= a - b - c$$

Use the properties of subtraction.

You can use the rules for adding and subtracting integers to solve problems.

Example 8

Carly has \$50 in a bank account. She writes a check for \$60 from the account. How much money does Carly have in her account after writing the check?

Strategy **Write a number sentence for the problem. Then solve.****Step 1**

Write a number sentence for the problem.

Let m represent the amount Carly has in her account after writing the check.

$$\$50 - \$60 = m$$

Step 2

Add the opposite of the number to be subtracted.

$$\$50 - \$60 = \$50 + (-\$60)$$

The integers being added have different signs.

Step 3

Find the difference of the absolute values of the integers.

$$|50| = 50$$

$$|-60| = 60$$

$$60 - 50 = 10$$

Step 4

Use the sign of the addend with the greater absolute value.

$|-60| > |50|$, so the sum is negative.

$$\$50 + (-\$60) = -\$10$$

Solution Carly has $-\$10$ in her account after writing the check.



Coached Example

The record low temperature for Albany, New York, was -28°F in January 1971.

The lowest temperature in U.S. history is 52°F lower than Albany's record low temperature. What is the lowest temperature in U.S. history?

Let l represent the lowest temperature in U.S. history.

Write a number sentence to represent the problem. _____

Is the subtrahend positive or negative? _____

Find the opposite of the subtrahend. _____

Add the opposite of the subtrahend to the minuend. _____

Both integers being added have a _____ sign.

Apply the rules for adding two integers.

Find the absolute value of the first addend. _____

Find the absolute value of the second addend. _____

Add the absolute values. _____

Use the sign of the addends in the sum. The sign for the sum is _____.

The lowest temperature in U.S. history is _____ $^{\circ}\text{F}$.

Multiply and Divide Integers



Getting the Idea

Use these rules to multiply two integers.

Rules for Multiplying Two Integers

Multiply the two integers as positive numbers.

Then find the sign of the product using these rules.

- If the signs of the two numbers are the same, the product is positive.
- If the signs of the two numbers are different, the product is negative.

Example 1

Multiply.

$$(-25) \cdot (-3) = \square$$

Strategy Apply the rules for multiplying two integers.

Step 1

Multiply the two integers as positive numbers.

$$25 \cdot 3 = 75$$

Step 2

Find the sign of the product.

The signs of the two numbers are the same, so the product is positive.

$$(-) \cdot (-) = (+)$$

The product is +75.

Solution $(-25) \cdot (-3) = 75$

Example 2

Marshall has \$12 automatically deducted from his checking account each month as a charitable donation. What is the total amount deducted from his account after 1 year?

Strategy Write an expression for the problem. Then solve.

Step 1

Write an expression to represent the problem.

The amount deducted each month can be represented by $-\$12$.

There are 12 months in 1 year.

So, find $-\$12 \cdot 12$.

Step 2 Multiply the two integers as positive numbers.

$$12 \cdot 12 = 144$$

Step 3 Find the sign of the product.

The signs of the two numbers are different, so the product is negative.

$$(-) \cdot (+) = (-)$$

The product is -144 .

Solution **The total amount deducted from Marshall's account is \$144.**

The **distributive property** applies the rules for multiplying and adding signed numbers.

The work below shows that $-(a + b) = -a - b$.

$$-(a + b) = -1(a + b)$$

$$= -1 \cdot (a + b)$$

Apply the distributive property.

$$= -1 \cdot (a) + -1 \cdot (b)$$

Multiply each addend by -1 .

$$= -a + (-b)$$

Apply the rules for adding integers.

$$= -a - b$$

Example 3

Write $-(a - b)$ as a sum.

Strategy **Use the distributive property.**

Step 1 Rewrite the expression to show the multiplication.

$$-(a - b) = -1 \cdot (a - b)$$

Step 2 Apply the distributive property.

$$-1 \cdot (a - b) = -1 \cdot (a) - (-1) \cdot (b)$$

Step 3 Multiply each addend by -1 .

$$-1 \cdot (a) - (-1) \cdot (b) = -a - (-b)$$

Step 4 Apply the rules for subtracting two integers.

The additive inverse of $-b$ is b .

$$-a - (-b) = -a + b$$

Solution **$-(a - b) = -a + b$**

Use these rules to divide two integers.

Rules for Dividing Two Integers

Divide the two integers as positive numbers.

Then find the sign of the quotient using these rules.

- If the signs of the two numbers are the same, the quotient is positive.
- If the signs of the two numbers are different, the quotient is negative.

All integers can be divided as long as the divisor is not zero.

Example 4

Divide.

$$(-32) \div (-8) = \square$$

Strategy **Apply the rules for dividing two integers.**

Step 1

Divide the two integers as positive numbers.

$$32 \div 8 = 4$$

Step 2

Find the sign of the quotient.

The signs of the two numbers are the same, so the quotient is positive.

$$(-) \div (-) = (+)$$

The quotient is $+4$.

Solution **$(-32) \div (-8) = 4$**

Example 5

The temperature fell 18°F in 3 hours. The temperature fell at the same rate every hour. How much did the temperature change each hour?

Strategy Write an expression for the problem. Then solve.

Step 1

Write an expression to represent the problem.

The temperature changed -18°F .

$\frac{(-18)}{3}$ or $-18 \div 3$ represents the temperature change per hour.

Step 2

Divide the two integers as positive numbers.

$$18 \div 3 = 6$$

Step 3

Find the sign of the product.

The signs of the two numbers are different, so the product is negative.

$$(-) \div (+) = (-)$$

The quotient is -6° .

Solution The temperature changed -6°F each hour.

Instead of using a multiplication sign, sometimes multiplication is shown by putting the factors in parentheses. For example, $(-3)(2) = -3 \times 2$.



Coached Example

What is the value of $(-5)(4)(-1)(-2)$?

Will the product of the first two integers be positive or negative? _____

$$-5 \cdot 4 = \underline{\hspace{2cm}}$$

When you multiply this product by the third integer, -1 , will the product be positive or negative? _____

Multiply the product of the first two integers by the third integer, -1 .

$$\underline{\hspace{2cm}} \cdot (-1) = \underline{\hspace{2cm}}$$

When you multiply this product by the fourth integer, -2 , will the product be positive or negative? _____

$$\underline{\hspace{2cm}} \cdot (-2) = \underline{\hspace{2cm}}$$

The value of $(-5)(4)(-1)(-2)$ is _____.

Add and Subtract Rational Numbers



Getting the Idea

You can use the rules for adding and subtracting positive and negative integers to add and subtract other positive and negative rational numbers.

Rules for Adding Two Rational Numbers

- When rational numbers have the same sign, add the absolute values and use the sign of the addends in the sum.
- When rational numbers have different signs, find the difference of their absolute values. Then use the sign of the addend with the greater absolute value in the sum.

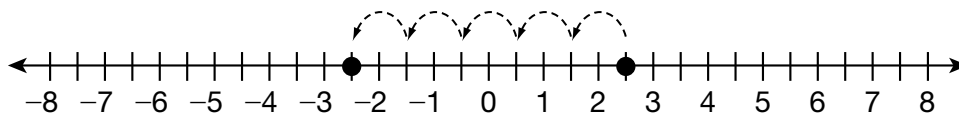
Example 1

Add.

$$2.5 + (-5) = \square$$

Strategy Use a number line to add.

Start at 2.5. Move 5 places to the left.



Solution $2.5 + (-5) = -2.5$

In real-world situations involving rational numbers, you may find the amount of money in a savings or checking account or net earnings. The **net** is the amount that remains after deductions and adjustments have been made.

Example 2

Elena had \$267.35 in her checking account when a check for \$280.50 was cashed. What is the balance in her account now?

Strategy **Write an expression for the problem. Then solve.**

Step 1

Write an expression to represent the problem.

The problem can be represented as $\$267.35 - \280.50 or as $\$267.35 + (-\$280.50)$.

Step 2

Find the absolute value of the two decimals.

$$|267.35| = 267.35$$

$$|-280.50| = 280.50$$

Step 3

Subtract the lesser absolute value from the greater absolute value.

$$280.5 - 267.35 = 13.15$$

Step 4

The difference should have the same sign as the decimal with the greater absolute value.

-280.50 has the greater absolute value, so the account has a negative balance.

Solution **The balance in the account is $-\$13.15$.**

To add or subtract fractions or mixed numbers with unlike denominators:

- Use the **least common denominator (LCD)** to find equivalent fractions with like denominators.
- Add or subtract, regrouping as needed.
- Write the answer in simplest form. You may need to rename an improper fraction as a mixed number.

Example 3

Add.

$$-\frac{5}{6} + \left(-\frac{4}{9}\right) = \square$$

Strategy Write equivalent rational numbers using the LCD. Then add.**Step 1**

Determine the LCD of the fractions.

The LCD of $\frac{5}{6}$ and $\frac{4}{9}$ is 18.**Step 2**

Write equivalent fractions with a denominator of 18.

$$-\frac{5}{6} \times \frac{3}{3} = -\frac{15}{18}$$

$$-\frac{4}{9} \times \frac{2}{2} = -\frac{8}{18}$$

Step 3

Add.

$$-\frac{15}{18} + \left(-\frac{8}{18}\right) = -\frac{23}{18}$$

Step 4

Rename the sum as a mixed number.

$$-\frac{23}{18} = -1\frac{5}{18}$$

Solution
$$-\frac{5}{6} + \left(-\frac{4}{9}\right) = -1\frac{5}{18}$$

You can use a number line to subtract rational numbers.

Example 4

Subtract.

$$-3\frac{1}{5} - 2\frac{3}{5} = \square$$

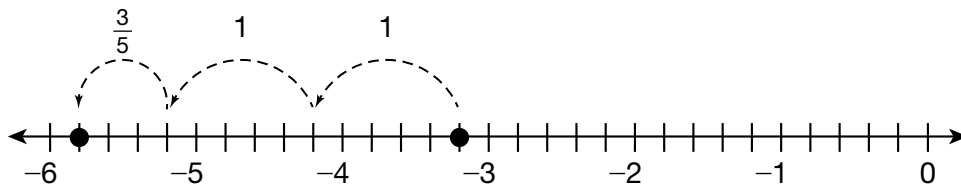
Strategy Rename as addition and use a number line.**Step 1**

Rewrite as addition.

$$-3\frac{1}{5} - 2\frac{3}{5} = -3\frac{1}{5} + \left(-2\frac{3}{5}\right)$$

Step 2

Use a number line to add.



Solution
$$-3\frac{1}{5} - 2\frac{3}{5} = -5\frac{4}{5}$$

Example 5

Subtract.

$$-0.7 - (-1.6) = \square$$

Strategy **Rename as addition and use a number line.**

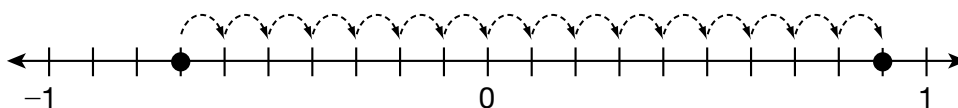
Step 1

Rewrite as addition.

$$-0.7 - (-1.6) = -0.7 + 1.6$$

Step 2

Use a number line to add.



Solution $-0.7 - (-1.6) = 0.9$

Example 6

Subtract.

$$-4\frac{1}{4} - 1\frac{7}{8} = \square$$

Strategy **Rename the fractional parts of the mixed numbers.**

Step 1

Rewrite as addition.

$$-4\frac{1}{4} - 1\frac{7}{8} = -4\frac{1}{4} + \left(-1\frac{7}{8}\right)$$

Step 2

Determine the LCD of the fractional parts.

The LCD of $\frac{1}{4}$ and $\frac{7}{8}$ is 8.

Step 3

Rename $-4\frac{1}{4}$ as a mixed number with 8 as the denominator.

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}, \text{ so } -4\frac{1}{4} = -4\frac{2}{8}$$

Step 4

Add.

$$-4\frac{2}{8} + \left(-1\frac{7}{8}\right) = -5\frac{9}{8}$$

$$-5\frac{9}{8} = -6\frac{1}{8}$$

Solution $-4\frac{1}{4} - 1\frac{7}{8} = -6\frac{1}{8}$



Coached Example

Genesis walked $2\frac{1}{4}$ miles south and then $1\frac{1}{2}$ miles north. Describe Genesis's location from her starting point.

Think of a number line. South is represented by _____ numbers and north is represented by _____ numbers.

Write a rational number to represent $2\frac{1}{4}$ miles south. _____

Write a rational number to represent $1\frac{1}{2}$ miles north. _____

Let f represent Genesis's location.

Write an addition equation to represent Genesis's location.

Which addend has a greater absolute value? _____

The sum has the sign of the _____ absolute value.

Is Genesis north or south from her starting location? _____

Solve your equation.

Genesis's location is _____ from her starting point.

Multiply and Divide Rational Numbers



Getting the Idea

Use these rules to help you multiply decimals:

- Multiply as you would with whole numbers.
- Count the total number of decimal places in the factors. The sum is the number of decimal places in the product.

Use these rules when multiplying two rational numbers.

- When both numbers have the same sign, the product is positive.
- When the numbers have different signs, the product is negative.

You can use the distributive property to break numbers into lesser numbers to compute with.

Example 1

Multiply.

$$-3.98 \times 20.5 = \square$$

Strategy Use the distributive property.

Step 1

Use the distributive property.

$$-3.98 \times 20.5 = (-3.98 \times 20) + (-3.98 \times 0.5)$$

Step 2

Find the partial products.

$$-3.98 \times 20 = -79.6$$

$$-3.98 \times 0.5 = -1.99$$

Step 3

Add the partial products.

$$-79.6 + (-1.99) = -81.59$$

Solution $-3.98 \times 20.5 = -81.59$

To divide a decimal by a whole number, place the decimal point in the quotient above the decimal point in the dividend. Then divide as you would with whole numbers.

Example 2

Divide.

$$-41.12 \div (-16) = \square$$

Strategy Determine the sign of the quotient and divide.

Step 1

Determine the sign of the quotient.

Both signs are negative, so the quotient will be positive.

You can now ignore the signs.

Step 2

Place the decimal point in the quotient above the decimal point in the dividend. Divide.

$$\begin{array}{r} 2.57 \\ 16 \overline{)41.12} \\ \underline{-32} \\ 91 \\ \underline{-80} \\ 112 \\ \underline{-112} \\ 0 \end{array}$$

Solution $-41.12 \div (-16) = 2.57$

Use these rules to divide a decimal by a decimal:

- Multiply the divisor by a **power of 10**, such as 10 or 100, to make it a whole number. Then multiply the dividend by the same power of 10.
- Divide as you would divide whole numbers.

Example 3

Kaz is competing in a 13.5-kilometer race. There will be water stops every 0.75-kilometer, including at the end of the race. How many water stops will there be in all?

Strategy **Multiply the divisor and the dividend by the same power of 10. Then divide.**

Step 1

Multiply the dividend and divisor by the same power of 10, so the divisor becomes a whole number.

The divisor 0.75 is in hundredths so multiply by 100.

$$0.75 \times 100 = 75$$

$$13.5 \times 100 = 1,350$$

Divide $1,350 \div 75$.

Step 2

Divide.

$$\begin{array}{r} 18 \\ 75 \overline{)1350} \\ \underline{-75} \\ 600 \\ \underline{-600} \\ 0 \end{array}$$

Solution **There will be 18 water stops in all.**

Sometimes, you will divide until the decimal terminates. Other times, it will be necessary to interpret a remainder.

Example 4

It costs \$0.36 to buy an eraser. Ms. Cole wants to buy as many erasers as she can for \$5.00. How many erasers can Ms. Cole buy?

Strategy **Multiply the divisor and the dividend by the same power of 10. Then divide.**

Step 1

Multiply the dividend and divisor by the same power of 10, so the divisor becomes a whole number.

The divisor 0.36 is in hundredths so multiply by 100.

$$0.36 \times 100 = 36$$

$$\$5 \times 100 = \$500$$

Divide $\$500 \div 36$

Step 2

Divide.

$$\begin{array}{r}
 13R32 \\
 36 \overline{)500} \\
 \underline{-36} \\
 140 \\
 \underline{-108} \\
 32
 \end{array}$$

Step 3

Interpret the remainder.

The quotient is 13 R32, or $13\frac{32}{36}$.Since it is not possible to buy $\frac{32}{36}$ of an eraser, ignore the remainder.**Solution** **Ms. Cole can buy 13 erasers with \$5.00.**

Use these rules to multiply fractions or mixed numbers.

- To multiply fractions, multiply the numerators. Then multiply the denominators. Write the answer in simplest form.
- To multiply mixed numbers, first rename them as improper fractions.

Example 5

Multiply.

$$-\frac{2}{3} \times \left(-\frac{7}{8}\right) = \square$$

Strategy**Determine the sign of the product. Then multiply.****Step 1**

Determine the sign of the product.

Both signs are negative, so the product will be positive.

You can now ignore the signs.

Step 2

Cancel like terms and multiply.

$$\begin{array}{c}
 1 \\
 \cancel{2}^1 \times \frac{7}{\cancel{8}_4} = \frac{7}{12}
 \end{array}$$

Solution

$$-\frac{2}{3} \times \left(-\frac{7}{8}\right) = \frac{7}{12}$$

Example 6

Multiply.

$$-4\frac{2}{3} \times 1\frac{5}{7} = \square$$

Strategy **Rename the mixed numbers as improper fractions. Multiply.**

Step 1

Determine the sign of the product.

The signs are different, so the product will be negative.

You can ignore the signs for now.

Step 2

Rename the mixed numbers as improper fractions.

$$4\frac{2}{3} = \frac{4 \times 3 + 2}{3} = \frac{14}{3}$$

$$1\frac{5}{7} = \frac{1 \times 7 + 5}{7} = \frac{12}{7}$$

Step 3

Cancel like terms and multiply.

$$\frac{\overset{2}{\cancel{14}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{4}{\cancel{12}}}{\underset{1}{\cancel{7}}} = \frac{8}{1} = 8$$

Step 4

Remember to put the negative sign back in the product.

Solution $-4\frac{2}{3} \times 1\frac{5}{7} = -8$

Use these rules when dividing two rational numbers.

- When both numbers have the same sign, the quotient is positive.
- When the numbers have difference signs, the quotient is negative.

Example 7

Divide.

$$-3\frac{1}{2} \div -4\frac{4}{5} = \square$$

Strategy **Rename the mixed numbers as improper fractions. Multiply.**

Step 1

Determine the sign of the quotient.

The signs are the same, so the quotient will be positive.

You can ignore the signs for now.

Step 2

Rename the mixed numbers as improper fractions.

$$3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$$

$$4\frac{4}{5} = \frac{4 \times 5 + 4}{5} = \frac{24}{5}$$

Step 3

Multiply by the reciprocal of the dividend.

There are no like terms to cancel.

$$\frac{7}{2} \div \frac{24}{5} = \frac{7}{2} \times \frac{5}{24} = \frac{35}{48}$$

Solution $-3\frac{1}{2} \div -4\frac{4}{5} = \frac{35}{48}$

**Coached Example**

Mr. Livio earns an hourly wage for every hour he works. Last week, he earned \$663.85 and worked for 35.5 hours. How much money does Mr. Livio earn per hour?

To solve the problem, divide 663.85 by _____ to find the amount of money Mr. Livio earns per hour.

Multiply the divisor, 35.5, by _____ to make it a whole number: _____

Multiply the dividend, 663.85, by that same power of 10: _____

Divide as you would with whole numbers.

$$355 \overline{)6,638.5}$$

Place a dollar sign and a decimal point in the quotient.

The answer is an amount of money, so be sure to give it two decimal places.

Mr. Livio earns _____ per hour.

Ratios and Rates



Getting the Idea

A **ratio** is a comparison of two numbers. Ratios can be written to compare a part to a part, a part to the whole, or the whole to a part. Each number in a ratio is called a **term**.

You can write a ratio in three ways:

- | | |
|------------------|---------------|
| 1. in words | 5 to 6 |
| 2. as a fraction | $\frac{5}{6}$ |
| 3. using a colon | 5:6 |

Example 1

For a certain shade of green paint, the paint store mixes 3 parts blue paint to 2 parts yellow paint. What is the ratio of blue paint to yellow paint?

Strategy Compare the number of blue parts to the number of yellow parts.

Step 1

Break down the paint mix.

blue = 3

yellow = 2

Step 2

Write the ratio of blue to yellow three ways.

In words 3 to 2

As a fraction $\frac{3}{2}$

With a colon 3:2

Solution The ratio of blue paint to yellow paint is 3 to 2, $\frac{3}{2}$, or 3:2.

A **rate** is a ratio that compares two quantities that have different units of measure.

A **unit rate** is a rate in which the second quantity in the comparison is 1 unit.

Example 2

Lazlo built 30 toy airplanes in 5 hours. What was his unit rate for building the airplanes?

Strategy Divide to find the unit rate.

Step 1

Write the rate as a fraction.

$$\frac{30 \text{ airplanes}}{5 \text{ hours}}$$

Step 2

Divide to find the unit rate.

$$30 \div 5 = 6$$

Solution Lazlo's unit rate was 6 toy airplanes per hour.

Example 3

Jen works for a florist. She worked 15 hours last week and earned \$112.50. At that rate, how much will she earn if she works for 10 hours?

Strategy Find the unit rate. Then multiply.

Step 1

Write the rate as a fraction.

$$\frac{\$112.50}{15 \text{ hours}}$$

Step 2

Divide to find the unit rate.

$$112.50 \div 15 = 7.5$$

Jen earns \$7.50 per hour.

Step 3

Multiply the unit rate by 10 hours.

$$\$7.50 \text{ per hour} \times 10 \text{ hours} = \$75.00$$

Solution Jen will earn \$75.00 if she works 10 hours.

Example 4

One lap around the path in a park is $\frac{1}{4}$ mile. It takes Andy $\frac{1}{6}$ hour to walk one lap. What is Andy's unit rate around the park?

Strategy Find the unit rate.

Step 1

Write the rate as a fraction.

In this case the rate is a complex fraction.

$$\frac{\frac{1}{4} \text{ mi}}{\frac{1}{6} \text{ hr}}$$

Step 2

Simplify the complex fraction.

$$\frac{\frac{1}{4}}{\frac{1}{6}} = \frac{1}{4} \div \frac{1}{6}$$

Step 3

Divide to find the unit rate.

Multiply by the reciprocal and simplify.

$$\frac{1}{4} \div \frac{1}{6} = \frac{1}{4} \times \frac{6}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Solution Andy's unit rate is $1\frac{1}{2}$ miles per hour.

Example 5

Holly's room is 12 feet long by 9 feet wide. The carpet she wants to put in the room costs \$4.50 per square foot. How much will it cost to carpet Holly's room?

Strategy **Multiply the area by the unit rate.**

Step 1

Find the area of the room.

Use the formula for the area of a rectangle: $\text{Area} = \text{length} \times \text{width}$.

$$A = \text{length} \times \text{width}$$

$$= 12 \text{ ft} \times 9 \text{ ft} = 108 \text{ sq ft}$$

Step 2

Multiply the area by the unit rate.

$$108 \times \$4.50 = \$486.00$$

Solution **It will cost \$486.00 to carpet Holly's room.**



Coached Example

If 5 tomatoes cost \$2.00, what is the unit price of the tomatoes? How much will a dozen tomatoes cost?

Write a ratio that compares the total cost to the number of tomatoes. _____

Divide to find the unit price. _____

To find the cost of a dozen tomatoes, multiply the unit price by _____.

$$\text{_____} \times \text{_____} = \text{_____}$$

The unit price of the tomatoes is _____ per tomato.

One dozen tomatoes will cost \$_____.

Proportions



Getting the Idea

A **proportion** is an equation that shows that two ratios are equivalent. For example, $\frac{1}{2} = \frac{5}{10}$ is a proportion. To tell if two ratios form a proportion, write each ratio in simplest form or use another common denominator. A common denominator can always be found by multiplying the denominators. If the ratios are equal, a proportion is formed.

Example 1

Determine if $\frac{10}{16}$ and $\frac{18}{28}$ forms a proportion.

Strategy Write each ratio in simplest form.

$$\begin{aligned}\frac{10}{16} \div \frac{2}{2} &= \frac{5}{8} \\ \frac{18}{28} \div \frac{2}{2} &= \frac{9}{14} \\ \frac{5}{8} &\neq \frac{9}{14}\end{aligned}$$

Solution The ratios $\frac{10}{16}$ and $\frac{18}{28}$ do not form a proportion.

A proportion can be solved by writing both ratios with a common denominator. This can be done by multiplying the denominators and then writing equivalent fractions. If one denominator is a factor of the other denominator, simply rename the fraction with the lesser denominator.

Example 2

Solve the proportion by using a common denominator.

$$\frac{a}{16} = \frac{21}{28}$$

Strategy Write equivalent fractions using like denominators.

Step 1

Multiply the denominators to find a common denominator.

$$16 \times 28 = 448$$

Step 2

Write equivalent fractions with 448 as the denominator.

$$\frac{a}{16} \times \frac{28}{28} = \frac{28a}{448}$$

$$\frac{21}{28} \times \frac{16}{16} = \frac{336}{448}$$

$$\text{So, } 28a = 336.$$

Step 3Divide to solve for a .

$$\frac{28a}{28} = \frac{336}{28}$$

$$a = 12$$

Solution **The solution is $a = 12$.**

You can use **cross multiplication** to solve a proportion. To cross multiply is to multiply the numerator of one fraction by the denominator of the other.

Example 3

Solve the proportion.

$$\frac{8}{12} = \frac{6}{x}$$

Strategy **Cross multiply to solve for x .****Step 1**

Multiply each numerator by the other denominator to find the cross products.

$$8 \times x = 12 \times 6$$

$$8x = 72$$

Step 2Divide to solve for x .

$$\frac{8x}{8} = \frac{72}{8}$$

$$x = 9$$

Solution **The solution is $x = 9$.****Example 4**What value of y makes this proportion true?

$$\frac{0.4}{y} = \frac{3.4}{10.2}$$

Strategy **Cross multiply to solve for y .****Step 1**

Multiply to find the cross products.

$$3.4 \times y = 0.4 \times 10.2$$

$$3.4y = 4.08$$

Step 2Divide to solve for y .

$$\frac{3.4y}{3.4} = \frac{4.08}{3.4}$$

$$y = 1.2$$

Solution **The solution is $y = 1.2$.**



Coached Example

What value of x makes this proportion true?

$$\frac{72}{90} = \frac{x}{25}$$

To cross multiply, multiply the _____ of each fraction by the _____ of the other fraction.

Write the factors for the cross products.

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

Multiply to find the cross products.

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Divide both sides by _____ to solve for x .

$$x = \underline{\hspace{2cm}}$$

Substituting the value _____ for x makes the proportion $\frac{72}{90} = \frac{x}{25}$ true.

Proportional Relationships



Getting the Idea

You can use proportions to solve problems. Proportional relationships, such as the number of miles driven at a constant speed and the amount of time spent driving, can be represented by equal ratios. Relationships that are not proportional, such as a person's age and height, cannot be represented by equal ratios.

Example 1

Derek counted 24 marshmallows in 3 servings of Marshy Morsels. At this rate, how many marshmallows are in 12 servings?

Strategy Write and solve a proportion.

Step 1

Set up a proportion.

Keep the units consistent.

$$\frac{24 \text{ marshmallows}}{3 \text{ servings}} = \frac{m \text{ marshmallows}}{12 \text{ servings}}$$

Step 2

Find a common denominator.

3 is a factor of 12.

Step 3

Rename $\frac{24}{3}$ as a fraction with 12 as the denominator.

$$\frac{24}{3} \times \frac{4}{4} = \frac{96}{12}$$

$$m = 96$$

Solution At this rate, there are 96 marshmallows in 12 servings.

In a proportional relationship, when one quantity increases, the other quantity also increases. The ratio of the two quantities stays constant in a proportional relationship. The constant ratio is also called the unit rate, or the **constant of proportionality**.

Example 2

A train travels 120 miles in 1.5 hours. At this rate, how many miles can it travel in 5 hours?

Strategy Find and use the unit rate.

Step 1

Write the rate as a fraction.

$$\frac{120 \text{ mi}}{1.5 \text{ hr}}$$

Step 2

Divide to find the unit rate, or the constant of proportionality.

$$120 \div 1.5 = 80$$

The unit rate is 80 mph.

Step 3

Write the distance equation.

$$\text{rate} \times \text{time} = \text{distance}$$

Step 4

Substitute the known values into the equation and solve for the distance.

$$\text{rate} = 80 \text{ mph}$$

$$\text{time} = 5 \text{ hr}$$

$$\text{rate} \times \text{time} = \text{distance}$$

$$80 \times 5 = 400 \text{ miles}$$

Solution At this rate, the train can travel 400 miles in 5 hours.

In Example 2, you could also have solved the problem by writing and solving a proportion.

$$\frac{120}{1.5} = \frac{x}{5}$$

Let x represent the distance traveled in 5 hours.

$$120 \times 5 = 1.5 \times x$$

Write the factors of the cross products.

$$600 = 1.5x$$

Find the cross products.

$$\frac{600}{1.5} = \frac{1.5x}{1.5}$$

Divide to solve for x .

$$400 = x$$

Some problems involving percents can be solved by writing and solving a proportion.

Example 3

18 is what percent of 60?

Strategy Write and solve a proportion.

Step 1

Let x represent the percent, which is unknown.

Write a proportion that compares 18 and 60 to $x\%$.

$$\frac{18}{60} = \frac{x}{100}$$

Step 2

Cross multiply.

$$\frac{18}{60} = \frac{x}{100}$$

$$60 \times x = 18 \times 100$$

$$60x = 1,800$$

$$\frac{60x}{60} = \frac{1,800}{60}$$

$$x = 30$$

Solution 18 is 30% of 60.



Coached Example

Mr. Collins is planning a party for his homeroom class. There are 30 students in his class. He wants each student to have a serving of 8 fluid ounces of juice. Each jug of juice contains 40 fluid ounces. At this rate, how many jugs of juice will he need for the party?

First find the unit rate, the number of servings of juice in each jug.

Write the number of fluid ounces for each student. _____

Write the number of fluid ounces in each jug. _____

To find the unit rate, write a _____ that compares the number of fluid ounces in each jug to the number of fluid ounces for each student. _____

Simplify the ratio to write the unit rate. _____ servings per jug

To find the number of jugs Mr. Collins needs, _____ the number of students in the class by the number of servings per jug.

$$\text{_____} \div \text{_____} = \text{_____}$$

Mr. Collins will need _____ jugs of juice for the party.

Represent Proportional Relationships



Getting the Idea

A **directly proportional** relationship has an equation of the form $y = kx$. It is a relationship between two quantities in which one is a constant multiple of the other. When one quantity changes, the other quantity changes by a constant factor, k . The constant factor k is the constant of proportionality.

Example 1

The function table below shows the relationship between the side lengths of a regular octagon and its perimeter.

Side Lengths, s (inches)	Perimeter, P (inches)
1	8
2	16
3	24
4	32
9	?

If a regular octagon has side lengths of 9 inches, what is its perimeter?

Strategy Write and solve an equation.

Step 1

Write an equation to represent the situation.

The perimeter is always 8 times the side length of a regular octagon.

So, 8 is the constant of proportionality.

$$P = 8s$$

Step 2

Substitute the side length of 9 for s and find the perimeter.

$$P = 8 \times 9 \text{ in.}$$

$$= 72 \text{ in.}$$

Solution A regular octagon with a side length of 9 inches has a perimeter of 72 inches.

A directly proportional relationship is a **linear relationship** because it forms a straight line when graphed. The graph of a proportional relationship is a straight line that passes through the **origin** at (0, 0). It will also pass through the point (1, k), where k is the constant of proportionality, or the unit rate.

Example 2

An empty swimming pool is being filled at a rate of 10 gallons per minute. Make a graph to display the amount of water in the pool each minute for 6 minutes.

Strategy Write an equation and create a function table to represent the situation.

Step 1

Write an equation to represent the situation.

Let x = the number of minutes and y = the number of gallons.

$$y = 10x$$

Step 2

Make a function table to show the number of gallons in the pool each minute.

At 0 minutes, when $x = 0$, there is no water going into the pool, so $y = 0$.

At 1 minute, when $x = 1$, the pool is filled with 10 gallons of water.

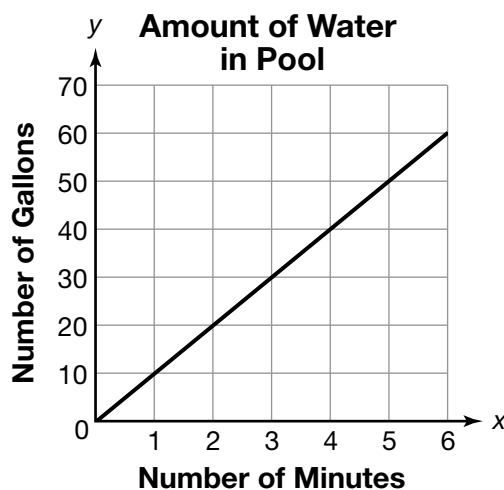
At 2 minutes, when $x = 2$, the pool is filled with 20 gallons of water.

Complete the rest of the table.

Number of Minutes (x)	0	1	2	3	4	5	6
Number of Gallons (y)	0	10	20	30	40	50	60

Step 3

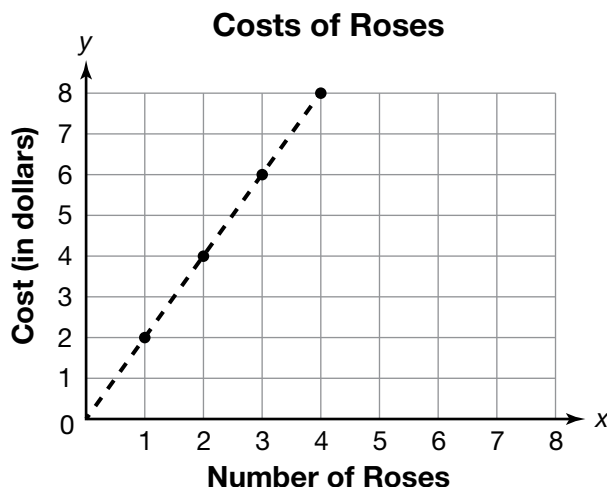
Make a line graph, using the ordered pairs from the function table.



Solution The graph is shown in Step 3.

Example 3

The graph below shows the amounts charged for purchasing different numbers of roses from a florist. Is there a proportional relationship between the number of roses bought and the cost? If so, what is the constant of proportionality and what does it mean in this context?



Strategy Analyze the graph to determine if the relationship is proportional.

Step 1

Think about the graph of a proportional relationship.

The graph of a proportional relationship is a straight line that passes through the origin. Since this graph matches that description, it shows a proportional relationship.

Step 2

Determine the constant of proportionality.

The graph must pass through the point $(1, k)$, where k is the constant of proportionality.

The graph passes through the point $(1, 2)$. So, k is 2.

Step 3

Determine what the constant of proportionality means in this context.

In this situation, the fact that $k = 2$ means that each rose costs \$2.

Solution The relationship is proportional and the constant of proportionality, 2, means that each rose costs \$2.



Coached Example

A movie theater charges \$16 for 2 tickets, and \$32 for 4 tickets. How much would it cost for 12 tickets? Make a graph to represent the situation.

Let x represent the number of tickets.

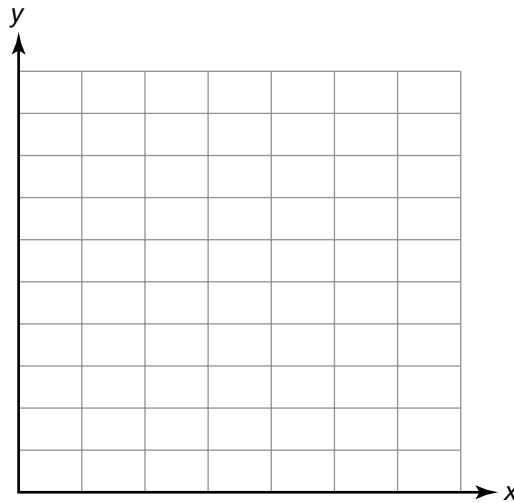
Let y represent the cost, in dollars.

Write an equation to represent the situation. _____

Complete the function table.

Number of Tickets (x)	0	2	4	6	8	10	12
Cost in Dollars, (y)							

Create a graph to show the values in the table.



The graph passes through the point $(1, \quad)$. So, \quad is the constant of proportionality, or the unit rate.

It would cost \quad for 12 people.

Write Algebraic Expressions



Getting the Idea

An **expression** is a statement that combines numbers, operation signs, and sometimes variables. An algebraic expression includes at least one variable. To write an algebraic expression from words, look for the relationship between the words and the numbers in the situation. This list can help you to translate many, but not all, math word problems.

Operation	Problem	Numerical Expression
addition	3 more than x the sum of x and 3 3 increased by x the total of x and 3 3 combined with x	$3 + x$
subtraction	3 minus x x fewer than 3 x less than 3 x subtracted from 3 3 decreased by x the difference of x from 3	$3 - x$
multiplication	3 times x 3 multiplied by x the product of 3 and x x groups of 3	$3x$
division	x partitioned into 3 equal groups x shared by 3 equally	$\frac{x}{3}$

Example 1

Write an expression to represent the phrase below.

2 less than the product of 7 and a number n

Strategy **Decompose the word expression into parts.**

Step 1

Look at the first part of the word expression.

“2 less” means to subtract 2.

2 will be subtracted from the second part of the word expression.

Step 2

Look at the second part of the word expression.

“The product of 7 and a number n ” means to multiply 7 and n or $7n$.

Step 3

Put the parts together to form one expression.

2 less than $7n$ means $7n - 2$.

Solution

The expression $7n - 2$ represents the phrase “2 less than the product of 7 and a number n .”

Example 2

The number of stamps in Ethan’s collection is 4 more than half the number of stamps in Helen’s collection.

Write an expression to show the number of stamps in Ethan’s collection.

Strategy

Decompose the word expression into parts.

Step 1

Look at the first part of the word expression.

“4 more” means to add 4.

Step 2

Look at the second part of the word expression.

Let h represent the number of stamps in Helen’s collection.

“Half the number” can mean to multiply $\frac{1}{2}$ or to divide by 2.

This can be represented by $\frac{h}{2}$ or $\frac{1}{2}h$.

Step 3

Put the two parts together to form one expression.

“4 more” than “half the number” can be represented by $4 + \frac{h}{2}$ or $4 + \frac{1}{2}h$.

Solution

There are different expressions that represent the stamps in Ethan’s collection. Two of them are $4 + \frac{h}{2}$ or $4 + \frac{1}{2}h$.

Example 3

Lucy babysat for 2 hours on Friday, 3 hours on Saturday, and 2.5 hours on Sunday. She earns d dollars per hour for babysitting.

Write an expression to represent her total earnings for the three babysitting jobs.

Strategy

Decompose the word expression into parts.

Step 1

Write what you know.

Lucy babysat three times for the same amount of money each time.

To find how much Lucy earned, add the hours and multiply the number of hours by her rate per hour.

Step 2

Add the hours Lucy worked.

$$2 + 3 + 2.5 = 7.5$$

Step 3

Write the expression.

Lucy worked 7.5 hours at a rate of d dollars per hour.

That can be shown as $7.5d$.

Solution The expression $7.5d$ represents Lucy's total earnings for the three jobs.



Coached Example

Kerrigan is k years old. Mia is twice as old as Kerrigan. William is 3 years younger than Mia.

Write an expression to represent William's age.

Mia is twice as old as Kerrigan.

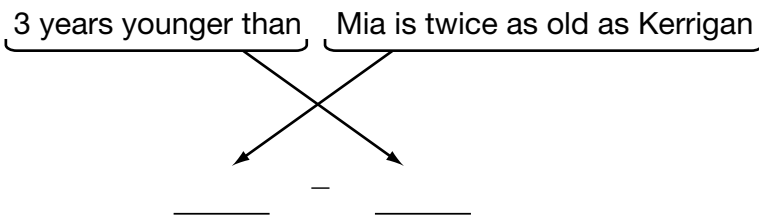
The word "twice" indicates you could _____ by 2.

William is 3 years younger than Mia.

The words "younger than" mean that William's age is _____ than Mia's age.

What operation should you use to show "younger than"? _____

Translate the words into an algebraic expression.



The expression _____ represents William's age.

Simplify and Evaluate Algebraic Expressions



Getting the Idea

To simplify a numerical expression, follow the **order of operations**.

Order of Operations

1. Perform operations inside parentheses or other grouping symbols.
2. Evaluate exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

An **exponent** tells how many times the **base** is used as a factor. For example, in 3^2 , 3 is the base and 2 is the exponent.

Example 1

Simplify this expression.

$$\frac{1}{2}(2^3 + 2)$$

Strategy Follow the order of operations.

Step 1

Perform operations within parentheses.

The expression within parentheses is $(2^3 + 2)$.

Evaluate the exponent first, then add.

$$2^3 + 2 =$$

$$8 + 2 = 10$$

$$\text{So, } \frac{1}{2}(2^3 + 2) = \frac{1}{2}(10).$$

Step 2

Multiply.

$$\frac{1}{2}(10) = \frac{1}{2} \times 10 = 5$$

Solution $\frac{1}{2}(2^3 + 2) = 5$

You can use number properties and like terms to help you simplify algebraic expressions.

Like terms are terms that contain the same variable(s) raised to the same power(s).

Commutative Properties	
commutative property of addition $a + b = b + a$	commutative property of multiplication $ab = ba$

Associative Properties	
associative property of addition $(a + b) + c = a + (b + c)$	associative property of multiplication $(a \times b) \times c = a \times (b \times c)$

Example 2

Simplify this expression.

$$(11k + 5) + 2k$$

Strategy Use number properties and like terms.

Step 1

Use the commutative property to reorder the first two terms.

$$(11k + 5) + 2k =$$

$$(5 + 11k) + 2k$$

$11k$ and $2k$ are like terms. The like terms are next to each other.

Step 2

Use the associative property to group like terms.

$$(5 + 11k) + 2k =$$

$$5 + (11k + 2k)$$

Step 3

Combine the like terms.

$$5 + (11k + 2k) =$$

$$5 + 13k$$

Solution The expression can be simplified as $5 + 13k$.

Example 3

Simplify this expression.

$$4s + 5t + (-3s) + 4t$$

Strategy Use the properties of addition.

Step 1

Use the commutative property to reorder the terms.

$$4s + 5t + (-3s) + 4t =$$

$$4s + (-3s) + 5t + 4t$$

Step 2

Use the associative property to group like terms and combine them.

$$\begin{aligned} 4s + (-3s) + 5t + 4t &= \\ [4s + (-3s)] + (5t + 4t) &= \\ s + 9t & \end{aligned}$$

Solution The expression can be simplified to $s + 9t$.

To expand an expression is to remove parentheses or brackets.

You can use the distributive property to expand an expression.

Distributive Properties	
distributive property over addition	distributive property over subtraction
$a(b + c) = ab + ac$	$a(b - c) = ab - ac$

Example 4

Simplify this expression.

$$2(4m + n) - 2n$$

Strategy

Use number properties and combine like terms.

Step 1

Expand the first part of the expression using the distributive property.

$$\begin{aligned} 2(4m + n) &= (2 \times 4m) + (2 \times n) \\ &= 8m + 2n \end{aligned}$$

Step 2

Rewrite the expression.

$$2(4m + n) - 2n = 8m + 2n - 2n$$

Step 3

Use the associative property to group and combine like terms.

$$\begin{aligned} 8m + (2n - 2n) &= \\ 8m + 0n &= 8m \end{aligned}$$

Solution The expression can be simplified to $8m$.

The opposite of expanding is factoring. You can also use the distributive property to help you factor an expression. An expression is completely factored when there are no more common factors among terms.

Example 5

Simplify and factor this expression.

$$6x + 3x + 15y + 12y$$

Strategy **Combine like terms. Then use the distributive property to find the GCF.**

Step 1

Combine like terms.

$$\begin{aligned} 6x + 3x + 15y + 12y &= \\ 9x + 27y \end{aligned}$$

Step 2

Find the GCF of the terms $9x$ and $27y$.

The GCF of $9x$ and $27y$ is 9.

Step 3

Factor 9 from each term in $9x + 27y$.

$$\begin{aligned} 9x + 27y &= \\ 9 \times x + 9 \times 3y &= 9(x + 3y) \end{aligned}$$

Solution **The simplified and factored expression is $9(x + 3y)$.**

To evaluate an algebraic expression, substitute the given values for the variables. Remember to follow the order of operations.

Example 6

Evaluate this expression when $a = 8$ and $b = -7$.

$$12 + 3a - b$$

Strategy **Substitute the value of each variable into the expression. Then evaluate.**

Step 1

Substitute 8 for a and -7 for b .

$$12 + 3a - b = 12 + 3(8) - (-7)$$

Step 2

Use the order of operations to simplify.

First, multiply and divide from left to right.

$$12 + 3(8) - (-7) = 12 + 24 - (-7)$$

Next, add and subtract from left to right.

$$\text{Add: } 12 + 24 - (-7) = 36 - (-7) = 36 + 7 = 43$$

Solution **The value of the expression is 43.**

Example 7

Evaluate this algebraic expression when $m = -3$ and $n = -4$.

$$m^2 + n$$

Strategy **Substitute the value of each variable into the expression. Then evaluate.**

Step 1

Substitute -3 for m and -4 for n .

$$m^2 + n = (-3)^2 + (-4)$$

Step 2

Use the order of operations to simplify.

First, evaluate the exponent.

$$(-3)^2 + (-4) =$$

$$(-3 \times -3) + (-4) =$$

$$9 + (-4)$$

Add.

$$9 + (-4) = 5$$

Solution **The value of the expression is 5.**

**Coached Example**

What is the value of this expression when $p = 8$ and $q = 5$?

$$\frac{16}{p} - 3q$$

Substitute _____ for p and _____ for q in the expression.

$$\frac{16}{p} - 3q = \underline{\hspace{2cm}}$$

Use the order of operations to simplify.

First, multiply and divide from left to right.

Now, add and subtract in order from left to right.

The value of the expression is _____.

Add and Subtract Algebraic Expressions



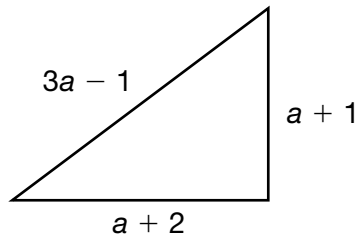
Getting the Idea

The same rules you learned for adding and subtracting rational numbers also apply to adding and subtracting algebraic expressions.

Number properties and the order of operations can also help you solve problems involving addition or subtraction of expressions.

Example 1

What is the perimeter of the triangle below?



Strategy Add the expressions for the 3 sides. Simplify the sum.

Step 1

Write an expression to represent the perimeter.

The perimeter of a figure is the sum of its side lengths.

$$(a + 1) + (a + 2) + (3a - 1)$$

Step 2

Use the commutative and associative properties to reorder and group like terms.

$$(a + 1) + (a + 2) + (3a - 1) =$$

$$a + 1 + a + 2 + 3a - 1 =$$

$$a + a + 3a + 1 + 2 - 1$$

Step 3

Add.

$$a + a + 3a + 1 + 2 - 1 =$$

$$(a + a + 3a) + (1 + 2 - 1) =$$

$$5a + 2$$

Solution The expression $5a + 2$ represents the perimeter of the triangle.

Remember that subtracting an integer is the same as adding its opposite. You can use the same strategy to subtract an algebraic expression.

Example 2

Subtract.

$$4x + 8 - (3 + 4y)$$

Strategy Find the opposite of the expression being subtracted.
Then add the opposite to the original expression.

Step 1

Find the opposite of the quantity being subtracted.

The opposite of $3 + 4y$ is $-(3 + 4y)$.

$-(3 + 4y)$ is the same as $-1(3 + 4y)$, so distribute -1 over the two terms.

$$-(3 + 4y) = (-3) - 4y$$

Step 2

Rewrite the problem as an addition problem by adding the opposite.

$$4x + 8 - (3 + 4y) =$$

$$4x + 8 + (-3) - 4y$$

Step 3

Use the commutative and associative properties to reorder and group like terms.

$$4x + 8 + (-3) - 4y = 4x - 4y + 8 + (-3)$$

Then add.

$$4x - 4y + 8 + (-3) = 4x - 4y + 5$$

Solution The difference is $4x - 4y + 5$.

Example 3

Drew baked c corn muffins. He brought $\frac{3}{4}$ of the corn muffins to the bake sale and gave $\frac{1}{8}$ of the muffins to his grandmother. How many muffins did Drew have left?

Strategy Translate the problem into an expression. Then simplify.

Step 1

Write an expression for the number of muffins Drew had left.

Drew baked c muffins. He brought $\frac{3}{4}$ of the muffins to the bake sale.

He also gave $\frac{1}{8}$ of the muffins to his grandmother.

This can be represented as $c - \frac{3}{4}c - \frac{1}{8}c$.

Step 2

Simplify the expression using a common denominator.

$$\begin{aligned}
 c - \frac{3}{4}c - \frac{1}{8}c &= \\
 1c - \frac{3}{4}c - \frac{1}{8}c &= \\
 \frac{8}{8}c - \frac{6}{8}c - \frac{1}{8}c &= \\
 \frac{(8 - 6 - 1)}{8}c &= \frac{1}{8}c
 \end{aligned}$$

Solution Drew had $\frac{1}{8}c$ muffins left.**Coached Example**

Carter bought a ruler for \$2 and a compass for x dollars. He paid for the items with a \$5 bill and y -dollar bills. How much did Carter receive in change? Write your answer in simplest form.

Translate the problem into an expression.

paid with a \$5 bill	and	y -dollar bills	-	a ruler for \$2	and	a compass for x dollars
↓		↓		↓		↓
_____	+	_____	-	(_____	+	_____)

Simplify the expression you wrote.

Find the opposite of the expression being subtracted.

The opposite of $2 + x$ is _____.Distribute the -1 over the two terms. _____.

Rewrite the problem as an addition problem by adding the opposite.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

Use number properties to reorder and group like terms. Then add.

$$\underline{\hspace{10cm}}$$

In simplest form, Carter received _____ in change.

Write Algebraic Equations



Getting the Idea

An **equation** is a mathematical sentence that contains an equal sign (=).

An algebraic equation contains at least one variable.

You may need to write equations to solve word problems.

Example 1

Nicholas has 28 coins. That is 5 more than his brother Sam has. Write an equation that represents s , the number of coins Sam has.

Strategy **Decompose the situation into two expressions.**

Step 1

Write what you know.

Nicholas has 28 coins.

Nicholas has 5 more coins than Sam.

Step 2

Translate the words into a number sentence.

$$\begin{array}{ccccc}
 \underbrace{28 \text{ coins}} & \underbrace{\text{is}} & \underbrace{5 \text{ more than Sam has}} & & \\
 \downarrow & \downarrow & \downarrow & & \\
 28 & = & s + 5 & &
 \end{array}$$

Solution The equation $28 = s + 5$ represents the number of coins Sam has.

Example 2

Mr. Edwards purchased 3 bags of potatoes. He bought 36 potatoes in all. Each bag contains the same number of potatoes. Write an equation that represents this situation.

Strategy **Decompose the situation into two expressions.**

Step 1

Write what you know.

Mr. Edwards bought 36 potatoes.

The potatoes came in 3 bags, each with an equal number of potatoes.

Step 2

Translate the words into an equation.

Let p = the number of potatoes in each bag.

$$3p = 36$$

Solution **The equation $3p = 36$ represents this situation.**

Example 3

Phoebe is 3 years less than half her brother's age. Phoebe is 13 years old. Her brother is b years old. Write an equation that could be used to find her brother's age.

Strategy **Decompose the situation into two expressions.**

Step 1

Write what you know.

Phoebe is 3 years less than $\frac{1}{2}$ her brother's age.

Phoebe is 13 years old.

Phoebe's brother is b years old.

Step 2

Decompose the situation.

Phoebe is 13 years old, so 13 goes on one side of the equation.

If Phoebe's brother is b years old, then Phoebe is $\frac{b}{2} - 3$ years old.

Step 3

Translate the words into an equation.

$$\frac{b}{2} - 3 = 13$$

Solution **The equation $\frac{b}{2} - 3 = 13$ could be used to find her brother's age.**

Example 4

Rafael's tennis racket cost 5% more than Carl's tennis racket. Rafael's racket cost \$126. Write an equation that could be used to find the cost of Carl's tennis racket.

Strategy Use mathematical sense to translate the words into an equation.

Step 1

Understand how the quantities are related.

Rafael's racket cost 5% more than 100% of the cost of Carl's racket.

Step 2

Write an expression for the cost of Rafael's racket.

Let c = the cost of Carl's tennis racket.

$5\%c + 100\%c = 0.05c + 1c = 1.05c$ or $105\%c$

The cost of Rafael's racket is 105% of the cost of Carl's racket.

Step 3

Write the equation for the cost of Rafael's racket.

$1.05c = 126$

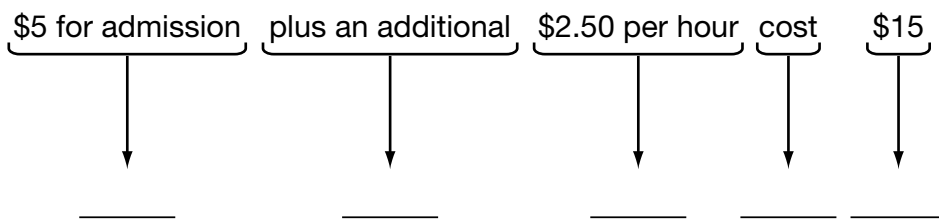
Solution The equation $1.05c = 126$ could be used to find the cost of Carl's tennis racket.



Coached Example

Nigel went to an ice rink and paid \$5 for admission plus an additional \$2.50 per hour to rent skates. The total cost was \$15. Write an equation that represents h , the number of hours for which Nigel rented skates.

Translate the words into a mathematical sentence.



The equation _____ represents the situation.

Solve Equations



Getting the Idea

To solve an equation, you must isolate the variable by using **inverse operations**. Inverse operations undo each other.

The first step to solving a two-step equation is to add or subtract to remove the **constant** to leave one side of the equation with a **coefficient** and variable. If the coefficient is an integer, the second step will be to divide to isolate the variable.

Example 1

What is the value of n in the equation $6n + 9 = 36$?

Strategy Use inverse operations to isolate the variable.

Step 1

Subtract 9 from both sides of the equation to remove the constant.

$$6n + 9 - 9 = 36 - 9$$

$$6n = 27$$

Step 2

Divide both sides of the equation by the coefficient to isolate the variable and write the solution in simplest form.

$$\frac{6n}{6} = \frac{27}{6}$$

$$n = 4\frac{3}{6} = 4\frac{1}{2}$$

Solution $n = 4\frac{1}{2}$

When the coefficient is a fraction, multiply both sides by the reciprocal of the coefficient.

Example 2

What is the value of z in the equation $\frac{2}{3}z - 6 = 15$?

Strategy Use inverse operations to isolate the variable.

Step 1

Add 6 to both sides of the equation to remove the constant.

$$\frac{2}{3}z - 6 + 6 = 15 + 6$$

$$\frac{2}{3}z = 21$$

Step 2

Multiply both sides of the equation by the reciprocal of the coefficient to isolate the variable.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{2}{3}z \times \frac{3}{2} = 21 \times \frac{3}{2}$$

$$z = \frac{63}{2} = 31\frac{1}{2}$$

Solution $z = 31\frac{1}{2}$

When a coefficient comes before a term in parentheses, use the distributive property to rename the equation.

Example 3

What is the value of b in the equation $4(b - 3) = 24$?

Strategy Use the distributive property to rename the equation.

Step 1

Use the distributive property to simplify the left side of the equation.

Multiply 4 by each term inside the parentheses.

$$\begin{aligned} 4(b - 3) &= 4 \cdot b + 4 \cdot -3 \\ &= 4b - 12 \end{aligned}$$

The equation is now $4b - 12 = 24$.

Step 2

Add 12 to both sides of the equation to remove the constant.

$$\begin{aligned} 4b - 12 + 12 &= 24 + 12 \\ 4b &= 36 \end{aligned}$$

Step 3

Divide both sides of the equation by the coefficient to isolate the variable.

$$\begin{aligned} 4b \div 4 &= 36 \div 4 \\ b &= 9 \end{aligned}$$

Solution $b = 9$

Example 4

What is the value of g in the equation $\frac{3}{4}(g - \frac{1}{2}) = 1\frac{7}{8}$?

Strategy Use the distributive property to rename the equation.

Step 1

Use the distributive property to simplify the left side of the equation.

Multiply $\frac{3}{4}$ by each term inside the parentheses.

$$\frac{3}{4}(g - \frac{1}{2}) = \frac{3}{4}g - \frac{3}{8}$$

The equation is now $\frac{3}{4}g - \frac{3}{8} = 1\frac{7}{8}$.

Step 2

Add $\frac{3}{8}$ to both sides of the equation to remove the constant.

$$\frac{3}{4}g - \frac{3}{8} + \frac{3}{8} = 1\frac{7}{8} + \frac{3}{8}$$

$$\frac{3}{4}g = 1\frac{10}{8} = 2\frac{1}{4}$$

Step 3

Multiply both sides of the equation by the reciprocal of the coefficient to isolate the variable.

Rename the mixed number as an improper fraction.

$$\frac{3}{4}g \cdot \frac{4}{3} = \frac{\overset{3}{\cancel{8}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{3}}}$$

$$g = 3$$

Solution $g = 3$

Use the rules for computing with rational numbers to solve equations involving negative numbers.

Example 5

What is the value of p in the equation $-4p - 8 = 24$?

Strategy **Use inverse operations to isolate the variable.**

Step 1

Add 8 to both sides of the equation to remove the constant.

$$-4p - 8 + 8 = 24 + 8$$

$$-4p = 32$$

Step 2

Divide both sides of the equation by the coefficient to isolate the variable.

The signs are different, so the solution will be negative.

$$\frac{-4p}{-4} = \frac{32}{-4}$$

$$p = -8$$

Solution $p = -8$

Use the distributive property for a negative coefficient as you would a positive coefficient.

Example 6

What is the value of q in the equation $-0.8(q - 0.5) = -2.6$?

Strategy **Use the distributive property to rename the equation.**

Step 1

Use the distributive property to simplify the left side of the equation.

Multiply -0.8 by each term inside the parentheses.

$$-0.8(q - 0.5) = -0.8q + 0.4$$

The equation is now $-0.8q + 0.4 = -2.6$.

Step 2

Subtract 0.4 from both sides of the equation to remove the constant.

$$-0.8q + 0.4 - 0.4 = -2.6 - 0.4$$

$$-0.8q = -3$$

Step 3

Divide both sides of the equation by the coefficient to isolate the variable.

$$\frac{-0.8q}{-0.8} = \frac{-3}{-0.8}$$

The signs are the same, so the solution will be positive.

$$q = 3.75$$

Solution $q = 3.75$



Coached Example

What is the solution for r in this equation?

$$-\frac{1}{2}\left(r - \frac{3}{4}\right) = \frac{-4}{5}$$

The first step is to use the _____ to simplify the left side of the equation.

Multiply both terms inside the parentheses by _____.

The left side of the equation is now _____.

The equation can be rewritten as _____.

Now you have a two-step equation to solve.

Subtract _____ from both sides of the equation. _____

What is the difference of the right side of the equation? _____

The equation is now _____.

Multiply both sides of the equation by the _____ of the coefficient.

Solve. _____

Write the solution in simplest form. $r =$ _____

Use Algebra to Solve Word Problems



Getting the Idea

One way to solve a word problem is arithmetically. Problem solving strategies can help you recognize the sequence of steps needed to solve a problem arithmetically.

Example 1

The art teacher has 6 packages of brushes and 8 single brushes. Each package has the same number of brushes. In all, he has 80 brushes for his students to use. How many brushes are in a package?

Strategy **Work backward to undo the sequence of operations in the problem.**

Step 1

Understand the problem.

There are 6 packages of brushes, each with the same number of brushes.

There are also 8 single brushes.

There are 80 brushes in all.

Step 2

Identify the first operation to undo.

The 8 single brushes are in addition to the packages of brushes.

Subtract 8 from the total of 80 to find how many brushes are in the packages in all.

$$80 - 8 = 72$$

There are 72 brushes in the packages in all.

Step 3

Identify the next operation to undo.

There are 72 brushes in the 6 packages. Since each package has the same number of brushes, divide to find the number in each package.

$$72 \div 6 = 12$$

Solution **There are 12 brushes in each package.**

Another way to solve a word problem is algebraically. First, define the variable in the problem situation. Next, write an equation to represent the situation. Then solve the equation using inverse operations and the properties of equality.

The properties of equality tell you that if you perform an operation to one side of an equation, you must perform the same operation to the other side of the equation to keep both sides equal.

When you solve an equation, you must isolate the variable on one side of the equation.

Example 2

Solve the problem in Example 1 algebraically.

Strategy **Write and solve an equation.**

Step 1

Write the problem as an equation.

Let x represent the number of brushes in each package.

There are 6 packages of brushes and 8 single brushes: $6x + 8$.

There are 80 brushes in all: 80.

$$6x + 8 = 80$$

Step 2

Solve the equation.

$$6x + 8 = 80$$

$$6x + 8 - 8 = 80 - 8$$

Subtract 8 from both sides.

$$6x = 72$$

Simplify.

$$\frac{6x}{6} = \frac{72}{6}$$

Divide both sides by 6.

$$x = 12$$

The solution is the same whether you solve the equation arithmetically or algebraically.

Example 3

Troy is 3 times as old as Mia's and Dan's ages combined. If Troy is 36 and Mia is 5, how old is Dan?

Strategy **Translate the problem into an algebraic equation. Then solve.**

Step 1

Translate the problem into an algebraic equation.

Let d represent Dan's age.

The total of Mia's and Dan's ages combined is $5 + d$.

Troy is 3 times the total. Troy is 36 years old.

$$3(5 + d) = 36$$

Step 2

Solve for d .

$$\frac{3(5 + d)}{3} = \frac{36}{3}$$

Divide both sides of the equation by 3.

$$5 + d = 12$$

Simplify.

$$5 - 5 + d = 12 - 5$$

Subtract 5 from both sides to isolate d .

$$d = 7$$

Solution **Dan is 7 years old.**

You can rename an expression to better relate how to solve a word problem. You may rename a percent to a fraction or a decimal or visa versa. Use whichever method is best for you.

Example 4

Ali was earning \$15 per hour before receiving a 10% raise. What is Ali's new hourly rate?

Strategy **Write and solve an equation.**

Step 1

Write an expression to represent Ali's hourly rate after her raise.

\$15 was 100% of her hourly rate.

Her rate increased by 10%, so add $100\% + 10\% = 110\%$.

Step 2

Find Ali's new hourly rate.

Rename 110% to 1.1 and multiply 1.1 times the original rate.

Let n represent Ali's new hourly rate.

$$n = 15 \times 1.1$$

$$n = 16.5$$

Solution **Ali's new hourly rate is \$16.50.**

Example 5

Aaron bought a stock at \$42.80 per share. The value of the stock dropped 20% in its first week. What is the value of the stock after one week?

Strategy **Write and solve an equation.**

Step 1

Write an expression to represent the value of Aaron's stock.

\$42.80 was 100% of the value of the stock when Aaron bought it.

The value decreased by 20%, so subtract $100\% - 20\% = 80\%$.

Step 2

Find the value of the stock.

Rename 80% to 0.8 and multiply 0.8 times the original value.

Let s represent the value of the stock after one week.

$$s = 42.8 \cdot 0.8$$

$$s = \$34.24$$

Solution **The value of the stock after one week is \$34.24.**

Example 6

Ron wants to place a painting that is $32\frac{1}{2}$ inches wide in the horizontal center of a wall that is $89\frac{1}{2}$ inches wide. How far from each corner of the wall should he place the painting for it to be centered?

Strategy Solve the problem algebraically.

Step 1

Translate the problem into an algebraic equation.

Let n represent the distance from each corner.

If the painting is centered, the distance from each corner plus the width of the painting is equal to the width of the wall.

$$32\frac{1}{2} + 2n = 89\frac{1}{2}$$

Step 2

Solve the equation for n .

$$32\frac{1}{2} - 32\frac{1}{2} + 2n = 89\frac{1}{2} - 32\frac{1}{2}$$

Subtract $32\frac{1}{2}$ from both sides.

$$2n = 57$$

Simplify.

$$\frac{2n}{2} = \frac{57}{2}$$

Divide both sides by 2.

$$n = 28\frac{1}{2}$$

Solution Ron should place the painting $28\frac{1}{2}$ inches from each corner.



Coached Example

The cost of the sneakers that Janeel wants to buy is \$84.20. This week those sneakers are 30% off. What is the sale price of the sneakers, s ?

What percent of the cost does \$84.20 represent? _____

Will Janeel pay for less than or more than 100% of the total cost? _____

What percent of the cost will Janeel pay? _____

Write an equation to represent how to find the sale price of the sneakers.

Solve your equation.

Janeel will spend _____ for the sneakers.

Inequalities

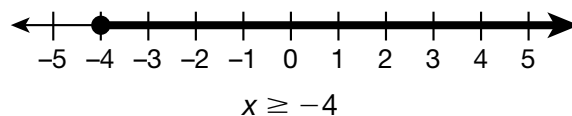
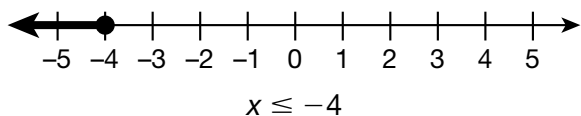
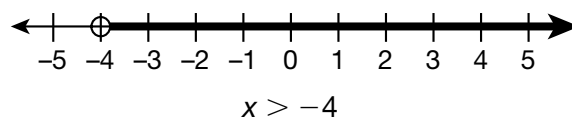
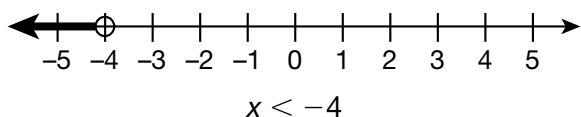


Getting the Idea

An **inequality** is a mathematical statement that two quantities are not equal. Inequalities are represented by using the symbols $>$ (**is greater than**), $<$ (**is less than**), \geq (**is greater than or equal to**), and \leq (**is less than or equal to**).

To solve an inequality, follow the same rules as when solving an equation. The solution of an inequality is a set of numbers called the **solution set**.

You can graph the solution set of an inequality on a number line. Draw a circle and an arrow to show all of the numbers that are part of the solution. If the circled number is part of the solution set, fill in the circle. If the circled number is not part of the solution set, leave the circle open. Here are some examples.



Example 1

Graph the solution set for this inequality.

$$2x - 7 < 11$$

Strategy Use inverse operations to isolate the variable.

Step 1

Add 7 to both sides of the inequality.

$$2x - 7 + 7 < 11 + 7$$

$$2x < 18$$

Step 2

Divide both sides of the inequality by 2.

$$\frac{2x}{2} < \frac{18}{2}$$

$$x < 9$$

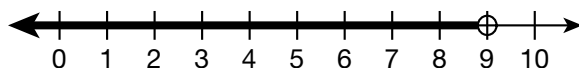
Step 3

Graph the solution set.

9 is not part of the solution set, so use an open dot.

Since x is less than 9, the arrow points left.

Solution



Example 2

Graph the solution set for this inequality.

$$\frac{3}{4}x + \frac{1}{4} \geq 2\frac{1}{2}$$

Strategy

Use inverse operations to isolate the variable.

Step 1

Subtract $\frac{1}{4}$ from both sides of the inequality.

$$\frac{3}{4}x + \frac{1}{4} - \frac{1}{4} \geq 2\frac{1}{2} - \frac{1}{4}$$

$$\frac{3}{4}x \geq 2\frac{1}{4}$$

Step 2

Multiply both sides by the reciprocal of $\frac{3}{4}$.

$$\frac{3}{4}x \cdot \frac{4}{3} \geq \frac{9}{4} \times \frac{4}{3}$$

$$x \geq 3$$

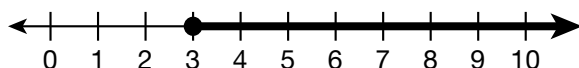
Step 3

Graph the solution set.

3 is part of the solution set, so use a closed dot.

Since x is greater than or equal to 3, the arrow points right.

Solution



When you multiply or divide by a negative number in an inequality, it reverses the sign.

For example, to find the solution set for x in $-2x > 4$, divide both sides by -2 .

This reverses the sign from $>$ to $<$. So the solution set would be $x < -2$.

Example 3

Find the solution set for $-4p + 6 > -10$.

Strategy

Use inverse operations to isolate the variable.

Step 1

Subtract 6 from both sides of the inequality.

$$-4p + 6 - 6 > -10 - 6$$

$$-4p > -16$$

Step 2

Divide both sides of the inequality by -4 .

$$\frac{-4p}{-4} > \frac{-16}{-4}$$

The sign reverses.

$$p < 4$$

Solution **The solution set is $p = 4$.**

You can solve problems by writing and solving an inequality.

Example 4

On Saturday, Maya read at least 5 fewer than 3 times as many pages in her book as she did on Friday. On Saturday, she read 58 pages. How many pages did Maya read on Friday?

Strategy **Translate the problem into an algebraic inequality. Then solve.**

Step 1

Translate the problem into an algebraic inequality.

Let n represent the number of pages Maya read on Friday.

“5 fewer than 3 times as many pages” translates to $3n - 5$.

Use the inequality sign \geq for “at least.”

$$3n - 5 \geq 58$$

Step 2

Solve the inequality using inverse operations.

$$3n - 5 + 5 \geq 58 + 5 \quad \text{Add 5 to both sides.}$$

$$3n \geq 63$$

$$\frac{3n}{3} \geq \frac{63}{3} \quad \text{Divide both sides by 3.}$$

$$n \geq 21$$

Step 3

Interpret the solution.

The solution set $n \geq 21$ means Maya read 21 pages or more on Friday.

Solution **Maya read at least 21 pages on Friday.**

Example 5

A taxi driver charges a flat fee of \$4 plus \$6 per mile. The tip is included in the mileage rate. Orlando only has \$22 to pay for a taxi ride. How many miles at most can Orlando ride in the taxi? Graph the solution set.

Strategy Translate the problem into an algebraic inequality. Then solve.

Step 1

Translate the problem into an algebraic inequality.

Let m represent the number of miles.

“Flat fee of \$4 plus \$6 per mile” translates to $4 + 6m$.

Use the inequality sign \leq for “at most.”

$$4 + 6m \leq 22$$

Step 2

Solve the inequality using inverse operations.

$$4 - 4 + 6m \leq 22 - 4 \quad \text{Subtract 4 from both sides.}$$

$$6m \leq 18$$

$$\frac{6m}{6} \leq \frac{18}{6} \quad \text{Divide both sides by 6.}$$

$$m \leq 3$$

Step 3

Interpret the solution.

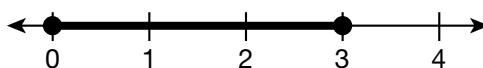
The solution set $m \leq 3$ means Orlando can ride 3 miles or less.

Step 4

Graph the solution set.

The graph must start at 0 since he cannot ride less than 0 miles.

Orlando can ride between 0 and 3 miles. So, draw closed circles on 0 and 3.



Solution Orlando can ride between 0 and 3 miles in the taxi.



Coached Example

A middle school is sponsoring a 5K Fun Run to raise money for the library. Each runner will receive a T-shirt. The T-shirts for the race cost \$345. Timers and other race equipment cost \$85. Local businesses donated \$50. If each runner pays \$15, how many people must enter the race for it to make a profit?

The expenses for the race are the _____, the timers, and other race equipment.

The total expenses are _____ + _____ = _____.

Let n represent the number of people _____.

Write an expression for the total amount raised by entry fees. _____

How much money was donated to help with expenses? _____

Write an expression for the total amount of money raised. _____

To make a profit, the amount raised by the entry fees plus the money donated must be _____ the total expenses for the race.

Write an inequality to show how many people must enter the race to make a profit.

Solve the inequality.

The solution of the inequality is a mixed number. Since you cannot have a fraction of a person, round the number up to the next whole number.

What is the least number of people who can enter for the race to make a profit? _____

At least _____ people must enter the race for it to make a profit.

Scale Drawings



Getting the Idea

A **scale drawing** is a representation of an actual object. The **scale** tells how to reduce or enlarge the dimensions of a scale drawing. Scale drawings are similar to and, therefore, proportional to the actual object.

Example 1

Ray went to visit the Great Pyramids. He learned that the base length of each triangular face is about 750 ft. He wants to make a scale drawing of the pyramids using the scale 1 in. = 150 ft. What will be the base length of his scale drawing?

Strategy Use the scale to write a proportion.

Step 1

Write the scale as a ratio.

The scale is 1 in. = 150 ft.

This means that, for each inch of the scale drawing, the actual length is 150 feet.

The ratio is $\frac{1 \text{ in.}}{150 \text{ ft.}}$

Step 2

Write a proportion using the ratio from Step 1.

Let x represent the base length of the scale drawing.

$$\frac{1 \text{ in.}}{150 \text{ ft.}} = \frac{\text{base length of scale drawing}}{\text{base length of actual pyramid}}$$

$$\frac{1}{150} = \frac{x}{750}$$

Step 3

Solve the proportion.

$$\frac{1}{150} = \frac{x}{750}$$

$$150 \times x = 750 \times 1$$

$$150x = 750$$

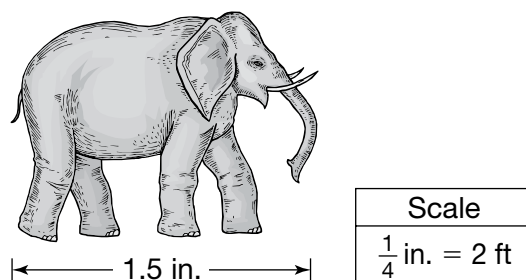
$$\frac{150x}{150} = \frac{750}{150}$$

$$x = 5$$

Solution The base length of Ray's scale drawing will be 5 inches.

Example 2

Loretta made a scale drawing of an elephant. What is the actual length of the elephant?



Strategy Use the scale to write a proportion.

Step 1

Write the scale as a ratio.

The scale is $\frac{1}{4}$ inch = 2 feet.

So the ratio is $\frac{\frac{1}{4} \text{ in.}}{2 \text{ ft}}$.

Step 2

Write a proportion.

Let x represent the actual length of the elephant.

$$\frac{\frac{1}{4} \text{ in.}}{2 \text{ ft}} = \frac{\text{scale drawing length}}{\text{actual length}}$$

$$\frac{\frac{1}{4}}{2} = \frac{1.5}{x}$$

Step 3

Solve the proportion.

$$\frac{\frac{1}{4}}{2} = \frac{1.5}{x}$$

$$\frac{1}{4} \times x = 2 \times 1.5$$

$$\frac{1}{4}x = 3$$

$$4 \times \frac{1}{4}x = 3 \times 4$$

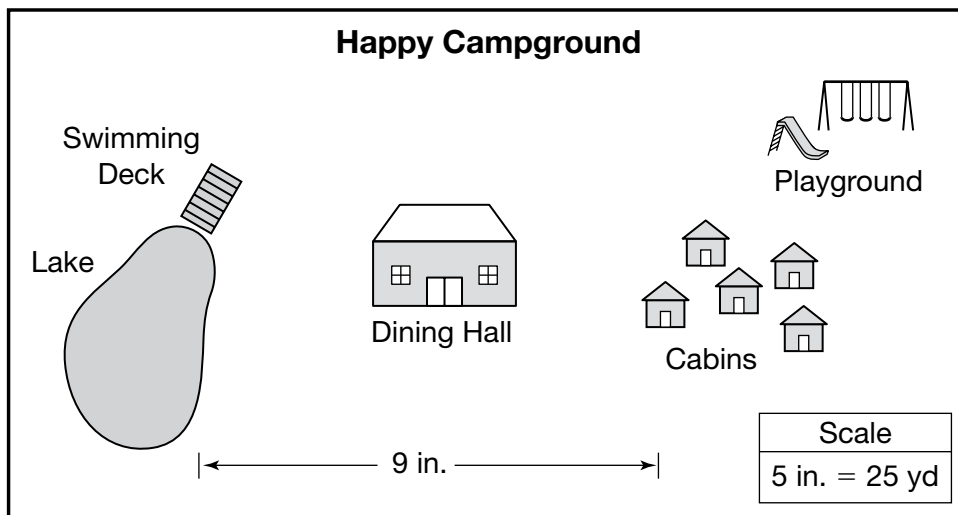
$$x = 12$$

Solution The actual length of the elephant is 12 feet.

A map is a type of scale drawing. You can use the scale on a map to find real-world distances, or use the scale to make an accurate map.

Example 3

The poster of Happy Campground shows the distance between the lake and the cabins.



What is the actual distance between the lake and the cabins?

Strategy Use the scale to write a proportion.

Step 1

Write the scale as a ratio.

The scale is 5 inches = 25 yards.

So, the ratio is $\frac{5 \text{ in.}}{25 \text{ yd}}$.

Step 2

Write a proportion.

Let x represent the actual distance.

$$\frac{5 \text{ in.}}{25 \text{ yd}} = \frac{\text{scale drawing distance}}{\text{actual distance}}$$

$$\frac{5}{25} = \frac{9}{x}$$

Step 3

Solve the proportion.

$$\frac{5}{25} = \frac{9}{x}$$

$$5 \times x = 25 \times 9$$

$$5x = 225$$

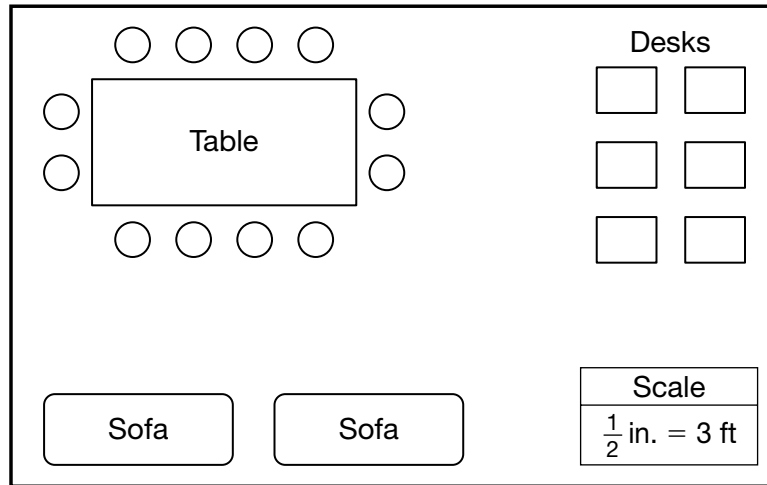
$$\frac{5x}{5} = \frac{225}{5}$$

$$x = 45$$

Solution The actual distance between the cabins and the lake is 45 yards.

Example 4

Drake made the scale drawing below of the lounge at a recreation center.



What is the actual area of the lounge?

Strategy Find the actual length and width of the lounge.

Step 1

Measure the length and width of the scale drawing.

The drawing is 4 inches long and 2.5 inches wide.

Step 2

Write a ratio of the scale.

The scale is $\frac{1}{2}$ inch = 3 feet.

So, the ratio is $\frac{\frac{1}{2} \text{ in.}}{3 \text{ ft}}$.

Step 3

Write and solve a proportion to find the actual length.

Let x represent the actual length of the lounge.

$$\frac{\frac{1}{2} \text{ in.}}{3 \text{ ft}} = \frac{\text{scale drawing length}}{\text{actual length}}$$

$$\frac{\frac{1}{2}}{3} = \frac{4}{x}$$

$$\frac{1}{2} \times x = 3 \times 4$$

$$\frac{1}{2}x = 12$$

$$2 \times \frac{1}{2}x = 12 \times 2$$

$$x = 24$$

The actual length of the lounge is 24 feet.

Step 4

Write and solve a proportion to find the actual width.

Let y represent the actual width of the lounge.

$$\frac{\frac{1}{2} \text{ in.}}{3 \text{ ft}} = \frac{\text{scale drawing width}}{\text{actual width}}$$

$$\frac{\frac{1}{2}}{3} = \frac{2.5}{y}$$

$$\frac{1}{2} \times y = 3 \times 2.5$$

$$\frac{1}{2}y = 7.5$$

$$2 \times \frac{1}{2}y = 7.5 \times 2$$

$$y = 15$$

The actual width of the lounge is 15 feet.

Step 5

Find the area of the lounge.

$$\text{Area} = \text{length} \times \text{width}$$

$$= 24 \text{ ft} \times 15 \text{ ft}$$

$$= 360 \text{ ft}^2$$

Solution The actual area of the lounge is 360 ft^2 .

**Coached Example**

Two cities are 420 miles apart. Kerri wants to draw a map that has a scale of $\frac{1}{2}$ inch = 50 miles. How far apart should Kerri draw the two cities on the map?

The scale is _____ inch = _____ miles.

Write a ratio of the scale as a fraction. _____

Let x represent the scale drawing distance.

Write a proportion to find the scale drawing distance. _____

Solve the proportion.

The scale drawing distance is _____ inches.

Kerri should draw the two cities _____ apart on the map.

Construct Geometric Shapes



Getting the Idea

Triangles can be constructed using simple tools such as a ruler and protractor, or using more complex tools such as computer drawing technology. When drawing triangles with a ruler and/or a protractor by hand, you may sometimes need to erase your work and start over again. It can involve some trial and error.

Acute triangles have 3 **acute angles**, a **right triangle** has 1 **right angle**, and an **obtuse triangle** has 1 **obtuse angle**. The sum of the angle measures of any triangle is 180° .

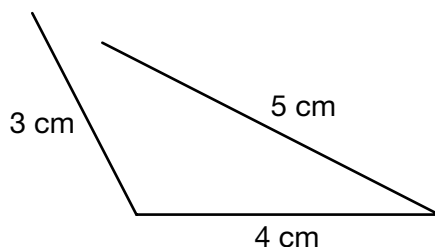
Example 1

Using a ruler, construct a triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters. What kind of triangle is it? Is it possible to draw another kind of triangle?

Strategy Use a ruler.

Step 1

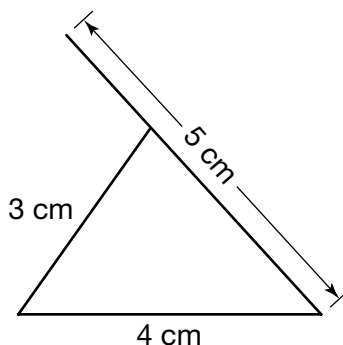
Try drawing a triangle with one obtuse angle—an obtuse triangle.



The figure is not closed. An obtuse triangle is not possible.

Step 2

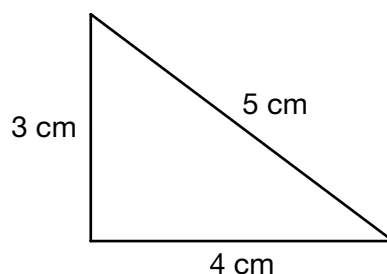
Try drawing a triangle with only acute angles—an acute triangle.



The endpoints do not meet. An acute triangle is not possible.

Step 3

Try drawing a triangle with one right angle—a right triangle.



You can draw one unique triangle with those side lengths, and it is a right triangle.

Solution **It is only possible to draw a right triangle with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters.**

When trying to construct a triangle with given side lengths or angle measures, there are several possibilities:

- The triangle may be uniquely defined. In other words, you may only be able to draw one triangle.
- The triangle may be ambiguously defined. That just means you may be able to draw more than one triangle.
- The triangle may be nonexistent. It may not be possible to draw a triangle with those measures.

The triangle in Example 1 is uniquely defined.

Example 2

Is it possible to construct a triangle with angles measuring 61° , 33° , and 86° ? If so, can you draw only a unique triangle or can you draw many different triangles?

Strategy Find the sum of the angle measures.

Step 1

Determine if the angle measures add to 180° .

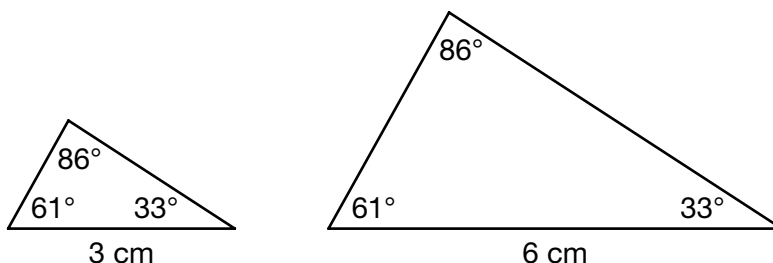
$$61^\circ + 33^\circ + 86^\circ \stackrel{?}{=} 180^\circ$$

$$180^\circ = 180^\circ \quad \checkmark$$

So, a triangle with these angle measures is possible.

Step 2

Use a protractor to draw one or more triangles with those angle measures.



The two triangles are **similar** to each other because they have the same angle measures.

It is possible to draw many different triangles with those angle measures.

Solution A triangle with angles measuring 61° , 33° , and 86° is **ambiguously defined** because no side lengths are mentioned. It is possible to draw many different similar triangles with those angle measures.

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example 3

Can you construct a triangle with sides measuring 5 inches, 8 inches, and 15 inches?

Strategy Determine if one length is greater than the sum of the other two lengths.

Step 1

Determine if one length is greater than the sum of the other two lengths.

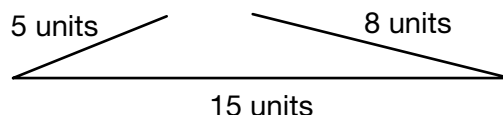
$$5 + 8 \stackrel{?}{>} 15$$

$13 < 15$, so the inequality $5 + 8 > 15$ is not true.

It is impossible to draw a triangle with those side lengths.

Step 2

Try to sketch a triangle with those dimensions so you can see why it is not possible.



There is no way to connect all three sides. It is impossible to draw a triangle with those side lengths.

Solution A triangle with sides measuring 5 inches, 8 inches, and 15 inches does not exist.



Coached Example

Is it possible to construct a triangle with sides measuring 8 feet, 9 feet, and 12 feet?

The sum of the lengths of any two sides of a triangle must be _____ than the length of the third side.

Determine if this triangle is possible or not.

Is $8 + 9 > 12$? _____

Is $8 + 12 > 9$? _____

Is $9 + 12 > 8$? _____

The inequalities above are all true, so it _____ possible to draw a triangle with side lengths of 8 feet, 9 feet, and 12 feet.

Cross Sections of Three-Dimensional Figures

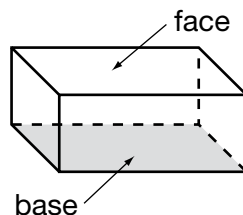


Getting the Idea

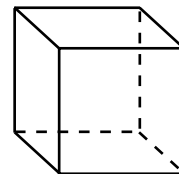
A **three-dimensional figure** (also called a **solid figure**) has length, width, and height. It is not flat. Some examples of three-dimensional figures are below.

A **prism** has a pair of **bases** that are **parallel**, congruent polygons. Its other **faces** are rectangles.

A **rectangular prism** has 6 faces that are rectangles.

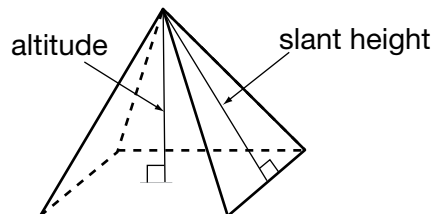


A **cube** is a rectangular prism with 6 square faces.

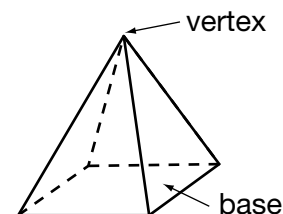


A **pyramid** has one base that is a polygon. Its other faces are triangles. The height of a pyramid is called its **altitude**, and the height of its lateral face is called its **slant height**.

A **rectangular pyramid** has a base that is a rectangle.



A **square pyramid** has a base that is a square.

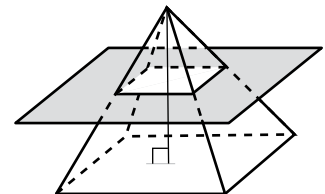


A three-dimensional figure can be sliced by a plane to show a two-dimensional view. This view is called a **cross section**.

Example 1

A square pyramid is sliced by a plane that is parallel to its base, as shown.

What is the shape of the cross section?



Strategy Visualize a plane, parallel to the base, slicing through the pyramid.

The cross section will have the same shape as the base.

It will be a square.

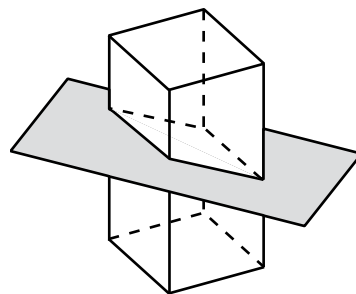


Solution The shape of the cross section is a square.

Example 2

A rectangular prism is cut by the slanted plane shown.

What is the shape of the cross section?



Strategy Visualize the prism being sliced by a thin piece of wire.

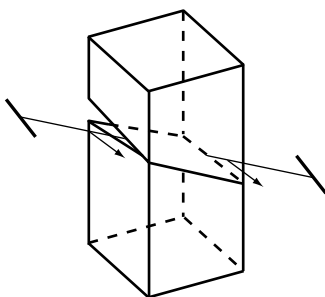
Step 1

Determine the angle at which the plane intersects the prism.

The plane is neither horizontal nor vertical to the faces of the prism.

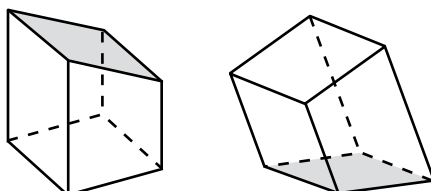
Step 2

Imagine slicing the rectangular prism with a piece of wire.



Step 3

The prism is now in two parts.



Step 4

Visualize the shape of the cross section.



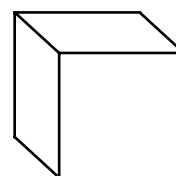
Solution The shape of the cross section is a parallelogram.

Example 3

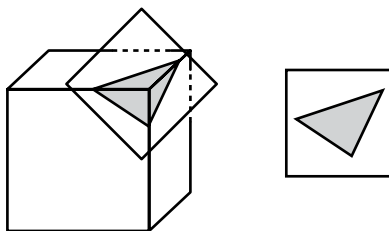
Look at this cube.

How can a plane slice the cube so that the cross section is a triangle?

Strategy Visualize using a plane to slice the cube to get a triangular cross section.



Slice through a corner of the cube with a plane.

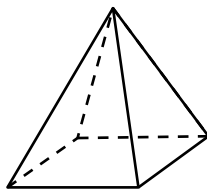


Solution The cross section of the cube is shown above.



Coached Example

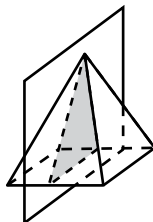
Nari will slice this pyramid with a plane that is perpendicular to the base and passes through the top vertex.



What is the shape of the cross section?

What does “perpendicular” mean?

Visualize slicing the prism with a plane that is perpendicular to the base and passes through the top vertex.



Make a sketch of the cross section in the space below.

The shape of the cross section is _____.

Circles



Getting the Idea

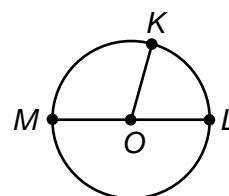
A **circle** is the set of all points in a plane that are the same distance from a given point called the **center**. A circle is named by its center.

A **radius** is the distance from the center of a circle to any point on the circle. \overline{OK} , \overline{OL} , and \overline{OM} are radii of circle O .

A **diameter** is the distance across a circle through its center.

The length of a diameter is always 2 times the length of a radius.

\overline{LM} is a diameter of circle O .



Circumference is the distance around a circle. The circumference of a circle is the product of its diameter and π , or πi . Use 3.14 or $\frac{22}{7}$ as approximations for π in computations.

The table below shows the formulas for finding the circumference and **area** of a circle.

Formulas	
circumference	$C = \pi d$ or $C = 2\pi r$
area	$A = \pi r^2$

Example 1

What is the approximate circumference of this circle? Use 3.14 for π .

Strategy Use the formula for the circumference of a circle.

Step 1

Write the formula for circumference when you know the radius.

$$C = 2\pi r$$

Step 2

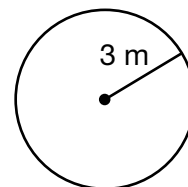
Substitute 3 for r and 3.14 for π . Then multiply.

$$C = 2\pi r$$

$$C \approx 2 \times 3.14 \times 3$$

$$C \approx 18.84$$

Note: Use \approx because 3.14 is an estimate.



Solution The circumference of the circle is about 18.84 meters.

Example 2

The circumference of a circle is 9π inches. What is the diameter of the circle?

Strategy Use the formula for the circumference of a circle.

Step 1

Write the formula for the circumference when you know the diameter.

$$C = \pi d$$

Step 2

Substitute 9π for C .

$$9\pi = \pi d$$

Step 3

Divide both sides of the equation by π .

$$9\pi = \pi d$$

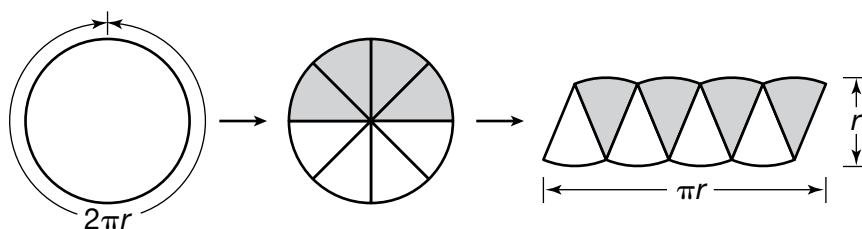
$$\frac{9\pi}{\pi} = \frac{\pi d}{\pi}$$

$$9 = d$$

Solution The diameter of the circle is 9 inches.

The formulas for the circumference of a circle and the area of a **parallelogram** can help you find a formula for the area of a circle.

Imagine cutting a circle into an equal number of pieces, such as 8 pieces. Arrange the pieces to form as close to a parallelogram as possible.



As you can see, the sides are not straight, so it is not a parallelogram. However, as the pieces of the circle get smaller, when arranged to make a parallelogram, the sides will be straight.

Since the circumference of a circle is $2\pi r$, the length of the parallelogram is $\frac{1}{2}$ the circumference. So, the length is $\frac{1}{2}C = \frac{1}{2} \times 2\pi r$, or πr .

The height of the parallelogram is about the same as the radius, r , of the circle. The area of a parallelogram is bh , so the area of the circle is $\pi r \times r$, or $A = \pi r^2$.

Example 3

A circle has a diameter of 8 inches. What is the approximate area of the circle? Use 3.14 for π .

Strategy **Use the formula for the area of a circle.**

Step 1

Use the diameter to find the radius.

The length of the radius is $\frac{1}{2}$ the length of the diameter.

The diameter is 8.

$8 \div 2 = 4$, so the radius is 4 inches.

Step 2

Write the formula for the area of a circle.

$$A = \pi r^2$$

Step 3

Substitute 4 for r and 3.14 for π . Solve.

$$A = \pi r^2$$

$A \approx 3.14 \times 4 \text{ in.} \times 4 \text{ in.}$ Again, use \approx because 3.14 is an estimate.

$$A \approx 50.24 \text{ in.}^2$$

Solution **The area of the circle is about 50.24 in.²**

Example 4

The area of a circle is 25π square centimeters. What is the radius of the circle?

Strategy **Use the formula for the area of a circle.**

Step 1

Write the formula for the area of a circle.

$$A = \pi r^2$$

Step 2

Substitute 25π for A .

$$25\pi = \pi r^2$$

Step 3

Divide both sides of the equation by π .

$$25\pi = \pi r^2$$

$$\frac{25\pi}{\pi} = \frac{\pi r^2}{\pi}$$

$$25 = r^2$$

Step 4

Take the square root of both sides of the equation to find the value of r .

$$25 = r^2$$

$$\sqrt{25} = \sqrt{r^2}$$

$$5 = r$$

Solution **When the area of a circle is 25π square centimeters, the radius is 5 centimeters.**



Coached Example

Philip is building a go-cart. The wheels he uses on the go-cart have a radius of 6 inches. What are the approximate circumference and the area of each wheel?

What is the formula for the circumference of a circle when the radius is given?

Use 3.14 for π and substitute the length of the _____ into the formula.

$$C \approx \underline{\hspace{2cm}}$$

Multiply.

$$C \approx \underline{\hspace{2cm}}$$

What is the formula for the area of a circle? _____

Use 3.14 for π and substitute the length of the _____ into the formula.

$$A \approx \underline{\hspace{2cm}}$$

Multiply.

$$A \approx \underline{\hspace{2cm}}$$

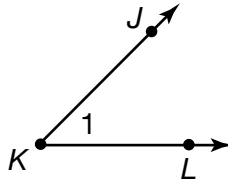
**The circumference of each wheel is about _____,
and the area is about _____.**

Angles



Getting the Idea

An **angle** is a geometric figure formed by two rays that have a common endpoint called the **vertex**. The angle below can be named $\angle 1$, $\angle JKL$, $\angle LKJ$, or $\angle K$.



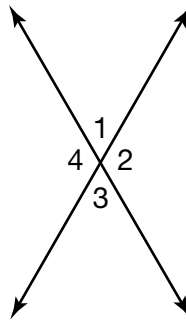
Angles are measured in degrees ($^\circ$) and can be classified by their angle measures.

An **acute angle** is an angle that measures less than 90° .

A **right angle** is an angle that measures 90° .

An **obtuse angle** is an angle that measures greater than 90° and less than 180° .

Intersecting lines can form some special angle pairs.



Adjacent angles are two angles with a side in common.

$\angle 1$ and $\angle 2$ are one set of adjacent angles.

Vertical angles are two non-adjacent angles, formed by intersecting lines, and are congruent.

$\angle 1$ and $\angle 3$ are one set of vertical angles.

Supplementary angles are two angles whose measures have a sum of 180° .

Supplementary angles that are adjacent angles have rays that form a line.

$\angle 4$ and $\angle 3$ are one set of supplementary angles.

Complementary angles are two angles whose measures have a sum of 90° .

To write the measure of an angle, you can use the abbreviation, m.

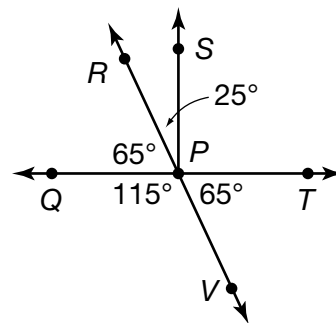
For example, “the measure of angle x” can be written as “m $\angle x$.”

Example 1

Look at the diagram on the right.

Find the following:

- a pair of complementary angles
- a pair of supplementary angles
- a pair of adjacent angles
- a pair of vertical angles



Strategy Use the definitions to identify the angle pairs.

Step 1

Find a pair of complementary angles.

Find two angles that have a sum of 90° .

$$m\angle QPR = 65^\circ \text{ and } m\angle RPS = 25^\circ.$$

$$65^\circ + 25^\circ = 90^\circ$$

Step 2

Find a pair of supplementary angles.

Find two angles that have a sum of 180° .

$$m\angle QPV = 115^\circ \text{ and } m\angle TPV = 65^\circ.$$

$$115^\circ + 65^\circ = 180^\circ$$

Another pair of supplementary angles is $\angle QPR$ and $\angle QPV$.

Step 3

Find a pair of adjacent angles.

Find two angles with a side in common.

$\angle QPR$ and $\angle RPS$ have \overrightarrow{PR} in common.

Other pairs of adjacent angles are $\angle RPS$ and $\angle SPT$, $\angle SPT$ and $\angle TPV$, $\angle TPV$ and $\angle QPV$, and $\angle QPV$ and $\angle QPR$.

Step 4

Find a pair of vertical angles.

Find two non-adjacent angles formed by intersecting lines.

\overrightarrow{RV} and \overrightarrow{QT} intersect at point P to form $\angle QPR$ and $\angle TPV$.

These angles are vertical angles, and they are congruent.

Solution $\angle QPR$ and $\angle RPS$ are a pair of complementary angles.

$\angle QPV$ and $\angle TPV$ are a pair of supplementary angles.

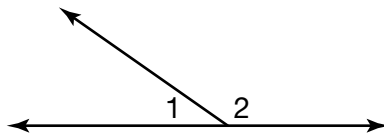
$\angle QPR$ and $\angle RPS$ are a pair of adjacent angles.

$\angle QPR$ and $\angle TPV$ are a pair of vertical angles.

Example 2

In the figure, the measure of $\angle 1$ is 35° .

What is the measure of $\angle 2$?



Strategy Look for a special angle pair.

Step 1

Decide what type of angles are $\angle 1$ and $\angle 2$.

The angles are adjacent angles that form a straight line.

So, $\angle 1$ and $\angle 2$ are supplementary angles.

Step 2

What angle measures do you know?

The measure of $\angle 1$ is 35° .

Supplementary angles have a sum of 180° .

Step 3

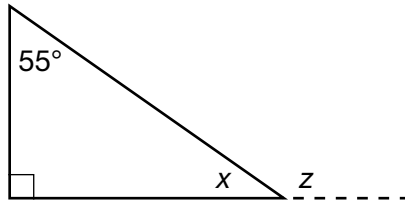
Subtract 35° from 180° to find the measure of $\angle 2$.

$$180^\circ - 35^\circ = 145^\circ$$

Solution The measure of $\angle 2$ is 145° .

Example 3

What are the measures of $\angle x$ and $\angle z$?



Strategy Look for special relationships between angles.

Step 1

Find the measure of $\angle x$.

The measures of the angles in a triangle have a sum of 180° .

The measures given are 55° and a right angle, which measures 90° .

Write an equation and solve for the measure of $\angle x$.

$$55^\circ + 90^\circ + m\angle x = 180^\circ$$

$$145^\circ + m\angle x = 180^\circ$$

$$145^\circ - 145^\circ + m\angle x = 180^\circ - 145^\circ$$

$$m\angle x = 35^\circ$$

Step 2

Identify the angle relationship of $\angle x$ and $\angle z$.

$\angle x$ is adjacent to $\angle z$. The two angles form a straight line.

Supplementary angles that are adjacent angles form a straight line.

So, $\angle x$ and $\angle z$ are supplementary angles.

Step 3

Recall the definition of supplementary angles.

Supplementary angles have a sum of 180° .

Step 4

Write an equation to find the measure of $\angle z$.

$$m\angle x + m\angle z = 180^\circ$$

$$35^\circ + m\angle z = 180^\circ$$

$$35^\circ - 35^\circ + m\angle z = 180^\circ - 35^\circ$$

$$m\angle z = 145^\circ$$

Solution The measure of $\angle x$ is 35° . The measure of $\angle z$ is 145° .



Coached Example

Two angles are complementary angles. One angle measures 59° .

The other angle measures $(4n - 1)^\circ$. What is the value of n ?

Complementary angles have a measure of _____.

The measure of one angle is _____, and the measure of the other angle is _____.

Write an equation for the total of the two angles.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 90$$

Solve the equation for n .

To find the measure of the unknown angle, substitute the value of n into $4n - 1$ and evaluate. _____

Check that the sum of the angle measures is 90° .

The value of n is _____.

Area



Getting the Idea

The **area** of a figure is the number of **square units** inside the figure.

Below are some formulas that can be used to find the areas of common polygons.

Formula	Diagram
Triangle $A = \frac{1}{2}bh$, where b represents the base length and h represents the height.	
Parallelogram $A = bh$, where b represents the base length and h represents the height.	
Rectangle $A = lw$, where l represents the length and w represents the width.	
Square $A = s^2$, where s represents the length of a side.	
Trapezoid $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 represent the base lengths and h represents the height.	

Example 1

What is the area of this trapezoid?

Strategy Use the formula for the area of a trapezoid.

Step 1

Write the formula for the area of a trapezoid.

$$A = \frac{1}{2}(b_1 + b_2)h$$

Step 2

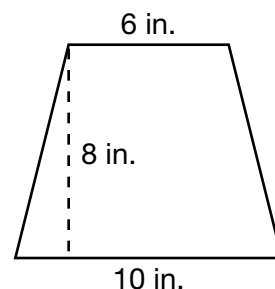
Substitute the known values in the formula and simplify.

$$\text{Let } b_1 = 10 \text{ in. and let } b_2 = 6 \text{ in.}$$

$$A = \frac{1}{2}(10 \text{ in.} + 6 \text{ in.}) \times 8 \text{ in.}$$

$$= \frac{1}{2}(16 \text{ in.}) \times 8 \text{ in.}$$

$$= 8 \text{ in.} \times 8 \text{ in.} = 64 \text{ in.}^2$$



Solution The area of the trapezoid is 64 in.^2

Example 2

Phillip drew the figure on the right to represent the design of his new garage. What is the area of the figure?

Strategy Divide the figure into smaller, familiar figures. Find the area of each figure. Then add to find the total area.

Step 1

Divide the figure into familiar figures.

The figure is divided into a triangle and a rectangle.

Step 2

Find the area of the triangle.

The height, h , of the triangle is 3 cm.

To determine the base length, subtract the two known horizontal lengths: $10 \text{ cm} - 6 \text{ cm} = 4$.

So, the base, b , of the triangle is 4 cm.

$$A \text{ of triangle} = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

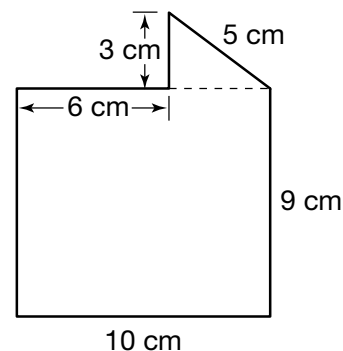
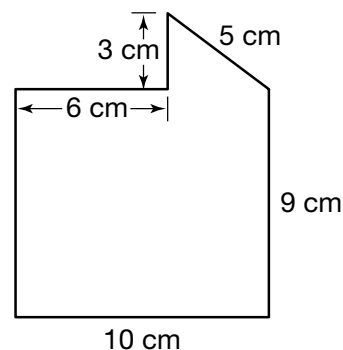
Step 3

Find the area of the rectangle.

The length, l , is 10 cm, and the width, w , is 9 cm.

$$A \text{ of rectangle} = lw$$

$$= 10 \times 9 = 90 \text{ cm}^2$$



Step 4

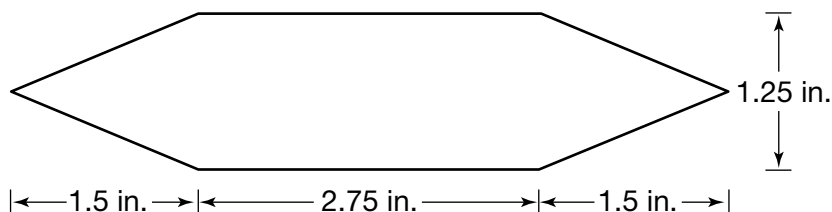
Add those areas to find the total area of the figure.

$$A \text{ of composite figure} = 6 + 90 = 96 \text{ cm}^2$$

Solution The area of the figure is 96 square centimeters.

Example 3

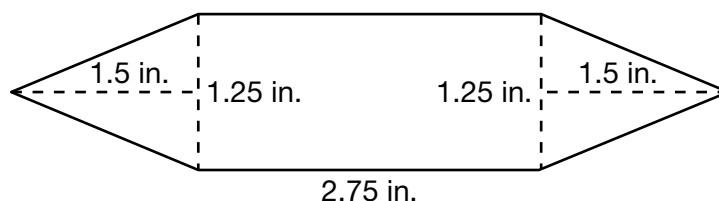
Jen is making a bracelet and needs beading for the front as shown in the diagram below. Jen has enough beads for 10 square inches. Is that enough?



Strategy Divide the figure into shapes whose area formulas you know.

Step 1

Divide the figure into 1 rectangle and 2 triangles.



Step 2

Find the area of the rectangle.

$$A = lw$$

$$A = 2.75 \times 1.25$$

$$= 3.4375 \text{ in.}^2$$

Step 3

Find the area of the 2 triangles.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 1.25 \times 1.5$$

$$= 0.9375 \text{ in.}^2$$

$$\text{There are 2 triangles: } 2 \times 0.9375 \text{ in.}^2 = 1.875 \text{ in.}^2$$

Step 4

Add to find the total area of the figure.

$$3.4375 \text{ in.}^2 + 1.875 \text{ in.}^2 = 5.3125 \text{ in.}^2$$

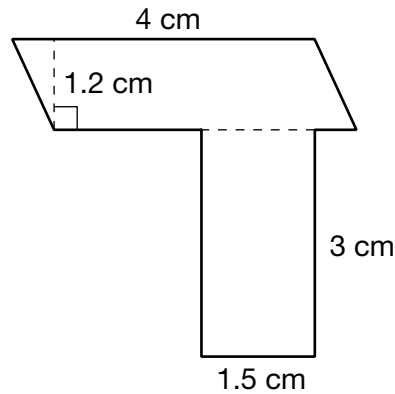
$$5.3125 < 10$$

Solution Jen has enough beads for 10 square inches.



Coached Example

Aster made a sticker in the shape shown below.



What is the area of the sticker?

A parallelogram and a _____ are combined to form the sticker.

What is the formula for the area of a parallelogram? _____

Find the area of the parallelogram.

What is the formula for the area of a rectangle? _____

Find the area of the rectangle.

Add to find the total area of the figure.

The area of the sticker is _____.

Surface Area

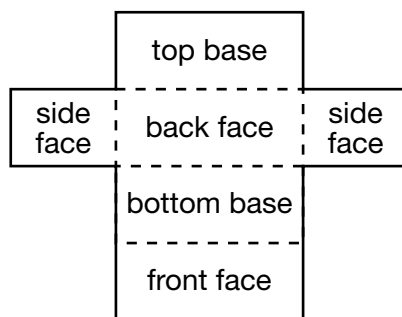
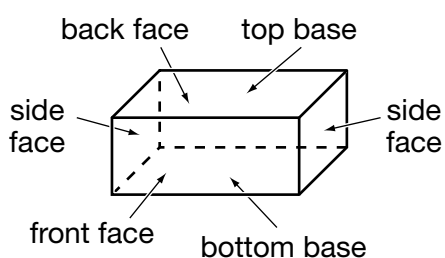


Getting the Idea

The **surface area**, measured in square units, of a solid figure is the sum of the areas of all the surfaces of the figure. You can calculate the surface area of a figure by finding the areas of all of its faces and then adding them.

Looking at a two-dimensional representation, called a **net**, of a solid figure may help you do this.

If the net below is folded along the dotted lines, a rectangular prism is formed.



The surface area of the rectangular prism is the total area of the 6 rectangular faces. The formula for the surface area of a rectangular prism is $SA = 2lw + 2lh + 2wh$, where l is length, w is width, and h is height.

Example 1

What is the surface area of this rectangular prism?

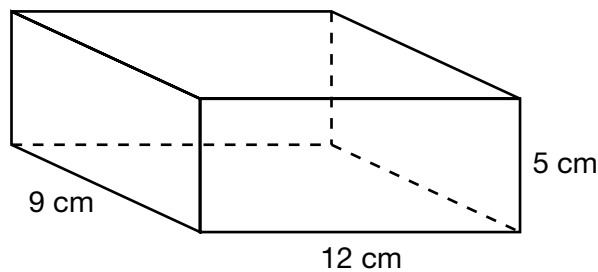
Strategy

Use the formula for the surface area of a rectangular prism.

$$SA = 2lw + 2lh + 2wh$$

$$SA = (2 \times 12 \text{ cm} \times 9 \text{ cm}) + (2 \times 12 \text{ cm} \times 5 \text{ cm}) + (2 \times 9 \text{ cm} \times 5 \text{ cm})$$

$$SA = 216 \text{ cm}^2 + 120 \text{ cm}^2 + 90 \text{ cm}^2 = 426 \text{ cm}^2$$



Solution

The surface area of the rectangular prism is 426 square centimeters.

Example 2

Mary Jane is going to wrap a box in the shape of rectangular prism that has a length of 15 inches, a width of 10 inches, and a height of 4 inches.

What is the minimum amount of wrapping paper she will need to cover the box?

Strategy Find the surface area of a rectangular prism.

Step 1

Substitute the values for the length, width, and height.

$$SA = 2lw + 2lh + 2wh$$

$$SA = (2 \times 15 \text{ in.} \times 10 \text{ in.}) + (2 \times 15 \text{ in.} \times 4 \text{ in.}) + (2 \times 10 \text{ in.} \times 4 \text{ in.})$$

Step 2

Multiply and add to find the surface area.

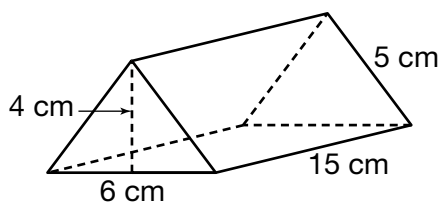
$$SA = 300 \text{ in.}^2 + 120 \text{ in.}^2 + 80 \text{ in.}^2 = 500 \text{ in.}^2$$

Solution The minimum amount of wrapping paper needed is 500 in.²

There is no special formula to find the surface area of a triangular prism. To find the surface area of a triangular prism, find the area of each face and add the areas.

Example 3

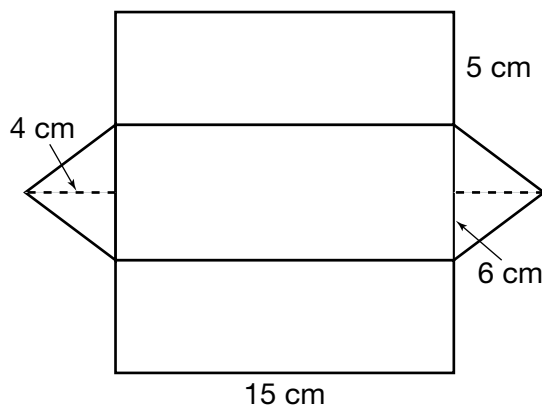
What is the surface area of this triangular prism?



Strategy Use a net to find the area of each of the faces. Then add the areas.

Step 1

Make a net of the triangular prism.



Step 2

Find the area of each of the triangles using the formula $A = \frac{1}{2}bh$.

$$A = \frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2$$

There are two triangles, so the area of the triangles is $2 \times 12 \text{ cm}^2 = 24 \text{ cm}^2$.

Step 3

Find the area of each of the rectangles using the formula $A = lw$.

$$\text{Top: } A = 15 \text{ cm} \times 5 \text{ cm} = 75 \text{ cm}^2$$

$$\text{Middle: } A = 15 \text{ cm} \times 6 \text{ cm} = 90 \text{ cm}^2$$

$$\text{Bottom: } A = 15 \text{ cm} \times 5 \text{ cm} = 75 \text{ cm}^2$$

Step 4

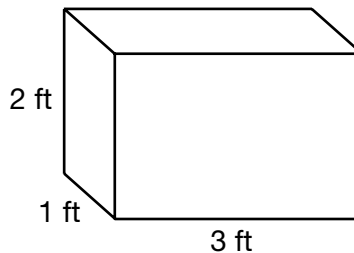
Add the areas.

$$24 \text{ cm}^2 + 75 \text{ cm}^2 + 90 \text{ cm}^2 + 75 \text{ cm}^2 = 264 \text{ cm}^2$$

Solution The surface area of the triangular prism is 264 square centimeters.

**Coached Example**

A toymaker will paint four sides of this toy chest. He will not paint the bottom or top surface. How many square feet of the chest will the toymaker paint?



You only need to find the areas of the surfaces that will be painted.

The front and back faces are rectangles that are 3 ft long and _____ ft high.

$$A \text{ of front face} = lw = 3 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ ft}^2$$

The area of the back face is also _____ ft^2 .

The left and right side faces are rectangles that are 1 ft long and _____ ft high.

$$A \text{ of left side face} = lw = 1 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The area of the right side face is also _____ ft^2 .

Add the areas of all four faces: _____ + _____ + _____ + _____ = _____

The toymaker will paint _____ square feet of the toy chest.

Volume



Getting the Idea

The **volume** of a solid figure is the number of **cubic units** that fit inside it. The formulas below can be used to calculate the volume of a rectangular prism and a cube.

Rectangular Prism

$V = lwh$, where l is the length, w is the width, and h is the height.

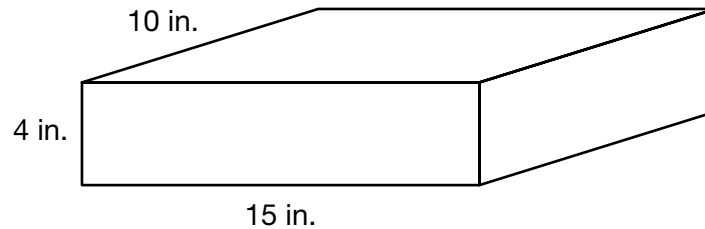
Cube

$V = e^3$, where e is the length of an edge

Example 1

A department store uses the box below for shirts.

What is the volume of the box?



Strategy Use the formula for the volume of a rectangular prism.

Step 1

The box is a rectangular prism. Write the formula for the volume.

$V = lwh$, where l is the length, w is the width, and h is the height.

Step 2

Identify the values for the variables.

The length is 15 in., the width is 10 in., and the height is 4 in.

So $l = 15$ in., $w = 10$ in., and $h = 4$ in.

Step 3

Substitute the values for the variables. Then multiply.

$$V = lwh$$

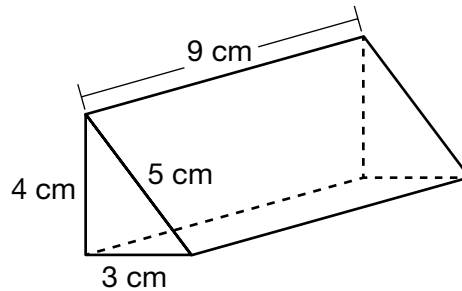
$$V = 15 \text{ in.} \times 10 \text{ in.} \times 4 \text{ in.} = 600 \text{ in.}^3$$

Solution The volume of the box is 600 cubic inches.

To find the volume of a triangular prism, use the formula $V = Bh$, where B is the area of the base.

Example 2

What is the volume of this triangular prism?



Strategy Use the formula for the volume of a triangular prism: $V = Bh$.

Step 1

Find the area of the base, which is the triangle.

$$\text{Use } A = \frac{1}{2}bh.$$

$$A = \frac{1}{2} \times 4 \text{ ft} \times 3 \text{ ft} = 6 \text{ ft}^2$$

Step 2

Multiply the area of the base times the height.

$$V = 6 \text{ ft}^2 \times 9 \text{ ft} = 54 \text{ ft}^3$$

Solution The volume of the triangular prism is 54 cubic feet.



Coached Example

Carol has a planter box that is in the shape of a cube. Each edge of the planter box measures 20 inches. What is the volume of Carol's planter box?

The formula for the volume of a cube is $V = \underline{\hspace{2cm}}$.

Substitute the values for the variables.

$$V = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

Multiply.

$$V = \underline{\hspace{2cm}} \text{ in.}^3$$

The planter box has a volume of $\underline{\hspace{2cm}}$ cubic inches.

Probability



Getting the Idea

Probability measures the chance of an event happening based on the number of the **possible outcomes**. Probability can be expressed as a fraction or a decimal from 0 to 1. A probability of 0 means that an event is impossible and a probability of 1 means that an event is certain. A probability close to 0 means an event is unlikely. A probability close to 1 means an event is very likely. A probability close to $\frac{1}{2}$ or 0.5 means an event is neither unlikely nor likely. You can also express a probability as a percent.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur (**favorable outcome**) to the number of possible outcomes. The probability, P , of an event, A , is:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.$$

Example 1

Josh is going to choose a random card from 13 cards. The cards are numbered from 1 to 13. What is the probability that he will choose a card with a number less than 4? Determine if the event is likely, unlikely, or neither.

Strategy Find the theoretical probability.

Step 1

Count the number of favorable outcomes.

There are 3 cards (1, 2, 3) with a number less than 4.

Step 2

Count the number of possible outcomes.

There are a total of 13 cards, each with the same chance of being drawn.

Step 3

Find the theoretical probability.

$$P(\text{card with a number less than 4}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{13}$$

Step 4

Determine if the event is likely or unlikely.

$\frac{3}{13}$ is closer to 0 than it is to 1, and it is less than $\frac{1}{2}$. So, the event is unlikely.

Solution

The probability of choosing a card with a number less than 4 is $\frac{3}{13}$. The event is unlikely.

You can use theoretical probability to make a prediction. Multiply the theoretical probability by the number of **trials**, or times the experiment is performed, to predict the number of favorable outcomes.

Example 2

Peter will roll a number cube, labeled 1 through 6, a total of 90 times. What is a good prediction for the number of times that the number cube will land on 5?

Strategy Find the number of possible outcomes and favorable outcomes.

Step 1

Find the number of possible outcomes.

There are 6 possible outcomes for the number cube.

Step 2

Find the number of favorable outcomes.

There is one 5 on the number cube.

Step 3

Write the theoretical probability in simplest form.

$$P(\text{rolling a 5}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 4

Multiply the probability by the number of trials.

$$\frac{1}{6} \times 90 = 90 \div 6 = 15$$

Solution A good prediction is that Peter will roll a 5 about fifteen times.

Experimental probability is the ratio of the total number of times the favorable outcome happens to the total number of trials, or times the experiment is performed. The experimental probability, P_e , of event A is:

$$P_e(A) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}.$$

Experimental probability is useful when you need to make predictions about an event. As the number of trials increases, the experimental probability gets closer to the theoretical probability.

Example 3

Minnie conducted an experiment with a spinner. The results are shown in the table.

Number	1	2	3	4	5	6
Times Landed	10	7	6	8	5	6

Based on the data, what is the probability that the spinner will land on 2 on the next spin?

Strategy Find the experimental probability.

Step 1

Find the number of trials.

$$10 + 7 + 6 + 8 + 5 + 6 = 42$$

Step 2

Find the number of favorable outcomes.

The spinner landed on 2 a total of 7 times.

Step 3

Write the experimental probability as a fraction in simplest form.

$$\frac{7}{42} = \frac{1}{6}$$

Solution

The experimental probability of the spinner landing on 2 on the next spin is $\frac{1}{6}$.

Example 4

Gavin rolls a number cube, labeled 1 to 6, a total of 40 times. The number 4 is rolled 8 times. What is the experimental probability of rolling a 4? What is the theoretical probability? Describe the difference between the two.

Strategy

Use the formulas for experimental probability and theoretical probability.

Step 1

Find the experimental probability.

The total number of trials is 40.

The number 4 is rolled 8 times, so the number of favorable outcomes is 8.

$$P_e(4) = \frac{\text{number of favorable outcomes}}{\text{total number of trials}} = \frac{8}{40} = \frac{1}{5}$$

Step 2

Find the theoretical probability.

The number of possible outcomes is 6.

There is only one 4 on a number cube, so the number of favorable outcomes is 1.

$$P_t(4) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{6}$$

Step 3

Compare the experimental probability and the theoretical probability.

The experimental probability is $\frac{1}{5}$, and the theoretical probability is $\frac{1}{6}$.

The theoretical probability shows the outcome you would expect.

The experimental probability shows the outcome that actually occurred during the experiment.

Solution

The experimental probability of rolling a 4 is $\frac{1}{5}$. This is greater than the theoretical probability of $\frac{1}{6}$.



Coached Example

The Skate Pro Company manufactures skateboards. They found 12 defective skateboards in a batch of 400. How many defective skateboards might they find in a batch of 1,200?

Find the experimental probability of a defective skateboard in simplest form.

There were _____ defective skateboards in a batch of _____.

Write the probability and express it in simplest form.

$$P(\text{defective skateboard}) = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Express the probability as a decimal. _____

Multiply the probability by 1,200.

$$1,200 \times \frac{\quad}{\quad} = \quad$$

The Skate Pro Company might expect to find _____ defective skateboards in a batch of 1,200.

Compound Events



Getting the Idea

A **compound event** is a combination of two or more events. Compound events can be dependent or independent. **Independent events** are two events in which the occurrence of the first event does not affect the probability of the occurrence of the second event.

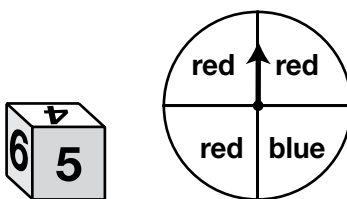
Dependent events are when the first event affects the outcome of the second event.

To find the probability of two independent events, multiply the probability of the first event by the probability of the second event.

$$P(\text{two independent events}) = P(\text{first event}) \times P(\text{second event})$$

Example 1

Adriana tosses a number cube with faces numbered 1 through 6 and spins the spinner shown below at the same time.



What is the probability of tossing a number greater than 2 on the cube and spinning red on the spinner?

Strategy Find the probability of each event and multiply them together.

Step 1

Find the probability of the number cube landing on a number greater than 2.

A number cube has 6 possible outcomes.

Four outcomes (3, 4, 5, 6) are greater than 2.

$$P(>2) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{4}{6} = \frac{2}{3}$$

Step 2

Find the probability of spinning red on the spinner.

Three of the 4 sections are labeled “red.”

$$P(\text{red}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{4}$$

Step 3

Multiply the two probabilities.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Solution The probability of the cube landing on a number greater than 2 and the spinner landing on red is $\frac{1}{2}$.

Example 2

Carlos has 3 comedies, 2 dramas, 2 concerts, and 1 horror movie in a DVD booklet. He will pick two DVDs from the booklet to play. What is the probability of Carlos picking a comedy DVD and then a drama DVD?

Strategy Find the probability of each event and multiply.

Step 1

Find the probability of picking a comedy DVD.

$$P(\text{comedy}) = \frac{3}{3 + 2 + 2 + 1} = \frac{3}{8}$$

Step 2

Find the probability of picking a drama DVD second.

There is now 1 less DVD to pick.

$$P(\text{drama}) = \frac{2}{7}$$

Step 3

Multiply the probabilities.

$$\frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$$

Solution The probability of picking a comedy DVD and then a drama DVD is $\frac{3}{28}$.

When you need to find the probability of a compound event, sometimes it is necessary to make a tree diagram, an organized list, or a table to find the number of possible outcomes.

You can also use the **fundamental counting principle** to find the number of possible outcomes. If event A can occur in m ways and event B can occur in n ways, then events A and B can occur in $m \times n$ ways.

Example 3

Cara is going to order the lunch special that consists of a sandwich, soup, and dessert for Emma and herself. The choices are shown below.

Sandwich	Soup	Dessert
Grilled Cheese Roast Beef Turkey	Split Pea Chicken Noodle	Ice Cream Fruit

Emma does not like chicken and turkey, but Cara does not know that. What is the probability that Emma will avoid turkey and chicken in her lunch?

Strategy **Make an organized list of the possible outcomes.**

Step 1

List the possible outcomes. Represent each food or beverage by using its first letter.

G-S-I, G-S-F, G-C-I, G-C-F

R-S-I, R-S-F, R-C-I, R-C-F

T-S-I, T-S-F, T-C-I, T-C-F

Step 2

Use the fundamental counting principle to check the number of possible outcomes.

$$3 \times 2 \times 2 = 12$$

Step 3

Find the combinations that do not include turkey (T) or chicken noodle soup (C).

G-S-I, G-S-F, G-C-I, G-C-F

R-S-I, R-S-F, R-C-I, R-C-F

T-S-I, T-S-F, T-C-I, T-C-F

Step 4

Write the probability.

$$\frac{4}{12} = \frac{1}{3}$$

Solution **The probability of Emma avoiding chicken and turkey in her lunch is $\frac{1}{3}$.**

A **simulation** is a way of acting out a problem by conducting experiments. The outcomes of a simulation are comparable to, but not the same as, what the actual outcomes would be. One way to conduct a simulation is to use random numbers. Random numbers can be generated by a computer or a random number table.

Example 4

Marco is shooting 70% at the free throw line. Conduct a simulation to find the probability that he will make at least one of his next two free throws.

Strategy Use a random number table.

Step 1

Set up your rules.

Since Marco is a 70% free throw shooter let 7 of the digits represent a made shot and 3 of the digits a missed shot.

Let 0-6 represent a made shot and 7-9 represent a missed shot.

Step 2

Use a random number generator. Try the first 20 digits.

46398803548565510177

Step 3

Break the random digits into twos.

If at least one of the two numbers is less than 7, then Marco made at least one of his shots.

46 39 88 03 54 85 65 51 01 77

In $\frac{8}{10}$ of the sets, Marco made at least one of the two free throws.

Solution The simulation states the probability of Marco making at least one of his next two free throws is $\frac{4}{5}$ or 80%.



Coached Example

Chris tosses two number cubes labeled 1 to 6. What is the probability of rolling double 4s?

Use the fundamental counting principle to find the number of outcomes.

How many faces does a number cube have? ____

How many possible outcomes are there for a number cube to land? ____

To find the number of possible outcomes, multiply ____ \times ____ = ____.

There are ____ possible outcomes.

How many different ways are there to toss doubles 4s? ____

What is the probability of rolling double 4s? ____

The probability of rolling double 4s is ____.

Samples



Getting the Idea

Statistics can be used to make generalizations about a population. A **population** is the group of interest. It is usually not possible to gather data from each member of a population, so the generalizations are often based upon a sample. A **sample** is a smaller group taken from the population.

Samples allow researchers to save time and money when gathering information. Samples are only useful if they are representative of the population. A **representative sample** is a portion of the population that is similar to the entire population. A **biased sample** is one in which some members of the population have a greater chance of being selected for the sample than other members. Because of bias, the sample does not fairly represent the population.

One way to gather information is by surveying the members of the sample. A **survey** is a question or set of questions used to gather **data**, or pieces of information. A survey can also be biased.

Example 1

Reggie thinks that more students in his school are right-handed than left-handed. He surveys the students in his class and finds that 23 of the 27 students are right-handed. Do the results of Reggie's survey support his inference that more students in his school are right-handed than left-handed?

Strategy **Use the definition of a representative sample to evaluate Reggie's sample.**

A sample should be representative of the population.

The population being studied is the students in Reggie's school.

The students in Reggie's class are representative of all students in his school.

More students in his class are right-handed than left-handed.

The results of the survey support his inference.

Solution **The results of Reggie's survey support his inference that more students in his school are right-handed than left-handed.**

Random samples are usually preferred when gathering information about a population. In a random sample, each individual in the population has an equal chance of being part of the sample.

Example 2

Collin asked every eighth student entering the school which of four subjects was his or her favorite. Can the results of Collin's survey be used to draw inferences about students' favorite subjects at the school?

Strategy **Decide if the sample is representative and the survey is unbiased.**

Step 1

Decide if the sample is a random sample.

In a random sample, each individual in the population has the same chance of being part of the sample. Each student entering the school has the same chance of being one of every eight students entering the school.

Step 2

Decide if the sample is representative of the population.

The students in the school are the population.

The random sample is representative of the school population.

Step 3

Decide if the survey is biased.

The results are only representative of the four subjects included in the survey. It is biased toward these four subjects since other subjects are not included. The results can only be used to draw inferences about student preferences for the four subjects included in the survey.

Solution **The results of Collin's survey can be used to draw inferences about student preferences for the four subjects included in the survey.**

You can use the results of a survey to make predictions about a population.

Example 3

The table below shows the results of Collin's survey from Example 2.

Favorite Subject

Subject	Number of Students
Math	15
Science	20
Language Arts	10
Social Studies	5

There are 400 students at Collin's school. How many students would you predict prefer language arts?

Strategy **Write and solve a proportion.**

Step 1

Add to find the total number of students Collin surveyed.

$$15 + 20 + 10 + 5 = 50$$

Step 2

Write a proportion.

Have each ratio show the number of students who prefer language arts to the total number of students. Let x represent all of the students at the school who prefer language arts.

$$\frac{10}{50} = \frac{x}{400}$$

Step 3

Cross multiply and solve for x .

$$\frac{10}{50} = \frac{x}{400}$$

$$10 \times 400 = 50 \times x$$

$$4,000 = 50x$$

$$x = 80$$

Solution **Out of 400 students, 80 students would probably say they prefer language arts.**

Example 4

Which two of the following samples are **not** good samples? Explain why.

- A. Every third shopper at a clothing store is asked whether he or she owns a pet.
- B. Every third shopper at a pet store is asked whether he or she owns a pet.
- C. At the beach in the summer, 150 people are asked to name their favorite vacation spot.
- D. A survey is mailed to 50 homes in a neighborhood, asking residents to name their favorite vacation spot.

Strategy **Read each sample description and decide whether it is random or biased.**

Step 1

In sample A, people shopping for clothes are chosen randomly to answer a question about pets, so this is a representative sample.

Step 2

Sample B is a biased sample. It is likely that most people who enter a pet store go there to purchase something for their pet.

Step 3

Sample C is biased since these people are probably already at a vacation spot.

Step 4

For sample D, the survey is randomly mailed to people in the neighborhood. The random sample is representative of the people in the neighborhood.

Solution **Samples B and C are not good samples because each sample is biased.**

The greater the number of participants in a survey, the closer the predictions will be to the actual choices of the population.

Example 5

Tory and Flavia each surveyed students in their school about how they would vote for the student council representative from the seventh grade. Tory surveyed the students in her homeroom. Flavia randomly surveyed 10 students from each of the five seventh-grade homeroom classes. Their results are shown in the tables below.

Tory's Results

Candidate	Number of Students
Timothy	8
Andrew	9
Lea	7

Flavia's Results

Candidate	Number of Students
Timothy	14
Andrew	21
Lea	15

Use their results to predict the winner of the election.

Step 1

Identify the population.

The population is the seventh graders at the school.

Step 2

Decide if the samples are representative samples.

Both samples appear to be representative samples.

Step 3

Compare the data in the tables.

In both tables, Andrew has the greatest number of votes.

Step 4

Use the results to predict the winner.

It appears likely that Andrew will win the election.

Notice that Andrew has a bigger lead in Flavia's table than in Tory's.

Since Flavia's sample size is greater than Tory's, her results make it seem more likely that Andrew will win than Tory's table. Her results also predict that Andrew will win by a larger margin than Tory's results.

Solution From the survey results, Andrew is the predicted winner.



Coached Example

Victoria randomly surveyed every tenth student who came to school on Monday. She asked each student to name his or her least favorite vegetable. The table below shows the results of her survey.

Least Favorite Vegetable

Vegetable	Number of Students
Broccoli	3
Asparagus	5
Spinach	12
Turnips	5

In a survey of 100 students at her school, how many students would you predict to choose spinach as their least favorite vegetable?

Victoria surveyed a total of _____ students.

Write a proportion in which each ratio shows the number of students who chose _____ as their least favorite vegetable to the _____ number of students.

Let s equal the number of students out of 100 who would choose spinach.

Cross multiply and solve for s .

$s =$ _____

In a survey of 100 students, _____ students would probably choose spinach as their least favorite vegetable.

Measures of Central Tendency



Getting the Idea

Measures of central tendency help to describe and interpret a data set. They are used to interpret the “average item” of a data set. The table below shows the measures of central tendency for this data set: 5, 1, 1, 6, and 7.

Measure of Central Tendency	Example
The mean is equal to the sum of the terms in a data set divided by the number of terms in the data set.	$\text{mean} = \frac{\text{sum of terms}}{\text{number of terms}}$ $\text{mean} = \frac{5 + 1 + 1 + 6 + 7}{5} = \frac{20}{5} = 4$
The median is the middle term in a data set ordered from least to greatest. If there is an even number of terms in a data set, the median is the mean of the two middle numbers.	<p>The data ordered from least to greatest are: 1, 1, <u>5</u>, 6, 7.</p> <p>The middle term, 5, is the median.</p>
The mode is the term or terms that appear most often in a data set. A data set may have no mode, one mode, or more than one mode.	<p>5, <u>1</u>, <u>1</u>, 6, 7</p> <p>The number 1 appears twice in the data set. Every other number appears only once. So, 1 is the mode.</p>

Example 1

The scores on a science quiz are: 7, 7, 9, 7, 10, 8, 6, 9, 10, and 7.

What are the mean, median, and mode of the science quiz scores?

Strategy Calculate each measure of central tendency.

Step 1

Find the mean.

Add the values and then divide the sum by the number of values.

$$7 + 7 + 9 + 7 + 10 + 8 + 6 + 9 + 10 + 7 = 80$$

$$80 \div 10 = 8$$

Step 2

Find the median.

Order the values from least to greatest.

$$6, 7, 7, 7, \underline{7}, \underline{8}, 9, 9, 10, 10$$

There is an even number of terms.

The two middle values are 7 and 8.

Find the mean of the two middle values.

$$(7 + 8) \div 2 = 15 \div 2 = 7.5$$

Step 3

Find the mode.

The value that occurs most frequently is 7.

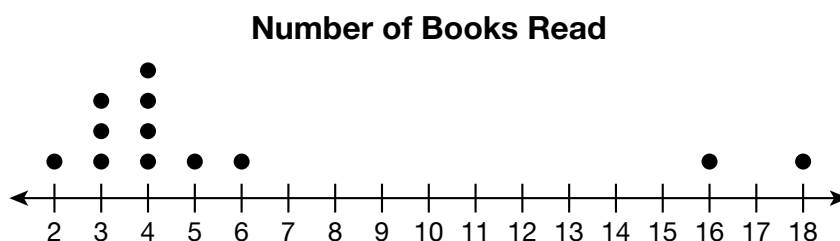
Solution **The mean is 8, the median is 7.5, and the mode is 7.**

To choose which measure of central tendency is most appropriate for a situation, look at the distribution of the ordered data from a sample. If there is a value that is much less or much greater than the other values in the set of data, the median or mode better represents the sample data than the mean. If the mode occurs at either extreme of the data, the mean or the median are better choices to represent the sample data.

Some data sets are shown on a **dot plot**, which is a display that uses a number line and dots to show data.

Example 2

Karen randomly surveyed some classmates to see how many books each of them read over the summer. The results of her survey are shown in the dot plot.



How does the mode compare to the median number of books read?

Strategy **Find the median and the mode. Compare the measures.**

Step 1

Find the median.

Use the data points shown on the dot plot.

Median: 2, 3, 3, 3, 4, **4**, **4**, 4, 5, 6, 16, 18

The two middle values are 4.

The median is 4.

Step 2

Find the mode.

Mode: 2, 3, 3, 3, **4**, **4**, **4**, **4**, 5, 6, 16, 18

The mode is 4.

Step 3

Compare the measures.

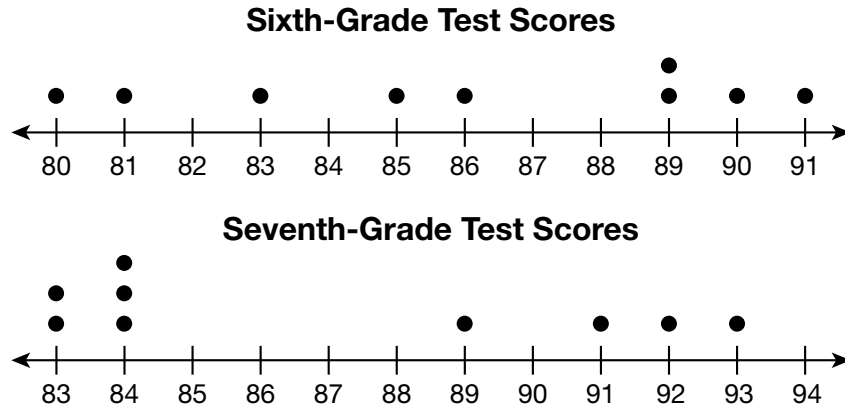
The median and the mode are the same.

Solution **The median and the mode are both 4.**

You can use measures of central tendency to make inferences about two populations.

Example 3

The dot plots show the test scores of students in Mr. Coen's sixth- and seventh-grade English classes.



Are the test scores of the seventh graders generally higher than the test scores of the sixth graders?

Strategy Find the measures of central tendency for each data set. Analyze and compare them.

Step 1

Find the mean, median, and mode of the sixth-grade test scores.

Use the data points shown on the dot plot.

$$\text{Mean: } \frac{80 + 81 + 83 + 85 + 86 + 89 + 89 + 90 + 91}{9} = \frac{774}{9} = 86$$

The mean is 86.

Median: 80, 81, 83, 85, **86**, 89, 89, 90, 91

The median is 86.

Mode: 80, 81, 83, 85, 86, **89, 89**, 90, 91

The mode is 89.

Step 2

Find the mean, median, and mode of the seventh-grade test scores.

$$\text{Mean: } \frac{83 + 83 + 84 + 84 + 84 + 89 + 91 + 92 + 93}{9} = \frac{783}{9} = 87$$

The mean is 87.

Median: 83, 83, 84, 84, **84**, 89, 91, 92, 93

The median is 84.

Mode: 83, 83, **84, 84, 84**, 89, 91, 92, 93

The mode is 84.

Step 3

Compare the measures of central tendency.

The sixth-grade mean is 86, and the seventh-grade mean is 87.

The seventh-grade mean is one point higher than the sixth-grade mean.

The sixth-grade median is 86, and the seventh-grade median is 84.

The sixth-grade median is two points higher than the seventh-grade median.

The sixth-grade mode is 89, and the seventh-grade mode is 84.

The sixth-grade mode is four points higher than the seventh-grade mode.

Step 4

Analyze the data.

All the data points are close together in both grades.

The mean, median, and mode for each data set are close to most of the data points.

The question does not ask for the most common test scores, so the mode is not the best measure to use to compare the data sets.

The sixth-grade median is slightly higher than the seventh-grade median, while the seventh-grade mean is slightly higher than the sixth-grade mean. The “average” test scores are close for both classes.

Solution

The test scores of the seventh graders are not generally higher than the test scores of the sixth graders.

Example 4

Students in Mrs. Becker's class and Mr. Roland's class sold boxes of popcorn for a school fund-raiser. The tables below show the number of boxes that each student sold.

Popcorn Boxes Sold

Mrs. Becker's Class				Mr. Roland's Class			
5	8	76	15	9	10	12	20
16	84	7	12	8	34	6	27
17	11	13		14	10	3	51

How does the average number of boxes sold by each class compare?

Strategy Find the measures of central tendency for each data set. Analyze and compare them.

Step 1

Find the mean, median, and mode of Mrs. Becker's class.

$$\text{Mean: } \frac{5 + 8 + 76 + 15 + 16 + 84 + 7 + 12 + 17 + 11 + 13}{11} = \frac{264}{11} = 24$$

Median: 5, 7, 8, 11, 12, **13**, 15, 16, 17, 76, 84

The median is 13.

There is no mode since no value appears more than once.

Step 2

Find the mean, median, and mode of Mr. Roland's class.

$$\text{Mean: } \frac{9 + 10 + 12 + 20 + 8 + 34 + 6 + 27 + 14 + 10 + 3 + 51}{12} = \frac{204}{12} = 17$$

Median: 3, 6, 8, 9, 10, **10**, **12**, 14, 20, 27, 34, 51

Find the mean of the two middle values, 10 and 12.

$$(10 + 12) \div 2 = 22 \div 2 = 11$$

The median is 11.

Mode: 3, 6, 8, 9, **10**, **10**, 12, 14, 20, 27, 34, 51

The mode is 10.

Step 3

Compare the measures of central tendency.

The mean for Mrs. Becker's class is greater than the mean for Mr. Roland's class.

The median for Mrs. Becker's class is greater than the median for Mr. Roland's class.

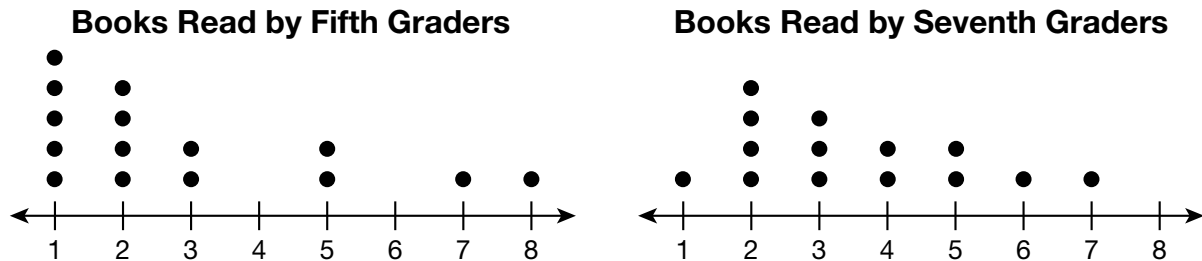
There is only one mode, so you cannot compare them.

Solution The average number of boxes sold by Mrs. Becker's students was greater than the average number sold by Mr. Roland's students.



Coached Example

The dot plots show the number of books read by fifth graders and by seventh graders during one month.



How does the average number of books read by the fifth graders compare to the average number of books read by the seventh graders?

Find the mean number of books read by fifth graders to the nearest tenth:

The mean is _____.

Find the median number of books read by fifth graders:

The median is _____.

Find the mean number of books read by seventh graders to the nearest tenth:

The mean is _____.

Find the median number of books read by seventh graders:

The median is _____.

The mean number of books read by the fifth graders is _____, while the mean number of books read by the seventh graders is _____.

The median number of books read by the fifth graders is _____, while the median number of books read by the seventh graders is _____.

The seventh graders read about an average of _____ book than the fifth graders.

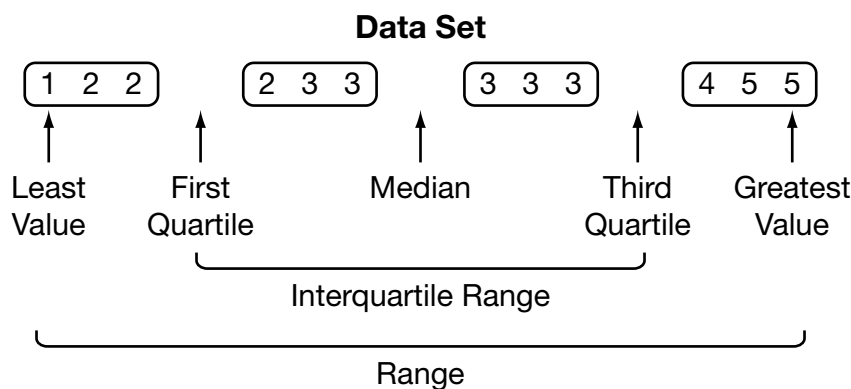
Measures of Variation



Getting the Idea

Instead of describing the center of a set of data by using the measures of central tendency, you may wish to describe the spread of a set of data. **Measures of variation** show how spread out or close together the data in a set are, or how much the data points vary.

When data are arranged from least to greatest, the median divides the data into two equal halves. The **first quartile** is the median of the data values that are less than the median. The **third quartile** is the median of the data values that are greater than the median. The **quartiles** and the median divide the data into four quarters. The **range** is the difference between the greatest value in a data set and the least value. The **interquartile range (IQR)**, is the difference between the third quartile and the first quartile. The range measures the spread of all the data. The IQR measures the spread of the two middle quarters of the data.



For the data set in the diagram, the median is 3. The first quartile is 2 (the mean of 2 and 2). The third quartile is 3.5 (the mean of 3 and 4). The range is $5 - 1$, or 4. The IQR is $3.5 - 2$, or 1.5.

In a data set, a number that is much less or much greater than the other numbers in the data set is an **outlier**. A data set may contain one or more outliers. An outlier will affect the range, but it will not affect the IQR. This is why the median is not as affected by outliers as the mean is.

Consider the following data set: 2, 3, 5, 6, 8, 10, 23.

The median is 6.

The first quartile is 3, which is the median of the values 2, 3, 5.

The third quartile is 10, which is the median of the values 8, 10, 23.

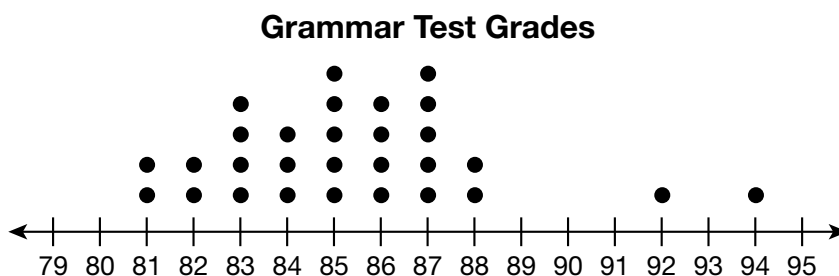
The range is 21: $23 - 2 = 21$.

The IQR is 7: $10 - 3 = 7$.

For this data set, 23 is an outlier. It is much greater than the other numbers in the set. The range is affected by this outlier. It suggests that there is a greater variability in the data, since it shows a greater spread, than the IQR suggests. The data vary by 21 from the least to greatest values, while they only vary by 7 away from the median, or the center of the data.

Example 1

The dot plot below shows the grades students received on a grammar test in Ms. Parsi's class.



What is the range in the grades? What is the median grade?

Strategy Use the dot plot.

Step 1 Find the greatest and least test grades.

The lowest grade is 81.

The highest grade is 94.

Step 2 Find the range.

Subtract the lowest grade from the highest grade.

$$\text{range} = 94 - 81 = 13$$

Step 3 Count the total number of test grades.

There are 29 grades.

Step 4 Find the median.

The median is the middle grade. It is halfway between the lowest and the highest grade.

Since there are 29 grades, there are 14 grades below the median and 14 grades above the median.

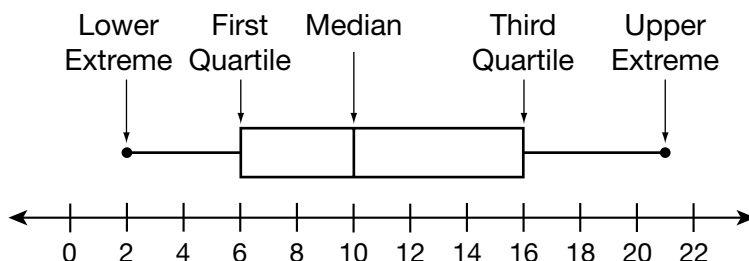
The median is the fifteenth grade in the ordered list of grades.

Start at 81 and count the dots until you reach the fifteenth dot.

This dot is at the grade of 85. So the median is 85.

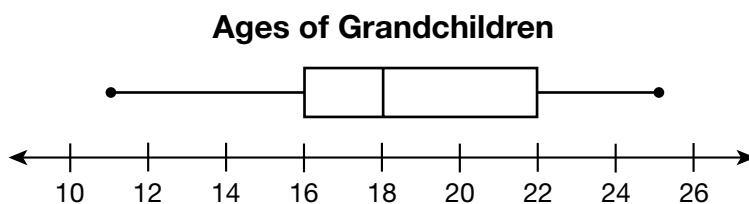
Solution The range in the grades is 13, and the median is 85.

A **box plot** is a method of visually displaying a distribution of data values by using the median, quartiles, and extremes (least and greatest values) of the data set. The box shows the middle 50% of the data.



Example 2

The box plot below shows the ages of Mr. Morehouse's grandchildren.



Find the median, first quartile, third quartile, and the IQR of their ages.

Strategy Use the box plot.

Step 1

Find the median.

The median is the middle value. On a box plot, it is represented by the vertical line inside the box.

The vertical line inside the box is above 18. The median is 18.

Step 2

Find the first quartile.

On a box plot, the first quartile is represented by the box's left vertical line.

The left vertical line is above 16. The first quartile is 16.

Step 3

Find the third quartile.

On a box plot, the third quartile is represented by the box's right vertical line.

The right vertical line is above 22. The third quartile is 22.

Step 4

Find the IQR.

$$\text{IQR} = \text{third quartile} - \text{first quartile} = 22 - 16 = 6$$

Solution

The median of the ages is 18. The first quartile is 16. The third quartile is 22. The IQR is 6.

Example 3

Below are the quiz scores from students in two different class sections.

Section 1: 7, 9, 9, 10, 8, 6, 8, 5, 5, 9, 10, 7, 8, 7, 9

Section 2: 7, 8, 9, 9, 8, 8, 7, 9, 9, 10, 8, 8, 7, 10, 8

Which section has greater variability in the scores?

Strategy **Compare the ranges and IQRs of the two class sections.**

Step 1

Find the range for Section 1.

Order the scores from least to greatest.

5, 5, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10

The highest score was 10. The lowest score was 5.

$$\text{Range} = 10 - 5 = 5$$

Step 2

Find the median and the quartiles for Section 1.

5, 5, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10

The median is 8, the first quartile is 7, and the third quartile is 9.

Step 3

Find the IQR for Section 1.

The IQR is the difference between the third and first quartiles.

$$\text{IQR} = 9 - 7 = 2$$

Step 4

Find the range for Section 2.

Order the scores from least to greatest.

7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10

$$\text{Range} = 10 - 7 = 3$$

Step 5

Find the median and the quartiles for Section 2.

7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10

The median is 8, the first quartile is 8, and the third quartile is 9.

Step 6

Find the interquartile range for Section 2.

$$\text{IQR} = 9 - 8 = 1$$

Step 7

Compare the range and IQR for each section.

The range for Section 1 is 5, and the IQR is 2.

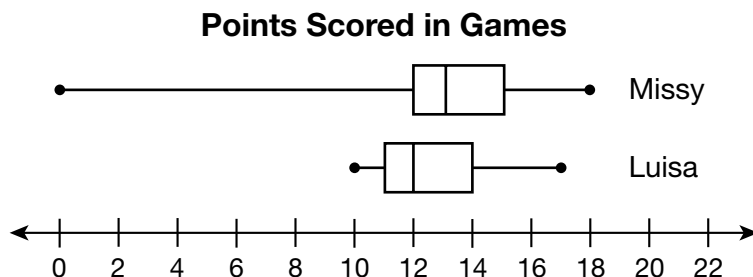
The range for Section 2 is 3, and the IQR is 1.

The range and IQR for Section 1 are greater than for Section 2.

Solution **Section 1 has greater variability in the quiz scores than Section 2.**

Example 4

The double box plot below shows the number of points scored in games by two basketball players on the same team.



Find the range and the IQR for each player. Who is the more consistent scorer?

Strategy Compare the range and IQR for each player.

Step 1

Find the range and the IQR for Missy.

Missy's highest score was 18. Her lowest score was 0.

$$\text{Range} = 18 - 0 = 18$$

$$\text{IQR} = 15 - 12 = 3$$

Step 2

Find the range and the IQR for Luisa.

Luisa's highest score was 17. Her lowest score was 10.

$$\text{Range} = 17 - 10 = 7$$

The third quartile score for Luisa is 14, and the first quartile score is 11.

$$\text{IQR} = 14 - 11 = 3$$

Step 3

Who is the more consistent scorer?

Missy's scores per game show a much greater range than Luisa's.

Notice that Missy's scores have an outlier. She scored 0 points during one game.

The outlier affects the range of her scores.

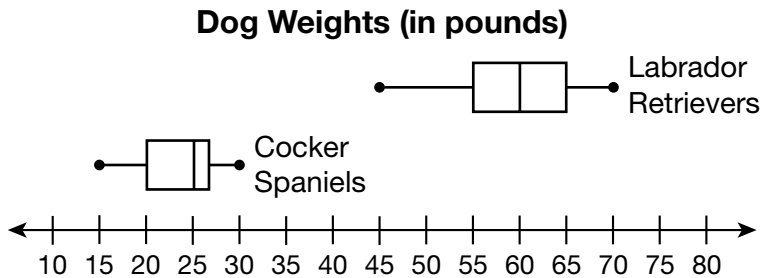
The IQR for both players is the same.

Solution Luisa may be a slightly more consistent scorer. Since the IQRs for Luisa and Missy are the same, it is likely that Missy is about as consistent a scorer as Luisa.



Coached Example

The double box plot below shows the weights, in pounds, of Labrador retrievers and cocker spaniels from a veterinarian's office.



Which type of dog shows greater variability in weight?

The least weight for the cocker spaniels is _____ pounds.

The greatest weight for the cocker spaniels is _____ pounds.

Find the range of the cocker spaniels' weights: _____ - _____ = _____.

The range of the weights of the cocker spaniels is _____ pounds.

The third quartile weight for the cocker spaniels is _____ pounds.

The first quartile weight for the cocker spaniels is _____ pounds.

Find the IQR for the cocker spaniels' weights: $27 - \text{_____} = \text{_____}$.

The IQR for the weights of the cocker spaniels is _____ pounds.

The least weight for the Labrador retrievers is _____ pounds.

The greatest weight for the Labrador retrievers is _____ pounds.

Find the range of the Labrador retrievers' weights: _____ - _____ = _____.

The range of the weights of the Labrador retrievers is _____ pounds.

The third quartile weight for the Labrador retrievers is _____ pounds.

The first quartile weight for the Labrador retrievers is _____ pounds.

Find the IQR for the Labrador retrievers' weights: _____ - _____ = _____.

The IQR for the weights of the Labrador retrievers is _____ pounds.

The range and the IQR for cocker spaniels are _____ than for Labrador retrievers.

_____ **have greater variability in weight**
than _____.

Mean Absolute Deviation



Getting the Idea

Another way to measure the variability of a data set is to measure variation from the mean. You do this by measuring how far each individual value is from the mean.

To measure variability away from the mean, first find the mean of the data set. Next, find the absolute value of the difference between the mean and each value of the data set. This gives the deviation of each value from the mean. Then find the sum of all the deviations and divide the sum by the number of values in the data set. The average of the absolute deviations from the mean is called the **mean absolute deviation (MAD)**.

Suppose the heights, in inches, of three plants, are: 18, 27, and 21.

Find the mean of the heights, in inches.

$$\text{mean} = \frac{18 + 27 + 21}{3} = \frac{66}{3} = 22$$

Find how each height differs from the mean height. This is the deviation from the mean.

$$\text{deviation of first value} = 18 - 22 = -4$$

$$\text{deviation of second value} = 27 - 22 = 5$$

$$\text{deviation of third value} = 21 - 22 = -1$$

Notice that the average deviation of the values will be zero.

So, take the absolute value of each deviation.

$$|-4| = 4$$

$$|5| = 5$$

$$|-1| = 1$$

Now find the average of the absolute deviations.

$$\text{MAD} = \frac{4 + 5 + 1}{3} = \frac{10}{3} = 3.\overline{3}$$

So, the plant heights vary by an average of $3.\overline{3}$ inches from the mean.

When the MAD is small, it means the data is bunched closely together. For the plant heights, the MAD is $3.\overline{3}$, which is relatively small. This makes sense since the plant heights are not very different. So, there is not much variability in the plant heights.

If the MAD is large, it means the data is spread out and has greater variability.

Example 1

Find the MAD for the following quiz scores: 6, 9, 6, 9, 8, and 10. The mean score on the quizzes is 8.

Strategy Find the deviation of each score from the mean score. Then find the absolute deviations to get the MAD.

Step 1

Find the deviation of each score from the mean score.

Subtract each score from the mean score to find the deviations.

$$6 - 8 = -2$$

$$9 - 8 = 1$$

$$6 - 8 = -2$$

$$9 - 8 = 1$$

$$8 - 8 = 0$$

$$10 - 8 = 2$$

Step 2

Find the absolute deviations, the absolute value of each deviation.

$$|-2| = 2$$

$$|1| = 1$$

$$|-2| = 2$$

$$|1| = 1$$

$$|0| = 0$$

$$|2| = 2$$

Step 3

Find the MAD.

Add the absolute deviations: $2 + 1 + 2 + 1 + 0 + 2 = 8$

There are 6 quiz scores in the set.

$$\text{MAD} = \frac{8}{6} = 1.\bar{3}$$

Solution The MAD is $1.\bar{3}$.

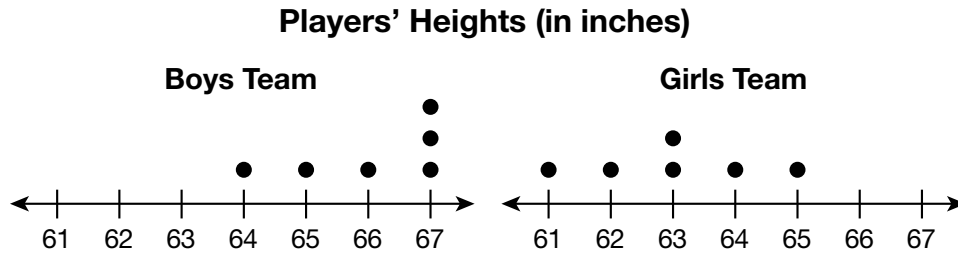
You can use the MAD to compare two populations.

Two sets with similar variability may overlap. To determine the level of overlap between two data sets, find the difference of the means divided by the quotient of the MAD.

If two sets of data have the same MAD, a quotient near 0 will mean that there is much or almost complete overlap. The greater the quotient the less overlap exists between the two sets.

Example 2

The heights of players on two volleyball teams are shown on the dot plots below.



Find the difference of the means divided by the quotient of the MADs to find the overlap.

Strategy Find the **MAD** of each team's heights.

Step 1

Find the mean of each team's heights in inches.

$$\text{Boys: } \frac{64 + 65 + 66 + 67 + 67 + 67}{6} = \frac{396}{6} = 66$$

$$\text{Girls: } \frac{61 + 62 + 63 + 63 + 64 + 65}{6} = \frac{378}{6} = 63$$

Step 2

Find the absolute deviation of each team.

For the boys, find the absolute value of the difference between each value and 66. For the girls find the absolute value of the difference between each value and 63.

Since you are finding the absolute values, the order of the subtraction does not matter.

Boys	Girls
$ 64 - 66 = 2$	$ 61 - 63 = 2$
$ 65 - 66 = 1$	$ 62 - 63 = 1$
$ 66 - 66 = 0$	$ 63 - 63 = 0$
$ 67 - 66 = 1$	$ 63 - 63 = 0$
$ 67 - 66 = 1$	$ 64 - 63 = 1$
$ 67 - 66 = 1$	$ 65 - 63 = 2$

Step 3

Find the MAD for each team.

$$\text{Boys: } \frac{2 + 1 + 0 + 1 + 1 + 1}{6} = \frac{6}{6} = 1$$

$$\text{Girls: } \frac{2 + 1 + 0 + 0 + 1 + 2}{6} = \frac{6}{6} = 1$$

The MADs are the same.

Step 4

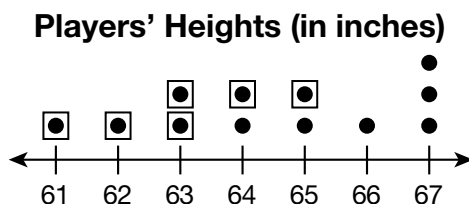
Divide the difference of the means by the quotient of the MAD.

$$\frac{66 - 63}{1} = 3$$

There will be some overlap.

Step 5

Combine the dot plots to check your answer.



Let the dots inside squares represent the girls.

Solution There is some overlap between the sets of data.

Example 3

The numbers of pages in books read by sixth- and seventh-grade students during one semester are shown below.

Sixth grade: 125, 132, 150, 137

Seventh grade: 198, 174, 208, 120

Compare the variability in the mean number of pages read by students in each grade.

Strategy

Find the MADs of the pages read by each grade.

Step 1

Find the MAD of the pages read by sixth graders.

$$\text{mean} = \frac{125 + 132 + 150 + 137}{4} = \frac{544}{4} = 136$$

Find the absolute deviations.

$$|125 - 136| = |-11| = 11 \quad |150 - 136| = |14| = 14$$

$$|132 - 136| = |-4| = 4 \quad |137 - 136| = |1| = 1$$

Find the MAD.

$$\text{MAD} = \frac{11 + 4 + 14 + 1}{4} = \frac{30}{4} = 7.5$$

Step 2

Find the MAD of the pages read by seventh graders.

$$\text{mean} = \frac{198 + 174 + 208 + 120}{4} = \frac{700}{4} = 175$$

Find the absolute deviations.

$$|198 - 175| = |23| = 23 \quad |208 - 175| = |33| = 33$$

$$|174 - 175| = |-1| = 1 \quad |120 - 175| = |-55| = 55$$

Find the MAD.

$$\text{mean absolute deviation} = \frac{23 + 1 + 33 + 55}{4} = \frac{112}{4} = 28$$

Step 3

Compare the MADs.

$$7.5 < 28$$

The MAD in the number of pages read by the sixth graders is much less than the MAD for the seventh graders. 28 is almost 4 times 7.5.

Solution The variability in the number of pages read by the seventh graders is almost 4 times the variability in the number of pages read by the sixth graders.

**Coached Example**

The weights, in pounds, of the dogs that boarded at a veterinarian's clinic over the weekend were: 43, 87, 12, 15, and 23.

Find the MAD of the weights of the dogs that boarded at the clinic.

Find the mean weight of the dogs that boarded at the clinic.

To find the deviations, _____ each weight from the mean weight.

$$\begin{array}{lll} 43 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 87 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 12 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \\ 15 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & 23 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} & \end{array}$$

To find the absolute deviations, find the _____ of each deviation.

$$\begin{array}{lll} |\underline{\hspace{1cm}}| = \underline{\hspace{1cm}} & |\underline{\hspace{1cm}}| = \underline{\hspace{1cm}} & |\underline{\hspace{1cm}}| = \underline{\hspace{1cm}} \\ |\underline{\hspace{1cm}}| = \underline{\hspace{1cm}} & |\underline{\hspace{1cm}}| = \underline{\hspace{1cm}} & \end{array}$$

Add the absolute deviations.

Find the average of the absolute deviations.

$$\text{mean absolute deviation} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

The MAD of the weights of the dogs is _____ pounds.

Make Predictions Using Data



Getting the Idea

You can use the data from a sample to make predictions about the population. It is important that the sample be representative of the population for the predictions to be reasonable.

Example 1

There are 60 students who take band classes at Mr. Tempo's school. Mr. Tempo surveyed 10 of those students to find out how long they practice their instruments each day. The survey was randomly distributed and anonymous. The results of the survey are shown below. The times are in minutes.

40, 25, 30, 40, 20, 15, 25, 30, 20, 25

Find the mean practice time for the sample. Predict the mean practice time of all the students who take band classes. Is the prediction reasonable?

Strategy Use the mean from the sample data to predict the mean for the population.

Step 1

Identify the sample and the population.

The students surveyed are the sample.

The population is all the students who take band classes.

Step 2

Find the mean for the sample data.

$$40 + 25 + 30 + 40 + 20 + 15 + 25 + 30 + 20 + 25 = 270$$

$$270 \div 10 = 27$$

The mean practice time is 27 minutes.

Step 3

Predict the mean for the population.

The mean practice time for the sample is 27 minutes.

The mean for the population should be about 27 minutes.

Step 4

Decide if the prediction is reasonable.

The sample was a random sample.

The size of the sample (10) is fairly large compared to the population (60).

The prediction is reasonable.

Solution The mean practice time for the sample is 27 minutes. The sample mean provides a reasonable prediction of the population's mean practice time.

Example 2

Mindy is the captain of the dance team at her school. She is running in a class election for class president. April surveyed the students on the dance team to see whom they planned to vote for in the election. The results of her survey are shown below.

Student	Number of votes
Mindy	18
Tobey	6
Roland	7

Based on the survey, predict who will win the class election. Is the prediction reasonable?

Strategy **Use the data to make a prediction. Evaluate the data to decide if the prediction is reasonable.**

Step 1

Use the data to make a prediction.

From the data in the table, the student with the greatest number of votes is Mindy.

Based on the data, Mindy should win the election.

Step 2

Evaluate the data to decide if the prediction is reasonable.

Mindy has quite a few more votes than either of the other students in the table.

However, Mindy is the captain of the dance team. All of the students who were surveyed are on the dance team. This suggests that they might be biased toward Mindy.

The prediction that Mindy will win does not seem reasonable.

Solution **Although the results of the survey suggest that Mindy will win, the survey is biased. The prediction that Mindy will win is not reasonable.**



Coached Example

There are 70 students taking a biology class. Eight of the students took a test one day early because they had to go to a track meet on the day of the test. Their test scores are shown below.

87, 84, 89, 91, 95, 73, 90, 87

The students are a representative sample of all the students taking biology. Predict the mean test score for all the students taking biology. Is the prediction reasonable?

What is the sample? _____

What is the population? _____

Find the mean of the sample to predict the mean of the _____.

Find the sum of the test scores. _____

Divide the sum by the number of scores in the sample. _____

The mean of the sample test scores is _____.

Based on the mean test score from the sample, the mean test score for the population should be about _____.

Is the prediction reasonable? _____

Explain. _____

The mean test score for all students taking biology should be about _____.

The sample mean provides a _____ prediction of the population test scores.

Compare Data Sets



Getting the Idea

You can compare the means, medians, modes, ranges, and interquartile ranges of two different data sets to draw conclusions about the data.

A **stem-and-leaf plot** is an arrangement that shows groups of data arranged by place value. The stems represent multiples of 10 and the leaves represent the ones place. If a value occurs more than once, it is listed each time it occurs. For example the number 64 would have a stem of 6 and a leaf of 4.

Example 1

Students at two middle schools sold reusable bags to raise money for Earth Day. Nellie surveyed a sample of ten students from each school to find out how many bags each student sold. Her data is shown in the stem-and-leaf plots.

Roosevelt Middle School	
Stem	Leaves
1	0 3 7 9
2	0 1 4 4
3	5
4	2

Key: 1 | 0 = 10 bags

Madison Middle School	
Stem	Leaves
1	2 4
2	1 3 6 8 9
3	0 0 4
4	

Key: 1 | 2 = 12 bags

Compare the measures of central tendency for the two schools.

What conclusions can you draw from the comparisons?

Strategy Find the mean, median, and mode of each data set. Then compare.

Step 1

Find the mean number of bags for Roosevelt Middle School.

The numbers of bags that students sold are listed below.

10, 13, 17, 19, 20, 21, 24, 24, 35, 42

$$\text{mean} = \frac{10 + 13 + 17 + 19 + 20 + 21 + 24 + 24 + 35 + 42}{10} = \frac{225}{10} = 22.5$$

Step 2

Find the median number of bags for Roosevelt Middle School.

The median is the middle number.

There are 10 values, so the median is the mean of the two middle values.

Find the mean of the fifth and sixth entries in the stem-and-leaf plot.

$$\text{median} = \frac{20 + 21}{2} = \frac{41}{2} = 20.5$$

Step 3

Find the mode number of bags for Roosevelt Middle School.

The mode is the value that occurs most often.

The mode is 24.

Step 4

Find the mean number of bags for Madison Middle School.

The numbers of bags that students sold are listed below.

12, 14, 21, 23, 26, 28, 29, 30, 30, 34

$$\text{mean} = \frac{12 + 14 + 21 + 23 + 26 + 28 + 29 + 30 + 30 + 34}{10} = \frac{247}{10} = 24.7$$

Step 5

Find the median number of bags for Madison Middle School.

Find the mean of the fifth and sixth entries in the stem-and-leaf plot.

$$\text{median} = \frac{26 + 28}{2} = \frac{54}{2} = 27$$

Step 6

Find the mode number of bags for Madison Middle School.

The mode is 30.

Step 7

Compare the measures of central tendency.

Compare the means: $24.7 - 22.5 = 2.2$

Compare the medians: $27 - 20.5 = 6.5$

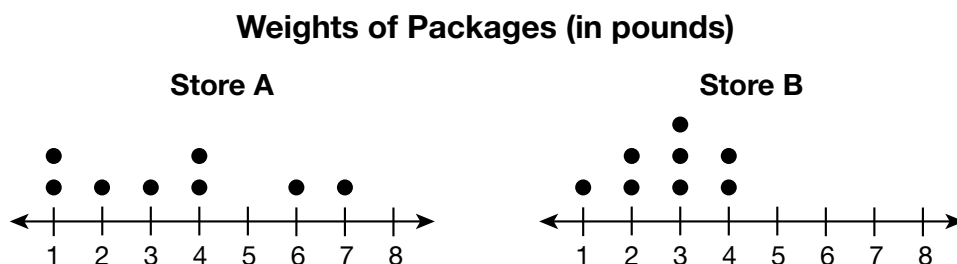
Compare the modes: $30 - 24 = 6$

Solution

Students at Madison Middle School sold an average of about 2 to 6 more bags than the students at Roosevelt Middle School.

Example 2

The dot plots show the weights of 8 packages that are waiting to be shipped from two stores owned by a large shipping company.



Compare the measures of central tendency for the two stores. What conclusions can you draw from the comparisons? What conclusions can you draw from the dot plots?

Strategy Find the mean, median, and mode of each data set. Then compare.

Step 1

Find the mean, median, and mode for Store A.

The weights of the packages at Store A are 1, 1, 2, 3, 4, 4, 6, and 7.

$$\text{mean} = \frac{1 + 1 + 2 + 3 + 4 + 4 + 6 + 7}{8} = \frac{28}{8} = 3.5$$

$$\text{median} = \frac{3 + 4}{2} = \frac{7}{2} = 3.5$$

$$\text{modes} = 1 \text{ and } 4$$

Step 2

Find the mean, median and mode for Store B.

The weights of the packages at Store B are 1, 2, 2, 3, 3, 3, 4, 4.

$$\text{mean} = \frac{1 + 2 + 2 + 3 + 3 + 3 + 4 + 4}{8} = \frac{22}{8} = 2.75$$

$$\text{median} = \frac{3 + 3}{2} = \frac{6}{2} = 3$$

$$\text{mode} = 3$$

Step 3

Compare the measures of central tendency.

$$\text{Compare the means: } 3.5 - 2.75 = 0.75$$

$$\text{Compare the medians: } 3.5 - 3 = 0.5$$

Compare the modes: You cannot compare the modes since one set of data has 1 mode and the other set has 2 modes.

The mean and median weights of the packages are 0.5 to 0.75 pound heavier at Store A than at Store B.

Step 4

Compare the dot plots.

From the dot plots, you can see that the weights of the packages at Store A are more spread out than at Store B. At Store B, the package weights are more closely clustered together.

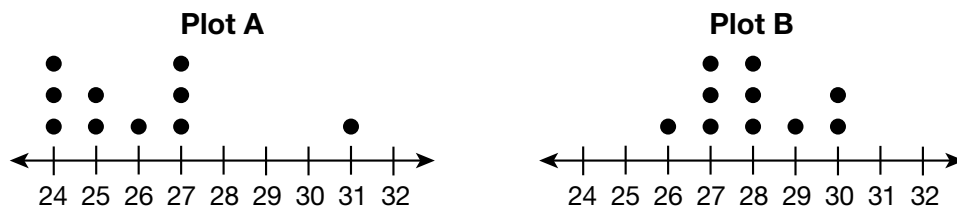
Solution

The mean and median weights of the packages are greater at Store A than at Store B. There is a greater variability in the weights of the packages at Store A than at Store B.

Example 3

The dot plots show the heights of sunflowers grown in two different plots on a farm.

Height of Sunflowers (in inches)



Make a visual comparison of the dot plots. What do they tell you about the average heights of the sunflowers on the two farms?

Compare the mean heights and the mean absolute deviations of the two plots. What conclusions can you draw about the variability of the heights?

Strategy

Examine and compare the two dot plots. Then, find the mean height and mean absolute deviation for each plot and compare these to your visual findings.

Step 1

Look at the dot plots and compare them visually.

Notice that the scale on both plots is the same; they both show numbers from 24 to 32.

In Plot A, the dots are all toward the left side of the plot, except for one value. They form a cluster from 24 to 27 inches and have a single outlier of 31 inches.

In Plot B, the dots are all in the middle, with the highest columns of dots on 27 and 28 inches.

In general, the heights appear to be higher in Plot B, and more variable in Plot A.

Step 2

Find the mean for Plot A.

The heights of the sunflowers in Plot A are 24, 24, 24, 25, 25, 26, 27, 27, 27, 31.

$$\text{mean} = \frac{24 + 24 + 24 + 25 + 25 + 26 + 27 + 27 + 27 + 31}{10} = \frac{260}{10} = 26$$

Step 3

Find the MAD for Plot A.

Subtract to find the deviations.

$$24 - 26 = -2 \quad 24 - 26 = -2 \quad 24 - 26 = -2$$

$$25 - 26 = -1 \quad 25 - 26 = -1 \quad 26 - 26 = 0$$

$$27 - 26 = 1 \quad 27 - 26 = 1 \quad 27 - 26 = 1 \quad 31 - 26 = 5$$

Add the absolute deviations: $2 + 2 + 2 + 1 + 1 + 0 + 1 + 1 + 1 + 5 = 16$

Find the MAD.

$$\text{MAD} = \frac{16}{10} = 1.6$$

Step 4

Find the mean for Plot B.

The heights of the sunflowers in Plot B are: 26, 27, 27, 27, 28, 28, 28, 29, 30, 30.

$$\text{mean} = \frac{26 + 27 + 27 + 27 + 28 + 28 + 28 + 29 + 30 + 30}{10} = \frac{280}{10} = 28$$

Step 5

Find the MAD for Plot B.

Subtract to find the deviations.

$$26 - 28 = -2 \quad 27 - 28 = -1 \quad 27 - 28 = -1 \quad 27 - 28 = -1$$

$$28 - 28 = 0 \quad 28 - 28 = 0 \quad 28 - 28 = 0$$

$$29 - 28 = 1 \quad 30 - 28 = 2 \quad 30 - 28 = 2$$

Add the absolute deviations: $2 + 1 + 1 + 1 + 0 + 0 + 0 + 1 + 2 + 2 = 10$

Find the MAD.

$$\text{MAD} = \frac{10}{10} = 1$$

Step 6

Compare the means and MADs to your visual findings for the dot plots.

The mean height of the sunflowers in Plot B is greater than the mean height of the sunflowers in Plot A. The deviation from the mean is greater in Plot A than in Plot B. These results support the conclusions that you drew in Step 1 from a visual comparison of the two dot plots.

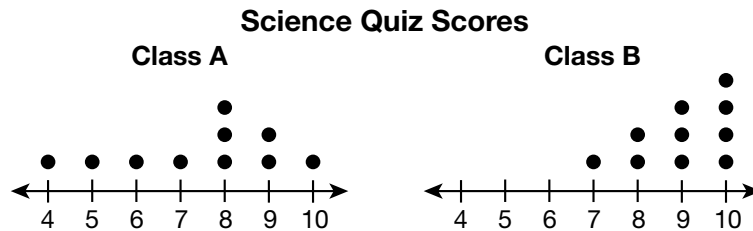
Solution

The mean height of the sunflowers in Plot B is greater than the mean height of the sunflowers in Plot A. There is a greater variability in the heights in Plot A than in Plot B.



Coached Example

The dot plots show the grades of students who took the same ten-question science quiz in two different classes.



Compare the measures of central tendency for each class. What conclusions can you draw from the comparisons?

What are the measures of central tendency? _____

What are the quiz scores for Class A? _____

What is the mean for Class A? _____

What is the median for Class A? _____

What is the mode for Class A? _____

What are the quiz scores for Class B? _____

What is the mean for Class B? _____

What is the median for Class B? _____

What is the mode for Class B? _____

Subtract to compare the means. _____

The mean score of Class A is about _____ than the mean score of Class B.

Subtract to compare the medians. _____

The median score of Class A is about _____ than the median score of Class B.

Subtract to compare the modes. _____

The mode score of Class A is about _____ than the mode score of Class B.

Look at the dot plots.

How does the variability of the scores from Class A compare to the variability of the scores from Class B? _____

The mean, median, and mode scores of Class A are _____ than those of Class B.

The variability of the scores from Class A _____ the variability of the scores from Class B.

Answer Key

Lesson 1

Coached Example

To convert a percent to a decimal, remove the percent sign and move the decimal point 2 places to the **left**.

The decimal **0.55** is equivalent to 55%.

To convert a percent to a fraction, write the digits after the decimal point as the numerator over a denominator of **100**.

What is the GCF of the numerator and denominator? **5**

Divide the numerator and denominator by **5**.

$$\text{Simplify. } \frac{55}{100} = \frac{55 \div 5}{100 \div 5} = \frac{11}{20}$$

55% written as a decimal is **0.55**. 55% written as a fraction is $\frac{11}{20}$.

Lesson 2

Coached Example

To solve this problem, add the amount of the **bill** and the amount of the **tip**.

The bill was **\$24.50**.

To find the amount of the tip, first change **15%** to a decimal.

$$15\% \rightarrow 0.15$$

Then multiply the decimal by the amount of the **bill**.

$$0.15 \times \$24.50 = \$3.675$$

Round the amount of the tip to the nearest cent.

$$\text{\$3.68}$$

Add the amount of the **bill** and the amount of the **tip**.

$$\text{\$24.50} + \text{\$3.68} = \text{\$28.18}$$

Angela and Sadie spent **\$28.18** for their bill and tip.

Lesson 3

Coached Example

Divide the **numerator** by the **denominator**.

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Does the decimal end? **yes**

The fraction $\frac{7}{8}$ can be expressed as a **terminating** decimal.

Lesson 4

Coached Example

Write a number sentence to represent this problem.

$$15 \div 1\frac{2}{3} = w$$

Rewrite 15 as an improper fraction. $\frac{15}{1}$

Rewrite $1\frac{2}{3}$ as an improper fraction. $\frac{5}{3}$

Rewrite the number sentence, using improper fractions.

$$\frac{15}{1} \div \frac{5}{3} = w$$

To divide fractions, multiply the dividend by the **reciprocal** of the divisor.

The reciprocal of the divisor is $\frac{3}{5}$.

Rewrite as a multiplication problem, using the reciprocal of the divisor.

$$\frac{15}{1} \div \frac{5}{3} = \frac{15}{1} \times \frac{3}{5}$$

Multiply.

$$\frac{15}{1} \times \frac{3}{5} = \frac{15 \times 3}{1 \times 5} = \frac{45}{5} \text{ or } \frac{9}{1}$$

Simplify the product. **9**

Mr. Camara has **9** pieces of wood.

Lesson 5

Coached Example

Write a number sentence to represent the problem.

$$-28 - 52 = I$$

Is the subtrahend positive or negative? **positive**

Find the opposite of the subtrahend. **-52**

Add the opposite of the subtrahend to the minuend.

$$-28 + (-52)$$

Both integers being added have a **negative** sign.

Find the absolute value of the first addend.

$$|-28| = 28$$

Find the absolute value of the second addend.

$$|-52| = 52$$

Add the absolute values.

$$28 + 52 = 80$$

Use the sign of the addends in the sum. The sign for the sum is **negative**.

The lowest temperature in U.S. history is -80°F .

Lesson 6

Coached Example

Will the product of the first two integers be positive or negative? **negative**

$$-5 \cdot 4 = -20$$

When you multiply this product by the third integer, -1 , will the product be positive or negative? **positive**

Multiply the product of the first two integers by the third integer, -1 .

$$-20 \cdot (-1) = 20$$

When you multiply this product by the fourth integer, -2 , will the product be positive or negative? **negative**

$$20 \cdot (-2) = -40$$

The value of $(-5)(4)(-1)(-2)$ is -40 .

Lesson 7

Coached Example

South is represented by **negative** numbers and north is represented by **positive** numbers.

Write a rational number to represent $2\frac{1}{4}$ miles south.

$$-2\frac{1}{4}$$

Write a rational number to represent $1\frac{1}{2}$ miles north.

$$1\frac{1}{2}$$

Write an addition equation to represent Genesis's

$$\text{location. } -2\frac{1}{4} + 1\frac{1}{2} = f$$

Which addend has a greater absolute value? $-2\frac{1}{4}$

The sum has the sign of the **greater** absolute value.

Is Genesis north or south from her starting location?

south

$$-2\frac{1}{4} + 1\frac{1}{2} = -\frac{3}{4}$$

Genesis's location is $\frac{3}{4}$ mile south from her starting point.

Lesson 8

Coached Example

To solve the problem, divide 663.85 by **35.5** to find the amount of money Mr. Livio earns per hour.

Multiply the divisor, 35.5, by **10**

to make it a whole number: $35.5 \times 10 = 355$

Multiply the dividend, 663.85, by that same power of 10: $663.85 \times 10 = 6638.5$

Divide as you would with whole numbers.

$$\begin{array}{r} 18.7 \\ 355 \overline{) 6,638.5} \\ \underline{-355} \\ 3088 \\ \underline{-2840} \\ 2485 \\ \underline{-2485} \\ 0 \end{array}$$

Mr. Livio earns **\$18.70** per hour.

Lesson 9

Coached Example

Write a ratio that compares the total cost to the number of tomatoes. $\frac{\$2.00}{5 \text{ tomatoes}}$

Divide to find the unit price.

$$\$2.00 \div 5 = \$0.40$$

To find the cost of a dozen tomatoes, multiply the unit price by **12**.

$$0.40 \times 12 = 4.8$$

The unit price of the tomatoes is **\$0.40** per tomato. One dozen tomatoes will cost **\$4.80**.

Lesson 10

Coached Example

To cross multiply, multiply the **numerator** of each fraction by the **denominator** of the other fraction.

Write the factors for the cross products.

$$72 \times 25 = 90 \times x$$

Multiply to find the cross products.

$$1,800 = 90x$$

Divide both sides by **90** to solve for x .

$$1,800 = 90x$$

$$\frac{1,800}{90} = \frac{90x}{90}$$

$$x = 20$$

Substituting the value **20** for x makes the proportion

$$\frac{72}{90} = \frac{x}{25} \text{ true.}$$

Lesson 11

Coached Example

Write the number of fluid ounces for each student. **8**

Write the number of fluid ounces in each jug. **40**

To find the unit rate, write a **ratio** that compares the number of fluid ounces in each jug to the number of fluid ounces for each student. $\frac{40}{8}$

Simplify the ratio to write the unit rate. **5** servings per jug

To find the number of jugs Mr. Collins needs, **divide** the number of students in the class by the number of servings per jug.

$$30 \div 5 = 6$$

Mr. Collins will need **6** jugs of juice for the party.

Lesson 12

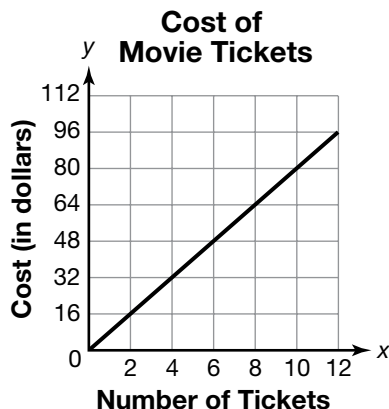
Coached Example

Write an equation to represent the situation. **$y = 8x$**

Number of Tickets (x)	0	2	4	6	8	10	12
Cost, in Dollars (y)	0	16	32	48	64	80	96

Create a graph to show the values in the table.

Graphs will vary depending on titles, labels, and scales used. Possible graph is shown.



The graph passes through the point **(1, 8)**. So, **8** is the constant of proportionality, or the unit rate.

It would cost **\$96** for 12 people.

Lesson 13

Coached Example

The word “twice” indicates you could **multiply** by 2.

The words “younger than” mean that William’s age is **less** than Mia’s age.

What operation should you use to show “younger than”? **subtraction**

$$2k - 3$$

The expression **$2k - 3$** represents William’s age.

Lesson 14

Coached Example

Substitute **8** for p and **5** for q in the expression.

$$\frac{16}{p} - 3q = \frac{16}{8} - 3 \times 5$$

$$\frac{16}{8} - 3 \times 5 =$$

$$2 - 3 \times 5 =$$

$$2 - 15$$

Now, add and subtract in order from left to right.

$$2 - 15 = -13$$

The value of the expression is **-13**.

Lesson 15

Coached Example

$$5 + y - (2 + x)$$

The opposite of $2 + x$ is **$-(2 + x)$** .

$$\text{Distribute the } -1 \text{ over the two terms. } -(2 + x) = (-2) - x$$

Rewrite the problem as an addition problem by adding the opposite.

$$5 + y + (-2) - x$$

Use number properties to reorder and group like terms. Then add.

$$5 + y + (-2) - x = 5 + (-2) + y - x = 3 + y - x$$

The expression **$3 + y - x$** represents the amount of change, in dollars, that Carter received.

Lesson 16

Coached Example

Translate the words into a mathematical sentence.

$$5 + 2.5h = 15$$

The equation **$5 + 2.5h = 15$** represents the situation.

Lesson 17

Coached Example

The first step is to use the **distributive property** to simplify the left side of the equation.

Multiply both terms inside the parentheses by $-\frac{1}{2}$.

The left side of the equation is now $-\frac{1}{2}r + \frac{3}{8}$.

The equation can be rewritten

$$\text{as } -\frac{1}{2}r + \frac{3}{8} = -\frac{4}{5}.$$

Subtract $\frac{3}{8}$ from both sides of the equation.

$$-\frac{1}{2}r + \frac{3}{8} - \frac{3}{8} = -\frac{4}{5} - \frac{3}{8}$$

What is the difference of the right side of the equation? $-\frac{47}{40}$

The equation is now $-\frac{1}{2}r = -\frac{47}{40}$

Multiply both sides of the equation by the **reciprocal** of the coefficient.

$$-\frac{1}{2}r \times (-2) = -\frac{47}{40} \times -2$$

$$r = 2\frac{7}{20}$$

Lesson 18

Coached Example

What percent of the cost does \$84.20 represent.
100%

Will Janeel pay for less than or more than 100% of the total cost? **less than**

What percent of the cost will Janeel pay? **70%**

Write an equation to represent how to find the sale price of the sneakers.

$$\text{\$84.2} \times 0.7 = s$$

Solve your equation.

$$\text{\$84.2} \times 0.7 = \text{\$58.94}$$

Janeel will spend **\\$58.94** for the sneakers.

Lesson 19

Coached Example

The expenses for the race are the **T-shirts**, the timers, and other race equipment.

The total expenses are **\\$345 + \\$85 = \\$430**.

Let n represent the number of people **who enter the race**.

Write an expression for the total amount raised by entry fees. **$15n$**

How much money was donated to help with expenses? **\\$50**

Write an expression for the total amount of money raised. **$15n + \$50$**

To make a profit, the amount raised by the entry fees plus the money donated must be **greater than** the total expenses for the race.

$$\textbf{15n + 50 > 430}$$

Solve the inequality.

$$\textbf{15n + 50 - 50 > 430 - 50}$$

$$\textbf{15n > 380}$$

$$\frac{\textbf{15n}}{\textbf{15}} > \frac{\textbf{380}}{\textbf{15}}$$

$$\textbf{n > 25}\frac{\textbf{1}}{\textbf{3}}$$

What is the least number of people who can enter for the race to make a profit? **26**

At least **26** people must enter the race for it to make a profit.

Lesson 20

Coached Example

The scale is $\frac{1}{2}$ inch = **50** miles.

Write a ratio of the scale. $\frac{\frac{1}{2} \text{ in.}}{50 \text{ mi}}$

Write a proportion to find the scale drawing

$$\text{distance. } \frac{\frac{1}{2} \text{ in.}}{50 \text{ mi}} = \frac{x \text{ in.}}{420 \text{ mi}}$$

Solve the proportion.

$$\frac{\frac{1}{2}}{50} = \frac{x}{420}$$

$$50 \times x = \frac{1}{2} \times 420$$

$$50x = 210$$

$$\frac{50x}{50} = \frac{210}{50}$$

$$x = 4.2$$

The scale drawing distance is **4.2** inches.

Kerri should draw the two cities **4.2 inches** apart on the map.

Lesson 21

Coached Example

The sum of the lengths of any two sides of a triangle must be **greater** than the length of the third side.

Is $8 + 9 > 12$? **Yes, $17 > 12$ is true.**

Is $8 + 12 > 9$? **Yes, $20 > 9$ is true.**

Is $9 + 12 > 8$? **Yes, $21 > 8$ is true.**

The inequalities above are all true, so it **is** possible to draw a triangle with side lengths of 8 feet, 9 feet, and 12 feet.

Lesson 22

Coached Example

What does “perpendicular” mean? Answers will vary. Possible answer: **The plane that is perpendicular to the base will slice through the pyramid straight from the top vertex down to the base, so the base of the pyramid and the plane will form right angles.**

Make a sketch of the cross section in the space below.

Check students’ work. Students should draw an isosceles triangle.

The shape of the cross section is an **isosceles triangle**.

Lesson 23

Coached Example

What is the formula for the circumference of a circle when the radius is given? **$C = 2\pi r$**

Use 3.14 for π and substitute the length of the **radius** into the formula.

$$C \approx 2 \times 3.14 \times 6 \text{ in.}$$

$$C \approx 37.68 \text{ in.}$$

What is the formula for the area of a circle? **$A = \pi r^2$**

Use 3.14 for π and substitute the length of the **radius** into the formula.

$$A \approx 3.14 \times (6 \text{ in.})^2$$

$$A \approx 113.04 \text{ in.}^2$$

The circumference of each wheel is about **37.68 in.**, and the area is about **113.04 in.²**.

Lesson 24

Coached Example

Complementary angles have a measure of **90°** .

The measure of one angle is **59°** , and the measure of the other angle is **$4n - 1^\circ$** .

Write an equation for the total of the two angles.

$$59 + 4n - 1 = 90$$

Solve the equation for n .

$$59 - 1 + 4n = 90$$

$$58 + 4n = 90$$

$$58 - 58 + 4n = 90 - 58$$

$$4n = 32$$

$$n = 8$$

To find the measure of the unknown angle, substitute the value of n into $4n - 1$ and evaluate.

$$4n - 1 = (4 \times 8) - 1 = 31^\circ$$

Check that the sum of the angle measures is 90° .

$$59^\circ + 31^\circ = 90^\circ$$

The value of n is **8**.

Lesson 25

Coached Example

A parallelogram and a **rectangle** are combined to form the sticker.

What is the formula for the area of a parallelogram?

$$A = bh$$

Find the area of the parallelogram.

$$A = 4 \text{ cm} \times 1.2 \text{ cm}$$

$$= 4.8 \text{ cm}^2$$

What is the formula for the area of a rectangle? **$A = lw$**

Find the area of the rectangle.

$$A = 1.5 \text{ cm} \times 3 \text{ cm}$$

$$= 4.5 \text{ cm}^2$$

Add to find the total area of the figure.

$$4.8 \text{ cm}^2 + 4.5 \text{ cm}^2 = 9.3 \text{ cm}^2$$

The area of the sticker is **9.3 cm^2** .

Lesson 26

Coached Example

The front and back faces are rectangles that are 3 ft long and 2 ft high.

$$A \text{ of front face} = lw = 3 \times 2 = 6 \text{ ft}^2$$

The area of the back face is also 6 ft².

The left and right side faces are rectangles that are 1 ft long and 2 ft high.

$$A \text{ of left side face} = lw = 1 \times 2 = 2 \text{ ft}^2$$

The area of the right side face is also 2 ft².

Add the areas of all four faces:

$$6 + 6 + 2 + 2 = 16 \text{ ft}^2$$

The toymaker will paint 16 square feet of the toy chest.

Lesson 27

Coached Example

The formula for the volume of a cube is $V = e^3$.

$$V = 20 \times 20 \times 20$$

$$V = 8,000 \text{ in.}^3$$

The planter box has a volume of 8,000 cubic inches.

Lesson 28

Coached Example

There were 12 defective skateboards in a batch of 400.

$$P(\text{defective skateboard}) = \frac{12}{400} = \frac{3}{100}$$

Express the probability as a decimal. 0.03

Multiply the probability by 1,200.

$$1,200 \times 0.03 = 36$$

The Skate Pro Company might expect to find 36 defective skateboards in a batch of 1,200.

Lesson 29

Coached Example

How many faces does a number cube have? 6

How many possible outcomes are there for a number cube to land? 6

To find the number of possible outcomes, multiply $6 \times 6 = 36$.

There are 36 possible outcomes.

How many different ways are there to toss doubles 4s? 1

What is the probability of rolling double 4s? $\frac{1}{36}$

The probability of rolling double 4s is $\frac{1}{36}$.

Lesson 30

Coached Example

Victoria surveyed a total of 25 students.

Write a proportion in which each ratio shows the number of students who chose **spinach** as their least favorite vegetable to the **total** number of students.

Let s equal the number of students out of 100 who would choose spinach.

$$\frac{12}{25} = \frac{s}{100}$$

Cross multiply and solve for s .

$$25 \times s = 12 \times 100$$

$$25s = 1200$$

$$s = 48$$

In a survey of 100 students, 48 students would probably choose spinach as their least favorite vegetable.

Lesson 31

Coached Example

Find the mean number of books read by fifth graders:

$$\frac{1 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 5 + 5 + 7 + 8}{15} = \frac{44}{15} = 2.\overline{93}$$

The mean is 2.9.

Find the median number of books read by fifth graders:

$$1, 1, 1, 1, 1, 1, 2, 2, \underline{2}, 2, 3, 3, 5, 5, 7, 8$$

The median is 2.

Find the mean number of books read by seventh graders:

$$\frac{1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 4 + 4 + 5 + 5 + 6 + 7}{14} = \frac{49}{14} = 3.5$$

The mean is 3.5.

Find the median number of books read by seventh graders:

1, 2, 2, 2, 2, 3, 3, 3, 4, 4, 5, 5, 6, 7

The median is **3**.

The mean number of books read by the fifth graders is **2.9**, while the mean number of books read by the seventh graders is **3.5**.

The median number of books read by the fifth graders is **2**, while the median number of books read by the seventh graders is **3**.

The seventh graders read about an average of **1 more** book than the fifth graders.

Lesson 32

Coached Example

The least weight for the cocker spaniels is **15** pounds.

The greatest weight for the cocker spaniels is **30** pounds.

Find the range of the cocker spaniels' weights:
30 – 15 = 15

The range of the weights of the cocker spaniels is **15** pounds.

The third quartile weight for the cocker spaniels is **27** pounds.

The first quartile weight for the cocker spaniels is **20** pounds.

Find the IQR for the cocker spaniels' weights:
27 – 20 = 7

The IQR for the weights of the cocker spaniels is **7** pounds.

The least weight for the Labrador retrievers is **45** pounds.

The greatest weight for the Labrador retrievers is **70** pounds.

Find the range of the Labrador retrievers' weights:
70 – 45 = 25

The range of the weights of the Labrador retrievers is **25** pounds.

The third quartile weight for the Labrador retrievers is **65** pounds.

The first quartile weight for the Labrador retrievers is **55** pounds.

Find the IQR for the Labrador retrievers' weights:
65 – 55 = 10

The IQR for the weights of the Labrador retrievers is **10** pounds.

The range and the IQR for cocker spaniels are **less** than for Labrador retrievers.

Labrador retrievers have greater variability in weight than **cocker spaniels**.

Lesson 33

Coached Example

Find the mean weight of the dogs that boarded at the clinic.

$$\frac{43 + 87 + 12 + 15 + 23}{5} = \frac{180}{5} = 36$$

To find the deviations, **subtract** each weight from the mean weight.

$$43 - 36 = 7$$

$$87 - 36 = 51$$

$$12 - 36 = -24$$

$$15 - 36 = -21$$

$$23 - 36 = -13$$

To find the absolute deviations, find the **absolute value** of each deviation.

$$|7| = 7 \quad |51| = 51 \quad |-24| = 24$$

$$|-21| = 21 \quad |-13| = 13$$

Add the absolute deviations.

$$7 + 51 + 24 + 21 + 13 = 116$$

$$\text{mean absolute deviation} = \frac{116}{5} = 23.2$$

The mean absolute deviation of the weights of the dogs is **23.2** pounds.

Lesson 34

Coached Example

What is the sample? **the test scores of the 8 students who took the test early**

What is the population? **the 70 students taking biology**

Find the mean of the sample to predict the mean of the **population**.

$$\text{Find the sum of the test scores. } 87 + 84 + 89 + 91 + 95 + 73 + 90 + 87 = 696$$

$$\text{Divide the sum by the number of scores in the sample. } \frac{696}{8} = 87$$

The mean of the sample test scores is **87**.

Based on the mean test score from the sample, the mean test score for the population should be about **87**.

Is the prediction reasonable? **yes**

Explain. **The prediction is reasonable because the sample is a representative sample and 8 out of 70 students is a big enough sample size.**

The mean test score for all students taking biology should be about **87**.

The sample mean provides a **reasonable** prediction of the population test scores.

Lesson 35

Coached Example

What are the measures of central tendency? **mean, median, and mode**

What are the quiz scores for Class A? **4, 5, 6, 7, 8, 8, 8, 9, 9, 10**

What is the mean for Class A? **7.4**

What is the median for Class A? **8**

What is the mode for Class A? **8**

What are the quiz scores for Class B? **7, 8, 8, 9, 9, 9, 10, 10, 10, 10**

What is the mean for Class B? **9**

What is the median for Class B? **9**

What is the mode for Class B? **10**

Subtract to compare the means. **$9 - 7.4 = 1.6$**

The mean score of Class A is about **1.6 points less** than the mean score of Class B.

Subtract to compare the medians. **$9 - 8 = 1$**

The median score of Class A is about **1 point less** than the median score of Class B.

Subtract to compare the modes. **$10 - 8 = 2$**

The mode score of Class A is about **2 points less** than the mode score of Class B.

How does the variability of the scores from Class A compare to the variability of the scores from Class B?
Class A has more variability than Class B.

The mean, median, and mode scores of Class A are **less** than those of Class B.

The variability of the scores from Class A **is greater than** the variability of the scores from Class B.