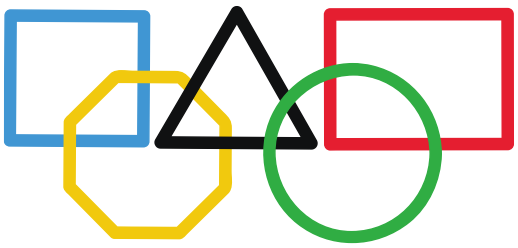

2020 INTERNATIONAL MATH CONTEST

Practice Problems for Russian Math Olympiad

Grade 3-4

2019 INTERNATIONAL MATH CONTEST

ID _____
Name _____
Grade _____
RSM Affiliation _____
Test Location _____

FINAL ROUND

Grades 3-4

RSM FOUNDATION

1 In the puzzle below, each card hides a digit. What digit is hidden under the card with the question mark?

$$20 + \boxed{?}\square + 19 = 100$$

2 Gary has 20 more candies than Mary. If Gary gives Mary 19 of his candies, Mary would have how many more candies than Gary?

3 Numbers were written in the twelve boxes shown, one number per box. For every four boxes in a row, the sum of their numbers was 12. Most of the numbers got erased over time, but three of them remain. What number was written in the last box on the right?

	0					1		2			

4 Four cats – Astro, Buttons, Calico, and Duchess – bought 20 mice altogether. Each of the four cats bought an odd number of mice, but none of them bought exactly 13 mice. Buttons bought more mice than Astro, fewer mice than Duchess, and as many mice as Calico. How many mice did Calico buy?

5 Natasha drew five straight lines (from border to border) on a triangular piece of paper. Then she cut the paper along all these lines and got several shapes. What is the largest number of sides one of Natasha's shapes could have?

6 A family has many children – brothers and sisters. Each of them wrote a statement about the family. Five of these statements are as follows:

- I have more brothers than sisters;
- I have more sisters than brothers;
- I have as many brothers as sisters;
- I have fewer sisters than brothers;
- I have fewer brothers than sisters.

What is the greatest possible number of these statements that can be true at the same time?

Please fold over on line. Write answers on back.

2019 INTERNATIONAL MATH CONTEST

ID _____
Name _____
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RSM Affiliation _____
Test Location _____

FINAL ROUND

Grades 3-4

RSM FOUNDATION

7

Dubbles the monster has twice as many ears as eyes, twice as many legs as arms, and twice as many tongues as noses. Overall he has 39 ears, eyes, legs, arms, tongues, and noses. How many ears, legs, and tongues does Dubbles the monster have altogether?

8

How many different counting numbers are there containing only odd digits such that for each of these numbers, the sum of all of its digits equals seven?

9

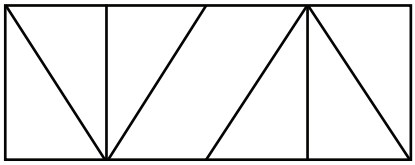
A square shape is divided into two non-overlapping rectangular shapes. Each of these two rectangular shapes is divided into three non-overlapping square shapes. Compute the sum of the perimeters of these six squares (in feet) if the perimeter of the original square is 60 feet. (The perimeter of a square is the sum of the lengths of all of its sides.)

10

In 2017, a long row of trees was planted in the empty RSM Garden. In 2018, a tree was planted between every two adjacent (next to each other) trees planted in the previous year. In 2019, a tree was planted between every two adjacent trees planted in the previous years, bringing the total number of trees in RSM Garden to 877. How many trees were planted in RSM Garden in 2018?

11

How many quadrilaterals of all sizes and positions are there in the diagram, including quadrilaterals that are made up of more than one shape? (A quadrilateral is a shape with four sides.)



12

Say that a counting number is "five-important" if it is a multiple of 5 and contains the digit 5. For instance, the numbers 125, 55, and 550 are five-important, but the numbers 59, 2019, and 2020 are not. How many different five-important numbers are there between 1 and 2019?

Please fold over on line. Write answers on back.

2019 International Math Contest

RSM Foundation

Final Round Answers

Problem/Grades	3-4
1	6
2	18
3	9
4	5
5	8
6	4
7	26
8	13
9	140
10	219
11	20
12	240

2018 INTERNATIONAL MATH CONTEST

ID _____
Name _____
Grade _____
RSM Affiliation _____
Test Location _____

FINAL ROUND

Grades 3-4

RSM FOUNDATION

1 Find the smallest five-digit number with exactly three odd digits if all five digits are different.

2 A bag contains 18 small red, 19 small black, 13 large black, and 15 large red T-shirts, and nothing else. What is the least number of T-shirts Alpa must take out of the bag (without looking at them) to be absolutely sure of having at least one T-shirt of each color among them?

3 From a piece of paper Felix cut out some squares and pentagons. He cut out a total of five shapes with 23 sides altogether. How many pentagons were among these shapes?

4 Leah said on her birthday: "Today I am exactly three times as old as I was four years ago." And that was true. In how many years can Leah truthfully say on her birthday: "Today I am exactly twice as old as I was four years ago"?

5 Alexa wrote a two-digit number on a piece of paper. To that number she added five different one-digit numbers, and got a sum of 134. What two-digit number did Alexa start from?

6 Peter was standing in a line of RSM students. There were twice as many students in front of him as behind him. Several students (more than 3 but fewer than 9) left the line but Peter did not. Then twice as many students were behind Peter as in front of him. How many students left the line?

7 Winnie-the-Pooh brought a bunch of balloons. Some were green, and the rest were red. Pooh gave some of the balloons to Eeyore, and the rest he gave to Piglet. Piglet got three more balloons then Eeyore did, and Eeyore got two more red balloons than the number of green balloons Piglet got. How many more red balloons than green balloons did Pooh bring?

Please fold over on line. Write answers on back.

2018 INTERNATIONAL MATH CONTEST

ID _____
Name _____
Grade _____
RSM Affiliation _____
Test Location _____

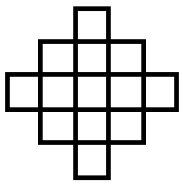
FINAL ROUND

Grades 3-4

RSM FOUNDATION

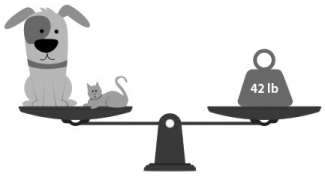
8

How many rectangles of all sizes and positions are there in the diagram, including rectangles that are made up of more than one shape? (Remember that every square counts as a rectangle.)



9

Two dogs weigh as much as five cats. A dog and a cat weigh 42 pounds altogether. A dog is heavier than a cat by how many pounds? Assume that all dogs weigh the same, and all cats weigh the same.



10

In Numberland there is a square, and all of the numbers live on its sides (some live on the corners). For each of the four sides, Olga calculated the sum of all of the numbers living on that side (including the corners), and she got the following results (in some order): 4, 8, 12, and 16. The sum of all of the numbers in Numberland equals 30. Find the sum of all of the numbers living on the corners of the square.

11

A rectangular shape is divided into four non-overlapping rectangular shapes as shown in the diagram. The perimeters of three of these four shapes are 20 feet, 18 feet, and 28 feet. What is the greatest possible perimeter (in feet) of the original rectangle? (The perimeter of a rectangle is the sum of the lengths of all of its sides.)

12

Rustam likes multiples of 5. For his RSM project he makes up numbers such that any two of the digits of any of his numbers, when put next to each other in some order, make a two-digit multiple of 5. His teacher calls such numbers Rustam's numbers. For example, the numbers 51 and 502 are Rustam's numbers, but the numbers 300 and 2018 are not. How many different three-digit Rustam's numbers with none of the digits greater than 5 are there?

Please fold over on line. Write answers on back.

2018 International Math Contest

RSM Foundation

Final Round Answers

Problem/Grades	3-4
1	10235
2	34
3	3
4	2
5	99
6	6
7	7
8	54
9	18
10	10
11	48
12	31

2017 INTERNATIONAL MATH CONTEST

First Name _____
Last Name _____
Grade _____
School _____
City _____
RSM Branch _____

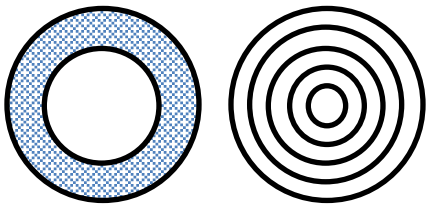
FINAL ROUND

Grades 3-4

RSM FOUNDATION

1 A princess is riding a horse. A bird is on her shoulder. The three of them together have how many more legs than heads?

2 A ring is a flat shape formed by an inner circle and an outer circle, as shown in the first diagram. How many rings of all sizes and types are there in the second diagram containing five circles?

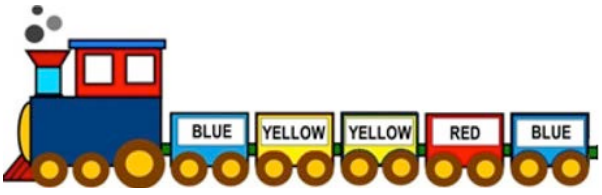


3 Mary and Jack are standing in line. Mary is the second in line, and Jack is the third from the end. There are 12 people in front of Jack. How many people are in line behind Mary?

4 Find the largest 6-digit number such that the sum of all its digits equals 40.

5 Yesterday Alice ate several candies and cookies, for a total of 12. Today she ate 3 fewer candies than yesterday, and twice as many cookies as yesterday, for a total of 14. How many candies did Alice eat yesterday?

6 In a very long toy train, the first and last cars were blue. After each blue car (except the last one), there were two yellow cars. After each pair of yellow cars, there was a red car. After each red car, there was a blue car. The first five train cars are shown in the picture. Oleg picked a car and recolored all cars in front of it green. Then Joyce picked a car and recolored all cars behind it green. What is the least possible number of non-green cars in the recolored toy train if it contains 7 more yellow cars than blue cars?



Please fold over on line. Write answers on back.

2017 INTERNATIONAL MATH CONTEST

First Name _____
Last Name _____
Grade _____
School _____
City _____
RSM Branch _____

FINAL ROUND

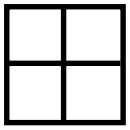
Grades 3–4

RSM FOUNDATION

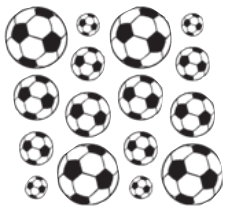
7 Ilya wrote the counting numbers from 1 to 50. He started this way: 123456789101112, and stopped when he wrote 50. How many odd digits did he write?

8 Anna really likes numbers and decided to collect them. She started her collection from the number 35, which was a birthday gift from RSM. After that, every week Anna added one more new number to the collection by selecting the smallest counting number which was neither a multiple nor a factor of any number already in the collection. What number was added to Anna's collection on week 10? Note that after 10 weeks the collection contained 11 different numbers.

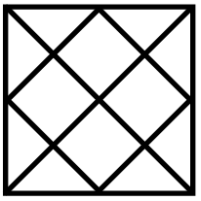
9 How many different ways are there to place four different digits from 1 to 4 inside the four square cells of a 2-by-2 grid (one digit per cell) such that for every pair of digits that are 1 apart (such as 2 and 3), their square cells share a side?



10 There are several balls in the RSM Sport Center. At least one of the balls is $1 cheaper than another one. At least one of the balls is $2 cheaper than another one. At least one of the balls is $3 cheaper than another one. At least one of the balls is $4 cheaper than another one. At least one of the balls is $6 cheaper than another one. At least one of the balls is $7 cheaper than another one.
What is the least possible number of balls in the RSM Sport Center?



11 How many more triangles (of all sizes and positions) than squares (of all sizes and positions) are there in the diagram?



12 There are six different cards (three red and three blue) with the letters R, S, M on them. Each card has exactly one letter, and each of these letters is on exactly two cards (one red and one blue). How many different ways are there to put all six cards in a row with letters face up and right-side up such that every card appears right next to another card with the same letter?

Please fold over on line. Write answers on back.

Answers:

Problem No.	Answer
1	5
2	10
3	13
4	999940
5	7
6	22
7	46
8	31
9	8
10	4
11	14
12	48

2016 INTERNATIONAL MATH CONTEST

First Name _____
Last Name _____
Grade _____
School _____
City _____
RSM Branch _____

FINAL ROUND

Grades 3-4

RSM FOUNDATION

1

In the puzzle below, each card hides a digit. What digit is hidden under the card with the question mark?

2

Eight kids are holding a total of 15 balloons. Some balloons are red, and the rest are blue. Nobody holds two or more balloons of the same color, and nobody shares a balloon. How many kids hold exactly one balloon each?

3

Grandma sent Jack a bag of candy. When Jack opened it, he found inside 8 large boxes of candy. Each of these large boxes had 6 smaller boxes of candy inside, and each of the smaller boxes had 10 candies. How many candy boxes of all sizes were in the bag?

4

A paper rectangle is folded once to get a 2 cm-by-3 cm rectangle. What is the greatest possible perimeter (in centimeters) of the original rectangle? (The perimeter of a rectangle is the sum of the lengths of all of its sides.)

5

RSM opened a new branch for puppies and kittens in Pawville. When the principal counted ears and tails of all 30 students, he discovered there were twice as many kittens' ears as puppies' tails. How many kittens were at the RSM-Pawville branch (if every animal had the usual number of body parts)?

6

Aurora made three paper triangles, four paper squares, and five paper octagons. Barbara made several paper pentagons. Aurora's shapes all together have as many sides as all Barbara's pentagons do. How many pentagons did Barbara make?

Please fold over on line. Write answers on back.

2016 INTERNATIONAL MATH CONTEST

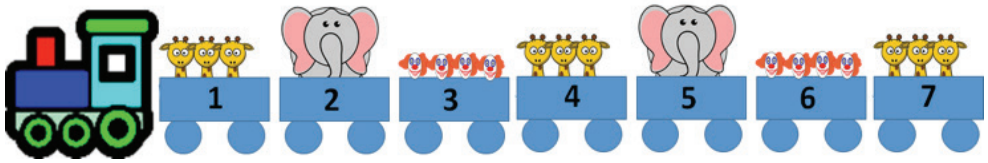
First Name _____
Last Name _____
Grade _____
School _____
City _____
RSM Branch _____

FINAL ROUND

Grades 3-4

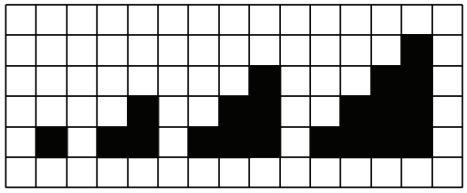
RSM FOUNDATION

7 A very long circus train is loaded with giraffes, clowns, and elephants. The first seven train cars are shown in the picture. If the pattern continues, how many beings will be riding in train car number 2016?



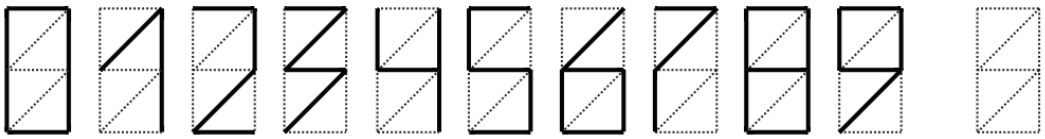
8 Ravi wrote (using white chalk) the number 123,456,789 on the board. Then he wrote (using yellow chalk) the number 20 above every odd digit on the board. Finally, he wrote (using yellow chalk) the number 16 below every white even digit on the board. How many even digits are on the board now?

9 In the diagram, each small square of the grid is one inch on a side. If the pattern continues, how many inches would the perimeter of the 504th shape be?

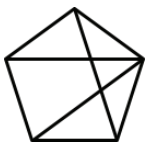


10 Mrs. Adder wrote some digits on the board. All of the digits were different. After she erased three of them, the remaining digits added up to 40. What is the product of the erased digits?

11 If the digits are all drawn by connecting the dots exactly as shown, a certain pair of the digits could fit upright within the same dotted rectangle without sharing any of the lines. Write the larger 2-digit number that uses both these digits.



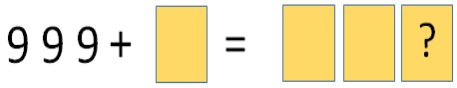
12 How many triangles of all sizes and positions are there in the diagram, including triangles that are made up of more than one shape?



Please fold over on line. Write answers on back.

2016 RSM Olympiad 3-4

1. In the puzzle below, each card hides a digit. What digit is hidden under the card with the question mark?



Answer: 9
Solution 1. Note that 999 is the largest 3-digit number. Therefore, if we add to it any 1-digit number except 0, the sum would have more than 3 digits. Thus, the only possibility is that we add 0. In this case the sum is 999, so the card with question mark hides digit 9.

Solution 2. One possibility for the statement partially hidden by cards is 999 + 0 = 999. In this case the card with the question mark hides digit 9. Since this possibility satisfies all the conditions of the problem, the answer is 9.

2. Eight kids are holding a total of 15 balloons. Some balloons are red, and the rest are blue. Nobody holds two or more balloons of the same color, and nobody shares a balloon. How many kids hold exactly one balloon each?
Answer: 1
Solution 1. There are only two colors of balloons and nobody holds two or more balloons of the same color. Therefore each kid holds at most two balloons. If each of the eight kids holds exactly two balloons (one red and one blue), we would have a total of 8×2 = 16 balloons. But they hold a total of 15 = 16 – 1 balloons, and nobody shares a balloon. This means that exactly one kid must hold just one balloon.

Solution 2. One possibility is the following: seven kids hold exactly two balloons each (one red and one blue), and one kid holds exactly one red balloon, for a total of 7×2 + 1 = 15 balloons. Since this possibility satisfies all the conditions of the problem, the answer is 1.

3. Grandma sent Jack a bag of candy. When Jack opened it, he found inside 8 large boxes of candy. Each of these large boxes had 6 smaller boxes of candy inside, and each of the smaller boxes had 10 candies. How many candy boxes of all sizes were in the bag?
Answer: 56
Solution. There were 8 large boxes of candy, with 6 smaller boxes per large box, for a total of 8×6 = 48 smaller boxes. Therefore there were 8 + 48 = 56 candy boxes of all sizes in the bag.

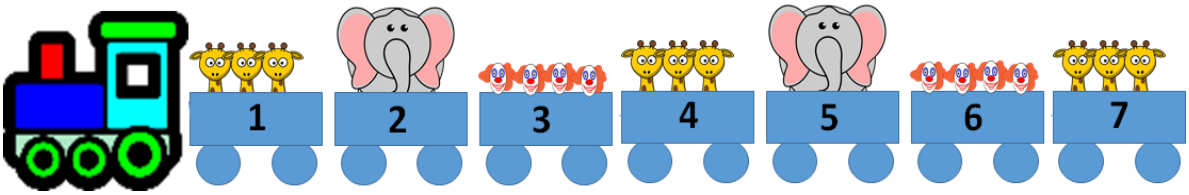
4. A paper rectangle is folded once to get a 2 cm-by-3 cm rectangle. What is the greatest possible perimeter (in centimeters) of the original rectangle? (The perimeter of a rectangle is the sum of the lengths of all of its sides.)
Answer: 16
Solution. Note that there are just two possibilities for the original rectangle. The first one is when one of its sides is 2 cm long, and the crease is along this side. In this case the longest possible adjacent side of the original rectangle is 2×3 = 6 cm long (twice the length of the folded side) and the greatest possible perimeter of the original rectangle is 2×(2 + 6) = 16 cm. The second possibility is when one of the sides of the original rectangle is 3 cm long, and the crease is along this side. In this case the longest possible

adjacent side of the original rectangle is 2×2= 4 cm long (twice the length of the folded side) and the greatest possible perimeter of the original rectangle is 2×(3 + 4) = 14 cm. Since 16 > 14, the answer is 16.

5. RSM opened a new branch for puppies and kittens in Pawville. When the principal counted ears and tails of all 30 students, he discovered there were twice as many kittens' ears as puppies' tails. How many kittens were at the RSM-Pawville branch (if every animal had the usual number of body parts)?
   Answer: 15
   Solution. Since kittens have two ears each, there were twice as many kittens' ears as kittens. Since puppies have one tail each, there were as many puppies' tails as puppies. So the number of kittens equals half the number of kittens' ears, and therefore the number of kittens equals the number of puppies' tails which equals the number of puppies. This means that the RSM-Pawville branch had the same number of puppies and kittens for a total of 30 students. Thus there were 15 (one half of 30) kittens at the RSM-Pawville branch.

6. Aurora made three paper triangles, four paper squares, and five paper octagons. Barbara made several paper pentagons. Aurora's shapes all together have as many sides as all Barbara's pentagons do. How many pentagons did Barbara make?
   Answer: 13
   Solution. Recall that a triangle has 3 sides, a square has 4 sides, a pentagon has 5 sides, and an octagon has 8 sides. Thus, Aurora's triangles have a total of 3×3 = 9 sides, her squares have a total of 4×4 = 16 sides, and her octagons have a total of 5×8 = 40 sides. Her shapes have a total of 9 + 16 + 40 = 65 sides. Barbara's pentagons all together have as many sides as all Aurora's shapes (65), so Barbara made 65÷5 = 13 pentagons.

7. A very long circus train is loaded with giraffes, clowns, and elephants. The first seven train cars are shown in the picture. If the pattern continues, how many beings will be riding in train car number 2016?



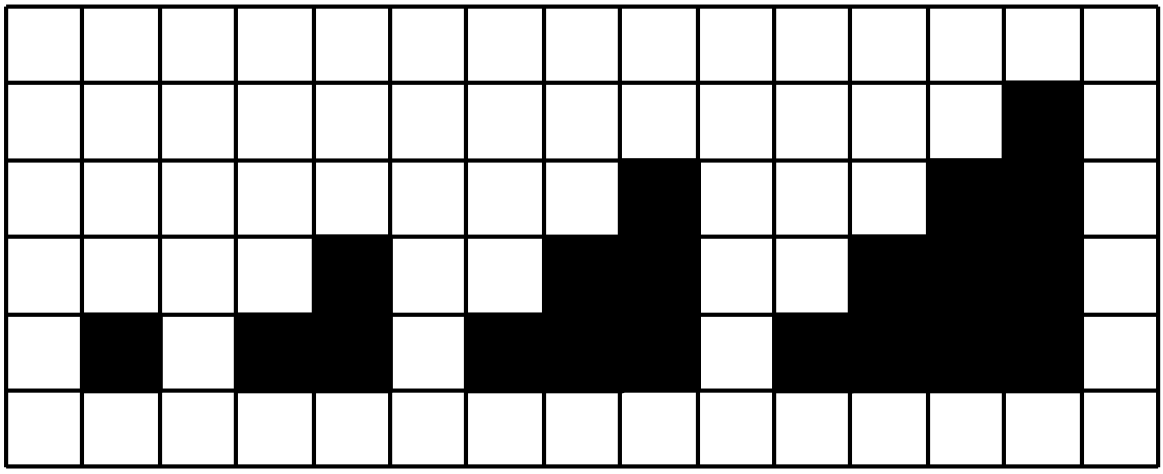
   Answer: 4
   Solution. The pattern repeats every three cars: three giraffes followed by an elephant followed by four clowns. Thus any car whose number is a multiple of 3 will have four clowns. Since 2016 = 672×3 is a multiple of 3, 4 beings (4 clowns) will be riding in train car number 2016.

8. Ravi wrote (using white chalk) the number 123,456,789 on the board. Then he wrote (using yellow chalk) the number 20 above every odd digit on the board. Finally, he wrote (using yellow chalk) the number 16 below every white even digit on the board. How many even digits are on the board now?
   Answer: 18

Solution 1. Ravi's initial number in white chalk contained 5 odd (1, 3, 5, 7, 9) and 4 even (2, 4, 6, 8) digits. For each of these 5 (white) odd digits, he wrote the number 20 in yellow above it. Since both digits 2 and 0 are even, Ravi wrote 10 yellow even digits (5 twos and 5 zeroes). For each of the 4 white even digits, he wrote the number 16 in yellow below it. Since 1 is odd and 6 is even, Ravi wrote 4 yellow odd digits (1s) and 4 more yellow even digits (6s). Thus the total number of even digits on the board now is 4 (white even digits from the original number) + 10 (yellow 2s and 0s) + 4 (yellow 6s) = 18.

Solution 2. Every white digit on the board is either even or odd. Ravi wrote the number 20 in yellow above every (white) odd digit on the board. Both digits 2 and 0 are even, so each white odd digit "owns" 2 even digits on the board. Then Ravi wrote the number 16 in yellow below every white even digit on the board. Only one of the digits 1 and 6 is even (6), so each white even digit also "owns" 2 even digits on the board (the one below it and itself). Thus, now there are twice as many even (white and yellow) digits on the board as white digits. Since Ravi wrote 9 white digits on the board, there are a total of $2\times9 = 18$ even digits on the board now.

9. In the diagram, each small square of the grid is one inch on a side. If the pattern continues, how many inches would the perimeter of the 504^{th} shape be?



Shape 1 Shape 2 Shape 3 Shape 4

Answer: 2016

Solution. For each shape, the length of its bottom side (in inches) is the same as the shape's number. The sum of the lengths of all of the shape's other horizontal sides is the same as the length of its bottom side. The length of the shape's right side (in inches) is the same as the shape's number. And the sum of the lengths of all of the shape's other vertical sides is the same as the length of its right side. Thus, for each shape its perimeter (in inches) is four times as large as the shape's number. Therefore, the perimeter of the 504^{th} shape would be $4\times504 = 2016$ inches.

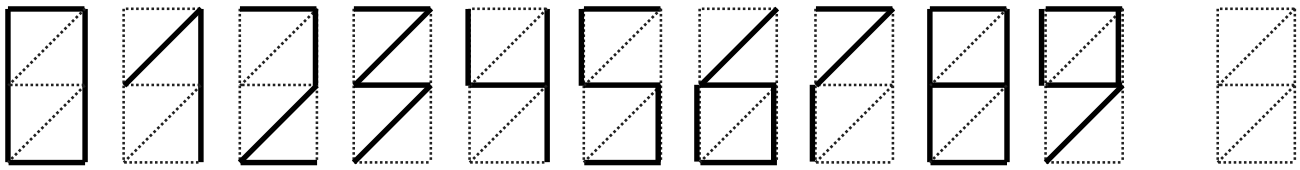
10. Mrs. Adder wrote some digits on the board. All of the digits were different. After she erased three of them, the remaining digits added up to 40. What is the product of the erased digits?

Answer: 0

Solution 1. Since the digits on the board were all different, and only ten different digits (from 0 to 9) exist, the sum of the digits before erasing must have been 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 or less. After erasing, the remaining digits added up to 40, therefore the sum of the three erased digits must have been 45 – 40 = 5 or less. Thus, one of the erased digits must be 0, otherwise the sum of the three different erased digits would be at least 1 + 2 + 3 = 6 which is greater than 5. Since one of the erased digits is 0, the product of the erased digits is 0 as well.

Solution 2. One possibility is the following: Mrs. Adder wrote all ten different digits (from 0 to 9) on the board, and then erased three digits 0, 2, and 3. In this case the remaining digits added up to 1 + 4 + 5 + 6 + 7 + 8 + 9 = 40, and the product of the erased digits is 0×2×3 = 0. Since this possibility satisfies all the conditions of the problem, the answer is 0.

11. If the digits are all drawn by connecting the dots exactly as shown, a certain pair of the digits could fit upright within the same dotted rectangle without sharing any of the lines. Write the larger 2-digit number that uses both these digits.



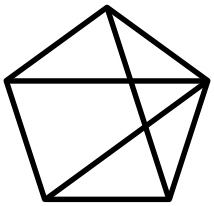
Answer: 74

Solution 1. By comparing 9 with each of the smaller digits, we see that no digit can fit in the same dotted rectangle with 9 without sharing any of the lines. By comparing 8 with each of the smaller digits, we see that 8 cannot be one of the digits either. By comparing 7 with each of the smaller digits, we find that 4 and 7 could fit upright within the same dotted rectangle without sharing any of the lines, so the pair is (4, 7), and the larger 2-digit number that uses both digits 4 and 7 is 74 (since 74 > 47).

Solution 2. Every digit drawn by connecting the dots exactly as shown has at least one line somewhere along the boundary of the dotted rectangle. Thus, if two digits could fit upright within the same dotted rectangle without sharing any of the lines, neither of these two digits is 0. Similarly, neither of them is 8. Every digit drawn by connecting the dots exactly as shown has a horizontal or a vertical line at the bottom half (which includes middle horizontal line) of the dotted rectangle. Thus, if two digits could fit upright within the same dotted rectangle without sharing any of the lines, neither of these two digits is 6. Similarly, neither of them is 9. Digit 5 contains all three possible horizontal lines. Other digits (except 1) contain at least one horizontal line each, and digits 1 and 5 share the bottom right vertical line. Thus, if two digits could fit upright within the same dotted rectangle without sharing any of the lines, neither of these two digits is 5. The remaining digits are 1, 2, 3, 4, and 7. Digit 3 contains both possible diagonal lines. Other remaining digits (except 4) contain at least one diagonal line each, and digits 3 and 4 share the middle horizontal line. Thus, if two digits could fit upright within the same dotted rectangle without sharing any of the lines, neither of these two digits is 3. The remaining digits are 1, 2, 4, and 7. But digits 1, 2, and 4 share the top right vertical line, so one of the digits in the pair must be 7. Digits 1 and 7 share the top diagonal line, and digits 2 and 7 share the top horizontal line. The only remaining possibility is digits 4 and 7. This pair

of digits indeed could fit upright within the same dotted rectangle without sharing any of the lines. There are just two 2-digit numbers each using both digits 4 and 7 (47 and 74), and the larger of them is 74.

12. How many triangles of all sizes and positions are there in the diagram, including triangles that are made up of more than one shape?



Answer: 11

Solution. The original shape, a pentagon, is divided into five small triangles and a quadrilateral that will be our building blocks. There are 5 triangles made up of exactly one building block each. There are 4 triangles made up of exactly two triangular building blocks each (take any two triangular building blocks that share a side), and 1 triangle made up of a triangular and a non-triangular building block. There is 1 triangle made up of exactly three (triangular) building blocks. And there is no triangle made up of four or more building blocks. Altogether there are $5 + 4 + 1 + 1 = 11$ triangles of all sizes and positions in the diagram.