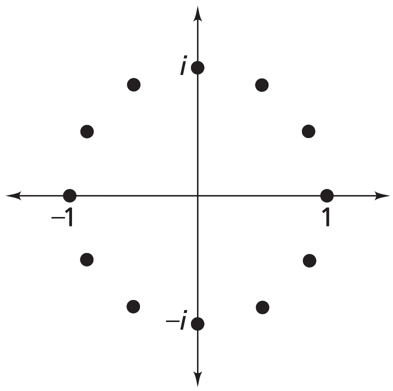
6. a. 1, 30° **b.** $n = 12$

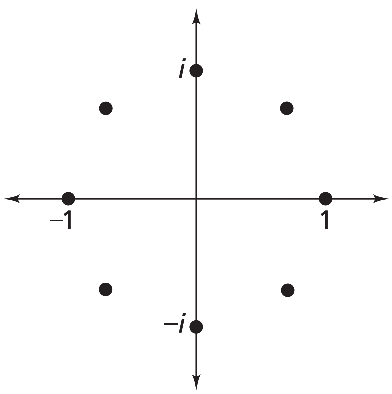
c.



7. a. The magnitude of each solution is 1. The directions going counterclockwise are 60°, 120°, 180°, 240°, 300°, and 360°.

b. $\pm 1, \pm i, \pm\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right), \pm\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

8.



Chapter 4

Lesson 4.2 Additional Practice

1. $x = 2, y = 0$

2. $x = 2, y = -2, z = -3$

3. $x = 0, y = 0, z = 0$

4. $x = 1, y = -4, z = 2, w = 3$

5. a. $2z + 2w = 3$
$2y + 2w = 1$
$2y + 2z = 0$

b. $-2z + 2w = 1$
$-2y + 2w = 3$
$2y + 2z = 0$

c. $y = -\frac{1}{2}, z = \frac{1}{2}, w = 1$

d. $x = 0, y = -\frac{1}{2}, z = \frac{1}{2}, w = 1$

6. $a = -2, b = 1, c = -3$

7. There is no solution to the system. The equations represent two lines that are parallel and therefore never intersect.

Lesson 4.3 Additional Practice

1. $\left(\begin{array}{cc|c} 1 & -2 & -8 \\ 2 & -1 & -7 \end{array}\right)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

2. $\left(\begin{array}{cc|c} 3 & 2 & 6 \\ -2 & 2 & 6 \end{array}\right)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

3. $\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 5 & -2 & 1 & 11 \\ 6 & -2 & -2 & 16 \end{array}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

4. $\left(\begin{array}{ccc|c} -1 & 6 & 1 & 18 \\ 1 & -3 & -2 & -16 \\ 7 & 5 & 1 & 0 \end{array}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$

5. 7, −3, and 11

6. $2x + 3y - 6z = 5$
$x - 3y - z = 0$
$6x + 2y - z = -2$

7. $\begin{pmatrix} -1 \\ 1 \\ 3 \\ 3 \end{pmatrix}$

8. a. The reduced form is

$$\left(\begin{array}{cc|c} 3 & 8 & 4 \\ \frac{3}{2} & 4 & 2 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 3 & 8 & 4 \\ 0 & 0 & 0 \end{array}\right).$$

The matrix on the right side represents this system.

$$3x + 8y = 4$$
$$0 = 0$$

Since $0 = 0$ is always true, the solutions to the system are all ordered pairs (x, y) on the line $3x + 8y = 4$. There are an infinite number of values that solve it.

In terms of *Algebra 1*, the two original equations are the same line and therefore have an infinite number of solutions.

b. The reduced form is

$$\left(\begin{array}{cc|c} -2 & 6 & 1 \\ 1 & -3 & 5 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 0 & 0 & 11 \\ 1 & -3 & 5 \end{array}\right).$$

The matrix on the right side represents this system.

$$0 = 11$$
$$x - 3y = 5$$

Since $0 = 11$ is always false, the system has no solutions.

In terms of *Algebra 1*, the two original equations are parallel lines. Since they do not intersect, there are no solutions.

Lesson 4.5 Additional Practice

1. a. 2×3
b. 3 **c.** 6 **d.** 5 **e.** 2

2. a. $\begin{pmatrix} 3 & 4 \\ 9 & 10 \\ 19 & 20 \end{pmatrix}$

b. $\begin{pmatrix} 3 & 4 & 5 & 6 \\ 9 & 10 & 11 & 12 \end{pmatrix}$

3. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \end{pmatrix}$

4. a. $\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$

b. $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

c. $\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

5. $z = -23x$

6. $z = 14x$

7. -4

8. a. \$938
b. The calculation is exactly parallel to Exercise 7. Let z = spending in dollars and y_1 = value in dollars of the pound, y_2 = value in dollars of the euro, and y_3 = value in dollars of the Swiss franc.
Then $z = 200y_1 + 320y_2 + 275y_3$ and $y_1 = 1.75$, $y_2 = 1.15$, and $y_3 = 0.80$. Solve for total spending in dollars by substituting y_1, y_2, and y_3 into the equation for z.

Lessons 4.6 and 4.7 Additional Practice

1. a. 44 **b.** undefined
c. $x + z$ **d.** 83
e. 0 **f.** 0

2. a. $(-2, -3)$ **b.** $(-a, b)$
c. $(1, 0, -1)$ **d.** $(0, 0, 0, 0)$

3. a. Let $A = (a_1, a_2, \dots, a_n)$ and $B = (b_1, b_2, \dots, b_n)$.
$A \cdot B = a_1b_1 + a_2b_2 + \cdots + a_nb_n$
$B \cdot A = b_1a_1 + b_2a_2 + \cdots + b_na_n$
Since real-number multiplication commutes, $a_1b_1 = b_1a_1$, $a_2b_2 = b_2a_2$, and so on. Therefore, $A \cdot B = B \cdot A$.

b. Let $A = (a_1, a_2, \cdots, a_n)$, $B = (b_1, b_2, \cdots, b_n)$, and $C = (c_1, c_2, \cdots, c_n)$.

$A \cdot (B + C)$
$= (a_1, a_2, \cdots, a_n) \cdot (b_1 + c_1, b_2 + c_2, \cdots, b_n + c_n)$
$= a_1(b_1 + c_1) + a_2(b_2 + c_2) + \cdots + a_n(b_n + c_n)$
$= (a_1b_1 + a_1c_1) + (a_2b_2 + a_2c_2) + \cdots + (a_nb_n + a_nc_n)$ [Distr. Prop. for real numbers]
$= A \cdot B + A \cdot C$

Similarly,
$(A + B) \cdot C$
$= (a_1 + b_1, a_2 + b_2, \cdots, a_n + b_n) \cdot (c_1, c_2, \cdots, c_n)$
$= (a_1 + b_1)c_1 + (a_2 + b_2)c_2 + \cdots + (a_n + b_n)c_n$
$= (a_1c_1 + b_1c_1) + (a_2c_2 + b_2c_2) + \cdots + (a_nc_n + b_nc_n)$
[Distr. Prop. for real numbers]
$= A \cdot C + B \cdot C$

4. a. $\begin{pmatrix} -1 & 0 \\ -3 & 2 \end{pmatrix}$ **b.** $\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$

c. $\begin{pmatrix} -6 & 6 \\ -6 & 6 \end{pmatrix}$ **d.** $\begin{pmatrix} -12 & 10 \end{pmatrix}$

e. $\begin{pmatrix} -15 & 15 \end{pmatrix}$

5. $AB = \begin{pmatrix} 15 \\ -12 \\ 1 \end{pmatrix}$, $AC = \begin{pmatrix} -9 \\ 10 \\ -9 \end{pmatrix}$,

$AD = \begin{pmatrix} 15 & -12 \\ -12 & 8 \\ 1 & 4 \end{pmatrix}$

6. a. 68 sixth grade students, 71 seventh grade students, 68 eighth grade students;

$$M\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 68 \\ 71 \\ 68 \end{pmatrix}$$

b. A: 47 students, B: 68 students, C: 64 students, D: 22 students, F: 6 students;
$(1 \;\; 1 \;\; 1)M = (47 \;\; 68 \;\; 64 \;\; 22 \;\; 6)$

c. 207 students; adding the results in part (a) will give the total enrollment. The same is true for the results in part (b).

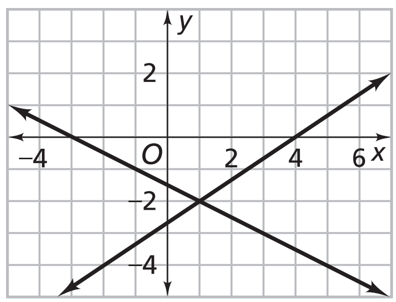
$68 + 71 + 68 = 207$ or
$47 + 68 + 64 + 22 + 6 = 207$.

In matrix form this is

$$((1 \;\; 1 \;\; 1)M)\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = (1 \;\; 1 \;\; 1)(M)\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

Lessons 4.8 and 4.9 Additional Practice

1. a. Graph the system of equations
$2x - 3y = 8$
$x + 2y = -3$.



The graphs intersect at the point $(1, -2)$, so the solution is $x = 1$ and $y = -2$.

b. Solving by Gaussian Elimination gives

$$\left(\begin{array}{cc|c} 2 & -3 & 8 \\ 1 & 2 & -3 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right).$$

The solution is $x = 1$ and $y = -2$.

c. The inverse is $M = \begin{pmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}$.

Solve by the inverse method.

$$\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix}\begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

The solution is $x = 1$ and $y = -2$.

2. $\begin{pmatrix} -2 & 1.5 \\ 1 & -0.5 \end{pmatrix}$ **3.** $\begin{pmatrix} -2 & 1.5 \\ 1 & -0.5 \end{pmatrix}$

4. $x_1 = 2, x_2 = 1, x_3 = 3$

5. a. $A = \begin{pmatrix} 2c \\ c \end{pmatrix}$ **b.** $A = \begin{pmatrix} 2c & 2d \\ c & d \end{pmatrix}$

6. Yes; by definition of a matrix sum, the ij entry of $A + A + A$ is $a_{ij} + a_{ij} + a_{ij}$. By real-number algebra, this is $3a_{ij}$. By definition of scalar multiplication, $3a_{ij}$ is the ij entry of $3A$. So $A + A + A = 3A$, since the two sides agree entry by entry. Alternately,

$$\begin{aligned} A + A + A &= 1A + 1A + 1A \\ &= (1 + 1 + 1)A \\ &= 3A, \end{aligned}$$

using the basic rules of matrices.

7. Yes; expand the left side of the equation.

$$\begin{aligned} &(X - I)^2 \\ &= (X - I)(X - I) && [\text{def. } (X - Y)^2] \\ &= X(X - I) - I(X - I) && [\text{Distr. Prop.}] \\ &= (XX - XI) + (-IX + II) && [\text{Distr. Prop.}] \\ &= X^2 - XI - IX + I^2 && [\text{def. } X^2] \\ &= X^2 - (XI + IX) + I^2 && [\text{Assoc. Prop.}] \\ &= X^2 - 2XI + I^2 && [I \text{ commutes.}] \end{aligned}$$

8. a. yes, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

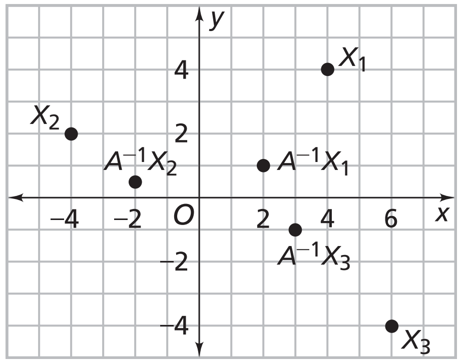
b. Any matrix of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ commutes under multiplication with $\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$.

Lesson 4.11 Additional Practice

1. a. The map dilates the plane by the factor 2 in the x-direction and the factor 4 in the y-direction.

b. The map dilates the plane by the factor $\frac{1}{2}$ in the x-direction and the factor $\frac{1}{4}$ in the y-direction;

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}.$$



2. a. A maps $(x, y) \mapsto (-x, -y)$. It rotates every point 180° counterclockwise around the origin.

b. B maps $(x, y) \mapsto (0, y)$. It projects every point horizontally onto the y-axis.

c. C maps $(x, y) \mapsto (0.25x, 0.25y)$. It scales every point by the factor 0.25.

3. a. image of L_1: $x = 1$, image of L_2: $x = 0$, image of L_3: $y = -2$, image of L_4: $y = 3$, image of L_5: $y = x$, image of L_6: $y = x + 1$

b. image of L_1: $x = 0$, image of L_2: $x = 0$, image of L_3: $(0, 2)$, image of L_4: $(0, -3)$, image of L_5: $x = 0$, image of L_6: $x = 0$

c. image of L_1–L_6: $y = x$

4. a. $\left(\frac{1}{3}, 1\right)$ **b.** $(-10, 0)$ **c.** $(8, -6)$

5. $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2 & 1 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

6. a. 360° rotation counterclockwise
b. $(x, y) \mapsto (-3x, -3y)$
c. $(x, y) \mapsto (x, -y)$
d. $(x, y) \mapsto (3x, -3y)$

Lessons 4.12 and 4.13 Additional Practice

1. Let $P(n) = \begin{cases} \begin{pmatrix} 200{,}000 \\ 200{,}000 \end{pmatrix} & \text{if } n = 0 \\ MP(n-1) & \text{if } n > 0 \end{cases}$,

where $M = \begin{array}{c|c} & \begin{array}{cc} N & S \end{array} \\ \begin{array}{c} N \\ S \end{array} & \begin{pmatrix} 0.96 & 0.06 \\ 0.04 & 0.94 \end{pmatrix} \end{array}$.

In the long run, $M^{\infty} = \begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}$, and

$P(\infty) = \begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}\begin{pmatrix} 200{,}000 \\ 200{,}000 \end{pmatrix}$
$= \begin{pmatrix} 240{,}000 \\ 160{,}000 \end{pmatrix}$. This means that the population distribution becomes stable at 240,000 people living north of downtown and 160,000 people living south of downtown. The people who live in each area may change, but the population will remain stable.

2. a. At the end of each week, of the trucks that start at location A, 60% return there, 30% are at B, and 10% are at C. Of the trucks that start at location B, 25% are at A, 45% return to B, and 30% are at C. Of the trucks that start at location C, 20% are at A, 15% are at B, and 65% return to C;

$M = \begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0.6 & 0.25 & 0.2 \\ 0.3 & 0.45 & 0.15 \\ 0.1 & 0.3 & 0.65 \end{pmatrix} \end{array}$.

b. Let $T(n) = \begin{pmatrix} a(n) \\ b(n) \\ c(n) \end{pmatrix} = M^n T(0)$ and

$T(0) = \begin{pmatrix} 50 \\ 50 \\ 50 \end{pmatrix}$. After two weeks,

$T(2) = \begin{pmatrix} 53.25 \\ 43.875 \\ 52.875 \end{pmatrix}$. After three weeks,

$T(3) = \begin{pmatrix} 53.5 \\ 43.65 \\ 52.85 \end{pmatrix}$. In the long run,

the distribution settles at $\begin{pmatrix} 53.63 \\ 43.63 \\ 52.73 \end{pmatrix}$.

3. a. Answers may vary. Check students' work.
b. $X = (312.5, 187.5)$

4. $G = \begin{array}{c|c} & \begin{array}{ccc} M & J & E \end{array} \\ \begin{array}{c} M \\ J \\ E \end{array} & \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix} \end{array}$

a. $\frac{1}{2}$

b. $\left(\frac{1}{2}\right)^6 = \frac{1}{64} \approx 0.016$

c. $\frac{1}{32} \approx 0.031$

5. a. $\frac{1}{8}$ **b.** $\frac{1}{24}$ **c.** $\frac{1}{96}$

6. a. -3 **b.** -24 **c.** 72
d. $-\frac{1}{3}$ **e.** $-\frac{1}{24}$

Chapter 5

Lessons 5.2 and 5.3 Additional Practice

1. a. 10,000 **b.** 60,466,176
c. $\frac{64}{125}$ **d.** 1