# Discovering <br> Geometry 

## Practice Your Skills with Answers

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## Introduction

The author and editors of Discovering Geometry: An Investigative Approach are aware of the importance of students developing geometry skills along with acquiring concepts through investigation. The student book includes many skill-based exercises. These Practice Your Skills worksheets provide problems similar to the introductory exercises in each lesson of Discovering Geometry. Like those exercises, these worksheets allow students to practice and reinforce the important procedures and skills developed in the lessons. Some of these problems provide non contextual skills practice. Others give students an opportunity to apply geometry concepts in fairly simple, straightforward contexts. Some are more complex problems that are broken down into small steps.

You might assign the Practice Your Skills worksheet for every lesson, or only for those lessons your students find particularly difficult. Or, you may wish to assign the worksheets on an individual basis, only to those students who need extra help. One worksheet has been provided for nearly every lesson. There are no worksheets for Chapter 0 , and the optional tessellation lessons have been combined into two worksheets. To save you the time and expense of copying pages, you can give students the inexpensive Practice Your Skills Student Workbook, which does not have answers. Though the copyright allows you to copy pages from Practice Your Skills with Answers for use with your students, the consumable Practice Your Skills Student Workbook should not be copied. Students, parents, and mentors can also download the student worksheets from flourishkh.com.

## Lesson 1.1•Building Blocks of Geometry

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-7, complete each statement. $\overline{P S}=3 \mathrm{~cm}$.


1. The midpoint of $\overline{P Q}$ is $\qquad$ .
2. $N Q=$ $\qquad$ .
3. Another name for $\overline{N S}$ is $\qquad$ .
4. $S$ is the $\qquad$ of $\overrightarrow{S Q}$.
5. $P$ is the midpoint of $\qquad$ .
6. $\overline{N S} \cong$ $\qquad$ .
7. Another name for $\overrightarrow{S N}$ is $\qquad$ .
8. Name all pairs of congruent segments in KLMN. Use the congruence symbol to write your answer.
9. $M(-4,8)$ is the midpoint of $\overline{D E}$. $D$ has coordinates $(6,1)$. Find the
 coordinates of $E$.

For Exercises 10 and 11, use a ruler to draw each figure. Label the figure and mark the congruent parts.
10. $\overline{A B}$ and $\overline{C D}$ with $M$ as the midpoint of both $\overline{A B}$ and $\overline{C D} . A B=6.4 \mathrm{~cm}$ and $C D=4.0 \mathrm{~cm} . A, B$, and $C$ are not collinear.
12. Sketch six points $A, B, C, D, E$, and $F$, no three of which are collinear. Name the lines defined by these points. How many lines are there?
11. $\overleftrightarrow{A B}$ and $\overline{C D}$. $C$ is the midpoint of $\overline{A B}$, with $A C=1.5 \mathrm{~cm} . D$, not on $\overrightarrow{A B}$, is the midpoint of $\overline{A E}$, with $A D=2 B C$.
13. In the figure below, $\{B, C, H, E\}$ is a set of four coplanar points. Name two other sets of four coplanar points. How many sets of four coplanar points are there?


Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-5, use the figure at right to complete each statement.

1. $A$ is the $\qquad$ of $\angle B A E$.
2. $\overrightarrow{A D}$ is the $\qquad$ of $\angle B A E$.
3. $\overrightarrow{A D}$ is a $\qquad$ of $\angle D A E$.
4. If $m \angle B A C=42^{\circ}$, then $m \angle C A E=$ $\qquad$ -.
5. $\angle D A B \cong$ $\qquad$ .

For Exercises 6-9, use your protractor to find the measure of each angle to the nearest degree.
6. $m \angle P R O$
7. $m \angle O R T$
8. $m \angle O$
9. $m \angle R T O$


For Exercises 10-12, use your protractor to draw and then label each angle with the given measure.
10. $m \angle M N O=15^{\circ}$
11. $m \angle R I G=90^{\circ}$
12. $m \angle z=160^{\circ}$

For Exercises 13-15, find the measure of the angle formed by the hands at each time.
13. 3:00
14. 4:00
15. 3:30

For Exercises 16 and 17, mark each figure with all the given information.

16. $m \angle A D B=90^{\circ}, A D=B D, \angle D A B \cong \angle D B A$

17. $m \angle R P Q=90^{\circ}, Q R=T Z, R T=Q Z, \angle Q \cong \angle T$


## Lesson 1.3•What's a Widget?

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-9, match each term with one of the items (a to i) below.
a.

b.

c.

d.

e.

f.

g.

h.

i.


1. $\qquad$ Vertical angles
2. $\qquad$ Obtuse angle
3. $\qquad$ Right angle
4. $\qquad$ Complementary angles
5. $\qquad$ Congruent angles
6. $\qquad$ Linear pair of angles
7. $\qquad$ Bisected angle
8. $\qquad$ Perpendicular lines
9. $\qquad$ Congruent segments
10. If $m \angle P=13^{\circ}, m \angle Q=77^{\circ}$, and $\angle Q$ and $\angle R$ are complementary, what can you conclude about $\angle P$ and $\angle R$ ? Explain your reasoning.

For Exercises 11-13, sketch, label, and mark a figure showing each property.
11. $\ell_{1} \| \ell_{2}, \ell_{2} \perp \ell_{3}$
12. $\overline{P Q} \perp \overrightarrow{P R}$
13. $\angle B A C \cong \angle X A Y, C X=B C$

## Lesson 1.4•Polygons

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-8, complete the table.

| Polygon name | Number of sides | Number of diagonals |
| :--- | :---: | :---: |
| 1. Triangle |  |  |
| 2. |  | 2 |
| 3. | 5 |  |
| 4. Hexagon |  |  |
| 5. Heptagon | 8 |  |
| 6. |  | 35 |
| 7. | 12 |  |
| 8. |  |  |

For Exercises 9 and 10, sketch and label each figure. Mark the congruences.
9. Concave pentagon $P E N T A$, with external diagonal $\overline{E T}$, and $\overline{T A} \cong \overline{P E}$.
10. Equilateral quadrilateral $Q U A D$, with $\angle Q \not \equiv \angle U$.

For Exercises 11-14, sketch and use hexagon $A B C D E F$.
11. Name the diagonals from $A$.
12. Name a pair of consecutive sides.
13. Name a pair of consecutive angles.
14. Name a pair of non-intersecting diagonals.

For Exercises 15-18, use the figures at right.
$M N O P Q \cong R S T U V$
15. $m \angle N=$ $\qquad$
16. $V R=$ $\qquad$
17. $m \angle P=$ $\qquad$
18. $O N=$ $\qquad$

19. The perimeter of a regular pentagon is 31 cm . Find the length of each side.

## Lesson 1.5•Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-5, use the figure at right. Name a pair of

1. Parallel segments
2. Perpendicular segments
3. Congruent segments
4. Supplementary angles
5. Linear angles


For Exercises 6 and 7, sketch, label, and mark each figure.
6. Isosceles obtuse triangle $T R I$ with vertex angle $T$.
7. Scalene right triangle $S C A$ with midpoints $L, M$, and $N$ on $\overline{S C}, \overline{C A}$, and $\overline{S A}$, respectively.

For Exercises 8 and 9, use your geometry tools to draw each figure.
8. Acute isosceles triangle $A C D$ with vertex angle $A$ measuring $40^{\circ}$.
9. Scalene right triangle $R G H$.

For Exercises 10-12, use the graph at right.
10. Locate $F$ so that $\triangle A B F$ is a right triangle.
11. Locate $D$ so that $\triangle A B D$ is an isosceles triangle.
12. Locate $G$ so that $\triangle A B G$ is scalene and not a right triangle.


Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-6, sketch, label, and mark each figure.

1. Parallelogram PGRA
2. Square $S Q R E$
3. Rhombus $R H O M$ with acute $\angle H$.
4. Trapezoid $T R A P$ with $\overline{T R} \| \overline{A P}, \overline{R E} \perp \overline{P A}$, and $P, E$, and $A$ collinear.
5. Kite KITE with $E K=K I$ and obtuse $\angle K$.
6. Rectangle $R A N G$ with perimeter $2 a+4 b$

For Exercises 7-10, name each polygon in the figure. Assume that the grid is square.
7. Square
9. Rhombus
8. Parallelogram
10. Kite


For Exercises 11-13, use the graph at right.
11. Locate $D$ so that $A B C D$ is a rectangle.
12. Locate $E$ so that $A B C E$ is a trapezoid.
13. Locate $G$ so that points $A, B, C$, and $G$ determine a parallelogram that is not a rectangle.


## Lesson 1.7•Circles

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises $1-4$, use the figure at right.

1. $m \widehat{Q R}=$ $\qquad$ 2. $m \widehat{P R}=$ $\qquad$
2. $m \widehat{P Q R}=$ $\qquad$
3. $m \widehat{Q P R}=$ $\qquad$
4. Sketch a circle with an inscribed pentagon.
5. A circle with center $(3,2)$ goes through $(-2,2)$. Give the coordinates of three other points on the circle.

6. Use a compass to construct a circle. Label the center $P$. Sketch two parallel tangents. Connect the points of tangency. What do you notice about the chord?

7. Sketch a circle with a circumscribed quadrilateral.
8. Use a compass, protractor, and straightedge to draw circle $O$ with diameter $\overline{A B}$; radius $\overline{O C}$ with $\overline{O C} \perp \overline{A B} ; \overline{O D}$, the angle bisector of $\angle A O C$, with $D$ on the circle; chords $\overline{A C}$ and $\overline{B C}$; and a tangent at $D$.
9. Use your compass and protractor to make an arc with measure $50^{\circ}$, an arc with measure $180^{\circ}$, and an arc with measure $290^{\circ}$. Label each arc with its measure.
10. Use your compass to construct two circles with different radii that intersect in two points. Label the centers $P$ and $Q$ and the points of intersection $A$ and $B$. Construct quadrilateral $P A Q B$. What type of quadrilateral is it?

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1 and 2, draw each figure.

1. A prism with a rectangular base.
2. A cylinder with base diameter greater than height.

For Exercises 3 and 4, sketch the three-dimensional figure formed by folding each net into a solid. Name the solid.
3.

4.


For Exercises 5 and 6, sketch the section formed when each solid is sliced by the plane as shown.
5.

6.

7. The prism below is built with $1-\mathrm{cm}$ cubes. How many cubes are completely hidden from sight, as seen from this angle?

8. Find the lengths of $x$ and $y$.


## Lesson 1.9•A Picture Is Worth a Thousand Words

Name $\qquad$ Period $\qquad$ Date $\qquad$

Read and reread each problem carefully, determining what information you are given and what it is that you trying to find.

1. A pair of parallel interstate gas and power lines run 10 meters apart and are equally distant from relay station $A$. The power company needs to locate a gas-monitoring point on one of the lines exactly 12 meters from relay station $A$. Draw a diagram showing the locus of possible locations.
2. The six members of the Senica High School math club are having a group photo taken for the yearbook. The photographer has asked the club to submit the height of each member so that he can quickly arrange them in order. The math club sent him the following information. Anica is 4 inches taller than Bruce. Charles is the same height as Ellen but an inch taller than Anica. Fred is midway between Bruce and Dora. Dora is 2 inches taller than Anica. Help out the photographer by arranging the club members in order from tallest to shortest.
3. Create a Venn diagram showing the relationships among triangles, acute triangles, isosceles triangles, and scalene triangles.
4. Sketch a possible net for each solid.
a.

b.

c.


## Lesson 2.1•Inductive Reasoning

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-7, use inductive reasoning to find the next two terms in each sequence.

1. $4,8,12,16$, $\qquad$ , $\qquad$
2. $400,200,100,50,25$, $\qquad$ ,
3. $\frac{1}{8}, \frac{2}{7}, \frac{1}{2}, \frac{4}{5}$, $\qquad$
$\qquad$
4. $-5,3,-2,1,-1,0$, $\qquad$ , $\qquad$
5. $360,180,120,90$, $\qquad$ ,
6. $1,3,9,27,81$, $\qquad$ -
7. $1,5,14,30,55$, $\qquad$ ,
For Exercises 8-10, use inductive reasoning to draw the next two shapes in each picture pattern.
8. 


9.



10.




For Exercises 11-13, use inductive reasoning to test each conjecture.
Decide if the conjecture seems true or false. If it seems false, give a counterexample.
11. The square of a number is larger than the number.
12. Every multiple of 11 is a "palindrome," that is, a number that reads the same forward and backward.
13. The difference of two consecutive square numbers is an odd number.

## Lesson 2.2 • Finding the nth Term

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-4, tell whether the rule is a linear function.
1.

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | ---: | ---: | ---: | ---: |
| $\boldsymbol{f}(\boldsymbol{n})$ | 8 | 15 | 22 | 29 | 36 |

2. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{g}(\boldsymbol{n})$ | 14 | 11 | 8 | 5 | 2 |

3. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{h}(\boldsymbol{n})$ | -9 | -6 | -2 | 3 | 9 |

4. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{j}(\boldsymbol{n})$ | $-\frac{3}{2}$ | -1 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

For Exercises 5 and 6, complete each table.
5.

| $n$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{f}(\boldsymbol{n})=\mathbf{7 n}-\mathbf{1 2}$ |  |  |  |  |  |

6. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{g}(\boldsymbol{n})=-\mathbf{8} \boldsymbol{n}-\mathbf{2}$ |  |  |  |  |  |

For Exercises 7-9, find the function rule for each sequence. Then find the 50th term in the sequence.
7.

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 | 6 | $\ldots$ | $n$ | $\cdots$ | 50 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| $\boldsymbol{f}(\boldsymbol{n})$ | 9 | 13 | 17 | 21 | 25 | 29 | $\ldots$ |  | $\cdots$ |  |

8. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 | 6 | $\ldots$ | $n$ | $\ldots$ | 50 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{g}(\boldsymbol{n})$ | 6 | 1 | -4 | -9 | -14 | -19 | $\ldots$ |  | $\ldots$ |  |

9. 

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 | 6 | $\ldots$ | $n$ | $\cdots$ | 50 |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| $\boldsymbol{h}(\boldsymbol{n})$ | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 | $\ldots$ |  | $\ldots$ |  |

10. Use the figures to complete the table.


| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 | $\cdots$ | $n$ | $\ldots$ | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number <br> of triangles | 1 | 5 | 9 |  |  | $\ldots$ |  | $\ldots$ |  |

11. Use the figures above to complete the table. Assume that the area of the first figure is 1 square unit.

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 | $\ldots$ | $n$ | $\ldots$ | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Area of <br> figure | 1 | 4 | 16 |  |  | $\ldots$ |  | $\ldots$ |  |

## Lesson 2.3•Mathematical Modeling

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Draw the next figure in this pattern.
a. How many small squares will there be in the 10th figure?
b. How many in the 25th figure?

c. What is the general function rule for this pattern?
2. If you toss a coin, you will get a head or a tail. Copy and complete the geometric model to show all possible results of three consecutive tosses.
a. How many sequences of results are possible?
b. How many sequences have exactly one tail?

c. Assuming a head or a tail is equally likely, what is the probability of getting exactly one tail in three tosses?
3. If there are 12 people sitting at a round table, how many different pairs of people can have conversations during dinner, assuming they can all talk to each other? What geometric figure can you use to model this situation?
4. Tournament games and results are often displayed using a geometric model. Two examples are shown below. Sketch a geometric model for a tournament involving 5 teams and a tournament involving 6 teams. Each team must have the same chance to win. Try to have as few games as possible in each tournament. Show the total number of games in each tournament. Name the teams $a, b, c \ldots$ and number the games $1,2,3 \ldots$


3 teams, 3 games (round robin)


4 teams, 3 games
(single elimination)

## Lesson 2.4 • Deductive Reasoning

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. $\triangle A B C$ is equilateral. Is $\triangle A B D$ equilateral? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?

2. $\angle A$ and $\angle D$ are complementary. $\angle A$ and $\angle E$ are supplementary. What can you conclude about $\angle D$ and $\angle E$ ? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?
3. Which figures in the last group are whatnots? What type of reasoning, inductive or deductive, do you use when solving this problem?



$\square$
a.

b.

c.



Whatnots


Not whatnots
d.

e.


Which are whatnots?
4. Solve each equation for $x$. Give a reason for each step in the process. What type of reasoning, inductive or deductive, do you use when solving these problems?
a. $4 x+3(2-x)=8-2 x$
b. $\frac{19-2(3 x-1)}{5}=x+2$
5. A sequence begins $-4,1,6,11 \ldots$
a. Give the next two terms in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?
b. Find a rule that generates the sequence. Then give the 50th term in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?

## Lesson 2.5•Angle Relationships

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-6, find each lettered angle measure without using a protractor.
1.

2.

3.

4.

5.

6.


For Exercises 7-10, tell whether each statement is always (A), sometimes $(\mathrm{S})$, or never $(\mathrm{N})$ true.
7. $\qquad$ The sum of the measures of two acute angles equals the measure of an obtuse angle.
8. $\qquad$ If $\angle X A Y$ and $\angle P A Q$ are vertical angles, then either $X, A$, and $P$ or $X, A$, and $Q$ are collinear.
9. $\qquad$ If two angles form a linear pair, then they are complementary.
10. $\qquad$ If a statement is true, then its converse is true.

For Exercises 11-15, fill in each blank to make a true statement.
11. If one angle of a linear pair is obtuse, then the other is $\qquad$ .
12. If $\angle A \cong \angle B$ and the supplement of $\angle B$ has measure $22^{\circ}$, then $m \angle A=$ $\qquad$ .
13. If $\angle P$ is a right angle and $\angle P$ and $\angle Q$ form a linear pair, then $m \angle Q$ is $\qquad$ .
14. If $\angle S$ and $\angle T$ are complementary and $\angle T$ and $\angle U$ are supplementary, then $\angle U$ is $\mathrm{a}(\mathrm{n})$ $\qquad$ angle.
15. Switching the "if" and "then" parts of a statement changes the statement to its $\qquad$ -

## Lesson 2.6•Special Angles on Parallel Lines

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-3, use your conjectures to find each angle measure.
1.

2.

3.


For Exercises 4-6, use your conjectures to determine whether $\ell_{1} \| \ell_{2}$, and explain why. If not enough information is given, write "cannot be determined."
4.

5.

6.

7. Find each angle measure.

8. Find $x$.

9. Find $x$ and $y$.


## Lesson 3.1 • Duplicating Segments and Angles

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, use the segments and angles below. Complete the constructions on a separate piece of paper.


1. Using only a compass and straightedge, duplicate each segment and angle. There is an arc in each angle to help you.
2. Construct a line segment with length $3 P Q-2 R S$.
3. Duplicate the two angles so that the angles have the same vertex and share a common side, and the nonshared side of one angle falls inside the other angle. Then use a protractor to measure the three angles you created. Write an equation relating their measures.
4. Use a compass and straightedge to construct an isosceles triangle with two sides congruent to $\overline{A B}$ and base congruent to $\overline{C D}$.

5. Repeat Exercise 4 with patty paper and a straightedge.
6. Construct an equilateral triangle with sides congruent to $\overline{C D}$.


## Lesson 3.2•Constructing Perpendicular Bisectors

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-6, construct the figures on a separate sheet of paper using only a compass and a straightedge.

1. Draw a segment and construct its perpendicular bisector.
2. Construct two congruent segments that are the perpendicular bisectors of each other. Form a quadrilateral by connecting the four endpoints. What type of quadrilateral does this seem to be?
3. Duplicate $\overline{A B}$. Then construct a segment with length $\frac{5}{4} A B$.

4. Draw a segment; label it $\overline{C M} \cdot \overline{C M}$ is a median of $\triangle A B C$. Construct $\triangle A B C$. Is $\triangle A B C$ unique? If not, construct a different triangle, $\triangle A^{\prime} B^{\prime} C$, also having $\overline{C M}$ as a median.
5. Draw a segment; label it $\overline{P Q} \cdot \overline{P Q}$ is a midsegment of $\triangle A B C$. Construct $\triangle A B C$. Is $\triangle A B C$ unique? If not, construct a different triangle, $\triangle A^{\prime} B^{\prime} C^{\prime}$, also having $\overline{P Q}$ as a midsegment.
6. Construct a right triangle. Label it $\triangle A B C$ with right angle $B$. Construct median $\overline{B D}$. Compare $B D, A D$, and $C D$.
7. Complete each statement as fully as possible.
a. $L$ is equidistant from $\qquad$ .
b. $M$ is equidistant from $\qquad$ .
c. $N$ is equidistant from $\qquad$ .
d. $O$ is equidistant from $\qquad$ .


## Lesson 3.3 • Constructing Perpendiculars to a Line

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises $1-5$, decide whether each statement is true or false. If the statement is false, explain why or give a counterexample.

1. In a triangle, an altitude is shorter than either side from the same vertex.
2. In a triangle, an altitude is shorter than the median from the same vertex.
3. In a triangle, if a perpendicular bisector of a side and an altitude coincide, then the triangle is isosceles.
4. Exactly one altitude lies outside a triangle.
5. The intersection of the perpendicular bisectors of the sides lies inside the triangle.

For Exercises 6 and 7, use patty paper. Attach your patty paper to your worksheet.
6. Construct a right triangle. Construct the altitude from the right angle to the opposite side.
7. Mark two points, $P$ and $Q$. Fold the paper to construct square $P Q R S$.

Use your compass and straightedge and the definition of distance to complete Exercises 8 and 9 on a separate sheet of paper.
8. Construct a rectangle with sides equal in length to $\overline{A B}$ and $\overline{C D}$.

9. Construct a large equilateral triangle. Let $P$ be any point inside the triangle. Construct $\overline{W X}$ equal in length to the sum of the distances from $P$ to each of the sides. Let $Q$ be any other point inside the triangle. Construct $\overline{Y Z}$ equal in length to the sum of the distances from $Q$ to each side. Compare $W X$ and $Y Z$.

## Lesson 3.4•Constructing Angle Bisectors

Name $\qquad$
$\qquad$ Date $\qquad$

1. Complete each statement as fully as possible.
a. $M$ is equidistant from $\qquad$ -.
b. $P$ is equidistant from $\qquad$ -
c. $Q$ is equidistant from $\qquad$ .
d. $R$ is equidistant from $\qquad$ .

2. If the converse of the Angle Bisector Conjecture is true, what can you conclude about this figure?

3. If $B E$ bisects $\angle A B D$, find $x$ and $m \angle A B E$.

4. Draw an obtuse angle. Use a compass and straightedge to construct the angle bisector. Draw another obtuse angle and fold to construct the angle bisector.
5. Draw a large triangle on patty paper. Fold to construct the three angle bisectors. What do you notice?

For Exercises 6 and 7, construct a figure with the given specifications using a straightedge and compass or patty paper. Use additional sheets of paper to show your work.
6. Using only your compass and straightedge, construct an isosceles right triangle.
7. Construct right triangle $R G H$ with right angle $R$. Construct median $\overline{R M}$, perpendicular $\overline{M N}$ from $M$ to $\overline{R G}$, and perpendicular $\overline{M O}$ from $M$ to $\overline{R H}$. Compare $R N$ and $G N$, and compare $R O$ and $H O$.

## Lesson 3.5•Constructing Parallel Lines

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-6, construct a figure with the given specifications using a straightedge and compass or patty paper. Use additional sheets of paper to show your work.

1. Draw a line and a point not on the line. Use a compass and straightedge to construct a line through the given point parallel to the given line.
2. Repeat Exercise 1, but draw the line and point on patty paper and fold to construct the parallel line.
3. Use a compass and straightedge to construct a parallelogram.
4. Use patty paper and a straightedge to construct an isosceles trapezoid.
5. Construct a rhombus with sides equal in length to $\overline{A B}$ and having an angle congruent to $\angle P$.

6. Construct trapezoid ZOID with $\overline{Z O}$ and $\overline{I D}$ as nonparallel sides and $A B$ as the distance between the parallel sides.


## Lesson 3.6•Construction Problems

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises $1-5$, construct a figure with the given specifications using either a compass and straightedge or patty paper. Use additional sheets of paper to show your work.

1. Construct kite KITE using these parts.

2. Construct a rectangle with perimeter the length of this segment.
3. Construct a rectangle with this segment as its diagonal.

4. Draw obtuse $\triangle O B T$. Construct and label the three altitudes $\overline{O U}, \overline{B S}$, and $\overline{T E}$.
5. Construct a triangle congruent to $\triangle A B C$. Describe your steps.


In Exercises 6-8, construct a triangle using the given parts. Then, if possible, construct a different (noncongruent) triangle using the same parts.
6.

7.

8.



## Lesson 3.7•Constructing Points of Concurrency

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1 and 2, make a sketch and explain how to find the answer.

1. A circular revolving sprinkler needs to be set up to water every part of a triangular garden. Where should the sprinkler be located so that it reaches all of the garden, but doesn't spray farther than necessary?
2. You need to supply electric power to three transformers, one on each of three roads enclosing a large triangular tract of land. Each transformer should be the same distance from the power-generation plant and as close to the plant as possible. Where should you build the power plant, and where should you locate each transformer?

For Exercises 3-5, construct a figure with the given specifications using a compass and straightedge. Use additional sheets of paper to show your work.
3. Draw an obtuse triangle. Construct the inscribed and the circumscribed circles.
4. Construct an equilateral triangle. Construct the inscribed and the circumscribed circles. How does this construction differ from Exercise 3?
5. Construct two obtuse, two acute, and two right triangles. Locate the circumcenter of each triangle. Make a conjecture about the relationship between the location of the circumcenter and the measure of the angles.

## Lesson 3.8 • The Centroid

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-3, use additional sheets of paper to show your work.

1. Draw a large acute triangle. Construct the centroid.
2. Construct a regular hexagon and locate its center of gravity.
3. Use a ruler and compass to find the center of gravity of a sheet-metal triangle with sides measuring $6 \mathrm{~cm}, 8 \mathrm{~cm}$, and 10 cm . How far is the center from each vertex, to the nearest tenth of a centimeter?
4. $\triangle A B C$ has vertices $A(9,12), B(-3,2)$, and $C(3,-2)$. Find the coordinates of the centroid.

5. $P L=24, Q C=10$, and $K C=7$. Find $P C, C L, Q M$, and $C R$.

6. Identify each statement as describing the incenter, circumcenter, orthocenter, or centroid.
a. $\qquad$ The point equally distant from the three sides of a triangle.
b. $\qquad$ The center of gravity of a thin metal triangle.
c. $\qquad$ The point equidistant from the three vertices.
d. $\qquad$ The intersection of the perpendicular bisectors of the sides of a triangle.
e. $\qquad$ The intersection of the altitudes of a triangle.
f. $\qquad$ The intersection of the angle bisectors of a triangle.
g. $\qquad$ The intersection of the medians of a triangle.

## Lesson 4.1• Triangle Sum Conjecture

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-9, determine the angle measures.

1. $p=$ $\qquad$ , $q=$ $\qquad$

2. $x=$ $\qquad$ , $y=$ $\qquad$
3. $a=$ $\qquad$ $b=$ $\qquad$

4. $r=$ $\qquad$ , $s=$ $\qquad$
$t=$ $\qquad$

5. $x=$ $\qquad$ , $y=$ $\qquad$
6. $y=$ $\qquad$

7. $s=$

$\qquad$
8. $m=$ $\qquad$

9. $m \angle P=$ $\qquad$

10. Find the measure of $\angle Q P T$.

11. Use the diagram to explain why $\angle A$ and $\angle B$ are complementary.

12. Find the sum of the measures of the marked angles.

13. Use the diagram to explain why $m \angle A+m \angle B=m \angle C+m \angle D$.


## Lesson 4.2•Properties of Isosceles Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the angle measures.

## 1. $m \angle T=$ <br> $\qquad$


2. $m \angle G=$ $\qquad$


In Exercises 4-6, find the measures.
4. $m \angle A=$ $\qquad$ , perimeter of $\triangle A B C=$ $\qquad$
5. The perimeter of $\triangle L M O$
is $536 \mathrm{~m} . L M=$ $\qquad$ , $m \angle M=$
$\qquad$

3. $x=$ $\qquad$

6. The perimeter of $\triangle Q R S$ is
$344 \mathrm{~cm} . m \angle Q=$ $\qquad$ —, QR = $\qquad$

7. a. Name the angle(s) congruent to $\angle D A B$.
b. Name the angle(s) congruent to $\angle A D B$.

c. What can you conclude about $\overline{A D}$ and $\overline{B C}$ ? Why?
8. $x=$ $\qquad$ , $y=$ $\qquad$

9. $P R=Q R$ and $Q S=R S$.

If $m \angle R S Q=120^{\circ}$, what is $m \angle Q P R$ ?

10. Use the diagram to explain why $\triangle P Q R$ is isosceles.


## Lesson 4.3•Triangle Inequalities

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1 and 2, determine whether it is possible to draw a triangle with sides of the given measures. If it is possible, write yes. If it is not possible, write no and make a sketch demonstrating why it is not possible.

1. $16 \mathrm{~cm}, 30 \mathrm{~cm}, 45 \mathrm{~cm}$
2. $9 \mathrm{~km}, 17 \mathrm{~km}, 28 \mathrm{~km}$
3. If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?

In Exercises 4-6, arrange the unknown measures in order from greatest to least.
4.

5.

6.

7. $x=$ $\qquad$

8. $x=$ $\qquad$
9. What's wrong with this picture?

10. Explain why $\triangle P Q S$ is isosceles.


In Exercises 11 and 12, use a compass and straightedge to construct a triangle with the given sides. If it is not possible, explain why not.
11.

12.


## Lesson 4.4•Are There Congruence Shortcuts?

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, name the conjecture that leads to each congruence.

1. $\triangle P A T \cong \triangle I M T$

2. $\triangle S I D \cong \triangle J A N$

3. $\overline{T S}$ bisects $\overline{M A}, \overline{M T} \cong \overline{A T}$, and $\triangle M S T \cong \triangle A S T$


In Exercises 4-9, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write "cannot be determined" and redraw the triangles so that they are clearly not congruent.
4. $M$ is the midpoint of $\overline{A B}$ and $\overline{P Q}$.
$\triangle A P M \cong \triangle$ $\qquad$

5. KITE is a kite with $K I=T I$.
$\triangle K I E \cong \triangle$

6. $\triangle A B C \cong$ $\qquad$
7. $\triangle M O N \cong$ $\qquad$
8. $\triangle S Q R \cong$

9. $\triangle T O P \cong$ $\qquad$



In Exercises 10-12, use a compass and a straightedge or patty paper and a straightedge to construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts. If it is not possible, explain why not.

12.


## Lesson 4.5 • Are There Other Congruence Shortcuts?

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, name a triangle congruent to the given triangle and state the congruence conjecture. If you cannot show any triangles to be congruent from the information given, write "cannot be determined" and explain why.

1. $\triangle P I T \cong \triangle$

2. $\triangle X V W \cong \triangle$

3. $\triangle A C N \cong \triangle$ $\qquad$
4. $\triangle E C D \cong \triangle$ $\qquad$

5. $\overline{P S}$ is the angle bisector of $\angle Q P R$.
$\triangle P Q S \cong \triangle$ $\qquad$


6. $E F G H$ is a parallelogram.
$G Q=E Q$.
$\triangle E Q L \cong \triangle$ $\qquad$

7. The perimeter of $\triangle Q R S$ is 350 cm .

Is $\triangle Q R S \cong \triangle M O L$ ? Explain.


8. The perimeter of $\triangle T U V$ is 95 cm .

Is $\triangle T U V \cong \triangle W X V$ ? Explain.


In Exercises 9 and 10, construct a triangle with the given parts. Then, if possible, construct a different (noncongruent) triangle with the same parts.
If it is not possible, explain why not.
9.

10.


## Lesson 4.6•Corresponding Parts of Congruent Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Give the shorthand name for each of the four triangle congruence conjectures.

In Exercises 2-5, use the figure at right to explain why each congruence is true. $W X Y Z$ is a parallelogram.
2. $\angle W X Z \cong \angle Y Z X$
3. $\angle W Z X \cong \angle Y X Z$

4. $\triangle W Z X \cong \triangle Y X Z$
5. $\angle W \cong \angle Y$

For Exercises 6 and 7, mark the figures with the given information. To demonstrate whether the segments or the angles indicated are congruent, determine that two triangles are congruent. Then state which conjecture proves them congruent.
6. $M$ is the midpoint of $\overline{W X}$ and
$\overline{Y Z}$. Is $\overline{Y W} \cong \overline{Z X}$ ? Why?

7. $\triangle A B C$ is isosceles and $\overline{C D}$ is the bisector of the vertex angle. Is $\overline{A D} \cong \overline{B D}$ ? Why?


In Exercises 8 and 9, use the figure at right to write a paragraph proof for each statement.
8. $\overline{D E} \cong \overline{C F}$
9. $\overline{E C} \cong \overline{F D}$

10. $T R A P$ is an isosceles trapezoid with $T P=R A$ and $\angle P T R \cong \angle A R T$. Write a paragraph proof explaining why $\overline{T A} \cong \overline{R P}$.


## Lesson 4.7 • Flowchart Thinking

Name $\qquad$ Period $\qquad$ Date $\qquad$

Complete the flowchart for each proof.

1. Given: $\overline{P Q} \| \overline{S R}$ and $\overline{P Q} \cong \overline{S R}$

Show: $\overline{S P} \cong \overline{Q R}$
Flowchart Proof

2. Given: Kite KITE with $\overline{K E} \cong \overline{K I}$

Show: $\overline{K T}$ bisects $\angle E K I$ and $\angle E T I$
Flowchart Proof

3. Given: $A B C D$ is a parallelogram

Show: $\angle A \cong \angle C$


## Flowchart Proof



## Lesson 4.8 • Proving Special Triangle Conjectures

Name $\qquad$
$\qquad$ Date $\qquad$

In Exercises 1-3, use the figure at right.

1. $\overline{C D}$ is a median, perimeter $\triangle A B C=60$, and $A C=22 . A D=$ $\qquad$
2. $\overline{C D}$ is an angle bisector, and $m \angle A=54^{\circ} . m \angle A C D=$ $\qquad$
3. $\overline{C D}$ is an altitude, perimeter $\triangle A B C=42, m \angle A C D=38^{\circ}$, and $A D=8$. $m \angle B=$ $\qquad$ , $C B=$ $\qquad$

4. $\triangle E Q U$ is equilateral.
$m \angle E=$ $\qquad$
5. $\triangle A N G$ is equiangular
and perimeter $\triangle A N G=51$.
$A N=$ $\qquad$
6. $\triangle A B C$ is equilateral, $\triangle A C D$ is isosceles with base $\overline{A C}$, perimeter $\triangle A B C=66$, and perimeter $\triangle A C D=82$. Perimeter $A B C D=$ $\qquad$

7. Complete a flowchart proof for this conjecture: In an isosceles triangle, the altitude from the vertex angle is the median to the base.
Given: Isosceles $\triangle A B C$ with $\overline{A C} \cong \overline{B C}$ and altitude $\overline{C D}$
Show: $\overline{C D}$ is a median
Flowchart Proof

8. Write a flowchart proof for this conjecture: In an isosceles triangle, the median to the base is also the angle bisector of the vertex angle.

Given: Isosceles $\triangle A B C$ with $\overline{A C} \cong \overline{B C}$ and median $\overline{C D}$
Show: $\overline{C D}$ bisects $\angle A C B$


## Lesson 5.1•Polygon Sum Conjecture

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1 and 2, find each lettered angle measure.

1. $a=$ $\qquad$ ,$b=$ $\qquad$ , $c=$ $\qquad$ —,
$d=$ $\qquad$ , $e=$ $\qquad$
2. $a=$ $\qquad$ , $b=$ $\qquad$ , $c=$ $\qquad$ —,
$d=$ $\qquad$ , $e=$ $\qquad$ , $f=$ $\qquad$

3. One exterior angle of a regular polygon measures $10^{\circ}$. What is the measure of each interior angle? How many sides does the polygon have?
4. The sum of the measures of the interior angles of a regular polygon is $2340^{\circ}$. How many sides does the polygon have?
5. $A B C D$ is a square. $A B E$ is an equilateral triangle.
$x=$ $\qquad$

6. $A B C D E$ is a regular pentagon. $A B F G$ is a square.

$$
x=
$$

$\qquad$

7. Use a protractor to draw pentagon $A B C D E$ with $m \angle A=85^{\circ}$, $m \angle B=125^{\circ}, m \angle C=110^{\circ}$, and $m \angle D=70^{\circ}$. What is $m \angle E$ ? Measure it, and check your work by calculating.

## Lesson 5.2•Exterior Angles of a Polygon

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. How many sides does a regular polygon have if each exterior angle measures $30^{\circ}$ ?
2. If the sum of the measures of the interior angles of a polygon equals the sum of the measures of its exterior angles, how many sides does it have?
3. How many sides does a polygon have if the sum of the measures of the interior angles is $3960^{\circ}$ ?
4. If the sum of the measures of the interior angles of a polygon is twice the sum of its exterior angles, how many sides does it have?

In Exercises 5-7, find each lettered angle measure.
5. $a=$ $\qquad$ , $b=$ $\qquad$

6. $a=$ $\qquad$ , $b=$ $\qquad$
7. $a=$ $\qquad$ , $b=$ $\qquad$ ,

$c=$ $\qquad$

8. Find each lettered angle measure.

$$
\begin{aligned}
& a= \\
& b= \\
& c= \\
& c= \\
& d=
\end{aligned}
$$


9. Construct an equiangular quadrilateral that is not regular.

## Lesson 5.3•Kite and Trapezoid Properties

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-4, find each lettered measure.

1. Perimeter $=116 . x=$

$\qquad$
2. $x=$ $\qquad$ , $y=$ $\qquad$

3. $x=$ $\qquad$ , $y=$ $\qquad$
4. $x=$ $\qquad$ , $y=$ $\qquad$

5. Perimeter $P Q R S=220 . P S=$ $\qquad$ 6. $b=2 a+1 . a>$


In Exercises 7 and 8, use the properties of kites and trapezoids to construct each figure. Use patty paper or a compass and a straightedge.
7. Construct an isosceles trapezoid given base $\overline{A B}, \angle B$, and distance between bases XY.

8. Construct kite $A B C D$ with $\overline{A B}, \overline{B C}$, and $\overline{B D}$.

9. Write a paragraph or flowchart proof of the Converse of the Isosceles

Trapezoid Conjecture. Hint: Draw $\overline{A E}$ parallel to $\overline{T P}$ with $E$ on $\overline{T R}$.
Given: Trapezoid $T R A P$ with $\angle T \cong \angle R$
Show: $\overline{T P} \cong \overline{R A}$


## Lesson 5.4•Properties of Midsegments

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, each figure shows a midsegment.

1. $a=$ $\qquad$ , $b=$
$c=$ $\qquad$

$\qquad$ ,
2. $x=$ $\qquad$ , $y=$ $\qquad$
$z=$ $\qquad$
3. $x=$ $\qquad$ , $y=$ $\qquad$
$z=$ $\qquad$

4. $X, Y$, and $Z$ are midpoints. Perimeter $\triangle P Q R=132, R Q=55$, and $P Z=20$.

Perimeter $\triangle X Y Z=$ $\qquad$
$P Q=$ $\qquad$
$Z X=$ $\qquad$

5. $\overline{M N}$ is the midsegment. Find the coordinates of $M$ and $N$. Find the slopes of $\overline{A B}$ and $\overline{M N}$.

6. Explain how to find the width of the lake from $A$ to $B$ using a tape measure, but without using a boat or getting your feet wet.

7. $M, N$, and $O$ are midpoints. What type of quadrilateral is $A M N O$ ? How do you know? Give a flowchart proof showing that $\triangle O N C \cong \triangle M B N$.

8. Give a paragraph or flowchart proof.

Given: $\triangle P Q R$ with $P D=D F=F H=H R$ and $Q E=E G=G I=I R$

Show: $\overline{H I}\|\overline{F G}\| \overline{D E} \| \overline{P Q}$


Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises $1-7, A B C D$ is a parallelogram.

1. Perimeter $A B C D=$

2. $A O=11$, and $B O=7$.
$A C=$ $\qquad$ , $B D=$ $\qquad$

3. Perimeter $A B C D=119$, and
4. $a=$ $\qquad$ , $b=$ $\qquad$
$c=$ $\qquad$
$c=$ $\qquad$
$\qquad$ ,
5. $a=$ $\qquad$ $b=$

$$
B C=24 . A B=
$$

3. Perimeter $A B C D=46$.
$A B=$ $\qquad$ , $B C=$ $\qquad$


4. Perimeter $A B C D=16 x-$ 12. $A D=$ $\qquad$ 8. Ball $B$ is struck at the same instant by two forces, $\vec{F}_{1}$ and $\vec{F}_{2}$. Show the resultant force on the ball.

5. Find each lettered angle measure.
$a=$ $\qquad$

$$
g=
$$

$\qquad$
$b=$

$$
h=
$$

$\qquad$
$\qquad$

$$
i=
$$

$\qquad$
$d=$ $\qquad$
$e=$
$k=$ $\qquad$
$f=$ $\qquad$

10. Construct a parallelogram with diagonals $\overline{A C}$ and $\overline{B D}$.

Is your parallelogram unique? If not, construct a different (noncongruent) parallelogram.


# Lesson 5.6 • Properties of Special Parallelograms 

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. $P Q R S$ is a rectangle and $O S=16$.
$O Q=$ $\qquad$
$m \angle Q R S=$ $\qquad$
$P R=$ $\qquad$

2. $K L M N$ is a square and $N M=8$.
$m \angle O K L=$ $\qquad$
$m \angle M O L=$ $\qquad$
Perimeter $K L M N=$ $\qquad$

3. $A B C D$ is a rhombus, $A D=11$, and $D O=6$.
$O B=$ $\qquad$ $B C=$ $\qquad$ $m \angle A O D=$ $\qquad$


In Exercises 4-11, match each description with all the terms that fit it.
a. Trapezoid
b. Isosceles triangle
c. Parallelogram
d. Rhombus
e. Kite
f. Rectangle
g. Square
h. All quadrilaterals
4. $\qquad$ Diagonals bisect each other.
5. $\qquad$ Diagonals are perpendicular.
6. $\qquad$ Diagonals are congruent.
7. $\qquad$ Measures of interior angles sum to $360^{\circ}$.
8. $\qquad$ Opposite sides are congruent.
9. $\qquad$ Opposite angles are congruent.
10. $\qquad$ Both diagonals bisect angles.
11. $\qquad$ Diagonals are perpendicular bisectors of each other.

In Exercises 12 and 13, graph the points and determine whether $A B C D$ is a trapezoid, parallelogram, rectangle, or none of these.
12. $A(-4,-1), B(0,-3), C(4,0), D(-1,5)$

13. $A(0,-3), B(-1,2), C(-3,4), D(-2,-1)$

14. Construct rectangle $A B C D$ with diagonal $\overline{A C}$ and $\angle C A B$.


## Lesson 5.7• Proving Quadrilateral Properties

Name $\qquad$ Period $\qquad$ Date $\qquad$

Write or complete each flowchart proof.

1. Given: $A B C D$ is a parallelogram and $\overline{A P} \cong \overline{Q C}$

Show: $\overline{A C}$ and $\overline{P Q}$ bisect each other


## Flowchart Proof


2. Given: Dart $A B C D$ with $\overline{A B} \cong \overline{B C}$ and $\overline{C D} \cong \overline{A D}$

Show: $\angle A \cong \angle C$

3. Show that the diagonals of a rhombus divide the rhombus into four congruent triangles.

Given: Rhombus $A B C D$
Show: $\triangle A B O \cong \triangle C B O \cong \triangle C D O \cong \triangle A D O$

4. Given: Parallelogram $A B C D, \overline{B Y} \perp \overline{A C}, \overline{D X} \perp \overline{A C}$

Show: $\overline{D X} \cong \overline{B Y}$


## Lesson 6.1• Tangent Properties

Name $\qquad$
$\qquad$ Date $\qquad$

1. Rays $r$ and $s$ are tangents. $w=$ $\qquad$

2. $\overleftrightarrow{A B}$ is tangent to both circles and $m \overline{A M C}=295^{\circ} . m \angle B Q X=$ $\qquad$

3. $\overleftrightarrow{P Q}$ is tangent to two externally tangent noncongruent circles, $M$ and $N$.
a. $m \angle N Q P=$ $\qquad$ , $m \angle M P Q=$ $\qquad$
b. What kind of quadrilateral is MNQP? Explain your reasoning.

4. $\overleftrightarrow{A T}$ is tangent to circle $P$. Find the equation of $\overleftrightarrow{A T}$.

5. $\overrightarrow{P A}, \overrightarrow{P B}, \overrightarrow{P C}$, and $\overrightarrow{P D}$ are tangents. Explain why $\overline{P A} \cong \overline{P D}$.

6. Circle $A$ has diameter 16.4 cm . Circle $B$ has diameter 6.7 cm .
a. If $A$ and $B$ are internally tangent, what is the distance between their centers?
b. If $A$ and $B$ are externally tangent, what is the distance between their centers?
7. Construct a circle, $P$. Pick a point, $A$, on the circle. Construct a tangent through $A$. Pick a point, $T$, on the tangent. Construct a second tangent to the circle through $T$.

## Lesson 6.2•Chord Properties

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, find each unknown or write "cannot be determined."

1. $a=$ $\qquad$ , $b=$ $\qquad$ ,
$c=$ $\qquad$

2. $w=$ $\qquad$ $x=$ $\qquad$ ,
3. $w=$ $\qquad$ , $x=$ $\qquad$
4. $x=$ $\qquad$ $y=$ $\qquad$ $y=$ $\qquad$ $y=$ $\qquad$

5. $\overline{A B} \cong \overline{A C} . A M O N$ is a
$\overline{\text { Justify your answer. }}$
6. What's wrong with this picture?
7. Find the coordinates of $P$ and $M$.

8. $m \overline{A B}=$ $m \overline{A B C}=$
$m \widehat{B A C}=$ $\qquad$ $m \overline{A C B}=$ $\qquad$

9. Trace part of a circle onto patty paper. Fold to find the center. Explain your method.


## Lesson 6.3•Arcs and Angles

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. $m \overline{X M}=80^{\circ}$
$m \angle X N M=$ $\qquad$ $m \widehat{X N}=$ $\qquad$
$m \widehat{M N}=$ $\qquad$

2. $\overleftrightarrow{A B}$ is a tangent.
$x=$ $\qquad$
$y=$ $\qquad$
$z=$ $\qquad$

3. $a=$ $\qquad$
$b=$ $\qquad$
$c=$ $\qquad$
4. $a=$ $\qquad$
$b=$ $\qquad$
$c=$ $\qquad$

5. $\overrightarrow{A B}$ and $\overrightarrow{A C}$ are tangents.
$x=$ $\qquad$

6. $\overline{A D}$ is a tangent. $\overline{A C}$ is a diameter.
$m \angle A=$ $\qquad$
$m \overline{A B}=$ $\qquad$
$m \angle C=$ $\qquad$
$m \widehat{C B}=$ $\qquad$

7. $m \overline{A D}=$ $\qquad$
$m \angle D=$ $\qquad$
$m \widehat{A B}=$ $\qquad$
$m \widehat{D A B}=$ $\qquad$

8. $p=$ $\qquad$

9. Find the lettered angle and arc measures. $\overrightarrow{A T}$ and $\overrightarrow{A Z}$ are tangents.
$a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$
$d=$ $\qquad$ $e=$ $\qquad$ $f=$ $\qquad$
$g=$ $\qquad$
$h=$ $\qquad$
$j=$ $\qquad$
$k=$ $\qquad$
$m=$ $\qquad$
$n=$ $\qquad$


## Lesson 6.4•Proving Circle Conjectures

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-4, complete each proof with a paragraph or a flowchart.

1. Given: Circles $O$ and $P$ are externally tangent, with common tangents $\overleftrightarrow{C D}$ and $\overleftrightarrow{A B}$
Show: $\overleftrightarrow{A B}$ bisects $\overline{C D}$ at $X$

2. Given: Circle $O$ with diameter $\overline{A B}$ and chord $\overline{A D} \cdot \overline{O E} \| \overline{A D}$.

Show: $\overparen{D E} \cong \overparen{B E}$

3. Given: $\overleftrightarrow{P Q}$ and $\overleftrightarrow{R S}$ are tangent to both circles.

Show: $\overline{P Q} \cong \overline{R S}$.

4. Prove the converse of the Chord Arcs Conjecture: If two arcs in a circle are congruent, then their chords are congruent. Hint: Draw radii.
Given: $\overline{A B} \cong \widehat{C D}$
Show: $\overline{A B} \cong \overline{C D}$


## Lesson 6.5 • The Circumference/Diameter Ratio

Name $\qquad$
$\qquad$ Date $\qquad$

In Exercises $1-4$, leave your answers in terms of $\pi$.

1. If $r=10.5 \mathrm{~cm}$, find $C$.
2. If $C=25 \pi \mathrm{~cm}$, find $r$.
3. What is the circumference of a circle whose radius is 30 cm ?
4. What is the diameter of a circle whose circumference is $24 \pi \mathrm{~cm}$ ?

In Exercises 5-9, round your answer to the nearest 0.1 unit. Use the symbol $\approx$ to show that your answer is an approximation.
5. If $d=9.6 \mathrm{~cm}$, find $C$.
6. If $C=132 \mathrm{~cm}$, find $d$ and $r$.
7. A dinner plate fits snugly in a square box with perimeter 48 inches. What is the circumference of the plate?
8. Four saucers are part of the same set as the dinner plate in Exercise 7. Each has a circumference of 15.7 inches. Will they fit, side by side, in the same square box? If so, how many inches will there be between the saucers for padding?
9. $\overleftrightarrow{A T}$ and $\overleftrightarrow{A S}$ are tangents. $A T=12 \mathrm{~cm}$. What is the circumference of circle $O$ ?

10. How can you use a large carpenter's square to find the circumference of a tree?

11. In order to increase the circumference of a circle from $16 \pi \mathrm{~cm}$ to $20 \pi \mathrm{~cm}$, by how much must the diameter increase?
$\qquad$ Date $\qquad$

1. Alfonzo's Pizzeria bakes olive pieces in the outer crust of its 20 -inch (diameter) pizza. There is at least one olive piece per inch of crust. How many olive pieces will you get in one slice of pizza? Assume the pizza is cut into eight slices.
2. To use the machine at right, you turn the crank, which turns the pulley wheel, which winds the rope and lifts the box. Through how many rotations must you turn the crank to lift the box 10 feet?
3. A satellite in geostationary orbit stays over the same spot on Earth.
 The satellite completes one orbit in the same time that Earth rotates once about its axis ( 23.93 hours). If the satellite's orbit has radius $4.23 \times 10^{7} \mathrm{~m}$, calculate the satellite's orbital speed (tangential velocity) in meters per second.
4. You want to decorate the side of a cylindrical can by coloring a rectangular piece of paper and wrapping it around the can. The paper is 19 cm by 29 cm . Find the two possible diameters of the can to the nearest 0.01 cm . Assume the paper fits exactly.
5. As you sit in your chair, you are whirling through space with Earth as it moves around the sun. If the average distance from Earth to the sun is $1.4957 \times 10^{11} \mathrm{~m}$ and Earth completes one revolution every 364.25 days, what is your "sitting" speed in space relative to the sun? Give your answer in km/h, rounded to the nearest $100 \mathrm{~km} / \mathrm{h}$.

## Lesson 6.7•Arc Length

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-10, leave your answers in terms of $\pi$.

1. Length of $\overline{A B}=$
$\qquad$
2. The circumference is $24 \pi$ and $m \overline{C D}=60^{\circ}$. Length
of $\widehat{C D}=$ $\qquad$

3. The length of $\widehat{E F}$ is $5 \pi$. Radius = $\qquad$

4. Length of $\overline{X Y}=$

5. The radius is 20 . Length of $\overline{A B}=$ $\qquad$

6. The circumference is $25 \pi$. Length of $\overline{A B}=$ $\qquad$

7. The diameter is 40 . Length of $\overline{A C}=$ $\qquad$

8. The length of $\overline{X Y}$ is $14 \pi$. Diameter $=$ $\qquad$

9. Length of $\overline{A B}=$ $\qquad$

10. A circle has an arc with measure $80^{\circ}$ and length $88 \pi$. What is the diameter of the circle?

## Exploration•Intersecting Secants, Tangents, and Chords

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. $x=$ $\qquad$

2. $\overrightarrow{F C}$ is tangent to circle $A$ at point $C$. $m \overline{D C}=\longrightarrow$

3. $\overleftrightarrow{E D}$ and $\overleftrightarrow{E C}$ are tangents.

$$
m \widehat{D C}=\longrightarrow, m \angle D E C=
$$


4. $\overleftrightarrow{C E}$ is a tangent, $m \widehat{B C}=150^{\circ}$
$m \angle B C E=$ $\qquad$ , $m \widehat{B A C}=$ $\qquad$

5. $x=$ $\qquad$ , $y=$ $\qquad$ , $z=$ $\qquad$
6. $x=$ $\qquad$ , $y=$ $\qquad$ , $z=$ $\qquad$

7. $\overrightarrow{A B}$ and $\overrightarrow{A C}$ are tangents.
$x=$ $\qquad$ , $y=$ $\qquad$ $z=$ $\qquad$
8. $\overleftrightarrow{A B}$ is a tangent, $m \angle A B C=75^{\circ}$
$x=$ $\qquad$ , $y=$ $\qquad$


## Lesson 7.1• Transformations and Symmetry

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, perform each transformation.

1. Reflect $\triangle T R I$ across line $\ell$.

2. Rotate PARL $270^{\circ}$ clockwise about $Q$.

3. Translate PENTA by the given vector.

4. $A B C D E$ and its reflected image, $A^{\prime} B^{\prime} C^{\prime} D^{\prime} E^{\prime}$, are shown below. Use construction tools to locate the line of reflection, $\ell$. Explain your method.


In Exercises 5-8, identify the type(s) of symmetry in each figure.
5. Equilateral triangle

6. Rectangle

7. Isosceles triangle

8. Square


In Exercises 9-12, draw each polygon and identify the type(s) of symmetry in each. Draw all lines of reflection and mark centers of rotation.
9. Rhombus
10. Parallelogram
11. Isosceles trapezoid
12. Square

Lesson 7.2 • Properties of Isometries

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, draw the image according to the rule and identify the type of transformation.

1. $(x, y) \rightarrow(-x,-y)$
2. $(x, y) \rightarrow(x-4, y+6)$
3. $(x, y) \rightarrow(4-x, y)$




In Exercises 4 and 5, the Harbour High Geometry Class is holding a Fence
Race. Contestants must touch each fence at some point as they run from $S$ to $F$. Use your geometry tools to draw the shortest possible race path.
4.

5.


In Exercises 6-8, complete the ordered pair rule that transforms each triangle to its image. Identify the transformation. Find all missing coordinates.
6. $(x, y) \rightarrow(\square)$

7. $(x, y) \rightarrow(\square, \quad-\quad)$

8. $(x, y) \rightarrow(\square, \quad$ _ $)$


## Lesson 7.3•Compositions of Transformations

Name $\qquad$
$\qquad$ Date $\qquad$

In Exercises 1-8, name the single transformation that can replace the composition of each set of multiple transformations.

1. Translation by $\langle+4,+1\rangle$, followed by $\langle+2,-3\rangle$, followed by $\langle-8,+7\rangle$
2. Rotation $60^{\circ}$ clockwise, followed by $80^{\circ}$ counterclockwise, followed by $25^{\circ}$ counterclockwise all about the same center of rotation
3. Reflection across vertical line $m$, followed by reflection across vertical line $n$, where $n$ is 8 units to the right of $m$
4. Reflection across vertical line $p$, followed by reflection across horizontal line $q$
5. Reflection across vertical line $n$, followed by reflection across vertical line $m$, where $n$ is 8 units to the right of $m$
6. Reflection across horizontal line $q$, followed by reflection across vertical line $p$
7. Translation by $\langle+6,0\rangle$, followed by reflection across the $y$-axis
8. Reflection across the $y$-axis, followed by translation by $\langle+6,0\rangle$

In Exercises 9-11, copy the figure onto your paper and use your geometry tools to perform the given transformation.
9. Locate $P^{\prime}$, the reflected image across $\overrightarrow{O R}$, and $P^{\prime \prime}$, the reflected image of $P^{\prime}$ across $\overrightarrow{O T}$. Find $m \angle R O T$ and give a single transformation that maps $P$ to $P^{\prime \prime}$.

10. Locate $P^{\prime}$, the reflected image across $k$, and $P^{\prime \prime}$, the reflected image of $P^{\prime}$ across $\ell$. Find the distance between $\ell$ and $k$ and give a single transformation that maps $P$ to $P^{\prime \prime}$.
11. Draw five glide-reflected images of the triangle.


## Lesson 7.4• Tessellations with Regular Polygons

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Find $n$.

2. Find $n$.

3. What is a regular tessellation? Sketch an example to illustrate your explanation.
4. What is a 1-uniform tiling? Sketch an example of a 1 -uniform tiling that is not a regular tessellation.
5. Give the numerical name for the tessellation at right.

6. Use your geometry tools to draw the $4.8^{2}$ tessellation.

## Lessons 7.5-7.8• Tessellations

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Trace the quadrilateral at right (or draw a similar one). Make the outline dark. Set another piece of paper on top of the quadrilateral and, by tracing, create a tessellation. (Hint: Trace vertices and use a straightedge to connect them.)

2. On dot paper, draw a small concave quadrilateral (vertices on dots).

Allow no more than three dots inside the figure. Tessellate the entire paper with your quadrilateral. Color and shade your tessellation.

3. In non-edge-to-edge tilings, the vertices of the polygons do not have to coincide, as in these wooden deck patterns. Use graph paper to create your own non-edge-to-edge tiling.

4. Use your geometry tools to draw a parallelogram. Draw squares on each side. Create a tessellation by duplicating your parallelogram and squares.

## Lesson 8.1•Areas of Rectangles and Parallelograms

Name $\qquad$
$\qquad$ Date $\qquad$

In Exercises $1-4$, find the area of the shaded region.
1.

2.

3.

4.

5. Rectangle $A B C D$ has area $2684 \mathrm{~m}^{2}$ and width 44 m . Find its length.
6. Draw a parallelogram with area $85 \mathrm{~cm}^{2}$ and an angle with measure $40^{\circ}$. Is your parallelogram unique? If not, draw a different one.
7. Find the area of $P Q R S$.

8. Find the area of $A B C D E F$.

9. Dana buys a piece of carpet that measures 20 square yards. Will she be able to completely cover a rectangular floor that measures 12 ft 6 in . by 16 ft 6 in.? Explain why or why not.

## Lesson 8.2•Areas of Triangles, Trapezoids, and Kites

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-4, solve for the unknown measures.

$\qquad$ _.
2. Area $=$ $\qquad$ .

3. Area $=126$ in $^{2}$
$b=$ $\qquad$ _.

5. Find the area of the shaded region.

6. $\overline{T P}$ is tangent to circles $M$ and $N . T P=16 \mathrm{~cm}$. The radius of $N$ is 7 cm and the radius of $M$ is 4 cm . Find the area of NMPT.

7. Find the area of $\triangle T R I$.

8. $A B C D$ is a parallelogram, $A B D E$ is a kite, $A D=18 \mathrm{~cm}$, and $B E=10 \mathrm{~cm}$. Find the area of $A B C D E$.

$\qquad$
$\qquad$ Date $\qquad$

1. A bundle of hardwood flooring contains $14 \frac{1}{2} \mathrm{ft}^{2}$ and costs $\$ 39.90$.
a. How many square feet of flooring is needed to cover the kitchen and family room? Exclude the fireplace, hearth, and tiled area.
b. You should buy 5\% extra flooring to account for waste. How many bundles of flooring should you buy? What will be the cost?

2. Bert's Bigtime Bakery has baked the world's largest chocolate cake. It is a rectangular sheet cake that is 600 cm by 400 cm by 180 cm high. Bert wants to apply frosting to the four sides and the top. How many liters of frosting does he need if 1 liter of frosting covers about $1200 \mathrm{~cm}^{2}$ ?
3. For a World Peace Day celebration the students at Cabot Junior/Senior High School are making a 6 m -by- 8 m flag. Each of the six grades will create a motif to honor the people of the six inhabited continents. Sketch three possible ways to divide the flag: one into six congruent triangles; one into six triangles with equal area but none congruent; and one into six congruent trapezoids. Give measurements or markings on your sketches so each class knows it has equal area.
4. Kit and Kat are building a kite for the big kite festival. Kit has already cut his sticks for the diagonals. He wants to position $P$ so that he will have maximum kite area. He asks Kat for advice. What should Kat tell him?


## Lesson 8.4•Areas of Regular Polygons

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, the polygons are regular.

1. $s=12 \mathrm{~cm}$
$a \approx 14.5 \mathrm{~cm}$
$A \approx$ $\qquad$
2. $s=4.2 \mathrm{~cm}$
$A \approx 197 \mathrm{~cm}^{2}$
$a \approx$ $\qquad$
3. $a=6 \mathrm{~cm}$ $A \approx 130.8 \mathrm{~cm}^{2}$
$p \approx$ $\qquad$

4. In a regular $n$-gon, $s=4.8 \mathrm{~cm}, a \approx 7.4 \mathrm{~cm}$, and $A \approx 177.6 \mathrm{~cm}^{2}$. Find $n$.
5. Draw a regular pentagon so that it has perimeter 20 cm . Use the Regular Polygon Area Conjecture and a centimeter ruler to find its approximate area.
6. Use a compass and straightedge to construct a regular octagon and its apothem. Use a centimeter ruler to measure its side length and apothem, and use the Regular Polygon Area Conjecture to find its approximate area.
7. Find the area of the shaded region between the square and the regular octagon. $s \approx 5 \mathrm{~cm} . r=3 \mathrm{~cm}$.


## Lesson 8.5•Areas of Circles

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-4, write your answers in terms of $\pi$.

1. If $r=9 \mathrm{~cm}, A=$ $\qquad$ .
2. If $d=6.4 \mathrm{~cm}, A=$ .
3. If $A=529 \pi \mathrm{~cm}^{2}, r=$ $\qquad$ .
4. If $C=36 \pi \mathrm{~cm}, A=$ $\qquad$ .

In Exercises 5-8, round your answers to the nearest 0.01 unit.
5. If $r=7.8 \mathrm{~cm}, A \approx$ $\qquad$ .
6. If $A=136.46, C \approx$
7. If $d=3.12, A \approx$ $\qquad$ 8. If $C=7.85, A \approx$ $\qquad$ .

For Exercises 9 and 10, refer to the figure of a circle inscribed in an equilateral triangle. Round your answers to the nearest 0.1 unit.
9. Find the area of the inscribed circle.
10. Find the area of the shaded region.


In Exercises 11 and 12, find the area of the shaded region. Write your answers in terms of $\pi$.
11. $A B C D$ is a square.

12. The three circles are tangent.


## Lesson 8.6•Any Way You Slice It

## Name

$\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, find the area of the shaded region. Write your answers in terms of $\pi$ and rounded to the nearest $0.01 \mathrm{~cm}^{2}$.
1.

2.

3.

4.

5.

6.

7. Shaded area is $40 \pi \mathrm{~cm}^{2}$.

Find $r$.

8. Shaded area is $54 \pi \mathrm{~cm}^{2}$.

Find $x$.

9. Shaded area is $51 \pi \mathrm{~cm}^{2}$. The diameter of the larger circle is 20 cm . Find $r$.

$\qquad$
$\qquad$ Date $\qquad$

In Exercises 1-8, find the surface area of each solid. All quadrilaterals are rectangles, and all measurements are in centimeters. Round your answers to the nearest $0.1 \mathrm{~cm}^{2}$.
1.

2.

3.


5. Base is a regular hexagon. $s=6, a \approx 5.2$, and $l=9$.

6.

7. Both bases are squares.

8. A square hole in a round peg

9. Ilsa is building a museum display case. The sides and bottom will be plywood and the top will be glass. Plywood comes in 4 ft -by- 8 ft sheets. How many sheets of plywood will she need to buy? Explain. Sketch a cutting pattern that will leave her with the largest single piece possible.


## Lesson 9.1•The Theorem of Pythagoras

Name $\qquad$ Period $\qquad$ Date $\qquad$

Give all answers rounded to the nearest 0.1 unit.

1. $a=$ $\qquad$

2. $p \approx$ $\qquad$

3. $x=$ $\qquad$

4. Area $=39 \mathrm{in}^{2}$
$h \approx$ $\qquad$

5. Find the area.

6. Find the coordinates of $C$ and the radius of circle $A$.

7. Find the area.

8. Base area $=16 \pi \mathrm{~cm}^{2}$ and slant height $=$ 3 cm . What's wrong with this picture?

9. $R S=3 \mathrm{~cm}$. Find $R V$.

10. Given $\triangle P Q R$, with $m \angle P=90^{\circ}, P Q=20$ in., and $P R=15$ in., find the area of $\triangle P Q R$, the length of the hypotenuse, and the altitude to the hypotenuse.

## Lesson 9.2•The Converse of the Pythagorean Theorem

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters. Give answers rounded to the nearest 0.01 cm .

In Exercises 1-4, determine whether a triangle with the given side lengths is a right triangle.

1. $76,120,98$
2. $221,204,85$
3. $5.0,1.4,4.8$
4. $80,82,18$
5. Find the area of $\triangle A B C$.

6. What's wrong with this picture?

7. Find $x$. Explain your method.

8. Find the area of $A B C D$.


In Exercises 9-11, determine whether $A B C D$ is a rectangle and justify your answer. If not enough information is given, write "cannot be determined."
9. $A B=3, B C=4$, and $A C=6$.
10. $A B=3, B C=4, D A=4$, and $A C=5$.

11. $A B=3, B C=4, C D=3, D A=4$, and $A C=B D$.

## Lesson 9.3 • Two Special Right Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

Give your answers in exact form unless otherwise indicated. All measurements are in centimeters.

In Exercises 1-3, find the unknown lengths.

## 1. $a=$ <br> $\qquad$


2. $a=$ $\qquad$ $b=$ $\qquad$
3. $a=$ $\qquad$ $b=$ $\qquad$

4. Find the area of rectangle $A B C D$.
5. Find the perimeter and area of KLMN.

6. $A C=\_, A B=$ $\qquad$ —, and area $\triangle A B C=$ $\qquad$ .

7. Find the area of an isosceles trapezoid if the bases have lengths 12 cm and 18 cm and the base angles have measure $60^{\circ}$.

In Exercises 8 and 9, find the coordinates of $C$.
8.

9.

10. Sketch and label a figure to demonstrate that $\sqrt{18}$ is equivalent to $3 \sqrt{2}$.

$\qquad$
$\qquad$ Date $\qquad$

1. A 20 ft ladder reaches a window 18 ft high. How far is the foot of the ladder from the base of the building? How far must the foot of the ladder be moved to lower the top of the ladder by 2 ft ?
2. Robin and Dovey have four pet pigeons that they train to race. They release the birds at Robin's house and then drive to Dovey's to collect them. To drive from Robin's to Dovey's, because of one-way streets, they go 3.1 km north, turn right and go 1.7 km east, turn left and go 2.3 km north, turn right and go 0.9 km east, turn left and go 1.2 km north, turn left and go 4.1 km west, and finally turn left and go 0.4 km south. How far do the pigeons have to fly to go directly from Robin's house to Dovey's house?
3. Hans needs to paint the 18 in.-wide trim around the roof eaves and gable ends of his house with 2 coats of paint. A quart can of paint covers $175 \mathrm{ft}^{2}$ and costs $\$ 9.75$. A gallon can of paint costs $\$ 27.95$. How much paint should Hans buy? Explain.

4. What are the dimensions of the largest $30^{\circ}-60^{\circ}-90^{\circ}$ triangle that will fit inside a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle with leg length 14 in.? Sketch your solution.

## Lesson 9.5 • Distance in Coordinate Geometry

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the distance between each pair of points.

1. $(-5,-5),(1,3)$
2. $(-11,-5),(5,7)$
3. $(8,-2),(-7,6)$

In Exercises 4 and 5, use the distance formula and the slope of segments to identify the type of quadrilateral. Explain your reasoning.
4. $A(-2,1), B(3,-2), C(8,1), D(3,4)$
5. $T(-3,-3), U(4,4), V(0,6), W(-5,1)$

For Exercises 6 and 7, use $\triangle A B C$ with coordinates $A(4,14), B(10,6)$, and $C(16,14)$.
6. Determine whether $\triangle A B C$ is scalene, isosceles, or equilateral. Find the perimeter of the triangle.
7. Find the midpoints $M$ and $N$ of $\overline{A B}$ and $\overline{A C}$, respectively. Find the slopes and lengths of $\overline{M N}$ and $\overline{B C}$. How do the slopes compare? How do the lengths compare?
8. Find the equation of the circle with center $(-1,5)$ and radius 2 .
9. Find the center and radius of the circle whose equation is $x^{2}+(y+2)^{2}=25$.
10. $P$ is the center of the circle. What's wrong with this picture?


## Lesson 9.6•Circles and the Pythagorean Theorem

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1 and 2, find the area of the shaded region in each figure. All measurements are in centimeters. Write your answers in terms of $\pi$ and rounded to the nearest $0.1 \mathrm{~cm}^{2}$.

1. $A O=5 . A C=8$.
2. Tangent $\overrightarrow{P T}, Q M=12, m \angle P=30^{\circ}$


3. $A P=63 \mathrm{~cm}$. Radius of circle $O=37 \mathrm{~cm}$. How far is $A$ from the circumference of the circle?

4. Two perpendicular chords with lengths 12.2 cm and 8.8 cm have a common endpoint. What is the area of the circle?
5. $A B C D$ is inscribed in a circle. $\overline{A C}$ is a diameter. If $A B=9.6 \mathrm{~cm}$, $B C=5.7 \mathrm{~cm}$, and $C D=3.1 \mathrm{~cm}$, find $A D$.
6. Find ST.

7. The coordinate of point $M$ is $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$.
Find the measure of $\angle A O M$.


## Lesson 10.1• The Geometry of Solids

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-14, refer to the figures below.


1. The cylinder is (oblique, right).
2. $\overline{O P}$ is $\qquad$ of the cylinder.
3. $\overline{T R}$ is $\qquad$ of the cylinder.
4. Circles $O$ and $P$ are $\qquad$ of the cylinder.
5. $\overline{P Q}$ is $\qquad$ of the cylinder.
6. The cone is (oblique, right).
7. Name the base of the cone.
8. Name the vertex of the cone.
9. Name the altitude of the cone.
10. Name a radius of the cone.
11. Name the type of prism.
12. Name the bases of the prism.
13. Name all lateral edges of the prism.
14. Name an altitude of the prism.

In Exercises 15-17, tell whether each statement is true or false. If the statement is false, give a counterexample or explain why it is false.
15. The axis of a cylinder is perpendicular to the base.
16. A rectangular prism has four faces.
17. The bases of a trapezoidal prism are trapezoids.

For Exercises 18 and 19, draw and label each solid. Use dashed lines to show the hidden edges.
18. A right triangular prism with height equal to the hypotenuse
19. An oblique trapezoidal pyramid

## Lesson 10.2 • Volume of Prisms and Cylinders

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the volume of each prism or cylinder. All measurements are in centimeters. Round your answers to the nearest 0.01 .

## 1. Right triangular prism


2. Right trapezoidal prism

3. Regular hexagonal prism


In Exercises 4-6, use algebra to express the volume of each solid.
4. Right rectangular prism

5. Right cylinder;
base circumference $=p \pi$

6. Right rectangular prism
and half of a cylinder

7. You need to build a set of solid cement steps for the entrance to your new house. How many cubic feet of cement do you need?


## Lesson 10.3 • Volume of Pyramids and Cones

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the volume of each solid. All measurements are in centimeters. Round your answers to two decimal places.

1. Rectangular pyramid; $O P=6$

2. Right hexagonal pyramid

3. Half of a right cone


In Exercises 4-6, use algebra to express the volume of each solid.
4.

5.

6. The solid generated by spinning $\triangle A B C$ about the axis


In Exercises 7-9, find the volume of each figure and tell which volume is larger.
7.

B.

8.

B.

9.
A.

B.

$\qquad$ Period $\qquad$ Date $\qquad$

1. A cone has volume $320 \mathrm{~cm}^{3}$ and height 16 cm . Find the radius of the base. Round your answer to the nearest 0.1 cm .
2. How many cubic inches are there in one cubic foot? Use your answer to help you with Exercises 3 and 4.
3. Jerry is packing cylindrical cans with diameter 6 in . and height 10 in . tightly into a box that measures 3 ft by 2 ft by 1 ft . All rows must contain the same number of cans. The cans can touch each other. He then fills all the empty space in the box with packing foam. How many cans can Jerry pack in one box? Find the volume of packing foam he uses. What percentage of the box's volume is filled by the foam?
4. A king-size waterbed mattress measures 72 in . by 84 in . by 9 in . Water weighs 62.4 pounds per cubic foot. An empty mattress weighs 35 pounds. How much does a full mattress weigh?
5. Square pyramid $A B C D E$, shown at right, is cut out of a cube with base $A B C D$ and shared edge $\overline{D E} . A B=2 \mathrm{~cm}$. Find the volume and surface area of the pyramid.
6. In Dingwall the town engineers have contracted for a new water storage tank. The tank is cylindrical with a base 25 ft in diameter and a height of 30 ft . One cubic foot holds about 7.5 gallons of water. About how many gallons will the new storage tank hold?
7. The North County Sand and Gravel Company stockpiles sand to use on the icy roads in the northern rural counties of the state. Sand is brought in by tandem trailers that carry $12 \mathrm{~m}^{3}$ each. The engineers know that when the pile of sand, which is in
 the shape of a cone, is 17 m across and 9 m high they will have
 enough for a normal winter. How many truckloads are needed to build the pile?

## Lesson 10.5 • Displacement and Density

Name $\qquad$
$\qquad$ Date $\qquad$

1. A stone is placed in a 5 cm -diameter graduated cylinder, causing the water level in the cylinder to rise 2.7 cm . What is the volume of the stone?
2. A 141 g steel marble is submerged in a rectangular prism with base 5 cm by 6 cm . The water rises 0.6 cm . What is the density of the steel?
3. A solid wood toy boat with a mass of 325 g raises the water level of a 50 cm -by -40 cm aquarium 0.3 cm . What is the density of the wood?
4. For Awards Night at Baddeck High School, the math club is designing small solid silver pyramids. The base of the pyramids will be a 2 in.-by- 2 in. square. The pyramids should not weigh more than $2 \frac{1}{2}$ pounds. One cubic foot of silver weighs 655 pounds. What is the maximum height of the pyramids?
5. While he hikes in the Gold Country of northern California, Sid dreams about the adventurers that walked the same trails years ago. He suddenly kicks a small bright yellowish nugget. Could it be gold? Sid quickly makes a balance scale using his walking stick and finds that the nugget has the same mass as the uneaten half of his 330 g nutrition bar. He then drops the stone into his water bottle, which has a 2.5 cm radius, and notes that the water level goes up 0.9 cm . Has Sid struck gold? Explain your reasoning. (Refer to the density chart in Lesson 10.5 in your book.)

## Lesson 10.6 • Volume of a Sphere

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, find the volume of each solid. All measurements are in centimeters. Write your answers in exact form and rounded to the nearest $0.1 \mathrm{~cm}^{3}$.
1.

2.

3.

4.

5.

6. Cylinder with hemisphere taken out of the top

7. A sphere has volume $221 \frac{5}{6} \pi \mathrm{~cm}^{3}$. What is its diameter?
8. The area of the base of a hemisphere is $225 \pi \mathrm{in}^{2}$. What is its volume?
9. Eight wooden spheres with radii 3 in . are packed snugly into a square box 12 in . on one side. The remaining space is filled with packing beads. What is the volume occupied by the packing beads? What percentage of the volume of the box is filled with beads?
10. The radius of Earth is about 6378 km , and the radius of Mercury is about 2440 km . About how many times greater is the volume of Earth than that of Mercury?

## Lesson 10.7 • Surface Area of a Sphere

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises $1-4$, find the volume and total surface area of each solid. All measurements are in centimeters. Round your answers to the nearest 0.1 cm .
1.

2.

3.

4.

5. If the surface area of a sphere is $48.3 \mathrm{~cm}^{2}$, find its diameter.
6. If the volume of a sphere is $635 \mathrm{~cm}^{3}$, find its surface area.
7. Lobster fishers in Maine often use spherical buoys to mark their lobster traps. Every year the buoys must be repainted. An average buoy has a 12 in . diameter, and an average fisher has about 500 buoys. A quart of marine paint covers $175 \mathrm{ft}^{2}$. How many quarts of paint does an average fisher need each year?

## Lesson 11.1•Similar Polygons

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters.

1. HAPIE $\sim$ NWYRS
$A P=$

$E I=$
$S N=$ $\qquad$
$Y R=$ $\qquad$

2. $Q U A D \sim S I M L$
$S L=$ $\qquad$
$M I=$ $\qquad$
$m \angle D=$ $\qquad$

$m \angle U=$ $\qquad$
$m \angle A=$ $\qquad$


In Exercises 3-6, decide whether or not the figures are similar. Explain why or why not.
3. $A B C D$ and $E F G H$

5. JKON and JKLM

7. Draw the dilation of $A B C D$ by a scale factor of $\frac{1}{2}$. What is the ratio of the perimeter of the dilated quadrilateral to the perimeter of the original quadrilateral?

4. $\triangle A B C$ and $\triangle A D E$

6. $A B C D$ and $A E F G$

8. Draw the dilation of $\triangle D E F$ by a scale factor of 2 . What is the ratio of the area of the dilated triangle to the area of the original triangle?


## Lesson $11.2 \cdot$ Similar Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters.

$$
\begin{aligned}
& \text { 1. } \triangle T A R \sim \triangle M A C \\
& M C=
\end{aligned}
$$


2. $\triangle X Y Z \sim \triangle Q R S$
$\angle Q \cong$ $\qquad$
$Q R=$ $\qquad$
$Q S=$ $\qquad$

$Q \overbrace{S}^{R}$
3. $\triangle A B C \sim \triangle E D C$
$\angle A \cong$ $\qquad$
$C D=$ $\qquad$
4. $\triangle T R S \sim \triangle T Q P$
$T S=$ $\qquad$
$Q P=$ $\qquad$
$A B=$ $\qquad$

For Exercises 5 and 6, refer to the figure at right.
5. Explain why $\triangle C A T$ and $\triangle D A G$ are similar.
6. $C A=$ $\qquad$


In Exercises 7-9, identify similar triangles and explain why they are similar.
7.

8.

9.


## Lesson $11.3 \cdot$ Indirect Measurement with Similar Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. At a certain time of day, a 6 ft man casts a 4 ft shadow. At the same time of day, how tall is a tree that casts an 18 ft shadow?
2. Driving through the mountains, Dale has to go up and over a high mountain pass. The road has a constant incline for $7 \frac{3}{4}$ miles to the top of the pass. Dale notices from a road sign that in the first mile he climbs 840 feet. How many feet does he climb in all?
3. Sunrise Road is 42 miles long between the edge of Moon Lake and Lake Road and 15 miles long between Lake Road and Sunset Road. Lake Road is 29 miles long. Find the length of Moon Lake.

4. Marta is standing 4 ft behind a fence 6 ft 6 in . tall. When she looks over the fence, she can just see the top edge of a building. She knows that the building is 32 ft 6 in . behind the fence. Her eyes are 5 ft from the ground. How tall is the building? Give your answer to the nearest half foot.

5. You need to add 5 supports under the ramp, in addition to the 3.6 m one, so that they are all equally spaced. How long should each support be? (One is drawn in for you.)


## Lesson $11.4 \cdot$ Corresponding Parts of Similar Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters.

1. $\triangle A B C \sim \triangle P R Q . M$ and $N$ are midpoints. Find $h$ and $j$.

2. The triangles are similar.

Find the length of each side of the smaller triangle to the nearest 0.01 .

4. Find $x$ and $y$.

6. Find $C B, C D$, and $A D$.


Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters unless otherwise indicated.

1. $\triangle A B C \sim \triangle D E F$. Area of $\triangle A B C=15 \mathrm{~cm}^{2}$. Area of $\triangle D E F=$ $\qquad$

2. $\frac{\text { Area of circle } O}{\text { Area of circle } P}=\frac{4}{9}$.
$a=$ $\qquad$

3. $\frac{\text { Area of square } S Q U A}{\text { Area of square } L R G E}=$ $\qquad$ 4. $\frac{\text { Area of circle } P}{\text { Area of circle } O}=$ $\qquad$ 5. $R E C T \sim A N G L$ $\frac{\text { Area of } R E C T}{\text { Area of ANGL }}=$ $\qquad$

4. The ratio of the corresponding midsegments of two similar trapezoids is $4: 5$. What is the ratio of their areas?
5. The ratio of the areas of two similar pentagons is $4: 9$. What is the ratio of their corresponding sides?
6. If $A B C D E \sim F G H I J, A C=6 \mathrm{~cm}, F H=10 \mathrm{~cm}$, and area of $A B C D E=320 \mathrm{~cm}^{2}$, then area of $F G H I J=$ $\qquad$ .
7. Stefan is helping his mother retile the kitchen floor. The tiles are 4 -by- 4 -inch squares. The kitchen is square, and the area of the floor is 144 square feet. Assuming the tiles fit snugly (don't worry about grout), how many tiles will be needed to cover the floor?

## Lesson 11.6 • Proportions with Volume

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters unless otherwise indicated.
In Exercises 1 and 2, decide whether or not the two solids are similar.
1.


2.

3. The triangular prisms are similar and the ratio of $a$ to $b$ is $\frac{5}{2}$.

Volume of large prism $=250 \mathrm{~cm}^{3}$
Volume of smaller prism $=$ $\qquad$

4. The right cylinders are similar and $r=10 \mathrm{~cm}$.

Volume of large cylinder $=64 \mathrm{~cm}$
Volume of small cylinder $=8 \mathrm{~cm}$
$R=$ $\qquad$

5. The corresponding heights of two similar cylinders is $2: 5$. What is the ratio of their volumes?
6. A rectangular prism aquarium holds 64 gallons of water. A similarly shaped aquarium holds 8 gallons of water. If a $1.5 \mathrm{ft}^{2}$ cover fits on the smaller tank, what is the area of a cover that will fit on the larger tank?

## Lesson 11.7• Proportional Segments Between Parallel Lines

Name $\qquad$ Period $\qquad$ Date $\qquad$

All measurements are in centimeters.

1. $x=$ $\qquad$

2. Is $\overline{X Y} \| \overline{B C}$ ?

3. Is $\overline{X Y} \| \overline{M K}$ ?

4. $N E=$

5. $P R=$ $\qquad$
$P Q=$ $\qquad$
$R I=$ $\qquad$

6. $x=$ $\qquad$
$y=$
7. $R S=$ $\qquad$
$E B=$ $\qquad$


8. $a=$ $\qquad$
$b=$ $\qquad$

9. $p=$ $\qquad$
$q=$ $\qquad$


## Lesson 12.1 • Trigonometric Ratios

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-4, give each answer as a fraction in terms of $p, q$, and $r$.

1. $\sin P=$ $\qquad$
2. $\cos P=$ $\qquad$
3. $\tan P=$ $\qquad$
4. $\sin Q=$ $\qquad$


In Exercises 5-8, give each answer as a decimal accurate to the nearest 0.001.
5. $\sin T=$ $\qquad$
6. $\cos T=$ $\qquad$
7. $\tan T=$ $\qquad$
8. $\sin R=$ $\qquad$


For Exercises 9-11, solve for $x$. Express each answer accurate to the nearest 0.01 .
9. $\cos 64^{\circ}=\frac{x}{28}$
10. $\sin 24^{\circ}=\frac{12.1}{x}$
11. $\tan 51^{\circ}=\frac{x}{14.8}$

For Exercises 12-14, find the measure of each angle to the nearest degree.
12. $\sin A=0.9455$
13. $\tan B=\frac{4}{3}$
14. $\cos C=0.8660$

For Exercises 15-17, write a trigonometric equation you can use to solve for the unknown value. Then find the value to the nearest 0.1.
15. $w \approx$ $\qquad$

16. $x \approx$ $\qquad$
17. $y \approx$ $\qquad$


For Exercises 18-20, find the value of each unknown to the nearest degree.
18. $a \approx$ $\qquad$
19. $t \approx$ $\qquad$
20. $z \approx$ $\qquad$



## Lesson 12.2•Problem Solving with Right Triangles

Name $\qquad$ Period $\qquad$ Date $\qquad$

For Exercises 1-3, find the area of each figure to the nearest square unit.

1. Area $\approx$

$\qquad$
2. Area $\approx$ $\qquad$

3. Area $\approx$ $\qquad$


For Exercises 4-9, find each unknown to the nearest tenth of a unit.
4. Area $=88 \mathrm{~cm}^{2}$
$x \approx$ $\qquad$
$\frac{4}{16 \mathrm{~cm}}$
5. $y \approx$ $\qquad$
6. $a \approx$ $\qquad$

7. $\overrightarrow{P S}$ and $\overrightarrow{P T}$ are tangents.

Diameter $\approx$ $\qquad$

8. Right cone
$\theta \approx$ $\qquad$
9. Right rectangular prism
$m \angle A B C=\beta \approx$ $\qquad$


In Exercises 10-12, give each answer to the nearest tenth of a unit.
10. A ladder 7 m long stands on level ground and makes a $73^{\circ}$ angle with the ground as it rests against a wall. How far from the wall is the base of the ladder?
11. To see the top of a building 1000 feet away, you look up $24^{\circ}$ from the horizontal. What is the height of the building?
12. A guy wire is anchored 12 feet from the base of a pole. The wire makes a $58^{\circ}$ angle with the ground. How long is the wire?

## Lesson 12.3• The Law of Sines

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find the area of each figure to the nearest square unit.

1. Area $\approx$ $\qquad$

2. Area $\approx$ $\qquad$
3. Area $\approx$ $\qquad$


In Exercises 4-6, find each length to the nearest centimeter. All lengths are in centimeters.
4. $m \approx$ $\qquad$

5. $p \approx$ $\qquad$

6. $q \approx$ $\qquad$


In Exercises 7-9, find the measure of each angle to the nearest degree.
7. $m \angle B \approx$ $\qquad$
$m \angle C \approx$ $\qquad$

8. $m \angle P \approx$ $\qquad$
9. $m \angle K \approx$ $\qquad$
$m \angle Q \approx$ $\qquad$
$m \angle M \approx$ $\qquad$

10. A large helium balloon is tethered to the ground by two taut lines. One line is 100 feet long and makes an $80^{\circ}$ angle with the ground. The second line makes a $40^{\circ}$ angle with the ground. How long is the second line, to the nearest foot? How far apart are the tethers?

## Lesson 12.4• The Law of Cosines

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-3, find each length to the nearest centimeter. All lengths are in centimeters.

1. $t \approx$ $\qquad$

2. $b \approx$ $\qquad$

3. $w \approx$ $\qquad$


In Exercises 4-6, find each angle measure to the nearest degree.
4. $m \angle A \approx$ $\qquad$
$m \angle B \approx$ $\qquad$
$m \angle C \approx$ $\qquad$

5. $m \angle A \approx$ $\qquad$
$m \angle P \approx$ $\qquad$
$m \angle S \approx$ $\qquad$

6. $m \angle S \approx$ $\qquad$
$m \angle U \approx$ $\qquad$
$m \angle V \approx$ $\qquad$

7. A circle with radius 12 in . has radii drawn to the endpoints of a 5 in. chord. What is the measure of the central angle?
8. A parallelogram has side lengths 22.5 cm and 47.8 cm . One angle measures $116^{\circ}$. What is the length of the shorter diagonal?
9. The diagonals of a parallelogram are 60 in . and 70 in . and intersect at an angle measuring $64^{\circ}$. Find the length of the shorter side of the parallelogram.

## Lesson $12.5 \cdot$ Problem Solving with Trigonometry

Name $\qquad$
$\qquad$ Date $\qquad$

1. While floating down a river with a $2.75 \mathrm{mi} / \mathrm{h}$ current, Alicia decides to swim directly toward the river bank. She can swim $0.75 \mathrm{mi} / \mathrm{h}$ in still water. What is the actual speed at which she moves toward the bank? At what angle will she approach the bank, measured with respect to the bank?
2. Find the measure of each angle to the nearest hundredth of a degree.

3. Two fire watchtowers 8.4 km apart spot a fire at the same time. Tower 1 reports the fire at a $36^{\circ}$ angle measure from its line of sight to Tower 2. Tower 2 reports a $68^{\circ}$ angle measure between the fire and Tower 1. How far is the fire from each tower?
4. Two airplanes leave O'Hare Airport in Chicago at the same time. One plane flies $280 \mathrm{mi} / \mathrm{h}$ at bearing $55^{\circ}$. The other plane flies $350 \mathrm{mi} / \mathrm{h}$ at bearing $128^{\circ}$. How far apart are the two planes after 2 hours 15 minutes?
5. Carla needs to fence her triangular plot of land. The angle between the two shorter sides measures $83^{\circ}$. The shortest side is 122 ft and the longest is 215 ft . How much fencing does Carla need? What is the area of her plot of land?

## Lesson 13.1• The Premises of Geometry

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Provide the missing property of equality or arithmetic as a reason for each step to solve the equation.

Solve for $x: 5(x-4)=2 x+17$
Solution: $\quad 5(x-4)=2 x+17$
a. $\qquad$

$$
\begin{aligned}
5 x-20 & =2 x+17 \\
3 x-20 & =17 \\
3 x & =37 \\
x & =\frac{37}{3}
\end{aligned}
$$

b. $\qquad$
c. $\qquad$
d. $\qquad$
e. $\qquad$

In Exercises 2-4, identify each statement as true or false. If the statement is true, tell which definition, property, or postulate supports your answer. If the statement is false, give a counterexample.
2. If $A M=B M$, then $M$ is the midpoint of $\overline{A B}$.
3. If $P$ is on $\overleftrightarrow{A B}$ and $D$ is not, then $m \angle A P D+m \angle B P D=180^{\circ}$.
4. If $\overline{P Q} \cong \overline{S T}$ and $\overline{P Q} \cong \overline{K L}$, then $S T=K L$.
5. Complete the flowchart proof.

Given: $\overline{A B}\|\overline{C D}, \overline{A P}\| \overline{C Q}, \overline{P B} \cong \overline{Q D}$
Show: $\overline{A B} \cong \overline{C D}$

Flowchart Proof



## Lesson 13.2•Planning a Geometry Proof

Name $\qquad$ Period $\qquad$ Date $\qquad$

For these exercises, you may use theorems added to your theorem list through the end of Lesson 13.2.

In Exercises 1-3, write a paragraph proof or a flowchart proof for each situation.

1. Given: $\overline{A B}\|\overline{C D}, \overline{A P}\| \overline{C Q}$

Show: $\angle P A B \cong \angle Q C D$

2. Given: $\overline{P Q} \| \overline{S T}, \angle Q P R \cong \angle S T U$

Show: $\overline{P R} \| \overline{U T}$

3. Given: Noncongruent, nonparallel segments $A B, B C$, and $A C$

Show: $x+y+z=180^{\circ}$


## Lesson 13.3•Triangle Proofs

Name $\qquad$ Period $\qquad$ Date $\qquad$

Write a proof for each situation. You may use theorems added to your theorem list through the end of Lesson 13.3.

1. Given: $\overline{X Y} \cong \overline{Z Y}, \overline{X Z} \perp \overline{W Y}$
Show: $\triangle W X Y \cong \triangle W Z Y$

2. Given: $\overline{C D} \perp \overline{A C}, \overline{B D} \perp \overline{A B}, \overline{C D} \cong \overline{B D}$
Show: $\triangle A B D \cong \triangle A C D$

3. Given: $\overline{M N} \cong \overline{Q M}, \overline{N O} \cong \overline{Q M}$, $P$ is the midpoint of $\overline{M O}$
Show: $\angle Q M N \cong \angle R O N$

4. Given: $\frac{\overline{A B}}{\overline{A B}} \cong \overline{B C}, \angle A C B \cong \angle E C D$, $\overline{A B} \perp \overline{B D}$
Show: $\overline{B D} \perp \overline{C E}$


## Lesson 13.4•Quadrilateral Proofs

Name $\qquad$ Period $\qquad$ Date $\qquad$

In Exercises 1-6, write a proof of each conjecture on a separate piece of paper. You may use theorems added to your theorem list through the end of Lesson 13.4.

1. The diagonals of a parallelogram bisect each other. (Parallelogram Diagonals Theorem)
2. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (Converse of the Parallelogram Diagonals Theorem)
3. The diagonals of a rhombus bisect each other and are perpendicular. (Rhombus Diagonals Theorem)
4. If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus. (Converse of the Rhombus Diagonals Theorem)
5. If the base angles on one base of a trapezoid are congruent, then the trapezoid is isosceles. (Converse of the Isosceles Trapezoid Theorem)
6. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles. (Converse of the Isosceles Trapezoid Diagonals Theorem)
In Exercises 7-9, decide if the statement is true or false. If it is true, prove it. If it is false, give a counterexample.
7. A quadrilateral with one pair of parallel sides and one pair of congruent angles is a parallelogram.
8. A quadrilateral with one pair of congruent opposite sides and one pair of parallel sides is a parallelogram.
9. A quadrilateral with one pair of parallel sides and one pair of congruent opposite angles is a parallelogram.

## Lesson 13.5•Indirect Proof

Name $\qquad$ Period $\qquad$ Date $\qquad$

1. Complete the indirect proof of the conjecture: In a triangle the side opposite the larger of two angles has a greater measure.

Given: $\triangle A B C$ with $m \angle A>m \angle B$
Show: $B C>A C$
Proof: Assume $\qquad$


Case 1: $B C=A C$
If $B C=A C$, then $\triangle A B C$ is $\qquad$ by $\qquad$ .
By $\qquad$ , $\angle A \cong \angle B$, which contradicts $\qquad$ .
So, $B C \neq A C$.
Case 2: $B C<A C$
If $B C<A C$, then it is possible to construct point $D$ on $\overline{C A}$ such that $\overline{C D} \cong \overline{C B}$, by the Segment Duplication Postulate. Construct $\overline{D B}$, by the Line Postulate. $\triangle D B C$ is $\qquad$ . Complete the proof.


In Exercises 2-5, write an indirect proof of each conjecture.
2. Given: $\overline{A D} \cong \overline{A B}, \overline{D C} \not \equiv \overline{B C}$

Show: $\angle D A C \not \equiv \angle B A C$

3. If two sides of a triangle are not congruent, then the angles opposite them are not congruent.
4. If two lines are parallel and a third line in the same plane intersects one of them, then it also intersects the other.

## Lesson 13.6•Circle Proofs

Name $\qquad$ Period $\qquad$ Date $\qquad$

Write a proof for each conjecture or situation. You may use theorems added to your theorem list through the end of Lesson 13.6.

1. If two chords in a circle are congruent, then their arcs are congruent.
2. Given: Regular pentagon $A B C D E$ inscribed in circle $O$, with diagonals $\overline{A C}$ and $\overline{A D}$
Show: $\overline{A C}$ and $\overline{A D}$ trisect $\angle B A E$

3. Given: Two circles externally tangent at $R$, common external tangent segment $\overline{T S}$
Show: $\angle T R S$ is a right angle

4. Given: Two circles internally tangent at $T$ with chords $\overline{T D}$ and $\overline{T B}$ of the larger circle intersecting the smaller circle at $C$ and $A$
Show: $\overline{A C} \| \overline{B D}$


## Lesson 13.7 • Similarity Proofs

Name $\qquad$ Period $\qquad$ Date $\qquad$

Write a proof for each situation. You may use theorems added to your theorem list through the end of Lesson 13.7.

1. Given: $\triangle A B C$ with $\angle A \cong \angle B C D$

Show: $B C^{2}=A B \cdot B D$

2. The diagonals of a trapezoid divide each other into segments with lengths in the same ratio as the lengths of the bases.
3. In a right triangle the product of the lengths of the two legs equals the product of the lengths of the hypotenuse and the altitude to the hypotenuse.
4. If a quadrilateral has one pair of opposite right angles and one pair of opposite congruent sides, then the quadrilateral is a rectangle.

## A N S W ERS

LESSON $1.1 \cdot$ Building Blocks of Geometry

1. $S$
2. 9 cm
3. $\overline{S N}$
4. endpoint
5. $\overline{N S}$
6. $\overline{P Q}$
7. $\overrightarrow{S P}$
8. $\overline{K N} \cong \overline{K L}, \overline{N M} \cong \overline{L M}, \overline{N O} \cong \overline{L O}$
9. $E(-14,15)$
10. 


11.

12. $\overleftrightarrow{A B}, \overleftrightarrow{A C}, \overleftrightarrow{A D}, \overleftrightarrow{A E}, \overleftrightarrow{A F}, \overleftrightarrow{B C}, \overleftrightarrow{B D}$, $\overleftrightarrow{B E}, \overleftrightarrow{B F}, \overleftrightarrow{C D}, \overleftrightarrow{C E}, \overleftrightarrow{C F}, \overleftrightarrow{D E}, \overleftrightarrow{D F}$, $\overleftrightarrow{E F}(15$ lines $)$

13. Possible coplanar set: $\{C, D, H, G\} ; 12$ different sets

## LESSON $1.2 \cdot$ Poolroom Math

1. vertex
2. bisector
3. side
4. $126^{\circ}$
5. $\angle D A E$
6. $133^{\circ}$
7. $47^{\circ}$
8. $63^{\circ}$
9. $70^{\circ}$
10. 


11.

12.

13. $90^{\circ}$
14. $120^{\circ}$
15. $75^{\circ}$
16.

17.


## LESSON 1.3•What's a Widget?

1. d
2. c
3. e
4. i
5. f
6. b
7. h
8. a
9. g
10. They have the same measure, $13^{\circ}$. Because $m \angle Q=77^{\circ}$, its complement has measure $13^{\circ}$. So $m \angle R=13^{\circ}$, which is the same as $m \angle P$.
11. 


12.

13.


## LESSON 1.4•Polygons

| Polygon <br> name | Number of <br> sides | Number of <br> diagonals |
| :--- | :---: | :---: |
| 1. Triangle | $\mathbf{3}$ | $\mathbf{0}$ |
| 2. Quadrilateral | $\mathbf{4}$ | 2 |
| 3. Pentagon | 5 | $\mathbf{5}$ |
| 4. Hexagon | $\mathbf{6}$ | $\mathbf{9}$ |
| 5. Heptagon | 7 | $\mathbf{1 4}$ |
| 6. Octagon | 8 | $\mathbf{2 0}$ |
| 7. Decagon | $\mathbf{1 0}$ | 35 |
| 8. Dodecagon | 12 | $\mathbf{5 4}$ |

9. 


11. $\overline{A C}, \overline{A D}, \overline{A E}$
12. Possible answer: $\overline{A B}$ and $\overline{B C}$
13. Possible answer: $\angle A$ and $\angle B$
14. Possible answer: $\overline{A C}$ and $\overline{F D}$
15. $82^{\circ}$
16. 7.2
17. $61^{\circ}$
18. 16.1
19. 6.2 cm

## LESSON $1.5 \cdot$ Triangles

For Exercises 1-7, answers will vary. Possible answers are shown.

1. $\overline{A B} \| \overline{G H}$
2. $\overline{E F} \perp \overline{B I}$
3. $\overline{C G} \cong \overline{F H}$
4. $\angle D E G$ and $\angle G E F$
5. $\angle D E G$ and $\angle G E F$
6. 


7.

8.

9.


For Exercises 10-12, answers may vary. Possible answers are shown.
10. $F(8,-2)$
11. $D(4,3)$
12. $G(10,-2)$

## LESSON $1.6 \cdot$ Special Quadrilaterals

1. 


2.

3.

4.

5.


For Exercises 6-10, 12, and 13, answers may vary. Possible answers are shown.
6.

7. $A C F D$
8. $E F H G$
9. BFJD
10. $B F H D$
11. $D(0,3)$
12. $E(0,5)$
13. $G(16,3)$

## LESSON 1.7 • Circles

1. $48^{\circ}$
2. $132^{\circ}$
3. $228^{\circ}$
4. $312^{\circ}$
5. 


6.

7. $(8,2) ;(3,7) ;(3,-3)$
8.

9. The chord goes through the center, $P$. (It is a diameter.)

10.

11. Kite


## LESSON $1.8 \cdot$ Space Geometry


3. Rectangular prism

2.

4. Pentagonal prism

5.

6.

7. 18 cubes
8. $x=2, y=1$

## LESSON 1.9•A Picture Is Worth a Thousand Words

1. 


2. Dora, Ellen, Charles, Anica, Fred, Bruce

3.

4. Possible answers:
a.

b.

c.


## LESSON 2.1•Inductive Reasoning

1. 20,24
2. $12 \frac{1}{2}, 6 \frac{1}{4}$
3. $\frac{5}{4}, 2$
4. $-1,-1$
5. 72,60
6. 243,729
7. 91,140
8. 


9.


10.

11. False; $\left(\frac{1}{2}\right)^{2}=\frac{1}{4}$
12. False; $11 \cdot 10=110,11 \cdot 12=132$
13. True

## LESSON 2.2•Finding the nth Term

1. Linear
2. Linear
3. Not linear
4. Linear
5. 

| $n$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(n)$ | -5 | 2 | 9 | 16 | 23 |

6. 

| $n$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $g(n)$ | $\mathbf{- 1 0}$ | $\mathbf{- 1 8}$ | $\mathbf{- 2 6}$ | $\mathbf{- 3 4}$ | $\mathbf{- 4 2}$ |

7. $f(n)=4 n+5 ; f(50)=205$
8. $f(n)=-5 n+11 ; f(50)=-239$
9. $f(n)=\frac{1}{2} n+6 ; f(50)=31$
10. 

| $n$ | 1 | 2 | 3 | 4 | 5 | $\ldots$ | $n$ | $\ldots$ | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of triangles | 1 | 5 | 9 | 13 | 17 | $\ldots$ | $4 n-3$ | $\ldots$ | 197 |

11. 

| $n$ | 1 | 2 | 3 | 4 | 5 | $\ldots$ | $n$ | $\ldots$ | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Area <br> of figure | 1 | 4 | 16 | 64 | 256 | $\ldots$ | $4^{n-1}$ | $\ldots$ | $4^{49}$ |

## LESSON 2.3•Mathematical Modeling

1. 


a. 240
b. 1350
c. $f(n)=2 n(n+2)$, or $f(n)=2 n^{2}+4 n$
2.

a. 8 sequences
b. 3 sequences have 1 tail.
c. $\frac{3}{8}$
3. 66 different pairs. Use a dodecagon showing sides and diagonals.
4. Answers will vary. Possible answers:


5 teams, 10 games


6 teams, 7 games


## LESSON 2.4•Deductive Reasoning

1. No. Explanations will vary. Sample explanation: Because $\triangle A B C$ is equilateral, $A B=B C$. Because $C$ lies between $B$ and $D, B D>B C$, so $B D$ is not equal to $A B$. Thus $\triangle A B D$ is not equilateral, by deductive reasoning.
2. Answers will vary. $m \angle E>m \angle D(m \angle E=$ $m \angle D+90^{\circ}$ ); deductive
3. a, e, f; inductive
4. Deductive
a. $4 x+3(2-x)=8-2 x \quad$ The original equation.

$$
\begin{array}{rlrl}
4 x+6-3 x & =8-2 x & & \text { Distributive property. } \\
x+6 & =8-2 x & & \text { Combining like terms. } \\
3 x+6 & =8 & & \text { Addition property of } \\
\text { equality. }
\end{array}
$$

$3 x=2 \quad \begin{aligned} & \text { Subtraction property of } \\ & \text { equality. }\end{aligned}$ equality.
$x=\frac{2}{3} \quad$ Division property of equality.
b. $\frac{19-2(3 x-1)}{5}=x+2 \quad$ The original equation.

$$
\begin{aligned}
& 19-2(3 x-1)=5(x+2) \text { Multiplication property } \\
& \text { of equality. } \\
& 19-6 x+2=5 x+10 \text { Distributive property. } \\
& 21-6 x=5 x+10 \quad \text { Combining like terms. } \\
& 21=11 x+10 \text { Addition property of } \\
& \text { equality. } \\
& 11=11 x \quad \text { Subtraction property of } \\
& \text { equality. } \\
& 1=x \quad \text { Division property of } \\
& \text { equality. }
\end{aligned}
$$

5. a. 16,21 ; inductive
b. $f(n)=5 n-9 ; 241$; deductive

## LESSON 2.5•Angle Relationships

1. $a=68^{\circ}, b=112^{\circ}, c=68^{\circ}$
2. $a=127^{\circ}$
3. $a=35^{\circ}, b=40^{\circ}, c=35^{\circ}, d=70^{\circ}$
4. $a=90^{\circ}, b=90^{\circ}, c=42^{\circ}, d=48^{\circ}, e=132^{\circ}$
5. $a=20^{\circ}, b=70^{\circ}, c=20^{\circ}, d=70^{\circ}, e=110^{\circ}$
6. $a=70^{\circ}, b=55^{\circ}, c=25^{\circ}$
7. Sometimes
8. Always
9. Never
10. Sometimes
11. acute
12. $158^{\circ}$
13. $90^{\circ}$
14. obtuse
15. converse

## LESSON 2.6•Special Angles on Parallel Lines

1. $a=54^{\circ}, b=54^{\circ}, c=54^{\circ}$
2. $a=115^{\circ}, b=65^{\circ}, c=115^{\circ}, d=65^{\circ}$
3. $a=72^{\circ}, b=126^{\circ}$
4. $\ell_{1} \| \ell_{2}$
5. $\ell_{1}| | \ell_{2}$
6. cannot be determined
7. $a=102^{\circ}, b=78^{\circ}, c=58^{\circ}, d=122^{\circ}, e=26^{\circ}$, $f=58^{\circ}$
8. $x=80^{\circ}$
9. $x=20^{\circ}, y=25^{\circ}$

## LESSON 3.1•Duplicating Segments and Angles

1. 


2. $X Y=3 P Q-2 R S$

3. Possible answer:

$$
128^{\circ}-35^{\circ}=93^{\circ}
$$


4.

5.

6.


## LESSON 3.2•Constructing Perpendicular Bisectors


3. $X Y=\frac{5}{4} A B$

4. $\triangle A B C$ is not unique.

5. $\triangle A B C$ is not unique.

6. $B D=A D=C D$

7. a. $A$ and $B$
b. $A, B$, and $C$
c. $A$ and $B$ and from $C$ and $D$ (but not from $B$ and C)
d. $A$ and $B$ and from $D$ and $E$

## LESSON 3.3 • Constructing Perpendiculars to a Line

1. False. The altitude from $A$ coincides with the side so it is not shorter.

2. False. In an isosceles triangle, an altitude and median coincide so they are of equal length.

3. True
4. False. In an acute triangle, all altitudes are inside. In a right triangle, one altitude is inside and two are sides. In an obtuse triangle, one altitude is inside and two are outside. There is no other possibility so exactly one altitude is never outside.
5. False. In an obtuse triangle, the intersection of the perpendicular bisectors is outside the triangle.
6. 


7.

8.

9. $W X=Y Z$


## LESSON 3.4•Constructing Angle Bisectors

1. a. $\ell_{1}$ and $\ell_{2}$
b. $\ell_{1}, \ell_{2}$, and $\ell_{3}$
c. $\ell_{2}, \ell_{3}$, and $\ell_{4}$
d. $\ell_{1}$ and $\ell_{2}$ and from $\ell_{3}$ and $\ell_{4}$
2. $\overrightarrow{A P}$ is the bisector of $\angle C A B$
3. $x=20^{\circ}, m \angle A B E=50^{\circ}$
4. 



5. They are concurrent.

6.

7. $R N=G N$ and $R O=H O$


## LESSON 3.5•Constructing Parallel Lines

1. 


2.

3.

4.

5.

6. Possible answer:


## LESSON 3.6•Construction Problems

1. Possible answer:

2. Possible answer:


3. Possible answers:

4. Possible answer:

5. Possible answer:

6. Possible answer:

7. Possible answer:


8. 



## LESSON 3.7•Constructing Points of Concurrency

1. Circumcenter
2. Locate the power-generation plant at the incenter. Locate each transformer at the foot of the perpendicular from the incenter to each side.
3. 


4. Possible answer: In the equilateral triangle, the centers of the inscribed and circumscribed circles are the same. In the obtuse triangle, one center is outside the triangle.

5. Possible answer: In an acute triangle, the circumcenter is inside the triangle. In a right triangle, it is on the hypotenuse. In an obtuse triangle, the circumcenter is outside the triangle. (Constructions not shown.)

## LESSON 3.8•The Centroid

1. 


2.

3. $C P=3.3 \mathrm{~cm}, C Q=5.7 \mathrm{~cm}, C R=4.8 \mathrm{~cm}$

4. $(3,4)$
5. $P C=16, C L=8, Q M=15, C R=14$
6. a. Incenter
b. Centroid
c. Circumcenter
d. Circumcenter
e. Orthocenter
f. Incenter
g. Centroid

## LESSON 4.1•Triangle Sum Conjecture

1. $p=67^{\circ}, q=15^{\circ}$
2. $x=82^{\circ}, y=81^{\circ}$
3. $a=78^{\circ}, b=29^{\circ}$
4. $r=40^{\circ}, s=40^{\circ}, t=100^{\circ}$
5. $x=31^{\circ}, y=64^{\circ}$
6. $y=145^{\circ}$
7. $s=28^{\circ}$
8. $m=72 \frac{1}{2}$ 。
9. $m \angle P=a$
10. $m \angle Q P T=135^{\circ}$
11. $720^{\circ}$
12. The sum of the measures of $\angle A$ and $\angle B$ is $90^{\circ}$ because $m \angle C$ is $90^{\circ}$ and all three angles must be $180^{\circ}$. So, $\angle A$ and $\angle B$ are complementary.
13. $m \angle B E A=m \angle C E D$ because they are vertical angles. Because the measures of all three angles in each triangle add to $180^{\circ}$, if equal measures are subtracted from each, what remains will be equal.
14. $m \angle T=64^{\circ}$
15. $m \angle G=45^{\circ}$
16. $x=125^{\circ}$
17. $m \angle A=39^{\circ}$, perimeter of $\triangle A B C=46 \mathrm{~cm}$
18. $L M=163 \mathrm{~m}, m \angle M=50^{\circ}$
19. $m \angle Q=44^{\circ}, Q R=125$
20. a. $\angle D A B \cong \angle A B D \cong \angle B D C \cong \angle B C D$
b. $\angle A D B \cong \angle C B D$
c. $\overline{A D} \| \overline{B C}$ by the Converse of the AIA Conjecture.
21. $x=21^{\circ}, y=16^{\circ}$
22. $m \angle Q P R=15^{\circ}$
23. $m \angle P R Q=55^{\circ}$ by VA, which makes $m \angle P=55^{\circ}$ by the Triangle Sum Conjecture. So, $\triangle P Q R$ is isosceles by the Converse of the Isosceles Triangle Conjecture.

## LESSON 4.3•Triangle Inequalities

1. Yes
2. No

3. $19<x<53$
4. $b>a>c$
5. $b>c>a$
6. $a>c=d>b$
7. $x=76^{\circ}$
8. $x=79^{\circ}$
9. The interior angle at $A$ is $60^{\circ}$. The interior angle at $B$ is $20^{\circ}$. But now the sum of the measures of the triangle is not $180^{\circ}$.
10. By the Exterior Angles Conjecture, $2 x=x+m \angle P Q S$. So, $m \angle P Q S=x$. So, by the Converse of the Isosceles Triangle Conjecture, $\triangle P Q S$ is isosceles.
11. Not possible. $A B+B C<A C$
12. 



LESSON 4.4•Are There Congruence Shortcuts?

1. SAA or ASA
2. SSS
3. SSS
4. $\triangle B Q M$ (SAS)
5. $\triangle$ TIE (SSS)
6. Cannot be determined, as shown by the figure.

7. $\triangle T N O$ (SAS)
8. Cannot be determined, as shown by the figure.

9. $\triangle D O G$ (SAS)
10. Only one triangle because of SSS.

11. Two possible triangles.

12. Only one triangle because of SAS.


## LESSON 4.5 • Are There Other Congruence Shortcuts?

1. Cannot be determined

2. $\triangle X Z Y$ (SAA)
3. $\triangle A C B$ (ASA or SAA)
4. $\triangle P R S$ (ASA)
5. $\triangle N R A(\mathrm{SAA})$
6. $\triangle G Q K$ (ASA or SAA)
7. Yes, $\triangle Q R S \cong \triangle M O L$ by SSS.
8. No, corresponding sides $\overline{T V}$ and $\overline{W V}$ are not congruent.
9. All triangles will be congruent by ASA. Possible triangle:

10. All triangles will be congruent by SAA. Possible procedure: Use $\angle A$ and $\angle C$ to construct $\angle B$ and then copy $\angle A$ and $\angle B$ at the ends of $\overline{A B}$.


## LESSON 4.6•Corresponding Parts of Congruent Triangles

1. SSS, SAS, ASA, SAA
2. $\overline{Y Z} \| \overline{W X}$, AIA Conjecture
3. $\overline{W Z} \| \overline{X Y}$, AIA Conjecture
4. ASA
5. СРСТС
6. $\triangle Y W M \cong \triangle Z X M$ by SAS. $\overline{Y W} \cong \overline{Z X}$ by CPCTC.
7. $\triangle A C D \cong \triangle B C D$ by SAS. $\overline{A D} \cong \overline{B D}$ by CPCTC.
8. Possible answer: $D E$ and $C F$ are both the distance between $\overleftrightarrow{D C}$ and $\overleftrightarrow{A B}$. Because the lines are parallel, the distances are equal. So, $\overline{D E} \cong \overline{C F}$.
9. Possible answer: $\overline{D E} \cong \overline{C F}$ (see Exercise 8 ). $\angle D E F \cong \angle C F E$ because both are right angles, $\overline{E F} \cong \overline{F E}$ because they are the same segment. So, $\triangle D E F \cong \triangle C F E$ by SAS. $\overline{E C} \cong \overline{F D}$ by CPCTC.
10. Possible answer: It is given that $T P=R A$ and $\angle P T R \cong \angle A R T$, and $\overline{T R} \cong \overline{R T}$ because they are the same segment. So $\triangle P T R \cong \angle A R T$ by SAS and $\overline{T A} \cong \overline{R P}$ by CPCTC.

## LESSON 4.7•Flowchart Thinking

1. (See flowchart proof at bottom of page 101.)
2. (See flowchart proof at bottom of page 101.)
3. (See flowchart proof at bottom of page 101.)

## LESSON 4.8• Proving Special Triangle Conjectures

1. $A D=8$
2. $m \angle A C D=36^{\circ}$
3. $m \angle B=52^{\circ}, C B=13$
4. $m \angle E=60^{\circ}$
5. $A N=17$
6. Perimeter $A B C D=104$
7. (See flowchart proof at bottom of page 102.)
8. Flowchart Proof


## LESSON 5.1• Polygon Sum Conjecture

1. $a=103^{\circ}, b=103^{\circ}, c=97^{\circ}, d=83^{\circ}, e=154^{\circ}$
2. $a=92^{\circ}, b=44^{\circ}, c=51^{\circ}, d=85^{\circ}, e=44^{\circ}, f=136^{\circ}$
3. $170^{\circ}$; 36 sides 4. 15 sides
4. $x=105^{\circ}$
5. $x=18^{\circ}$
6. $m \angle E=150^{\circ}$


## LESSON 5.2•Exterior Angles of a Polygon

1. 12 sides
2. 24 sides
3. 4 sides
4. 6 sides
5. $a=64^{\circ}, b=138 \frac{2}{3}^{\circ}$
6. $a=102^{\circ}, b=9^{\circ}$
7. $a=156^{\circ}, b=132^{\circ}, c=108^{\circ}$
8. $a=135^{\circ}, b=40^{\circ}, c=105^{\circ}, d=135^{\circ}$
9. 



Lesson 4.7, Exercises 1, 2, 3
1.

2.

3.


## LESSON 5.3 - Kite and Trapezoid Properties

1. $x=30$
2. $x=124^{\circ}, y=56^{\circ}$
3. $x=64^{\circ}, y=43^{\circ}$
4. $x=12^{\circ}, y=49^{\circ}$
5. $P S=33$
6. $a>11$
7. 


8.

9. Possible answer:

Paragraph proof: Draw $\overline{A E} \| \overline{P T}$ with $E$ on $\overline{T R}$. $T E A P$ is a parallelogram. $\angle T \cong \angle A E R$ because they are corresponding angles of parallel lines. $\angle T \cong \angle R$ because it is given, so $\angle A E R \cong \angle R$, because both are congruent to $\angle T$. Therefore, $\triangle A E R$ is isosceles by the Converse of the Isosceles Triangle Conjecture. $\overline{T P} \cong \overline{E A}$ because they are opposite sides of a parallelogram and $\overline{A R} \cong \overline{E A}$ because $\triangle A E R$ is isosceles. Therefore, $\overline{T P} \cong \overline{R A}$ because both are congruent to $\overline{E A}$.

## LESSON 5.4 • Properties of Midsegments

1. $a=89^{\circ}, b=54^{\circ}, c=91^{\circ}$
2. $x=21, y=7, z=32$
3. $x=17, y=11, z=6.5$
4. Perimeter $\triangle X Y Z=66, P Q=37, Z X=27.5$
5. $M(12,6), N(14.5,2)$; slope $\overline{A B}=-1.6$, slope $\overline{M N}=-1.6$
6. Pick a point $P$ from which $A$ and $B$ can be viewed over land. Measure $A P$ and $B P$ and find the midpoints $M$ and $N . A B=2 M N$.

7. $A M N O$ is a parallelogram. By the Triangle Midsegment Conjecture, $\overline{O N} \| \overline{A M}$ and $\overline{M N} \| \overline{A O}$.
Flowchart Proof

8. Paragraph proof: Looking at $\triangle F G R, \overline{H I} \| \overline{F G}$ by the Triangle Midsegment Conjecture. Looking at $\triangle P Q R, \overline{F G} \| \overline{P Q}$ for the same reason. Because $\overline{F G} \| \overline{P Q}$, quadrilateral $F G Q P$ is a trapezoid and $\overline{D E}$ is the midsegment, so it is parallel to $\overline{F G}$ and $\overline{P Q}$. Therefore, $\overline{H I}\|\overline{F G}\| \overline{D E} \| \overline{P Q}$.

Lesson 4.8, Exercise 7


1. Perimeter $A B C D=82 \mathrm{~cm}$
2. $A C=22, B D=14$
3. $A B=16, B C=7$
4. $a=51^{\circ}, b=48^{\circ}, c=70^{\circ}$
5. $A B=35.5$
6. $a=41^{\circ}, b=86^{\circ}, c=53^{\circ}$
7. $A D=75$
8. 


9. $a=38^{\circ}, b=142^{\circ}, c=142^{\circ}, d=38^{\circ}, e=142^{\circ}$, $f=38^{\circ}, g=52^{\circ}, h=12^{\circ}, i=61^{\circ}, j=81^{\circ}, k=61^{\circ}$
10. No


LESSON 5.6•Properties of Special Parallelograms

1. $O Q=16, m \angle Q R S=90^{\circ}, P R=32$
2. $m \angle O K L=45^{\circ}, m \angle M O L=90^{\circ}$, perimeter $K L M N=32$
3. $O B=6, B C=11, m \angle A O D=90^{\circ}$
4. c, d, f, g 5. d, e, g
5. f, g
6. h
7. c, d, f, g
8. c, d, f, g
9. d, g
10. d, g
11. None
12. Parallelogram
13. 



## LESSON 5.7• Proving Quadrilateral Properties

1. (See flowchart proof at bottom of page.)

## 2. Flowchart Proof


3. Flowchart Proof


## Lesson 5.7, Exercise 1


4. Flowchart Proof


## LESSON 6.1 • Tangent Properties

1. $w=126^{\circ}$
2. $m \angle B Q X=65^{\circ}$
3. a. $m \angle N Q P=90^{\circ}, m \angle M P Q=90^{\circ}$
b. Trapezoid. Possible explanation: $\overline{M P}$ and $\overline{N Q}$ are both perpendicular to $\overline{P Q}$, so they are parallel to each other. The distance from $M$ to $\overline{P Q}$ is $M P$, and the distance from $N$ to $\overline{P Q}$ is $N Q$. But the two circles are not congruent, so $M P \neq N Q$. Therefore, $\overline{M N}$ is not a constant distance from $\overline{P Q}$ and they are not parallel. Exactly one pair of sides is parallel, so $M N Q P$ is a trapezoid.
4. $y=-\frac{1}{3} x+10$
5. Possible answer: Tangent segments from a point to a circle are congruent. So, $\overline{P A} \cong \overline{P B}, \overline{P B} \cong \overline{P C}$, and $\overline{P C} \cong \overline{P D}$. Therefore, $\overline{P A} \cong \overline{P D}$.
6. a. 4.85 cm
b. 11.55 cm
7. 



## LESSON 6.2•Chord Properties

1. $a=95^{\circ}, b=85^{\circ}, c=47.5^{\circ}$
2. $v$ cannot be determined, $w=90^{\circ}$
3. $z=45^{\circ}$
4. $w=100^{\circ}, x=50^{\circ}, y=110^{\circ}$
5. $w=49^{\circ}, x=122.5^{\circ}, y=65.5^{\circ}$
6. $x=16 \mathrm{~cm}, y$ cannot be determined
7. Kite. Possible explanation: $\overline{O M} \cong \overline{O N}$ because congruent chords $\overline{A B}$ and $\overline{A C}$ are the same distance from the center. $\overline{A M} \cong \overline{A N}$ because they are halves of congruent chords. So, $A M O N$ has two pairs of adjacent congruent sides and is a kite.
8. The perpendicular segment from the center of the circle bisects the chord, so the chord has length 12 units. But the diameter of the circle is 12 units, and the chord cannot be as long as the diameter because it doesn't pass through the center of the circle.
9. $P(0,1), M(4,2)$
10. $m \widehat{A B}=49^{\circ}, m \overline{A B C}=253^{\circ}, m \widehat{B A C}=156^{\circ}$, $m \widehat{A C B}=311^{\circ}$
11. Possible answer: Fold and crease to match the endpoints of the arc. The crease is the perpendicular bisector of the chord connecting the endpoints. Fold and crease so that one endpoint falls on any other point on the arc. The crease is the perpendicular bisector of the chord between the two matching points. The center is the intersection of the two creases.


## LESSON 6.3•Arcs and Angles

1. $m \angle X N M=40^{\circ}, m \overline{X N}=180^{\circ}, m \overline{M N}=100^{\circ}$
2. $x=120^{\circ}, y=60^{\circ}, z=120^{\circ}$
3. $a=90^{\circ}, b=55^{\circ}, c=35^{\circ}$
4. $a=50^{\circ}, b=60^{\circ}, c=70^{\circ}$
5. $x=140^{\circ}$
6. $m \angle A=90^{\circ}, m \widehat{A B}=72^{\circ}, m \angle C=36^{\circ}, m \widehat{C B}=108^{\circ}$
7. $m \overline{A D}=140^{\circ}, m \angle D=30^{\circ}, m \widehat{A B}=60^{\circ}$, $m \widehat{D A B}=200^{\circ}$
8. $p=128^{\circ}, q=87^{\circ}, r=58^{\circ}, s=87^{\circ}$
9. $a=50^{\circ}, b=50^{\circ}, c=80^{\circ}, d=50^{\circ}, e=130^{\circ}$, $f=90^{\circ}, g=50^{\circ}, h=50^{\circ}, j=90^{\circ}, k=40^{\circ}$, $m=80^{\circ}, n=50^{\circ}$

## LESSON 6.4 • Proving Circle Conjectures

## 1. Flowchart Proof


2. Angles are numbered for reference.


## Paragraph Proof

It is given that $\overline{O E} \| \overline{A D}$, so $\angle 2 \cong \angle 1$ by the CA Conjecture. Because $\overline{O A}$ and $\overline{O D}$ are radii, they are congruent, so $\triangle A O D$ is isosceles. Therefore $\angle 4 \cong \angle 1$ by the IT Conjecture. Both $\angle 2$ and $\angle 4$ are congruent to $\angle 1$, so $\angle 2 \cong \angle 4$. By the AIA Conjecture, $\angle 4 \cong \angle 3$, so $\angle 2 \cong \angle 3$. The measure of an arc equals the measure of its central angle, so because their central angles are congruent, $\widetilde{D E} \cong \widehat{B E}$.

## 3. Flowchart Proof



## 4. Flowchart Proof

Construct radii $\overline{A O}, \overline{O B}, \overline{O C}$, and $\overline{O D}$.


## LESSON 6.5 • The Circumference/Diameter Ratio

1. $C=21 \pi \mathrm{~cm}$
2. $r=12.5 \mathrm{~cm}$
3. $C=60 \pi \mathrm{~cm}$
4. $d=24 \mathrm{~cm}$
5. $C \approx 30.2 \mathrm{~cm}$
6. $d \approx 42.0 \mathrm{~cm}, r \approx 21.0 \mathrm{~cm}$
7. $C \approx 37.7 \mathrm{in}$.
8. Yes; about 2.0 in.

9. $C \approx 75.4 \mathrm{~cm}$
10. Press the square against the tree as shown. Measure the tangent segment on the square. The tangent segment is the same length as the radius. Use $C=2 \pi r$ to find the circumference.

11. 4 cm

## LESSON 6.6 • Around the World

1. At least 7 olive pieces
2. About 2.5 rotations
3. $\frac{\left(2 \pi \cdot 4.23 \cdot 10^{7}\right)}{(60 \cdot 60 \cdot 23.93)} \approx 3085 \mathrm{~m} / \mathrm{s}$ (about $3 \mathrm{~km} / \mathrm{s}$ or just under $2 \mathrm{mi} / \mathrm{s}$ )
4. 6.05 cm or 9.23 cm
5. Sitting speed $=\frac{\left(2 \pi \cdot 1.4957 \cdot 10^{11} / 10^{3}\right)}{(364.25 \cdot 24)}$ $\approx 107,500 \mathrm{~km} / \mathrm{h}$

## LESSON $6.7 \cdot$ Arc Length

1. $4 \pi$
2. $4 \pi$
3. 30
4. $\frac{35 \pi}{9}$
5. $\frac{80 \pi}{9}$
6. $6.25 \pi$ or $\frac{25 \pi}{4}$
7. $\frac{100 \pi}{9}$
8. 31.5
9. $22 \pi$
10. 396

## EXPLORATION • Intersection Secants, Tangents, and

 Chords1. $x=21^{\circ}$
2. $m \widehat{D C}=70^{\circ}, m \widehat{E D}=150^{\circ}$
3. $m \widehat{D C}=114^{\circ}, m \angle D E C=66^{\circ}$
4. $m \angle B C E=75^{\circ}, m \overline{B A C}=210^{\circ}$
5. $x=80^{\circ}, y=110^{\circ}, z=141^{\circ}$
6. $x=34^{\circ}, y=150^{\circ}, z=122^{\circ}$
7. $x=112^{\circ}, y=68^{\circ}, z=53^{\circ}$
8. $x=28^{\circ}, y=34.5^{\circ}$

## LESSON 7.1•Transformations and Symmetry

1. 


2.

3.

4. Possible answers: The two points where the figure and the image intersect determine $\ell$. Or connect any two corresponding points and construct the perpendicular bisector, which is $\ell$.

5. 3-fold rotational symmetry, 3 lines of reflection
6. 2-fold rotational symmetry
7. 1 line of reflection
8. 1 line of reflection
9. 2-fold rotational symmetry, 2 lines of reflection

10. 2-fold rotational symmetry

11. 1 line of reflection

12. 4-fold rotational symmetry, 4 lines of reflection


## LESSON 7.2 • Properties of Isometries

1. Rotation

2. Translation

3. Reflection

4. 


5.

6. $(x, y) \rightarrow(x+13, y+6)$; translation; $B^{\prime}(8,8)$, $C(-8,-4)$
7. $(x, y) \rightarrow(-x, y)$; reflection across the $y$-axis; $P^{\prime}(7,-3), R(-4,5)$
8. $(x, y) \rightarrow(y, x)$; reflection across the line $y=x$; $T^{\prime}(7,0), R(0,3)$

## LESSON 7.3 - Compositions of Transformations

1. Translation by $\langle-2,+5\rangle$
2. Rotation $45^{\circ}$ counterclockwise
3. Translation by $\langle+16,0\rangle$
4. Rotation $180^{\circ}$ about the intersection of the two lines
5. Translation by $\langle-16,0\rangle$
6. Rotation $180^{\circ}$ about the intersection of the two lines
7. Reflection across the line $x=-3$
8. Reflection across the line $x=3$
9. $m \angle R O T=50^{\circ}$; rotation $100^{\circ}$ clockwise about $O$

10. Translation 2 cm along the line perpendicular to $k$ and $\ell$ in the direction from $k$ to $\ell$.

11. 


$\square \square$


## LESSON 7.4•Tessellations with Regular Polygons

1. $n=15$
2. $n=20$
3. Possible answer: A regular tessellation is a tessellation in which the tiles are congruent regular polygons whose edges exactly match.

4. Possible answer: A 1-uniform tiling is a tessellation in which all vertices are identical.

5. $3.4^{2} \cdot 6 / 3.6 .3 .6$
6. 



## LESSONS 7.5-7.8•Tessellations

1. 


2. Sample answer:

3. Sample answer:

4. Sample answer:


LESSON 8.1•Areas of Rectangles and Parallelograms

1. $112 \mathrm{~cm}^{2}$
2. $7.5 \mathrm{~cm}^{2}$
3. $110 \mathrm{~cm}^{2}$
4. $81 \mathrm{~cm}^{2}$
5. 61 m
6. No. Possible answer:

7. 88 units $^{2}$
8. 72 units $^{2}$
9. No. Carpet area is $20 \mathrm{yd}^{2}=180 \mathrm{ft}^{2}$. Room area is $(21.5 \mathrm{ft})(16.5 \mathrm{ft})=206.25 \mathrm{ft}^{2}$. Dana will be $26 \frac{1}{4} \mathrm{ft}^{2}$ short.

## LESSON 8.2•Areas of Triangles, Trapezoids, and Kites

1. 16 ft
2. $20 \mathrm{~cm}^{2}$
3. $b=12 \mathrm{in}$.
4. $A D=4.8 \mathrm{~cm}$
5. $40 \mathrm{~cm}^{2}$
6. $88 \mathrm{~cm}^{2}$
7. 54 units $^{2}$
8. $135 \mathrm{~cm}^{2}$

## LESSON 8.3•Area Problems

1. a. $549.5 \mathrm{ft}^{2}$
b. 40 bundles; $\$ 1596.00$
2. 500 L
3. Possible answer:

4. It is too late to change the area. The length of the diagonals determines the area.

## LESSON 8.4•Areas of Regular Polygons

1. $A \approx 696 \mathrm{~cm}^{2}$
2. $a \approx 7.8 \mathrm{~cm}$
3. $p \approx 43.6 \mathrm{~cm}$
4. $n=10$
5. $s=4 \mathrm{~cm}, a \approx 2.8 \mathrm{~cm}, A \approx 28 \mathrm{~cm}^{2}$

6. Possible answer ( $s$ will vary): $s \approx 3.1 \mathrm{~cm}, a \approx 3.7 \mathrm{~cm}$, $A \approx 45.9 \mathrm{~cm}^{2}$

7. Approximately $31.5 \mathrm{~cm}^{2}$ : area of square $=36$; area of square within angle $=\frac{3}{8} \cdot 36=13.5$; area of octagon $\approx 120$; area of octagon within angle $\approx$ $\frac{3}{8} \cdot 120 \approx 45$; shaded area $\approx 45-13.5 \approx 31.5 \mathrm{~cm}^{2}$

## LESSON 8.5•Areas of Circles

1. $81 \pi \mathrm{~cm}^{2}$
2. $10.24 \pi \mathrm{~cm}^{2}$
3. 23 cm
4. $324 \pi \mathrm{~cm}^{2}$
5. $191.13 \mathrm{~cm}^{2}$
6. 41.41 cm
7. $7.65 \mathrm{~cm}^{2}$
8. $4.90 \mathrm{~cm}^{2}$
9. $51.3 \mathrm{~cm}^{2}$
10. 33.5 or $33.6 \mathrm{~cm}^{2}$
11. $(64 \pi-128)$ square units
12. $25 \pi \mathrm{~cm}^{2}$

## LESSON 8.6•Any Way You Slice It

1. $\frac{25 \pi}{12} \mathrm{~cm}^{2} \approx 6.54 \mathrm{~cm}^{2}$
2. $\frac{32 \pi}{3} \mathrm{~cm}^{2} \approx 33.51 \mathrm{~cm}^{2}$
3. $12 \pi \mathrm{~cm}^{2} \approx 37.70 \mathrm{~cm}^{2}$
4. $(16 \pi-32) \mathrm{cm}^{2} \approx 18.27 \mathrm{~cm}^{2}$
5. $13.5 \pi \mathrm{~cm}^{2} \approx 42.41 \mathrm{~cm}^{2}$
6. $10 \pi \mathrm{~cm}^{2} \approx 31.42 \mathrm{~cm}^{2}$
7. $r=10 \mathrm{~cm}$
8. $x=135^{\circ}$
9. $r=7 \mathrm{~cm}$

## LESSON 8.7 • Surface Area

1. $136 \mathrm{~cm}^{2}$
2. $240 \mathrm{~cm}^{2}$
3. $558.1 \mathrm{~cm}^{2}$
4. $796.4 \mathrm{~cm}^{2}$
5. $255.6 \mathrm{~cm}^{2}$
6. $356 \mathrm{~cm}^{2}$
7. $468 \mathrm{~cm}^{2}$
8. $1055.6 \mathrm{~cm}^{2}$
9. 1 sheet: front rectangle: $3 \cdot 1 \frac{1}{2}=4 \frac{1}{2}$; back rectangle: $3 \cdot 2 \frac{1}{2}=7 \frac{1}{2}$; bottom rectangle: $3 \cdot 2=6$; side trapezoids: $2\left(2 \cdot \frac{2 \frac{1}{2}+1 \frac{1}{2}}{2}\right)=8$; total $=26 \mathrm{ft}^{2}$. Area of 1 sheet $=4 \cdot 8=32 \mathrm{ft}^{2}$. Possible pattern:


## LESSON 9.1•The Theorem of Pythagoras

1. $a=21 \mathrm{~cm}$
2. $p \approx 23.9 \mathrm{~cm}$
3. $x=8 \mathrm{ft}$
4. $h \approx 14.3 \mathrm{in}$.
5. Area $\approx 19.0 \mathrm{ft}^{2}$
6. $C(11,-1) ; r=5$
7. Area $\approx 49.7 \mathrm{~cm}^{2}$
8. $R V \approx 15.4 \mathrm{~cm}$
9. If the base area is $16 \pi \mathrm{~cm}^{2}$, then the radius is 4 cm . The radius is a leg of the right triangle; the slant height is the hypotenuse. The leg cannot be longer than the hypotenuse.
10. Area $=150 \mathrm{in}^{2}$; hypotenuse $Q R=25$ in.; altitude to the hypotenuse $=12 \mathrm{in}$.

LESSON 9.2•The Converse of the Pythagorean Theorem

1. No
2. Yes
3. Yes
4. Yes
5. Area $\approx 21.22 \mathrm{~cm}^{2}$
6. The top triangle is equilateral, so half its side length is 2.5 . A triangle with sides $2.5,6$, and 6.5 is a right triangle because $2.5^{2}+6^{2}=6.5^{2}$. So, the angle marked $95^{\circ}$ should be $90^{\circ}$.
7. $x \approx 44.45 \mathrm{~cm}$. By the Converse of the Pythagorean Theorem, $\triangle A D C$ is a right triangle, and $\angle A D C$ is a right angle. $\angle A D C$ and $\angle B D C$ are supplementary, so $\angle B D C$ is also a right triangle. Use the Pythagorean Theorem to find $x$.
8. $129.6 \mathrm{~cm}^{2}$

9. No. Because $A B^{2}+B C^{2} \neq A C^{2}, \angle B$ of $\triangle A B C$ is not a right angle.
10. Cannot be determined. The length of $C D$ is unknown. One possible quadrilateral is shown.

11. Yes. Using SSS, $\triangle A B C \cong \triangle B A D \cong \triangle C D A \cong$ $\triangle D C B$. That means that the four angles of the quadrilateral are all congruent by СРСТС. Because the four angles must sum to $360^{\circ}$ and they are all congruent, they must be right angles. So, $A B C D$ is a rectangle.

## LESSON 9.3•Two Special Right Triangles

1. $a=14 \sqrt{2} \mathrm{~cm}$
2. $a=12 \mathrm{~cm}, b=24 \mathrm{~cm}$
3. $a=12 \mathrm{~cm}, b=6 \sqrt{3} \mathrm{~cm}$
4. $64 \sqrt{3} \mathrm{~cm}^{2}$
5. Perimeter $=32+6 \sqrt{2}+6 \sqrt{3} \mathrm{~cm}$; area $=60+18 \sqrt{3} \mathrm{~cm}^{2}$
6. $A C=30 \sqrt{2} \mathrm{~cm} ; A B=30+30 \sqrt{3} \mathrm{~cm}$; area $=450+450 \sqrt{3} \mathrm{~cm}^{2}$
7. $45 \sqrt{3} \mathrm{~cm}^{2}$
8. $C\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
9. $C(-6 \sqrt{3},-6)$
10. Possible answer:


## LESSON 9.4•Story Problems

1. The foot is about 8.7 ft away from the base of the building. To lower it by 2 ft , move the foot an additional 3.3 ft away from the base of the building.
2. About 6.4 km

3. 149.5 linear feet of trim must be painted, or 224.3 feet ${ }^{2}$. Two coats means $448.6 \mathrm{ft}^{2}$ of coverage. Just over $2 \frac{1}{2}$ quarts of paint is needed. If Hans buys 3 quarts, he would have almost $\frac{1}{2}$ quart left. It is slightly cheaper to buy 1 gallon and have about $1 \frac{1}{2}$ quarts left. The choice is one of money versus conserving. Students may notice that the eaves extend beyond the exterior walls of the house and adjust their answer accordingly.
4. 14 in., $\frac{14}{\sqrt{3}}$ in. $\approx 8.08$ in., $\frac{28}{\sqrt{3}}$ in. $\approx 16.17$ in.


## LESSON 9.5•Distance in Coordinate Geometry

1. 10 units
2. 20 units
3. 17 units
4. $A B C D$ is a rhombus: All sides $=\sqrt{34}$, slope $\overline{A B}=-\frac{3}{5}$, slope $\overline{B C}=\frac{3}{5}$, so $\angle B$ is not a right angle, and $A B C D$ is not a square.
5. TUVW is an isosceles trapezoid: $\overline{T U}$ and $\overline{V W}$ have slope 1 , so they are parallel. $\overline{U V}$ and $\overline{T W}$ have length $\sqrt{20}$ and are not parallel (slope $\overline{U V}=-\frac{1}{2}$, slope $\overline{T W}=-2$ ).
6. Isosceles; perimeter $=32$ units
7. $M(7,10)$; $N(10,14)$; slope $\overline{M N}=\frac{4}{3}$; slope $\overline{B C}=\frac{4}{3}$; $M N=5 ; B C=10$; the slopes are equal; $M N=\frac{1}{2} B C$.
8. $(x+1)^{2}+(y-5)^{2}=4$
9. Center $(0,-2), r=5$
10. The distances from the center to the three points on the circle are not all the same: $A P=\sqrt{61}$, $B P=\sqrt{61}, C P=\sqrt{52}$
11. $(25 \pi-24) \mathrm{cm}^{2}$, or about $54.5 \mathrm{~cm}^{2}$
12. $(72 \sqrt{3}-24 \pi) \mathrm{cm}^{2}$, or about $49.3 \mathrm{~cm}^{2}$
13. $(\sqrt{5338}-37) \mathrm{cm} \approx 36.1 \mathrm{~cm}$
14. Area $=56.57 \pi \mathrm{~cm} \approx 177.7 \mathrm{~cm}^{2}$
15. $A D=\sqrt{115.04} \mathrm{~cm} \approx 10.7 \mathrm{~cm}$
16. $S T=9 \sqrt{3} \approx 15.6$
17. $150^{\circ}$

## LESSON 10.1•The Geometry of Solids

1. oblique
2. the axis
3. the altitude
4. bases
5. a radius
6. right
7. Circle $C$
8. $A$
9. $\overline{A C}$ or $A C$
10. $\overline{B C}$ or $B C$
11. Right pentagonal prism
12. ABCDE and FGHIJ
13. $\overline{A F}, \overline{B G}, \overline{C H}, \overline{D I}, \overline{E J}$
14. Any of $\overline{A F}, \overline{B G}, \overline{C H}, \overline{D I}, \overline{E J}$ or their lengths
15. False. The axis is not perpendicular to the base in an oblique cylinder.
16. False. A rectangular prism has six faces. Four are called lateral faces and two are called bases.
17. True
18. 


19.


## LESSON 10.2•Volume of Prisms and Cylinders

1. $232.16 \mathrm{~cm}^{3}$
2. $144 \mathrm{~cm}^{3}$
3. $415.69 \mathrm{~cm}^{3}$
4. $V=4 x y(2 x+3)$, or $8 x^{2} y+12 x y$
5. $V=\frac{1}{4} p^{2} h \pi$
6. $V=\left(6+\frac{1}{2} \pi\right) x^{2} y$
7. $6 \mathrm{ft}^{3}$
8. $80 \mathrm{~cm}^{3}$
9. $209.14 \mathrm{~cm}^{3}$
10. $615.75 \mathrm{~cm}^{3}$
11. $V=840 x^{3}$
12. $V=\frac{8}{3} \pi a^{2} b$
13. $V=4 \pi x y^{2}$
14. A: $128 \pi$ cubic units, $\mathrm{B}: 144 \pi$ cubic units. $\mathbf{B}$ is larger.
15. A: 5 cubic units, B: 5 cubic units.

They have equal volumes.
9. A: $9 \pi x$ cubic units, B: $27 \pi x$ cubic units. $\mathbf{B}$ is larger.

## LESSON 10.4 • Volume Problems

1. 4.4 cm
2. $1728 \mathrm{in}^{3}$
3. 24 cans; $3582 \mathrm{in}^{3}=2.07 \mathrm{ft}^{3} ; 34.6 \%$
4. 2000.6 lb (about 1 ton)
5. Note that $\overline{A E} \perp \overline{A B}$ and $\overline{E C} \perp \overline{B C} . V=\frac{8}{3} \mathrm{~cm}^{3}$; $S A=(8+4 \sqrt{2}) \mathrm{cm}^{2} \approx 13.7 \mathrm{~cm}^{2}$
6. About 110,447 gallons
7. 57 truckloads

## LESSON $10.5 \cdot$ Displacement and Density

All answers are approximate.

1. $53.0 \mathrm{~cm}^{3}$
2. $7.83 \mathrm{~g} / \mathrm{cm}^{3}$
3. $0.54 \mathrm{~g} / \mathrm{cm}^{3}$
4. 4.94 in .
5. No, it's not gold (or at least not pure gold). The mass of the nugget is 165 g , and the volume is $17.67 \mathrm{~cm}^{3}$, so the density is $9.34 \mathrm{~g} / \mathrm{cm}^{3}$. Pure gold has density $19.3 \mathrm{~g} / \mathrm{cm}^{3}$.

## LESSON 10.6 • Volume of a Sphere

1. $288 \pi \mathrm{~cm}^{3}$, or about $904.8 \mathrm{~cm}^{3}$
2. $18 \pi \mathrm{~cm}^{3}$, or about $56.5 \mathrm{~cm}^{3}$
3. $72 \pi \mathrm{~cm}^{3}$, or about $226.2 \mathrm{~cm}^{3}$
4. $\frac{28}{3} \pi \mathrm{~cm}^{3}$, or about $29.3 \mathrm{~cm}^{3}$
5. $432 \pi \mathrm{~cm}^{3}$, or about $1357.2 \mathrm{~cm}^{3}$
6. $\frac{304}{3} \pi \mathrm{~cm}^{3}$, or about $318.3 \mathrm{~cm}^{3}$
7. 11 cm
8. $2250 \pi$ in $^{3} \approx 7068.6$ in $^{3}$
9. $823.2 \mathrm{in}^{3} ; 47.6 \%$
10. 17.86

## LESSON 10.7•Surface Area of a Sphere

1. $V=1563.5 \mathrm{~cm}^{3} ; S=651.4 \mathrm{~cm}^{2}$
2. $V=184.3 \mathrm{~cm}^{3} ; S=163.4 \mathrm{~cm}^{2}$
3. $V=890.1 \mathrm{~cm}^{3} ; S=486.9 \mathrm{~cm}^{2}$
4. $V=34.1 \mathrm{~cm}^{3} ; S=61.1 \mathrm{~cm}^{2}$
5. About 3.9 cm
6. About $357.3 \mathrm{~cm}^{2}$
7. 9 quarts

## LESSON 11.1•Similar Polygons

1. $A P=8 \mathrm{~cm} ; E I=7 \mathrm{~cm} ; S N=15 \mathrm{~cm} ; Y R=12 \mathrm{~cm}$
2. $S L=5.2 \mathrm{~cm} ; M I=10 \mathrm{~cm} ; m \angle D=120^{\circ}$;
$m \angle U=85^{\circ} ; m \angle A=80^{\circ}$
3. Yes. All corresponding angles are congruent. Both figures are parallelograms, so opposite sides within each parallelogram are equal. The corresponding sides are proportional $\left(\frac{15}{5}=\frac{9}{3}\right)$.
4. Yes. Corresponding angles are congruent by the CA Conjecture. Corresponding sides are proportional $\left(\frac{2}{4}=\frac{3}{6}=\frac{4}{8}\right)$.
5. No. $\frac{6}{18} \neq \frac{8}{22}$.
6. Yes. All angles are right angles, so corresponding angles are congruent. The corresponding side lengths have the ratio $\frac{4}{7}$, so corresponding side lengths are proportional.
7. $\frac{1}{2}$

8. 4 to 1


## LESSON 11.2•Similar Triangles

1. $M C=10.5 \mathrm{~cm}$
2. $\angle Q \cong \angle X ; Q R=4.8 \mathrm{~cm} ; Q S=11.2 \mathrm{~cm}$
3. $\angle A \cong \angle E ; C D=13.5 \mathrm{~cm} ; A B=10 \mathrm{~cm}$
4. $T S=15 \mathrm{~cm} ; Q P=51 \mathrm{~cm}$
5. AA Similarity Conjecture
6. $C A=64 \mathrm{~cm}$
7. $\triangle A B C \sim \triangle E D C$. Possible explanation: $\angle A \cong \angle E$ and $\angle B \cong \angle D$ by AIA, so by the AA Similarity Conjecture, the triangles are similar.
8. $\triangle P Q R \sim \triangle S T R$. Possible explanation: $\angle P \cong \angle S$ and $\angle Q \cong \angle T$ because each pair is inscribed in the same arc, so by the AA Similarity Conjecture, the triangles are similar.
9. $\triangle M L K \sim \triangle N O K$. Possible explanation: $\angle M L K \cong \angle N O K$ by $C A$ and $\angle K \cong \angle K$ because they are the same angle, so by the AA Similarity Conjecture, the two triangles are similar.

## LESSON 11.3•Indirect Measurement with Similar Triangles

1. 27 ft
2. 6510 ft
3. 110.2 mi
4. About 18.5 ft
5. $0.6 \mathrm{~m}, 1.2 \mathrm{~m}, 1.8 \mathrm{~m}, 2.4 \mathrm{~m}$, and 3.0 m

## LESSON 11.4•Corresponding Parts of Similar Triangles

1. $h=0.9 \mathrm{~cm} ; j=4.0 \mathrm{~cm}$
2. $3.75 \mathrm{~cm}, 4.50 \mathrm{~cm}, 5.60 \mathrm{~cm}$
3. $W X=13 \frac{5}{7} \approx 13.7 \mathrm{~cm} ; A D=21 \mathrm{~cm} ; D B=12 \mathrm{~cm}$; $Y Z=8 \mathrm{~cm} ; X Z=6 \frac{6}{7} \approx 6.9 \mathrm{~cm}$
4. $x=\frac{50}{13} \approx 3.85 \mathrm{~cm} ; y=\frac{80}{13} \approx 6.15 \mathrm{~cm}$
5. $a=8 \mathrm{~cm} ; b=3.2 \mathrm{~cm} ; c=2.8 \mathrm{~cm}$
6. $C B=24 \mathrm{~cm} ; C D=5.25 \mathrm{~cm} ; A D=8.75 \mathrm{~cm}$

## LESSON 11.5•Proportions with Area

1. $5.4 \mathrm{~cm}^{2}$
2. 4 cm
3. $\frac{9}{25}$
4. $\frac{36}{1}$
5. $\frac{25}{4}$
6. $16: 25$
7. $2: 3$
8. $888 \frac{8}{9} \mathrm{~cm}^{2}$
9. 1296 tiles

## LESSON 11.6• Proportions with Volume

1. Yes
2. No
3. $16 \mathrm{~cm}^{3}$
4. 20 cm
5. $8: 125$
6. $6 \mathrm{ft}^{2}$

## LESSON 11.7• Proportional Segments Between <br> Parallel Lines

1. $x=12 \mathrm{~cm}$
2. Yes
3. No
4. $N E=31.25 \mathrm{~cm}$
5. $P R=6 \mathrm{~cm} ; P Q=4 \mathrm{~cm} ; R I=12 \mathrm{~cm}$
6. $a=9 \mathrm{~cm} ; b=18 \mathrm{~cm}$
7. $R S=22.5 \mathrm{~cm}, E B=20 \mathrm{~cm}$
8. $x=20 \mathrm{~cm} ; y=7.2 \mathrm{~cm}$
9. $p=\frac{16}{3}=5 . \overline{3} \mathrm{~cm} ; q=\frac{8}{3}=2 . \overline{6} \mathrm{~cm}$

## LESSON 12.1• Trigonometric Ratios

1. $\sin P=\frac{p}{r}$
2. $\cos P=\frac{q}{r}$
3. $\tan P=\frac{p}{q}$
4. $\sin Q=\frac{q}{r}$
5. $\sin T=0.800$
6. $\cos T=0.600$
7. $\tan T \approx 1.333$
8. $\sin R=0.600$
9. $x \approx 12.27$
10. $x \approx 29.75$
11. $x \approx 18.28$
12. $m \angle A \approx 71^{\circ}$
13. $m \angle B \approx 53^{\circ}$
14. $m \angle C \approx 30^{\circ}$
15. $\sin 40^{\circ}=\frac{w}{28} ; w \approx 18.0 \mathrm{~cm}$
16. $\sin 28^{\circ}=\frac{x}{14} ; x \approx 7.4 \mathrm{~cm}$
17. $\cos 17^{\circ}=\frac{73}{y} ; y \approx 76.3 \mathrm{~cm}$
18. $a \approx 28^{\circ}$
19. $t \approx 47^{\circ}$
20. $z \approx 76^{\circ}$

LESSON 12.2• Problem Solving with Right Triangles

1. Area $\approx 2 \mathrm{~cm}^{2}$
2. Area $\approx 325 \mathrm{ft}^{2}$
3. Area $\approx 109 \mathrm{in}^{2}$
4. $x \approx 54.0^{\circ}$
5. $y \approx 31.3^{\circ}$
6. $a \approx 7.6 \mathrm{in}$.
7. Diameter $\approx 20.5 \mathrm{~cm}$
8. $\theta \approx 45.2^{\circ}$
9. $\beta \approx 28.3^{\circ}$
10. About 2.0 m
11. About 445.2 ft
12. About 22.6 ft

## LESSON 12.3•The Law of Sines

1. Area $\approx 46 \mathrm{~cm}^{2}$
2. Area $\approx 24 \mathrm{~m}^{2}$
3. Area $\approx 45 \mathrm{ft}^{2}$
4. $m \approx 14 \mathrm{~cm}$
5. $p \approx 17 \mathrm{~cm}$
6. $q \approx 13 \mathrm{~cm}$
7. $m \angle B \approx 66^{\circ}, m \angle C \approx 33^{\circ}$
8. $m \angle P \approx 37^{\circ}, m \angle Q \approx 95^{\circ}$
9. $m \angle K \approx 81^{\circ}, m \angle M \approx 21^{\circ}$
10. Second line: about 153 ft , between tethers: about 135 ft

## LESSON 12.4•The Law of Cosines

1. $t \approx 13 \mathrm{~cm}$
2. $b \approx 67 \mathrm{~cm}$
3. $w \approx 34 \mathrm{~cm}$
4. $m \angle A \approx 76^{\circ}, m \angle B \approx 45^{\circ}, m \angle C \approx 59^{\circ}$
5. $m \angle A \approx 77^{\circ}, m \angle P \approx 66^{\circ}, m \angle S \approx 37^{\circ}$
6. $m \angle S \approx 46^{\circ}, m \angle U \approx 85^{\circ}, m \angle V \approx 49^{\circ}$
7. About $24^{\circ}$
8. About 43.0 cm
9. About 34.7 in.

## LESSON $12.5 \cdot$ Problem Solving with Trigonometry

1. About $2.85 \mathrm{mi} / \mathrm{h}$; about $15^{\circ}$
2. $m \angle A \approx 50.64^{\circ}, m \angle B \approx 59.70^{\circ}, m \angle C \approx 69.66^{\circ}$
3. About 8.0 km from Tower $1,5.1 \mathrm{~km}$ from Tower 2
4. About 853 miles
5. About 530 ft of fencing; about $11,656 \mathrm{ft}^{2}$

## LESSON 13.1•The Premises of Geometry

1. a. Given
b. Distributive property
c. Subtraction property
d. Addition property
e. Division property
2. False

3. False

4. True; transitive property of congruence and definition of congruence
5. 



## LESSON 13.2• Planning a Geometry Proof

Proofs may vary.

## 1. Flowchart Proof



## 2. Flowchart Proof



## 3. Flowchart Proof



## LESSON 13.3•Triangle Proofs

Proofs may vary.

2. Proof:

Statement

1. $\overline{C D} \cong \overline{B D}$
2. $\overline{B D} \perp \overline{A B}$

Reason
3. $\overline{C D} \perp \overline{A C}$
4. $\overline{A D}$ is bisector of $\angle C A B$
5. $\angle C A D \cong \angle B A D$
6. $\angle A C D$ is a right angle
7. $\angle A B D$ is a right angle
8. $\angle A C D \cong \angle A B D$
9. $\triangle A B D \cong \triangle A C D$

## 3. Flowchart Proof



## 4. Proof:

Statement

1. $\overline{A B} \cong \overline{B C}$
2. $\triangle A B C$ is isosceles
3. $\angle A \cong \angle A C B$
4. $\angle A C B \cong \angle D C E$

## Reason

1. Given
2. Definition of isosceles triangle
3. IT Theorem
4. Given
5. $\angle A \cong \angle D C E$
6. Transitivity
7. $\overline{A B} \| \overline{C E}$
8. $\angle A B D \cong \angle C E D$
9. $\overline{A B} \perp \overline{B D}$
10. Converse of CA Postulate
11. $\angle A B D$ is a right angle
12. $\angle C E D$ is a right angle
13. CA Postulate
14. Given
15. $\overline{B D} \perp \overline{C E}$
16. Definition of perpendicular
17. Definition of right angle, transitivity
18. Definition of perpendicular

## LESSON 13.4•Quadrilateral Proofs

Proofs may vary.

1. Given: $A B C D$ is a parallelogram
Show: $\overline{A C}$ and $\overline{B D}$ bisect each other at $M$


Flowchart Proof

2. Given: $D M=B M$,
$A M=C M$
Show: $A B C D$ is a parallelogram


## Proof:

## Statement

## Reason

1. $D M=B M$
2. Given
3. $\overline{D M} \cong \overline{B M}$
4. Definition of congruence
5. $A M=C M$
6. Given
7. $\overline{A M} \cong \overline{C M}$
8. $\angle D M A \cong \angle B M C$
9. $\triangle A M D \cong \triangle C M B$
10. SA
11. $\angle D A C \cong \angle B C A$
12. $\overline{A D} \| \overline{B C}$
13. СРСТС
14. Converse of AIA

Theorem
9. $\angle D M C \cong \angle B M A$
9. VA Theorem
10. $\triangle D M C \cong \triangle B M A$
10. SAS Postulate
11. $\angle C D B \cong \angle A B D$
11. СРСТС
12. $\overline{D C} \| \overline{A B}$
13. $A B C D$ is a parallelogram
12. Converse of AIA Theorem
13. Definition of parallelogram
3. Given: $A B C D$ is a rhombus

Show: $\overline{A C}$ and $\overline{B D}$ bisect each other at $M$ and $\overline{A C} \perp \overline{B D}$


Flowchart Proof

4. Given: $\overline{A C}$ and $\overline{B D}$ bisect each other at $M$ and $\overline{A C} \perp \overline{B D}$
Show: $A B C D$ is a rhombus Flowchart Proof

(See flowchart at bottom of page.)
5. Given: $A B C D$ is a trapezoid with $\overline{A B} \| \overline{C D}$ and $\angle A \cong \angle B$

Show: $A B C D$ is isosceles Proof:

## Statement

1. $A B C D$ is a trapezoid with $\overline{A B} \| \overline{C D}$
2. Construct $\overline{C E} \| \overline{A D}$
3. $A E C D$ is a parallelogram
4. $\overline{A D} \cong \overline{C E}$
5. $\angle A \cong \angle B E C$
6. $\angle A \cong \angle B$
7. $\angle B E C \cong \angle B$
8. $\triangle E C B$ is isosceles
9. $\overline{E C} \cong \overline{C B}$
10. $\overline{A D} \cong \overline{C B}$
11. $A B C D$ is isosceles


## Reason

1. Given
2. Parallel Postulate
3. Definition of parallelogram
4. Opposite Sides
Congruent Theorem
5. CA Postulate
6. Given
7. Transitivity
8. Converse of IT Theorem
9. Definition of isosceles triangle
10. Transitivity
11. Definition of isosceles trapezoid
12. Given: $A B C D$ is a trapezoid with $\overline{A B} \| \overline{C D}$ and $\overline{A C} \cong \overline{B D}$
Show: $A B C D$ is isosceles

Proof:

## Statement

1. $A B C D$ is a trapezoid with $\overline{A B} \| \overline{C D}$
2. Construct $\overleftrightarrow{B E} \| \overline{A C}$
3. $\overleftrightarrow{D C}$ and $\overleftrightarrow{B E}$ intersect at $F$
4. $A B F C$ is a parallelogram
5. $\overline{A C} \cong \overline{B F}$
6. $\overline{A C} \cong \overline{B D}$
7. $\overline{B F} \cong \overline{B D}$
8. $\triangle D F B$ is isosceles
9. $\angle D F B \cong \angle F D B$
10. $\angle C A B \cong \angle D F B$
11. $\angle F D B \cong \angle D B A$
12. $\angle C A B \cong \angle D B A$
13. $\overline{A B} \cong \overline{A B}$
14. $\triangle A C B \cong \triangle B D A$
15. $\overline{A D} \cong \overline{B C}$
16. $A B C D$ is isosceles


## Reason

1. Given
2. Parallel Postulate
3. Line Intersection Postulate
4. Definition of parallelogram
5. Opposite Sides Congruent Theorem
6. Given
7. Transitivity
8. Definition of isosceles triangle
9. IT Theorem
10. Opposite Angles Theorem
11. AIA Theorem
12. Transitivity
13. Reflexive property
14. SAS Postulate
15. СРСТС
16. Definition of isosceles trapezoid

## Lesson 13.4, Exercise 4


7. False

8. False

9. True

Given: $A B C D$ with
$\overline{A B} \| \overline{C D}$ and $\angle A \cong \angle C$
Show: $A B C D$ is a parallelogram


Flowchart Proof


## LESSON 13.5•Indirect Proof

Proofs may vary.

1. Assume $B C \leq A C$

Case 1: If $B C=A C$, then $\triangle A B C$ is isosceles, by the definition of isosceles. By the IT Theorem, $\angle A \cong \angle B$, which contradicts the given that $m \angle A>m \angle B$. So, $B C \neq A C$.
Case 2: $\triangle D B C$ is isosceles.


By the Exterior Angle Theorem, $m \angle 1+m \angle 2$
$=m \angle 4$, so $m \angle 1<m \angle 4$.
By the Angle Sum Postulate, $m \angle 2+m \angle 3$
$=m \angle A B C$, so $m \angle 3<m \angle A B C$. But $\triangle D B C$ is isosceles, so $m \angle 4=m \angle 3$ by the IT Theorem.

So, by transitivity, $m \angle 1<m \angle 4=m \angle 3<m \angle A B C$, or $m \angle 1<m \angle A B C$, which contradicts the given that $m \angle A>m \angle B$. So, $B C \nless A C$.

Therefore the assumption, $B C \leq A C$, is false, so $B C>A C$.
2. Paragraph Proof: Assume $\angle D A C \cong \angle B A C$

It is given that $\overline{A D} \cong \overline{A B}$. By the reflexive property $\overline{A C} \cong \overline{A C}$. So by SAS, $\triangle A D C \cong \triangle A B C$. Then $\overline{D C}$ $\cong \overline{B C}$ by СРСТС. But this contradicts the given that $\overline{D C} \not \equiv \overline{B C}$. So $\angle D A C \not \equiv \angle B A C$.
3. Given: $\triangle A B C$ with $\overline{A B} \not \equiv \overline{B C}$

Show: $\angle C \nRightarrow \angle A$
Paragraph Proof: Assume $\angle C \cong \angle A$


If $\angle C \cong \angle A$, then by the Converse of the IT Theorem, $\triangle A B C$ is isosceles and $\overline{A B} \cong \overline{B C}$. But this contradicts the given that $\overline{A B} \not \equiv \overline{B C}$. Therefore, $\angle C \not \equiv \angle A$.
4. Given: Coplanar lines $k, \ell$, and $m, k \| \ell$, and $m$ intersecting $k$


Show: $m$ intersects $\ell$
Paragraph Proof: Assume $m$ does not intersect $\ell$ If $m$ does not intersect $\ell$, then by the definition of parallel, $m \| \ell$. But because $k \| \ell$, by the Parallel Transitivity Theorem, $k \| m$. This contradicts the given that $m$ intersects $k$. Therefore, $m$ intersects $\ell$.

## LESSON 13.6 • Circle Proofs

1. Given: Circle $O$ with
$\overline{A B} \cong \overline{C D}$
Show: $\widehat{A B} \cong \widehat{C D}$

Flowchart Proof

2. Paragraph Proof: Chords $\overline{B C}, \overline{C D}$, and $\overline{D E}$ are congruent because the pentagon is regular. By the proof in Exercise 1, the arcs $\overline{B C}, \overline{C D}$, and $\widehat{D E}$ are congruent and therefore have the same measure. $m \angle E A D=\frac{1}{2} m \widehat{D E}$ by the Inscribed Angles Intercepting Arcs Theorem. Similarly, $m \angle D A C=\frac{1}{2} m \widetilde{D C}$ and $m \angle B A C=\frac{1}{2} m \widehat{B C}$. By transitivity and algebra, the three angles have the same measure. So, by the definition of trisect, the diagonals trisect $\angle B A E$.
3. Paragraph Proof: Construct the common internal tangent $\overrightarrow{R U}$ (Line Postulate, definition of tangent). Label the intersection of the tangent and $\overline{T S}$ as $U$.

$\overline{T U} \cong \overline{R U} \cong \overline{S U}$ by the Tangent Segments Theorem. $\triangle T U R$ is isosceles by definition because $\overline{T U} \cong \overline{R U}$. So, by the IT Theorem, $\angle T \cong \angle T R U$. Call this angle measure $x . \triangle S U R$ is isosceles because $\overline{R U} \cong \overline{S U}$, and by the IT Theorem, $\angle S \cong \angle U R S$. Call this angle measure $y$. The angle measures of $\triangle T R S$ are then $x, y$, and $(x+y)$. By the Triangle Sum Theorem, $x+y+(x+y)=180^{\circ}$. By algebra (combining like terms and dividing by 2 ), $x+y$ $=90^{\circ}$. But $m \angle T R S=x+y$, so by transitivity and the definition of right angle, $\angle T R S$ is a right angle.
4. Paragraph Proof: Construct tangent $\overleftrightarrow{T P}$ (Line Postulate, definition of tangent). $\angle P T D$ and $\angle T A C$ both have the same intercepted arc, $\overparen{T C}$. Similarly, $\angle P T D$ and $\angle T B D$ have the same intercepted arc, $\overline{T D}$. So, by transitivity, the Inscribed Angles Intercepting Arcs Theorem, and algebra, $\angle T A C$ and $\angle T B D$ are congruent. Therefore, by the Converse of the CA Postulate, $\overline{A C} \| \overline{B D}$.


## LESSON 13.7 • Similarity Proofs

1. Flowchart Proof

2. Given: Trapezoid $A B C D$ with $\overline{A B} \| \overline{C D}$, and $\overline{A C}$ and $\overline{B D}$ intersecting at $E$
 Show: $\frac{D E}{B E}=\frac{C E}{A E}=\frac{D C}{A B}$
Flowchart Proof

3. Given: $\triangle A B C$ with $\angle A C B$ right, $\overline{C D} \perp \overline{A B}$

Show: $A C \cdot B C=A B \cdot C D$


Flowchart Proof

4. Given: $A B C D$ with right angles $A$ and $C, \overline{A B} \cong \overline{D C}$

Show: $A B C D$ is a rectangle


## Proof:

## Statement

1. Construct $\overline{D B}$
2. $\angle A$ and $\angle C$ are right angles
3. $\angle A \cong \angle C$
4. $\overline{A B} \cong \overline{D C}$
5. $\overline{D B} \cong \overline{D B}$
6. $\triangle D B A \cong \triangle B D C$
7. $\angle D B A \cong \angle B D C$
8. $m \angle D B A=m \angle B D C$
9. $m \angle A D B+m \angle D B A$
$+m \angle A=180^{\circ}$
10. $m \angle A=90^{\circ}$
11. $m \angle A D B+m \angle D B A$ $=90^{\circ}$
12. $m \angle A D B+m \angle B D C$
$=90^{\circ}$
13. $m \angle A D B+m \angle B D C$
$=m \angle A D C$
14. $m \angle A D C=90^{\circ}$
15. $m \angle C=90^{\circ}$
16. $m \angle A+m \angle A B C$
$+m \angle C+m \angle A D C$
$=360^{\circ}$
17. $m \angle A B C=90^{\circ}$
18. $\angle A \cong \angle A B C \cong \angle C$ $\cong \angle A D C$
19. $A B C D$ is a rectangle

## Reason

1. Line Postulate
2. Given
3. Right Angles Are Congruent Theorem
4. Given
5. Reflexive property
6. HL Congruence Theorem
7. СРСТС
8. Definition of congruence
9. Triangle Sum Theorem
10. Definition of right angle
11. Subtraction property
12. Substitution
13. Angle Addition Postulate
14. Transitivity
15. Definition of right angle
16. Quadrilateral Sum Theorem
17. Substitution property and subtraction property
18. Definition of congruence
19. Four Congruent Angles Rectangle Theorem
