Pre-AP Algebra 2
Unit 4 - Lesson 3 – Complex Numbers

Objectives: The students will be able to:
- extend the real numbers to the complex numbers by using the closure property.
- define *i* as √-1 and plot complex numbers on the complex plane.
- perform operations on complex numbers, including finding absolute value.

Materials: DO NOW; pairwork; homework 4-3

Time	Activity
5 min	**Review Homework** Show the answers to hw #4-2 on the overhead. Students correct their answers. Pass around the tally sheets.
10 min	**Homework Presentations** Review the top 2 or 3 problems.
15 min	**DO NOW**: My Favorite No – Display the 3 questions and have students answer them on notecards. Collect notecards to see where students are on square roots. Choose the "best" wrong answer to discuss. Solve for *x:* *(leave radicals in simplest form – no decimals)* (1) $x^2 = 36$ (2) $x^2 = 48$ (3) $(x-2)^2 = 50$ *Solutions:* *(1)* $x = \pm 6$ *(2)* $x = \pm 4\sqrt{3}$ *(3)* $x = 2 \pm 5\sqrt{2}$
35 min	**Direct Instruction** **Part 1:** **Background:** What does √16 equal? It is 4, because $(4)^2 = 16$. It could also be -4, because $(-4)^2 = 16$. What does √-16 equal? $(?)^2 = -16$. *What real number goes in the parentheses?* Any real number times itself will be **positive**, so no real number works. Remember, the real numbers are **not closed** under square roots! **Concepts:** *Mathematicians don't like to be told that things are impossible. When we hit a roadblock, we invent our way across it and make new math concepts. And then, we amazingly find new applications of these concepts in real life. For example, complex numbers are used in electronics engineering to describe the flow of current through a circuit.* • What number squared is -1? This can be represented: $(?)^2 = -1$ • The answer is *i*. We have the definition $i^2 = -1$, which means that *i* = √-1. • *i* is called an **imaginary number** (as opposed to a real number). • A **complex number** is one that has both a real *and* an imaginary part: **a + b*i*** • The complex plane is made up of a real axis (horizontal) and an imaginary axis (vertical) **Examples:** 1) Is each number real, imaginary, or complex? -7*i*, 12, 1 + 3*i* 2) Plot the following numbers on the complex plane: -5 + *i*, 3.5*i*, -2, 2 – 3*i* 3) Solve $2x^2 + 55 = 5$ **Part 2** **Background:** • Difference of Squares Pattern: $(a - b)(a + b) = a^2 - b^2$

	• Notice: the middle term always cancels out. • *We will use this pattern today, and many more times in Algebra. It is important to memorize it.* **Concepts:** 1) Addition and Subtraction • Combine like terms, just as we do with polynomial addition and subtraction 2) Multiplication • Use the distributive property, just like polynomials. • When simplifying, always replace i^2 with -1. • a + bi and a – bi are called **complex conjugates**. When you multiply them, the product is always **real** only. (Multiply it out to verify.) 3) Division • Write as a fraction. • Multiply top and bottom by the complex conjugate of the denominator. 4) Absolute Value • Definition: distance from the complex number to the origin. • Plot the point, draw a right triangle, and use the Pythagorean Theorem. • In general, if $z = a + bi,\ \lvert z \rvert = \sqrt{a^2 + b^2}$. • *Note: Every complex number **z** has an **infinite number** of other complex numbers with the same absolute value – think of drawing a circle with the origin as the center and |z| as the radius.* **Examples:** 1) (2 – 5i) – (3 – i) 2) a) -2i(4 – 10i) b) (3 + 4i)(3 – 4i) 3) (5 + 3i) ÷ (1 – 2i) 4) a) |-3i| b) |4 – 4i| c) Name two other complex numbers with the same absolute value as in b).
15 min	**Pairwork** (if time) Students work in pairs. Each person works on one of the two columns. The problems with the same numbers have the same solution.

Solve for *x:*

(leave radicals in simplest form – no decimals)

(1) $x^2 = 36$

(2) $x^2 = 48$

(3) $(x-2)^2 = 50$

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Lesson 4-3 – Notes

Part 1: Background Information

What does √16 equal? Why?

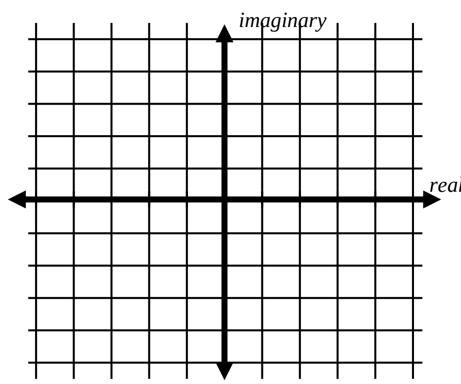
What does √-16 equal? Why?

Concepts

Examples

1) Is each number real, imaginary, or complex?

-7i 12 1 + 3i



2) Plot the following numbers on the complex plane:

A =-5 + i B = 3.5i C = -2 D = 2 – 3i

3) Solve $2x^2 + 55 = 5$

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Lesson 4-3 – Notes

Part 2: Background:

- Difference of Squares Pattern: $(a - b)(a + b) = a^2 - b^2$
- Notice: the middle term always cancels out.

We will use this pattern today, and many more times in Algebra. It is important to memorize it.

Concepts:

1) Addition and Subtraction

- Combine like terms, just as we do with polynomial addition and subtraction

Ex: $(2 - 5i) - (3 - i)$

2) Multiplication

- Use the distributive property, just like polynomials.
- When simplifying, always replace i^2 with -1.
- $a + bi$ and $a - bi$ are called **complex conjugates**. When you multiply them, the product is always **real** only. (Multiply it out to verify.)

Examples:

$-2i(4 - 10i)$ $(3 + 4i)(3 - 4i)$

3) Division

- Write as a fraction.
- Multiply top and bottom by the complex conjugate of the denominator.

Ex: $(5 + 3i) \div (1 - 2i)$

4) Absolute Value

- Definition: distance from the complex number to the origin.
- Plot the point, draw a right triangle, and use the Pythagorean Theorem.
- In general, if $z = a + bi,\ |z| = \sqrt{a^2 + b^2}$.
- *Note: Every complex number **z** has an **infinite number** of other complex numbers with the same absolute value – think of drawing a circle with the origin as the center and |z| as the radius.*

Examples:

a) $|-3i|$

b) $|4 - 4i|$

c) Name two other complex numbers with the same absolute value as in b).

Operations on Complex Numbers

Simplify each expression. Remember to replace i^2 with -1. Write complex numbers in standard form: a + bi.

1) $(4-i)+(3+2i)$

2) $(7-5i)-(1-5i)$

3) $6-(-2+9i)+(-8+4i)$

4) $5i(-2+i)$

5) $(7-4i)(-1+2i)$

6) Multiply $(6-7i)$ by its complex conjugate.

7) $\dfrac{4}{3-i}$

8) $\dfrac{3+6i}{1+2i}$

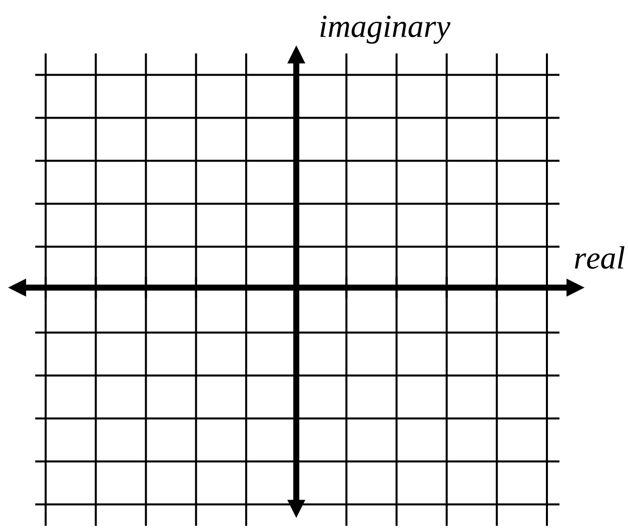
Plot each number on the complex plane (using its given letter). Then, find the absolute value of each number.

1) c = -4i $\qquad$ $|-4i| =$

2) d = 3 + 3i $\qquad$ $|3+3i| =$

3) e = -2.5 $\qquad$ $|-2.5| =$

4) f = -2 + 5i $\qquad$ $|-2+5i| =$



5) List three other complex numbers that have the same absolute value as -2 + 5i, and plot them on the complex plane (label them f_2, f_3, and f_4).

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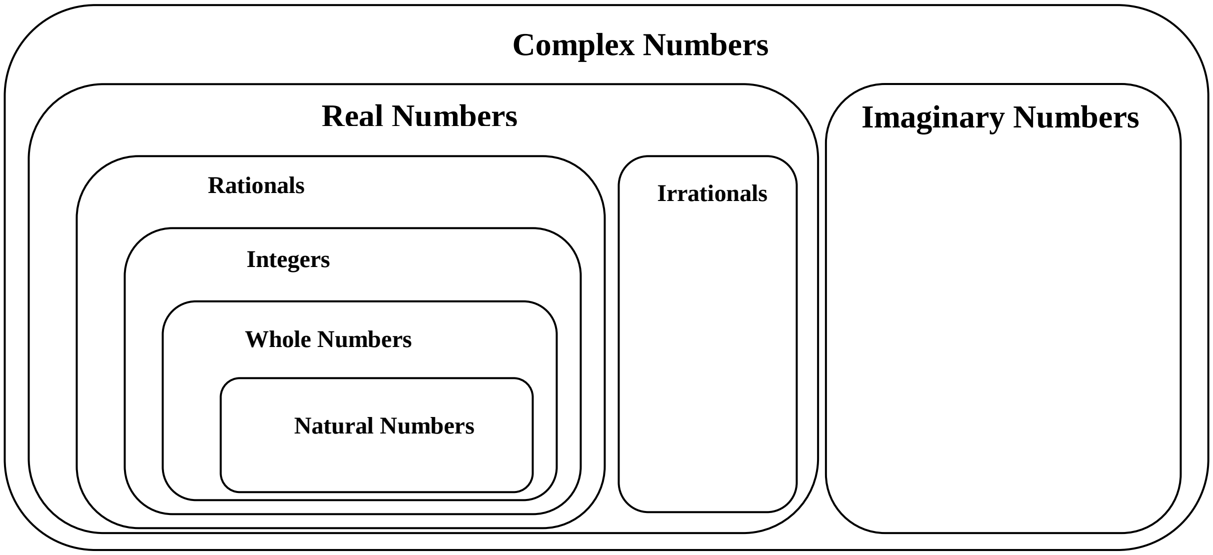
Lesson 4-3 Homework

Homework #4-3: Life is Complex

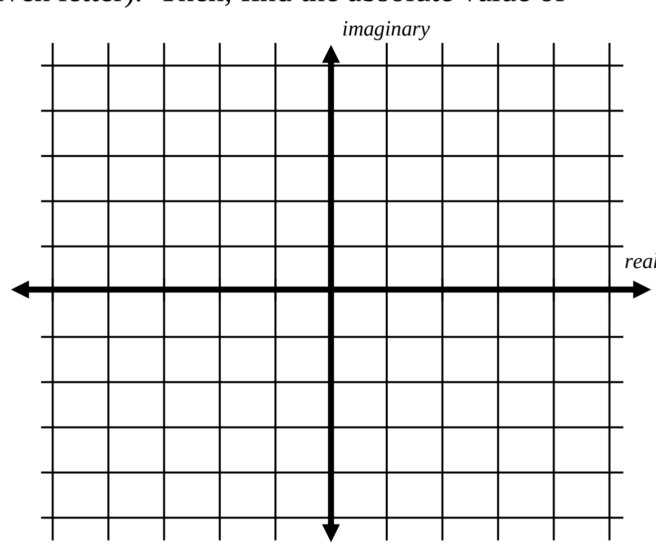
Do scratch work on binder paper stapled to this sheet; write answers on this sheet

1) Fill in each number into its appropriate place on the Venn Diagram:

$$5i,\quad 3+2i,\quad \sqrt{2},\quad \sqrt{-16},\quad -7.56,\quad 0,\quad 3\pi,\quad 15,\quad \frac{7}{11},\quad i^2,\quad -\sqrt{4},\quad 4-\frac{1}{2}i$$



2) Plot each number on the complex plane (using its given letter). Then, find the absolute value of each number.



a) $2 + 3i$ $\qquad |2 + 3i| =$

b) -2.5 $\qquad |-2.5| =$

c) $3i$ $\qquad |3i| =$

d) $3 - 4i$ $\qquad |3 - 4i| =$

e) List 2 other complex numbers that have the same absolute value as $3 - 4i$

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Lesson 4-3 Homework

3) Simplify each expression. Remember to replace i^2 with -1. Write complex numbers in standard form: a + bi.

a) $(5-4i)+(-2-6i)$

b) $(-10+i)-(-4+6i)$

c) $5-(3-4i)-(2+4i)$

d) $(5i-4)(-6i)$

e) $(10-5i)(5-10i)$

f) Multiply $\left(\frac{3}{4}-\frac{1}{5}i\right)$ by its complex conjugate.

g) $\frac{1}{c+di}=$

h) $\frac{3+6i}{1+2i}$

i) $|-6.3i|$

j) $|-4-8i|$

4) Solve each of the following quadratic equations. Some of the answers will be real, and others will be imaginary. Always give simplified, exact answers. Don't forget that each time you take a square root, there is a positive and a negative answer.

a) $x^2+144=0$

b) $10+3x^2=130$

c) $x^2-7=4x^2+5$

d) $2(x^2+7)=x^2+10$

e) $3x^2-8=5x^2-158$

Bonus (+2 Points)

Solve $-3(x+6)^2-7=0$. Write exact, fully simplified solutions to get credit.