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Chapter 1 Review NOTES \& BASIC EXERCISES

This review package is based on Chapter 1 Polynomial Functions from the Pre-calculus 12 student workbook and textbook.

- Read over the given notes for each section or group of sections.
- Complete the sample review questions for each section or group of sections, and check your answers with those in the answer key.
(NOTE: these are basic review questions, which may be a bit easier than similar ones on the test.)
- Ensure that you have completed the exercises assigned in previous classes.
- Also, complete the listed review questions from the Pre-calculus 12 textbook.


### 1.1 Horizontal and Vertical Translations

### 1.2 Stretches and Reflections

### 1.3 Combining Transformations

- When transformations are applied to a function, an equation of a function like $y=f(x)$ can become $y=a f(b(x-h))+k$, where $a, b, h$, and $k$ are real numbers.
- If $a \neq \pm 1$ then the graph will be vertically stretched about the $x$-axis by a factor of $|a|$. Also, If $a$ is negative, then the graph wIII be reflected in the $x$-axis.
- If $b \neq \pm 1$ then the graph will be horizontally stretched about the $y$-axis by a factor of $\left|\frac{1}{b}\right|$. Also, If $b$ is negative, then the graph wIII be reflected in the $y$-axis.
- If $h \neq 0$ then the graph will be horizontally shifted $h$ units to the right if $h$ is positive, and $h$ units to the left if $h$ is negative.
- If $k \neq 0$ then the graph will be horizontally shifted $k$ units upwards if $k$ is positive, and $k$ units downwards if $k$ is negative.
- The mapping of points on the graph of $y=f(x)$ to the graph of $y=a f(b(x-h))+k$ will look like $(x, y) \rightarrow\left(\frac{1}{b} x+h, a y+k\right)$

1. To the right is a grid with the graph of $y=f(x)$. On the same grid draw the graphs of the following functions.
$y=f(x)+3$
$y=f(x-9)$
$y=f(x+1)-5$

2. To the right is a grid with the graph of $y=g(x)$. On the same grid draw the graphs of the following functions.
$y=-g(x)$
$y=g(-x)$
$y=-g(-x)$

3. To the right is a grid with the graph of $y=h(x)$. On the same grid draw the graphs of the following functions.
$y=-\frac{1}{2} h(x-3)+1$
$y-5=3 h(4-2 x)$

4. To the right is a grid with the graphs of $y=f(x)$ and a version with transformations, $y=a f(b(x-h))+k$. Rewrite $y=a f(b(x-h))+k$ with numbers that reflect the actual transformations.

5. List all of the transformations that would have to be done to $y=f(x)$ to get $y=\frac{3}{2} f\left(-\frac{1}{4}(x-7)\right)+1$. Also, create a mapping that describes the combine transformations.
6. List all of the transformations that would have to be done to $y=g(x)$ to get $y+5=-2.5 g(9+2 x)$. Also, create a mapping that describes the combined transformations.
7. Suppose that point $(4,-5)$ is on the graph of $y=h(x)$. What point will it become on the graph of $y=2 h\left(-\frac{1}{2}(x-3)\right)+10$.
8. Suppose that point $(8,3)$ is on the graph of $y=f(x)$. What point will it get mapped onto on the graph of $y=-3 f(-10+4 x)-8$.
9. Suppose that $f(x)=5 x-2$. Write the transformed function $2 f(-2 x+4)+1$ in simplest form.
10. Suppose that $g(x)=2 x^{2}-3 x$. Write the transformed function $-3 g(6-x)-4$ in simplest form.

### 1.4 Inverse of a Relation

- The mapping $(x, y) \rightarrow(y, x)$ describes going from a relation to the inverse of the relation.
- The graph of an inverse relation will be the graph of the original relation reflected in the line $y=x$.
- The range of the inverse relation of a function will be the same as the domain of the original function, and the domain of the inverse relation of a function will be the same as the range of the original function.
- If the inverse relation of a function $f$ is also a function, then it is called an inverse function and will be referred to as $f^{-1}$.

1. To the right is a grid with the graph of $y=f(x)$. On the same grid draw the graph of the inverse relation of function $f$. Is the inverse also a function?

2. To the right is a grid with the graph of $y=g(x)$. On the same grid draw the graph of $y=g^{-1}(x)$. Also, write out the domain and range of both $g$ and $g^{-1}$.

3. To the right is a grid with the graph of $y=g(x)$. On the same grid draw the graph of $y=2 g^{-1}(-x-5)+1$.


## Chapter 1 Exercises Assigned in Previous Classes

- From pages 12 to 15 in 1.1 Vertical and Horizontal Translations:
\#1, \#2, \#3, \#4, \#5, \#6, \#7, \#8, \#9, \#10, \#11, \#12, \#13, \#16, and (optionally \#18 and \#19).
- From pages 28 to 31 in 1.2 Stretches and Reflections:
\#1, \#2, \#3, \#5, \#6, \#7, \#8, \#9, \#10, \#12, and \#13 (and, optionally, \#14 and \#15).
- From pages 38 to 41 in 1.3 Combining Transformations:
\#1, \#2, \#3, \#4, \#5, \#6 (b,d,e), \#7 (b,d,f), \#8, \#9 (b,e,f), \#10, \#11 (b), \#12, \#14, and try \#15
- From pages 51 to 54 in 1.4 Inverse Relations:
\#1, \#2, \#3, \#4, \#5, \#6, \#7, \#8, \#9 (a, c,e), \#10 (a), \#11, \#12 (c,e), \#13 (b,e), \#14, \#15, \#16, and try \#19.


## Other Chapter 1 Review Exercises Worth Doing

- Complete the Chapter 1 Review exercises \#1 to \# 17 on pages 56 and 57 of the Pre-calculus 12 textbook.
- Also work on the Chapter 1 Practice Test exercises \#1 to \#15 on pages 58 and 59 of the Pre-calculus 12 textbook.


## ANSWER KEY

### 1.1 Horizontal and Vertical Translations

1.2 Stretches and Reflections
1.3 Combining Transformations
1.

2.

3.

4. $y=-\frac{1}{2} f\left(\frac{1}{2}(x+3)\right)-1$
5. $f$ must be vertically stretched about the $x$-axis by a factor of $\frac{3}{2}$, then horizontally stretched about the $y$-axis by a factor of 4 , then reflected in the $x$-axis, and then shifted 7 units to the right and one unit upward.
Mapping: $(x, y) \rightarrow\left(-4 x+7, \frac{3}{2} y+1\right)$
6. Rewrite $y+5=-2.5 g(9+2 x)$ as $y=-2.5 g(2(x+4.5))-5$.
$g$ must be vertically stretched about the $x$-axis by a factor of 2.5 , then horizontally stretched about the $y$-axis by a factor of $\frac{1}{2}$, then reflected in the $y$-axis, and then shifted 4.5 units to the left and five units downward.
Mapping: $(x, y) \rightarrow\left(\frac{1}{2} x-4.5,-2.5 y-5\right)$
7. $(4,-5)$ will become $(-5,0)$.
8. Rewrite $y=-3 f(-10+4 x)-8$ as $y=-3 f(4(x-2.5))-8$.
$(8,3)$ will get mapped onto $(4.5,-17)$.
9. $2 f(-2 x+4)+1=2(5(-2 x+4)-2)+1=-20 x+37$
10. $-3 g(6-x)-4=-3\left(2(6-x)^{2}-3(6-x)\right)-4=-6 x^{2}+63 x-162$

### 1.4 Inverse of a Relation

1. 


2.

3.


No, the inverse is not a function.

$$
\begin{aligned}
& \text { Domain of } g=\{x \mid-1 \leq x \leq 3, x \in \square\} \\
& \text { Range of } g=\{y \mid-4 \leq y \leq-2, y \in \square\} \\
& \text { Domain of } g^{-1}=\{x \mid-4 \leq x \leq-2, x \in \square\} \\
& \text { Range of } g^{-1}=\{y \mid-1 \leq y \leq 3, y \in \square\}
\end{aligned}
$$

