PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

RATIONAL EXPONENTS AND RADICALS

Rules: $x^m \cdot x^n =$

$(x^m)^n =$

$\frac{x^m}{x^n}$

$\left(\frac{x}{y}\right)^m =$

$(xy)^m =$

$x^{1/n} =$

$\sqrt[m]{x^n} =$

Negative exponents:

$x^{-1} =$ $\qquad$ $\frac{1}{x^{-1}} =$ $\qquad$ $\left(\frac{x}{y}\right)^{-1} =$

Evaluate:

$125^{1/3} =$ $\qquad$ $27^{2/3} =$ $\qquad$ $32^{-6/5} =$ $\qquad$ $3^4 \cdot 3^x =$

Write using <u>rational</u> exponents.

$\frac{1}{\sqrt{1-x^2}} =$ $\qquad$ $\sqrt[4]{27x^6y^7} =$ $\qquad$ $\sqrt[3]{4a^4b^2} \cdot \sqrt[3]{4a^3b^4} =$

Write expressing the answer in <u>radical</u> form.

$25^{\frac{1}{2}}\, r^{\frac{1}{4}} s^{\frac{5}{4}} =$ $\qquad$ $\frac{x^{-\frac{3}{2}}}{x^2} =$ $\qquad$ $\left(\frac{u^{-2}}{v^{-4}}\right)^{-\frac{1}{2}} =$

Anything to the 0 power is ___.

HOMEWORK FOR RATIONAL EXPONENTS AND RADICALS

$\sqrt[3]{125}$ $3^{-4} \cdot 3^{8}$ $\sqrt[4]{16^{2}}$ $\left(5^{\frac{3}{4}}\right)^{4}$ $\left(169^{\frac{1}{2}}\right)^{0}$

$\left(8^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ $\left(\sqrt[3]{216}\right)^{2}$ $(3^{-1} + 3^{-2})^{-1}$ $81^{\frac{1}{2}} - 81^{-\frac{1}{2}}$

$\dfrac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}}$ $\left(\sqrt[3]{343}\right)^{-2}$ $\dfrac{27}{27^{\frac{2}{3}}}$

Express using Rational Exponents:

$\sqrt{a^{6}b^{3}}$ $\sqrt[6]{b^{3}}$ $\sqrt{25a^{4}b^{10}}$ $\sqrt[3]{125a^{2}b^{3}}$ $\sqrt[3]{64s^{9}t^{15}}$

$\sqrt[4]{24a^{12}b^{16}}$ $\sqrt{169x^{5}}$ $\sqrt[5]{32x^{5}y^{8}}$ $\sqrt[5]{15x^{3}y^{15}}$

Express using Radicals:

$64^{\frac{1}{6}}$ $x^{\frac{2}{3}}$ $4^{\frac{1}{3}}a^{\frac{2}{3}}y^{\frac{4}{3}}$ $x^{\frac{4}{7}}y^{\frac{3}{7}}$ $(rt^{2})^{\frac{1}{5}}v^{\frac{3}{5}}$

$a^{\frac{1}{6}}b^{\frac{4}{6}}c^{\frac{3}{6}}$ $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ $15x^{\frac{1}{3}}y^{\frac{1}{5}}$ $(x^{10}y^{2})^{\frac{1}{5}}a^{\frac{2}{5}}$

Simplify:

$4x^{2}(4x)^{-2}$ $((2x)^{4})^{-2}$ $(4y^{4})^{\frac{3}{2}}$ $\sqrt{a^{3}b^{2}} \cdot \sqrt{a^{4}b^{5}}$ $(5ac)^{\frac{1}{3}}(a^{2}c^{3})^{\frac{1}{3}}$

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

MORE RATIONAL EXPONENTS AND RADICALS

Changing bases: **Rewrite each expression using the base indicated.**

$27^{4x} = \ 3$ $\qquad$ $5^{6x} = 25$ $\qquad$ $32^{.5x} = 2$

Adding and Subtracting Radicals:

$\sqrt{3} - 4\sqrt{3} + \sqrt{27} =$ $\qquad$ $\sqrt[3]{16} - \sqrt[3]{54} + \sqrt[3]{128} =$

Multiplying Radicals:

$3\sqrt{5} \cdot 5\sqrt{8} =$ $\qquad$ $(1 - \sqrt{3})(5 + 2\sqrt{3}) =$

$(4 - \sqrt{6})(4 + \sqrt{6})$

$\frac{4}{\sqrt{2}}$ $\qquad$ $\frac{5}{3 - \sqrt{5}}$

Assignment: Worksheets

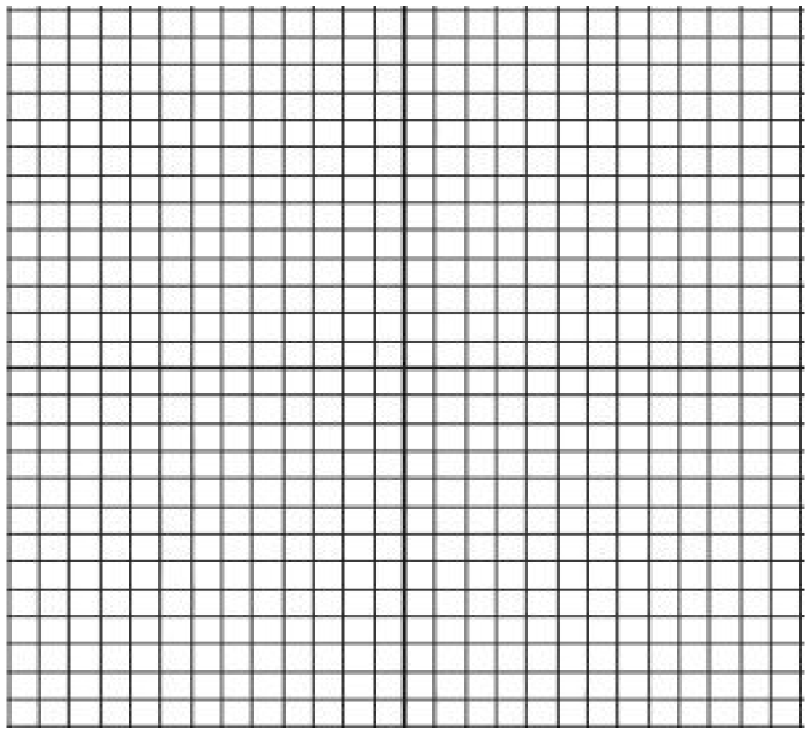
PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Exponential Functions

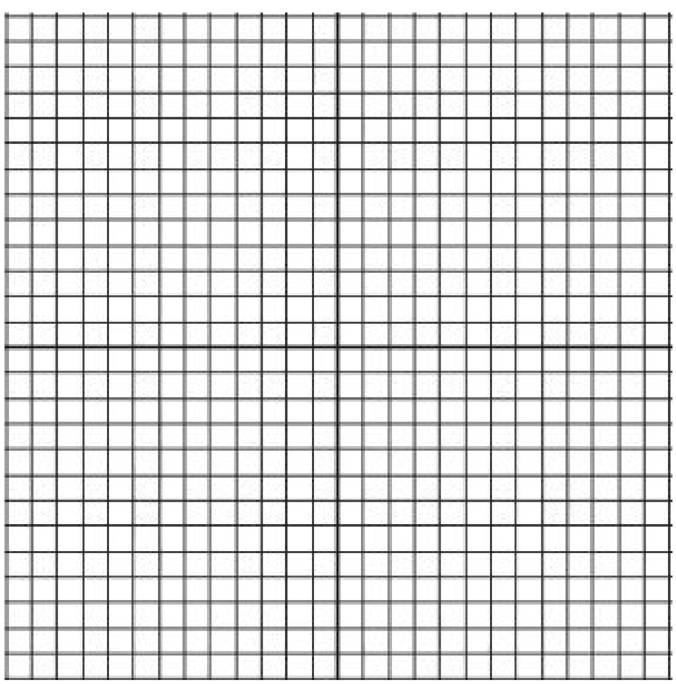
Exponential Function Form: $y = k \cdot a^x$

x	y or f(x)

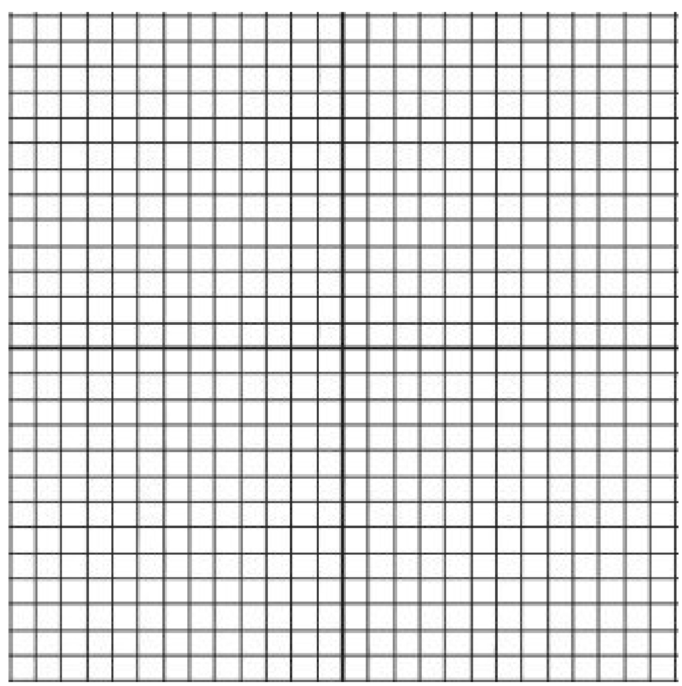
$y = 2^x$ **Domain:** **Range:**



$y = 2^{-x}$ **Domain:** **Range:**

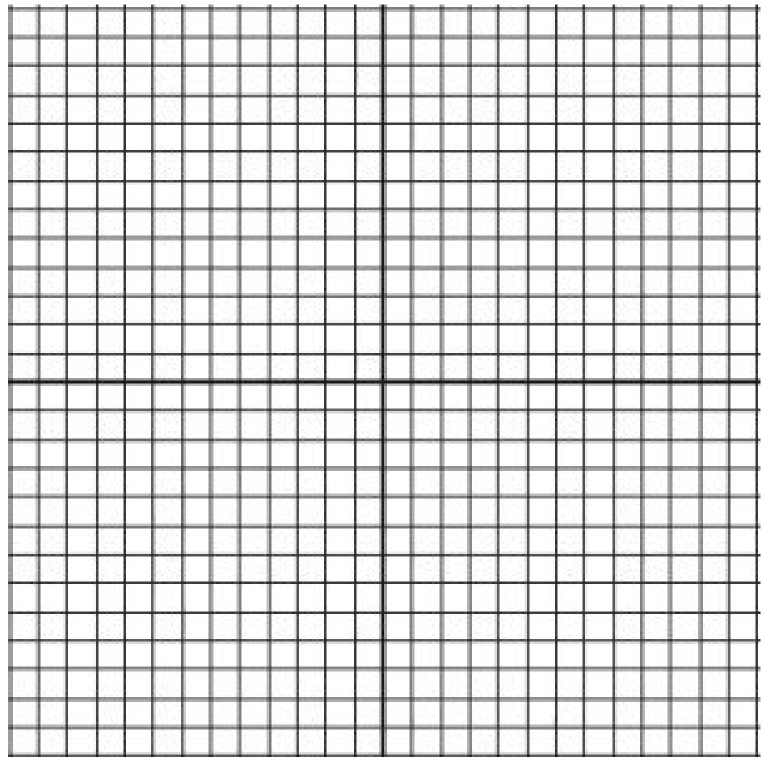


$y = -2^x$ **Domain:** **Range:**



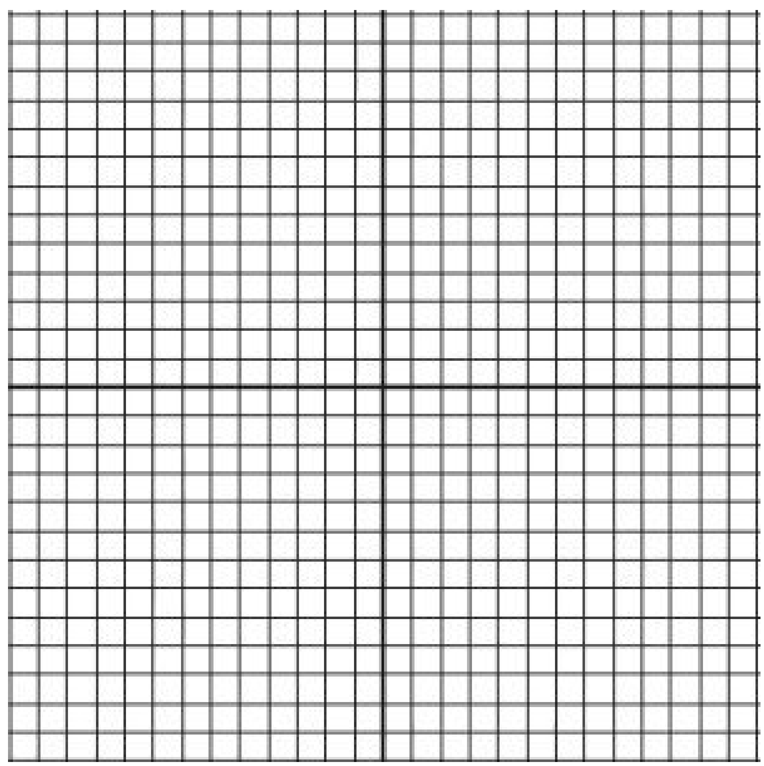
$y = 3^{x+1} - 2$ **Domain:** **Range:**

x	y or f(x)



$y = -2 \cdot 3^{x}$ **Domain:** **Range:**

x	y or f(x)



Assignment: Worksheet

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Logarithmic Functions

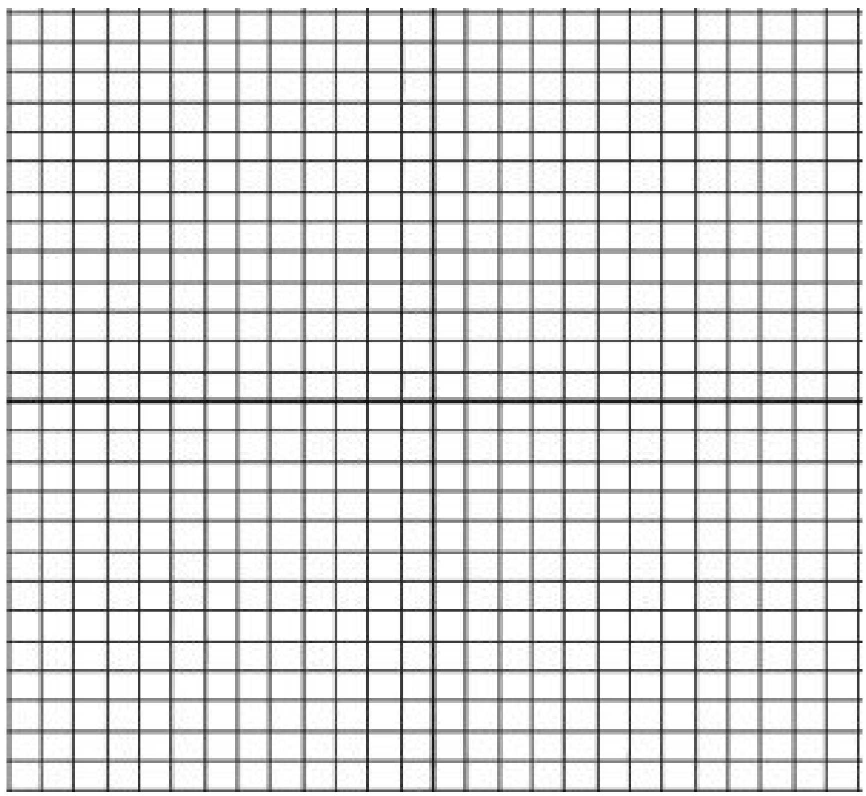
Exponential

$y = 2^x$ **Domain:** **Range:**

Logarithmic

$y = \log$ **Domain:** **Range:**

x	y or f(x)



Converting exponentials to logarithms:

$27^{2/3} = 9$

$5^{-3} = \frac{1}{125}$

Converting logarithms to exponentials:

$\log_8 2 = \frac{1}{3}$

$\log_2 32 = 5$

Evaluate the following expressions:

$\log_3 27 =$

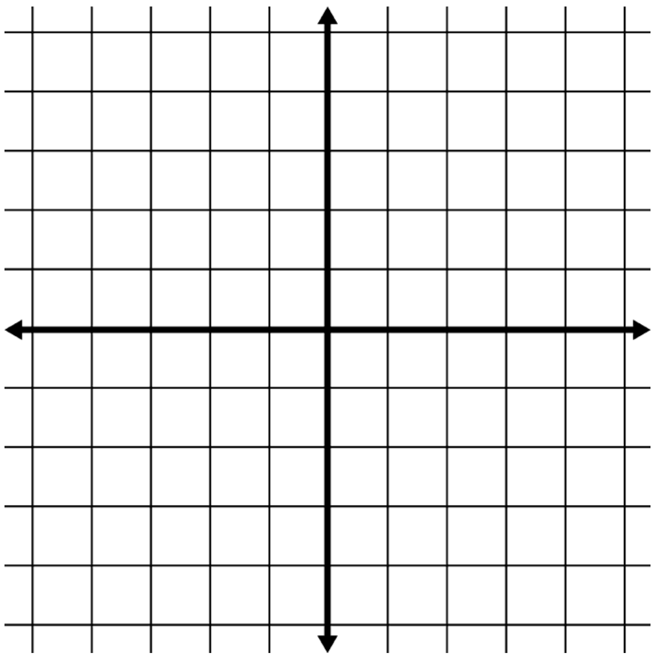
$\log_{10} .001 =$

$\log_{\frac{1}{2}} 64 =$

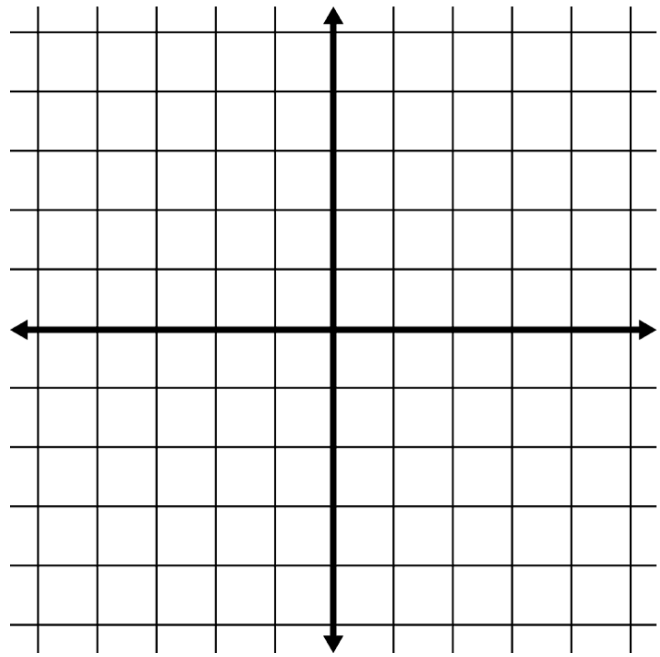
$\log_3 3^x$ $\log_{5x}(5x)^4 =$ $6^{\log_6 x} =$ $12x^{\log_{12x} 9}$

Graphing Logarithmic Functions:

$y = -\log_3(x+2)$ **D:** **R:**



$y = \log_2(x-3) + 2$ **D:** **R:**



HOMEWORK FOR LOGARITHMIC FUNCTIONS

Write in logarithmic form.

$2^4 = 16$ $\quad$ $5^{-2} = \frac{1}{25}$ $\quad$ $3^{-3} = \frac{1}{27}$

$10^6 = 1,000,000$ $\quad$ $8^{-\frac{2}{3}} = \frac{1}{4}$ $\quad$ $4^0 = 1$

Write in Exponential form.

$\log_2 8 = 3$ $\quad$ $\log_5 125 = 3$ $\quad$ $\log_{10} 10,000 = 4$

$\log_7 \frac{1}{2401} = -4$ $\quad$ $\log_8 2 = \frac{1}{3}$ $\quad$ $\log_{\sqrt{6}} 36 = 4$

Evaluate each expression.

$\log_9 9^6$ $\quad$ $\log_{10} 0.01$ $\quad$ $12^{\log_{12} 5}$ $\quad$ $\log_2 \frac{1}{16}$

$\log_6 6^5$ $\quad$ $\log_8 16$ $\quad$ $\log_a a^{10}$ $\quad$ $\log_{11} 11$

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

The Laws of Logarithms

Recall the Laws of Exponents:

$x^m \cdot x^n =$

$\frac{x^m}{x^n} =$

The Law of Logarithms:

$\log(m \cdot n) =$

$\log\left(\frac{m}{n}\right) =$

One special Law: $\log x^m =$

A Logarithm No-No!

$$\log(3^2 + 5) =$$

Using the Laws of Logarithms:

$\log\frac{9}{8} =$

$\log 36$

$\log \sqrt[3]{81} =$

Going the other way:

$\log(3) + 4\log x =$

$2\log 3 - 4\log 2 =$

$\log 5 - (2\log x + \log 7) =$

Breaking down equations with logarithms:

$\log\left(\frac{x^3}{(x-5)}\right) =$

$\log(x^2 \cdot \sqrt[3]{(x-1)} =$

Homework for Laws of Logarithms

Solve each equation:

$\log_x 49 = 2$

$\log_5 .04 = x$

$\log_6(4x+4) = \log_6 64$

$\log_3 3x = \log_3 36$

$\log_x 16 = -4$

$\log_6 216 = x$

$\log_{10} \sqrt[3]{10 = x}$

$\log_2 4 + \log_2 6 = \log_2 x$

$2\log_6 4 - \frac{1}{4}\log_6 16 = \log_6 x$

$3\log_7 4 - 4\log_7 3 = \log_7 x$

$\log_3 12 - \log_3 x = \log_3 3$

$\log_4(x-3) + \log_4(x+3) = 2$

$\log_6 x = \frac{1}{2}\log_6 9 + \frac{1}{3}\log_6 27$

$\log_9 5x = \log_9 6 + \log_9(x-2)$

$\log_{10} x + \log_{10} x + \log_{10} x = \log_{10} 8$

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Solving Exponential and Logarithmic Equations

Using the Calculator: The Common Logarithm:

The Conversion Equation: $\log_a x =$

Convert the following logarithms and then find the value:

$\log_5 7$ =

$\log_3(-2)$

$\log_{.25} 32$ =

Solving Exponential Equations:

$x^{\frac{5}{3}} = 32$

$5 = 3 + 4a^{-\frac{1}{6}}$

Solving Exponential Equations using a Calculator:

$5^{2x} = 147$

$3^{x-1} = 17$

$6^{x+2} = 4^x$

Solving Logarithmic Equations without a Calculator:

$\log_b 5 = -\frac{1}{3}$

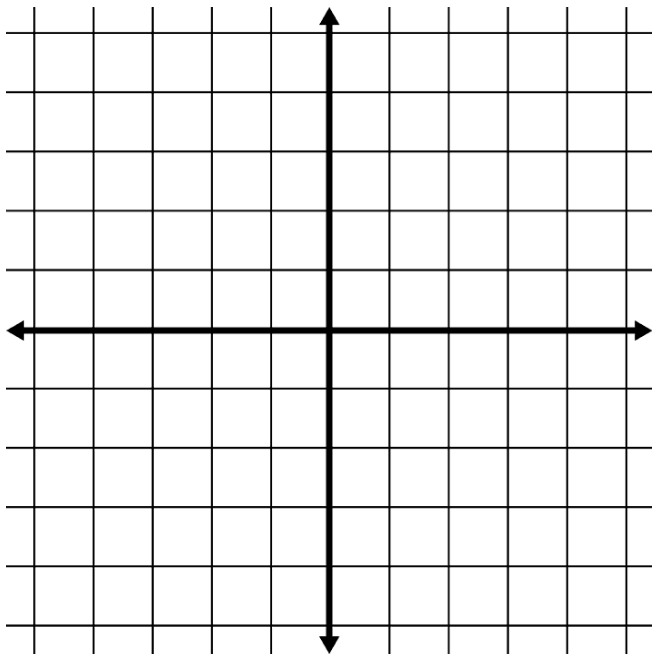
$\log_3(4x+5) - \log_3(3-2x) = 2$

Assignment: Worksheets

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

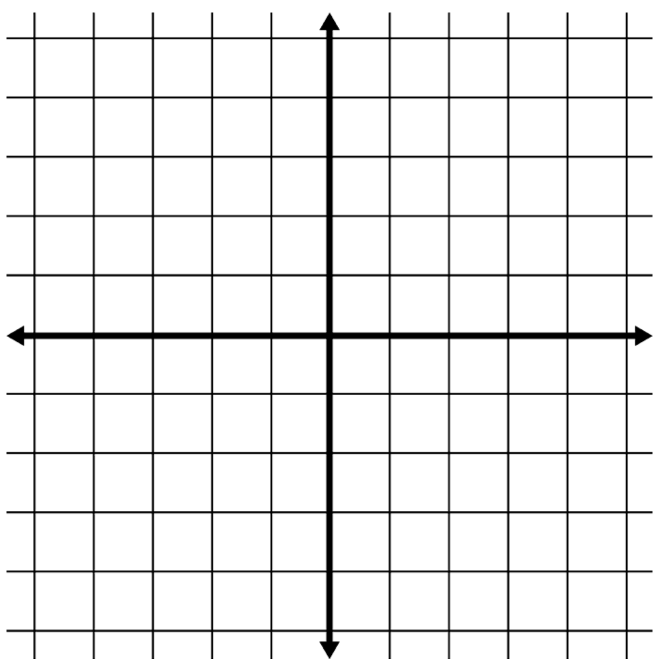
The Exponential and Natural Logarithms

Recall the graphs of $y = a^x$ and its inverse $y = \log_a x$.



The most famous of the exponential values is _____ which is approximately ______.

The graph of y = **Its Inverse is ______________.**



Some key facts:

ln 1 = ____ **ln e = ____** **ln 0 = ____**

Solving Equations:

$e^x = 13$

$3e^{2x-1} = 64.2$

$2\ln x + \ln 3 = 5$

$\ln x - 3\ln 4 = 2$

Assignment:

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EXPONENTIAL GROWTH AND DECAY

(Day One)

When something goes up in value, it is said to ______________________________________,
When something goes down in value, it is said to ___________________________________.

Exponential Growth By a Fixed Rate:

$y = y_0\,(1 + r)^t$ **y is the ________________, y_0 is the ___________________**

Calculating Amounts when the Interest is Compounded.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ **n represents:**

Semi-annually: **Quarterly:** **Monthly:**

Example: **Investor Sam decides to sock $85,000 into an annuity account that yields 4% quarterly. How much can he expect to gain in the next 10 years?**

Sam will decide to retire when the account reaches $200,000. How many years must he wait?

Example: Mr. Bashore purchased a rare copy of a Pre-Calculus Textbook online 10 years ago at a cost of $12,000. It is now worth $19,100. Assuming a steady rate of growth, what was the yearly rate of appreciation?

<u>Exponential Decay By a Fixed Rate:</u>

$$\boldsymbol{y = y_0\,(1 - r)^t}$$

<u>Example</u>: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of the caffeine to be eliminated from a person's body?

How long will it take for 90% of this caffeine to be eliminated from a person's body?

Assignment: Worksheet

PRE-CALCULUS EXPONENTIAL AND LOGARITHMIC FUNCTIONS

EXPONENTIAL GROWTH AND DECAY

(Day Two)

A model preferred by scientists for exponential growth is: $y = y \quad e$

Where y is the _____________, y_0 is the ________________, and k is a _________________

This model is used to calculate population growth, appreciation or any growth when a fixed rate is _________ given:

Example: You are the mayor of a small city with a population of 17,500, up from 10,600 from 10 years ago. Your water and sewage system is only capable of handling a population of 25,000 people. When will you as mayor, have to have a new system in place?

You must first find *k*:

Now the time *t* can be found:

In banking, when interest is compounded _____________________________, the model is:

$$A =$$

Example: Your wealthy uncle invested a sum of money in an account that was compounded continuously on the day you were born, to be collected on your 18th birthday. If you received $10,000 on your birthday, what was the initial investment?

A model preferred by scientists to describe decay is:

$$y = y \quad e$$

This model is used in problems involving radioactive decay (__________________________) of substances. This is the time period it takes for a radioactive substance to decay into half of its mass.

<u>Example:</u> The half-life of Carbon-14 is 5,760 years. What is the value of <u>*k*</u> for Carbon-14?

$$y = y_0 \, e^{-kt}$$

A paleontologist examining the bones of a woolly mammoth estimates that they contain only 10% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

Assignment: Worksheet

Exponential Growth and Decay Worksheet

Day One

1. For graduation, you receive a new car that was purchased for $25,000. It is expected to depreciate at a rate of 20% per year. What will be its value when you graduate from college in 4 years?

2. In 1915, the population of Westphalia was 120. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in 2015?

3. Bacteria growing on a doorknob increased from 20 after being disinfected to 2000. How many hours did it take for the culture to do this if the growth rate was 85%?

4. Miserly Mike invested $100 in an account where the interest rate was compounded quarterly at a rate of 4.1%. He planned to let the money "ride" for 60 years. How much does the old curmudgeon have when he decides to finally cash in?

5. Discount Larry's is finally going out of business. Each week Larry plans on lowering his prices by 20%. About how many weeks must Tightwad Tim wait for a $500 HD TV to drop to below $100?

6. How many years will it take your money to double if you put it in a bank that compounds interest semi-annually at an annual rate of 5.1%?

7. You buy a rare Bob Ross painting for $25,000. It is expected to increase in value at a rate of 3.25% each year. How much will the painting be worth in 15 years?

How many years will it take the painting to triple in value?

Exponential Growth and Decay Worksheet

Day Two

1. Radium 226 is used to treat cancer. It is also used in fluorescent paint. It decomposes radioactively and has a half-life of 1800 years. Find the constant k for Radium 226.

2. A major highway was constructed 5 years ago to accommodate 40,000 cars per day. At the time of construction, the rate was 25,000 per day. Today, 31,000 cars are recorded each day. If this growth continues, when will upgrades be needed?

3. Assuming the upgrade will be made. How many more cars should be planned for per day in order to meet the additional demand in the next 10 years?

4. You run a pizza chain that sold 45,000 pizzas last year. This year, you decided to not advertise you pizzas. You noticed that sales this year dropped to 37,000 pizzas. Find the rate of decline in sales (k) this year.

5. Find the projected sales of pizzas in 3 years if you don't advertise in the above problem.

6. You will have to declare bankruptcy if you don't sell 15,000 pizzas. If you fail to advertise, when can you expect to be foreclosed on?

7. DDT is an insecticide used in agriculture that is sometimes absorbed by plants that animals and humans eat. DDT decomposes to 10% of its original amount every 5 years in the mud at the bottom of a lake. Find the value of k in the decay formula.

8. In 1972, the US Environmental Protection Agency banned the use of DDT. In what year will the DDT in lakes fall below 25%.

9. How long will it take your money to double if it is compounded continuously at a rate of 4%