Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2018

Manitoba

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error). **Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).**				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 T1

Pierre pushes his car into a garage. The radius of a tire on his car is 22 cm. Determine the distance travelled by his car if the tire rotated a total of 1000°.

Solution

$$\theta = (1000)\left(\frac{\pi}{180}\right)$$

$$= \frac{50\pi}{9}$$

1 mark for conversion

$$s = \theta r$$

$$s = \left(\frac{50\pi}{9}\right)(22)$$

$$= \frac{1100\pi}{9}\text{ cm}$$

or

$$s = 383.972 \text{ cm}$$

1 mark for substitution

2 marks

Exemplar 1

$S = \theta r \qquad \theta = 1000 \times \frac{\pi}{180} \approx 17.4533$

$S = \left(\frac{1000\pi}{180}\right)(22)$

$S \approx 34.907$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 3
E5 (units of measure omitted in final answer)

Exemplar 2

$S = \theta r$

$S = 1000(22)$

$S = \underline{22\,000} \text{ cm} \quad \text{or } 220 \text{ m}$

1 out of 2

+ 1 mark for substitution

Exemplar 3

$r = 11 \text{ cm}$

$S = \theta r$

$S = \frac{1000\pi}{180} \cdot 11$

$\boxed{S = 191.986 \text{ cm}}$

1½ out of 2
award full marks
– ½ mark for procedural error in line 1

Question 2 R10

Solve, algebraically.

$$7^{\frac{x}{2}} = 85$$

Solution

$\log 7^{\frac{x}{2}} = \log 85$ ½ mark for applying logarithms

$\frac{x}{2}\log 7 = \log 85$ 1 mark for power law

$x\log 7 = 2\log 85$

$x = \frac{2\log 85}{\log 7}$

$x = 4.566\ 142$ ½ mark for evaluating quotient of logarithms

$x = 4.566$

2 marks

Exemplar 1

$$\log_7 85 = \frac{x}{2}$$

$$\frac{\log 85}{\log 7} = \frac{x}{2}$$

$$x = 2\left(\frac{\log 85}{\log 7}\right)$$

$$\boxed{x = 4.566}$$

2 out of 2

Exemplar 2

$$\log 7^{\frac{x}{2}} = \log 85$$

$$\left(\frac{x}{2}\right)\log 7 = \log 85$$

$$x \log 7 - 2 \log 7 = \log 85$$

$$x \log 7 = \log 85 + 2 \log 7$$

$$x = \frac{\log 85 + 2 \log 7}{\log 7}$$

$$x = 4.283$$

1 out of 2

award full marks
– 1 mark for concept error in line 3

Question 3 T5

Solve, algebraically, over the interval $\left[0, 2\pi\right)$.

$$\sin x\left(\sec x + 3\right) = 0$$

Solution

$\sin x = 0$	$\sec x = -3$	½ mark for solving for $\sin x$ ½ mark for solving for $\sec x$
	$\cos x = -\frac{1}{3}$	1 mark for reciprocal
	$x_r = 1.230\ 959$	
$x = 0,\ \pi$	$x = 1.911,\ 4.373$	2 marks for values of x (1 mark for each branch)

4 marks

Exemplar 1

$\sin x \left(\sec x + 3\right) = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\sec x = -3$

$\cos x = \frac{-1}{3}$

$\theta = 0.944956946$

$Q\text{II}: x = \pi - \theta r$

$x = \pi - 0.944956946$

$x = 2.197$

$Q\text{III}: x = \pi + \theta r$

$x = \pi + 0.944956946$

$x = 4.087$

$\boxed{x = 0,\ 2.197,\ \pi,\ 4.087,\ 2\pi}$

3½ out of 4

award full marks
– ½ mark for procedural error in line 4
E3 (variable introduced without being defined in line 4)
E8 (answer outside the given domain)

Exemplar 2

$$\sin x \left(\frac{1}{\cos x} + 3\right) = 0$$

$$\sin x = 0 \qquad \frac{1}{\cos x} + 3 = 0$$

$$x = 0 \qquad \cos x = -3$$

NO SOLUTION

3 out of 4

+ 1 mark for reciprocal
+ ½ mark for solving for $\sin x$
+ ½ mark for value of x
+ 1 mark for no solution (consistent with error)

Exemplar 3

$$\sin x \left(\frac{1}{\sin x} + 3\right) = 0$$

$$\sin x = 0 \qquad \sin x = -\frac{1}{3}$$

$$0, \pi, \cancel{2\pi} \qquad x_R = 0.3398$$

$$x = 3.481$$

$$x = 5.943$$

3 out of 4

award full marks
– 1 mark for concept error in line 1
E7 (notation error in line 3)

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Question 4 R10

Brahim invests \$2500 at an annual interest rate of 6.75% compounded monthly. Determine, algebraically, how many years it will take for his investment to reach an amount of \$10 500.

Use the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = the amount of the investment after t years

P = the principal of the investment

r = the annual interest rate (as a decimal)

n = the number of compounding periods per year

t = the length of the investment in years

Solution

$$10\ 500 = 2500\left(1 + \frac{0.0675}{12}\right)^{12t}$$ ½ mark for substitution

$$4.2 = (1.005\ 625)^{12t}$$ ½ mark for simplification

$$\log 4.2 = 12t \log 1.005\ 625$$ ½ mark for applying logarithms
1 mark for power law

$$\frac{\log 4.2}{12 \log 1.005\ 625} = t$$

$$21.320\ 250 = t$$

$$21.320 \text{ years} = t$$ ½ mark for evaluating quotient of logarithms

3 marks

Exemplar 1

$A = 10\,500$

$P = 2500$

$r = 6.75$

$n = 12$

$t = ?$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{10\,500}{2\,500} = \frac{2500\left(1 + \frac{(6.75)}{(12)}\right)^{(12)t}}{2500}$$

$$4.2 = 1 + \left(\frac{6.75}{12}\right)^{12t}$$

$$4.2 = 1 + (0.5625)^{12t}$$

$$4.2 = (1.5625)^{12t}$$

$$\log 4.2 = \log 1.5625^{12t}$$

$$\frac{\log 4.2}{\log 1.5625} = \frac{12t \cancel{\log 1.5625}}{\cancel{\log 1.5625}}$$

$$t = 0.268 \text{ years}$$

2½ out of 3

+ ½ mark for simplification
+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for evaluating quotient of logarithms
E4 (missing brackets but still implied in lines 3 and 4)

Exemplar 2

$10\,500 = 2500\left(1 + \frac{0.675}{12}\right)^{(12)t}$ $r = 0.0675$

$\frac{10500}{2500} = \frac{2500\,(1.05625)^{12t}}{2500}$

$4.2 = 1.05625^{12t}$

$4.2 = 1.928415722^{t}$

$\log 4.2 = t \log 1.928415722$ ∴ It would take 2 years

$0.623249290 = 0.285200663\,t$

$t = 2.165$

3 out of 3

award full marks
E7 (transcription error in line 1)
E6 (rounding error in final answer)

Exemplar 3

$10{,}500 = 2500\left(1 + \frac{0.0675}{12}\right)^{12t}$

$\frac{10{,}500 = 2500(1.005625)^{12t}}{2500}$

$4.2 = 1.005625^{12t}$

$\frac{\log 4.2 = 12t \log 1.005625}{\log 1.005625}$

2½ out of 3

+ ½ mark for substitution
+ ½ mark for simplification
+ ½ mark for applying logarithms
+ 1 mark for power law
E7 (notation error in lines 2 and 4)

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Question 5 P3

There are 13 adults and 18 children who can be selected to go on a trip. Determine the number of ways 4 adults and 7 children can be selected if Sandra, one of the adults, must be selected.

Solution

${}_{1}C_{1} \bullet {}_{12}C_{3} \bullet {}_{18}C_{7}$	1 mark for ${}_{12}C_{3}$ ½ mark for ${}_{18}C_{7}$
7 001 280	½ mark for product of combinations

2 marks

Note:

- ${}_{1}C_{1}$ does not need to be shown.

Exemplar 1

$_{12}C_4 \times {}_{18}C_7 \times {}_1C_1 = 15\ 752\ 860$

1 out of 2

+ ½ mark for $_{18}C_7$
+ ½ mark for product of combinations

Exemplar 2

$_{12}C_3$ $\quad$ $_{18}C_7$ $\quad = 32044$ ways

220 $\quad$ 31824

1½ out of 2

+ 1 mark for $_{12}C_3$
+ ½ mark for $_{18}C_7$

Exemplar 3

$\underset{\text{Sandra}}{\underline{1}} \cdot \underline{12} \cdot \underline{11} \cdot \underline{10} \times \underline{18} \cdot \underline{17} \cdot \underline{16} \cdot \underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} =$

$= 2.117 \times 10^{11}$ ways

1 out of 2

award full marks
– 1 mark for concept error (using permutations instead of combinations)

In the binomial expansion of $\left(\frac{2}{x^2} - x^3\right)^9$, determine and simplify the 6th term.

Solution

$t_6 = {}_9C_5\left(\frac{2}{x^2}\right)^4\left(-x^3\right)^5$ 2 marks (1 mark for ${}_9C_5$; ½ mark for each consistent factor)

$t_6 = 126\left(\frac{16}{x^8}\right)\left(-x^{15}\right)$

$t_6 = -2016x^7$ 1 mark for simplification (½ mark for coefficient; ½ mark for exponent)

3 marks

Exemplar 1

$$t_{k+1} = {}_nC_k\, x^{n-k} y^k$$
$$t_{5+1} = ({}_9C_5)\left(\frac{2}{x^2}\right)^{9-5}(-y^3)^5$$
$$t_6 = (126)\left(\frac{16}{x^4}\right)(-x^{15})$$
$$t_6 = \frac{-2016x^{15}}{x^4}$$
$$\boxed{t_6 = -2016x^{11}}$$

2½ out of 3

+ 1 mark for ${}_9C_5$
+ 1 mark for consistent factors
+ ½ mark for simplification of coefficient

Exemplar 2

$$t_6 = t_{k+1}$$
$$k = 5$$
$$t_6 = {}_9C_6 \cdot \left(\frac{2}{x^2}\right)^3 \cdot (-x^3)^6$$
$$t_6 = \frac{9!}{3!6!} \cdot \frac{8}{x^6} \cdot -x^{18}$$
$$t_6 = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{3! \cdot \cancel{6!}} \cdot \frac{-8x^{18}}{x^6}$$
$$t_6 = 84 \cdot -8x^{12}$$
$$\boxed{t_6 = -672x^{12}}$$

1½ out of 3

+ 1 mark for consistent factors
+ ½ mark for simplification of exponent

Question 7 R1

Given that $f(x) = \{(-1, 0), (0, 2), (1, -3), (2, 4)\}$, evaluate $f(f(0))$.

Solution

$f(f(0))$

$f(2)$ ½ mark for $f(0) = 2$

4 ½ mark for $f(f(0))$

1 mark

Exemplar 1

$f(0) = -1$

$f(-1) = 0$

½ out of 1

+ ½ mark for $f\left(f(0)\right)$ (consistent with error)

Exemplar 2

$f(f(0))$

$f(2)$

½ out of 1

+ ½ mark for $f(0) = 2$

Question 8 T2

The point $\left(-\frac{5}{6}, b\right)$ is on the unit circle and is in quadrant III.

Determine the exact value of b.

Solution

$$\left(-\frac{5}{6}\right)^2 + b^2 = 1$$ ½ mark for substitution

$$b^2 = 1 - \frac{25}{36}$$

$$b^2 = \frac{11}{36}$$

$$b = \pm\frac{\sqrt{11}}{6}$$

$$b = -\frac{\sqrt{11}}{6}$$ ½ mark for the exact value of b

1 mark

Exemplar 1

$x = -5 \quad r = 6 \quad y = -\sqrt{11}$

$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y^2 = 36 - 25$

$y^2 = 11$

$y = \pm\sqrt{11}$

$P(\theta) = \left(\frac{-5}{6}, \frac{-\sqrt{11}}{6}\right)$

1 out of 1

Exemplar 2

$r^2 = x^2 + y^2$

$1^2 = \left(-\frac{5}{6}\right)^2 + b^2$

$1 = \frac{25}{36} + b^2$

$1 - \frac{25}{36} = b^2$

$\sqrt{\frac{11}{36}} = \sqrt{b^{\cancel{2}}}$

$\frac{\sqrt{11}}{6} = b$

½ out of 1

+ ½ mark for substitution

Exemplar 3

$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y^2 = 1 - \left(-\frac{5}{6}\right)^2$$
$$= 1 - \frac{25}{36}$$
$$= \frac{36}{36} - \frac{25}{36}$$
$$= \frac{11}{36}$$
$$\therefore y = \pm\frac{\sqrt{11}}{6}$$
$$y = -\frac{\sqrt{11}}{6}$$

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 4

$$\left(-\frac{5}{6}\right)^2 + b^2 = 1$$
$$-\frac{25}{36} + b^2 = 1$$
$$b^2 = 1 + \frac{25}{36}$$
$$b = -\frac{\sqrt{61}}{6}$$

½ out of 1

award full marks
– ½ mark for arithmetic error in line 2
E1 (impossible solution not rejected in final answer)

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Question 9 P4

Given the following row of Pascal's Triangle, determine the values of the next row.

1 6 15 20 15 6 1

Solution

1 7 21 35 35 21 7 1 **1 mark**

Exemplar 1

7 21 35 35 21 7

0 out of 1

Exemplar 2

1 7 21 35 21 7 1

0 out of 1

Question 10 R3, R5

The following transformations are applied to $f(x)$, resulting in a new function, $g(x)$.

- reflection over the x-axis
- vertical stretch by a factor of 3
- horizontal stretch by a factor of 4

State the equation of $g(x)$ in terms of $f(x)$.

Solution

$$g(x) = \underline{-3f\left(\frac{1}{4}x\right)}$$

1 mark for vertical reflection
1 mark for vertical stretch
1 mark for horizontal stretch

3 marks

Exemplar 1

$g(x) = 3 - f(4x)$

1 out of 3

+ 1 mark for vertical reflection

Exemplar 2

$g(x) = 3g(-\frac{1}{4}x)$

1½ out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal stretch
– ½ mark for procedural error (g instead of f)

Exemplar 3

$f(x) = a(b(x-h))+k$

$a = -3$
$b = \frac{1}{4}$

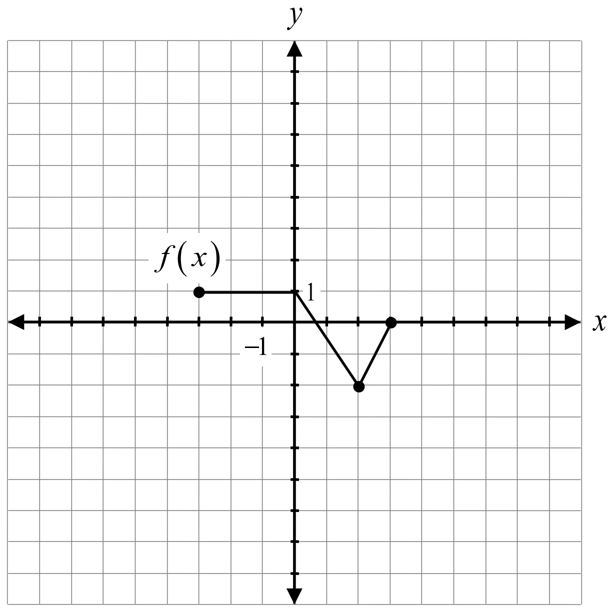
$g(x) = -3(\frac{1}{4}(x))$

2 out of 3

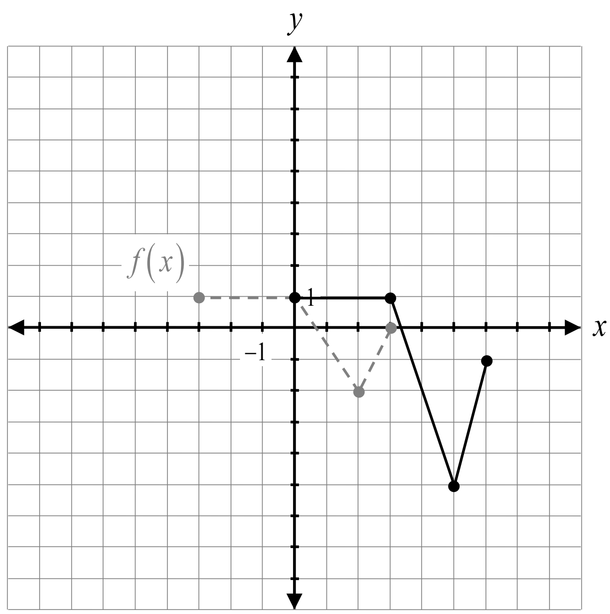
award full marks
– 1 mark for concept error (omitting f)

Question 11 R4

Given the graph of $f(x)$, sketch the graph of $y+1=2f(x-3)$.



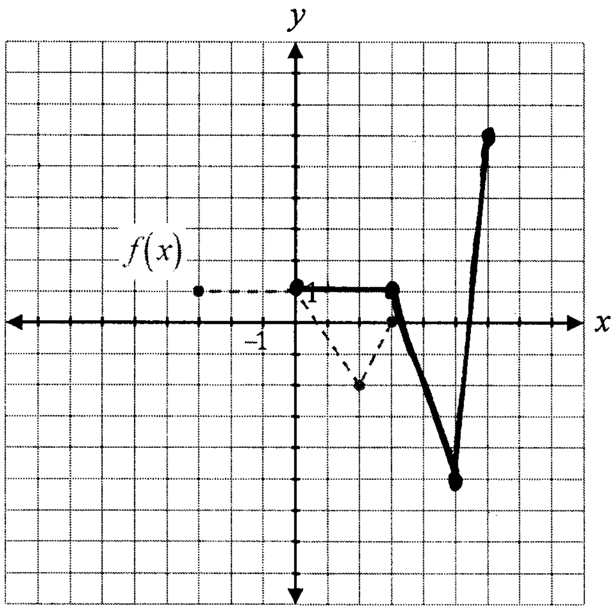
Solution



1 mark for vertical stretch
1 mark for horizontal translation
1 mark for vertical translation

3 marks

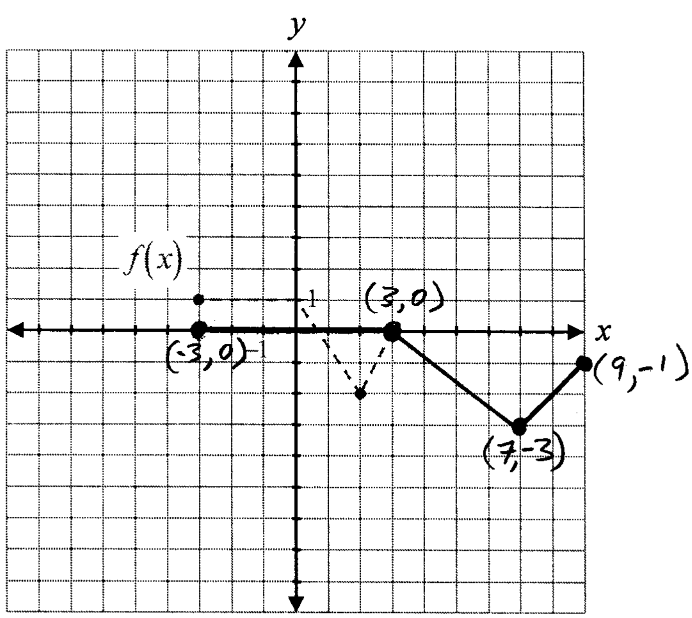
Exemplar 1



2½ out of 3

award full marks
– ½ mark for procedural error (one incorrect point)

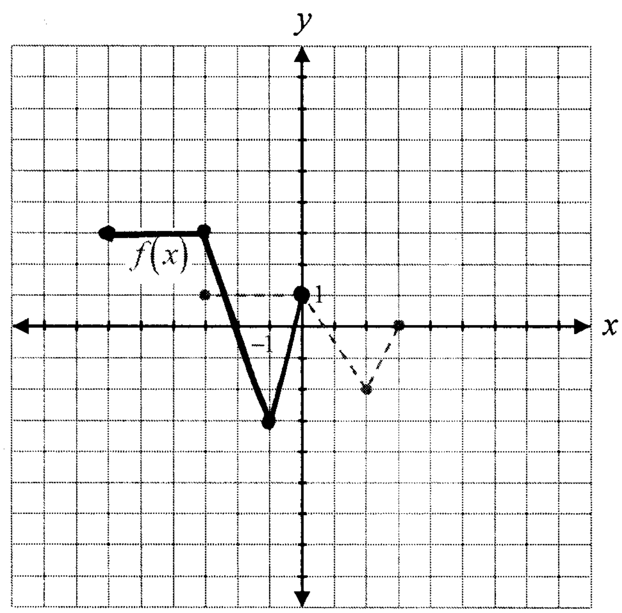
Exemplar 2



2 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 3



1 out of 3

+ 1 mark for vertical stretch

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Question 12 R14

State the equation of the horizontal asymptote of $f(x) = \dfrac{2x^2 - 3x + 5}{4x^2 + 2x - 7}$.

Solution

$y = \dfrac{1}{2}$

1 mark

Exemplar 1

$$= \frac{2}{4}$$

H.A $= \frac{1}{2}$

½ out of 1

award full marks
– ½ mark for procedural error

Exemplar 2

$$\frac{2}{4} = 2$$

$y = 2$

½ out of 1

award full marks
– ½ mark for arithmetic error

Exemplar 3

$y = 2$

0 out of 1

Exemplar 4

Horizontal Asymptote $x = \frac{1}{2}$

0 out of 1

Given that $(x+4)$ is one of the factors of $p(x) = x^3 + 6x^2 - 32$, express $p(x)$ in completely factored form.

Solution

$$\begin{array}{r|rrrr} -4 & 1 & 6 & 0 & -32 \\ & \downarrow & -4 & -8 & 32 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

½ mark for $x = -4$

1 mark for synthetic division (or any equivalent strategy)

$p(x) = (x+4)\left(x^2 + 2x - 8\right)$

$p(x) = \underline{(x+4)(x+4)(x-2)}$

½ mark for consistent product of factors

or

2 marks

$p(x) = \underline{(x+4)^2(x-2)}$

Exemplar 1

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
x+4 & x^3 + 6x^2 - 0x - 32 \\
 & \underline{x^3 + 4x^2} \\
 & \quad 2x^2 - 0x \\
 & \quad \underline{2x^2 + 8x} \\
 & \qquad -8x - 32 \\
 & \qquad \underline{-8x - 32} \\
 & \qquad\qquad 0
\end{array}
$$

$x^2 + 2x - 8$

$(x+4)(x-2)$

$p(x) =$ $(x+4)(x-2)$

1½ out of 2

+ ½ mark for $x = -4$
+ 1 mark for equivalent strategy

Exemplar 2

$$
\begin{array}{r|rrrr}
-4 & 1 & 6 & 0 & -32 \\
 & & -4 & -8 & 32 \\
\hline
 & 1 & 2 & -8 & 0
\end{array}
$$

$p(x) =$ $(x^2 + 2x - 8)(x+4)$

1½ out of 2

+ ½ mark for $x = -4$
+ 1 mark for synthetic division

Exemplar 3

$$
\begin{array}{r|rrrr}
-4 & 1 & 6 & 0 & -32 \\
 & \downarrow & -4 & -8 & 32 \\
\hline
 & 1 & 2 & -8 & 0
\end{array}
$$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4 \quad x=2$

$p(x) =$ $x=-4 \quad x=2$

1½ out of 2

+ ½ mark for $x = -4$
+ 1 mark for synthetic division

Exemplar 4

$$
\begin{array}{r|rrr}
-4 & 1 & 6 & -32 \\
 & \downarrow & -4 & -8 \\
\hline
 & 1 & 2 & -40
\end{array}
$$

$P(x) = (x^2+2x-40)(x+4)$

$p(x) =$ $(x^2+2x-40)(x+4)$

---

1 out of 2

+ ½ mark for $x = -4$
+ 1 mark for synthetic division
– ½ mark for procedural error in line 1

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Question 14 T6

Prove the identity for all permissible values of θ.

$$\frac{2\cos^2\theta}{1-\cot\theta} = \frac{\sin 2\theta}{\tan\theta - 1}$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{2\cos^2\theta}{1-\dfrac{\cos\theta}{\sin\theta}}$	$\dfrac{2\sin\theta\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-1}$
$\dfrac{2\cos^2\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}$	$\dfrac{2\sin\theta\cos\theta}{\dfrac{\sin\theta-\cos\theta}{\cos\theta}}$
$\dfrac{2\cos^2\theta\sin\theta}{\sin\theta-\cos\theta}$	$\dfrac{2\sin\theta\cos^2\theta}{\sin\theta-\cos\theta}$

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Solution

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{2\cos^2\theta}{1-\cot\theta}$	$\dfrac{2\sin\theta\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-1}$
	$\dfrac{2\sin\theta\cos\theta}{\dfrac{\sin\theta-\cos\theta}{\cos\theta}}$
	$\dfrac{2\sin\theta\cos^2\theta}{\sin\theta-\cos\theta}$
	$\dfrac{\sin\theta\left(2\cos^2\theta\right)}{\sin\theta\left(1-\dfrac{\cos\theta}{\sin\theta}\right)}$
	$\dfrac{2\cos^2\theta}{1-\cot\theta}$

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Solution

Method 3

Left-Hand Side	Right-Hand Side
$\dfrac{2\cos^2\theta}{1-\cot\theta}$	$\dfrac{\sin 2\theta}{\tan\theta-1}$
$\dfrac{2\cos^2\theta}{1-\dfrac{1}{\tan\theta}}$	
$\dfrac{2\cos^2\theta}{\dfrac{\tan\theta-1}{\tan\theta}}$	
$\dfrac{2\cos^2\theta}{\tan\theta-1}(\tan\theta)$	
$\dfrac{2\cos^2\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)}{\tan\theta-1}$	
$\dfrac{2\cos\theta\sin\theta}{\tan\theta-1}$	
$\dfrac{\sin 2\theta}{\tan\theta-1}$	

1 mark for algebraic strategies
1 mark for logical process to prove the identity
1 mark for correct substitution of identities

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\rightarrow \dfrac{2\cos^2\theta}{1-\cot\theta}$	$\rightarrow \dfrac{\sin 2\theta}{\tan\theta - 1}$
$\rightarrow \dfrac{2\cos^2\theta}{\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}}$	$\rightarrow \dfrac{\sin 2\theta}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}$
$\rightarrow 2\cos^2\theta\left(\dfrac{\sin\theta}{\sin\theta\cos\theta}\right)$	$\rightarrow \dfrac{\sin 2\theta}{\frac{\sin\theta\cdot\cos\theta}{\cos\theta}}$
$\rightarrow \dfrac{2\cos^2\theta\sin\theta}{\sin\theta\cos\theta}$	$\rightarrow \dfrac{2\sin\theta\cos\theta(\cos\theta)}{\sin\theta\cos\theta}$
$\rightarrow \dfrac{2\cos\theta\sin\theta}{\sin\theta}$	
	$\rightarrow 2\cos\theta$
$\rightarrow 2\cos\theta$	

$\therefore$ LHS = RHS

2 out of 3

+ 1 mark for correct substitution of identities
+ 1 mark for logical process to prove the identity

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{2\cos^2\theta}{1-\cot\theta}$	$\dfrac{\sin 2\theta}{\tan\theta - 1}$
$\dfrac{2\cos^2\theta}{\frac{\sin\theta}{\sin\theta} \quad \frac{\cos\theta}{\sin\theta}}$	$\dfrac{2\sin\theta\cos\theta}{\tan\theta-1}$
	$\dfrac{2\sin\theta\cos\theta}{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\cos\theta}}$
	$\dfrac{2\sin\theta\cos\theta}{\frac{\sin\theta-\cos\theta}{\cos\theta}}$

2 out of 3

+ 1 mark for correct substitution of identities
+ 1 mark for algebraic strategies

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Question 15 P1

Restaurant A has 5 types of hamburgers, 2 types of french fries, and 10 types of drinks.
Restaurant B has 4 types of hamburgers, 5 types of french fries, and 6 types of drinks.

If a meal is made up of a hamburger, french fries, and a drink, justify which restaurant offers a greater variety of meals.

Solution

Restaurant A: $5 \bullet 2 \bullet 10 = 100$ meals
Restaurant B: $4 \bullet 5 \bullet 6 \ \ = 120$ meals

Restaurant B offers a greater variety of meals. **1 mark**

Exemplar 1

A $(5)(2)(10) = 1000$

B $(5)(6)(4) = 120$

A has more variety.

½ out of 1

award full marks
– ½ mark for arithmetic error in line 1

Exemplar 2

Restaurant B

0 out of 1

Question 16 R8

Express $\log_7(2x-5)+2\log_7 3$ as a single logarithm.

Solution

$\log_7(2x-5)+\log_7 3^2$ 1 mark for power law

$\log_7(2x-5)+\log_7 9$

$\log_7(9(2x-5))$ 1 mark for product law

or **2 marks**

$\log_7(18x-45)$

Exemplar 1

$$\log_7(2x-5) + \log_7 3^2$$

$$\log_7 (2x-5)(9)$$

$$\log_7\ 18x-45$$

2 out of 2

award full marks
E7 (notation error in lines 2 and 3)

Exemplar 2

$$\log_7(2x-5) + \log_7 3^2$$
$$\log_7((2x-5)(6))$$
$$\log\ (12x-30)$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 2
E7 (transcription error in line 3)

Exemplar 3

$$\log_7 2x - \log_7 5 + \log_7 9$$
$$\log_7\left(\frac{(2x)(9)}{5}\right)$$

1 out of 2

award full marks
– 1 mark for concept error

Question 17 P2

Explain why 11! is not the total number of 11-letter arrangements that can be made from the word CELEBRATION.

Solution

The total number of possible arrangements is half of 11! because switching the two Es creates duplicate arrangements.

1 mark

Exemplar 1

because celebration has 2 E's so you have to consider that!

$$\frac{11!}{2!}\boxed{= 19958400}$$

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

there are duplicate E's

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

Because the letter E repeats we subtract 2!

0 out of 1

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
18	C	R13
19	B	P2
20	D	T3
21	D	R7
22	C	T4
23	A	R12
24	B	P4
25	A	R2
26	B	R11

Question 18 R13

Given $f(x) = x - 1$, identify a point on the graph of $y = \sqrt{f(x)}$.

a) $(0, -1)$

b) $(3, 2)$

c) $(1, 0)$

d) $(0, 1)$

Question 19 P2

Identify the total number of possible arrangements of 6 adults and 4 children seated in a row if the children must sit together.

a) $6!4!$

b) $7!4!$

c) $10!$

d) $6!$

Question 20 T3

Identify the exact value of $\sec\left(-\frac{7\pi}{3}\right)$.

a) -2

b) $-\frac{2}{\sqrt{3}}$

c) $\frac{2}{\sqrt{3}}$

d) 2

Question 21 R7

Given $\log_x\left(\frac{1}{25}\right) = -2,$ identify the value of x.

a) -5

b) $-\frac{1}{5}$

c) $\frac{1}{5}$

d) 5

Question 22 T4

Identify the equation for all of the asymptotes on the graph of $y = \tan x$.

a) $x = k\pi,\ k \in \mathbb{Z}$

b) $x = 2k\pi,\ k \in \mathbb{Z}$

c) $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

d) $x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

Question 23 R12

If $p(x) = 3(m)(x+1)^2$ is a cubic function with a y-intercept of $-12,$ identify the missing factor, m.

a) $m = x - 4$

b) $m = x + 4$

c) $m = x + 12$

d) $m = x - 12$

Question 24 P4

Identify the number of negative terms in the binomial expansion of $(x-y)^5$.

a) 2

b) 3

c) 5

d) 6

Question 25 R2

Given $f(x)=x^2$, identify which equation represents the graph of $y=f(x)$ after a translation of 5 units to the left.

a) $y=(x+5)^2$

b) $y=(x-5)^2$

c) $y=x^2-5$

d) $y=x^2+5$

Question 26 R11

When a polynomial, $p(x)$, is divided by $(x-7)$, the remainder is 24. Identify the only statement that must be true.

a) $x=7$ is a zero of $p(x)$

b) $p(7)=24$

c) $x=24$ is a zero of $p(x)$

d) the y-intercept is 24

Question 27 R14

Given $f(x) = \dfrac{(2x+1)(x-8)}{(x-8)(x+4)}$, state the equation(s) of the vertical asymptote(s).

Solution

$x = -4$ **1 mark**

Exemplar 1

V.A.: $y = -4$

0 out of 1

Exemplar 2

$$f(x) = \frac{(2x+1)}{x+4}$$

$x \neq 8, -4$

0 out of 1

Exemplar 3

$x \neq -4$

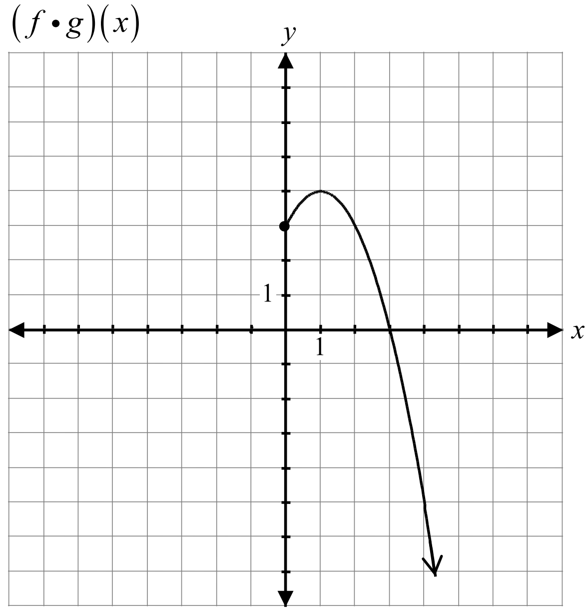
1 out of 1

E7 (notation error)

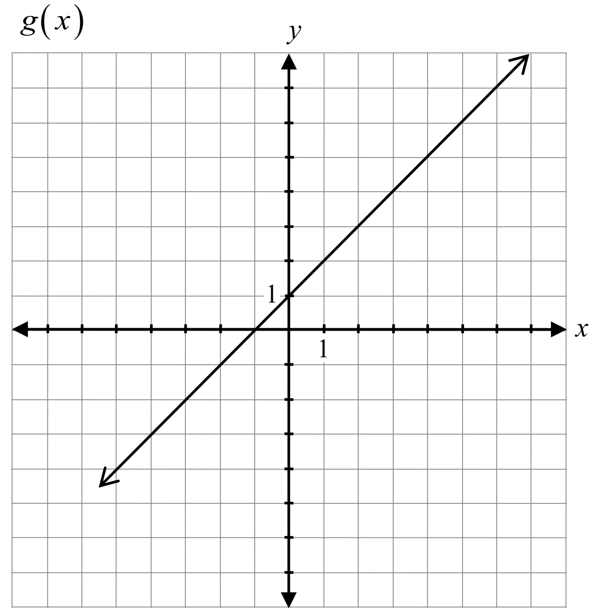
Question 28 R1

Given the graphs of $(f \bullet g)(x)$ and $g(x)$, sketch the graph of $f(x)$.

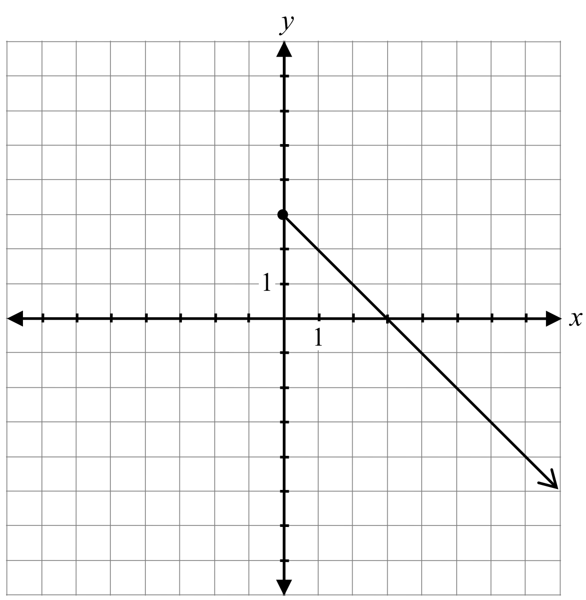
$(f \bullet g)(x)$



$g(x)$



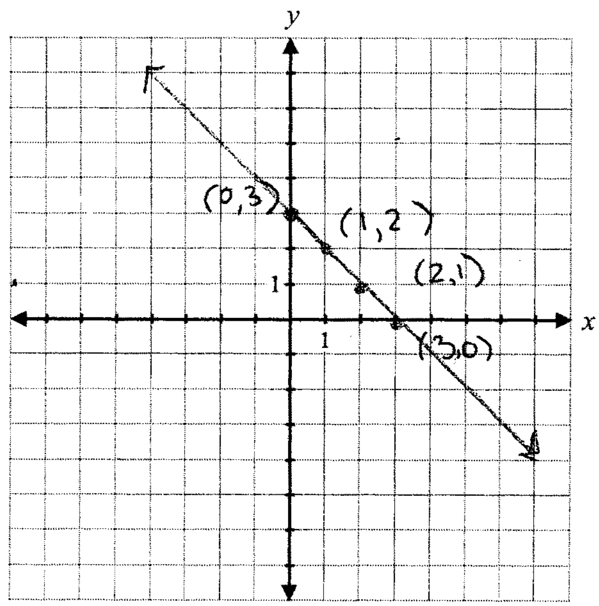
Solution



1 mark for operation of division
1 mark for restricted domain

2 marks

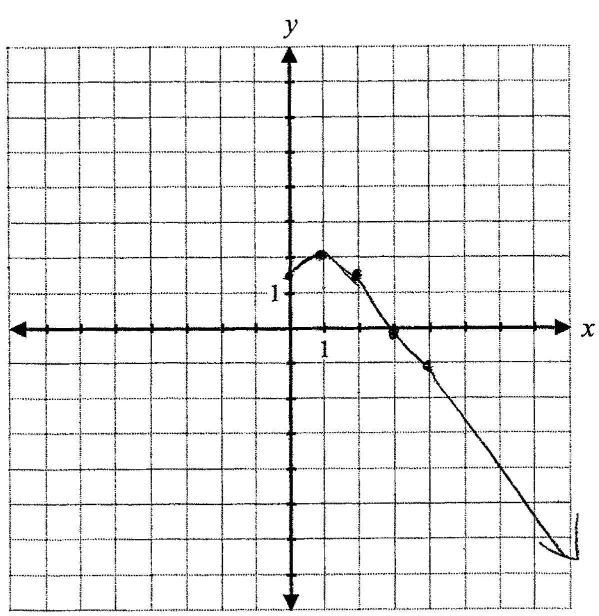
Exemplar 1



1 out of 2

+ 1 mark for operation of division

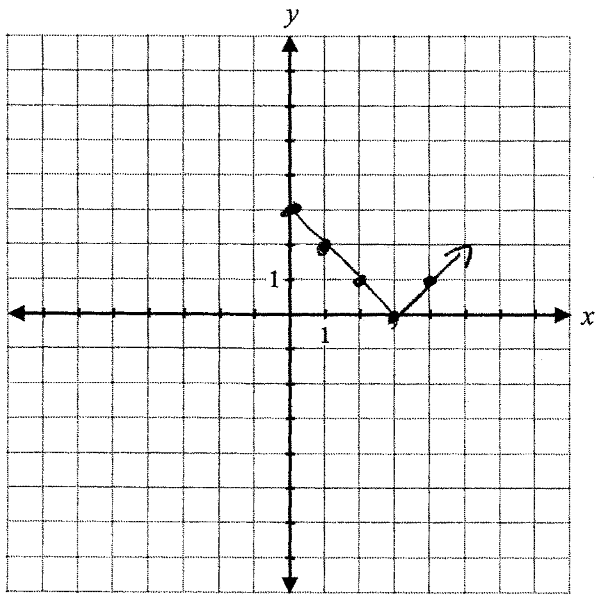
Exemplar 2



1 out of 2

+ 1 mark for restricted domain

Exemplar 3



1½ out of 2

award full marks
– ½ mark for procedural error (one incorrect point)

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Question 29 R11

Brian was asked to state the zeros of the polynomial $p(x) = (x+2)(x-5)(x-1)$.

Brian's response:

Zeros: $(x+2), (x-5), (x-1)$

Explain why his response is incorrect.

Solution

Brian stated the factors instead of the zeros.

1 mark

Exemplar 1

The zeros are $(x+2)(x-5)(x-1)$ after they have been solved:

$x+2=0$ $\quad$ $x-5=0$ $\quad$ $x-1=0$

$x=-2$ $\quad$ $x=5$ $\quad$ $x=1$

0 out of 1

Exemplar 2

the 0's are what make the inner value of the brakets 0

$x=-2, 5, 1$ would be correct

1 out of 1

Question 30 P3

Simplify ${}_{n+3}C_2$.

Solution

$$\frac{(n+3)!}{(n+3-2)!2!}$$ ½ mark for substitution

$$\frac{(n+3)!}{(n+1)!2!}$$

$$\frac{(n+3)(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}2}$$ 1 mark for factorial expansion

$$\frac{(n+3)(n+2)}{2}$$ ½ mark for simplification of factorials

or

2 marks

$$\frac{n^2+5n+6}{2}$$

Exemplar 1

$$\frac{(n+3)!}{((n+3)-2)!2!}$$

$$\frac{(n+3)!}{(n+1)!\ 2}$$

$$\frac{\cancel{(n+3)!}}{(n+1)(n+2)\cancel{(n+3)!}2}$$

$$\frac{1}{(n+1)(n+2)2}$$

1 out of 2

+ ½ mark for substitution
+ ½ mark for simplification of factorials

Exemplar 2

$$\frac{n+3\ !}{2!((n+3)-2)!}$$

$$\frac{n+3!}{2!(n+1)!}$$

$$\frac{(n+3)(n+2)\cancel{(n+1)!}}{2!\ \cancel{(n+1)!}}$$

$$\frac{n^2+5n+6}{2!}$$

1½ out of 2

+ ½ mark for substitution
+ 1 mark for factorial expansion
E4 (missing brackets but still implied in lines 1 and 2)

Exemplar 3

$$\frac{(n+3)!}{(n+3-2)!\,2!}$$

$$\frac{(n+3)!}{(n+1)!\,2!}$$

$$\frac{(n+3)(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}\;2!}$$

$$\frac{(n+3)(n+2)}{\cancel{2}} = 0 \cdot 2$$

$n = -3 \qquad n = -2$

1½ out of 2

award full marks
– ½ mark for procedural error (solving for *n*)
E1 (impossible solution not rejected in final answer)

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Question 31 T5

Verify that the equation $2\cos^2 x = \sin x + 1$ is true for $x = \frac{\pi}{6}$.

Solution

Left-Hand Side	Right-Hand Side	
$2\cos^2\left(\frac{\pi}{6}\right)$	$\sin\left(\frac{\pi}{6}\right)+1$	½ mark for substitution
$2\left(\frac{\sqrt{3}}{2}\right)^2$	$\frac{1}{2}+1$	1 mark for exact values (½ mark for each)
$2\left(\frac{3}{4}\right)$		
$\frac{3}{2}$	$\frac{3}{2}$	½ mark for simplification

$\therefore$ LHS = RHS

2 marks

Exemplar 1

$$2\cos^2 x = \sin x + 1$$

$$2\cos^2 x \underset{-1}{} = \sin x \cancel{+1}_{-1}$$

$$2\cos^2 x - 1 = \sin x$$

$$\cos 2x = \sin x$$

$$\cos 2\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{6}$$

$$\cos \frac{\pi}{3} = \sin \frac{\pi}{6}$$

$$\frac{1}{2} = \frac{1}{2} \quad \checkmark$$

2 out of 2

Exemplar 2

$2\cos^2 x = \sin x + 1$ is not true for $x = \pi/6$

$$2\cos^2 x = \sin x + 1$$

$$2\cos^2(\pi/6) = \sin(\pi/6) + 1$$

$$2(\sqrt{3}/2)^2 = (1/2) + 1$$

$$2(3/4) = 3/2$$

$$6/8 \neq 3/2$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 5

Exemplar 3

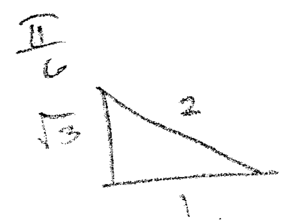
$2\cos^2 x = \sin x + 1$

$2\cos^2 x - \sin x - 1 = 0$

$2(1 - \sin^2 x) - \sin x - 1 = 0$

$2 - 2\sin^2 x - \sin x - 1 = 0$

$\dfrac{-2\sin^2 x}{-1} - \dfrac{\sin x}{-1} + \dfrac{1}{-1} = \dfrac{0}{-1}$



$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Yes, it is true.

2 out of 2

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Question 32 R12

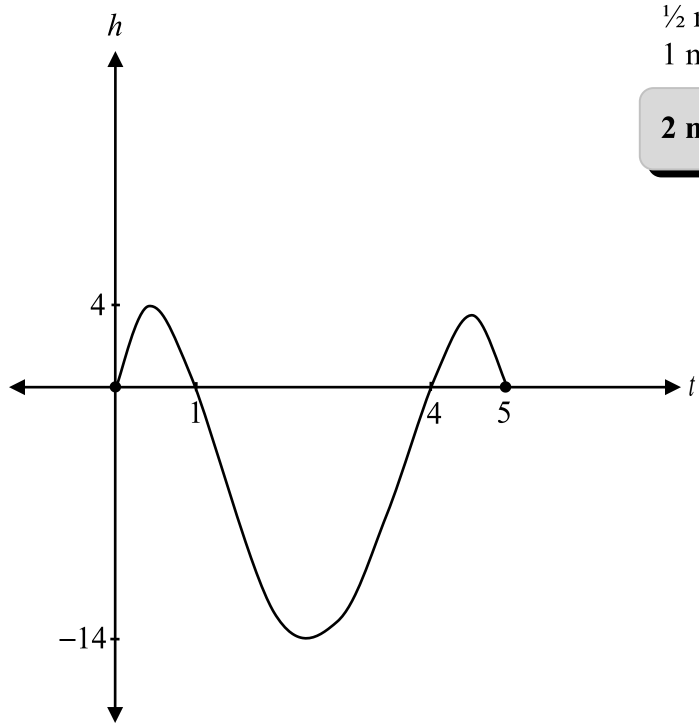
The height of a fish jumping out of the water can be modelled by the function $h(t) = -t(t-1)(t-4)(t-5)$ where $h(t)$ is the height of the fish above or below the water in cm, and t is the time in seconds, $t \geq 0$.

a) Sketch a graph representing the height of the fish with respect to time over the interval $[0, 5]$.

b) State, from the graph in a), the total number of seconds that the fish is above the water.

Solution

a)



½ mark for *h*-intercept
½ mark for other *t*-intercepts
1 mark for vertical reflection of a quartic function

2 marks

b) 2 seconds

1 mark for time consistent with graph in a)

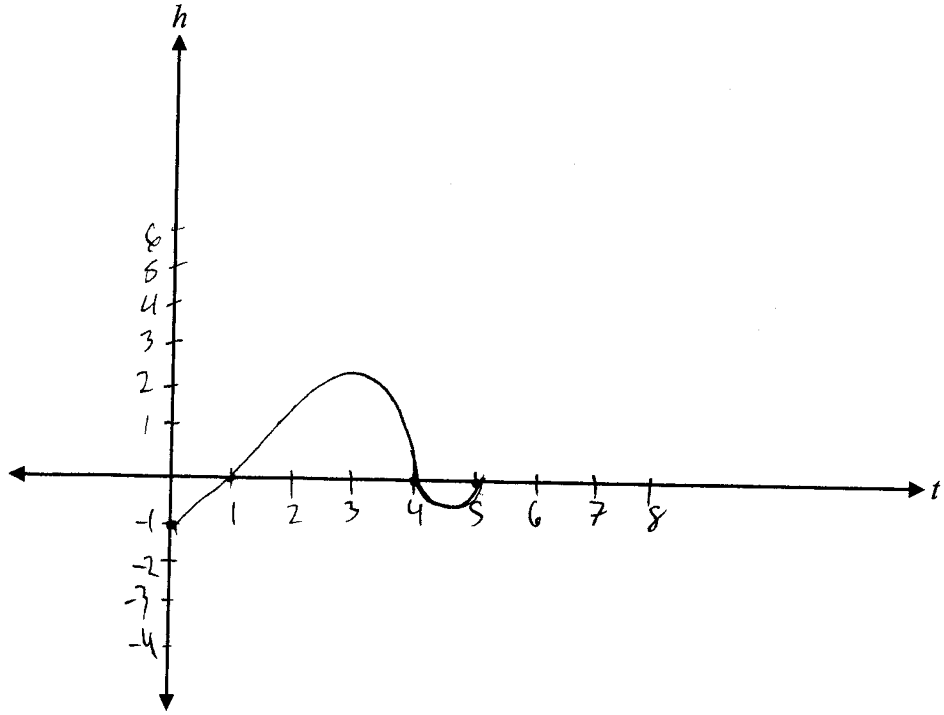
1 mark

Note:

- Scale values on *h*-axis are not required.

Exemplar 1

a)



½ out of 2

+ ½ mark for other t-intercepts

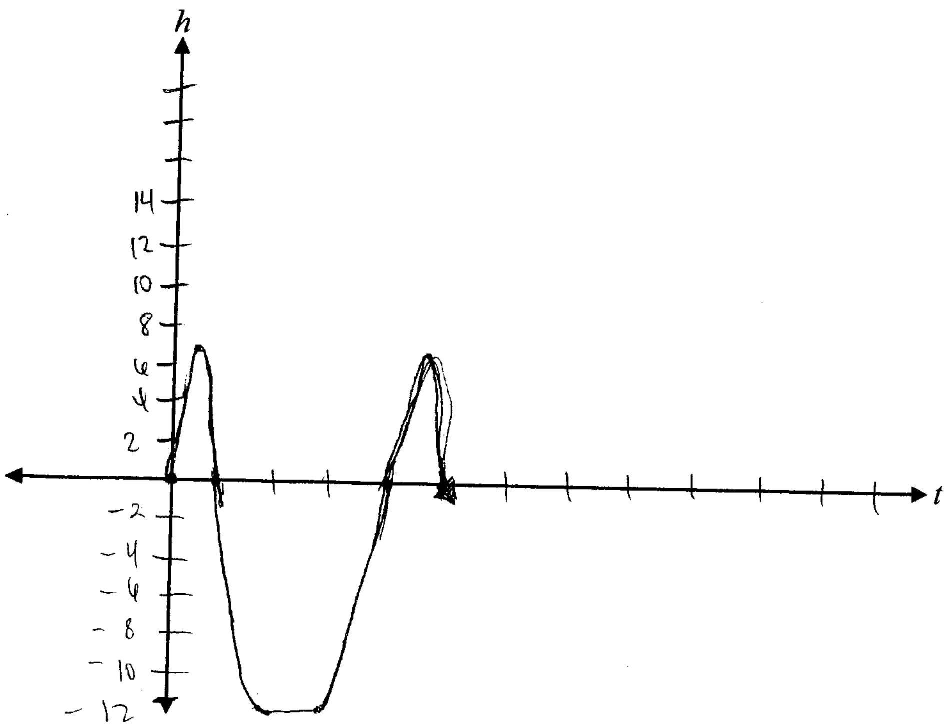
b)

2 seconds

0 out of 1

Exemplar 2

a)



2 out of 2

award full marks
E9 (scale values on t-axis not indicated)
E8 (answer outside the given domain)

b)

2 seconds.

1 out of 1

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Question 33 R9

Describe the behaviour of the graph of $y = 5^x + 4$ as it approaches $y = 4$.

Solution

There is a horizontal asymptote at $y = 4$, so the graph approaches $y = 4$ without ever touching it.

1 mark

Exemplar 1

- The graph will not pass through y=4 since it is an asymptote

1 out of 1

Exemplar 2

The y value will come close to 4 but won't actually reach 4. it will grow smaller and smaller but it will never reach 0

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 3

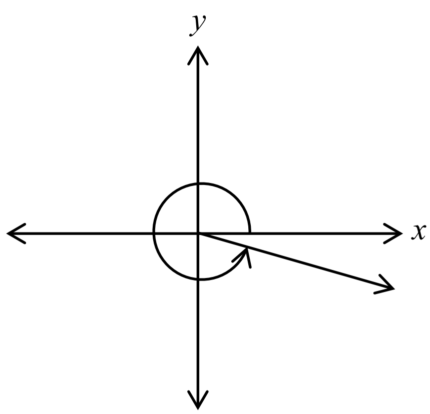
When it approaches y = 4, the graph will get very close to the vertical asymptote there, and as it gets closer, the x values will increase rapidly.

0 out of 1

Question 34 T1

Sketch the angle of 6 radians in standard position.

Solution



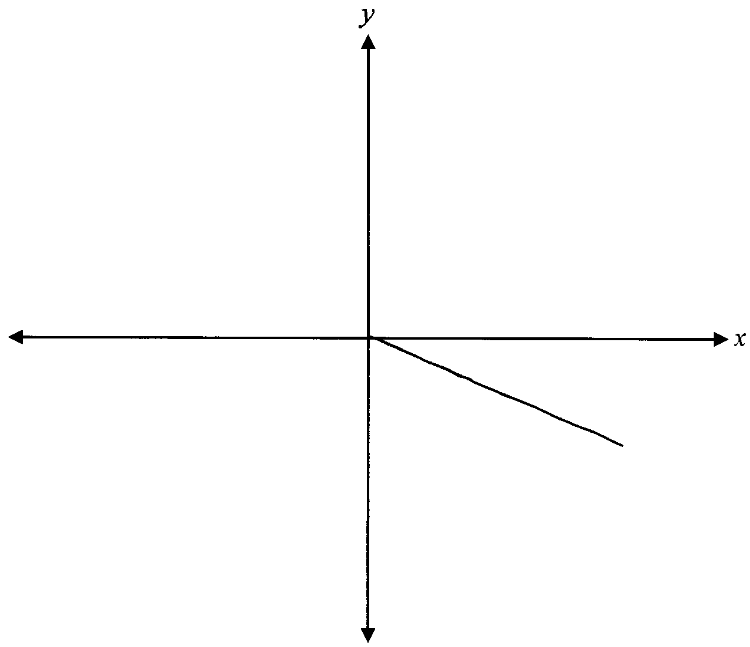
½ mark for an angle in quadrant IV
½ mark for an appropriate angle

1 mark

Note:

- If the directional arrow is not indicated, deduct an E1 error (final answer not stated).

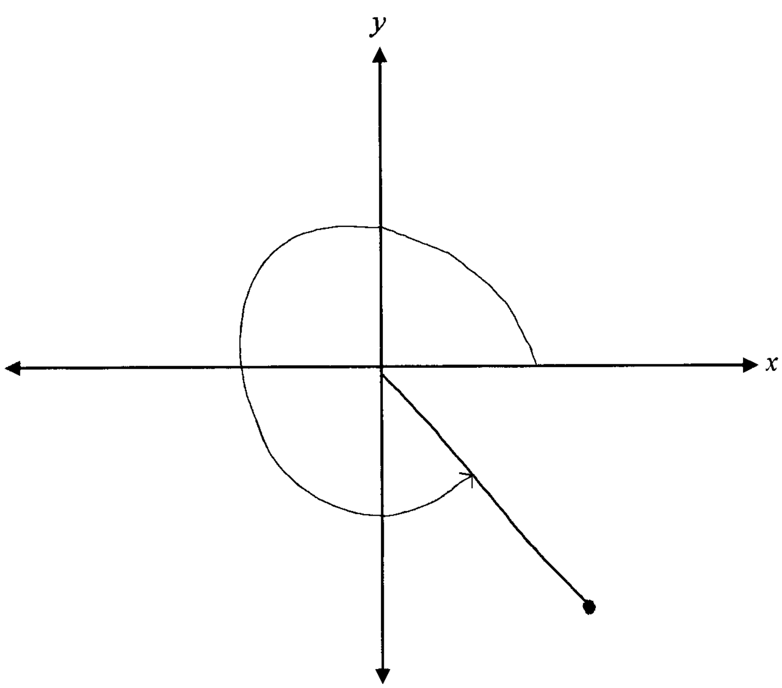
Exemplar 1



1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 2



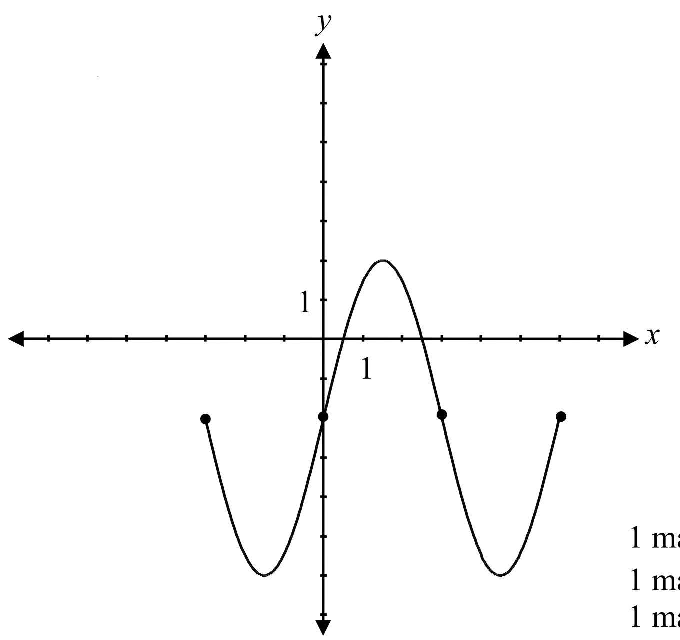
½ out of 1

+ ½ mark for an angle in quadrant IV

Question 35 T4

Sketch the graph of the function $y = 4\sin\left(\frac{\pi}{3}x\right) - 2$ over the domain $[-3, 6]$.

Solution



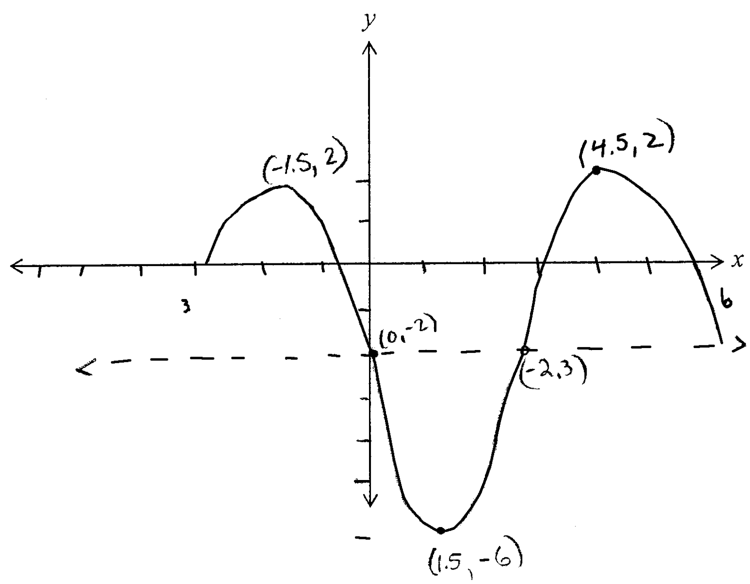
1 mark for shape of $y = \sin x$
1 mark for amplitude
1 mark for period
1 mark for vertical translation

4 marks

Note:

- Deduct ½ mark for procedural error for not completing the domain of $[-3, 6]$.
- If period mark not awarded, do not deduct for E9 error (scale value on x-axis not indicated).

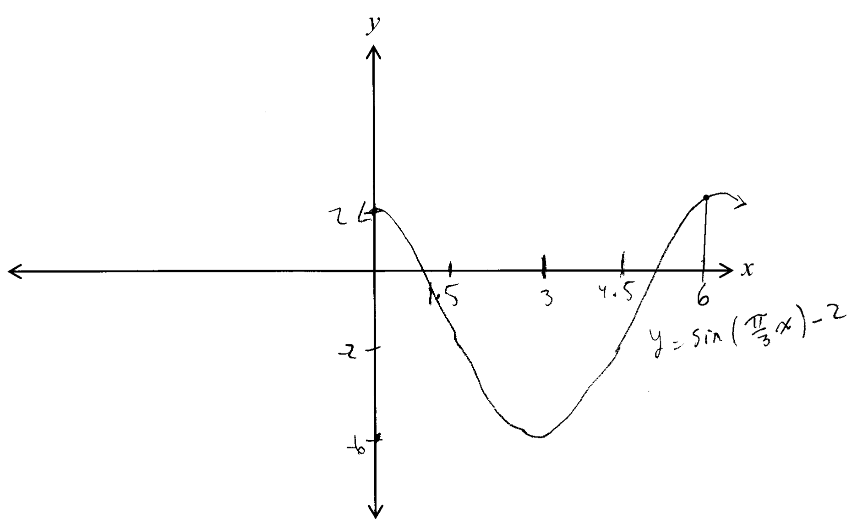
Exemplar 1



2½ out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for vertical translation
– ½ mark for procedural error (not completing the domain of $[-3, 6]$)

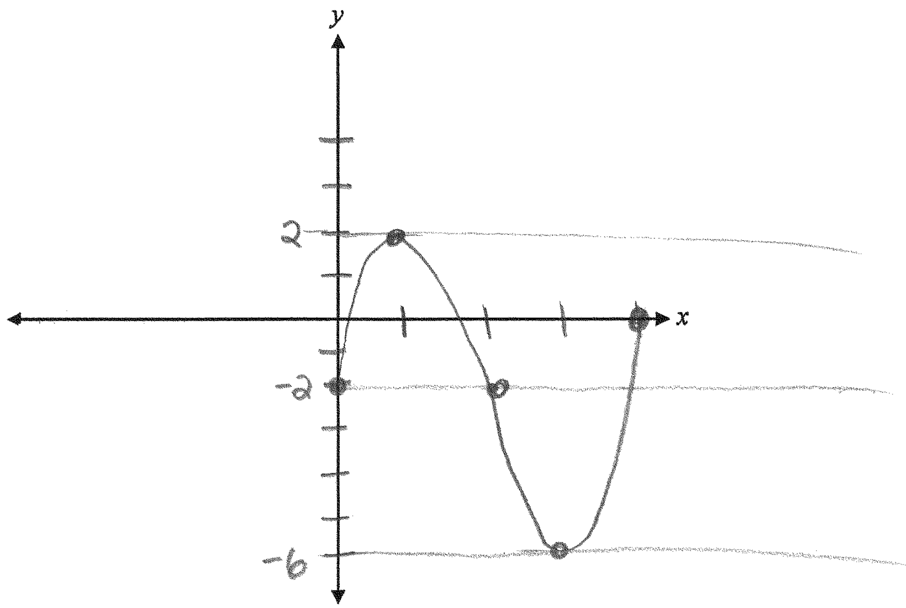
Exemplar 2



2½ out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for vertical translation
– ½ mark for procedural error (not completing the domain of $[-3, 6]$)

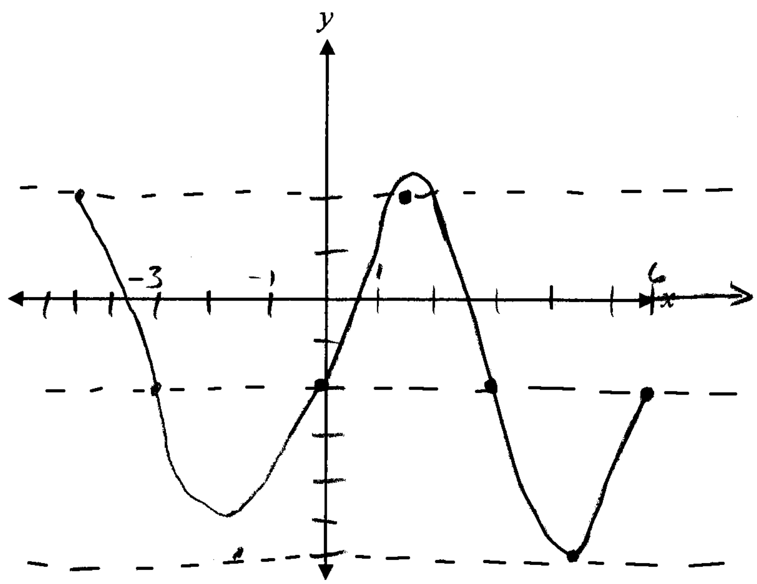
Exemplar 3



2 out of 4

+ 1 mark for shape of $y = \sin x$
+ 1 mark for amplitude
+ 1 mark for vertical translation
– ½ mark for procedural error (one incorrect point)
– ½ mark for procedural error (not completing the domain of $[-3, 6]$)

Exemplar 4



3 out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for vertical translation
E8 (answer outside the given domain)
E9 (scale values on *y*-axis not indicated)

Question 36 R1, R14

Given $f(x) = 3x - 12$ and $g(x) = x - 4$,

a) determine the equation of $h(x) = \left(\frac{f}{g}\right)(x)$.

b) describe what the non-permissible value represents on the graph of $h(x)$.

Solution

a) $h(x) = \frac{3x-12}{x-4}$

1 mark for division of $\left(\frac{f}{g}\right)(x)$

1 mark

$$h(x) = \frac{3\cancel{(x-4)}}{\cancel{(x-4)}}$$

$$h(x) = \underline{3,\ x \neq 4}$$

b) The non-permissible value is a point of discontinuity (hole).

1 mark

Exemplar 1

a)

$$h(x) = \frac{3x - 12}{x - 4}$$

$$h(x) = \frac{3\cancel{(x-4)}}{\cancel{x-4}}$$

$$h(x) = 3$$

1 out of 1

b)

The point at x = 4 doesn't exist.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 2

a)

$$h(x) = \frac{3x - 12}{x - 4}$$

1 out of 1

b) $x \neq 4$

0 out of 1

Question 37 R13

Determine an equation of a radical function, $f(x)$, with a domain of $x \geq 5$ and a range of $y \geq -2$.

Solution

$f(x) = \underline{\sqrt{x-5} - 2}$

1 mark for horizontal translation
1 mark for vertical translation

2 marks

Note:

- Other equations are possible.

Exemplar 1

$$f(x) = (x-5) - 2$$

1 out of 2

award full marks
– 1 mark for concept error (not writing $f(x)$ as a radical function)

Exemplar 2

$$\sqrt{x+5} - 2$$

$f(x) =$ ______________________________

1 out of 2

+ 1 mark for vertical translation
E2 (changing an equation to an expression)

Question 38 T6

Determine the exact value of $\cos\left(\frac{17\pi}{12}\right)$.

Solution

$$\cos\left(\frac{17\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$$

1 mark for substitution into correct identity

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

2 marks for exact values (½ mark for each)

3 marks

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

Note:

- Other combinations are possible.
- Deduct a maximum ½ mark for arithmetic errors in simplification.

Exemplar 1

$$\cos\left(\frac{8\pi}{12} + \frac{9\pi}{12}\right)$$

$$\cos\left(\overset{a}{\frac{2\pi}{3}} + \overset{b}{\frac{3\pi}{4}}\right)$$

$$(\cos a \cos b) - (\sin a \sin b)$$

$$\left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{2} - \left(-\frac{\sqrt{2}}{2}\right)$$

1 out of 3

+ 1 mark for substitution into correct identity

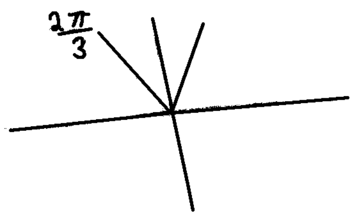
+ ½ mark for the value of $\sin\frac{3\pi}{4}$

– ½ mark for arithmetic error in line 5

Exemplar 2

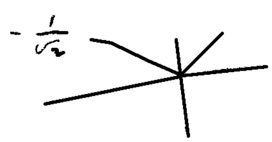
$$\frac{8\pi}{12} + \frac{9\pi}{12} = \frac{17\pi}{12}$$

$$\frac{2\pi}{3} + \frac{3\pi}{4}$$



$$\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{3\pi}{4}\right)$$

$$= -\frac{1}{2}\frac{(\sqrt{2})}{(\sqrt{2})} + -\frac{1}{\sqrt{2}}\frac{(2)}{(2)}$$



$$= \frac{-\sqrt{2}}{2\sqrt{2}} + \frac{-2}{2\sqrt{2}}$$

$$= \frac{-2-\sqrt{2}}{2\sqrt{2}}$$

1 out of 3

+ 1 mark for values of $\cos\frac{2\pi}{3}$ and $\cos\frac{3\pi}{4}$

Exemplar 3

$$\cos\frac{17\pi}{12} = \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$$

$$\cos\left(-\frac{\sqrt{2}}{2}\right)\cos\left(\frac{-1}{2}\right) - \sin\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)$$

$$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

2½ out of 3

award full marks
– ½ mark for procedural error in line 2
E2 (changing an equation to an expression in lines 2 and 3)
E1 (final answer not stated)

Exemplar 4

$$\cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$$

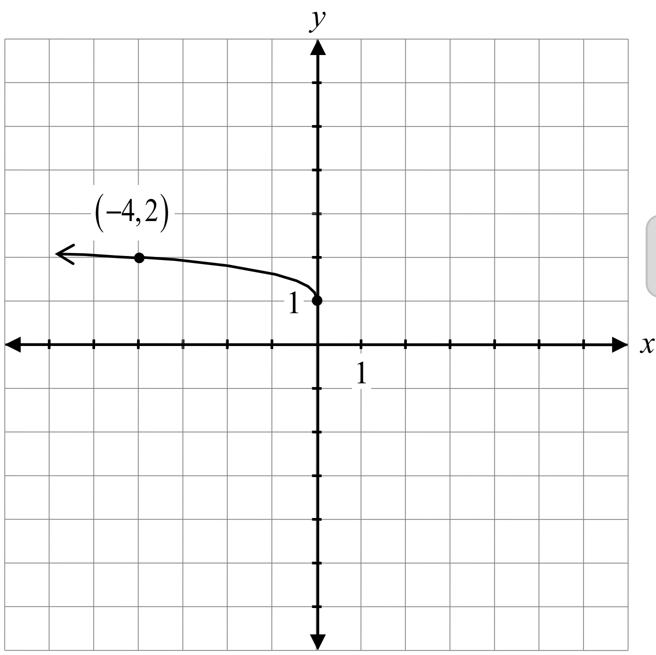
$$\cos\left(-\frac{\sqrt{2}}{2}\right)\cos\left(\frac{-1}{2}\right) - \sin\left(\frac{\sqrt{2}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)$$

2 out of 3

award full marks
– 1 mark for concept error in line 2

Sketch the graph of $y = \frac{1}{2}\sqrt{-x} + 1$.

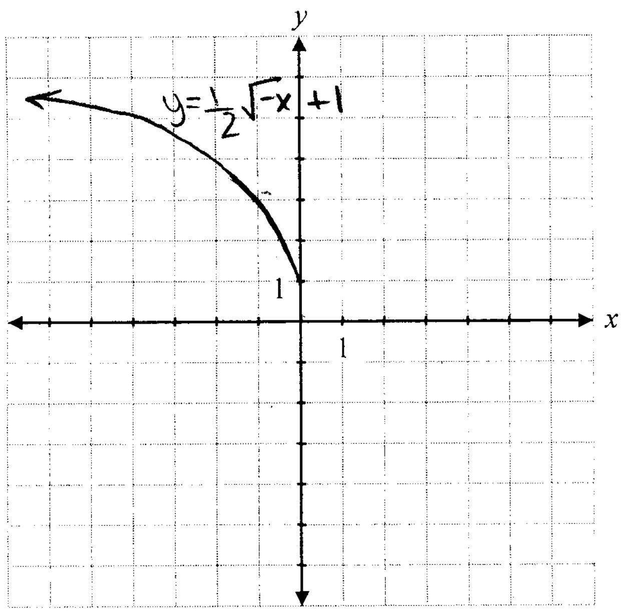
Solution



1 mark for shape of a radical function
1 mark for vertical compression
1 mark for horizontal reflection
1 mark for vertical translation

4 marks

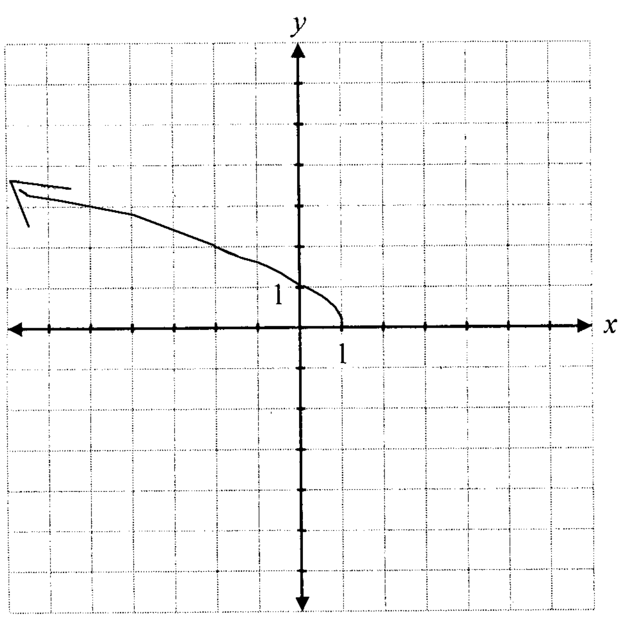
Exemplar 1



3 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for horizontal reflection
+ 1 mark for vertical translation

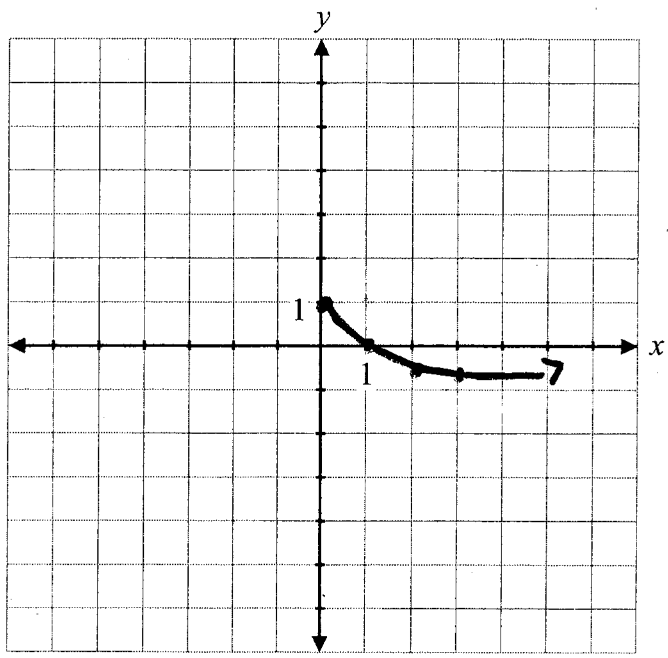
Exemplar 2



2 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for horizontal reflection

Exemplar 3



2 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for vertical translation

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Question 40 R6

Given $f(x) = \frac{3x}{4} + 9$, determine the equation of $f^{-1}(x)$.

Solution

Let $y = f(x)$

$$y = \frac{3x}{4} + 9$$

$$x = \frac{3y}{4} + 9$$ 1 mark for switching x and y values

$$4x = 3y + 36$$

$$3y = 4x - 36$$

$$y = \frac{4}{3}x - 12$$ ½ mark for solving for y

$$f^{-1}(x) = \frac{4x}{3} - 12$$ ½ mark for writing equation of $f^{-1}(x)$

2 marks

Exemplar 1

$$f(x) = \frac{3x}{4} + 9$$

$$y = \frac{3x}{4} + 9$$

$$x = \frac{3y}{4} + 9$$

$$4x = 3y + 9$$

$$\frac{4x-9}{3} = \frac{3y}{3}$$

$$\frac{4x-9}{3} = y$$

$$\frac{4x-9}{3} = f^{-1}(x)$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 4

Exemplar 2

$$f(x) = y$$

$$y = \frac{3x}{4} + 9$$

$$x = \frac{3y}{4} + 9$$

$$x - 9 = \frac{3y}{4}$$

$$\frac{4x-36}{3} = \frac{\cancel{3}y}{\cancel{3}}$$

$$\frac{4x-36}{3} = y$$

1½ out of 2

+ 1 mark for switching *x* and *y* values
+ ½ mark for solving for *y*

Question 41 R12

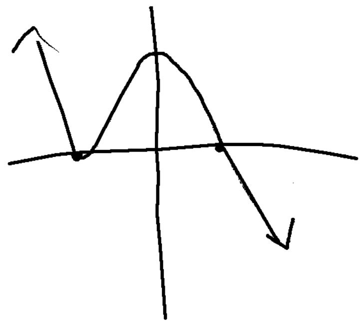
Describe the end behaviour of the polynomial function $p(x) = -(x-2)(x+3)^2$.

Solution

The graph rises as x approaches negative infinity and falls as x approaches positive infinity.

1 mark

Exemplar 1



The graph will end in quadrant 4.

0 out of 1

Exemplar 2

- Both ends go seperate ways.

0 out of 1

Exemplar 3

Starts in quadrant 2 and goes to quadrant 4.

1 out of 1

Exemplar 4

One end goes up and one end goes down

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Question 42 T3

Given $\csc\theta = -4$ and θ is in quadrant IV,

a) determine the exact value of $\cos\theta$.

b) determine the exact value of $\cot\theta$.

Solution

a) $\sin\theta = -\frac{1}{4}$ 1 mark for reciprocal

$$1^2 - \left(-\frac{1}{4}\right)^2 = \cos^2\theta$$

$$\frac{15}{16} = \cos^2\theta$$

$$\pm\frac{\sqrt{15}}{4} = \cos\theta$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

1 mark for $\cos\theta$ (½ mark for quadrant; ½ mark for value)

2 marks

b)

$$\cot\theta = \frac{\frac{\sqrt{15}}{4}}{-\frac{1}{4}}$$

$$= -\sqrt{15}$$

1 mark for $\cot\theta$ consistent with answer in a)
(½ mark for quadrant; ½ mark for value)

1 mark

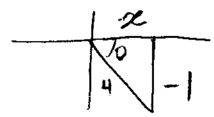
Exemplar 1

a)

$$\csc\theta = -4$$

$$\frac{1}{\sin\theta} = -4$$

$$\sin\theta = \frac{1}{-4}$$



$$x = \sqrt{4^2 - (-1)^2}$$

$$x = \sqrt{4 - 1}$$

$$x = \pm\sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{4}$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 5

b)

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{\sqrt{3}}{4} \div -\frac{1}{4}$$

$$\cot\theta = \frac{\sqrt{3}}{4} \times -4$$

$$\cot\theta = -\sqrt{3}$$

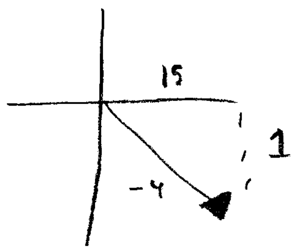
1 out of 1

answer consistent with answer in a)

Exemplar 2

a)

$\frac{1}{\sin\theta} = -4 \quad \sin\theta = \frac{1}{-4}$



$\boxed{\cos\theta = \frac{15}{-4}}$

$1^2 + x^2 = (-4)^2$

$\overset{-1}{1} + x^2 = \overset{-1}{16}$

$\sqrt{x^2} = \pm\sqrt{15}$

$x = 15$

1½ out of 2

+ 1 mark for reciprocal
+ ½ mark for value of $\cos\theta$
E7 (transcription error in line 6)
E1 (impossible solution not rejected in final answer)

b) $\cot\theta = \dfrac{\frac{15}{-4}}{\frac{1}{-4}}$ $\qquad \cot\theta = \left(\frac{15}{-4}\right)\left(\frac{-4}{1}\right)$

$\cot\theta = \cancel{\frac{-60}{-4}}$

$\boxed{\cot\theta = 15}$

1 out of 1

answer consistent with answer in a)

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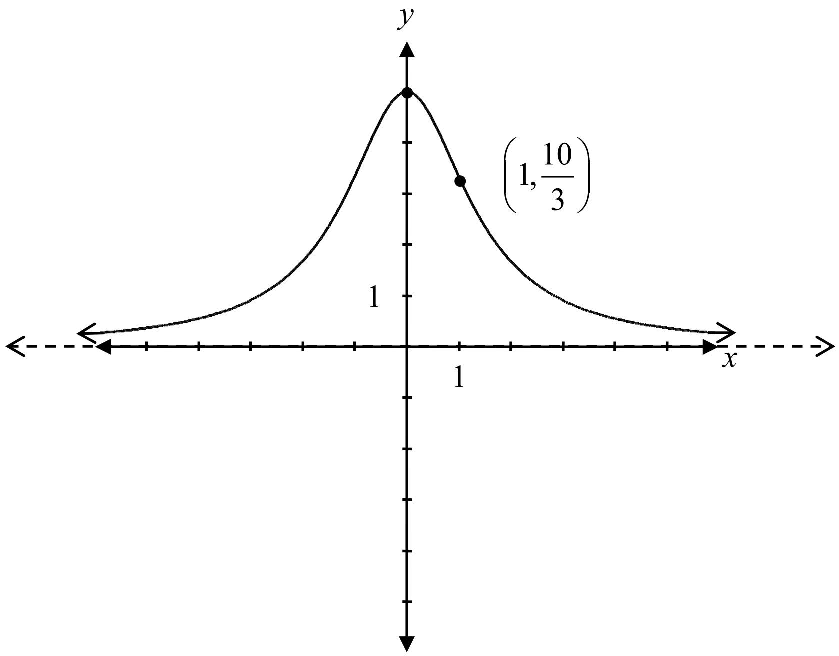
Question 43 R14

Sketch the graph of the function $f(x) = \dfrac{10}{x^2 + 2}$.

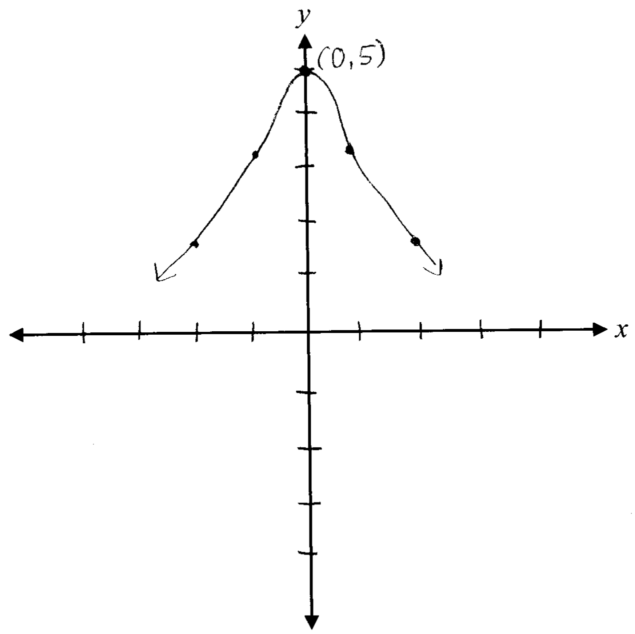
Solution



1 mark for asymptotic behaviour approaching $y = 0$
1 mark for shape (½ mark for graph left of y-axis; ½ mark for graph right of y-axis)

2 marks

Exemplar 1



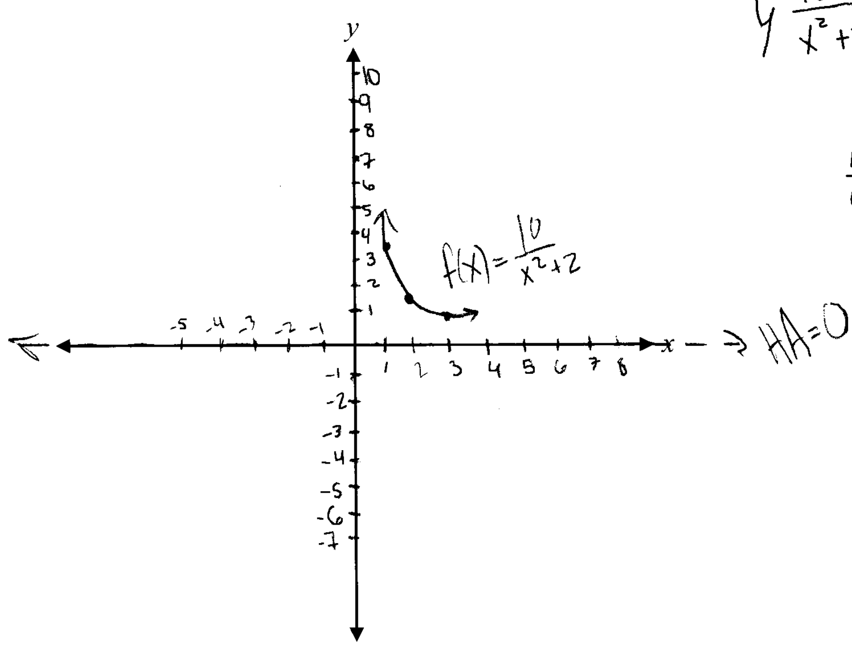
x	y
−2	5/3
−1	10/3
0	5
1	10/3
2	5/3

1 out of 2

+ 1 mark for shape

Exemplar 2

$y \frac{10}{x^2+2}$ $x=1$



$\frac{10}{3}=3.1$

$\frac{10}{6}=1.4$

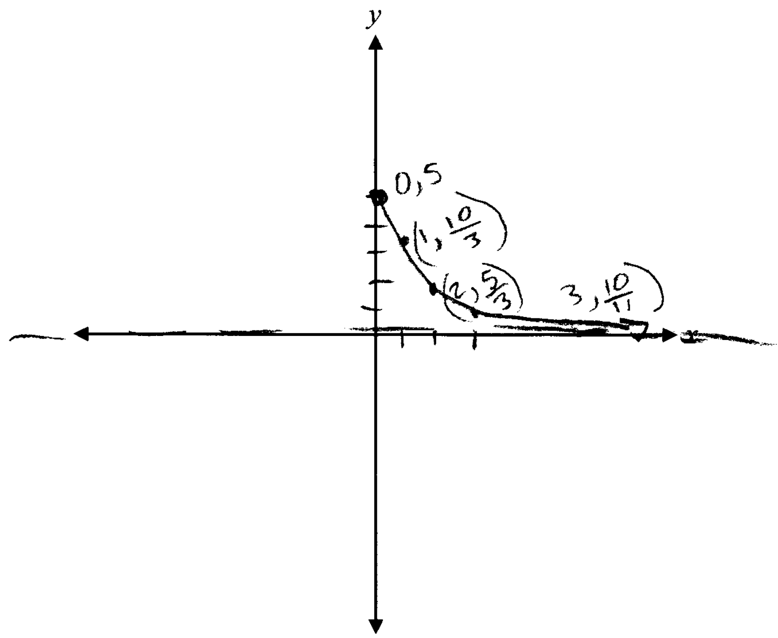
$\frac{10}{11}=1.1$

x	
1	3.1
2	1.4
3	1.1

1 out of 2

+ 1 mark for asymptotic behaviour approaching $y = 0$

Exemplar 3



1½ out of 2

+ 1 mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph right of y-axis

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Question 44 R10

Explain why only one of the following equations could be solved algebraically without using logarithms.

$3^{5x} = 6^{2x-1}$ or $16^{2x+3} = \left(\frac{1}{2}\right)^{4x-5}$

Solution

The equation $16^{2x+3} = \left(\frac{1}{2}\right)^{4x-5}$ can be solved without the use of logarithms because 16 and $\frac{1}{2}$ can be changed to a common base of 2.

1 mark

Exemplar 1

This can be solved algebraically because you can get the same base of 3 and then drop the base to solve.

0 out of 1

Exemplar 2

$16^{2x+3} = \frac{1}{2}^{4x-5}$ could be because you could multiply both sides by 2 and then have one side equal to 1.

0 out of 1

Exemplar 3

$3^{5x} = 6^{2x-1}$ can't ~~is~~ be solved algebraically without using logarithms.

$$\cancel{2}\ 2^{4(2x+3)} = 2^{-1(4x-5)}$$

$$4(2x+3) = -4x+5$$

$$8x+12 = -4x+5$$

$$12x = -7$$

$$x = -\frac{7}{12}$$

$16^{2x+3} = \left(\frac{1}{2}\right)^{4x-5}$ can be solved algebraically without using logarithms.

0 out of 1

Question 45 R1

Given a graph of $y = f(x)$, describe how to sketch the graph of $y = |f(x)|$.

Solution

Reflect all the points with negative y-values over the x-axis. **1 mark**

Exemplar 1

absolute graphs never go negative y's

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 2

all negative values become positive

½ out of 1

award full marks
– ½ mark for lack of clarity in description

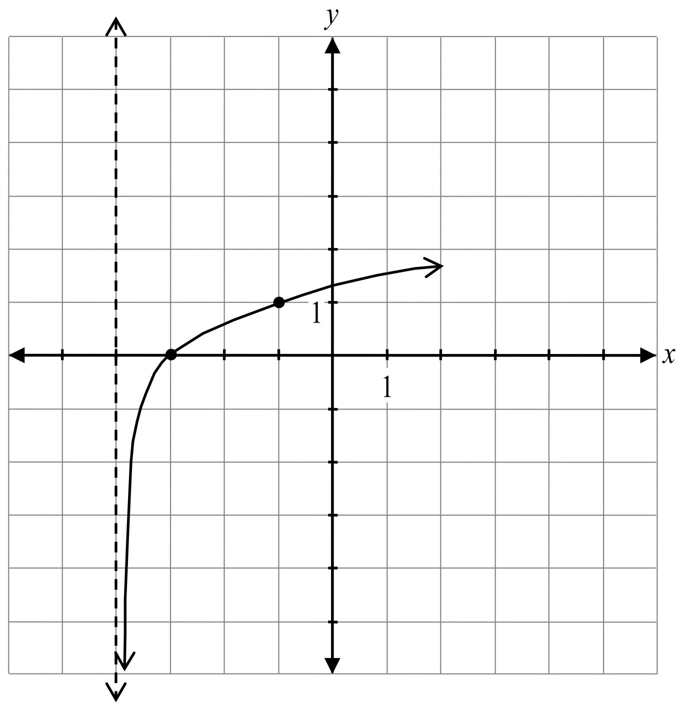
Exemplar 3

The graph $|f(x)|$ can be sketched by taking the absolute value of the y-coordinate of $f(x)$ while the x remains the same.

1 out of 1

Sketch the graph of $y = \log_3 (x + 4)$.

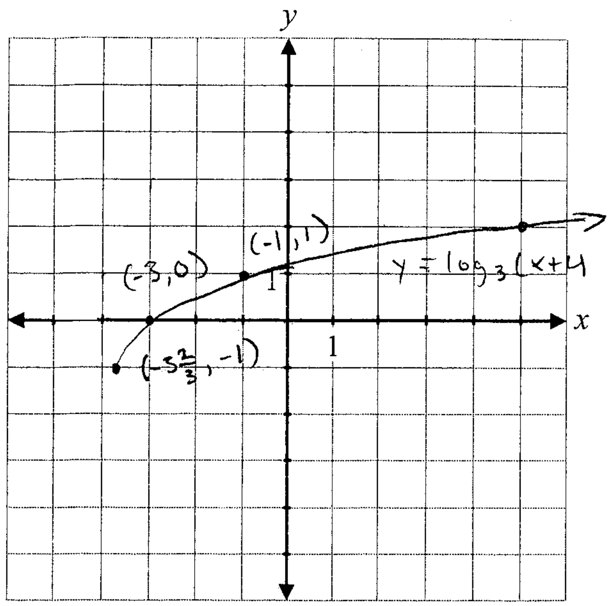
Solution



1 mark for increasing logarithmic function
1 mark for asymptotic behaviour approaching $x = -4$

2 marks

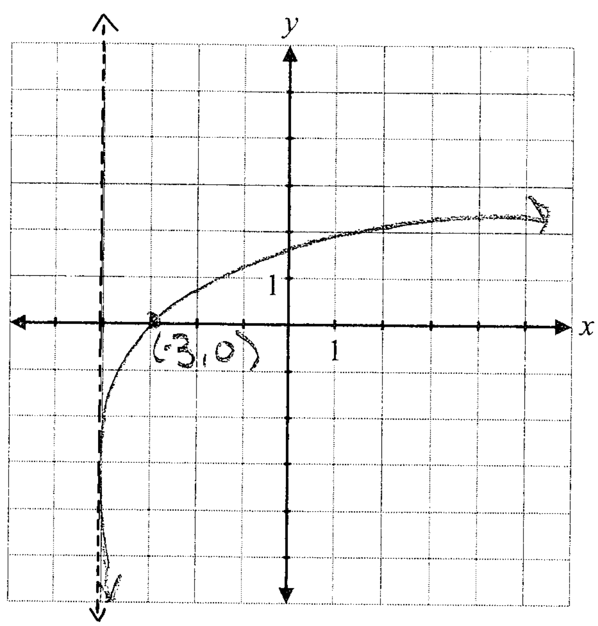
Exemplar 1



1 out of 2

+ 1 mark for increasing logarithmic function
E9 (endpoints or arrowheads omitted or incorrect)

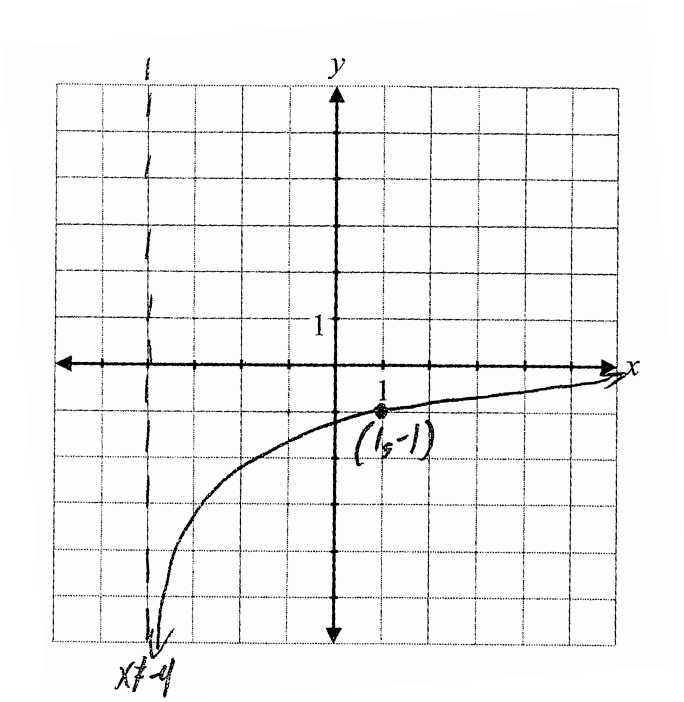
Exemplar 2



1½ out of 2

award full marks
– ½ mark for procedural error (not showing second point)
E10 (graph crosses asymptote)

Exemplar 3



1 out of 2

+ 1 mark for asymptotic behaviour approaching $x = -4$

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Question 47 R10

Solve, algebraically.

$$\log x + \log 4 - \log(x-2) = \log 5$$

Solution

Method 1

$$\log\left(\frac{4x}{x-2}\right) = \log 5$$

1 mark for product law
1 mark for quotient law

$$\frac{4x}{x-2} = 5$$

½ mark for equating arguments

$$4x = 5(x-2)$$

$$4x = 5x - 10$$

$$10 = x$$

½ mark for solving for x

3 marks

Method 2

$$\log x + \log 4 - \log(x-2) - \log 5 = 0$$

$$\log\left(\frac{4x}{5(x-2)}\right) = 0$$

1 mark for product law
1 mark for quotient law

$$10^0 = \frac{4x}{5(x-2)}$$

½ mark for converting to exponential form

$$5(x-2) = 4x$$

$$5x - 10 = 4x$$

$$x = 10$$

½ mark for solving for x

3 marks

Exemplar 1

$$\log\left(\frac{x+4}{x-2}\right) = \log 5$$

$$\frac{x+4}{x-2} = 5$$

$$x+4 = 5(x-2)$$

$$x+4 = 5x-10$$

$$\frac{-4x}{-4} = \frac{-14}{-4}$$

$$\boxed{x = 7/2}$$

2 out of 3

+ 1 mark for quotient law
+ ½ mark for equating arguments
+ ½ mark for solving for *x*

Exemplar 2

$$\frac{\log(4x)}{\log(x-2)} = \log 5$$

1 out of 3

+ 1 mark for product law
+ 1 mark for quotient law
– 1 mark for concept error (not written as a single logarithm)

Exemplar 3

$$\cancel{\log}\frac{4x}{x-2} = \cancel{\log}5$$

$$\frac{4x}{x-2} = 5$$

$$4x = 5x - 10$$

$$\frac{9x}{9} = \frac{-10}{9}$$

$$x = -10/9$$

2 out of 3

award full marks
– ½ mark for procedural error in line 1
– ½ mark for arithmetic error in line 4
E1 (impossible solution not rejected in the final answer)

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Question 48 T5

Given $\sin\theta = \frac{1}{2}$, determine all possible values of θ over the interval $[-2\pi, 2\pi]$.

Solution

$\theta = -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

1 mark for positive values of θ (½ mark for each)
1 mark for negative values of θ (½ mark for each)

2 marks

Exemplar 1

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, -\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

1 out of 2

+ 1 mark for consistent negative values of θ

Exemplar 2

$\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{6}\quad -\frac{5\pi}{6}$

1 out of 2

+ 1 mark for positive values of θ

Exemplar 3

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

2 out of 2

award full marks
E8 (answer outside the given domain)

Exemplar 4

$\theta = \frac{\pi}{6} + 2\pi k,\ k \in \mathbb{Z}$

$\theta = \frac{5\pi}{6} + 2\pi k,\ k \in \mathbb{Z}$

2 out of 2

award full marks
E8 (answer outside the given domain)

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Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allowed for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

Question(s) affected: ______________________________

Action taken or rationale for assigning marks: ______________________________

Follow-up: ______________________________

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Decision: ______________________________

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Marker's Signature: ______________________________

Principal's Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
7	R1	1
10	R3, R5	3
11	R4	3
25	R2	1
28	R1	2
36a)	R1	1
40	R6	2
45	R1	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
8	T2	1
20	T3	1
22	T4	1
34	T1	1
35	T4	4
42a)	T3	2
42b)	T3	1
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
5	P3	2
6	P4	3
9	P4	1
15	P1	1
17	P2	1
19	P2	1
24	P4	1
30	P3	2
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
13	R11	2
23	R12	1
26	R11	1
29	R11	1
32a)	R12	2
32b)	R12	1
41	R12	1

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
3	T5	4
14	T6	3
31	T5	2
38	T6	3
48	T5	2
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
2	R10	2
4	R10	3
16	R8	2
21	R7	1
33	R9	1
44	R10	1
46	R9	2
47	R10	3
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
12	R14	1
18	R13	1
27	R14	1
36b)	R14	1
37	R13	2
39	R13	4
43	R14	2