Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2019

Manitoba

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Booklet 1. June 2019

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Manitoba Education and Training
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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time Required to Complete the Test: 3 hours

Additional Time Allowed: 30 minutes

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	—	17	34
Booklet 2	9	23	56
Total	9	40	**90**

* The first 4 questions with the symbol  in *Booklet 1* require a scientific calculator. You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.

No marks will be awarded for work done on this page.

Formula Sheet

$s = \theta r$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$

$\log_a(M^n) = n\log_a M$

$P(n,r)$ or ${}_nP_r = \dfrac{n!}{(n-r)!}$

$C(n,r)$ or ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

$t_{k+1} = {}_nC_k a^{n-k}b^k$

For $ax^2 + bx + c = 0$,

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Describe: Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

State: Give an answer without an explanation or justification.

Verify: Establish the truth of a statement by substitution or comparison.

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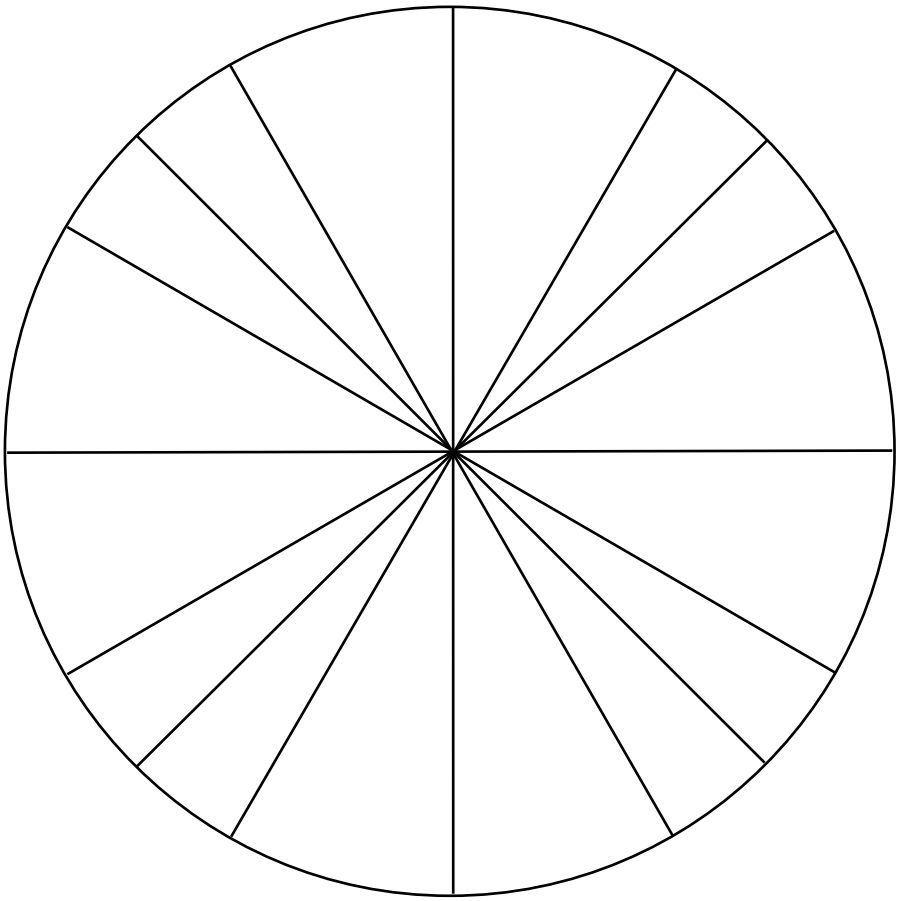

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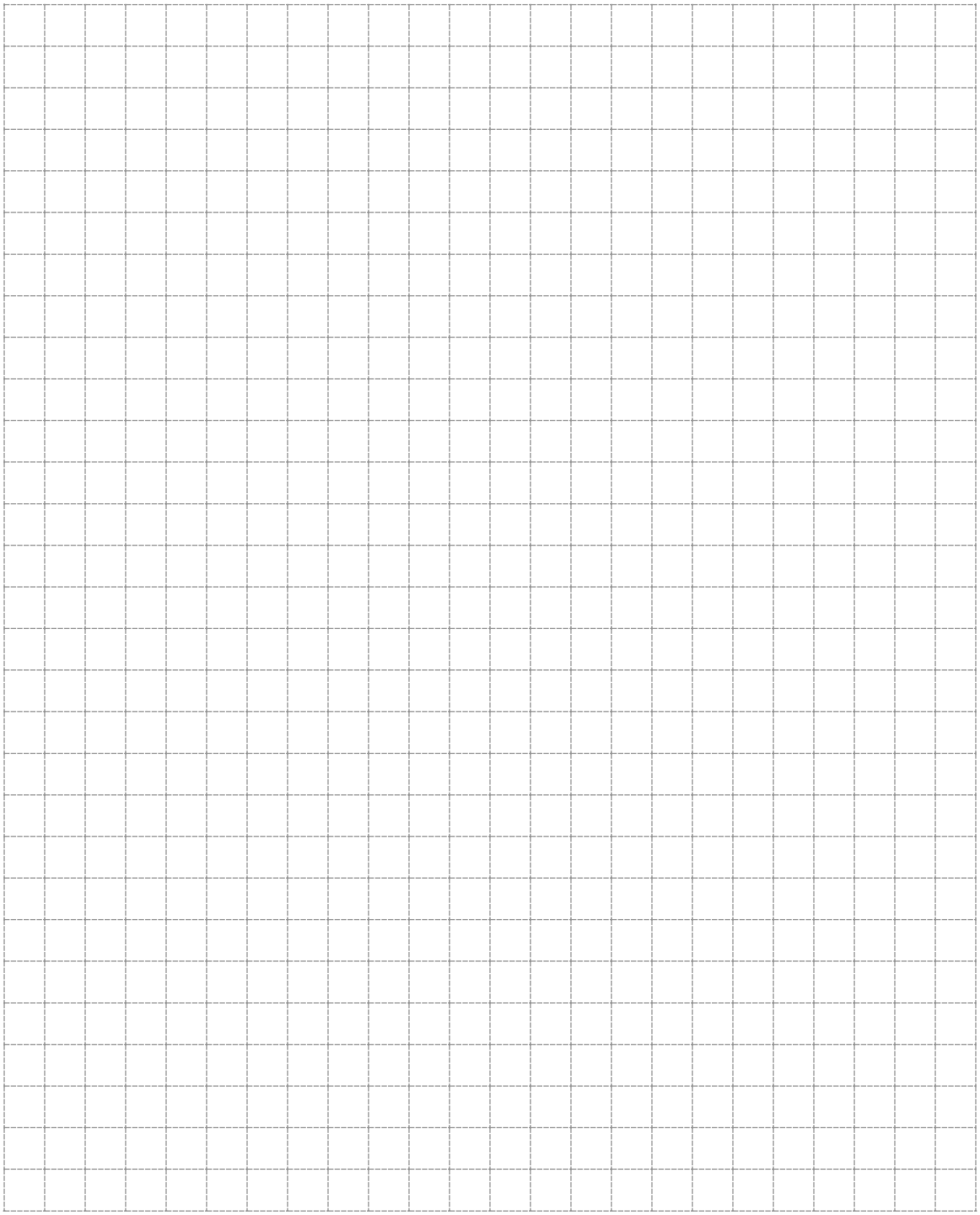


Question 1  2 marks 101

Avery has 4 adventure books, 5 mystery books, and 1 comic book.

Determine the number of ways he can arrange all of the books on his shelf if each type of book must be grouped together.

Question 2  4 marks 102

Solve the following equation, algebraically, over the interval $[0, 2\pi]$.

$$5\cos^2\theta - \cos\theta - \sin^2\theta = 0$$

Question 3  3 marks 103

Given that $-6048x^2y^5$ is the sixth term in the expansion of $(3x-m)^7$, determine m.

Question 4  a) 2 marks b) 1 mark

104
105

A series of blood tests measures the concentration of a prescribed drug. This concentration decreases according to the formula $A = Pe^{rt}$ where:

- A is the concentration at time t
- P is the initial concentration
- r is the rate of change
- t is the time, in hours, after the first blood test

The initial concentration is 3.8900 units/mL. Three hours later, the concentration is 1.7505 units/mL.

a) Determine the rate of change, r, algebraically.

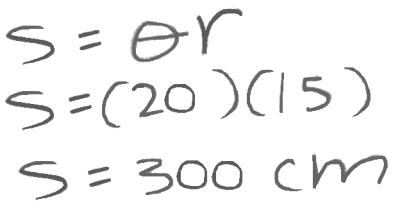
b) Determine the concentration of the prescribed drug four hours after the initial concentration of 3.8900 units/mL was measured. Express the answer correct to 4 decimal places.

Note: A calculator is not required for the remaining test questions.

Question 5 — 1 mark — 106

Ariane uses the formula $s = \theta r$ to determine the arc length of a circle that has a central angle of $20°$ and a radius of 15 cm.

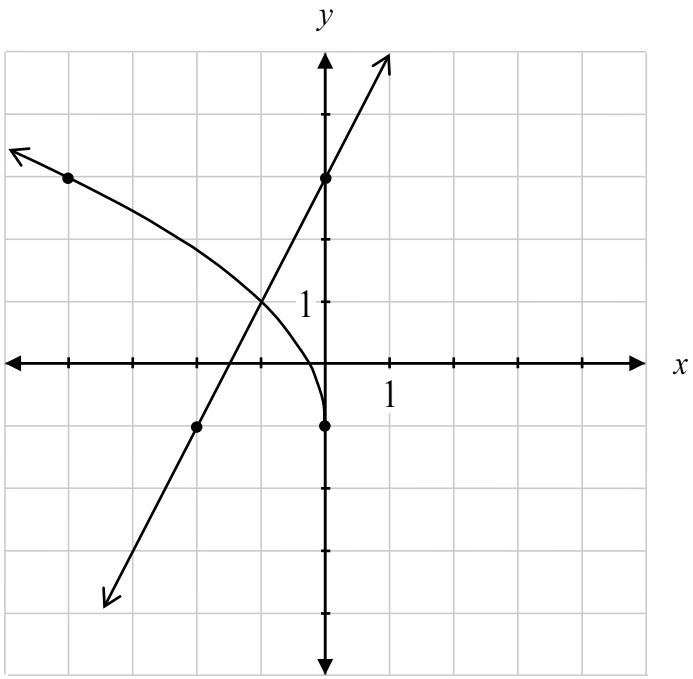
Below is Ariane's work:



Describe her error.

Question 6 — 1 mark — 107

Using the graphs below, state the solution of the equation $2x + 3 = 2\sqrt{-x} - 1$.



Question 7 — 3 marks — 108

Solve, algebraically.

$$\log_2(x+3) = 5 - \log_2(x-1)$$

Question 8 — 1 mark — 109

Explain why the value of n must be greater than or equal to the value of r, when using ${}_nC_r$.

Question 9 — 3 marks — 110

Given that $y = |x|$, determine the equation of the resulting function, $g(x)$, after the following transformations:

- reflection in the x-axis
- vertical translation 5 units down
- horizontal stretch by a factor of 3

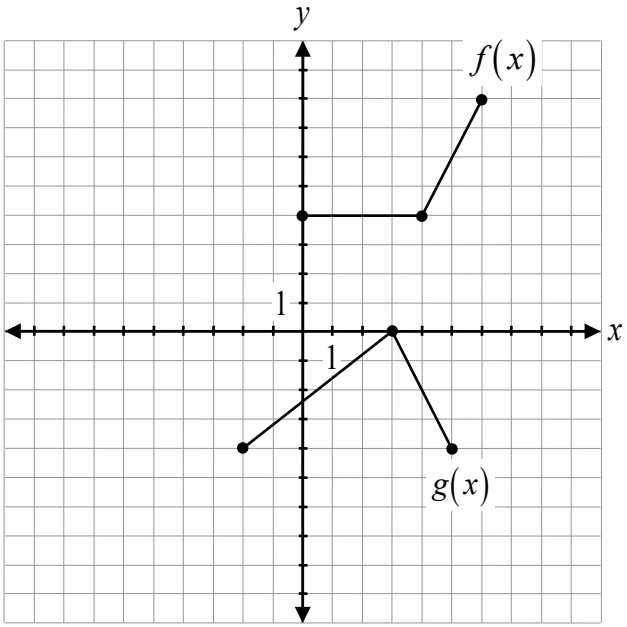
$g(x) =$ _____

Question 10 — 1 mark 111

Explain why the graph of $y = \dfrac{x-1}{x^2+x-6}$ has a horizontal asymptote at $y = 0$.

Question 11 — 1 mark 112

Given the graphs of $f(x)$ and $g(x)$, evaluate $g\big(f(2)\big)$.



Question 12 — 1 mark

Kennedy was asked to solve the equation $\tan\theta = 1$ over all real numbers.

Below is Kennedy's solution:

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

Describe her error.

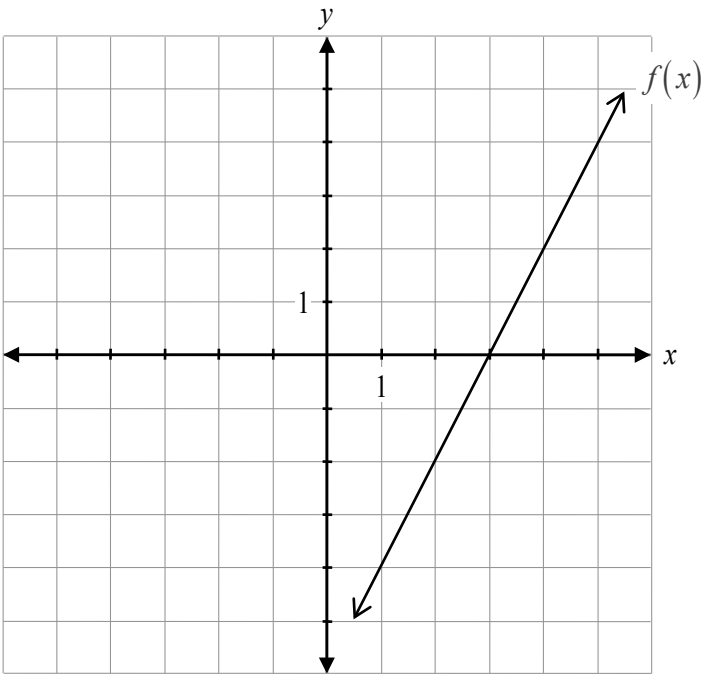
Question 13 — 3 marks 114

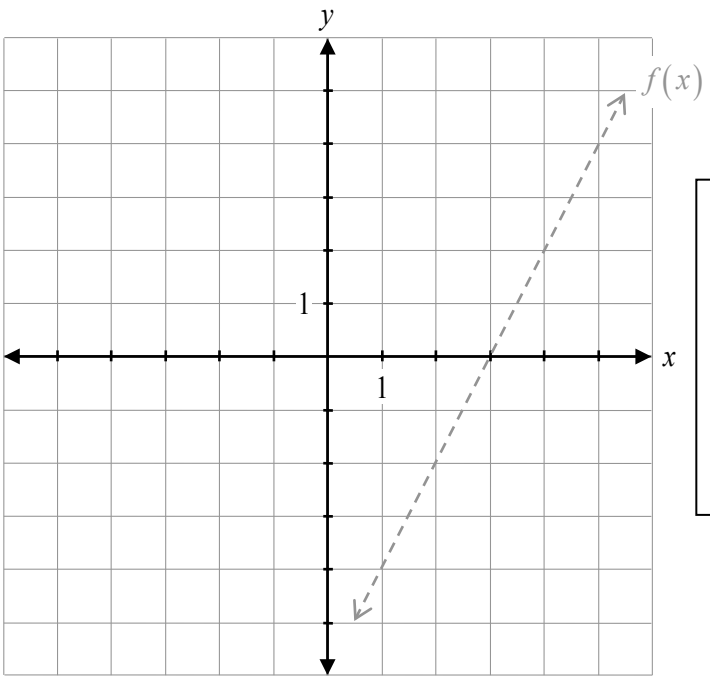
Solve, algebraically.

$${}_nC_3 = 3\left({}_nP_2\right)$$

Question 14 — 2 marks 115

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.





The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 15

3 marks 116

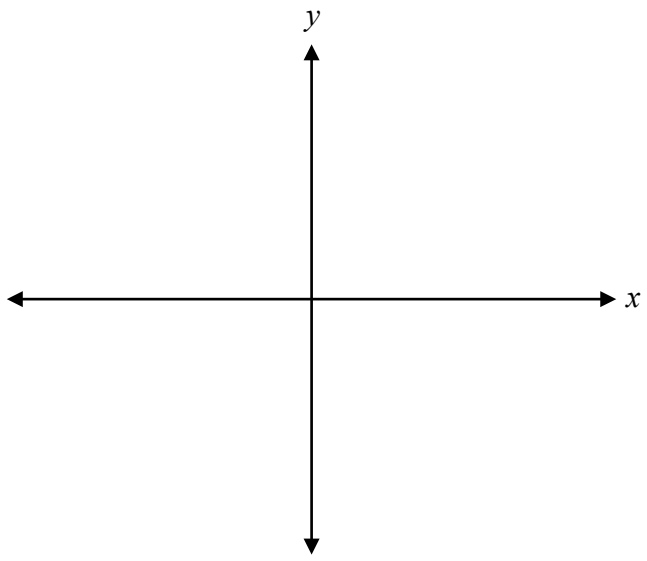
Prove the identity for all permissible values of θ.

$$\frac{\sec\theta - \tan\theta\sin\theta}{\tan\theta\sin\theta} = \csc^2\theta - 1$$

Left-Hand Side	Right-Hand Side

Question 16 — 1 mark 117

Sketch the angle of $-\frac{\pi}{12}$ radians in standard position.



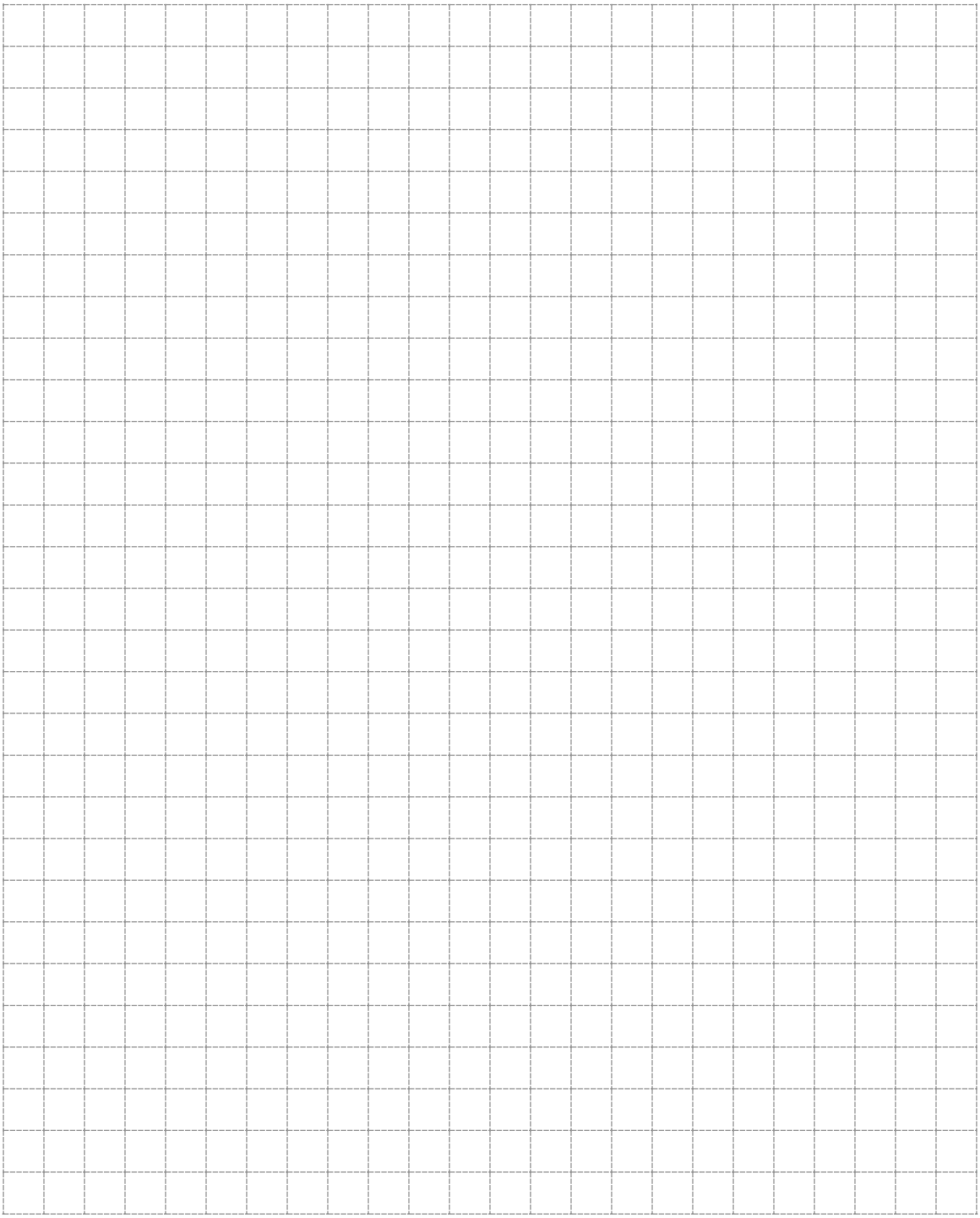
Question 17 — 1 mark 118

Given that $h(x) = 2x^2 - 7x - 15,$ determine possible equations of the functions $f(x)$ and $g(x)$ if $h(x) = f(x) \bullet g(x)$.

$f(x) =$ _____

$g(x) =$ _____

No marks will be awarded for work done on this page.



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