Name _____

Pre-Calculus Second Semester Exam Review

Chapter 8 and 7.5

1) Determine $2\mathbf{u} + \mathbf{v}$ if $\mathbf{u} = \langle -4, 3 \rangle$ and $\mathbf{v} = \langle -5, 12 \rangle$.

2) Determine the magnitude of vector $\langle -5, 12 \rangle$.

3) Evaluate $|\boldsymbol{a} + \boldsymbol{b}|$ if $\boldsymbol{a} = \langle 2, -1 \rangle$ and $\boldsymbol{b} = \langle 4, 0 \rangle$.

4) Determine $\boldsymbol{u} \cdot \boldsymbol{v}$ if $\boldsymbol{u} = \langle -2, 8 \rangle$ and $\boldsymbol{v} = \langle 3, 1 \rangle$.

5) Determine if vectors $\langle 3, 6 \rangle$ and $\langle 4, -2 \rangle$ are perpendicular.

6) Determine if vectors $\langle 1, -5 \rangle$ and $\langle 5, -1 \rangle$ are perpendicular.

7) Determine the angle between vectors $\langle -4, 3 \rangle$ and $\langle -5, 12 \rangle$.

8) Find the component form of $\overrightarrow{CD}$ with initial point $C\langle 10, -1 \rangle$ and terminal point $D\langle 12, -5 \rangle$.

9) Find the vertical component of the vector $\boldsymbol{v}$ with a magnitude of 120 feet and a direction angle of $30°$ from the horizontal.

10) Find the component form of the vector $\boldsymbol{v}$ with a magnitude of 500 feet and a direction angle of $120°$ from the horizontal.

11) F_1 is a force of 3N pulling an object due north and F_2 is a force of 5N pulling an object 60° east of north. Determine the direction (measured clockwise from north) and magnitude of the resultant force.

12) A plane is on a course of S80°W at a speed of 320 mph. What are the north-south and east-west components of the plane's velocity vector?

13) Write the pair of parametric equations $x = 3\sin\theta$ and $y = 3\cos\theta$ in rectangular form.

14) Write the pair of parametric equations $x = \cos\theta$ and $y = 4\sin\theta$ in rectangular form.

Chapter 10

For each of the following sequences, determine the value of a_5 and a_6 and write a rule for a_n. State whether the sequence is arithmetic, geometric or neither.

15) 23, -23, 23, -23, …

16) 101, 91, 81, 71, …

17) 4, -16, 64, -256, …

18) 120, 60, 30, 15, …

19) 3, 6, 11, 18, …

20) 3, 7, 11, 15, …

Determine the first four terms of the following sequences. State whether the sequence is arithmetic, geometric or neither.

21) $a_1 = 12 \, and \, a_n = 3a_{n-1} - 4$

22) $a_n = n + \frac{1}{n}$

23) $a_n = 3n + 2$

24) $a_n = 3^n$

25) $a_1 = 5, a_2 = 8, a_n = a_{n-1} + a_{n-2}$

Evaluate the following sums.

26) $\sum_{k=1}^{5} 3k-2$

27) $\sum_{k=1}^{4} 2\cdot 3^{k-1}$

28) $\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1}$

Write the following using sigma notation.

29) 1 + 8 + 27 + 64 + 125

30) 5 + 10 + 15 + 20

31) $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$

Determine the specified sum/term for each of the following series.

32) Determine the 10^{th} partial sum for the arithmetic series with $a_1 = 5 \text{and} a_{10} = 41$.

33) Determine the 4^{th} partial sum for the geometric series with $a_1 = 1 \text{and} a_2 = 4$.

34) Determine the 9^{th} partial sum for the geometric series with $a_1 = 6$ and $r = -2$.

35) S_{50} for the series with $a_1 = 4$ and $a_n = a_{n-1} + 2$

36) Determine a_{60} of an arithmetic sequence with $a_4 = 7$ and $a_7 = 13$

37) Determine a_6 of a geometric sequence with $a_1 = 16$ and $a_4 = 54$

38) An arithmetic series has first term 78 and common difference 3.2. Determine the thirtieth term and thirtieth partial sum.

39) An arithmetic series has $a_1 = 56$ and $a_3 = 66$. Determine a_2 and S_{100}.

40) A geometric series has $a_1 = 100$ and $a_2 = 90$. Determine the value of $a_3, a_4,$ and a_5. Evaluate S_{20}. Determine the limit to which the partial sums converge.

41) An arithmetic series has first term 13 and tenth term 31. What is the twentieth term?

42) What are the next two terms of the sequence $\frac{1}{3},\frac{2}{9},\frac{3}{27},\frac{4}{81},\ldots$? Is this sequence arithmetic or geometric or neither?

43) Write using sigma notation and determine the sum: $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\frac{1}{625}+\ldots$

44) Evaluate S_{20} for the geometric series 100 + 88 + 77.44 +… Also, determine the sum, S, of this series.

45) Explain why the geometric series 10 + 20 + 40 + 80 + … does not converge.

46) What is the fifth term in the expansion $(2x-3y)^4$?

47) The expression $x^5+10x^4y+40x^3y^2+80x^2y^3+80xy^4+32y^5$ is the expansion a binomial $(a+b)^n$. Determine a, b, and n.

Chapter 6

Evaluate.

48) $\begin{vmatrix} 5 & -2 \\ 4 & 6 \end{vmatrix}$

Perform the following operations, if possible. Use the matrices given below.

$A = \begin{bmatrix} -1 & a \\ 0 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ -1 \\ b \\ 2 \end{bmatrix}$ $C = \begin{bmatrix} 7 & -2 \\ 0 & c \\ -3 & d \end{bmatrix}$ $D = \begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix}$

49) A + B

53) CD

50) 2A – D

54) AD

51) D – C

55) BC

52) A^{-1}

Solve for the variables.

56) $3\begin{bmatrix} x+4 \\ y-2 \end{bmatrix}+\begin{bmatrix} 5 \\ -4 \end{bmatrix}=\begin{bmatrix} 8 \\ 2 \end{bmatrix}$

57) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & -1 & 4 \\ 3 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ x \end{bmatrix}=\begin{bmatrix} 5 \\ y \\ -1 \end{bmatrix}$

58) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\mathbf{X}=\begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}$

59) $\begin{bmatrix} 9 & 7 \\ 5 & 4 \end{bmatrix}\mathbf{X}=\begin{bmatrix} 1 & 0 & -2 \\ 4 & 1 & 3 \end{bmatrix}$

60) $\begin{bmatrix} 6 & 2 \\ 4 & -2 \end{bmatrix}\mathbf{X}-\begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}=\begin{bmatrix} 6 & 4 \\ 8 & -2 \end{bmatrix}$

61) What is the augmented matrix for the system $\begin{matrix} 2x - y = 24 \\ x + 6y = 52 \end{matrix}$?

Solve each system using matrices.

62) $\begin{matrix} 2x - 5y = 4 \\ x - 3y = 1 \end{matrix}$

63) $\begin{matrix} 3x - 7y = -16 \\ -2x + 3y = 9 \end{matrix}$

64) $\begin{matrix} x + 3y - 2z = -11 \\ -3x + 10y - 7z = -13 \\ -2x - 6y + 5z = 27 \end{matrix}$

65) $\begin{matrix} x - 3y + 2z = -6 \\ -3x - 10z = 6 \\ 5x + 6y - 2z = 14 \end{matrix}$

Determine the inverse of the following matrices, if it exists.

66) $\begin{bmatrix} 10 & -2 \\ -6 & 1 \end{bmatrix}$

67) $\begin{bmatrix} 4 & -9 \\ 0 & 7 \end{bmatrix}$

68) $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$

69) $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

Chapter 12 & Section 2-5

Evaluate the limit or state that it does not exist.

70) $\lim_{x\to-\infty}\dfrac{3x^3-8x^2+6}{9-15x^5}$

71) $\lim_{x\to6}\dfrac{x^2-36}{x^2+4x-60}$

72) $\lim_{x\to3}\dfrac{x-3}{x^2-9}$

73) $\lim_{x\to2^+}\dfrac{3x+1}{2-x}$

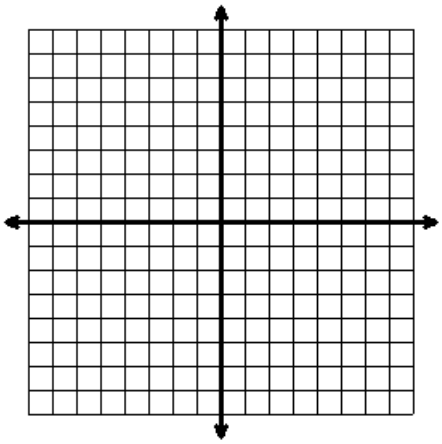
74) $\lim_{x\to-5}\dfrac{1}{x+5}$

75) $\lim_{n\to\infty}\dfrac{3n^2-n-2}{2n^2+5n}$

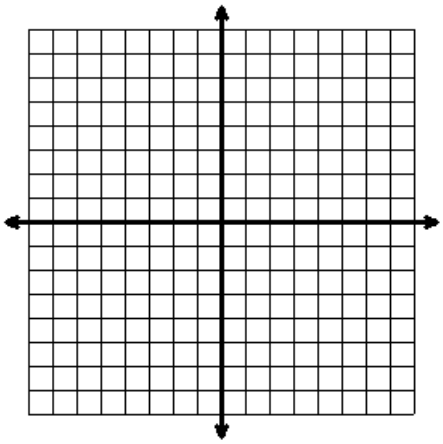
76) $\lim_{x\to4}\dfrac{\sqrt{x}-2}{x-4}$

Sketch the graph. Name the intercepts, asymptotes and list any discontinuities.

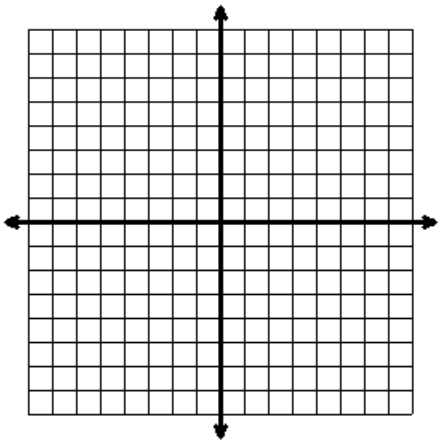
77) $y=\dfrac{x^2+x}{3x-6}$



78) $y=\dfrac{8}{x^2-9}$



79) $y=\dfrac{x^2+x-6}{x^2-4}$



Chapter 9

80) Determine two other polar representations of the following points.

a) $\left(-4,100^\circ\right)$ b) $\left(2,\pi\right)$ c) $\left(-3,225^\circ\right)$

81) Change to polar coordinates.

a) (0,-3) b) (-4,-4)

82) Change to rectangular coordinates.

a) $\left(4,240^\circ\right)$ b) $\left(-3,\frac{\pi}{2}\right)$ c) $\left(4,-\frac{\pi}{4}\right)$

83) Convert the polar equation to rectangular form.

a) $r=10\sec\theta$ b) $r=3\cos\theta$

84) Convert the rectangular equation to polar form.

a) x = 10 b) 3x + 2y = 7 c) $x^2+y^2=16$

85) Express the complex number in polar form.

a) $-2-2i$ b) $3i$ c) $1-i\sqrt{3}$

86) Express the complex number in rectangular form.

a) $4\left(\cos 240^\circ + i\sin 240^\circ\right)$ b) $6\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

87) Evaluate the product in polar form and convert your answer to rectangular form:

$\left(4\left(\cos 60^\circ + i\sin 60^\circ\right)\right)\left(3\left(\cos 30^\circ + i\sin 30^\circ\right)\right)$

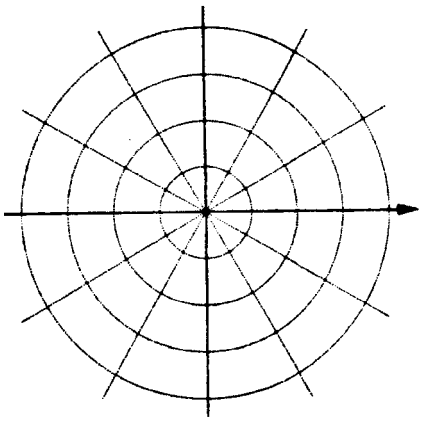
88) Plot the points on a polar graph.

a) $A\left(2, 60^\circ\right)$

b) $B\left(-4, 300^\circ\right)$

c) $C\left(4, -\pi\right)$

d) $D\left(-1, \frac{5\pi}{3}\right)$



89) Expand and simplify: $(3+3i)^4$

90) Given $z_1 = 4\left(\cos 30^\circ + i \sin 30^\circ\right)^\circ$ and $z_2 = \frac{1}{8}\left(\cos 120^\circ + i \sin 120^\circ\right)$. Determine each of the following in polar form.

a) $z_1 z_2$ b) $\left(z_1\right)^7$ c) $\left(z_2\right)^2$

91) Determine all solutions to the equation $x^4 + 81 = 0$. Leave answers in polar form.

92) Determine the fifth roots of $-i$. Leave answers in polar form.

93) Determine the cube roots of $4\left(\cos 60^\circ + i \sin 60^\circ\right)$ in polar form.