Pre-Calculus/Trigonometry

Standard 1

Relations and Functions

CORE STANDARD

Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

CORE STANDARD

Logarithmic and Exponential Functions

Define and find inverse functions. Verify whether two given functions are inverses of each other. Solve problems involving logarithmic and exponential functions by using the laws of logarithms and understand why those properties are true.

[Standard Indicators: PC.1.7, PC.3.2]

PC.1.1 Use paper and pencil methods and technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions. Use these graphs to solve problems, and translate among verbal, tabular, graphical and symbolic representations of functions by using technology as appropriate.

Example: Draw the graphs of the functions $y = x^5 - 2x^3 - 5x^2$, $y = \frac{2x-1}{3x+2}$, $y = \sqrt{(x+2)(x-5)}$ and $f(x) = \sin^{-1}x$.

PC.1.2 Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions represented symbolically or graphically, using technology as appropriate.

Example: Let $R(x) = \frac{1}{\sqrt{x-2}}$. Find the domain of $R(x)$ (i.e., the values of x for which $R(x)$ is defined). Also, find the range and asymptotes of $R(x)$.

PC.1.3 Solve word problems that can be modeled using functions and equations.

Example: You are on the committee for planning the prom and need to decide what to charge for tickets. Last year you charged $5.00 and 400 people bought tickets. Earlier experiences suggest that for every 20-cent decrease in price you will sell 20 extra tickets. Use a spreadsheet and write a function to show how the amount of money in ticket sales depends on the number of 20-cent decreases in price. Construct a graph that shows the price and gross receipts. What ticket price maximizes revenue?

PC.1.4 Recognize and describe continuity, end behavior, asymptotes, symmetry and limits and connect these concepts to graphs of functions.

Example: Determine the numbers a and b so that the following function is continuous:

x^2 if $x \leq 1$

$f(x) = ax + b$ if $1 < x < 2$

$5 - x$ if $x \geq 2$

PC.1.5 Find, interpret and graph the sum, difference, product and quotient (when it exists) of two functions and indicate the relevant domain and range of the resulting function.

Example: Find $(f + g)(x)$ if $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x}{x-1}$. State the domain of $(f + g)(x)$.

PC.1.6 Find the composition of two functions and determine the domain and the range of the composite function. Conversely, when given a function, find two other functions for which the composition is the given one.

Example: If $h(x) = (2x+3)^4$, find functions f and g so that $f \circ g = h$.

PC.1.7 Define and find inverse functions, their domains and their ranges. Verify symbolically and graphically whether two given functions are inverses of each other.

Example: Find the inverse function of $h(x) = (x - 2)^3$.

PC.1.8 Apply transformations to functions and interpret the results of these transformations verbally, graphically and numerically.

Example: Explain how you can obtain the graph of $g(x) = -|2(x + 3)^2 - 2|$ from the graph of $f(x) = x^2$.

Standard 2

Conics

CORE STANDARD

Conic Sections

Derive equations for conic sections. Graph conic sections by hand by completing the square and find foci, centers, asymptotes, eccentricity, axes and vertices as appropriate.

[Standard Indicators: PC.2.1, PC.2.2]

PC.2.1 Derive equations for conic sections and use the equations that have been found.

Example: Derive an equation for the ellipse with foci at (-1, 0) and (1, 0) that contains the point (0, 2).

PC.2.2 Graph conic sections with axes of symmetry parallel to the coordinate axes by hand, by completing the square, and find the foci, center, asymptotes, eccentricity, axes and vertices (as appropriate).

Example: Graph $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{9} = 1$. Find its foci, centers, asymptotes, eccentricity, axes and vertices (as appropriate).

Standard 3

Logarithmic and Exponential Functions

CORE STANDARD

Graphing Functions

Use paper and pencil methods and graphing technology to

graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

CORE STANDARD

Logarithmic and Exponential Functions

Define and find inverse functions. Verify whether two given functions are inverses of each other. Solve problems involving logarithmic and exponential functions by using the laws of logarithms and understand why those properties are true.

[Standard Indicators: PC.1.7, PC.3.2]

PC.3.1 Compare and contrast symbolically and graphically $y = e^x$ with other exponential functions.

Example: Graph $y = e^x$, $y = 3^x$ and $y = 2^{-x}$. Show how to rewrite 3^x and 2^{-x} as e^{kx} for certain values of k.

PC.3.2 Define the logarithmic function g(x) = $log_a\ x$ as the inverse of the exponential function $f(x) = a^x$. Apply the inverse relationship between exponential and logarithmic functions and apply the laws of logarithms to solve problems.

Example: Simplify the expression $e^{\ln 8}$.

PC.3.3 Analyze, describe and sketch graphs of logarithmic and exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: For the function $l(x) = \log_{10}(x - 4)$, find its domain, range, x-intercept and asymptote, and sketch the graph.

PC.3.4 Solve problems that can be modeled using logarithmic and exponential functions. Interpret the solutions and determine whether the solutions are reasonable.

Example: The amount A of a radioactive element (in gm) after t years is given by the formula: $A(t)$ =

$100e^{-0.02t}$. Find t when the amount is 50 gm, 25 gm and 12.5 gm. What do you notice about these time periods?

Standard 4
Trigonometry

CORE STANDARD

Unit Circle

Define sine and cosine using the unit circle. Convert between degree and radian measures. Use the values of the sine, cosine and tangent functions at 0, $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$ radians and their multiples.

[Standard Indicators: PC.4.4, PC.4.5, PC.4.6]

CORE STANDARD

Trigonometric Functions

Define and analyze trigonometric functions, including inverse functions. Solve problems involving trigonometric functions and prove trigonometric identities.

[Standard Indicators: PC.4.8, PC.4.9, PC.4.10, PC.4.11]

CORE STANDARD

Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

PC.4.1 Define and use the trigonometric ratios cotangent, secant and cosecant in terms of angles of right triangles.

Example: Use the relationships among the lengths of the sides of a 30°- 60° right triangle to find the exact value of the secant of 30°.

PC.4.2 Model and solve problems involving triangles using trigonometric ratios.

Example: Find the area of ΔABC if a, which is the side opposite angle A, measures 5 units; b, which is the side opposite angle B, measures 8 units; and angle C , which is the angle opposite c, measures 30°.

PC.4.3 Develop and use the laws of sines and cosines to solve problems.

Example: You want to determine the location of a water tower by taking measurements from two positions three miles apart. From the first position, the angle between the water tower and the second position is 78º. From the second position, the angle between the water tower and the first position is 53º. How far is the water tower from each position?

PC.4.4 Define sine and cosine using the unit circle.

Example: Find the acute angle A for which $\sin 150^\circ = \sin A$.

PC.4.5 Develop and use radian measures of angles, measure angles in degrees and radians, and convert between degree and radian measures.

Examples:

- Convert 90º, 45º and 30º to radians.
- Find the length of an arc subtended by an angle of $\frac{5\pi}{6}$ radians on a circle of radius 5 cm.

PC.4.6 Deduce geometrically and use the value of the sine, cosine and tangent functions at $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ radians and their multiples.

Example: Find the values of $\cos \pi/2$, $\tan 3\pi/4$, $\csc 2\pi/3$, $\sin^{-1} -\sqrt{3}/2$ and $\sin 3\pi$.

PC.4.7 Make connections among right triangle ratios, trigonometric functions and the coordinate function on the unit circle.

Example: Angle A is a 60º angle of a right triangle with a hypotenuse of length 14 and a shortest side of length 7. Find the exact sine, cosine, and tangent of

angle *A*. Find the real numbers x, $0 < x < 2\pi$, with exactly the same sine, cosine and tangent values.

PC.4.8 Analyze and graph trigonometric functions, including the translation of these trigonometric functions. Describe their characteristics (i.e., spread, amplitude, zeros, symmetry, phase, shift, vertical shift, frequency).

Example: Draw the graph of $y = 5 + \sin(x - \pi/3)$.

PC.4.9 Define, analyze and graph inverse trigonometric functions and find the values of inverse trigonometric functions.

Example: Graph $f(x) = \sin^{-1}x$.

PC.4.10 Solve problems that can be modeled using trigonometric functions, interpret the solutions and determine whether the solutions are reasonable.

Example: In Indiana, the length of a day in hours varies through the year, usually with the longest day of about 14 hours on June 21 and the shortest day of about 10 hours on December 21. Model this situation with a sine function, by giving both the graph of this function and its formula. Find another day that is as long as July 4 by using your model.

PC.4.11 Derive the fundamental Pythagorean trigonometric identities; sum and difference identities; half-angle and double-angle identities; and the secant, cosecant and cotangent functions. Use these identities to verify other identities and simplify trigonometric expressions.

Example: Find the acute angle between the lines given by $y = 2x$ and $y = 3x$.

PC.4.12 Solve trigonometric equations and interpret solutions graphically.

Example: Solve $3 \sin 2x = 1$ for x between 0 and 2π.

Standard 5
Polar Coordinates and Complex Numbers

CORE STANDARD

Polar Coordinates and Complex Numbers

Define and use polar coordinates and complex numbers. Graph equations in the polar coordinate plane. Use their relation to trigonometric functions to solve problems.

[Standard Indicators: PC.5.1, PC.5.2, PC.5.3, PC.5.4]

PC.5.1 Define and use polar coordinates and relate polar coordinates to Cartesian coordinates.

Example: Convert the polar coordinate $(2, \pi/3)$ to Cartesian coordinates.

PC.5.2 Represent equations given in Cartesian coordinates in terms of polar coordinates.

Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates.

PC.5.3 Graph equations in the polar coordinate plane.

Example: Graph $y = 1 - \cos \theta$.

PC.5.4 Define complex numbers, convert complex numbers to polar form and multiply complex numbers in polar form.

Example: Write $3 + 3i$ and $2 - 4i$ in trigonometric form and then multiply the results.

PC.5.5 Prove and use De Moivre's Theorem.

Example: Simplify $(1 - i)^{23}$.

Standard 6

Sequences and Series

CORE STANDARD

Sequences and Series

Define arithmetic and geometric sequences and series. Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series. Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge and recognize an infinite series as the limit of a sequence of partial sums. Use series to solve problems. Derive the binomial theorem by combinatorics.

[Standard Indicators: PC.6.1, PC.6.2, PC.6.3]

PC.6.1 Define arithmetic and geometric sequences and series.

Example: Write an example and explain the differences among each of the following sequences and series: arithmetic sequence, geometric sequence, arithmetic series and geometric series.

PC.6.2 Derive and use formulas for finding the general term for arithmetic and geometric sequences.

Example: Write the general term formula for the arithmetic sequence with initial term two and common difference three.

PC.6.3 Develop, prove and use sum formulas for arithmetic series and for finite and infinite geometric series.

Example: Find the sum of the infinite geometric series $8 + 4 + 2 + \ldots$.

PC.6.4 Generate a sequence by using recursion.

Example: Write the first five terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Observe a pattern of even and odd terms for this sequence. Prove that your observation is correct.

PC.6.5 Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge. Recognize an infinite series as the limit of a sequence of partial sums.

Example: Find the limit as $n \to \infty$ of the sequence $\frac{2n-1}{3n+2}$ and the limit as $x \to 5$ of the function $\frac{x^2-5^2}{x-5}$.

PC.6.6 Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable.

Example: You put one grain of rice on the square of a chess board the first day, two on a second square the next day and continue to double the number of grains of rice you place each day through the 64^{th} day. How many grains of rice need to be used on the 64^{th} square on the 64^{th} day? How many grains of rice were used all together?

PC.6.7 Derive the binomial theorem by combinatorics.

Example: Give an algebraic and combinatorial proof of the binomial theorem if a and b are real numbers and n is a positive integer.

Standard 7
Vectors and Parametric Equations

PC.7.1 Define vectors as objects having magnitude and direction. Represent vectors geometrically.

Example: Graph the vector that goes from point p = (3, 2) to the point q = (5, 6).

PC.7.2 Use parametric equations to represent situations involving motion in the plane.

Example: Car A is traveling east at 40 mph, and Car B is traveling north at 30 mph. Both are heading toward the same intersection. Car A is five miles from the intersection when car B is four miles from the intersection. Write the parametric equations that describe the position of each car as a function of time.

PC.7.3 Convert between a pair of parametric equations and an equation in x and y

Example: Given the parametric equations $x = 3t^2$ and $y = t + 1$, find an equation relating x and y.

PC.7.4 Analyze planar curves, including those given in parametric form.

Example: Describe the curve that is defined by the parametric equations $x = 5 \cos t$ and $y = 5 \sin t$ for $0 \le t \le \pi$

PC.7.5 Model and solve problems using parametric equations.

Example: For the problem in Standard 7.2, write a formula for the distance between the cars as a function of time and then find when the cars are the closest.

Standard 8
Data Analysis

PC.8.1 Find linear models by using median fit and least squares regression methods. Decide which among several linear models gives a better fit. Interpret the slope in terms of the original context.

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PC.8.2 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a "best-fit" line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods

of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics

curriculum.

- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
    - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
    - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or not: Creating a high school diploma that counts.* Washington, DC: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.