# PRE-CALCULUS TRIGONOMETRY UNIT <br> Basic Angles 

## Standard Position:

## Initial Side:

## Terminal Side:



## Positive vs. Negative Angles:

## Degree:

## Going Greek

When representing angles using variables, it is traditional to use Greek letters. Here is a list of commonly encountered Greek letters.

| $\theta$ | $\varphi$ or $\phi$ | $\alpha$ | $\beta$ | $\gamma$ |
| :--- | :--- | :--- | :--- | :--- |
| theta | phi | alpha | beta | gamma |

## Basic Angles:



Radian Measure: When it comes to doing problems involving calculations with angles, using degrees can be confusing. Using radian measure is the preferred method. radian measure $=\frac{s}{r}$


Converting from degrees to radian measure:

## Converting from radian measure to degrees:



## Coterminal Angles:

Positive:
Negative:


Arc length: $\quad$ Using the formula: $\quad \theta=-$

## Converting the above formula:

## Area of a Circular Sector:

$$
\text { Area of sector }=\frac{1}{2} \theta r^{2}
$$

An automatic lawn sprinkler sprays a distance of 20 feet while rotating 30 degrees. What is the area of the sector of grass the sprinkler waters?


## Reference Angles:



Convert each measure from degrees to radians or from radians to degrees.

1. $-\frac{3 \pi}{2}$
2. $450^{\circ}$
3. $\frac{5 \pi}{18}$
4. $-200^{\circ}$
5. $\frac{7 \pi}{4}$
6. $-\frac{11 \pi}{6}$
7. $350^{\circ}$
8. $\frac{7 \pi}{20}$
9. $12^{\circ}$
10. $\frac{13 \pi}{10}$
11. $222^{\circ}$
12. $-105^{\circ}$

Find the measure of the reference angle for each given angle.
14. $\theta=475^{\circ}$
15. $\theta=212^{\circ}$
18. $\theta=-96^{\circ}$
16. $\theta=-115^{\circ}$
17. $\theta=740^{\circ}$
19. $\theta=-401^{\circ}$
20. $\theta=320^{\circ}$
23. $\theta=-850^{\circ}$
21. $\theta=-722^{\circ}$
22. $\theta=292^{\circ}$
24. $\theta=1000^{\circ}$
25. $\theta=-1000^{\circ}$


On a circle of radius 7 miles, find the length of the arc that subtends a central angle of 5 radians.

On a circle of radius 12 cm , find the length of the arc that subtends a central angle of 120 degrees.

Find the distance along an arc on the surface of the Earth that subtends a central angle of 5 minutes ( 1 minute $=1 / 60$ degree ). The radius of the Earth is 3960 miles.

On a circle of radius 6 feet, what angle in degrees would subtend an arc of length 3 feet?

A sector of a circle has a central angle of $45^{\circ}$. Find the area of the sector if the radius of the circle is 6 cm .

A pendulum is 18 feet long. Its central angle is $44^{\circ}$. The pendulum makes one back and forth swing every 12 seconds. To the nearest foot, how far does the pendulum swing each minute?

# PRE-CALCULUS TRIGONOMETRY UNIT 

Finding Exact Values of Trig Functions

## 6 Basic Trig Functions:

$\sin \theta=$ $\cos \theta=$ $\sec \theta=$
$\csc \theta=$

$$
5
$$

$\tan \theta=$
$\operatorname{ctn} \theta=$

Find the Exact Values of the following:

$$
\cos \frac{2 \pi}{3}
$$


$\tan -\frac{5 \pi}{4}$

csc $\frac{7 \pi}{6}$


## Quadrantal Angles:

Graphs of Sine, Cosine and Tangent:

$$
y=\sin x
$$




Find the Exact Values of the following:

## Homework for Finding Exact Trig Values

For each of the angles in the unit circle below, state the angle in degrees, the radian measure, and the coordinate of the point. Start with the quadrantal, then progress to the first quadrant angles, then 2nd, etc.


Find the exact values of the following:
$\sec \left(\frac{-5 \pi}{6}\right)$
$\operatorname{ctn}\left(\frac{7 \pi}{4}\right)$

$\csc \left(\frac{-3 \pi}{2}\right)$

$\tan \left(\frac{11 \pi}{4}\right)$


Which quadrant does the following functions lie?
$\sin \alpha<0$ and $\cos \alpha<0$
$\tan \alpha<0$ and $\cos \alpha>0$

$$
\sin \alpha>0 \text { and } \cos \alpha<0
$$

$\boldsymbol{\operatorname { c s c }} \boldsymbol{\alpha}<0$ and $\operatorname{ctn} \alpha<0$
$\sin \alpha<0$ and $\cos \alpha>0$
$\sec \alpha>0$ and $\tan \alpha>0$

## PRE-CALCULUS TRIGONOMETRY UNIT

More on Finding Exact Values of Trig Functions
So far, we have been exploring trig values using a $\qquad$ circle. In other words, a circle whose radius is $\qquad$ . But what about circles with other radii?


The point $(-3,4)$ lies on a circle with a radius of 5 at some angle $\boldsymbol{\beta}$. Find the all of the trig functions that correspond to the angle.


The coordinates of a point on a Circle with a given radius $r$, at an angle $\boldsymbol{\theta}$ :

$$
\begin{aligned}
& x=r \cos \theta \\
& y=
\end{aligned}
$$

Recall the Pythagorean Theorem:
Using this theorem along with the new values for x and y above:

This is called the $\qquad$ Identity

Example: If $\sin \theta=\frac{3}{7}$ and $\theta$ is in the second quadrant, find $\cos (\theta)$.


Find the coordinates of the point on a circle of radius 6 at an angle of $\frac{\pi}{4}$. Using the coordinates formulas: $x=$ $\qquad$ and $\mathrm{y}=$ $\qquad$ .

Find the coordinates of the point on a circle of radius 5 at an angle of $\frac{5 \pi}{3}$.

A distress signal is sent from a sailboat during a storm, but the transmission is unclear and the rescue boat sitting at the marina cannot determine the sailboat's location. Using high powered radar, they determine the distress signal is coming from a distance of 20 miles at an angle of 225 degrees from the marina. How many miles east/west and north/south of the rescue boat is the stranded sailboat?


## Homework for More on Finding Exact Values of Trig Functions

The point $P$ is on the unit circle. If the $y$-coordinate of $P$ is $\frac{3}{5}$, and $P$ is in quadrant II, find the $x$ coordinate.

If $\cos (\theta)=\frac{1}{7}$ and $\theta$ is in the $4^{\text {th }}$ quadrant, find $\sin (\theta)$.



If $\sin (\theta)=\frac{3}{8}$ and $\theta$ is in the $2^{\text {nd }}$ quadrant, find $\cos (\theta)$.


For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute sine and cosine of the angle.
a. $\frac{5 \pi}{4}$
b. $\frac{7 \pi}{6}$
c. $\frac{5 \pi}{3}$
d. $\frac{3 \pi}{4}$





Give exact values for $\sin (\theta)$ and $\cos (\theta)$ for each of these angles.
a. $-\frac{3 \pi}{4}$
b. $\frac{23 \pi}{6}$
c. $-\frac{\pi}{2}$
d. $5 \pi$





Find the coordinates of the point on a circle with radius 15 corresponding to an angle of $220^{\circ}$.

# PRE-CALCULUS TRIGONOMETRY UNIT 

Right Triangle Trigonometry
Given a right triangle with an angle of $\theta$

$$
\begin{aligned}
& \sin (\theta)=\frac{\text { opposite }}{\text { hypotenuse }} \\
& \cos (\theta)=\frac{\text { adjacent }}{\text { hypotenuse }} \\
& \tan (\theta)=\frac{\text { opposite }}{\text { adjacent }}
\end{aligned}
$$



Given the right triangle, find the following:
$\sin \alpha=$
$\cos \alpha=$
$\tan \alpha=$
$\sin \beta=$
$\cos \beta=$
$\tan \beta=$


The cofunction identities for sine and cosine

$$
\cos (\theta)=\sin \left(\frac{\pi}{2}-\theta\right) \quad \sin (\theta)=\cos \left(\frac{\pi}{2}-\theta\right)
$$

A right triangle has an angle of $\frac{\pi}{3}$ and a hypotenuse is 20 cm . Find the unknown sides and angles.

Using a calculator to find trigonometric values and angles. You must be aware of the mode that your calculator is in!
$\sin 32^{\circ} 35^{\prime}$
$\tan 77^{\circ} 18^{\prime}$
$\cos \theta=.5678$
$\tan \theta=5.7$

To find the height of a tree, a person walks to a point 30 feet from the base of the tree, and measures the angle from the ground to the top of the tree to be 57 degrees. Find the height of the tree.


A person standing on the roof of a 100 foot building is looking towards a skyscraper a few blocks away, wondering how tall it is. She measures the angle of declination from the roof of the building to the base of the skyscraper to be 20 degrees and the angle of inclination to the top of the skyscraper to be 42 degrees.


## Homework for Right Triangle Trigonometry

In each of the triangles below, find $\sin (A), \cos (A), \tan (A), \sec (A), \csc (A), \cot (A)$.


In each of the following triangles, solve for the unknown sides and angles.


A 33 - ft ladder leans against a building so that the angle between the ground and the ladder is $80^{\circ}$. How high does the ladder reach up the side of the building?

The angle of elevation to the top of a building in New York is found to be 9 degrees from the ground at a distance of 1 mile from the base of the building. Using this information, find the height of the building.

A radio tower is located 400 feet from a building. From a window in the building, a person determines that the angle of elevation to the top of the tower is $36^{\circ}$ and that the angle of depression to the bottom of the tower is $23^{\circ}$. How tall is the tower?

There is an antenna on the top of a building. From a location 300 feet from the base of the building, the angle of elevation to the top of the building is measured to be $40^{\circ}$. From the same location, the angle of elevation to the top of the antenna is measured to be $43^{\circ}$. Find the height of the antenna.


## PRE-CALCULUS TRIGONOMETRY UNIT

Inverse Trigonometric Functions
Basic Graphs of Sine, Cosine and Tangent:




Because we are discussing the Inverse Functions, it is important to restrict the domains of the Trig Functions in order to make them $\qquad$ functions.

The domain of sine is: cosine:
tangent:
The range of sine is:

The Notations of Inverse Functions:
Arcs:

$\cos ^{-1}(x)$

$\tan ^{-1}(x)$


Note the relationships between functions and their inverses:

Domains and Ranges of Inverse Trig Functions:
a) $\sin ^{-1}\left(\frac{1}{2}\right)$
b) $\sin ^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
c) $\cos ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
d) $\tan ^{-1}(1)$




$\qquad$ of the functions!
$\sin ^{-1}(-1)$
b) $\tan ^{-1}(-1)$
c) $\cos ^{-1}(-1)$

Solve for the missing angle:


Compound Trig Expressions: (2 trig functions compositioned:)
Evaluate $\sin ^{-1}\left(\cos \left(\frac{13 \pi}{6}\right)\right)$.

Find an exact value for $\sin \left(\cos ^{-1}\left(\frac{4}{5}\right)\right)$

Find an exact value for $\sin \left(\tan ^{-1}\left(\frac{7}{4}\right)\right)$.
$\cos \left[\arctan (1)-\tan ^{-1}(-1)=\right.$

## Homework for Inverse Trigonometric Functions

Evaluate the following expressions:
$\sin ^{-1}\left(\frac{\sqrt{2}}{2}\right)$
$\arcsin \left(-\frac{1}{2}\right)$

$\arccos \left(-\frac{\sqrt{3}}{2}\right)$


$\arctan (\sqrt{3})$




Find the value for theta:


Evaluate the following expressions:

$$
\begin{equation*}
\cos \left(\cos ^{-1}\right. \tag{1}
\end{equation*}
$$

$\tan ^{-1}\left(\tan \left(\frac{\pi}{3}\right)\right)$
$\csc \left(\arcsin \left(\frac{1}{2}\right)\right)$



$\sin \left(\arccos \left(\frac{\sqrt{3}}{2}\right)\right)$
$\sin ^{-1}\left(\cos \left(\frac{4 \pi}{3}\right)\right)$
$\cos \left(\tan ^{-1}\left(\frac{1}{4}\right)\right)$



$\tan \left[\sin ^{-1}\left(\frac{1}{2}\right)+\cos ^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$
$\tan ^{-1}\left[\cos \left(\frac{\pi}{3}\right)-\sin \left(\frac{-\pi}{6}\right)\right]$




Find an angle between $0^{\circ}$ and $360^{\circ}$ that is coterminal with a $451^{\circ}$ angle.

Find an angle between 0 and $2 \pi$ that is coterminal with the angle $\frac{17 \pi}{3}$.

On a pizza with radius of 8 in., find the length of the crust that subtends a central angle of 1 radian.

Find the distance along an arc on the surface of the Earth that subtends a central angle of 12 minutes ( 1 minute $=1 / 60$ degree). The radius of the Earth is 3960 miles.

On a circle of radius 10 feet, what angle in degrees would subtend an arc of length 2 feet?

A slice of pie has a central angle of 60 degrees. Find the area of the sector if the radius of the pie is 6 inches.

Find the quadrant in which the terminal point determined by $t$ lies if:
a. $\sin \mathrm{t}<0$ and $\cos \mathrm{t}<0$
b $\sin t>0$ and $\cos t<0$

If $\cos \theta=\frac{2}{9}$ and $\theta$ lies in the 4th quadrant, find $\sin \theta$.

For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute the sine, cosine and tangent.
a. $120^{\circ}$
b. $315^{\circ}$
c. $\frac{5 \pi}{3}$
d. $\frac{3 \pi}{4}$





Find the coordinates of the point on a circle with radius of 12 corresponding to an angle of 240 degrees.

If $\boldsymbol{\operatorname { s e c }}(\boldsymbol{\beta})=-\frac{7}{3}$ and $\frac{\pi}{2}<\beta<\pi$, find $\tan (\boldsymbol{\beta})$ and $\sin (\boldsymbol{\beta})$.


If $\alpha=\frac{7 \pi}{4}$, find the exact values for $\sec (\alpha), \csc (\alpha), \tan (\alpha), \operatorname{ctn}(\alpha)$.


Evaluate:
a. $\sec \left(150^{\circ}\right)$
b. $\csc \left(\frac{-5 \pi}{6}\right)$
c. $\tan \left(135^{\circ}\right)$




Using SohCahToa, find the missing sides and angle.


A 28-ft ladder leans against a building so that the angle between the ground and the ladder is $\mathbf{8 0}$ degrees. How high does the ladder reach up the side of the building.

A radio tower is located 325 feet from a building. From a window in the building, a person determines the angle of elevation to the top of the tower is 43 degrees and that the angle of depression to the bottom of the tower is 31 degrees. How tall is the tower?

Find the length of side $x$.


Evaluate:
$\sin ^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
$\arccos \left(-\frac{\sqrt{2}}{2}\right)$



Find the missing angle.


Evaluate the following expressions:
$\cos ^{-1}\left(\tan \left(\frac{\pi}{4}\right)\right)$
$\tan \left(\arccos \frac{4}{9}\right)$
$\sin ^{-1}\left[\sin (-\pi)+\cos \left(\frac{2 \pi}{3}\right)\right]$





