PreCalc Final Exam Review

Multiple Choice

Identify the choice that best completes the statement or answers the question.

- 1. Determine which binomial is *not* a factor of $4x^4 21x^3 46x^2 + 219x + 180$. a. x + 4 b. x + 3 c. x - 5 d. 4x + 3
- 2. Which of the following sums has the largest value?
 - a. $\sum_{k=1}^{100} (k+2)$ b. $\sum_{k=1}^{100} 2k$ c. $\sum_{k=1}^{100} 2^{k}$ d. $\sum_{k=1}^{100} (\frac{1}{2})^{k}$

The sum of the geometric series $\frac{1}{4} + \frac{1}{2} + 1 + \dots + 2048$ is

- a. 2047.75 b. 2049.75 c. 4095.75 d. 4096.75
- 4. Evaluate the related series of the sequence 22, 28, 34, 40, 46.

a. 108 b. 147 c. 170 d. 216 e. 256

5. Determine the number of terms in the geometric series $-4 + 16 - 64 + 256 \dots$, $S_n = 52428$.

a. 6 b. 7 c. 8 d. 9 e. 10

- 6. What kind of symmetry does $y = \sqrt{x-2}$ have? a. x-axis b. y-axis c. origin d. none
- 7. If $f(x) = x^2 + 2x 1$ and g(x) = 3x 1, find f(g(2)):

Short Answer

- 10. Suppose f(x) = 4x 2 and g(x) = -2x + 1. Find the value of $\frac{f(5)}{g(-3)}$.
- 11. Find the zeros of y = x(x 3)(x 2). Then graph the equation.

Use Pascal's Triangle to expand the binomial.

- 15. $(d + 3)^{7}$
- 16. Use the Binomial Theorem to expand $(d 3b)^3$.

Write the equation in logarithmic form.

17. 64 = 1,296

8. If $f(x) = x^2 + 4x - 2$ and $g(x) = \sqrt{x+1}$, find $(g \circ f')(3)$: a. 19 b. 2 c. 38 d. $\sqrt{20}$

a. 34 b. 7 c. 20 d. 5

- 9. Find the inverse of the following function: $f(x) = \frac{x-1}{x+2}$ a. $f^{-1}(x) = \frac{x+2}{x-1}$ b. $f^{-1}(x) = \frac{x-2}{x+1}$ c. $f^{-1}(x) = \frac{-2x-1}{x-1}$ d. $f^{-1}(x) = \frac{x-2}{x+1}$
- 12. Write a polynomial function in standard form with zeros at 5, -4, and 1.
- 13. Divide $3x^3 3x^2 4x + 3$ by x + 3.

Solve the equation by graphing.

14. $x^2 + 7x + 19 = 0$

Evaluate the logarithm.

The pH of a liquid is a measure of how acidic or basic it is. The concentration of hydrogen ions in a liquid is labeled $[H^+]$. Use the formula $pH = -\log[H^+]$ to answer questions about pH.

- 19. Find the pH level, to the nearest tenth, of a liquid with $[H^+]$ about 6.5×10^{-3} .
- 20. Write the equation $\log_{32} 8 = \frac{3}{5}$ in exponential form.
 - Write the expression as a single logarithm. Find any points of discontinuity for the rational function.

25.
$$y = \frac{(x+6)(x+2)(x+8)}{(x+9)(x+7)}$$

26. Describe the vertical asymptote(s) and hole(s) for (x - 5)(x - 2)

the graph of $y = \frac{(x-5)(x-2)}{(x-2)(x+4)}$.

- 27. Write an equation of a parabola with a vertex at the origin and a focus at (-2, 0).
- 28. Identify the vertex, focus, and directrix of the graph of $y = \frac{1}{8} (x - 2)^2 + 5$.
- 29. Write an equation in standard form for the circle.
- 30. Graph $(x + 4)^2 + (y 7)^2 = 49$.
- 31. Find the foci of the ellipse with the equation $18x^2 + 36y^2 = 648$. Graph the ellipse.
- 32. Write an equation of an ellipse with center (3, −3), vertical major axis of length 12, and minor axis of length 6.
- 34. Graph the relation.

$$\left\{ \left(0, 3\right), \left(2, 4\frac{1}{2}\right), \left(1\frac{1}{2}, 5\right), \left(4, -1\right), \left(-4, 4\right) \right\}$$

- 35. Is the relation {(−2, 5), (−1, 5), (−1, 4), (−1, −3), (−2, 0)} a function? Explain.
- 36. Graph $y = 2x^2 7$.

- 21. $5 \log_{\theta} q + 2 \log_{\theta} y$
- 22. log₃4 log₃2

Sketch the asymptotes and graph the function.

23.
$$y = \frac{2}{x+2} - 3$$

24. Write an equation for the translation of $y = \frac{4}{x}$ that has the asymptotes x = 7 and y = 6.



33. Write an equation of an ellipse with center (3, 4), horizontal major axis of length 16, and minor axis of length 10.





37. Graph $y = x^2 + 3x + 2$. Identify the vertex and the axis of symmetry.



- 38. Graph $y = 3x^2 12x + 13$. What is the minimum value of the function?
- 39. Use the graph of $y = (x 3)^2 + 5$.
 - If you translate the parabola to the right 2 units and down 7 units, what is the equation of a. the new parabola in vertex form?
 - If you translate the original parabola to the left 2 units and up 7 units, what is the b. equation of the new parabola in vertex form?
 - How could you translate the new parabola in part (a) to get the new parabola in part (b)? c.
- 40. Suppose you cut a small square from a square of fabric as shown in the diagram. Write an expression for the remaining shaded area. Factor the expression.



- 41. Without graphing, determine whether the function $y = (5.2)^{x}$ represents exponential growth or exponential decay.
- 42. Without graphing, determine whether the function $y = 7\left(\frac{2}{3}\right)^{*}$ represents exponential growth or exponential decay.

- 43. Consider the sequence 16, -8, 4, -2, 1, ...
 - Describe the pattern formed in the sequence. a.
 - b. Find the next three terms.

In an arithmetic sequence, the third term is 11 and the eighth term is 46. Find the sequence.

44.

How many terms are in each sequence?

	a) 2, 6, 10, , 94	b) -9, -4, 1, , 171	c) 4, 15, 26,, 213
45.	d) 18, 13, 8, , -102	e) 5, 12, 19, , 222	f) 42, 29, 16,, -673

Find the missing terms in the arithmetic sequence:

	a) _, 9, 16, _, _;	b) _, _, 8, 2, _;	c) 12, _, 22, _, _;
46.	d) 3, _, _, 24, _;	e) _, 4, _, _, -8;	f) 15, _, _, _, _21

Essay

47. Suppose you invest \$580 at 10% compounded continuously. a. Write an exponential function to model the amount in your investment account. **b.** Explain what each value in the function model represents. c. In how many years will the total reach \$3600? Show your work.

Other

- 48. Describe the vertical-line test for a graph and tell how it can determine whether a graph represents a function.
- 49. What are multiple zeros? Explain how you can tell if a function has multiple zeros.
- 50. Use division to prove that x = 3 is a real zero of $\gamma = -x^3 + 9x^2 - 38x + 60.$
- 51. In a particular region of a national park, there are currently 330 deer, and the population is increasing at an annual rate of 11%. a. Write an exponential function to model the deer

population.

b. Explain what each value in the model represents.

c. Predict the number of deer that will be in the region after five years. Show your work.

REF: 2-1 Relations and Functions

TOP: 2-1 Example 6

PreCalc Final Exam Review Answer Section

MULTIPLE CHOICE

- 1. ANS: A PTS: 1 REF: 6-3 Dividing Polynomials DIF: L4 OBJ: 6-3.1 Using Long Division TOP: 6-3 Example 2 KEY: division of polynomials | polynomial | factoring a polynomial
- 2. ANS: C PTS: 1
- 3. ANS: C PTS: 1
- 4. ANS: C PTS: 1
- 5. ANS: C PTS: 1
- 6. ANS: D PTS: 1
- 7. ANS: A PTS: 1
- 8. ANS: D PTS: 1
- 9. ANS: C PTS: 1

SHORT ANSWER

10. ANS:

 2^{4}_{7}

- PTS: 1 DIF: L3 OBJ: 2-1.2 Identifying Functions
- KEY: function notation
- 11. ANS:



REF: 6-2 Polynomials and Linear Factors PTS: 1 DIF: L2 OBJ: 6-2.2 Factors and Zeros of a Polynomial Function STA: TX TEKS 2A.1A TOP: 6-2 Example 4

Name_____

This review counts as a quiz grade. You must show work to receive credit. Due at the BEGINNING of your final. KEY: Zero Product Property | polynomial function | zeros of a polynomial function | graphing 12. ANS: $f(x) = x^3 - 2x^2 - 19x + 20$ DIF: L2 REF: 6-2 Polynomials and Linear Factors PTS: 1 OBJ: 6-2.2 Factors and Zeros of a Polynomial Function STA: TX TEKS 2A.1A TOP: 6-2 Example 5 KEY: polynomial function | standard form of a polynomial | zeros of a polynomial function 13. ANS: $3x^2 - 12x + 32$, R - 93 PTS: 1 DIF: L2 **REF: 6-3 Dividing Polynomials** OBJ: 6-3.1 Using Long Division TOP: 6-3 Example 1 KEY: polynomial | division of polynomials 14. ANS: no solution PTS: 1 DIF: L4 **REF: 6-4 Solving Polynomial Equations** OBJ: 6-4.1 Solving Equations by Graphing STA: TX TEKS 2A.2A TOP: 6-4 Example 1 KEY: graphing | graphing calculator | solving equations | no solutions | polynomial function 15. ANS: $d^7 + 21d^6 + 189d^5 + 945d^4 + 2835d^3 + 5103d^2 + 5103d + 2187$ PTS: 1 DIF: L2 REF: 6-8 The Binomial Theorem OBJ: 6-8.1 Binomial Expansion and Pascal's Triangle TOP: 6-8 Example 1 KEY: Pascal's Triangle | binomial expansion 16. ANS: $d^3 - 9d^2b + 27db^2 - 27b^3$ REF: 6-8 The Binomial Theorem PTS: 1 DIF: L2 OBJ: 6-8.2 The Binomial Theorem TOP: 6-8 Example 3 KEY: Pascal's Triangle | binomial expansion 17. ANS: $\log_6 1, 296 = 4$ REF: 8-3 Logarithmic Functions as Inverses PTS: 1 DIF: L3 OBJ: 8-3.1 Writing and Evaluating Logarithmic Expressions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.4C | TX TEKS 2A.11A | TX TEKS 2A.11B TOP: 8-3 Example 2 KEY: logarithm | logarithmic form 18. ANS: -4 DIF: L4 REF: 8-3 Logarithmic Functions as Inverses PTS: 1 OBJ: 8-3.1 Writing and Evaluating Logarithmic Expressions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.4C | TX TEKS 2A.11A | TX TEKS 2A.11B TOP: 8-3 Example 3 KEY: evaluating logarithms 19. ANS: 2.2

Name

This review counts as a quiz grade. You must show work to receive credit. Due at the BEGINNING of your final. PTS: 1 DIF: L4 **REF: 8-3 Logarithmic Functions as Inverses** OBJ: 8-3.1 Writing and Evaluating Logarithmic Expressions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.4C | TX TEKS 2A.11A | TX TEKS 2A.11B TOP: 8-3 Example 4 KEY: logarithm | problem solving 20. ANS: $32^{5} = 8$ PTS: 1 DIF: L4 **REF: 8-3 Logarithmic Functions as Inverses** OBJ: 8-3.1 Writing and Evaluating Logarithmic Expressions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.4C | TX TEKS 2A.11A | TX TEKS 2A.11B KEY: logarithmic form | logarithm | exponential form TOP: 8-3 Example 3 21. ANS: $\log_{\lambda}(q^5y^2)$ PTS: 1 DIF: L4 **REF: 8-4 Properties of Logarithms** OBJ: 8-4.1 Using the Properties of Logarithms STA: TX TEKS 2A.2A TOP: 8-4 Example 2 KEY: properties of logarithms | logarithm | Product Property of Logarithms | Power Property of Logarithms 22. ANS: log, 2 PTS: 1 DIF: L3 **REF: 8-4 Properties of Logarithms** OBJ: 8-4.1 Using the Properties of Logarithms STA: TX TEKS 2A.2A TOP: 8-4 Example 2 KEY: properties of logarithms | simplifying a logarithm | Quotient Property of Logarithms 23. ANS: DIF: L2 PTS: 1 **REF: 9-2 The Reciprocal Function Family** OBJ: 9-2.2 Graphing Translations of Reciprocal Functions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.10A | TX TEKS 2A.10G

TOP: 9-2 Example 4KEY: graphing | asymptote

24. ANS:

 $y = \frac{4}{r-7} + 6$ PTS: 1 DIF: L2 **REF: 9-2 The Reciprocal Function Family** OBJ: 9-2.2 Graphing Translations of Reciprocal Functions STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.10A | TX TEKS 2A.10G TOP: 9-2 Example 5 KEY: asymptote | translation 25. ANS: x = -9, x = -7PTS: 1 DIF: L2 **REF: 9-3 Rational Functions and Their Graphs OBJ:** 9-3.1 Properties of Rational Functions STA: TX TEKS 2A.10A | TX TEKS 2A.10B | TX TEKS 2A.10C TOP: 9-3 Example 1 KEY: rational function | point of discontinuity 26. ANS: asymptote: x = -4 and hole: x = 2PTS: 1 DIF: L2 REF: 9-3 Rational Functions and Their Graphs **OBJ:** 9-3.1 Properties of Rational Functions STA: TX TEKS 2A.10A | TX TEKS 2A.10B | TX TEKS 2A.10C TOP: 9-3 Example 2 KEY: asymptote | vertical asymptote | rational function | graphing | hole in the graph of a function 27. ANS: $x = -\frac{1}{8}y^2$ PTS: 1 DIF: L2 REF: 10-2 Parabolas OBJ: 10-2.1 Writing the Equation of a Parabola STA: TX TEKS 2A.5B | TX TEKS 2A.5C | TX TEKS 2A.5D | TX TEKS 2A.5E TOP: 10-2 Example 2 KEY: equation of a parabola | focus of a parabola | parabola | vertex of a parabola 28. ANS: vertex (2, 5), focus (2, 7), directrix at y = 3PTS: 1 DIF: L2 REF: 10-2 Parabolas OBJ: 10-2.2 Graphing Parabolas STA: TX TEKS 2A.5B | TX TEKS 2A.5C | TX TEKS 2A.5D | TX TEKS 2A.5E TOP: 10-2 Example 5 KEY: directrix | equation of a parabola | focus of a parabola | parabola | graphing | vertex of a parabola 29. ANS: $(x+1)^{2} + (y+3)^{2} = 4$ PTS: 1 DIF: L2 REF: 10-3 Circles OBJ: 10-3.1 Writing the Equation of a Circle STA: TX TEKS 2A.5B | TX TEKS 2A.5D TOP: 10-3 Example 3 KEY: circle | equation of a circle | graphing | center of a circle | radius 30. ANS:



PTS: 1 DIF: L2 REF: 10-3 Circles

OBJ: 10-3.2 Using the Center and Radius of a Circle STA: TX TEKS 2A.5B | TX TEKS 2A.5D

- TOP: 10-3 Example 5
- KEY: center of a circle | circle | equation of a circle | radius | translation
- 31. ANS:



REF: 10-4 Ellipses PTS: 1 DIF: L2

OBJ: 10-4.2 Finding and Using the Foci of an Ellipse STA: TX TEKS 2A.5B | TX TEKS 2A.5D

TOP: 10-4 Example 3

KEY: ellipse | equation of an ellipse | foci of an ellipse | major axis of an ellipse | minor axis of an ellipse 32. ANS:

 $\frac{(x-3)^2}{9} + \frac{(y+3)^2}{36} = 1$

PTS: 1 DIF: L2 **REF: 10-6 Translating Conic Sections**

OBJ: 10-6.1 Writing Equations of Translated Conic Sections

STA: TX TEKS 2A.5B | TX TEKS 2A.5D | TX TEKS 2A.5E TOP: 10-6 Example 1

KEY: conic sections | co-vertex of an ellipse | equation of an ellipse | major axis of an ellipse | minor axis of an

ellipse | translation | vertex of an ellipse

.

33. ANS:

$$\frac{(x-3)^2}{64} + \frac{(y-4)^2}{25} = 1$$

PTS: 1 DIF: L3 REF: 10-6 Translating Conic Sections

OBJ: 10-6.1 Writing Equations of Translated Conic Sections

STA: TX TEKS 2A.5B | TX TEKS 2A.5D | TX TEKS 2A.5E TOP: 10-6 Example 1

KEY: conic sections | co-vertex of an ellipse | ellipse | equation of an ellipse | major axis of an ellipse | minor axis of an ellipse | translation | vertex of an ellipse

34. ANS:



PTS:	1	DIF: L2
ODI	a 1 1 a	11 D 1

OBJ: 2-1.1 Graphing Relations

KEY: relation | graphing | ordered pair

REF: 2-1 Relations and Functions TOP: 2-1 Example 1

35. ANS:

No; a domain value corresponds to two or more range values.

PTS:	1 DIF: L2	REF:	2-1 Relations and Functions
OBJ:	2-1.1 Graphing Relations	TOP:	2-1 Example 4
KEY:	relation domain range		

36. ANS:





STA: TX TEKS 2A.1A | TX TEKS 2A.5C | TX TEKS 2A.6B | TX TEKS 2A.8A TOP: 5-2 Example 1



DIF: L2

REF: 5-2 Properties of Parabolas

- OBJ: 5-2.1 Graphing Parabolas
 - STA: TX TEKS 2A.1A | TX TEKS 2A.5C | TX TEKS 2A.6B | TX TEKS 2A.8A
 - TOP: 5-2 Example 2 KEY: quadratic function | vertex of a parabola | axis of symmetry
- 38. ANS:

PTS: 1



minimum: 1

PTS:1DIF:L2REF:5-2 Properties of ParabolasOBJ:5-2.2 Finding Maximum and Minimum ValuesSTA:TX TEKS 2A.1A | TX TEKS 2A.5C | TX TEKS 2A.6B | TX TEKS 2A.8ATOP:5.2 Finamela 2

- TOP:5-2 Example 3KEY: quadratic function | minimum value
- 39. ANS:
 - **a.** $y = (x 5)^2 2$
 - **b.** $\gamma = (x-1)^2 + 12$
 - **c.** left 4 units, up 14 units

PTS: 1 DIF: L3 REF: 5-3 Translating Parabolas

OBJ: 5-3.1 Using Vertex Form

STA: TX TEKS 2A.4A | TX TEKS 2A.4B | TX TEKS 2A.6C | TX TEKS 2A.7A | TX TEKS 2A.7B | TX TEKS 2A.8A TOP: 5-3 Example 2

KEY: parabola | translation | vertex form | problem solving | word problem | multi-part question

40. ANS:

 $x^2 - 9; (x - 3)(x + 3)$

PTS: 1 DIF: L2 REF: 5-4 Factoring Quadratic Expressions
OBJ: 5-4.2 Factoring Special Expressions
STA: TX TEKS 2A.2A | TX TEKS 2A.8A | TX TEKS 2A.8D TOP: 5-4 Example 8
KEY: difference of two squares | factoring a difference of two squares
41. ANS:

exponential growth

PTS:	1	DIF: L3	REF: 8-1 Exploring Exponential Models

OBJ: 8-1.2 Exponential Decay

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STA: TX TEKS 2A.4A | TX TEKS 2A.11F | TX TEKS 2A.1B | TX TEKS 2A.11B
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TOP:8-1 Example 4KEY:exponential function | exponential growth | reasoning
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42. ANS:

exponential decay

This review counts as a quiz grade. You must show work to receive credit. Due at the BEGINNING of your final. OBJ: 8-1.2 Exponential Decay STA: TX TEKS 2A.4A | TX TEKS 2A.11F | TX TEKS 2A.1B | TX TEKS 2A.11B TOP: 8-1 Example 4 KEY: exponential decay | exponential function | reasoning 43. ANS: **a.** Divide the previous term by -2. $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$ b. **REF: 11-1 Mathematical Patterns** PTS: 1 DIF: L2 OBJ: 11-1.1 Identifying Mathematical Patterns TOP: 11-1 Example 1 KEY: sequence | pattern | multi-part question 44. ANS: -3, 4, 11, 18... PTS: 1 45. ANS: a) 24 b) 37 c) 20 d) 25 e) 32 f) 56 PTS: 1 46. ANS: a)2,23,30 b)20,14,-4 c)17,27,32 d)10,17,31 e)8,0,-4 f)6,-3,-12 PTS: 1

ESSAY

47. ANS:

- [4] **a.** $y = 580e^{0.1t}$
 - **b.** In the model, the coefficient of e is 580, the original investment. The formula for continuously compounded interest uses the number e raised to the power rt, where r is the rate as a decimal, in this case 0.1, and t is the time in years.
 - c. To find the number of years to reach \$3600, substitute 3600 into the model.

 $y = 580e^{0.1t}$ $3600 = 580e^{0.1t}$ $6.2 = e^{0.1t}$ $\ln 6.2 = 0.1t$ $t = \frac{\ln 6.2}{0.1}$

Name

This review counts as a quiz grade. You must show work to receive credit. Due at the BEGINNING of your final.

Dividing and rounding to the nearest year, $t \approx 18$. The investment will reach \$3600 in about 18 years.

- [3] one error in computation or incomplete explanation
- [2] two errors in computation or no explanation
- [1] one correct answer with no explanation
- PTS: 1 DIF: L4 REF: 8-6 Natural Logarithms
- OBJ: 8-6.2 Natural Logarithmic and Exponential Equations
- STA: TX TEKS 2A.2A | TX TEKS 2A.11C | TX TEKS 2A.11D

KEY: compounding continuously | exponential function | exponential growth | extended response | graphing | interest rates | percent | problem solving | the number e | writing in math | rubric-based question

OTHER

48. ANS:

Answers may vary. Sample: Imagine drawing a vertical line through every point on the graph. If no line intersects the graph more than once, the graph is a function. It works because if a vertical line intersects the graph twice, then two points have the same *x*-value but two different *y*-values.

PTS:	1	DIF:	L3	REF:	2-1	Relations and Functions

OBJ: 2-1.2 Identifying Functions TOP: 2-1 Example 5

KEY: vertical-line test | graphing | writing in math | reasoning

49. ANS:

If a linear factor of a polynomial is repeated, then the zero is repeated and the function has multiple zeros. To determine whether a function has a multiple zero, factor the polynomial. If a factor is repeated in the factored expression, then it is a multiple zero.

PTS: 1 DIF: L3 REF: 6-2 Polynomials and Linear Factors

OBJ: 6-2.2 Factors and Zeros of a Polynomial Function STA: TX TEKS 2A.1A

TOP: 6-2 Example 6

KEY: reasoning | factoring a polynomial | multiple zero | polynomial function | zeros of a polynomial function | writing in math

50. ANS:

 $-x^3 + 9x^2 - 38x + 60 \div (x - 3) = -x^2 + 6x - 20$ with no remainder, so x = 3 is a real zero of the function.

PTS: 1 DIF: L3 REF: 6-3 Dividing Polynomials

OBJ: 6-3.2 Using Synthetic Division TOP: 6-3 Example 5

KEY: division of polynomials | polynomial | polynomial function | root of a function | writing in math

51. ANS:

a. $y = 330(1.11)^x$

b. In the model, 330 represents the initial population of deer. The growth factor is represented by 1 + 0.11 or 1.11. **c.** To predict the number of deer present after 5 years, substitute 5 into the function and evaluate.

 $y = 330(1.11)^x$ function model $y = 330(1.11)^5$ Substitute 5 for x. $y \approx 556$ Use a calculator.

There will be about 556 deer in the region.

PTS:	1 DIF: L4	REF:	8-1 Exploring Exponential Models
OBJ:	8-1.2 Exponential Decay		

STA: TX TEKS 2A.4A | TX TEKS 2A.11F | TX TEKS 2A.1B | TX TEKS 2A.11B

TOP: 8-1 Example 2

KEY: exponential decay | exponential function | multi-part question | percent | problem solving | writing in math