Precalculus

Final Exam Review

2014 – 2015

You must show work to receive credit!

- **This review covers the major topics in the material that will be tested on the final exam. It is not necessarily all inclusive and additional study and problem solving practice may be required to fully prepare for the final exam.**

- **Place answers in the blanks, when provided.**
- **Use additional paper, if necessary.**
- **Calculators may be used; however, the final exam will have non-calculator portions. Therefore, prudence suggests you prepare with and without a calculator so you can handle any contingency.**

Name:______________________

Period:______

Due Date: _________________

The answers on this paper are my own. I did not receive nor give aid on this exam. I have complied with all aspects of the LCHS Honor Code.

Student signature ____ Date ________________________

True-False

1. Arithmetic sequences have a common ratio.
2. The graph of a rational function can never cross a vertical asymptote.
3. If interest is compounded <u>*continuously*</u>, use $A = Pe^{nt}$.
4. The graph of $y = a^x$ will always go through the points (0, 1), (1, a), and $\left(-1, \frac{1}{a}\right)$.
5. The function $f(x) = 5x^3 + 17$ is a one-to-one function.
6. The graph of the function $f(x) = \dfrac{4x^3 - x^2 + 5}{6x^3 + 5x^2 - 8x + 10}$ has an oblique asymptote.
7. 7.5, 5, 2.5, 0, ... is an example of an arithmetic sequence.
8. $\ln e^x = x$
9. $\dfrac{\log x}{\log y} = \log x - \log y$
10. The triangle described by $a = 10, b = 14,$ and $A = 50°$, has 2 solutions.
11. The range of a function is the same as the domain of the function's inverse.
12. π radians = $360°$
13. All reference angles must be between $0°$ and $90°$
14. The Ambiguous Case for solving a triangle is when given angle-angle-side (AAS).
15. Given side-angle-side (SAS) of a triangle, use the Law of Cosines to solve it.
16. If interest is compounded <u>*continuously*</u>, use $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
17. The unit circle has a radius of 2.
18. The natural base is 10.
19. The common log (log) is the inverse of the natural log (ln).
20. $y = \log x$ has a vertical asymptote at $x = 0$.
21. $\ln e^x = x$

Multiple Choice

22. Solve for x: $25^{x^2} = 5^{3x-1}$

a. 1 b. $\frac{1}{2}$ c. $1, \frac{1}{2}$ d. no solution

23. Evaluate $(81)^{-\frac{1}{4}}$

a. 3 b. –3 c. $\frac{1}{3}$ d. $-\frac{1}{3}$

24. Express in expanded form: $\log_2 \sqrt[4]{3x^5y^7}$

a. $-4[3\log_2 + 5\log_2 x + 7\log_2 y]$ b. $\frac{1}{4}[5\log_2 3x + 7\log_2 y]$

c. $\frac{1}{4}[\log_2 3 + 5\log_2 x + 7\log_2 y]$ d. $-\frac{1}{4}[\log_2 3 + 5\log_2 x + \log_2 y]$

25. Solve the equation: $\sqrt{x+5} - x = -1$

a. 4 b. –1 c. 4, –1 d. no solution

26. Express the logarithmic equation as an exponential equation and solve: $\log_5 \frac{1}{125} = x$

a. $125^x = \frac{1}{5},\ x = -3$ b. $\left(\frac{1}{5}\right)^x = 125,\ x = \frac{1}{3}$

c. $x^5 = \frac{1}{125},\ x = -3$ d. $5^x = \frac{1}{125},\ x = -3$

27. Solve $\frac{e^x}{e^x - 2} = 3$

a. $\ln 3$ b. $\ln 3e$ c. $\ln \frac{3}{2}$ d. $\ln \frac{2}{3}$

28. The vertex of $y = x^2 - 2x + 5$ is:

a. (–1, 4) b. (1, –4) c. (1, 4) d. (–1, –4)

29. Which type of function *cannot* have an asymptote?

a. polynomial b. rational c. exponential d. logarithmic

30. Determine a polynomial of lowest degree with real coefficients that has $2i$ and 3 as roots.

a. $x^2 + x - 6$ b. $x^2 + x + 6$

c. $x^3 + 3x^2 - 4x + 12$ d. $x^3 - 3x^2 + 4x - 12$

31. Determine any points of discontinuity for $f(x) = \dfrac{x(x-5)}{(x-3)(x-5)}$

a. 0 b. 3 c. 3, 5 d. 0, 3, 5

32. Solve $e^{2x} - 4e^x = 0$ for x.

a. 0, 4 b. 4 c. ln 0, ln 4 d. ln 4

33. Evaluate the expression $\dbinom{157}{3}$

a. $\dfrac{157!}{154!}$ b. 154 c. 3,796,260 d. 632,710

34. The 5th term in the expansion of $(4x+3)^5$.

a. $2160x^2$ b. $1620x$ c. $540x$ d. 1215

35. Expand the expression $(x-9)^5$ using the Binomial Theorem.

a. $x^5 - 45x^4 + 1620x^3 - 14,580x^2 + 32,805x - 9$ c. $x^5 - 45x^4 + 810x^3 - 7290x^2 + 32,805x - 59,049$

b. $x^5 - 45x^4 + 810x^3 - 7290x^2 + 32,805x - 9$ d. $x^5 - 45x^4 + 1620x^3 - 14,580x^2 + 32,805x - 59,049$

36. The triangle described by A = 35°, $a = 6$, and $b = 12$ has _____ solutions.

a. 0 b. 1 c. 2 d. infinite

37. The size P of a small herbivore population at time t (in years) obeys the function $P(t) = 700e^{0.18t}$ if they have enough food and the predator population stays constant. After how many years will the population reach 2800?

a. 16.91 yrs b. 7.7 yrs c. 42.5 yrs d. 13.25 yrs

38. The half-life of a radioactive element is 130 days, but your sample will not be useful to you after 80% of the radioactive nuclei originally present have disintegrated. About how many days can you use the sample?

a. 302 b. 287 c. 297 d. 312

39. Let $f(x)=\dfrac{x+3}{x-1}$ and $g(x)=\dfrac{x^2-2}{x}$, the domain of $g(f(x))$ is

a. $\mathbb{R}$ b. $\mathbb{R}, x\neq 0,-1,2$ c. $\mathbb{R}, x\neq -3,1$ d. $\mathbb{R}, x\neq 0,1$

40. The sum of the first 10 terms of the sequence $3\left(\dfrac{2}{3}\right)^{N+1}$.

a. 1.767 b. 3.931 c. 8.844 d. 4092

41. The domain of $f(x)=\dfrac{1}{x^2+1}$ is.

a. all real numbers b. $x\geq -1$ c. $x\neq -1$ d. $x>-1$

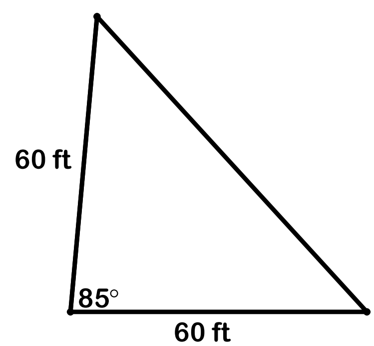
42. The inverse of $f(x)=(x+2)^3-3$ is $f^{-1}(x)=$

a. $\sqrt[3]{x+3}+2$ b. $\sqrt[3]{x-3}+2$ c. $\sqrt[3]{x+5}$ d. $\sqrt[3]{x+3}-2$

43. Let $f(x)=2x+3$ and $g(x)=x^2-2x+3$, $g(f(x))=$

a. $2x^2-4x+9$ b. $4x^2+8x+18$ c. $4x^2+8x+6$ d. $4x^2-4x+6$

44. A corner of McCormick Park occupies a triangular area that faces two streets that meet at an angle measuring 85°. The sides of the area facing the streets are each 60 feet in length (see diagram). The park's landscaper wants to plant begonias around the edges of the triangular area. Find the perimeter of the triangular area to the nearest foot.



a. 125 ft b. 240 ft c. 180 ft d. 201 ft

45. Find the domain of $f(x)=\log(x-5)$

a. $x>0$ b. $x<5$ c. $x>5$ d. all real numbers

46. Identify the x and y –intercepts, if any, of the equation $y = \frac{-1}{x+1} + 4$

a. $x-\text{int}: -1$ $y-\text{int}: \text{None}$

b. $x-\text{int}: \text{None}$ $y-\text{int}: 3$

c. $x-\text{int}: -\frac{3}{4}$ $y-\text{int}: 3$

d. $x-\text{int}: -1$ $y-\text{int}: 4$

47. Evaluate: $\sum_{n=1}^{\infty} 5\left(\frac{5}{2}\right)^{n-1}$

a. 5

b. $-\frac{10}{3}$

c. $-\frac{3}{10}$

d. not possible

48. The sum of the infinite geometric sequence $a_n = 4\left(-\frac{3}{4}\right)^{n-1}$ is

a. $\frac{7}{16}$

b. $\frac{16}{7}$

c. 4

d. not possible

49. Solve $\log_{25} 2x = \frac{3}{2}$ for x.

a. 125

b. 62.5

c. 625

d. 250

Free Response. Calculators may be used. Show all your work. Not all problems require a calculator. Round answers to *three decimal places* unless otherwise directed.

50. If you invest \$1000 an account that pays 7.5% interest compounded quarterly, how much will you have in the account after 15 years? How much would you have if the same account was continuously compounded?

51. Determine whether $x+1$ is a linear factor of $4x^2 - 2x - 9$ yes or no

52. Express $x^3 - x^2 - 6x$ as a *product of linear factors*. ____________________

53. Determine any intercepts or asymptotes for $g(x) = \dfrac{x-5}{x+12}$.

x-intercept: (____, ____) horizontal asymptote: $y =$ ________

y-intercept: (____, ____) vertical asymptote: $x=$ ________

54. Simplify $\left(x^{\frac{1}{3}} y^{-2} z^{\frac{5}{8}}\right)^{-3} \left(x^5 y^{-1} z^{\frac{1}{8}}\right)^2$. Use positive exponents. ____________________

55. Solve $\log_8 2x = \dfrac{5}{3}$ for *x*.

56. Write as the sum and/or difference of logs. Express powers as factors.

$$\ln\left(\frac{z(x+y)^{1/3}}{y^2}\right)$$

57. Express as a single logarithm and simplify.

$$\frac{1}{5}\log 32 + 2\log(x-1) - 3\log(x+1)$$

58. Identify all ***real zeros*** of $f(x) = 2x^4 + x^3 - 3$. x = ____________________

59. Solve $\sqrt{5-x} = 2x - \sqrt[3]{4x}$ for *x*. (Hint: Graph it.) $x =$ ____________________

60. Solve $3\sin x = x + \frac{\pi}{6}$ where $0 < x < 2\pi$. Use *radian mode* and round to *three decimal places*.

x = ____________________

61. If \$2000 is invested in an account that compounds interest quarterly at 7.5%, what will be the account balance in 8 years?

Balance: ________________

62. Solve $6\log x = 7.332$ for x.

x = ____________________

63. Solve $32^{4x+3} = 16^{3x+4}$ for x.

x = ____________________

64. Find the coefficient of the x^6 term of the expansion of $(x+3)^8$.

coefficient = _____________

65. Find the 7^{th} term of the expansion of $(2x+1)^{10}$.

7^{th} term = _______________

66. Assume that the half-life of Carbon-14 is 5600 years. Find the age (to the nearest year) of a wooden axe in which the amount of Carbon-14 is 30% of what it originally had.

age = ________________

67. Find the amount owed at the end of 8 years if \$5000 is loaned at a rate of 5% compounded monthly.

amount = _______________

68. The formula

$$D = 8e^{-0.6h}$$

can be used to find the number of milligrams D of a certain drug that is in a patient's bloodstream h hours after the drug has been administered. The drug is to be administered again when the amount in the bloodstream reaches 4 milligrams. What is the time between injections?

time = ________________

69. Express as a single logarithm and simplify.

$$\ln\left(\frac{x^2-4x-32}{x-7}\right) - \ln\left(\frac{x^2-3x-28}{x+7}\right) + \ln\left(x^2-16x+64\right), \quad a>0$$

70. Write as the sum and/or difference of logs. Express powers as factors.

$$\ln\left(\frac{(x+6)(x-5)}{(x-8)^4}\right)^{5/2}, \quad x>5$$

71. In ΔABC, A = 47°, B = 56°, and c = 14, find b. (2 pts)

72. How many years will it take for your investment to triple in value if it is placed in an account that pays 12% interest and is compounded continuously?

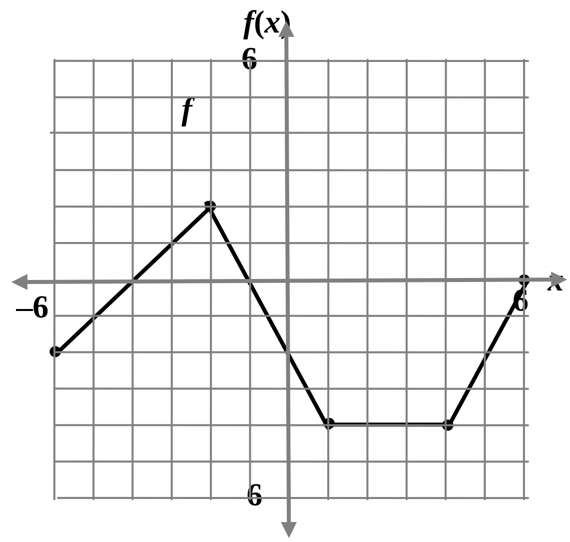
t = ____________________

73. A culture of bacteria obeys the law of uninhibited growth. If 500 bacteria are present initially and there are 800 after 1 hour, how many will be present after 5 hours?

74. Determine if the sequence is arithmetic, geometric, or neither. If arithmetic or geometric, write the explicit rule.

a. 1, 4, 9, 16, 25, …	b. –3, 6, –12, 24, –48, …	c. 10, 16, 22, 28, 34, …

75. A triangle has side lengths of 12 feet, 18 feet, and 22 feet. What is the area of the triangle?



76. Given the graph of $f(x)$ is shown. Give all values which appear to satisfy the following conditions.
Use interval notation where appropriate.

a. domain of $f(x)$: ______________________

b. range of $f(x)$: ______________________

c. all zeros of $f(x)$: ___________

d. y-intercept of $f(x)$: _________

e. if $f(x) = 2$, then x = _________

f. interval(s) over which $f(x)$ is constant ______________________

g. interval(s) over which $f(x)$ is increasing ______________________

h. interval(s) over which $f(x)$ is decreasing ______________________

i. Give a justification for calling the relation graphed above a function.

k. Is the inverse of $f(x)$ also a function? Justify your answer.

Advanced Algebra/Precalculus Final Exam Review Formula Sheet

Name:______________________

Law of Sines & Cosines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

Area of Triangles:

$$\text{Area} = \frac{1}{2}bc\sin A$$

Heron's Formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{(a+b+c)}{2}$$

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous Interest Formula

$$A = Pe^{rt}$$

Exponential Growth

$$N(t) = N_0 e^{kt}$$

Exponential Decay

$$A(t) = A_0 e^{kt}$$

Arithmetic Sequences and Series

Explicit: $a_n = a_1 + (n-1)d$

Sum: $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$

$S_n = \frac{n}{2}\left(a_1 + a_n\right)$

Geometric Sequences and Series

Explicit: $a_n = a_1 r^{n-1}$

Sum: $S_n = a_1\frac{1-r^n}{1-r}, \quad r \neq 0, 1$

$\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r}, \quad |r| < 1$

Binomial Theorem

$$\binom{n}{n-j} a^{n-j} x^j$$