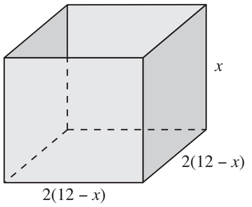
Chapter 12

Section 12.1 *(page 860)*

Vocabulary Check *(page 860)*

1. limit **2.** oscillates **3.** direct substitution

1. (a)



(b) $V = lwh$

$= 2(12 - x) \cdot 2(12 - x) \cdot x$

$= 4x(12 - x)^2$

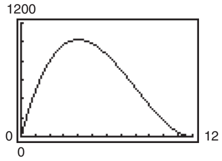
(c)

x	3	3.5	3.9	4
V	972	1011.5	1023.5	1024

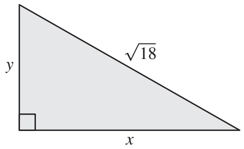
x	4.1	4.5	5
V	1023.5	1012.5	980

$\lim_{x \to 4} V = 1024$

(d)



2. (a)



(b) $A = \frac{1}{2}bh$

$= \frac{1}{2}xy$

$= \frac{1}{2}x\sqrt{18 - x^2}$

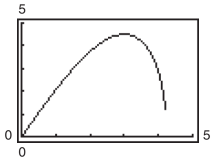
(c)

x	2	2.5	2.9	3
A	3.7417	4.2848	4.4903	4.5

x	3.1	3.5	4
A	4.4897	4.1964	2.8284

$\lim_{x \to 3} A = 4.5$

(d)



3.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	13.5	13.95	13.995	14	14.005	14.05	14.5

14; Yes

4.

x	1.9	1.99	1.999	2
$f(x)$	−1.090	−1.010	−1.001	−1

x	2.001	2.01	2.1
$f(x)$	−0.999	−0.990	−0.890

−1; Yes

5.

x	2.9	2.99	2.999	3
$f(x)$	0.1695	0.1669	0.1667	Error

x	3.001	3.01	3.1
$f(x)$	0.1666	0.1664	0.1639

$\frac{1}{6}$; No

6.

x	−1.1	−1.01	−1.001	−1
$f(x)$	−0.3226	−0.3322	−0.3332	Error

x	−0.999	−0.99	−0.9
$f(x)$	−0.3334	−0.3344	−0.3448

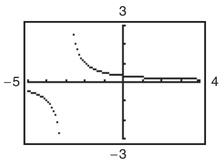
$-\frac{1}{3}$; No

7.

x	0.9	0.99	0.999	1
$f(x)$	0.2564	0.2506	0.2501	Error

x	1.001	1.01	1.1
$f(x)$	0.2499	0.2494	0.2439

$\frac{1}{4}$;



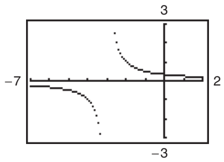
(Continued)

8.

x	-2.1	-2.01	-2.001	-2
$f(x)$	1.1111	1.0101	1.001	Error

x	-1.999	-1.99	-1.9
$f(x)$	0.999	0.9901	0.9091

1;

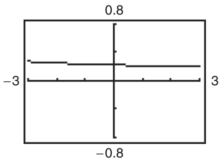


9.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.2247	0.2237	0.2236	Error

x	0.001	0.01	0.1
$f(x)$	0.2236	0.2235	0.2225

0.2236;

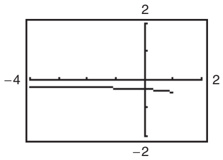


10.

x	-3.1	-3.01	-3.001	-3
$f(x)$	-0.2485	-0.2498	-0.25	Error

x	-2.999	-2.99	-2.9
$f(x)$	-0.25	-0.2502	-0.2516

$-\frac{1}{4}$;

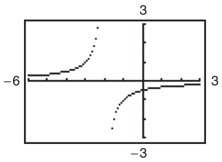


11.

x	-4.1	-4.01	-4.001	-4
$f(x)$	0.4762	0.4975	0.4998	Error

x	-3.999	-3.99	-3.9
$f(x)$	0.5003	0.5025	0.5263

$\frac{1}{2}$;

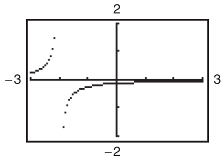


12.

x	1.9	1.99	1.999	2
$f(x)$	-0.0641	-0.0627	-0.0625	Error

x	2.001	2.01	2.1
$f(x)$	-0.0625	-0.0623	-0.061

$-\frac{1}{16}$;

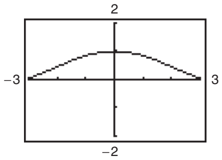


13.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9983	0.99998	0.9999998	Error

x	0.001	0.01	0.1
$f(x)$	0.9999998	0.99998	0.9983

1;

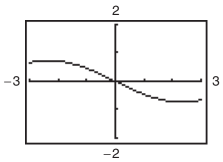


14.

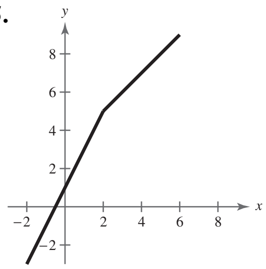
x	-0.1	-0.01	-0.001	0
$f(x)$	0.050	0.005	0.0005	Error

x	0.001	0.01	0.1
$f(x)$	-0.0005	-0.005	-0.05

0;

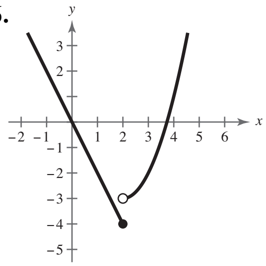


15.



5

16.



Limit does not exist.

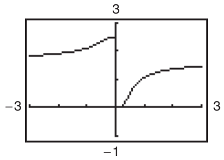
17. 13 **18.** 12 **19.** Does not exist. Answers will vary.

20. Does not exist. Answers will vary.

(Continued)

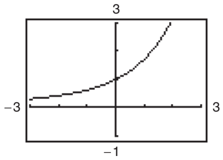
21. Does not exist. Answers will vary. **22.** -1

23.



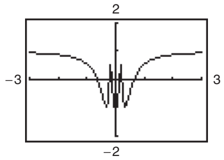
No. Answers will vary.

24.



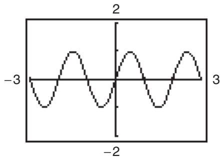
Yes

25.



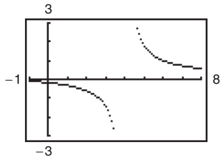
No. Answers will vary.

26.



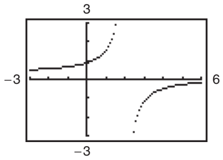
Yes

27.



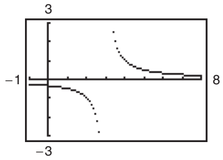
No. Answers will vary.

28.



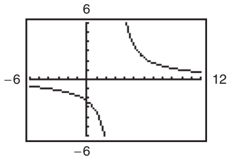
No. Answers will vary.

29.



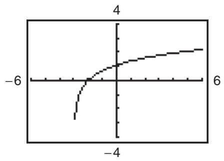
Yes will vary.

30.



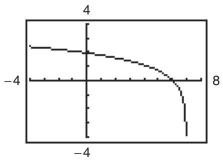
No. Answers

31.



Yes

32.



Yes

33. (a) -12 (b) 9 (c) $\frac{1}{2}$ (d) $\sqrt{3}$

34. (a) 9 (b) -60 (c) $-\frac{1}{2}$ (d) $\frac{\sqrt{5}}{5}$

35. (a) 8 (b) $\frac{3}{8}$ (c) 3 (d) $-\frac{61}{8}$

36. (a) 2 (b) 0 (c) 0 (d) -2 **37.** -15

38. 6 **39.** 7 **40.** 9 **41.** -3 **42.** -2

43. $-\frac{9}{10}$ **44.** $\frac{1}{9}$ **45.** $\frac{7}{13}$ **46.** $\frac{10}{3}$ **47.** 1

48. 2 **49.** $\frac{35}{3}$ **50.** $\frac{3}{4}$ **51.** $e^3 \approx 20.09$ **52.** 1

53. 0 **54.** 0 **55.** $\frac{\pi}{6}$ **56.** $\frac{\pi}{3}$ **57.** True

58. True, provided the individual limits exist.

59. (a) and (b) Answers will vary.

60. Answers will vary.

61. (a) No. The function may approach different values from the right and left of 2. For example,

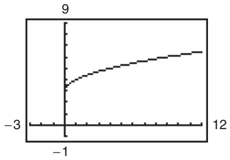
$$f(x) = \begin{cases} 0, & x < 2 \\ 4, & x = 2 \\ 6, & x > 2 \end{cases}$$

implies $f(2) = 4$, but $\lim_{x \to 2} f(x) \neq 4$.

(b) No. The function may approach 4 as x approaches 2, but the function could be undefined at $x = 2$. For example, in the function $f(x) = \dfrac{4 \sin(x - 2)}{x - 2}$, the limit is 4 as x approaches 2, but $f(2)$ is not defined.

62. As a function's x-value approaches 5 from both the right and left sides, its corresponding output values approach 12.

63. (a)



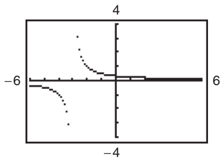
6

(b) Domain: all real numbers x such that $x \geq 0$

(c) Domain: all real numbers x such that $x \geq 0$ except $x = 9$

(d) It may not be clear from a graph that a function is not defined at a single point. Examining a function graphically and algebraically ensures that you will find all points at which the function is not defined.

64. (a)



$\frac{1}{6}$

(b) Domain: all real numbers x except $x = -3$

(c) Domain: all real numbers x except $x = \pm 3$

(d) It may not be clear from a graph that a function is not defined at a single point. Examining a function graphically and algebraically ensures that you will find all points at which the function is not defined.

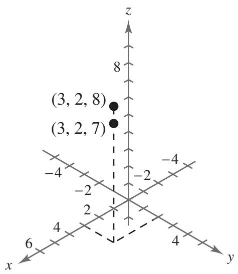
65. $-\frac{1}{3}, x \neq 5$ **66.** $-(x + 9), x \neq 9$

67. $\frac{5x + 4}{5x + 2}, x \neq \frac{1}{3}$ **68.** $\frac{x - 6}{x - 1}, x \neq 6$

69. $\frac{x^2 - 3x + 9}{x - 2}, x \neq -3$ **70.** $\frac{x^2 + 2x + 4}{x + 2}, x \neq 2$

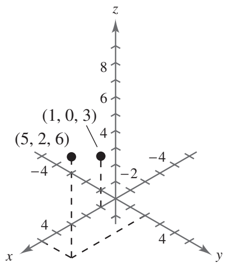
(Continued)

71. (a)



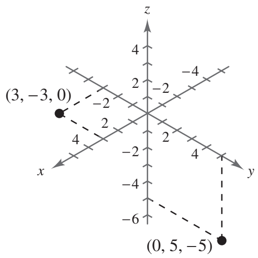
(b) 1 (c) $\left(3, 2, \frac{15}{2}\right)$

72. (a)



(b) $\sqrt{29}$ (c) $\left(3, 1, \frac{9}{2}\right)$

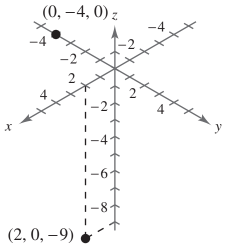
73. (a)



(b) $7\sqrt{2}$

(c) $\left(\frac{3}{2}, 1, -\frac{5}{2}\right)$

74. (a)



(b) $\sqrt{101}$

(c) $\left(1, -2, -\frac{9}{2}\right)$

Section 12.2 *(page 870)*

Vocabulary Check *(page 870)*

1. dividing out technique **2.** indeterminate form
3. one-sided limit **4.** difference quotient

1. (a) 1 (b) 3 (c) 5
$g_2(x) = -2x + 1$

2. (a) -5 (b) -3 (c) 0
$h_2(x) = x - 3$

3. (a) 2 (b) 0 (c) 0
$g_2(x) = x(x + 1)$

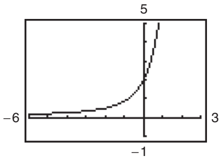
4. (a) 0 (b) 1 (c) -2
$f_2(x) = x - 1$

5. $\frac{1}{12}$ **6.** $-\frac{1}{10}$ **7.** 4 **8.** -5 **9.** 12 **10.** 27

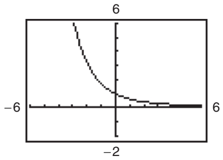
11. 80 **12.** 4 **13.** $\frac{\sqrt{5}}{10}$ **14.** $-\frac{\sqrt{7}}{14}$ **15.** $\frac{\sqrt{3}}{6}$

16. $\frac{1}{4}$ **17.** 1 **18.** $-\frac{1}{6}$ **19.** $\frac{1}{4}$ **20.** $\frac{1}{8}$ **21.** -1

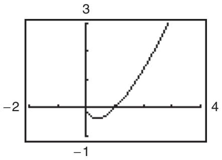
22. $\frac{1}{16}$ **23.** $-\frac{1}{16}$ **24.** $-\frac{1}{4}$ **25.** Does not exist **26.** 0

27.


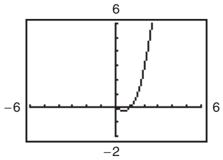
2.000

28.


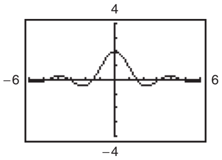
1.000

29.


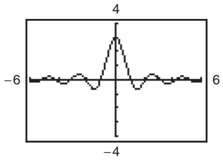
0

30.


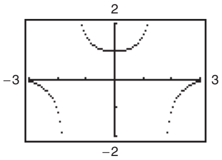
0

31.


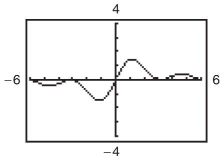
2.000

32.


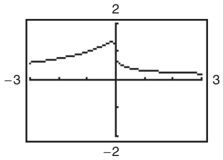
3.000

33.


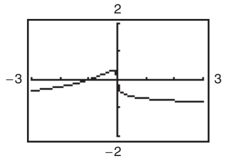
1.000

34.


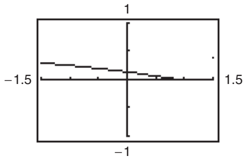
0

35.


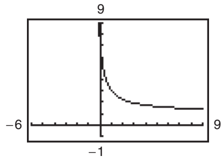
0.333

36.


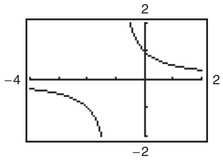
-0.667

37.


0.135

38.


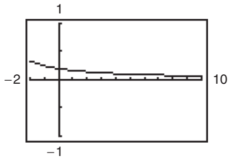
7.389

39. (a)


(b)

x	0.9	0.99	0.999	0.9999
$f(x)$	0.5263	0.5025	0.5003	0.5000

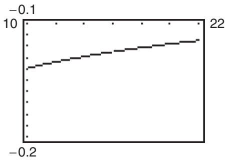
(c) $\lim_{x\to 1^-} \frac{(x-1)}{(x^2-1)} = 0.5$

40. (a)


(b)

x	5.1	5.01	5.001	5.0001
$f(x)$	0.09901	0.09990	0.09999	0.099999

(c) $\lim_{x\to 5^+} \frac{5-x}{25-x^2} = 0.1$

41. (a)


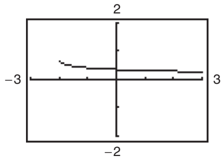
(b)

x	16.1	16.01	16.001	16.0001
$f(x)$	-0.12481	-0.12498	-0.124998	-0.1249998

(c) $\lim_{x\to 16^+} \frac{4-\sqrt{x}}{x-16} = -0.125$

(Continued)

42. (a)

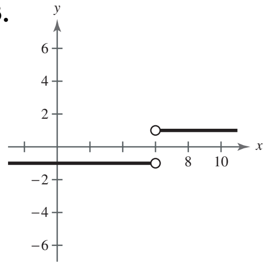


(b)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	0.35809	0.353996	0.353598	0.353558

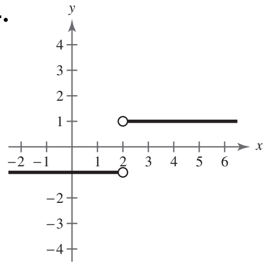
(c) $\lim_{x\to 0^-} \dfrac{\sqrt{x+2}-\sqrt{2}}{x} = \dfrac{\sqrt{2}}{4}$

43.



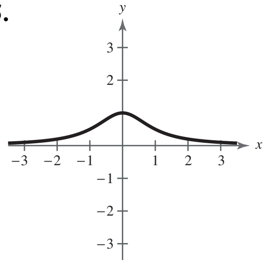
The limit does not exist.

44.



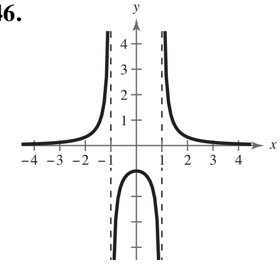
The limit does not exist.

45.



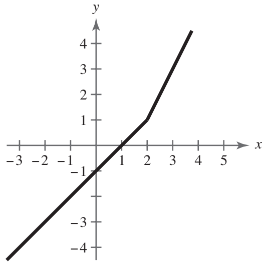
$\lim_{x\to 1} \dfrac{1}{x^2+1} = \dfrac{1}{2}$

46.



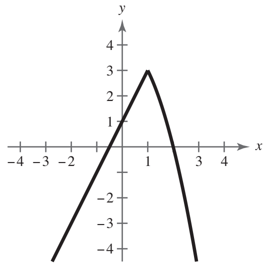
The limit does not exist.

47.



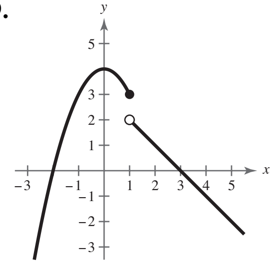
$\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x-1, & x \le 2 \\ 2x-3, & x > 2 \end{cases} = 1$

48.



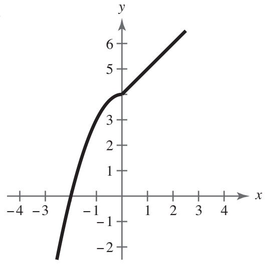
3

49.



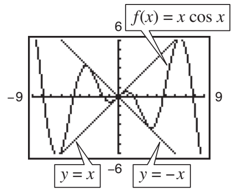
The limit does not exist.

50.



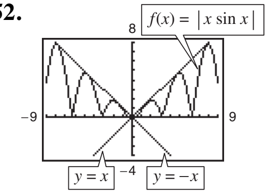
4

51.



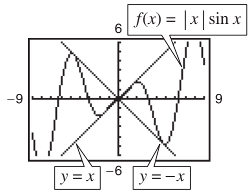
$\lim_{x\to 0} x\cos x = 0$

52.



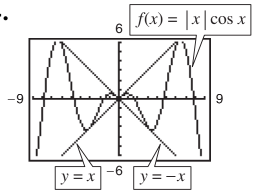
$\lim_{x\to 0} |x\sin x| = 0$

53.



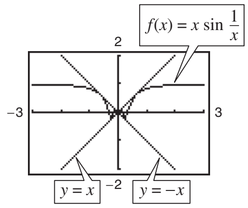
$\lim_{x\to 0} |x|\sin x = 0$

54.



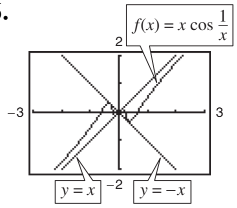
$\lim_{x\to 0} |x|\cos x = 0$

55.



$\lim_{x\to 0} x\sin\dfrac{1}{x} = 0$

56.



$\lim_{x\to 0} x\cos\dfrac{1}{x} = 0$

57. Limit (a) can be evaluated by direct substitution.
(a) 0 (b) 1

58. Limit (a) can be evaluated by direct substitution.
(a) 0 (b) 0

(Continued)

59. 3 **60.** -6 **61.** $\frac{1}{2\sqrt{x}}$ **62.** $\frac{1}{2\sqrt{x-2}}$

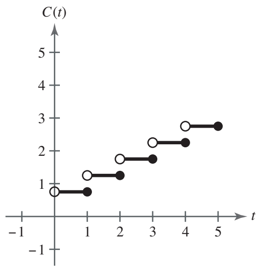
63. $2x - 3$ **64.** $-2x - 2$ **65.** $-\frac{1}{(x+2)^2}$

66. $-\frac{1}{(x-1)^2}$ **67.** -32 feet per second

68. -64 feet per second **69.** Answers will vary.

70. Answers will vary.

71. (a)



(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	1.75	2.25	2.25	2.25	2.25	2.25	2.25

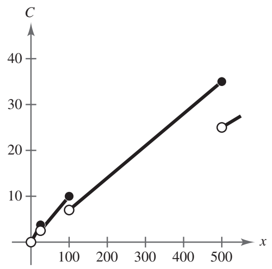
$\lim_{t \to 3.5} C(t) = 2.25$

(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	1.25	1.75	1.75	1.75	2.25	2.25	2.25

The limit does not exist.

72. (a)



(b) (i) 2.25 (ii) 9.9 (iii) 21.35

(c) Answers will vary.

x	24.99	24.999	25	25.001	25.01
$\lim_{x \to 25} C(x)$	3.7485	3.74985	DNE	2.5001	2.501

x	99.99	99.999	100	100.001	100.01
$\lim_{x \to 100} C(x)$	9.999	9.9999	DNE	7.00007	7.0007

x	499.99	499.999	500	500.001	500.01
$\lim_{x \to 500} C(x)$	34.9993	34.99993	DNE	25.00005	25.0005

(d) The graph shows that the one-sided limits are not equal, which can be seen when the graph "jumps" at $x = 25, 100$, and 500.

73. True **74.** False. Exercise 49 is a counter example.

75. (a) and (b) Graphs will vary.

76. Answers will vary. **77.** $x - 2y - 26 = 0$

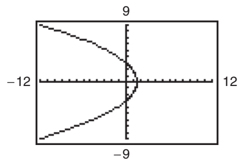
78. $2y - x + 3 = 0$ **79.** $s = 2.125\pi$ inches

80. $s = 14.59\pi$ millimeters

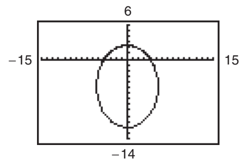
81. $A = 30.375\pi$ square centimeters

82. $A = 2.601\pi$ square feet

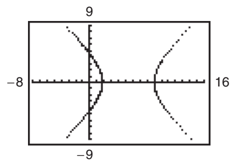
83. Parabola



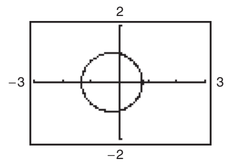
84. Ellipse



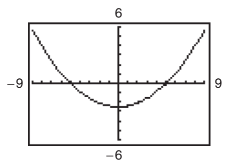
85. Hyperbola



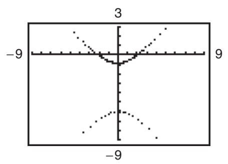
86. Ellipse



87. Parabola



88. Hyperbola



89. Orthogonal **90.** Neither

91. Parallel **92.** Neither

Section 12.3 *(page 880)*

Vocabulary Check *(page 880)*

1. Calculus **2.** tangent line **3.** secant line
4. limit of a difference quotient **5.** derivative

1. 0 **2.** -1 **3.** $\frac{1}{2}$ **4.** -2 **5.** 2 **6.** -2
7. -2 **8.** 2 **9.** -1 **10.** $-\frac{1}{4}$ **11.** $\frac{1}{6}$ **12.** $\frac{1}{6}$

13. $m = -2x$; (a) 0 (b) 2

14. $m = 3x^2$; (a) 3 (b) 12

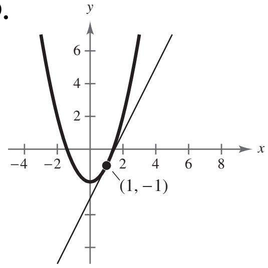
15. $m = -\dfrac{1}{(x+4)^2}$; (a) $-\dfrac{1}{16}$ (b) $-\dfrac{1}{4}$

16. $m = -\dfrac{1}{(x+2)^2}$; (a) $-\dfrac{1}{4}$ (b) -1

17. $m = \dfrac{1}{2\sqrt{x-1}}$; (a) $\dfrac{1}{4}$ (b) $\dfrac{1}{6}$

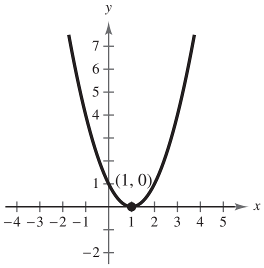
18. $m = \dfrac{1}{2\sqrt{x-4}}$; (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$

19.



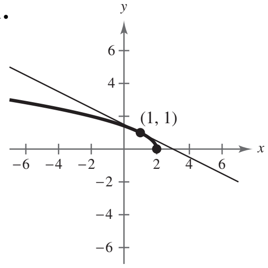
2

20.



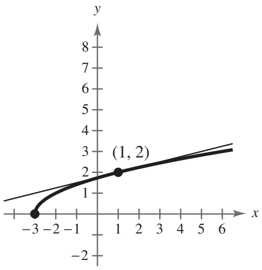
0

21.



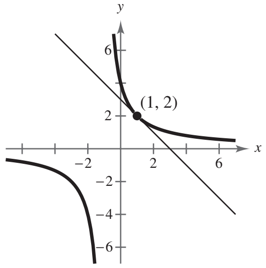
$-\frac{1}{2}$

22.



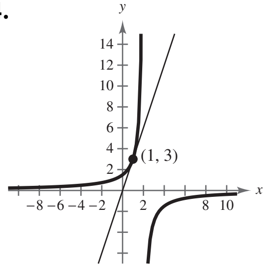
$\frac{1}{4}$

23.



-1

24.



3

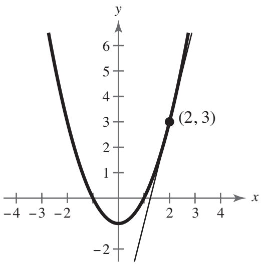
25. 0 **26.** 0 **27.** $-\frac{1}{3}$ **28.** -5

29. $-6x$ **30.** $2x - 3$ **31.** $-\dfrac{2}{x^3}$ **32.** $-\dfrac{3}{x^4}$

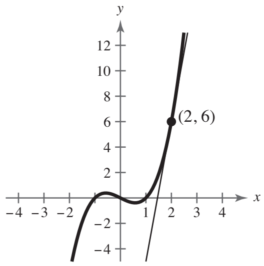
33. $-\dfrac{1}{2(x-9)^{3/2}}$

34. $-\dfrac{1}{2(s+1)^{3/2}}$

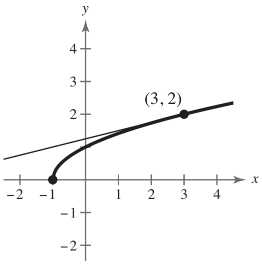
35. (a) 4
(b) $y = 4x - 5$
(c)



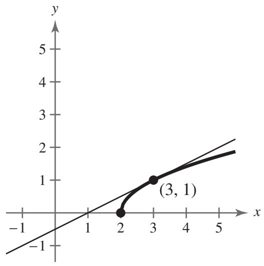
36. (a) 11
(b) $y = 11x - 16$
(c)



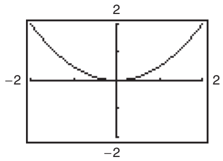
37. (a) $\frac{1}{4}$ (b) $y = \frac{1}{4}x + \frac{5}{4}$
(c)



38. (a) $\frac{1}{2}$ (b) $y = \frac{1}{2}x - \frac{1}{2}$
(c)



39.

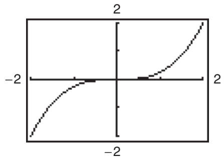


They appear to be the same.

x	-2	-1.5	-1	-0.5	0
$f(x)$	2	1.125	0.5	0.125	0
$f'(x)$	-2	-1.5	-1	-0.5	0

x	0.5	1	1.5	2
$f(x)$	0.125	0.5	1.125	2
$f'(x)$	0.5	1	1.5	2

40.



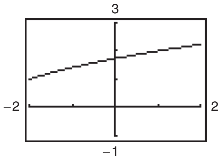
They appear to be the same.

x	-2	-1.5	-1	-0.5	0
$f(x)$	-2	-0.844	-0.25	-0.031	0
$f'(x)$	3	1.688	0.75	0.188	0

(Continued)

x	0.5	1	1.5	2
$f(x)$	0.031	0.25	0.844	2
$f'(x)$	0.188	0.75	1.688	3

41.

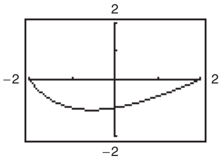


x	−2	−1.5	−1	−0.5	0
$f(x)$	1	1.225	1.414	1.581	1.732
$f'(x)$	0.5	0.408	0.354	0.316	0.289

x	0.5	1	1.5	2
$f(x)$	1.871	2	2.121	2.236
$f'(x)$	0.267	0.25	0.236	0.224

They appear to be the same.

42.



x	−2	−1.5	−1	−0.5	0
$f(x)$	0	−0.7	−1	−1.071	−1
$f'(x)$	−2	−0.92	−0.333	0.020	0.25

x	0.5	1	1.5	2
$f(x)$	−0.833	−0.6	−0.318	0
$f'(x)$	0.407	0.52	0.603	0.667

They appear to be the same.

43. $y = -x + 1$ **44.** $y = -2x$ **45.** $y = -6x \pm 8$

46. $y = -\frac{1}{2}x - \frac{1}{16}$

47. $f'(x) = 2x - 4$; horizontal tangent at $(2, -1)$

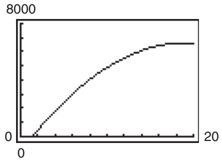
48. $f'(x) = 3x^2 + 3$; no horizontal tangents

49. $f'(x) = 9x^2 - 9$; horizontal tangents at $(-1, 6)$ and $(1, -6)$

50. $f'(x) = 12x^3 + 12x^2$; horizontal tangents at $(0, 0)$ and $(-1, -1)$

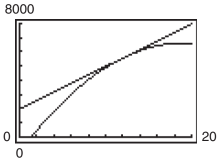
51. (a) $y = -21.048x^2 + 804.47x - 1054.5$

(b)



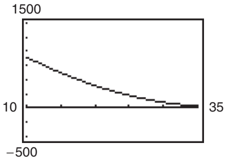
Slope when $x = 12$ is 299.31. This represents $299.31 million and the rate of change of revenue in 2002.

(c)



52. (a) $N(p) = 1.04p^2 - 81.5p + 1613$

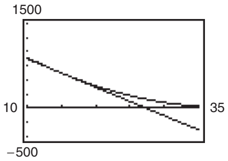
(b)

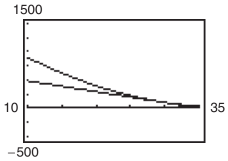


$p = \$15$: -50

$p = \$30$: -19

(c) $p = \$15$ $p = \$30$





The slopes $(-50.3$ and $-19.1)$ given by the graphing utility are close to the estimates $(-50$ and $-19)$.

(d) The rate of decrease in sales decreases as the price increases.

53. Answers will vary. **54.** Answers will vary.

55. True. The slope is dependent on x.

56. False. Tangent lines to noncircular graphs can intersect the graph at more than one point.

57. b **58.** a **59.** d **60.** c

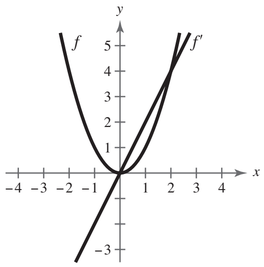
61. Answers will vary. Example: a sketch of any linear function with positive slope

62. Answers will vary. Example: a sketch of any linear function with negative slope

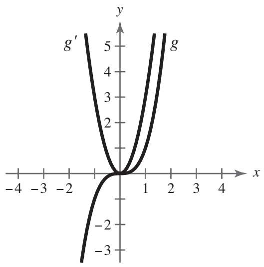
63. Answers will vary. Example: a sketch of any quadratic function of the form $y = a(x - 1)^2 + k$, where $a > 0$.

(Continued)

64. (a)

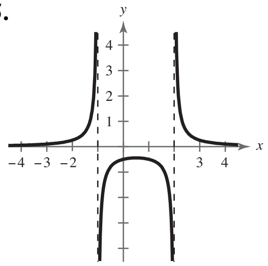


(b)

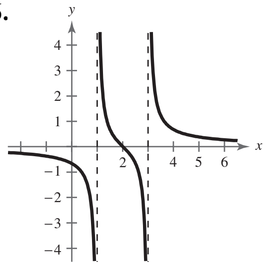


(c) Answers will vary.
$h'(x) = nx^{n-1}$

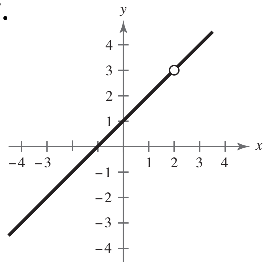
65.



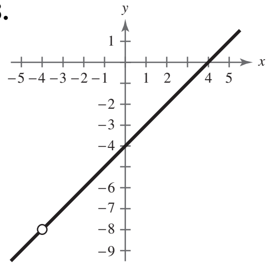
66.



67.



68.



69. $\langle -2, 3, -1 \rangle$

70. $\langle 0, 42, 0 \rangle$

71. $\langle 0, 0, -36 \rangle$

72. $\langle -140, -46, 57 \rangle$

Section 12.4 *(page 889)*

Vocabulary Check *(page 889)*

1. limit, infinity **2.** converge **3.** diverge

1. c **2.** a **3.** d **4.** b **5.** 0 **6.** 0

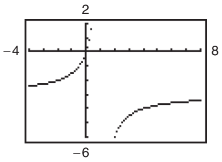
7. -1 **8.** -1.2 **9.** 2 **10.** -2 **11.** -3

12. 0.75 **13.** Does not exist **14.** Does not exist

15. $-\frac{3}{2}$ **16.** -0.25 **17.** -1 **18.** 2 **19.** -4

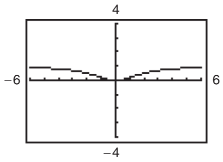
20. 9 **21.** -5 **22.** $\frac{7}{2}$

23.



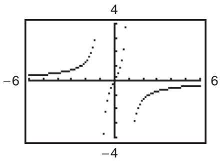
Horizontal asymptote: $y = -3$

24.



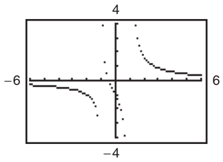
Horizontal asymptote: $y = 1$

25.



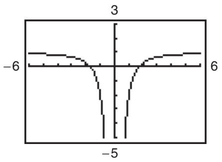
Horizontal asymptote: $y = 0$

26.



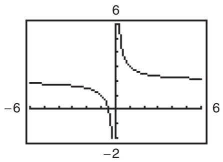
Horizontal asymptote: $y = 0$

27.



Horizontal asymptote: $y = 1$

28.



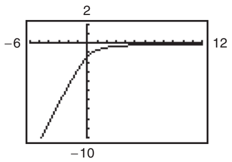
Horizontal asymptote: $y = 2$

29. (a)

x	10^0	10^1	10^2	10^3
$f(x)$	-0.7321	-0.0995	-0.0100	-0.0010

x	10^4	10^5	10^6
$f(x)$	-1.0×10^{-4}	-1.0×10^{-5}	-1.0×10^{-6}

(b)



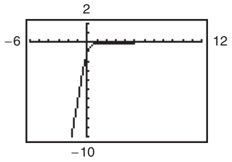
$\lim_{x\to\infty} f(x) = 0$

30. (a)

x	10^0	10^1	10^2	10^3
$f(x)$	-0.162	-0.0167	-0.00167	-1.67×10^{-4}

x	10^4	10^5	10^6
$f(x)$	-1.7×10^{-5}	-1.7×10^{-6}	-2×10^{-7}

(b)



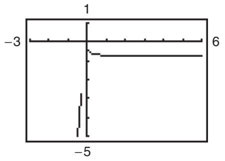
$\lim_{x\to\infty} f(x) = 0$

31. (a)

x	10^0	10^1	10^2	10^3
$f(x)$	-0.7082	-0.7454	-0.7495	-0.74995

x	10^4	10^5	10^6
$f(x)$	-0.749995	-0.7499995	-0.75

(b)

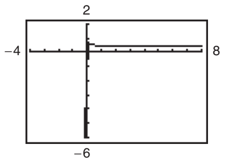


$\lim_{x\to\infty} f(x) = -\frac{3}{4}$

32. (a)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.508	0.5008	0.50008	0.5	0.5	0.5	0.5

(b)



$\lim_{x\to\infty} f(x) = \frac{1}{2}$

33. $1, \frac{3}{5}, \frac{2}{5}, \frac{5}{17}, \frac{3}{13}$

Limit: 0

34. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}$

Limit: 0

35. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$

Limit: $\frac{1}{2}$

36. $\frac{3}{4}, \frac{7}{5}, \frac{11}{6}, \frac{15}{7}, \frac{19}{8}$

Limit: 4

37. $\frac{1}{5}, \frac{1}{2}, \frac{9}{11}, \frac{8}{7}, \frac{25}{17}$

Limit does not exist.

38. $\frac{5}{2}, \frac{17}{4}, \frac{37}{6}, \frac{65}{8}, \frac{101}{10}$

Limit does not exist.

39. 2, 3, 4, 5, 6

Limit does not exist.

40. $\frac{1}{12}, \frac{1}{42}, \frac{1}{90}, \frac{1}{156}, \frac{1}{240}$

Limit: 0

(Continued)

41. $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$
Limit: 0

42. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}$
Limit: 0

43. $\lim_{n\to\infty} a_n = \frac{3}{2}$

n	10^0	10^1	10^2	10^3
a_n	2	1.55	1.505	1.5005

n	10^4	10^5	10^6
a_n	1.50005	1.500005	1.5000005

44. $\lim_{n\to\infty} a_n = 12$

n	10^0	10^1	10^2	10^3
a_n	20	12.8	12.08	12.008

n	10^4	10^5	10^6
a_n	12.0008	12.00008	12.000008

45. $\lim_{n\to\infty} a_n = \frac{16}{3}$

n	10^0	10^1	10^2	10^3
a_n	16	6.16	5.4136	5.3413

n	10^4	10^5	10^6
a_n	5.3341	5.33341	5.333341

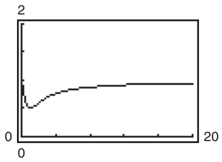
46. $\lim_{n\to\infty} a_n = \frac{3}{4}$

n	10^0	10^1	10^2	10^3
a_n	1	0.7975	0.754975	0.75049975

n	10^4	10^5	10^6
a_n	0.75005	0.750005	0.7500005

47. (a) 1

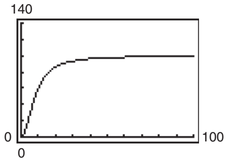
(b)



(c) Over a long period of time, the level of the oxygen in the pond is normal.

48. (a) 100

(b)



(c) A student's average typing speed will not exceed 100 words per minute.

49. (a) $\overline{C}(x) = \dfrac{13.50x + 45{,}750}{x}$

(b) \$471; \$59.25

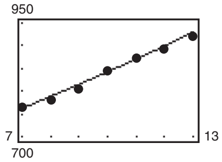
(c) \$13.50. As the number of PDAs gets very large, the average cost approaches \$13.50.

50. (a) $\overline{C}(x) = \dfrac{1.25x + 10{,}500}{x}$

(b) \$106.25, \$11.75

(c) \$1.25; As the number of tons of material gets very large, the average cost to recycle approaches \$1.25.

51. (a)

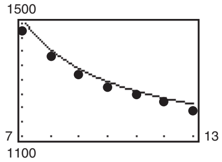


The model is a good fit to the data.

(b) 1032 dollars

(c) When t is slightly greater than 45, a vertical asymptote is found.

52. (a)



The model is a good fit to the data.

(b) 1174 thousand

(c) 1048.78 thousand. The least number of military reserves available will not go below 1048.78 thousand. Answers will vary.

53. False. Graph $y = \dfrac{x^2}{x+1}$.

54. False. The limit does not exist.

55. True. If the sequence converges, then the limit exists.

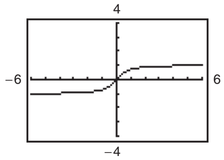
56. False. The limit is the ratio of the coefficients of the highest-powered terms.

57. Let $f(x) = \dfrac{1}{x^2}$, $g(x) = \dfrac{1}{x^2}$, and $c = 0$. Now

$\lim_{x\to 0} \dfrac{1}{x^2} = \infty$, and $\lim_{x\to 0}[f(x) - g(x)] = 0$.

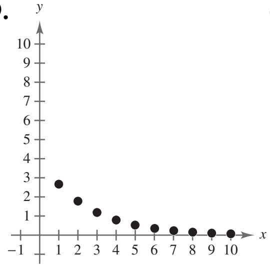
(Continued)

58.



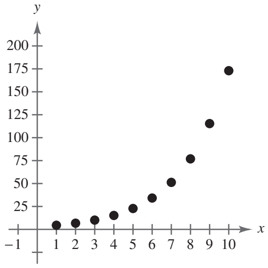
Two horizontal asymptotes: $y = -1$ and $y = 1$

59.



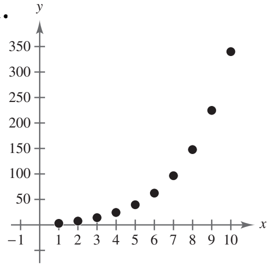
Converges to 0

60.



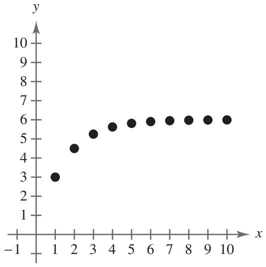
Diverges

61.



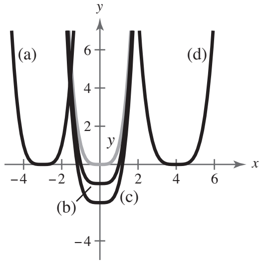
Diverges

62.

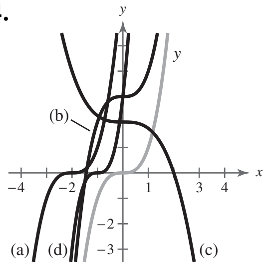


Converges to 6

63.



64.



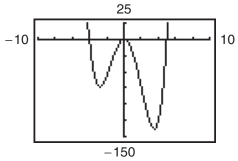
65. $x^2 + 2x + 1$

66. $2x^3 + 4x^2 - 2x - 8 + \dfrac{-10x + 7}{x^2 - 2x + 1}$

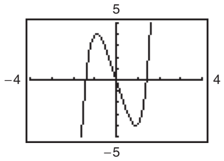
67. $x^3 + 5x^2 - 3 - \dfrac{2}{3x + 2}$

68. $2x^2 + 11x + 14$

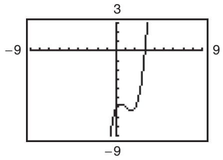
69. $-4, 5, 0, 0$



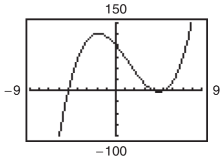
70. $0, \pm\sqrt{2}$



71. 3



72. $4, \pm 5$



73. 60 **74.** 150 **75.** 150 **76.** 5.87909

Section 12.5 *(page 898)*

Vocabulary Check *(page 898)*

1. $\frac{n(n+1)}{2}$ **2.** $\frac{n^2(n+1)^2}{4}$ **3.** area

1. 420 **2.** 9455 **3.** 44,140
4. 2600 **5.** 5850 **6.** 1870

7. (a) $S(n) = \frac{n^2 + 2n + 1}{4n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	1	0.3025	0.25503	0.25050	0.25005

(c) Limit: $\frac{1}{4}$

8. (a) $S(n) = \frac{n + 1}{2n}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	1	0.55	0.505	0.5005	0.50005

(c) Limit: $\frac{1}{2}$

9. (a) $S(n) = \frac{2n^2 + 3n + 7}{2n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	6	1.185	1.0154	1.0015	1.00015

(c) Limit: 1

10. (a) $S(n) = \frac{n + 4}{n}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	5	1.4	1.04	1.004	1.0004

(c) Limit: 1

11. (a) $S(n) = \frac{14n^2 + 3n + 1}{6n^3}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	3	0.2385	0.02338	0.002334	0.000233

(c) Limit: 0

12. (a) $S(n) = \frac{2n - 1}{n}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	1	1.9	1.99	1.999	1.9999

(c) Limit: 2

13. (a) $S(n) = \frac{4n^2 - 3n - 1}{6n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	0	0.615	0.66165	0.666167	0.666617

(c) Limit: $\frac{2}{3}$

14. (a) $S(n) = \frac{16n^2 + 18n + 2}{3n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	12	5.94	5.3934	5.33933	5.33393

(c) Limit: $\frac{16}{3}$

15. 14.25 square units **16.** 3.25 square units

17. 1.2656 square units **18.** 3.125 square units

19.

n	4	8	20	50
Approximate area	18	21	22.8	23.52

20.

n	4	8	20	50
Approximate area	14.344	16.242	17.314	17.728

21.

n	4	8	20	50
Approximate area	3.52	2.85	2.48	2.34

22.

n	4	8	20	50
Approximate area	7.113	7.614	7.8895	7.994

23. 3 square units **24.** 10 square units

25. 2 square units **26.** $\frac{39}{2}$ square units

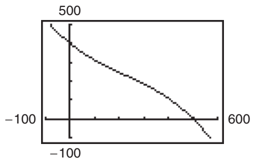
27. $\frac{10}{3}$ square units **28.** $\frac{7}{3}$ square units

29. $\frac{17}{4}$ square units **30.** $\frac{513}{4}$ square units

31. $\frac{3}{4}$ square unit **32.** 4 square units

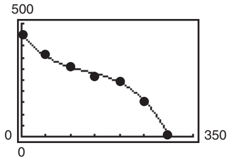
33. $\frac{51}{4}$ square units **34.** $\frac{2}{3}$ square unit

35.



Area is 105,208.33 square feet ≈ 2.4153 acres

(Continued)

36. (a)
$y = (-4.088889 \times 10^{-5})x^3 + 0.0162x^2 - 2.67x + 453$
(b)


(c) 78,750 square feet

37. True

38. False. The exact area of a region is given by the limit of the sum of n rectangles as n approaches infinity.

39. Answers will vary.

40. c

41. $n\pi, \frac{\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$

42. $\frac{\pi}{2} + n\pi, \frac{5\pi}{4} + 2n\pi, \frac{7\pi}{4} + 2n\pi$

43. $n\pi$

44. $\frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi$

45. $\frac{\pi}{2} + n\pi$

46. $\frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

47. $\langle 24, -30 \rangle$

48. 18

49. $\sqrt{5} - 2$

50. 36

Review Exercises *(page 901)*

1.

x	2.9	2.99	2.999	3
$f(x)$	16.4	16.94	16.994	17

x	3.001	3.01	3.1
$f(x)$	17.006	17.06	17.6

$\lim_{x\to 3} (6x - 1) = 17$; the limit can be reached.

2.

x	1.9	1.99	1.999	2
$f(x)$	0.1299	0.1255	0.1250	?

x	2.001	2.01	2.1
$f(x)$	0.1250	0.1245	0.1205

$\lim_{x\to 2} \frac{x - 2}{3x^2 - 4x - 4} = \frac{1}{8}$; the limit cannot be reached.

3. 2 **4.** Limit does not exist. **5.** 2 **6.** 3

7. (a) 64 (b) 7 (c) 20 (d) $\frac{4}{5}$

8. (a) 3 (b) $\frac{3}{2}$ (c) 324 (d) 3

9. 5 **10.** $\sqrt{6}$ **11.** $\frac{3}{10}$ **12.** 7 **13.** 0

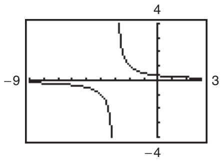
14. 0 **15.** 11 **16.** 5 **17.** 77 **18.** -91

19. $\frac{10}{3}$ **20.** $\frac{11}{7}$ **21.** $-\frac{1}{4}$ **22.** 6 **23.** $\frac{1}{15}$

24. $-\frac{1}{7}$ **25.** $-\frac{1}{3}$ **26.** 6 **27.** -1 **28.** -1

29. $\frac{1}{4}$ **30.** $\frac{1}{6}$ **31.** $\frac{1}{4}$ **32.** $\frac{\sqrt{3}}{6}$

33. (a)



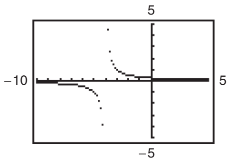
$\frac{1}{6}$

(b)

x	2.9	2.99	3	3.01	3.1
$f(x)$	0.1695	0.1669	Error	0.1664	0.1639

0.166

34. (a)



0.1250

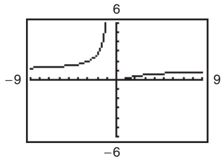
(b)

x	3.9	3.99	3.999	4
$f(x)$	0.12658	0.12516	0.12502	Error

x	4.001	4.01	4.1
$f(x)$	0.12498	0.12484	0.12346

0.1250

35. (a)



Limit does not exist.

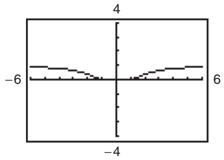
(b)

x	−0.1	−0.01	−0.001	0
$f(x)$	4.85E8	7.2E86	Error	Error

x	0.001	0.01	0.1
$f(x)$	0	1E−87	2.1E−9

Limit does not exist.

36. (a)



0

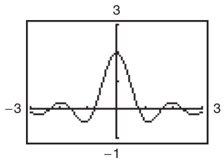
(b)

x	−0.1	−0.01	−0.001	0
$f(x)$	0	0	0	Error

x	0.001	0.01	0.1
$f(x)$	0	0	0

0

37. (a)



2

(b)

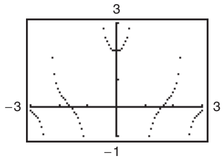
x	−0.1	−0.01	−0.001	0
$f(x)$	1.94709	1.99947	2	Error

x	0.001	0.01	0.1
$f(x)$	2	1.99947	1.94709

2

(Continued)

38. (a)



2

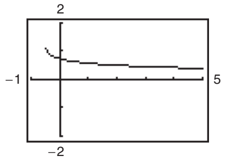
(b)

x	-0.1	-0.01	-0.001	0
$f(x)$	2.02710	2.00027	2.000003	Error

x	0.001	0.01	0.1
$f(x)$	2.000003	2.00027	2.02710

2

39. (a)



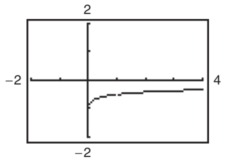
≈ 0.575

(b)

x	1.1	1.01	1.001	1.0001
$f(x)$	0.5680	0.5764	0.5773	0.5773

$\approx 0.577 \left(\text{Actual limit is } \dfrac{\sqrt{3}}{3}.\right)$

40. (a)



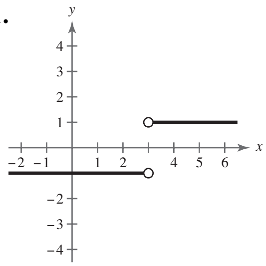
$-\frac{1}{2}$

(b)

x	1.1	1.01	1.001	1.0001
$f(x)$	-0.4881	-0.4988	-0.4999	-0.5000

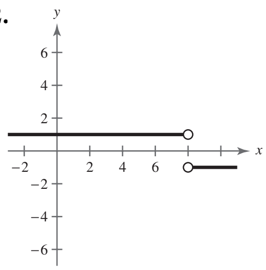
$-\frac{1}{2}$

41.



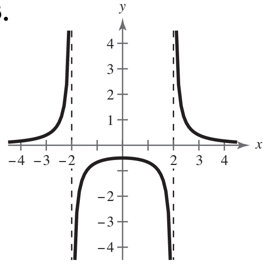
Limit does not exist.

42.



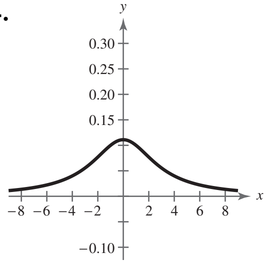
Limit does not exist.

43.



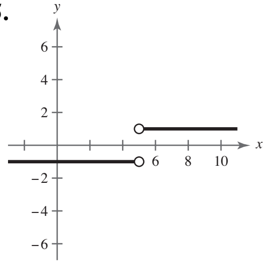
Limit does not exist.

44.



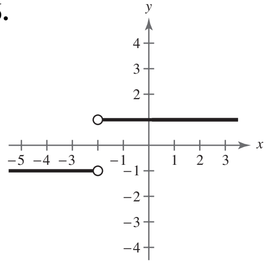
$\frac{1}{18}$

45.



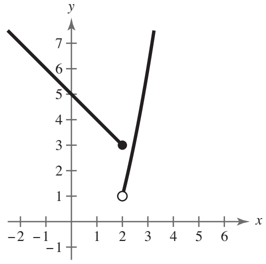
Limit does not exist.

46.



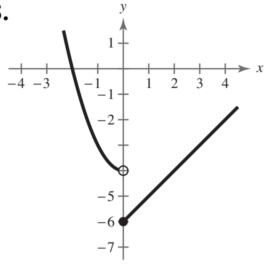
Limit does not exist.

47.



Limit does not exist.

48.

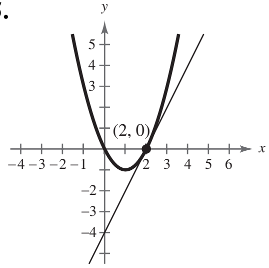


Limit does not exist.

49. 4 **50.** -2 **51.** $3 - 2x$

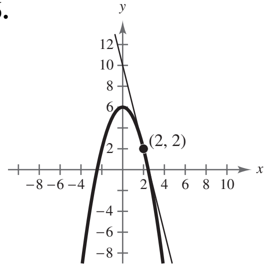
52. $2x - 5$ **53.** 2 **54.** 0

55.



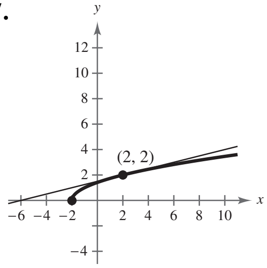
2

56.



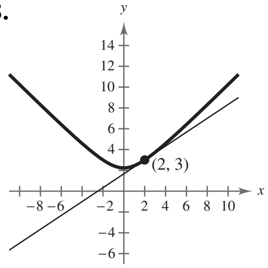
-4

57.



$\frac{1}{4}$

58.



$\frac{2}{3}$

(Continued)

59. $m = 2x - 4$; (a) -4 (b) 6

60. $m = x^3$; (a) -8 (b) 1

61. $m = -\dfrac{4}{(x-6)^2}$; (a) -4 (b) -1

62. $m = \dfrac{1}{2\sqrt{x}}$; (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

63. $f'(x) = 0$ **64.** $g'(x) = 0$ **65.** $h'(x) = -\frac{1}{2}$

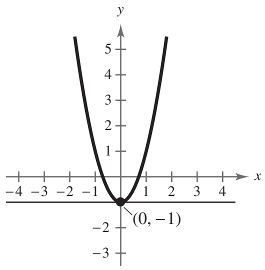
66. $f'(x) = 3$ **67.** $g'(x) = 4x$ **68.** $f'(x) = -3x^2 + 4$

69. $f'(t) = \dfrac{1}{2\sqrt{t+5}}$ **70.** $f'(x) = \dfrac{1}{2(12-x)^{3/2}}$

71. $g'(s) = -\dfrac{4}{(s+5)^2}$ **72.** $g'(t) = \dfrac{6}{(5-t)^2}$

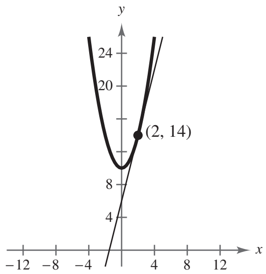
73. (a) 0 (b) $y = -1$

(c)



74. (a) 4 (b) $y = 4x + 6$

(c)



75. 2 **76.** $\frac{1}{2}$

77. 0 **78.** 0

79. Limit does not exist. **80.** Limit does not exist.

81. 3 **82.** 0

83. $\frac{2}{3}, 1, \frac{8}{7}, \frac{11}{9}, \frac{14}{11}$
Limit: $\frac{3}{2}$

84. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}$
Limit: 0

85. $-\frac{1}{2}, -\frac{9}{8}, -\frac{7}{6}, -\frac{37}{32}, -\frac{57}{50}$
Limit: -1

86. $-2, 1, 2, \frac{5}{2}, \frac{14}{5}$
Limit: 4

87. (a) $S(n) = \dfrac{(n+1)(5n+4)}{6n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	3	0.99	0.8484	0.8348	0.8335

(c) Limit: $\frac{5}{6}$

88. (a) $S(n) = -\dfrac{3(n+1)(n+9)}{4n^2}$

(b)

n	10^0	10^1	10^2	10^3	10^4
$S(n)$	-15	-1.5675	-0.8257	-0.7575	-0.7508

(c) Limit: $-\frac{3}{4}$

89. $\frac{27}{4} \approx 6.75$ square units

90. $\frac{113}{32} \approx 3.5313$ square units

91.

n	4	8	20	50
Approximate area	7.5	6.375	5.74	5.4944

92.

n	4	8	20	50
Approximate area	10	10.5	10.64	10.6624

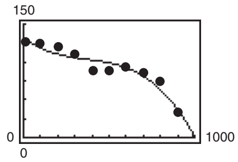
93. 50 square units **94.** 9 square units

95. 15 square units **96.** $\frac{4}{3}$ square units

97. $\frac{4}{3}$ square units **98.** $\frac{32}{3}$ square units

99. (a) $y = (-3.376068 \times 10^{-7})x^3 + (3.7529 \times 10^{-4})x^2 - 0.17x + 132$

(b)



(c) $\approx 87{,}695.0$ square feet (Answers will vary.)

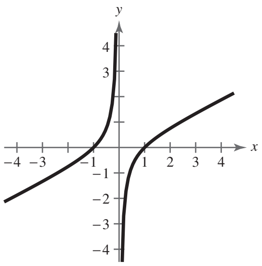
100. True. $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$, if both limits exist.

101. False. The limit of the rational function as x approaches ∞ does not exist.

102. Answers will vary.

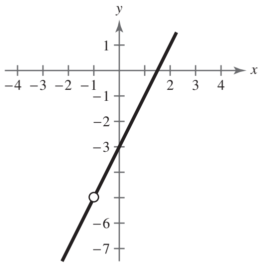
Chapter Test (page 904)

1.



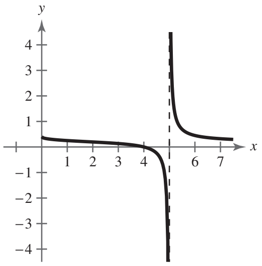
$$\lim_{x \to -2} \frac{x^2 - 1}{2x} = -\frac{3}{4}$$

2.



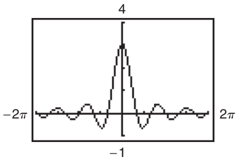
$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1} = -5$$

3.



The limit does not exist.

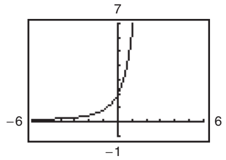
4.



$$\lim_{x \to 0} \frac{\sin 3x}{x} \approx 3$$

x	-0.02	-0.01	0	0.01	0.02
$f(x)$	2.9982	2.9996	Error	2.9996	2.9982

5.



$$\lim_{x \to 0} \frac{e^{2x} - 1}{x} \approx 2$$

x	-0.004	-0.003	-0.002	-0.001	0
$f(x)$	1.9920	1.9940	1.9960	1.9980	Error

6. (a) $m = 6x - 5; 7$ (b) $m = 6x^2 + 6; 12$

7. $f'(x) = -\frac{3}{4}$

8. $f'(x) = 4x + 4$

9. $f'(x) = -\dfrac{1}{(x + 3)^2}$

10. 0

11. -3

12. Does not exist

13. $0, \frac{3}{4}, \frac{14}{19}, \frac{12}{17}, \frac{36}{53}$

Limit: $\frac{1}{2}$

14. $0, 1, 0, \frac{1}{2}, 0$

Limit: 0

15. $\frac{25}{2}$ square units

16. 8 square units

17. $\frac{3}{4}$ square unit

18. (a) $y = 8.79x^2 - 6.2x - 0.4$

(b) 81.7 feet per second

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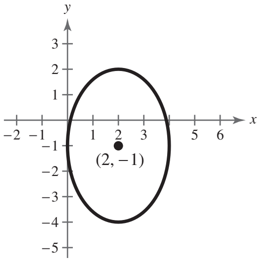
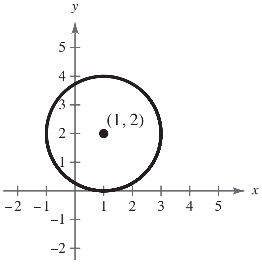
Cumulative Test for Chapters 10–12 *(page 905)*

1. Ellipse

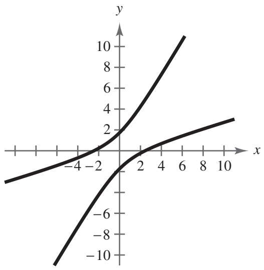


2. Circle

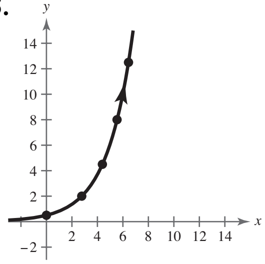


3. $\dfrac{x^2}{1} + \dfrac{(y-2)^2}{4} = 1$

4. $\theta = 37.98°$

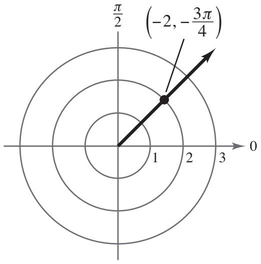


5.



The corresponding rectangular equation is $y = \sqrt{e^x}/2$.

6.

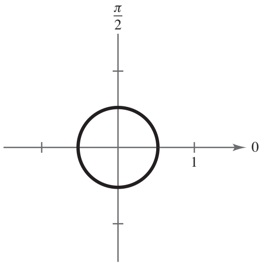


$\left(-2, \dfrac{5\pi}{4}\right), \left(2, -\dfrac{7\pi}{4}\right), \left(2, \dfrac{\pi}{4}\right)$

7. $-8r\cos\theta - 3r\sin\theta + 5 = 0$ or $r = \dfrac{5}{8\cos\theta + 3\sin\theta}$

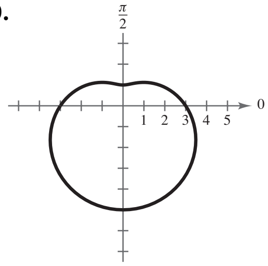
8. $9x^2 + 20x - 16y^2 + 4 = 0$

9.



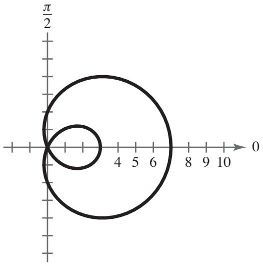
Circle

10.



Dimpled limaçon

11.



Limaçon with an open loop

12. $(-6, 1, 3)$ **13.** $(0, -4, 0)$ **14.** $\sqrt{149}$

15. 3, 4, 5

$3^2 + 4^2 \stackrel{?}{=} 5^2$

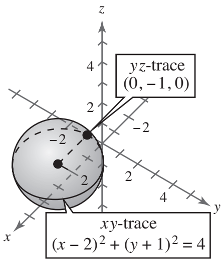
$9 + 16 \stackrel{?}{=} 25$

$25 = 25$

16. $\left(-1, 2, \frac{1}{2}\right)$

17. $(x-2)^2 + (y-2)^2 + (z-4)^2 = 24$

18.



19. $\mathbf{u} \cdot \mathbf{v} = -38$

$\mathbf{u} \times \mathbf{v} = \langle -18, -6, -14 \rangle$

20. Neither **21.** Orthogonal

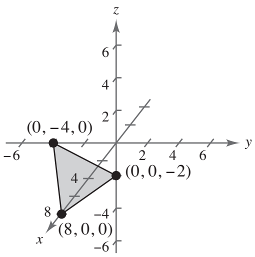
22. Parallel **23.** 12 cubic units

24. (a) $x = -2 + 7t, y = 3 + 5t, z = 25t$

(b) $\dfrac{x+2}{7} = \dfrac{y-3}{5} = \dfrac{z}{25}$

25. $75x + 50y - 31z = 0$

26.



27. $\dfrac{\sqrt{30}}{2} \approx 2.74$

28. 84.26° **29.** 4 **30.** $-\frac{1}{3}$ **31.** $\frac{1}{14}$

32. $\frac{1}{4}$ **33.** -1 **34.** Limit does not exist.

35. $m = -2x; -2$ **36.** $m = \frac{1}{2}(x+3)^{-1/2}; \frac{1}{2}$

37. $m = -(x+3)^{-2}; -\frac{1}{16}$ **38.** $m = 4x^3; -4$

39. Limit does not exist. **40.** -7 **41.** 3

42. 0 **43.** $-42{,}875$ **44.** 8190 **45.** 672,880

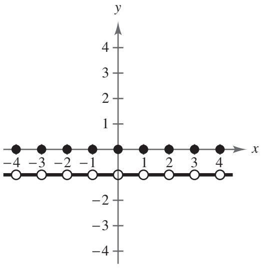
46. $A = 10.5$ square units **47.** $A \approx 1.566$ square units

48. $\frac{3}{4}$ square unit **49.** 18 square units **50.** $\frac{2}{3}$ square unit

P.S. Problem Solving *(page 908)*

1. (a) g_1, g_4 (b) g_1, g_3, g_4 (c) g_1, g_4

2.



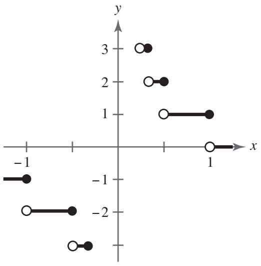
The graph jumps at every integer.

(a) $f(1) = 0, f(0) = 0, f\left(\frac{1}{2}\right) = -1, f(-2.7) = -1$

(b) $\lim_{x\to 1^-} f(x) = -1,\ \lim_{x\to 1^+} f(x) = -1,$

$\lim_{x\to 1/2} f(x) = -1$

3.



(a) $f\left(\frac{1}{4}\right) = \llbracket 4 \rrbracket = 4$ (b) $\lim_{x\to 1^-} f(x) = 1$

$f(3) = \llbracket \frac{1}{3} \rrbracket = 0$ $\lim_{x\to 1^+} f(x) = 0$

$f(1) = \llbracket 1 \rrbracket = 1$ $\lim_{x\to (1/2)^-} f(x) = 2$

$\lim_{x\to (1/2)^+} f(x) = 1$

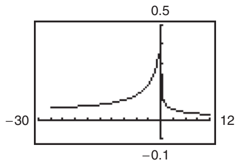
4. (a) $\frac{4}{3}$ (b) $y = -\frac{3}{4}x + \frac{25}{4}$ (c) $m_x = \frac{\sqrt{25 - x^2} - 4}{x - 3}$

(d) $-\frac{3}{4}$; the slopes are the same.

5. $a = 3, b = 6$

6. (a) Domain: $x \geq -27,\ x \neq 1$

(b)

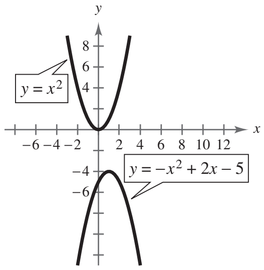


(c) ≈ 0.0714 (d) $\frac{1}{12}$

7. $\lim_{x\to 0} f(x)$ does not exist. No matter how close x is to 0, there are still an infinite number of rational and irrational numbers, so $\lim_{x\to 0} f(x)$ does not exist.

$\lim_{x\to 0} g(x) = 0$. When x is close to 0, both parts of the function are close to 0.

8.



Tangent lines: $y = -2x - 1$ and $y = 4x - 4$

9. $y = 1 + 3\sqrt{x}$

10. (a) Tangent line: $y = 4x - 4$

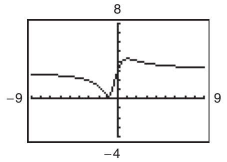
(b) Normal line: $y = -\frac{1}{4}x + \frac{9}{2}$

Intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$; normal line: $x = 0$

11. (a) $d(m) = \frac{|-3m - 3|}{\sqrt{m^2 + 1}}$

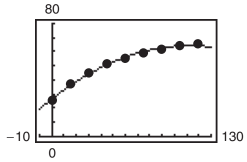
(b)



(c) $\lim_{x\to\infty} d(m) = 3,\ \lim_{x\to -\infty} d(m) = 3$. This indicates that the distance between the point and the line approaches 3 as the slope approaches positive or negative infinity.

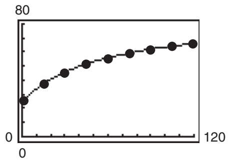
12. (a) $T_1 = -0.00303t^2 + 0.677t + 26.56$

(b)



The function is a good fit to the data.

(c)



(d) $T_1(0) = 26.56, T_2(0) = 25.017$ (e) $\lim_{x\to\infty} T_2 = 86$

(e) The furnace reaches a maximum temperature of 86°C. No, this type of analysis is not possible using T_1 because as t gets larger, T_1 approaches negative infinity.

13. The error was probably due to the calculator being in degree mode rather than radian mode.

(Continued)

14. (a)

Perimeter $\triangle PAO = 1 + \sqrt{(x^2 - 1)^2 + x^2} + \sqrt{x^4 + x^2}$

Perimeter $\triangle PBO = 1 + \sqrt{x^4 + (x - 1)^2} + \sqrt{x^4 + x^2}$

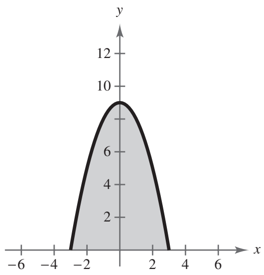
(b)

x	4	2	1
Perimeter $\triangle PAO$	33.0166	9.0777	3.4142
Perimeter $\triangle PBO$	33.7712	9.5952	3.4142
$r(x)$	0.9777	0.9461	1.0000

x	0.1	0.01
Perimeter $\triangle PAO$	2.0955	2.0100
Perimeter $\triangle PBO$	2.0006	2.0000
$r(x)$	1.0474	1.005

(c) 1

15. (a)



(b) $A = 36$

(c) Base $= 6$,

height $= 9$;

Area $= \frac{2}{3}bh = 36$