Precautionary saving and un-anchored expectations

Grimaud, Alex

TU Wien

5 July 2021

Precautionary saving and un-anchored expectations[*]

Alex GRIMAUD[†]

2021-07-05

Abstract

This paper revisits monetary policy in a heterogeneous agents new Keynesian (HANK) model where agents use adaptive learning (AL) in order to form their expectations. Due to the households' finite heterogeneity triggered by idiosyncratic unemployment risk, the model is subject to micro-founded heterogeneous expectations that are not anchored to the rational expectation path. Households experience different histories which has non-trivial consequences on their individual AL processes. In this model, supply shocks generate precautionary saving and possible long-lasting disinflationary traps associated with excess saving. Dovish policies focused on closing the output gap dampen the learning effects which is in contradiction with previously established representative agent under learning results. Price level targeting appears to resolve most of the problem by better anchoring long run expectations of future utility flows.

Keywords: Adaptive learning, supply shocks, precautionary saving, heterogeneous expectations, HANK and price level targeting.

***JEL codes***: E25, E31, E52 and E70.

1   Introduction

The very large supply shock generated by the Covid-19 pandemic and its containment measures have revitalized the debate around their impact on inflation and consumption decisions (see *e.g.* Blanchard 2020, Goodhart & Pradhan 2020).[1] The data show an increase in saving and a drop in consumption (see Figure 1). As pointed out by Guerrieri, Lorenzoni, Straub & Werning (2020), negative supply shocks' dynamics in simple representative agent New Keynesian (RANK) models are at odds with empirical data and the insights of Keynes (1936). In those models, adverse supply shocks generate inflationary responses thanks to positive prices adjustments due to the low supply not being able to match the steady demand; and the increase in marginal cost. In order to smooth its consumption, the representative household is reducing its saving.

In this paper, I show that in an imperfect unemployment insurance economy under adaptive learning, negative supply shocks while being inflationary at first trigger long-lasting disinflationary periods characterized by excess saving and low interest rate. Even if a more aggressive response from monetary policy to output and a more *dovish* stance on inflation appear to neutralize the excess volatility generated by the unanchored expectations *w.r.t* rational expectations, only price level targeting (PLT) enables the model to converge back quickly to the steady state of inflation and consumption.

I develop a model based on the truncated histories of heterogeneous households in line with Challe & Ragot (2016), Challe, Matheron, Ragot & Rubio-

---

[1]In this paper, supply shocks are defined as productivity shocks.
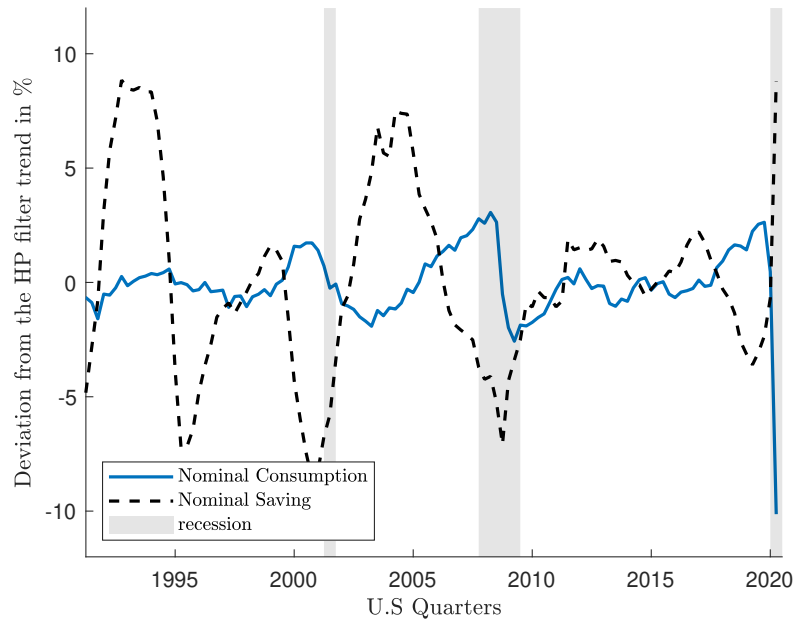
Figure 1: Nominal consumption and saving dynamic in the US (in deviation from the HP filtered trend)

Ramirez (2017), Ragot (2018) and Le Grand & Ragot (2020). This extension is a simple discrete time heterogeneous agents New Keynesian (HANK) model with sticky prices and uninsured idiosyncratic unemployment risk. First, it is possible to derive the steady-state distribution of wealth based on the recursive structure of the households' first order conditions (FOC) in the absence of aggregate shocks by following their idiosyncratic histories given an initial endowment. Then, it is possible to obtain a finite partition of households by truncating their idiosyncratic histories. Finally their FOC conditions are projected and the linearised model can be solve with aggregate shocks. The uninsured unemployment risk creates a precautionary motive for saving.

2

In this context, the novelty of this paper is to introduce adaptive learning in a simple discrete time HANK model with a finite households partition in sequential competitive equilibrium. Heterogeneity in wealth holdings and individual histories has important effects on the expectations of households and per consequence on macroeconomic dynamics. Indeed, aggregate shocks have asymmetric effects on households' decisions but also on their learning process. For instance, an $i.i.d$ shock on the real rate would effect more the consumption decision of an agent with a large holding of asset than an agent with no asset. Nonetheless, the expectations of richer agents with high consumption and asset holding have a larger feedback toward the aggregate economy. Therefore, wealthy households not anchored to the rational expectation solution might revise their forecast strategy based on this new experience in a different way than poorer households with little to no impact on the aggregate economy. In this model, the asymmetric effect of shocks generates asymmetric responses by the learning dynamics driving the expectations. Poorer households tend to easily loose their anchorage to the rational expectations solution.

The adaptive learning expectations formation process used is the standard recursive least square (RLS) learning formalized by Marcet & Sargent (1989) and Evans & Honkapohja (2001). Agents are assumed to forecast *as well as good econometricians.* In this model, agents hold beliefs about the economy. Those beliefs are consistent in their form with the rational expectations solution - *called the Minimum State Variables (MSV) solution form* - but not necessarily with the value of the rational expectations solution. Agents revise their beliefs about the MSV solution by minimizing the square forecast error of

their forecasting strategy. In this context, introducing AL in a sequential competitive equilibrium model relates to the seminal work of Grandmont (1977) on temporary equilibrium where subjective heterogeneous agents' expectations map into a general equilibrium economy.

The first result of this paper is that learning properties are strengthened in the HANK set-up in comparison with RANK models. Assuming endogeneity between unemployment risk and productivity, the propagations of supply shocks are increased by the learning. Facing negative supply shock, precautionary saving is enlarged $w.r.t$ the rational expectations benchmark which triggers deflationary pressure through the demand channel but also the marginal cost channel due to the excess capital supply. The model eventually converges back to its steady state after a long disinflationary period.

Exploring the monetary policy options, it appears that a stronger stance on the output and lower one on inflation in the reaction function neutralizes the excess volatility with respect to rational expectations HANK benchmarks. This result is due to the monetary policy decreasing the magnitude of the income risk generated by the uninsured unemployment risk per consequence the precautionary motive of saving. By decreasing the volatility of output, the more aggressive monetary policy relative to the output gap generates smaller forecast errors for individual consumptions. It also better anchors individual consumption forecasts to the rational expectations HANK solution thanks to smoother consumption patterns. This results in a HANK set-up contradict previous ones in a RANK model under adaptive learning by the literature where inflation stabilization ought to be the main priority of monetary policy

4

in order to achieve stable dynamics (see, Orphanides & Williams 2008, Ascari, Florio & Gobbi 2017, Eusepi & Preston 2018). Yet, they are consistent with previous results established in HANK under rational expectations which emphasis on output stabilization (Kaplan, Moll & Violante 2018).

Nonetheless, excess saving periods following supply shocks cannot be prevented by a more aggressive monetary policy *w.r.t* output gap. A counterfactual policy scenario where the central bank would target price level and not inflation appears to solve this issue by anchoring better expected future discounted utility flows. Indeed, contrary to inflation targeting (IT), PLT targets inflation rate on average and enables agents to expect a smoother future income, and a smaller recession, thus avoids large amount of precautionary saving by letting an overshoot in inflation after a disinflation period.

Related literature. Following the development of the Euler based consumption equation by Hall (1978) and its generalization within the NK framework (see, e.g., Woodford 2003, Galí 2015), the main channel of monetary policy has been the inter-temporal substitution effect based on rational expectations of discounted future consumption by the real interest rate. The higher real interest rate is, the higher the positive impact of current saving on future discounted consumption will be. Thus, an increase in the real rate will increase saving rate for the optimizing household; therefore decreases consumption, expected consumption and labour supply which have a positive effect on marginal cost. Lower demand and marginal cost decrease inflation which furthermore

increases the expected real rate.[2]

Against this backdrop, there has been increasing evidence of heterogeneity in households' responses to shock, uncertainty and economic conditions (see e.g., Berger & Vavra 2015, Kaplan & Violante 2018, Crawley & Kuchler 2018). In order to answer to those new data, the literature has been developing heterogeneous agents then HANK models (see e.g., Krusell & Smith 1998, Challe et al. 2017, Kaplan et al. 2018, Ahn, Kaplan, Moll, Winberry & Wolf 2018, Bilbiie 2019). In those models, households are subject to idiosyncratic heterogeneity in their quantity and quality of assets holding, risk or preference for instance. Some of those models acknowledge the existence of idiosyncratic shocks while other rely purely on heterogeneity on the initial endowment. As observed empirically, households with few or no financial assets appear not to respond to change in the real interest rate. Moreover, the permanent income hypothesis generates asymmetric responses to change in income when long-run heterogeneity in income and wealth holdings is considered. In those models, the role of expected income in monetary policy transition is enhanced.

The weakening of the inter-temporal substitution effects and the increasing one of the expected income effect enhance the role of future utility stream expectations in the consumption decisions, especially in the presence of idiosyncratic risks. Yet, current HANK models rely exclusively on the rational expectations hypothesis for their solution methods. Assuming non-rational expectations appears yet to be an unexplored hypothesis in this literature. Moreover, there exists extensive survey data (see e.g., Carroll 2003, Branch

---

[2]In the appendix, Figure 12b and Figure 12a display this phenomenon under rational expectation and adaptive learning.

2004b, Del Negro & Eusepi 2011, Malmendier & Nagel 2016) and laboratory evidences (see, Hommes 2011, 2020, Assenza, Heemeijer, Hommes & Massaro 2019) of heterogeneous expectations that are non consistent with the representative agent rational expectations hypothesis. Against those facts, it exists an important literature based on the hypothesis that economic agents do not know the rational expectations solution but learn to forecast in the most accurate manner based on past data (see *e.g*, Evans & Honkapohja 2001, 2003b, Woodford 2013). This hypothesis has non trivial impact on optimal monetary policy design (see *e.g*, Orphanides & Williams 2008, Williams 2010, Ascari et al. 2017, Eusepi & Preston 2018, Honkapohja & Mitra 2019). To the best of my knowledge, New Keynesian models under adaptive learning have always been implemented using a representative agent consumption framework (see, e.g., Evans & Honkapohja 2003a, Eusepi & Preston 2011, Slobodyan & Wouters 2012).

Nonetheless, it is necessary to acknowledge Evans & Ramey (1992) for introducing heterogeneous firms and updating cost, Gobbi & Grazzini (2019) developing an agent based approach but no wealth heterogeneity and Honkapohja & Mitra (2006) and Radke & Wicknig (2020) for using overlapping generation setups which can be envisioned as heterogeneous agents New Keynesian model under adaptive learning. Finally, Giusto (2014) proves that the Krusell & Smith (1998) equilibrium is learnable un RLS.

It is also necessary to point out that this paper differs from previous heterogeneous expectations models such as Branch & McGough (2010), Massaro (2013) and Arifovic, Bullard & Kostyshyna (2013). Those models are based on

7

an optimizing representative agent hypothesis where the heterogeneous expectations are combined in an aggregated representative expectation whereas this paper is based on heterogeneous New Keynesian agents with heterogeneous individual expectations.

Finally this paper relates to the literature on the optimal policy with regards to supply shocks. Ravn & Sterk (2020), Challe (2020) and Den Haan, Rendahl & Riegler (2018) investigate the optimal monetary policy in a rational expectations framework similar to this one, with a richer search and matching labour market, where supply shocks lead to precautionary saving. Guerrieri et al. (2020) entangle the disinflationary effect of large negative supply shock in a multi-sectors New Keynesian economy. Those papers emphasize the role of monetary easing in responses to those shocks.

The paper proceeds as follows: In Section 2, I develop the HANK model; the solution methods are presented in Section 3; the dynamic properties of the model are analysed in Section 4; Section 5 discusses the effects of different policy exercises; and Section 6 concludes.

2   The model

Time $t = 1, 2, ...$ is discrete. The economy is populated by a continuum of agents of measure i, distributed on an interval $\jmath$ according to measure $\ell(.)$. Following the literature, it is assumed that the law of large numbers holds. The economy is on a sequential competitive equilibrium and is described as a collection of individual allocations (c_t^i, l_t^i, a_t^i), aggregate quantities (K_t, L_t, Y_t) and

prices processes (π_t, i_t, W_t, Z_t). Given an initial wealth distribution $(a^i_{-1})_{i \in J}$ and an initial value of aggregate capital stock $K_{-1} = \int_i a^i_{-1} \ell(di)$ it is possible to: solve the agents' optimization programs, clear the markets for good, labour and capital and solve for consistent prices.

2.1   The heterogeneous households problem

The household side is defined by the utility function $U(c, l)$ in the form of the Greenwood-Hercowitz-Huffman (GHH) (see, Greenwood, Hercowitz & Huffman 1988) where households choose their consumption c and l supply. This functional form exhibits no wealth effect for the labour supply and therefore greatly simplifies our model by reducing to one the number of labour supply equation in the later part of the paper. The utility reads as:

$$U_t(c_t, l_t)_t = u\left(c_t - \chi^{-1}\frac{l_t^{1+1/\varphi}}{1+1/\varphi}\right),\tag{1}$$

where $\varphi > 0$ is the Frish elasticity of labour supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}^+ \to \mathbb{R}$ is twice continuously derivable , increasing, and concave. As a reminder, GHH follows the following form:

$$\begin{cases} U_t(c_t, l_t)_t = \frac{1}{1-\sigma}(c_t - \frac{l_t^{1+1/\varphi}}{\chi(1+1/\varphi)})^{1-\sigma} \text{ if } \sigma \neq 1; \\ U_t(c_t, l_t)_t = \log(c_t - \frac{l_t^{1+1/\varphi}}{\chi(1+1/\varphi)}) \text{ if } \sigma = 1; \end{cases}\tag{2}$$

where $\sigma > 0$ is the inter-temporal elasticity of substitution.

Agents have additive inter-temporal preferences with a discount factor $\beta > 0$. They optimize their individual consumption c_t and labour supply

9

l_t streams using inter-temporal utility criterion $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$. I consider a simplified set-up based on Krueger, Mitman & Perri (2016) and Ragot (2018) where households face an idiosyncratic unemployment risk in order to create heterogeneity in wealth and consumption decisions. At the beginning of each period, each agent faces an exogenous employment risk denoted e_t^i. $\mathcal{E} = \{e, u\}$ denotes the set of possible employment statuses. An agent with $e_t^i = e$ is considered as employed and free to choose her labour supply l_t^i. An agent with $e_t^i = u$ is considered as unemployed, cannot work and will suffer from a fixed disutility reflecting unemployment cost.[3] The history of idiosyncratic states until t is written $e^{i,t} = (e_0^i...e_t^i)$. The employment dynamic follows a discrete Markov process from the transition matrix $M_t \in [0,1]^{2x2}$. The job separation rate in t is written as $\Pi_{eu} = 1 - \Pi_{ee}$ and the job finding rate is symmetrically denoted as $\Pi_{ue,t} = 1 - \Pi_{uu,t}$. Hence, the transition matrix across employment situation is

$$M_t = \begin{bmatrix} 1 - \Pi_{eu} & \Pi_{eu} \\ \Pi_{ue,t} & 1 - \Pi_{ue,t} \end{bmatrix}, \tag{3}$$

with $\Pi_{eu,t}$ the probability to transition from unemployment to employment comoves with respect to productivity shock in this fashion

$$\Pi_{ue,t} = \Pi_{ue}^{SS} + \nu \varepsilon_t^p. \tag{4}$$

The productivity is following a basic a AR(1) exogenous shock $e_t^{\varepsilon^r} = e^{\rho^p \varepsilon_{t-1}^p + \vartheta_t^p}$. This design is similar to the empirical finding of Shimer (2005) and the search and matching literature where the job separation rate is almost constant over

---

[3]or home production

time and unemployment dynamics is explained by variation in the job finding rate. Here, an increase in productivity generates an increase in job finding.[4] The total share of employed and unemployed agent are respectively defined as $S_{e,t}$ and $S_{u,t}$ with $S_{e,t} + S_{u,t} = 1$ and at the steady-state satisfies $1 - \overline{\Pi}_{eu} = \overline{\Pi}_{uu} = \frac{(1-\Pi_{eu})S_u}{1-S_u}$.

We can write the budget constraint of the agents in the following fashion

$$c_t^i + a_t^i = (1 - \delta + Z_t)a_{t-1}^i + 1_{e_t^i=e}l_t^i W_t + \Delta_t^i, \tag{5}$$

where $1_{e_t^i=e}$ is a function equal to 1 when the agent is employed and 0 in the opposite case. Thus, $1_{e_t^i=e}l_t^i W_t$ is a notation for the expected net wage. a_t^i is the net individual asset holding, $0 < \delta < 1$ the depreciation rate and Z_t is the dividend paid by the firm in order to rent the capital from the households. Δ_t^i is a net transfer from the risk sharing agreement between agents with similar idiosyncratic histories after N periods.

I should note that no arbitrage condition on the financial market between risk free bonds and capital enables me to write

$$\mathbb{E}_t^* \frac{i_t}{\pi_{t+1}} = \mathbb{E}_t^* Z_{t+1} - \delta - 1, \tag{6}$$

with i_t the nominal interest rate, π_t the inflation rate and $\frac{i_t}{\mathbb{E}_t^* \pi_{t+1}}$ the real interest rate. $\mathbb{E}_t^{*=\{RE,AL\}}$ is the expectation operator that can be rational $* = \{RE\}$ or follows Recursive Least Square (RLS) learning $* = \{AL\}$.

---

[4]This setup could be envisioned as a special case of the search and matching literature (Christoffel & Kuester 2008) where the matching probability is 1 and wages are flexible.

I considered now an household $i \in \jmath$. She can save in an asset that pay a dividend Z_t. She is subject to the borrowing constraint such as her asset holding should be greater than a threshold $-\bar{a} = 0$. In $t = 0$, the household chooses consumption $c_t^i \geq 0$, labour supply $l_t^i \geq 0$ and saving plan $a_t^i \geq 0$ that maximize inter-temporal utility over an infinite horizon, subject to a budget constraint and the previous borrowing limit. If the household is credit constrained $a_t^i = 0$, she is said to belong the set $i \in \mathcal{C}$ of credit constrain agent. For a given a_{t-1}^i, the problem is written

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \quad \mathbb{E}_t^* \sum_{t=0}^{\infty} \beta^t U \left(c_t^i - \chi^{-1} \frac{l_t^{i,1+1/\varphi}}{1 + 1/\varphi} \right), \tag{7}$$

$$st. \ c_t^i + a_t^i = (1 - \delta + Z_t) a_{t-1}^i + 1_{e_t^i = e} l_t^i W_t,$$

$$st. \ a_t^i = \geq -\bar{a}, \tag{8}$$

β is the discount factor and v_t^i the Lagrange multiplier of the credit constraint of agent i.[5] The first order conditions for the employed agents boiled down to

$$U' \left(c_t^i - \chi^{-1} \frac{l_t^{i,1+1/\varphi}}{1 + 1/\varphi} \right) = \beta \mathbb{E}_t^* \left[\frac{i_t}{\pi_{t+1}} U' \left(c_{t+1}^i - \chi^{-1} \frac{l_{t+1}^{i,1+1/\varphi}}{1 + 1/\varphi} \right) \right] + v_t^i, \tag{9}$$

$$l_t^{i,1/\varphi} = \chi W_t 1_{e_t^i = e} \tag{10}$$

[5]The Lagrange multiplier is null while i is not credit constrained

and for the unemployed agents:

$$U'\left(c_t^i - \chi^{-1}\frac{l_t^{i,1+1/\varphi}}{1+1/\varphi}\right) = \beta\mathbb{E}_t^*\left[\frac{i_t}{\pi_{t+1}}U'\left(c_{t+1}^i - \chi^{-1}\frac{\varsigma^{i,1+1/\varphi}}{1+1/\varphi}\right)\right] + v_t^i, \quad (11)$$

with ς the disutility implied by labour search or/and home production.

The aggregation for the model economy is straightforward: first, financial market clearing implies that total sum of individual asset holdings equals the aggregate capital stock

$$\int_i a_t^i \ell(di) = K_t; \quad (12)$$

the labour is only supply by employed agents, thus aggregate labour supply L_t can be written as

$$\int_i l_t^i \ell(di) = L_t; \quad (13)$$

and aggregate consumption C_t is the total sum of individual consumptions

$$\int_i c_t^i \ell(di) = C_t. \quad (14)$$

Finally, using the transition matrix M_t we can express the aggregate law of motion for the employed $S_{e,t}$ and unemployed $S_{u,t}$ agents as follow

$$S_{u,t} = 1 - S_{e,t} = \Pi_{eu}S_{e,t-1} + (1 - \Pi_{ue,t})S_{u,t-1}. \quad (15)$$

2.2 A truncated history model

Following Ragot (2018) and Challe et al. (2017), I generate a discrete time finite partition, HANK model based on the truncated idiosyncratic histories

13

of households. This method is appealing for adaptive learning implementation for two reasons: first, it enables to implement the adaptive learning algorithm in a canonical way and avoid the complication of working with continuous time; second, it allows for an explicit expression of expectation of each individual parts of the household partition and avoid the creation of complex abstract state variables.

At any date t, each agent $i \in \jmath$ is characterized by her personal history of idiosyncratic unemployment risk realization $e^{i,t} = (e^i_t, e^i_{t-1}, e^i_{t-2}...)$. The main intuition is to sort agents in a finite number of bins following their idiosyncratic unemployment history. Nonetheless, agent being infinitely living, the number of idiosyncratic histories is infinite which would lead to an infinite number of consumption bins. To overcome this issue, I impose $1 < N < +\infty$, a truncation of the idiosyncratic histories considered in the model.[6] Per consequence, every history bin $\hbar$ is defined by a limited set of idiosyncratic realisations $\hbar \Leftrightarrow e^{\hbar,t} = (e^\hbar_t, e^\hbar_{t-1}...e^\hbar_{t-N-1})$

I defined a set of bins as a partition of the idiosyncratic histories of the total population. A partition $\mathcal{H}$ is a finite collection of sets of idiosyncratic histories such that at any date t, a idiosyncratic history e^t, truncated after N periods, belongs to only one element $\hbar$ of the partition $\mathcal{H}$. An element $\hbar \in \mathcal{H}$ will be called a history bin and represents individual histories. In this paper, an agent belongs to $\hbar \in \mathcal{H}$ at date t if her idiosyncratic employment history e^t is the same that the one defining $\hbar$. When an agent i is in bin $\hbar$ corresponding

---

[6]The $N = 0$ case is the RANK case where there was no idiosyncratic realisation and only one history bin exists. Thus, there is no probability for an agent to transition from one bin to another.

to an history $e^{i,N}$ in $t - 1$, then probability that she switches to another bin $\tilde{\hslash}$ with history $i,\tilde{e}^N$ in t is denoted by $\Pi^{\mathcal{E}}_{\tilde{e}^{i,N},e^{i,N},t}$ with

$$\Pi^{\mathcal{E}}_{e^{i,N},\tilde{e}^{i,N},t} = \Pi^{\mathcal{E}}_{\hslash,\tilde{\hslash},t}. \tag{16}$$

The probability $\Pi_{\hslash,\tilde{\hslash},,t}$ is the transition probability of moving from idiosyncratic history $\tilde{e}$ corresponding to bin $\tilde{\hslash}$ to the history e corresponding to bin $\hslash$.

It is important to note that partitioning the households following their idiosyncratic histories generates a large number of bins of very heterogeneous size. The number of bins follows a geometric progression as function of the number of the idiosyncratic states $\mathcal{E} = \{e, u\}$ to the power of the number of periods considered N.

I now consider a partition $\mathcal{H}$ containing a finite set of history bin $\hslash \in \mathcal{H}$ of N periods. The size of a bin $\hslash \in \mathcal{H}$ in t corresponds to the measure of agents i with a idiosyncratic history e^t belonging to bin $\hslash$. Bin size $\hslash$ in t are denoted by $S_{\hslash,t}$. Bin size $S_{\hslash,t}$ boils down to

$$S_{\hslash,t} = \sum_{\tilde{\hslash} \in \mathcal{H}} \Pi^{\mathcal{E}}_{\hslash\tilde{\hslash},t} S_{\tilde{\hslash},t-1}, \tag{17}$$

which simply denotes that the size of bin $\hslash$ in t is equal to the total number of households from other history bins $\tilde{\hslash}$ in $t - 1$ transitioning to this bin $\hslash$ in t.

In order to achieve similar preference within each history bin $\hslash \in \mathcal{H}$, I assume a pooling mechanism of wealth as a risk sharing arrangement between every members of the same history bin - see the term Δ^i_t in Equation 5.[7] This

---

[7]Ragot (2018), Challe et al. (2017) and the appendix in Le Grand & Ragot (2020) offer

wealth pooling leads to homogeneous preferences and policy functions for the agents within the same history bin.
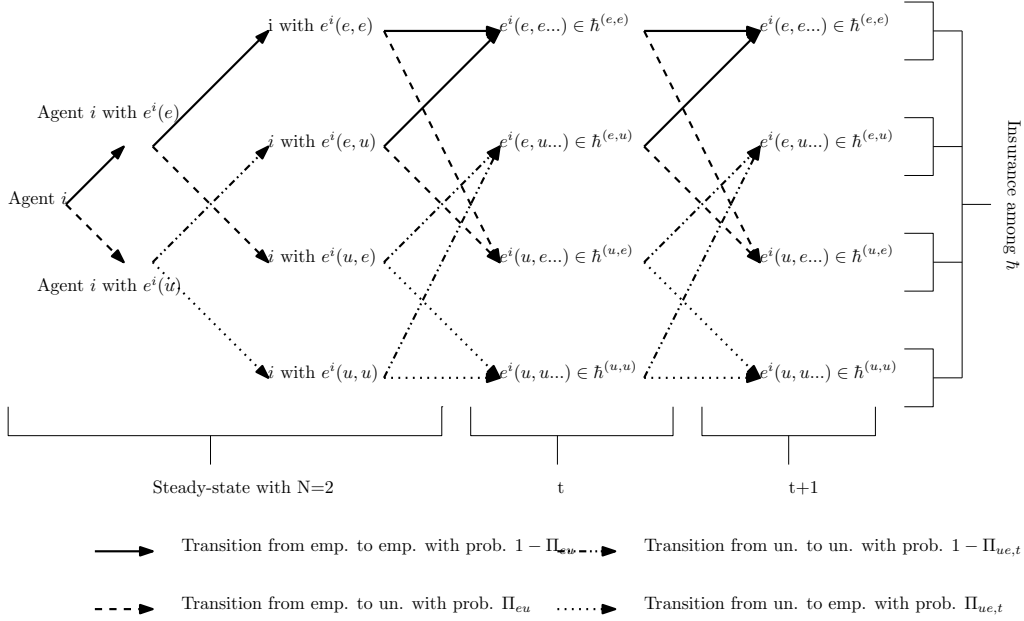

Figure 2: Idiosyncratic dynamics in the HANK model with $N = 2$

For the sake of clarity, Figure 2 details the internal structure of the heterogeneous agent model in a truncated history model with $N = 2$. It is possible to see from Figure 2 what consumption bins $\tilde{\hbar}$ are possible continuation of $\hbar$ and how the pooling mechanism assures homogeneity in preference. With $N = 2$, households in bin $\hbar \Leftrightarrow e^{\hbar,t} = (e, e)$ can transition in $t + 1$ to $\tilde{\hbar} \Leftrightarrow e^{\tilde{\hbar},t} = (u, e)$ with a probability Π_{eu} and stays in $\hbar \Leftrightarrow e^{\hbar,t} = (e, e)$ with a probability Π_{ee}. Symmetrically, households in bin $\hbar \Leftrightarrow e^{\hbar,t} = (u, e)$ can transition in $t + 1$ to

---

different justifications to achieve similar preference within history bins. As in Lucas (1975), one justification is that the agents with the same idiosyncratic history for the last N periods belong to a *family* and are located in the same *island*. They pool their resources and the optimal decision is taken by the *family head*. The other one is just a perfect risk sharing arrangement between all members of the same history bin.

$\tilde{\hbar} \Leftrightarrow e^{\tilde{\hbar},t} = (u,u)$ with a probability $\Pi_{uu,t+1}$ and transfer in $\hat{\hbar} \Leftrightarrow e^{\hat{\hbar},t} = (e,u)$ with a probability $\Pi_{ue,t+1}$.

Finally, it is important to acknowledge the timing of the model. At the beginning of the period, exogenous aggregate shock happens. Then the unemployment risk of all agents is realized. Agents transition to their new history bin and pool their wealth together. Afterwards, agents form their expectations about the future states of the economy. Finally, the model is solved. If the model is under adaptive learning, agents observe their forecast error and update their forecasting rule before next period. Figure 3 summarizes the timing of event in the HANK model under adaptive learning.
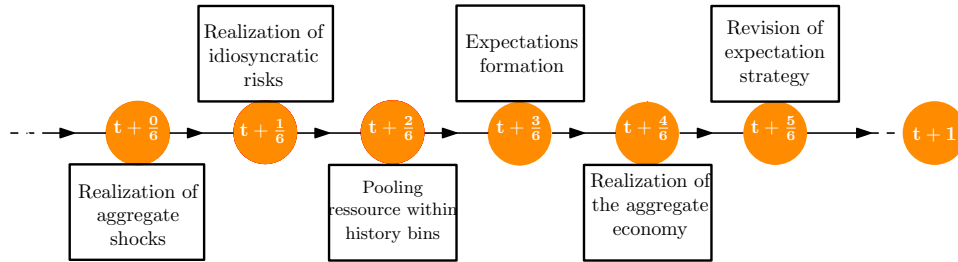

Figure 3: Intra-period timing of events in the HANK model under adaptive learning

2.3 The simulated model

After discussing the construction of the model and its micro-foundations, the next subsection presents the simulated model used in this paper.

The steady-state. The paper uses Le Grand & Ragot (2020) routine to computed the parameters and the transition probability of the projected individual consumptions bins' equations of the model. This uses the steady-state equilibrium in the absence of aggregate shocks of the model. Aiyagari (1994) and Huggett (1993) following Hopenhayn & Prescott (1992) have demonstrated that given the initial wealth and idiosyncratic shocks distribution, it is possible to characterized the steady-state wealth distribution of this model in $[-\bar{a}, +\infty]$ in the absence of aggregate shocks.

In order to achieve this distribution, the utility function is iterated through a guess and try algorithm in order to directly obtain stable policy rules over every asset levels of an exponential grid of 500 points. Given an initial endowment and an interest rate it is possible to simulate the steady state distribution under idiosyncratic risk and then aggregate saving and labour supply which leads to a new equilibrium aggregate rate. The process is iterated until the initial interest rate generates a distribution which lead to the same interest rate

At any moment t, the beginning of the period wealth holding of agent i in a_{t-1}^i can be seen as a function of the realisation idiosyncratic history $e^{i,t}$ up to date t. Thus $a_t^i = a(e^{i,t-1})$ defines a mapping between an initial wealth holding, idiosyncratic history realizations and the beginning of the period wealth to unique bin $\hbar$ such that $a(e^{t-1}) \in \hbar$. Setting an exogenous number period N in the idiosyncratic history, it is possible to write a the model which is consistent with the above mentioned steady state distribution as well as with the idiosyncratic shock realisation during the N periods.

The dynamic system. The model consider a finite partition of idiosyncratic histories $\mathcal{H}$[8]. First the Euler equations for all bin $\hbar \in \mathcal{H}$ write as follow

$$\forall \hbar \in \mathcal{H} \setminus \mathcal{C}, \xi_\hbar U'(c_{\hbar,t}, l_{\hbar,t}) = \beta \mathbb{E}_t^* \left[\frac{i_t}{\pi_{t+1}} \sum_{\tilde{\hbar} \in \mathcal{H}} \Pi_{\hbar\tilde{\hbar},t+1}^{\mathcal{E}} \xi_{\tilde{\hbar}}^{\mathcal{E}} U'\left(c_{\tilde{\hbar},t+1}, l_{\tilde{\hbar},t+1}\right) \right], \quad (18)$$

where U' is the marginal utility, $\xi_\hbar$ is a *preference shifter*, a coefficient correcting for consumption elasticity and level across the distribution and the non-linearity of the utility function.[9] In the end, this function is fairly simple, households bin $\hbar$ are trying to smooth their utility over infinite horizon. The agents are forecasting their expected utility according to their probability to fall into the different consumption bins. The existence of unemployment risk and poorer household generates precautionary saving in order to smooth the utility flow. This equation is at the core of this paper. The discounted expected utility stream depends on expected inflation, expected transitions probabilities depending on the state of the labour market which itself a function of the productivity, expected labour supply and expected consumptions of every consumption bins that the households in bin $\hbar$ can transition to. Per consequence, consumption decisions follow the expected discounted stream of labour and consumption depending on the expected probability to transition

---

[8]see Le Grand & Ragot (2020) for longer discussion about the difference between the *true* representation of the projected model and its approximation.

[9]Those coefficients are computing using the Le Grand & Ragot (2020) routine. See Le Grand & Ragot (2020) for a discussion about the computation of this model.

to different idiosyncratic states. The FOC for constrained bins reads

$$\forall \hbar \in \mathcal{C}, a_{h,t} = -\bar{a}. \tag{19}$$

For the agents subject to the borrowing constraint, considering the fact that they have no saving, they fall into the "hand-to-mouth" category where they will consume all their endowment without any consideration for future consumption.[10] The FOC condition for labour supply means that it is only a function of net wage for the employed agents,

$$\forall \hbar \in \mathcal{H}, l_{\hbar,t} = (\chi W_t 1_{e_\hbar=e})^\varphi. \tag{20}$$

The bin width resource is equal to total wage (for employed agents) W_t, plus the total discounted assets a_{t-1} and dividend Z_t held in $t-1$ by agents staying and transferring to the bin $\hbar$. Resources are meant to at least be equal to total current consumption and investment according to[11]

$$\forall \hbar \in \mathcal{H}, c_{\hbar,t} + a_{\hbar,t} = (1 - \delta + Z_t) \sum_{\tilde{\hbar} \in \mathcal{H}} \Pi^{\mathcal{E}}_{\tilde{\hbar}\hbar,t-1} \frac{S_{\tilde{\hbar},t}}{S_{\hbar,t}} a_{\tilde{\hbar},t-1} + 1_{e_\hbar=e} l_{\hbar,t} W_t. \tag{21}$$

Aggregation of all bins means that the total asset holdings, labour inputs

---

[10]The truncated histories method assumes the constrained bins are always constrained and vice-versa. This is a reasonable assumption considering the fact that the model is linearised around a steady-state and is subject to only small shocks. In the current calibration, there are no credit constrained agents.

[11]In order not to complicate the bin-width resource constraint and deal with the distribution of price adjustment costs over the distribution of households, it is assumed that they are negligible. The results are robust to a lump-sum cost applied to all history bins. This is not an issue considering that the first order approximation of the prices adjustment costs in zero inflation steady state model are always equal to zero.

and consumptions are respectively equal to the capital stocks, aggregate labour supply and aggregate consumption. Formally we can write

$$K_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} a_{\hbar,t}, \tag{22}$$

$$L_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} l_{\hbar,t}, \tag{23}$$

$$C_t = \sum_{\hbar \in \mathcal{H}} S_{\hbar,t} c_{\hbar,t}. \tag{24}$$

The rest of the model follows the standard representative firm New Keynesian set-up developed with the following equations. The production function is a simple Cobb-Douglas function with exogenous productivity and aggregate capital and aggregate labour as inputs,

$$Y_t = e^{\varepsilon_t^p} K_{t-1}^{\alpha} L_t^{1-\alpha}. \tag{25}$$

The representative firm minimizes wages through the following FOC,

$$W_t = (1 - \alpha) e^{\varepsilon_t^p} K_{t-1}^{\alpha} L_t^{-\alpha}, \tag{26}$$

and the of marginal cost is,

$$mc_t = \frac{1}{e^{\varepsilon_t^p}} \left(\frac{Z_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1 - \alpha}\right)^{1-\alpha}, \tag{27}$$

which is an inverse function of productivity shock. The representative firm sets its prices under the constraint of quadratic adjustment menu costs. Per

21

consequence, the New Keynesian Phillips curve satisfies:

$$0 = 1 - (1 - mc_t)\epsilon - \psi(\pi_t - 1)\pi_t + \psi\beta\mathbb{E}_t^*[(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}]. \qquad (28)$$

The clearing of the financial market forces a no-arbitrage condition between bonds and assets,

$$\mathbb{E}_t^*\frac{i_t}{\pi_{t+1}} = \mathbb{E}_t^* Z_{t+1} - \delta - 1.$$

The policy rate is set by the central bank and is subject to a standard Taylor Rule as below,

$$i_t - \bar{i} = \phi^\pi(\pi_t - \bar{\pi}) + \phi^y(\frac{Y_t - \overline{Y}}{\overline{Y}}) + \varepsilon_t^r. \qquad (29)$$

with ε_t^r an AR(1) exogenous process representing shocks on the nominal rate. Finally, it exists two exogenous stochastic AR(1) processes that shock the supply and demand sides of the economy as

$$\varepsilon_t^r = \rho^r \varepsilon_{t-1}^r + \vartheta_t^r. \qquad (30)$$

$$\varepsilon_t^p = \rho^p \varepsilon_{t-1}^p + \vartheta_t^p.$$

3 Solving the model under different expectation operators

In order to solve the model, I perform trough Dynare (Juillard et al. 1996) a first order linearisation of the model around the steady state. Therefore, in this paper I denote with an $[\hat{\ }]$ the log-linearised transformation of a variable.

For the sake of clarity, I recapitulate the explicit difference between the RANK version (see Appendix A) and the HANK model with respect to expectations. From Appendix A it is possible to define $\hat{x}^{e,RANK}$ the forward looking variables in the RANK model as

$$\hat{x}^{e,RANK} = [\hat{Y}_t, \hat{\pi}_t, \hat{Z}_t, \hat{L}_t, \hat{C}_t]. \tag{31}$$

On the other hand, the heterogeneity in the HANK model generated by the heterogeneous Euler Equations 18 creates a larger forward looking variable vector $\hat{x}^{e,HANK}$ such as

$$
\begin{aligned}
\hat{x}^{e,HANK} = & [\hat{Y}_t, \hat{\pi}_t, \hat{Z}_t, \hat{l}_t, \quad \ldots \\
& \hat{c}_{\hbar=1,t}, \hat{c}_{\hbar=2,t} \quad [\ldots] \quad , \hat{c}_{\hbar=\mathcal{E}^N,t}, \quad \ldots \\
& \Pi^{\mathcal{E}}_{\hbar=1\tilde{\hbar}=1,t}, \Pi^{\mathcal{E}}_{\hbar=1\tilde{\hbar}=2,t}, \quad \ldots \\
& \Pi^{\mathcal{E}}_{\hbar=2\tilde{\hbar}=1,t}, \Pi^{\mathcal{E}}_{\hbar=2\tilde{\hbar}=3,t}, \quad \ldots \\
& [\ldots] \\
& \Pi^{\mathcal{E}}_{\hbar=\mathcal{E}^N\tilde{\hbar}=\mathcal{E}^N-1,t}, \Pi^{\mathcal{E}}_{\hbar=\mathcal{E}^N\tilde{\hbar}=\mathcal{E}^N,t}].
\end{aligned}
\tag{32}
$$

In the absence of a representative consumption state variable $\hat{C}_t$, the HANK

23

model introduces through the heterogeneous households multiple forward look-ing variables for expected disaggregated utility flows. The new forward looking variables are the set of all history bins expected consumptions $\hat{c}_{\hbar,t}$ and expected transition probabilities between all consumption bins $\Pi_{\hbar,t}^{\mathcal{E}}$. All of those vari-ables are used in the set of Equations 18. Per consequence, it should be clear that the heterogeneous expectations come from the heterogeneity in house-holds. By symmetry, the number of state variables $\hat{x}_t$ in the HANK model is also much larger than in the RANK version which has an impact on the MSV size and form.

3.1 Rational Expectations

I present first the rational expectations solution. Following Uhlig (1995), I can collapse the state variables vector as $\hat{x}_t = [\hat{Y}_t, \hat{\pi}_t, \hat{i}_t, \hat{W}_t, ...]$. I define $\hat{z}_t = [\varepsilon_t^r, \varepsilon_t^p]$ and $\hat{u}_t = [\vartheta_t^r, \vartheta_t^p]$ as respectively the shock processes and the exogenous variables. The state space representation of the linearised model boils down to

$$A_0 + A_1\hat{x}_{t-1} + A_2\hat{x}_t + A_2\mathbb{E}_t^*\hat{x}_{t+1} + A_3\hat{z}_t = 0. \tag{33}$$

Then, assuming rational expectation and determinacy we can write the MSV of the model as

$$\hat{x}_t = A + P\hat{x}_{t-1} + Q\hat{z}_t \tag{34}$$

I use Dynare (Juillard et al. 1996) on Matlab to automatize the computation process and deduce A, P, Q trough perturbation methods. Having deduced

24

the model's solution, I can now iterate the expression one step ahead

$$\hat{x}_{t+1} = A + P\hat{x}_t + X z_{t+1} \Leftrightarrow \hat{x}_{t+1} = \alpha + P\hat{x}_t + X(\rho z_t + u_{t+1}),$$

with u_t the exogenous i.i.d process. Thus I write $\mathbb{E}_t^{RE}(u_{t+1}) = 0$ and deduce

$$\mathbb{E}_t^{RE}\hat{x}_{t+1} = A + P\hat{x}_t + Q(\rho\hat{z}_t + 0).$$

Then, with the iteration $E_t^{RE}\hat{x}_{t+1}$ and $\hat{x}_t$,

$$\mathbb{E}_t^{RE}\hat{x}_{t+1} = (I + P)A + P^2 x_{t-1} + (PQ + \rho)\hat{z}_t).$$

Assuming that agents form expectation for the forward looking variables of the model denoted $\hat{x}_t^e$ and by using the MSV solution 34, we deduce

$$\mathbb{E}_t^{RE}\hat{x}_{t+1}^e = \alpha + \beta\hat{x}_{t-1} + \gamma z_t, \tag{35}$$

with:

$$\begin{cases} \alpha = (I + P)A = 0 \\ \beta = P^2 \\ \gamma = PQ + \rho \end{cases}$$

3.2 Adaptive Learning

In this section, I present how to solve the model under adaptive learning based on the same notation as the rational expectations solution. The intuition behind this feature is that agents know the specification of the rational expectations MSV solution but do not know the values of the parameters. It is important to highlight that in this paper under AL and RE, the agents observe the whole set of past state variable and exogenous shocks realizations.[12] They try to learn the structural parameters values from past realizations and the feedback loop between expectations formation and realizations of the model can drive the model out of rational expectations dynamics. Following Evans & Honkapohja (2001), it is possible to rewrite the model MSV solution in the following way

$$\hat{x}_t^e = a_{t-1} + b_{t-1}\hat{x}_{t-1} + c_{t-1}z_t, \tag{36}$$

The subscript $t-1$ means that the forecasting coefficients are subject to change following what is called in the literature the perceived law of motion with the information set available in $t - 1$. I now define $\phi_t = [a_t', vec(b_t, c_t)']'$ as the beliefs matrix and $M_t = \{1, x_{t-1}', z_t'\}$ the moments matrix available. I follow the econometric learning hypothesis, where the law of motion of those matrices follows a constant gain Recursive Least Square (RLS) process which is standard

---

[12]It exists models such as Branch (2004a), Hommes & Zhu (2014) or Hommes, Mavromatis & Ozden (2020) which make less assumption about the specification of the expectations solution and the information set available under learning. In this paper I chose to stick to the textbook learning process.

in the literature,

$$\phi_t = \phi_{t-1} + gR_t^{-1}M_{t-1}(\hat{x}_t^e - M_{t-1}'\phi_{t-1}), \tag{37}$$

$$R_t = R_{t-1} + g(M_{t-1}M_{t-1}' - R_{t-1}). \tag{38}$$

Those equations describe the updating process of the beliefs by the model's agents. Here, agents minimize their forecasts' square error based on past data. The small $0 < g$ means that the gain coefficient is constant and enables the learning to slowly discount the importance of past observations over time. I implement a ridge regression device that will trigger when near-singular moment in the variance-covariance matrix appears which would have lead to inaccuracy in the inversion as in Slobodyan & Wouters (2012). I initialize the moment matrix R_0 and the beliefs matrix ϕ_0 using variance/covariance matrix generated by the rational expectation solution.[13]

Finally, iterating (36), it is possible to write

$$\mathbb{E}_t^{AL}\hat{x}_{t+1}^e = (I + b_{t-1})a_{t-1} + b_{t-1}^2\hat{x}_{t-1} + (b_{t-1}c_{t-1} + \rho)z_t. \tag{39}$$

Then, plugging back (39) into (33), I can simulate the model dynamic under constant gain recursive least square learning.

---

[13]If the model is initialized at the REE solution and there is no gain in the learning, rational expectations and adaptive learning dynamics are the same because adaptive learning agents are at the rational expectations equilibrium.

4 Model dynamics

In this section, I present the dynamic response from the model to aggregate shocks. This section first discusses the calibration of the model and then presents supply side shock impulse response function (IRF). IRFs for a demand/nominal rate shock is displayed in the Appendix C.

4.1 Calibration

I choose a discount factor of $\beta = 0.99$ (Woodford 2003) and an inter-temporal elasticity of substitution of $\sigma = 1.5$ consistent with Smets & Wouters (2007), the Taylor rule coefficient on inflation $\phi^\pi = 1.50$ and output gap $\phi^y = 0.125$ are standard. The price elasticity of demand is $\epsilon = 10$ (Smets & Wouters 2007) and the menu cost $\psi = 50$ is consistent with fairly flexible prices but allows for a large determinacy zone. $\alpha = 0.2$ is consistent with Smets & Wouters (2007) estimate. I assume a zero inflation target $\bar{\pi} = 1$ as in Woodford (2003). The same calibration $\delta = 0.025$ as in Smets & Wouters (2007) is used for the depreciation rate of capital. I use $\varphi = 0.5$ for the Frish elasticity which is consistent with Ragot (2018), Le Grand & Ragot (2020) and Smets & Wouters (2007) and literature on HANK model with GHH utility function. Finally I scale labour supply with $\chi = 0.04$ as in Le Grand & Ragot (2020).

In order to generate large heterogeneity between agents without introducing more idiosyncratic states, I set the unemployment rate at the steady state $S_u = 10\%$. I then use the estimated job finding probability $\Pi_{ue}^{SS} = 0.8$ by Krueger et al. (2016) which means that unemployed households have a 80%

probability to exit unemployment every quarter at the steady state. I define $\nu = 10$ in the realm of the literature which implies than any increase of 1% of productivity increase by 0.1 the probability to exit unemployment. There are no standard calibration for the disutility generated by unemployment, I set $\varsigma = \frac{1}{2}\bar{l}$ to be equal to half the steady state labour supply of employed agent. This calibration enables the model to avoid negative consumption for poor unemployed agents and reasonable level of aggregate capital and investment. I set the borrowing constrain to $-a = 0$ as in Ragot (2018) to avoid agent with negative wealth and have a one to one mapping between aggregate savings and capital stock.

In order for the model to yield realistic dynamic, I calibrate the shock processes with $\{\sigma(\vartheta_t^R), \rho^r, \sigma(\vartheta_t^p), \rho^p\} = \{0.01, 0.8, 0.01, 0.8\}$ which are values within the boundaries of the literature. Results are robust to different calibration, especially shock calibrations.

I truncate the idiosyncratic history after 2 quarters $N = 2$. This leads to the creation of a partition $\mathcal{H}$ of 4 different history bins $\hbar$ with $\hbar^{(e,e)} = \{e_t, e_{t-1}\}$ and $\hbar^{(u,u)} = \{u_t, u_{t-1}\}$ being respectively the richest and poorest bins. At the steady-state, $\hbar^{(e,e)}$ and $\hbar^{(u,e)}$ respectively include 82% and 8% of the households. Those history bins can be considered as representative of the employed and unemployed agents populations. The number N before truncation is motivated by the trade-off between the need to have enriched dynamics, tractability, computational speed and the accuracy problems encountered while inverting the very large variance-covariance matrix of the state variables during the learning process. The results are robust to larger N but less tractable.

Finally, I set $g = 0.01$ which is within the bound of the literature (Williams 2010). This value allows us to display relatively large variation between the RE and AL economy without a high risk of loosing stability. Results are robust within the bounds of the literature.

4.2   Impulse response functions from a supply shock

In this model, household heterogeneity and the endogenous dynamic of the unemployment risk make the supply shock propagation richer. Indeed when productivity decreases, the job finding probability decreases and unemployment rate and duration increase. I present here the dynamic of the HANK model with respect to the RANK counterpart under adaptive learning *i.e* in the absence of idiosyncratic risk (see Appendix A for the detailed equations of the RANK model). The HANK model is simulated under rational expectations and adaptive learning in order to disentangle the effect generated by the learning, the heterogeneity and the interaction between both features.

Figure 4 presents the IRFs of the HANK and RANK models under rational expectations and adaptive learning to a negative supply side shock represented by -1% productivity $AR(1)$ process. Demand sided shocks have fairly similar propagation. IRFs of a nominal rate shocks are displayed in Appendix C. In the RANK model, with this calibration, and under this fairly small gain, there is only small difference between the rational expectations and adaptive learning simulations. Therefore, the RANK-RE responses are not displayed.

In order to be exhaustive, the panels j, k, l and m in Figure 4 present the consumption behaviours of all history bins. $\hbar^{(e,e)}$ and $\hbar^{(e,u)}$ are the employed

households history bins and respectively include at the steady-state, 82% and 8% of the households. $\hbar^{(u,e)}$ and $\hbar^{(u,u)}$ are the unemployed households history bins and respectively include at the steady-state, 8% and 2% of the households. For the sake of clarity, in Figures 5, I present under RE and AL the expectations contribution to consumption of $\hbar^{(e,e)}$ which include 80% of the total households population and 88% of the employed population.[14]

The RANK AL model presents the standard response to a negative supply shock. The decrease in productivity creates a drop in supply and increases the marginal cost while demand is relatively steady. Thus, the representative firm increases its price in order to equilibrate the supply and demand sides (Figure 4-a). The change in expected inflation decreases the real rate (Figure 4-f). Reacting to the surge in inflation the CB increases nominal rate (Figure 4-a). This rate hike depresses demand and tempers down the prices hike by the supply side Figure 4-c). Per consequence, the representative household cuts down on saving (Figure 4-e), consumption (Figure 4 c) and labour (Figure 4-g) until the productivity level is back to its steady state. To sum up, a negative supply shock in a RANK model is inflationary even though it creates output drop.

The HANK model under rational expectations presents already a different dynamic. First of all, the decrease in productivity implies a lower job finding rate and thus a higher unemployment (Figure 4 h) and lower aggregate labour supply by households (Figure 4 g). Even though initially the supply side effects are the same, the aggregate demand behaves very differently (Fig-

---

[14]In Appendix B, I display expectations contribution to the representative household consumption.

ure 4-f). First, the consumption drop twice as much and the aggregate saving by household does not drop as much (Figure 4 e and f). This is due to the precautionary saving implied in the households' FOCs (see Equation 18). Indeed, the increased probability and duration of unemployment triggers precautionary saving in order to smooth future utility flows. Precautionary saving and demand contraction appear to be larger for richer households than the richer ones (Figure 4 k and m). This is logical because richer households' income is less a function of dividends and saving and more a function of wages and labour which are adversely affected by the shock. Moreover being richer, it is easier for them to allocate part of their income into future utility/saving.

Looking at Figure 5b , it is possible to observe that the main driver of consumption drop for employed agents is an expected drop in their individual consumption (in purple) through an expected drop of consumption in case of unemployment and employment. Another phenomenon is the effect of expected labour condition (in red) which initially generates an increase in consumption (less poor agents find employment and join the consumption bin) and then a decrease due to the longer expected unemployment duration. The expected labour supply (in blue), drive consumption up due to the construction of the GHH utility function.

The HANK model under adaptive learning broadly exhibits on the short run the same responses than its rational expectations counterpart. Nonetheless, the aggregate consumption drops more (see Figure 4-f) while labour supply are comparable (see Figure 4 g) which leads to a large increase in individual saving and aggregate capital (see Figure 4-e). This drop in consumption w.r.t

to the RE counterpart is the most important within the employed agent bin (see Figure 4 m). Figure 5a displays the consumption of $\hbar_{(e,e)}$, the history bin which represents 80% of the households and 88.88% of the employed households at the steady state. It can be consider as a good proxy for employed agents behavior.

Figure 5a shows that employed agents under AL initially struggle to learn their level of consumption. On the medium run, it is possible to observe after 20 quarters a negative contribution from expected labour supply which is higher than the RE counterpart (see Figure 5b). This higher expected labour supply is more than compensate on the longer run by an higher than the steady state expected consumption. Looking at the consumption of $\hbar_{(u,e)}$ (Figure 4-k)), the history bin which represents 8% of the households and 88.88% of the unemployed households at the steady state. It is possible to see the same phenomenon.

On the medium run and at the aggregate level, expectations loose their anchorage to the rational expectations path and drift toward a disinflationary transient state that can last a very long time (see Figure 4-a). The model eventually converges back to its steady state after over 800 periods. What happens in the model is that the lower consumption expectations generates a lower aggregate consumption that leads to excess aggregate capital accumulation (see Figure 4-e). This pushes down marginal cost and wage and pushes up labour supply (see Figure 4-g). All those aggregate effects are disinflationary.

5 Monetary policy implications

After describing the implications of supply side shocks in a HANK model under adaptive learning, the purpose of this section is to investigate the monetary policy consequences of those modelling choices and the change with respect to a HANK model under rational expectations and a RANK model under adaptive learning.

Monetary policy regime: Calibration	Standard IT $\phi^\pi = 1.5 \ \phi^y = 0.125$	Hawkish IT $\phi^\pi = 2.50 \ \phi^y = 0$	Dovish IT $\phi^\pi = 1 \ \phi^y = 1$	PLT $\phi^p = 0.25 \ \phi^y = 1$
Inflation Variance $var\left(\hat{\pi}_t\right)$:				
RANK-AL	9.6906 (0.0151)	1.1679 (0.0018)	356.5283 (0.3770)	15.6856 (0.0116)
HANK-RE	8.8518 (0.0127)	1.1303 (0.0017)	31.2210 (0.0386)	9.1640 (0.0070)
HANK-AL	**9.3797** (0.0141)	**1.1379** (0.0018)	**34.2460** (0.0446)	**9.3199** (0.0070)
Output Gap Variance $var\left(\hat{Y}_t\right)$:				
RANK-AL	6.4486 (0.0118)	6.5042 (0.0122)	3.9548 (0.0050)	6.8125 (0.0014)
HANK-RE	18.1107 (0.0353)	16.9978 (0.0308)	7.0714 (0.0069)	8.3490 (0.0104)
HANK-AL	**20.5742** (0.0445)	**20.5339** (0.0491)	**7.0660** (0.0069)	**9.9830** (0.0154)
Aggregate Consumption Variance $var\left(\hat{C}_t\right)$:				
RANK-AL	3.4068 (0.0077)	2.3378 (0.0070)	34.6332 (0.0359)	2.0657 (0.0074)
HANK-RE	12.3378 (0.0371)	8.7402 (0.0022)	5.9171 (0.0011)	2.7393 (0.001)
HANK-AL	**16.6570** (0.0486)	**14.8344** (0.0408)	**4.3015** (0.0108)	**2.3605** (0.0073)
Employed Consumption Variance $var\left(\hat{c}_{h(eu),t}\right)$:				
HANK-RE	12.0004 (0.0370)	8.5259 (0.0224)	5.2813 (0.0088)	2.7210 (0.0099)
HANK-AL	**16.3411** (0.0477)	**14.4165** (0.0383)	**4.0961** (0.008)	**2.5191** (0.0064)
Unemployed Consumption Variance $var\left(\hat{c}_{h(ue),t}\right)$:				
HANK-RE	22.8162 (0.0563)	19.1556 (0.0415)	4.2169 (0.008)	4.3296 (0.0098)
HANK-AL	**33.2964** (0.0914)	**47.2806** (0.2435)	**3.9644** (0.0076)	**17.3357** (0.0848)
Capital Variance $var\left(\hat{K}_t\right)$:				
RANK-AL	6.9852 (0.0279)	6.8444 (0.0255)	5.7805 (0.0013)	9.2484 (0.0038)
HANK-RE	41.8095 (0.1750)	15.5756 (0.0642)	67.7670 (0.1215)	43.9973 (0.1184)
HANK-AL	**64.8356** (0.3333)	**43.5664** (0.3444)	**70.1603** (0.1274)	**64.5969** (0.2671)
Excess forecast Error HANK, AL w.r.t RE:				
Excess error $\mathbb{E}^{AL}C_{h(ee),t+1}$	**+0.3151%**	**+0.6419%**	**+ 0.7129%**	**+0.4959%**
Excess error $\mathbb{E}^{AL}C_{h(ue),t+1}$	**+0.8922%**	**+0.2841%**	**+0.2158%**	**+0.2914%**

Notes: Every moments of the table is the result of 10,000 Monte-Carlo simulations over 400 periods. No crashes happens in any simulations. Excess errors are the differences between the average absolute forecast error over each Monte-Carlo simulation for the forecast of the different consumption bin.

Table 1: Moments under different CB's reaction function calibrations

Primarily, it is important to observe the models statistical moments under a standard inflation targeting (IT) regime. The first column of Table 1

presents the business cycle statistics of the HANK-AL model with respect to the RANK-AL and HANK-RE models with a standard calibration of the CB's reaction function. Relative to the RANK-AL model, both HANK models exhibit higher volatility in output, aggregate consumption and aggregate capital. Those effects come from the fact that households from the HANK models use capital as a way to smooth their consumption and insured themselves again expected idiosyncratic future drop in income due to unemployment risk. Per consequence, contrary to the RANK household which uses its capital as the adjustment variable of the resource constraint, HANK households keep a buffer of capital as an insurance mechanism against idiosyncratic risk which is evolving over time. For the sake of clarity, I include in the table the consumptions $\hbar_{(e,e)}$[15] and $\hbar_{(u,e)}$[16] variances and their excess forecast error. Unemployed agents' consumption volatility is higher than employed due to their lower saving buffer and income (only dividends) which doesn't allow them to smooth their consumption as much as employed agents.

With respect to the HANK under rational expectations, the HANK under adaptive learning exhibits a higher volatility in all its state variables. The most striking ones, are the ones driven by the households sides where the variance in capital stock is almost 50% as big as in the rational expectations counterpart. This excess variance in aggregate capital is due to the excess volatility in de-aggregated consumptions.

This excess volatility of is driven by the learning dynamic. Both un-

---

[15]This history bin represents 80% of the households and 88.88% of the employed households at the steady state. It is a good proxy for employed agents behaviour.

[16]This history bin represents 8% of the households and 88.88% of the unemployed households at the steady state. It is a good proxy for unemployed agents behaviour.

employed and employed agents are subject to a 50% increase consumption variances. On the last two lines of the tables, we can observe that average consumption forecasts errors are larger under learning than under rational expectations. The excess error is larger for unemployed agent which explain its larger increase in volatility.

5.1 The ambiguous effects of monetary policy

In Figure 6 a and b, I observe the evolutions of the output and inflation variances over the parameter space of the Taylor rule. In Appendix D, it is possible to observe the excess volatility of the HANK-AL model *w.r.t* the other models. It is obvious that in all models, there exist trades-offs between inflation and output gap stabilization *i.e* the higher/lower the reaction to inflation ϕ^π/output ϕ^y the more stable inflation is and the more volatile output gap is. Nonetheless, in the HANK-RE and HANK-AL, it is crucial to remark that the top left hand zones of both maps *i.e* where the monetary policy reacts aggressively to the output gap and lightly to inflation; is not the high volatility inflation unstable zone like the RANK-AL and RANK-RE model. It is striking to note that a *dovish* monetary policy stance appears to be a viable low volatility scenario. Moreover, it is the zone where the HANK-AL variances are the closest to the HANK-RE scenario (see Figure 6, third column of Table 1 and Appendix D) and excess forecast error from the HANK-AL model is the smallest.

On the contrary, the RANK-AL model yields, in Figure 6 e and f, the standard result that only an aggressive monetary policy relative to the inflation

36

generates low inflation variance. This result is in line with Orphanides & Williams (2008), Ascari et al. (2017) and Eusepi & Preston (2018) analysis where a conservative CB should be preferred under adaptive learning in a RANK model.[17] The trade-off between output and inflation stabilization is very strong in the RANK adaptive learning model and low volatility from the output gap means an very high volatility of inflation; and volatile inflation is hard to learn due the self referential nature of the New Keynesian Phillips Curve.

In the second and third columns of Table 1, I present two cases of non standard calibration for monetary policy. The first one is name *hawkish* with $\{\phi^\pi, \phi^y\} = \{2.5, 0\}$, a harsh response to inflation deviation and the other one is *dovish* with $\{\phi^\pi, \phi^y\} = \{1, 1\}$ a strong reaction to output gap. Figure 6 summarizes the results for the whole monetary policy space of the different models. In the *hawkish* case, the change in all three models are fairly similar. The inflation is less volatile in the same magnitude. Output is more relatively unchanged even though the difference between the HANK-RE and HANK-AL models increases. The gap between the HANK-RE and HANK-AL (see appendix D and column 2 of Table 1). Output (+70%) and especially aggregate capital variances (+186%) are very different. It can be explained by the fact that the increase in the volatility of the nominal rate makes the complex expected utility streams of Equations 18 harder to be forecasted. In Table 1, it is possible to observe a large increase in forecast error for employed agent in the HANK-AL model with respect to the HANK-RE. The decrease in forecast

---

[17]The inflation/output gap stabilization sacrifice ratio is said to be larger in the RANK-AL than in the RANK-RE model.

error by the unemployed agents is due to stabler dividend streams generated by a stable real rate.

On the other hand, the *dovish* policy creates a large difference between the HANKs and RANK-AL models. Inflation in the RANK-AL model explodes *w.r.t* to the standard Taylor rule calibration case. The only large difference between the HANK-AL and HANK-RE is the aggregate capital and consumption decisions' variances. Nonetheless those ones are similar in magnitude to the standard calibration scenario which make up for the case that a more *dovish* monetary policy is not harder to learn for heterogeneous households. From the last lines of Table 1, it is possible to observe that the *dovish* policy seems easier to forecast under adaptive learning for richer households and harder for the poorer ones. Yet, rich households make for larger parts of aggregate capital and consumption. Thus, thanks to the feedback loop, this has a stabilizing effect. Nonetheless those results make-up for the case that stability in HANK models is driven through the stability of expected consumption channel and not through the fast adjustment of the discounting process.[18] Per consequence, it explains also why the discrepancy between the HANK-RE and HANK-AL is so small. Indeed stabler expected consumptions and discounting process ease the complex learning dynamic of heterogeneous consumption bins. Finally, it is noteworthy to point that the overall and desegregated volatilities of consumption in the HANK-AL are smaller than under RE. This puzzling data-point might be generated by a drift from the RE perceived law of motion to a less volatile one under learning.

---

[18]Kaplan et al. (2018) reach the same conclusion with a different HANK implementation.

5.2 A price level targeting experiment

Despite some promising results, the alternative policy does not appear to solve the slow convergence issue. Hawkish policies while avoiding long disinflation increase the magnitude of large precautionary saving and low consumption periods after a supply shock under learning. In the meanwhile, dovish policy despite smaller difference between the rational case creates large precautionary saving, high variance in inflation and deflation on the short run due to monetary easing.

A burgeoning strand of the literature (see e.g Williams 2010, Honkapohja & Mitra 2019) has been developed in order to analysis possible ways to escape deflationary trap at the Effective Lower Bound (ELB) through PLT when agents are learning. PLT appears to be an efficient way to drive expectations out of those sunspots driven liquidity traps. Despite this paper's model not including the ELB, it can still generate disinflationary episodes and PLT appears to be a promising policy treatment to avoid those.

I introduce a PLT reaction function instead of the canonical IT rule. Based on the definition of inflation $\pi_t = \frac{P_t}{P_{t-1}}$, it is possible to write:

$$P_t = \pi_t P_{t-1}. \tag{40}$$

The PLT rule is then implemented. The reaction function reads as:

$$i_t - \bar{i} = \phi^p \left(\frac{P_t - \overline{P}}{\overline{P}} \right) + \phi^y \left(\frac{Y_t - \overline{Y}}{\overline{Y}} \right) + \varepsilon_t^r, \tag{41}$$

with $\{\phi^p, \phi^y\} = \{0.25, 1\}$ in line with previous works under learning by Williams (2010) or Honkapohja & Mitra (2019). The intuition behind PLT is that the CB commits to *make-up* for the past under/overshoot of inflation by generating a symmetric over/undershoot. In a zero inflation steady state model like this one, this implies keeping the price level at an arbitrary level (here $\overline{P} = 100$).

The last column of Table 1 presents the main statistical moments of this policy experiment and Figures 7 displays the results for a large range of the parameters space. First we can observe that PLT reduces inflation's variance in the HANK cases but not in the RANK (*w.r.t* to the benchmark calibration). This effect is due to the fact that prices adjustment by the representative firm become pointless if output is stable enough. In fact, they will trigger a symmetric price adjustment in the opposite direction. Yet in all cases and model, regarding inflation, it is less efficient than the *hawkish* calibration.

The PLT treatment increases the volatility of output in the RANK-AL (and thus inflation) but not in the HANK models. As seen in Figure 7e and Figure 7f, in the RANK-AL model, the trade-off between inflation and output stabilization is stronger. Per consequence, the current PLT calibration trades output stability for price/inflation volatility. In the HANK models, inflation is more driven by the output volatility and the trade-off is less clear. This why a decline in the output volatility is possible. This due to the same effect discussed previously: in a same fashion as the *dovish* policy, PLT generates a stabler future discounted expected utility flow and thus decreases precautionary saving and consumption variances. PLT by averaging inflation over time smooths

40

expected utility streams much more than the IT framework. By smoothing utility stream the model is able to reduce precautionary saving by generating a symmetric boom after a bust. Finally, PLT dramatically reduces excess forecast error for unemployed agent while the employed ones stays around the benchmark level.

Observing, Figure 8 the most striking fact is the change in the policy rate reaction. While under IT the reaction function leads to a tightening, PLT eases which is very inflationary and then quickly deflationary for a dozen of quarters. Nonetheless, the decrease in nominal and real rate boost consumption for employed agent (see Figure 8 j and l). It seems that unemployed consumptions decisions tend to drift less from the rational expectations solution under PLT (see Figure 8 j and l). This is due to the smoother and stabler dynamic implied by the more aggressive policy generated in case of deflationary trap. It seems that those stabler dynamics are also much easier to learn for unemployed agent and their excess forecast errors implied by the adaptive learning are greatly reduced relative to all the other policy scenarios (see Table 1). In Figure 8 we can see than after the initial productivity shock and following policy rate cut, aggregate consumption and saving converge much faster to their steady state. Consumption in the HANK-AL model under PLT converges back to the steady state in about 200 periods. It is a long time but much shorter than the HANK-AL under IT and the deviation is smaller. Poorer households appear to be better anchored in their saving and consumption decisions thanks to smoother expected utility streams. Richer employed households are better guided by the expected change in the real rate which allows for less drift in

41

the PLM and ALM.

6   Conclusion

In the wake of the Covid-19 Crisis, I investigate the implication of supply shocks when heterogeneous households are subject to an imperfect unemployment insurance market and form their expectations through learning. This paper is built on a HANK model based on the truncated idiosyncratic histories of heterogeneous households. Expectations are explicitly modelled through an adaptive learning system based on the RLS algorithm.

The model shows that negative supply shocks can trigger very long disinflationary episodes characterized by excess precautionary saving and depressed consumption. On one hand, monetary policy focused on inflation tends to increase the difference between the adaptive learning model and its counterpart. On the other hand, Taylor rules with more emphasis on output gap deviation decrease the discrepancy between the rational and the learning model. Those results are not in line with the literature with representative agent models under learning which favours more inflation oriented policies. PLT appears to be a policy that increases speed of convergence, limits the forecast errors generated by the AL and enhances expected consumption anchorage to the RE solution at the disaggregated level.

This insight from the model points toward a disinflationary episode in the medium run following the Covid shock. Commitment to very loose monetary policy and some kind of PLT or average inflation targeting might help to

lessen the aftermath of the shock. Considering the existence of this model, future work more grounded in empirical macroeconomics and expectations data could lead to interesting development in forecasting and business cycle analysis. Deviating from the canonical RLS learning toward more complex form of learning such as sample autocorrelation or restricted perceptions learning could also yield interesting results in such model where the MSV solution and the number of state variables are so large.
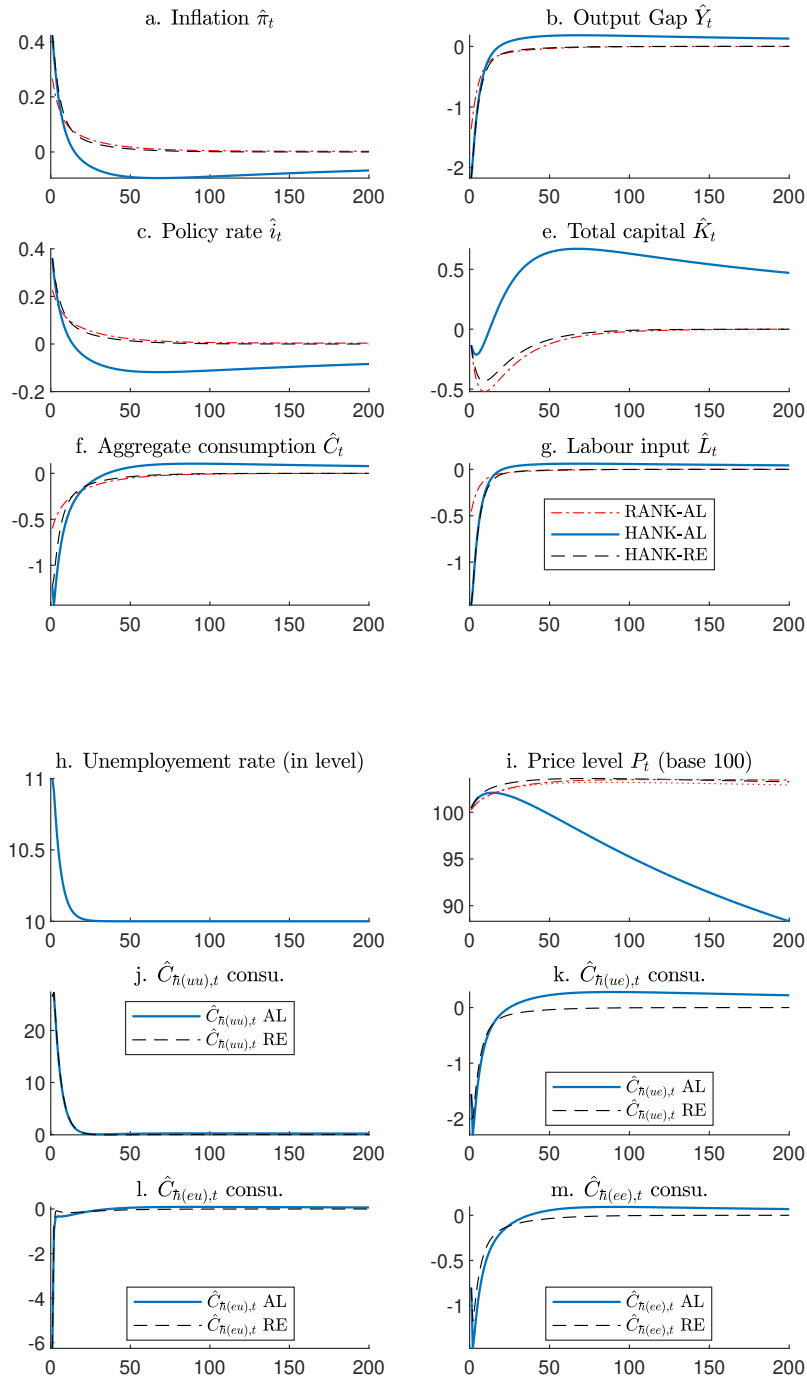
a. Inflation $\hat{\pi}_t$

b. Output Gap $\hat{Y}_t$

c. Policy rate $\hat{i}_t$

e. Total capital $\hat{K}_t$

f. Aggregate consumption $\hat{C}_t$

g. Labour input $\hat{L}_t$

- - - RANK-AL
——— HANK-AL
- - - HANK-RE

h. Unemployement rate (in level)

i. Price level P_t (base 100)

j. $\hat{C}_{\hbar(uu),t}$ consu.

——— $\hat{C}_{\hbar(uu),t}$ AL
- - - $\hat{C}_{\hbar(uu),t}$ RE

k. $\hat{C}_{\hbar(ue),t}$ consu.

——— $\hat{C}_{\hbar(ue),t}$ AL
- - - $\hat{C}_{\hbar(ue),t}$ RE

l. $\hat{C}_{\hbar(eu),t}$ consu.

——— $\hat{C}_{\hbar(eu),t}$ AL
- - - $\hat{C}_{\hbar(eu),t}$ RE

m. $\hat{C}_{\hbar(ee),t}$ consu.

——— $\hat{C}_{\hbar(ee),t}$ AL
- - - $\hat{C}_{\hbar(ee),t}$ RE

<u>Notes</u>: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level.

Figure 4: Deflationary episode in the HANK model under learning (in blue) after a -1% productivity shock
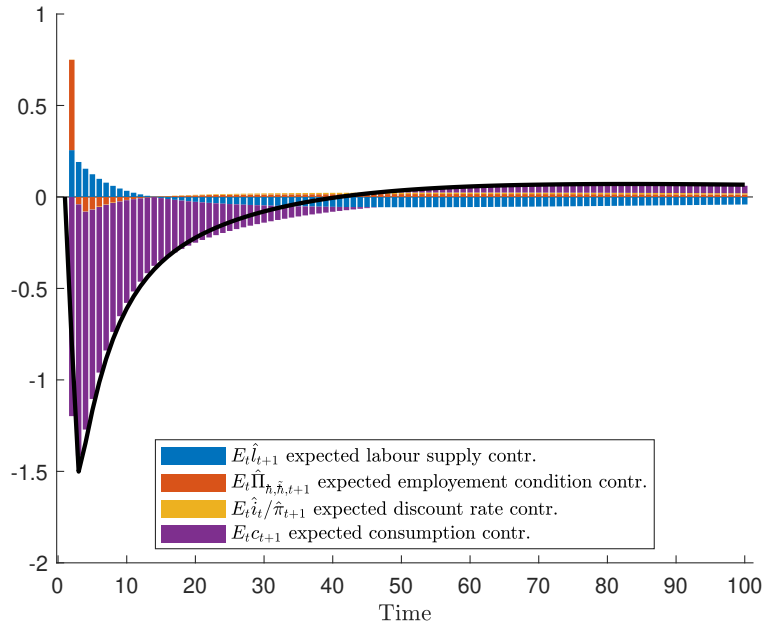
44

(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK after a -1% productivity shock
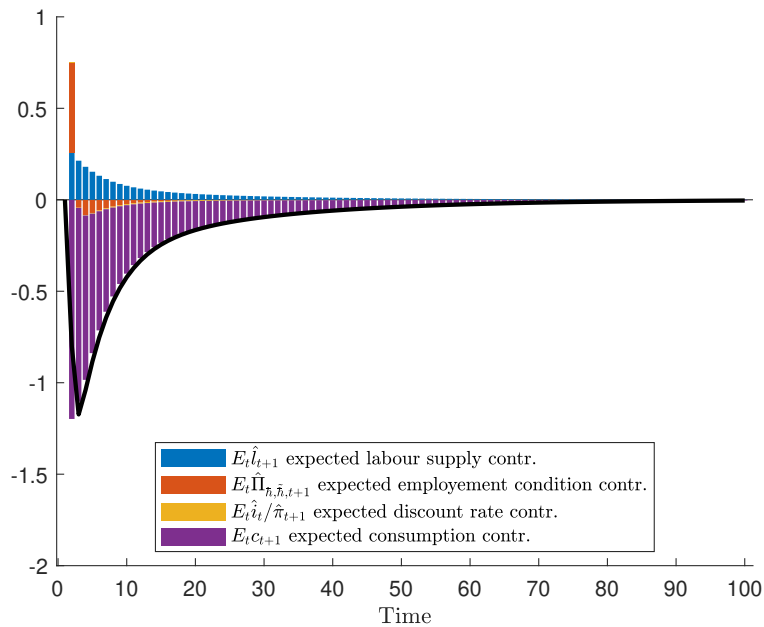
(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK after a -1% productivity shock

<u>Notes</u>: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the history bin with agents employed in the last two periods. At the steady states 82% of the agents are in this bin.

45

Figure 5: Expectation contribution of the employed agents' consumption after a -1% productivity shock
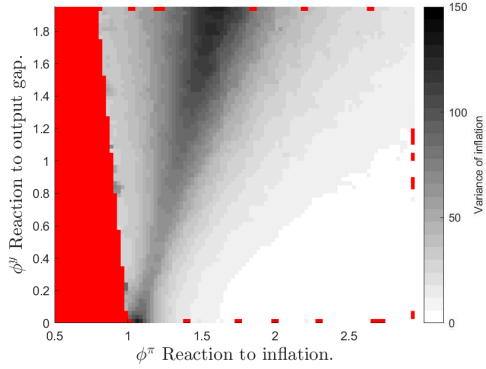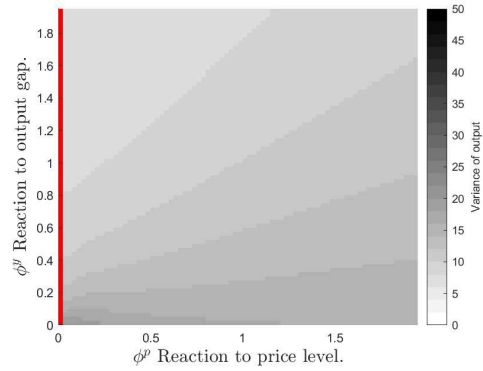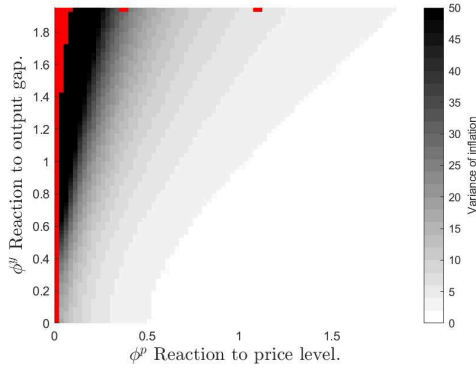
(a) Variance in inflation HANK AL   (b) Variance in output HANK AL
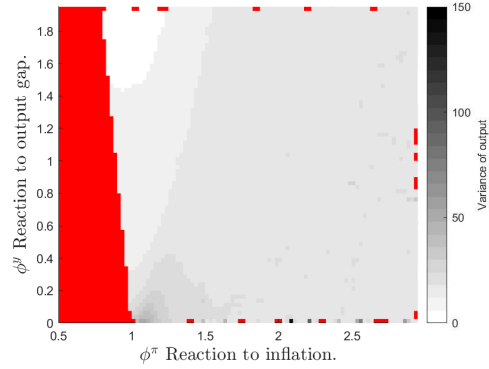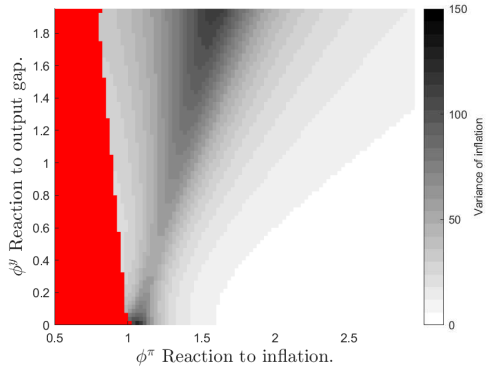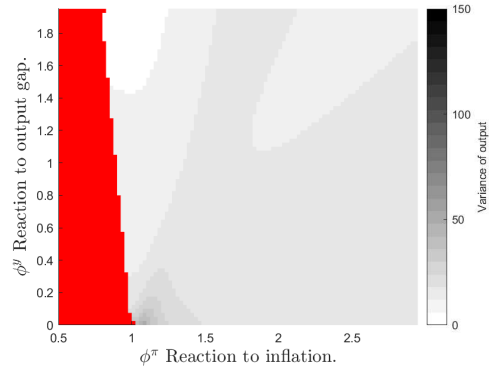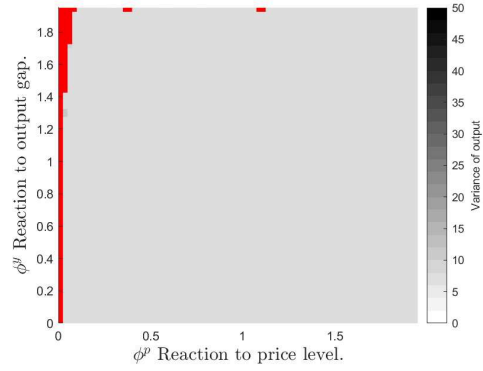
(c) Variance in inflation HANK RE   (d) Variance in output HANK RE
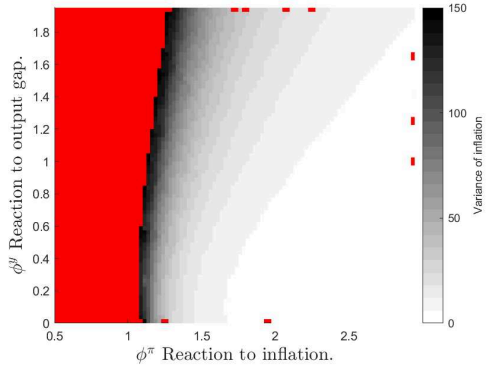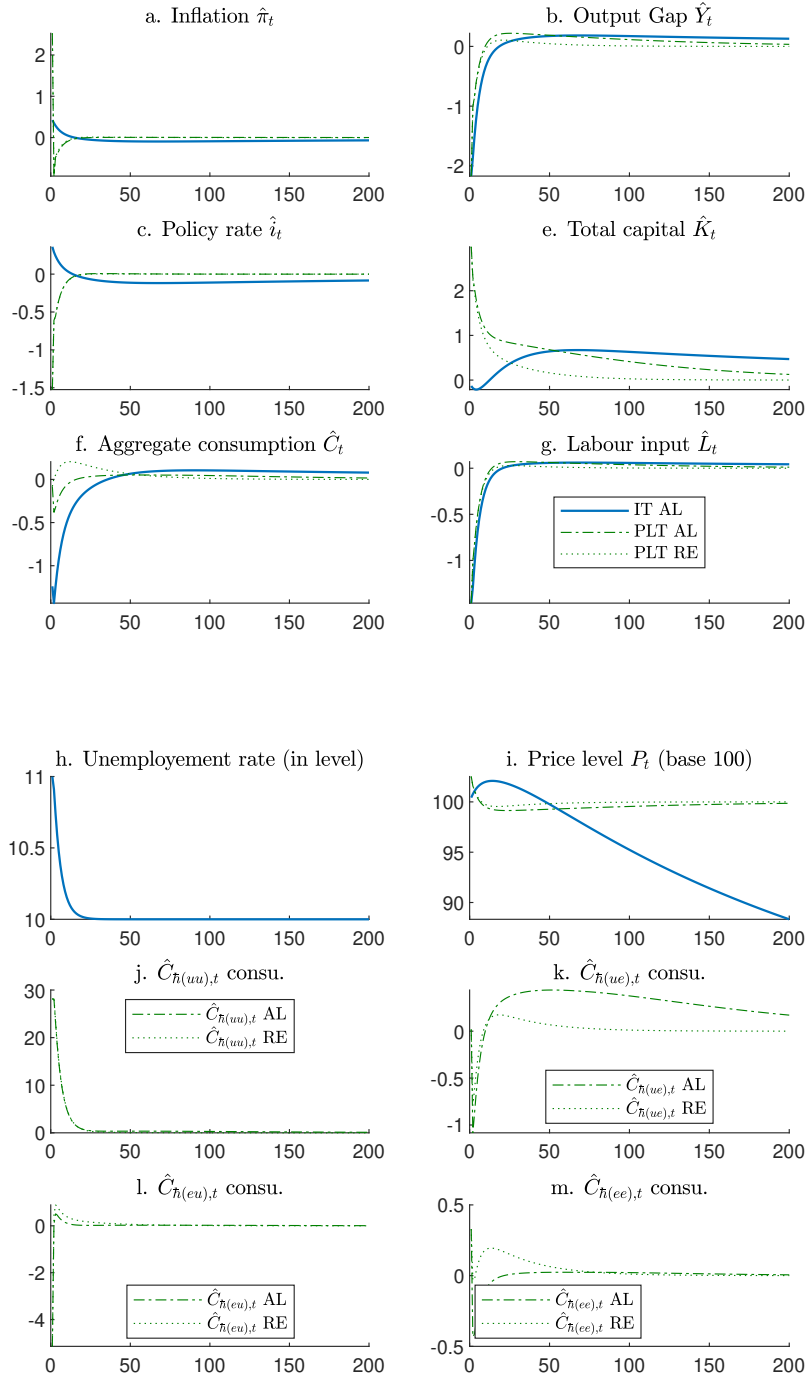
(e) Variance in inflation RANK AL   (f) Variance in output RANK AL
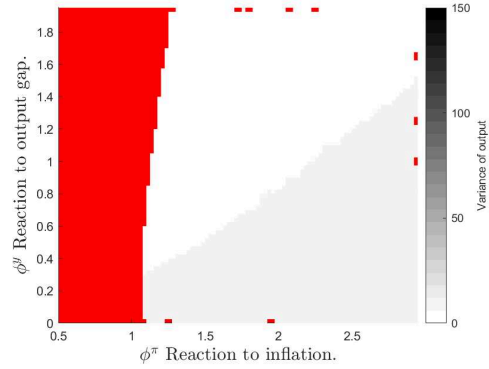
<u>Notes</u>: Every data points of the map is the mean result of 100 Monte-Carlo simulations (800,000 total) over 200 periods. The red zone is where respectively the probability of the time series to crash is over 20% in the adaptive learning models and indeterminate zone in the rational expectations model. In the RANK model, the red zone is also where inflation o' output variances are over 150 (in order to keep the same legend). A crash is defined when the standard deviation of output or inflation is more than 4 times its rational expectations counterpart. For the adaptive learning models, the noise from the map is filtered out by taking for a point $x[i,j]$ the median value between $filter(x[i,j]) = median\{x[i,j], x[i-1,j], x[i-1,j-1], x[i,j-1], x[i+1,j], x[i+1,j+1], x[i,j+1], x[i-1,j+1], x[i+1,j-1]\}$

Figure 6: Policy trade-off over the monetary policy space in the HANK-AL, HANK-RE and RANK-AL models under IT
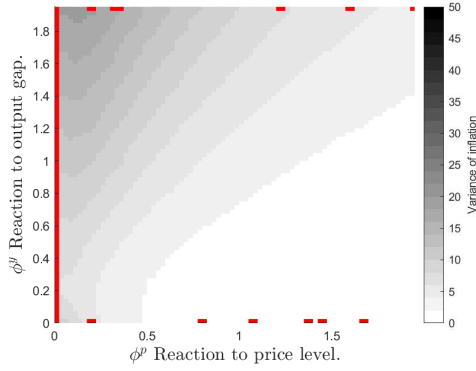
(a) Variance in inflation HANK AL
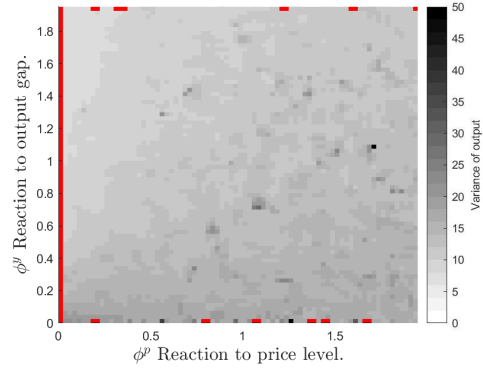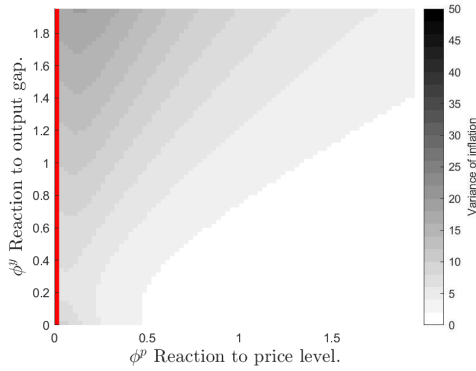
(b) Variance in output HANK AL

(c) Variance in inflation HANK RE

(d) Variance in output HANK RE

(e) Variance in inflation RANK AL

(f) Variance in output RANK AL

<u>Notes</u>: Every data points of the map is the mean result of 100 Monte-Carlo simulations (200,000 total) over 200 periods. The red zone is where respectively the probability of the time series to crash is over 10% in the adaptive learning models and indeterminate zone in the rational expectations model. In the RANK model, the red zone is also where inflation or output variance is over 50 (in order to keep the same legend). A crash is defined when the standard deviation of output or inflation is more than 4 times its rational expectations counterpart. For the adaptive learning models, the noise from the map is filtered out by taking for a point $x[i,j]$ the median value between $filter(x[i,j]) = median\{x[i,j], x[i-1,j], x[i-1,j-1], x[i,j-1], x[i+1,j], x[i+1,j+1], x[i,j+1], x[i-1,j+1], x[i+1,j-1]\}$

Figure 7: Policy trade-off over the monetary policy space in the HANK-AL, HANK-RE and RANK-AL models under PLT

a. Inflation $\hat{\pi}_t$
b. Output Gap $\hat{Y}_t$
c. Policy rate $\hat{i}_t$
e. Total capital $\hat{K}_t$
f. Aggregate consumption $\hat{C}_t$
g. Labour input $\hat{L}_t$

IT AL
PLT AL
PLT RE

h. Unemployement rate (in level)
i. Price level P_t (base 100)
j. $\hat{C}_{\hbar(uu),t}$ consu.
k. $\hat{C}_{\hbar(ue),t}$ consu.

$\hat{C}_{\hbar(uu),t}$ AL
$\hat{C}_{\hbar(uu),t}$ RE

$\hat{C}_{\hbar(ue),t}$ AL
$\hat{C}_{\hbar(ue),t}$ RE

l. $\hat{C}_{\hbar(eu),t}$ consu.
m. $\hat{C}_{\hbar(ee),t}$ consu.

$\hat{C}_{\hbar(eu),t}$ AL
$\hat{C}_{\hbar(eu),t}$ RE

$\hat{C}_{\hbar(ee),t}$ AL
$\hat{C}_{\hbar(ee),t}$ RE

Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level.

Figure 8: Convergence after a -1% productivity shock under PLT (in green)
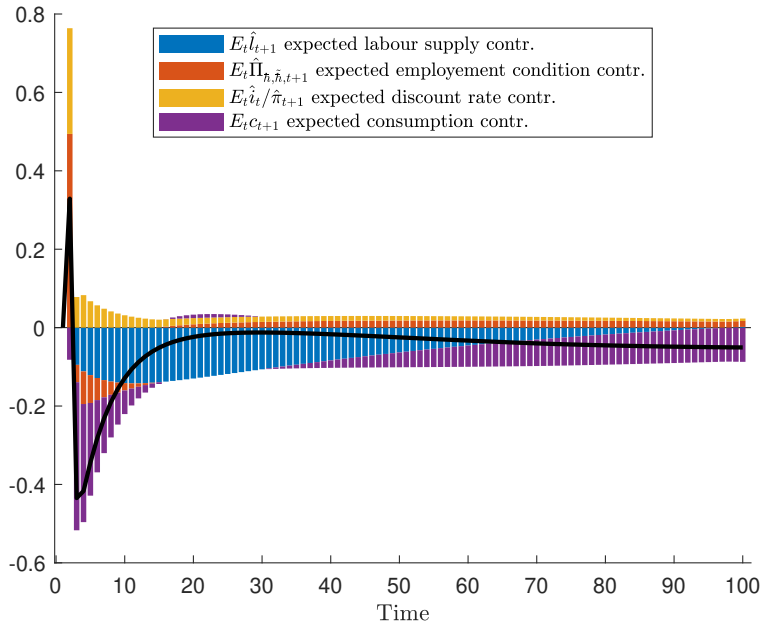
(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK after a -1% productivity shock under PLT
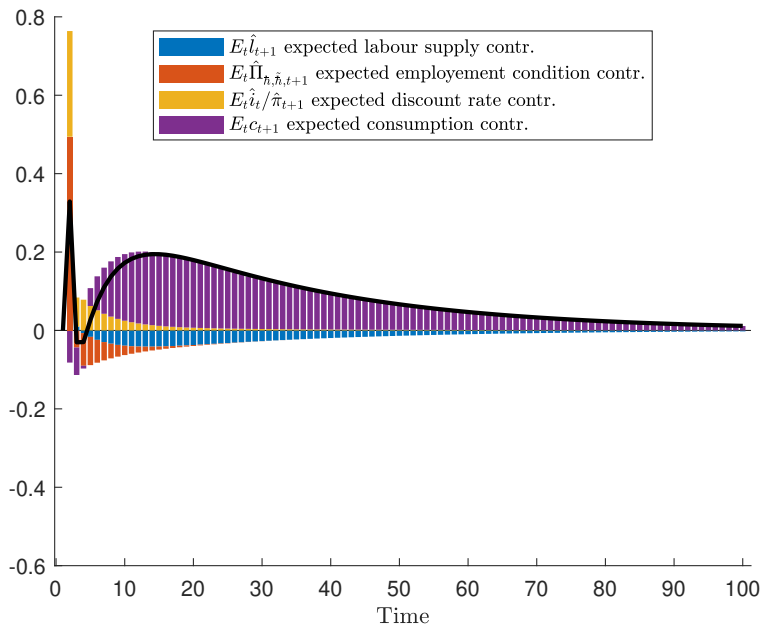
(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK after a -1% productivity shock under PLT

<u>Notes</u>: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the history bin with agents employed in the last two periods.

At the steady states 82% of the agents are in this bin.

Figure 9: Expectation contribution of the employed agents' consumption after a -1% productivity shock under PLT

References

Ahn, S., Kaplan, G., Moll, B., Winberry, T. & Wolf, C. (2018), 'When inequality matters for macro and macro matters for inequality', *NBER macroeconomics annual* **32**(1), 1–75.

Aiyagari, S. R. (1994), 'Uninsured idiosyncratic risk and aggregate saving', *Quarterly Journal of Economics* **109**(3), 659–684.

Arifovic, J., Bullard, J. & Kostyshyna, O. (2013), 'Social learning and monetary policy rules', *The Economic Journal* **123**(567), 38–76.

Ascari, G., Florio, A. & Gobbi, A. (2017), 'Transparency, expectations anchoring and inflation target', *European Economic Review* **91**, 261–273.

Assenza, T., Heemeijer, P., Hommes, C. & Massaro, D. (2019), 'Managing self-organization of expectations through monetary policy: A macro experiment', *Journal of Monetary Economics* .

Berger, D. & Vavra, J. (2015), 'Consumption dynamics during recessions', *Econometrica* **83**(1), 101–154.

Bilbiie, F. O. (2019), 'The new Keynesian cross', *Journal of Monetary Economics* .

Blanchard, O. (2020), 'Is there deflation or inflation in our future?'.
  URL: *https://voxeu.org/article/there-deflation-or-inflation-our-future*

Branch, W. A. (2004*a*), 'Restricted perceptions equilibria and learning in macroeconomics', *Post walrasian macroeconomics: Beyond the dynamic stochastic general equilibrium model* pp. 135–160.

Branch, W. A. (2004*b*), 'The theory of rationally heterogeneous expectations: Evidence from survey data on inflation expectations', *Economic Journal* **114**(497), 592–621.

Branch, W. A. & McGough, B. (2010), 'Dynamic predictor selection in a new Keynesian model with heterogeneous expectations', *Journal of Economic Dynamics and Control* **34**(8), 1492–1508.

Carroll, C. D. (2003), 'Macroeconomic expectations of households and professional forecasters', *Quarterly Journal of economics* **118**(1), 269–298.

Challe, E. (2020), 'Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy', *American Economic Journal: Macroeconomics* **12**(2), 241–83.

Challe, E., Matheron, J., Ragot, X. & Rubio-Ramirez, J. F. (2017), 'Precautionary saving and aggregate demand', *Quantitative Economics* **8**(2), 435–478.

Challe, E. & Ragot, X. (2016), 'Precautionary saving over the business cycle', *Economic Journal* **126**(590), 135–164.

Christoffel, K. & Kuester, K. (2008), 'Resuscitating the wage channel in models with unemployment fluctuations', *Journal of Monetary Economics* **55**(5), 865–887.

Crawley, E. & Kuchler, A. (2018), Consumption heterogeneity: Micro drivers and macro implications, Technical report, Danmarks Nationalbank Working Papers.

Del Negro, M. & Eusepi, S. (2011), 'Fitting observed inflation expectations', *Journal of Economic Dynamics and Control* **35**(12), 2105–2131.

Den Haan, W. J., Rendahl, P. & Riegler, M. (2018), 'Unemployment (fears) and deflationary spirals', *Journal of the European Economic Association* **16**(5), 1281–1349.

Eusepi, S. & Preston, B. (2011), 'Expectations, learning, and business cycle fluctuations', *American Economic Review* **101**(6), 2844–72.

Eusepi, S. & Preston, B. (2018), 'The science of monetary policy: An imperfect knowledge perspective', *Journal of Economic Literature* **56**(1), 3–59.

Evans, G. W. & Honkapohja, S. (2001), *Learning and Expectations in Macroeconomics*, Princeton University Press.

Evans, G. W. & Honkapohja, S. (2003*a*), 'Expectations and the stability problem for optimal monetary policies', *Review of Economic Studies* **70**(4), 807–824.

Evans, G. W. & Honkapohja, S. (2003*b*), 'Policy Interaction, Learning and the Fiscal Theory of Prices', *SSRN Electronic Journal* (0136848), 1–34.

Evans, G. W. & Ramey, G. (1992), 'Expectation calculation and macroeconomic dynamics', *American Economic Review* pp. 207–224.

Galí, J. (2015), *Monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework and its applications*, Princeton University Press.

Giusto, A. (2014), 'Adaptive learning and distributional dynamics in an incomplete markets model', *Journal of Economic Dynamics and Control* **40**, 317–333.

Gobbi, A. & Grazzini, J. (2019), 'A basic New Keynesian DSGE model with dispersed information: An agent-based approach', *Journal of Economic Behavior & Organization* **157**, 101–116.

Goodhart, C. & Pradhan, M. (2020), 'Future imperfect after coronavirus'.
URL: *https://voxeu.org/article/future-imperfect-after-coronavirus*

Grandmont, J. M. (1977), 'Temporary general equilibrium theory', *Econometrica* pp. 535–572.

Greenwood, J., Hercowitz, Z. & Huffman, G. W. (1988), 'Investment, capacity utilization, and the real business cycle', *American Economic Review* pp. 402–417.

Guerrieri, V., Lorenzoni, G., Straub, L. & Werning, I. (2020), Macroeconomic implications of covid-19: Can negative supply shocks cause demand shortages?, Technical report, National Bureau of Economic Research.

Hall, R. E. (1978), 'Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence', *Journal of political economy* **86**(6), 971–987.

Hommes, C. (2011), 'The heterogeneous expectations hypothesis: Some evidence from the lab', *Journal of Economic Dynamics and Control* **35**(1), 1–24.

Hommes, C. H. (2020), 'Behavioral & experimental macroeconomics and policy analysis: a complex systems approach', *Journal of Economic Literature* **Forthcoming**.

Hommes, C., Mavromatis, K. & Ozden, T. (2020), 'Expectations and learning: a horse-race in a medium-scale dsge model'.

Hommes, C. & Zhu, M. (2014), 'Behavioral learning equilibria', *Journal of Economic Theory* **150**, 778–814.

Honkapohja, S. & Mitra, K. (2006), 'Learning stability in economies with heterogeneous agents', *Review of Economic Dynamics* **9**(2), 284–309.

Honkapohja, S. & Mitra, K. (2019), 'Price level targeting with evolving credibility', *Journal of Monetary Economics* .

Hopenhayn, H. A. & Prescott, E. C. (1992), 'Stochastic monotonicity and stationary distributions for dynamic economies', *Econometrica* pp. 1387–1406.

Huggett, M. (1993), 'The risk-free rate in heterogeneous-agent incomplete-insurance economies', *Journal of economic Dynamics and Control* **17**(5-6), 953–969.

Juillard, M. et al. (1996), *Dynare: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm*, Vol. 9602, CEPREMAP Paris.

Kaplan, G., Moll, B. & Violante, G. L. (2018), 'Monetary policy according to HANK', *American Economic Review* **108**(3), 697–743.

Kaplan, G. & Violante, G. L. (2018), 'Microeconomic Heterogeneity and Macroeconomic Shocks', *Journal of Economic Perspectives* **32**(3), 167–94.

Keynes, J. M. (1936), *The general theory of employment, interest and money. London, 1936.*

Krueger, D., Mitman, K. & Perri, F. (2016), On the distribution of the welfare losses of large recessions, Technical report, National Bureau of Economic Research.

Krusell, P. & Smith, Jr, A. A. (1998), 'Income and wealth heterogeneity in the macroeconomy', *Journal of Political Economy* **106**(5), 867–896.

Le Grand, F. & Ragot, X. (2020), 'Managing inequality over business cycles: Optimal policies with heterogeneous agents and aggregate shocks'.

Lucas, R. (1975), 'An equilibrium model of the business cycle', *Journal of Political Economy* pp. 1113–1144.

Malmendier, U. & Nagel, S. (2016), 'Learning from inflation experiences', *Quarterly Journal of Economics* **131**(1), 53–87.

Marcet, A. & Sargent, T. J. (1989), 'Convergence of least-squares learning in environments with hidden state variables and private information', *Journal of Political Economy* **97**(6), 1306–1322.

Massaro, D. (2013), 'Heterogeneous expectations in monetary DSGE models', *Journal of Economic Dynamics and Control* **37**(3), 680–692.

Orphanides, A. & Williams, J. C. (2008), 'Learning, expectations formation, and the pitfalls of optimal control monetary policy', *Journal of Monetary Economics* **55**, S80–S96.

Radke, L. & Wicknig, F. (2020), 'Experience-based heterogeneity in expectations and monetary policy'.

Ragot, X. (2018), Heterogeneous agents in the macroeconomy: reduced-heterogeneity representations, *in* 'Handbook of Computational Economics', Vol. 4, Elsevier, pp. 215–253.

Ravn, M. O. & Sterk, V. (2020), 'Macroeconomic Fluctuations with HANK and SAM: an Analytical Approach', *Journal of the European Economic Association* .

Shimer, R. (2005), 'The cyclical behavior of equilibrium unemployment and vacancies', *American economic review* **95**(1), 25–49.

Slobodyan, S. & Wouters, R. (2012), 'Learning in an estimated medium-scale DSGE model', *Journal of Economic Dynamics and Control* **36**(1), 26–46.

Smets, F. & Wouters, R. (2007), 'Shocks and frictions in US business cycles: A Bayesian DSGE approach', *American Economic Review* **97**, 586–606.

Uhlig, H. (1995), 'A toolkit for analyzing nonlinear dynamic stochastic models easily'.

Williams, J. C. (2010), 'Monetary policy in a low inflation economy with learning', *Economic Review-Federal Reserve Bank of San Francisco* p. 1.

Woodford, M. (2003), *Interest and prices: Foundations of a theory of monetary policy*, Princeton University Press.

Woodford, M. (2013), 'Macroeconomic analysis without the rational expectations hypothesis', *Annual Review of Economics* **5**(1), 303–346.

A The RANK model

In this appendix, the equations describing the RANK model are displayed.

$$U'_t(C_t, L_t) = \beta \mathbb{E}^*_t [\frac{i_t}{\pi_{t+1}} U'_{t+1}(C_{t+1}, L_{t+1})], \tag{42}$$

$$L_t = (\chi W_t)^\varphi, \tag{43}$$

$$C_t + K_t = Y_t + (1 - \delta)K_{t-1} - \frac{\psi}{2}(\pi_t - 1)^2, \tag{44}$$

$$0 = 1 - (1 - mc_t)\epsilon - \psi(\pi_t - 1)\pi_t + \psi\beta\mathbb{E}^*_t(\{\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\}, \tag{45}$$

$$W_t = (1 - \alpha)e^{\varepsilon^p_t}K^\alpha_{t-1}L^{-\alpha}_t, \tag{46}$$

$$Y_{j,t} = e^{\varepsilon^s_t}K^\alpha_{t-1}L^{1-\alpha}_t, \tag{47}$$

$$mc_t = \frac{1}{e^{\varepsilon^s_t}}(\frac{Z_t}{\alpha})^\alpha(\frac{W_t}{1-\alpha})^{1-\alpha}, \tag{48}$$

$$\mathbb{E}^*_t \frac{i_t}{\pi_{t+1}} = \mathbb{E}^*_t Z_{t+1} - \delta - 1, \tag{49}$$

$$i_t - \bar{i} = \phi^\pi(\pi_t - \bar{\pi}) + \phi^y(\frac{Y_t - \overline{Y}}{\overline{Y}}) + \varepsilon^r_t, \tag{50}$$

$$\varepsilon^r_t = \rho^r\varepsilon^r_{t-1} + \vartheta^r_t, \tag{51}$$

$$\varepsilon^p_t = \rho^p\varepsilon^p_{t-1} + \vartheta^p_t. \tag{52}$$

B   Expectation contribution of the RANK consumption after a productivity shock
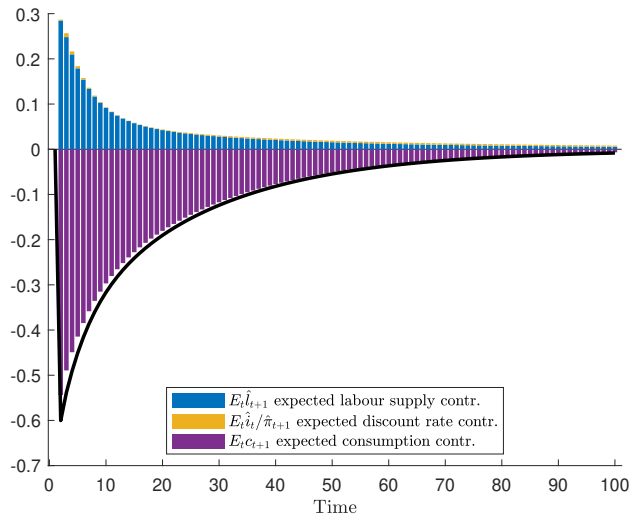
(a) Expectations contribution to $\hat{C}_t$ under AL in the RANK after a -1% productivity shock
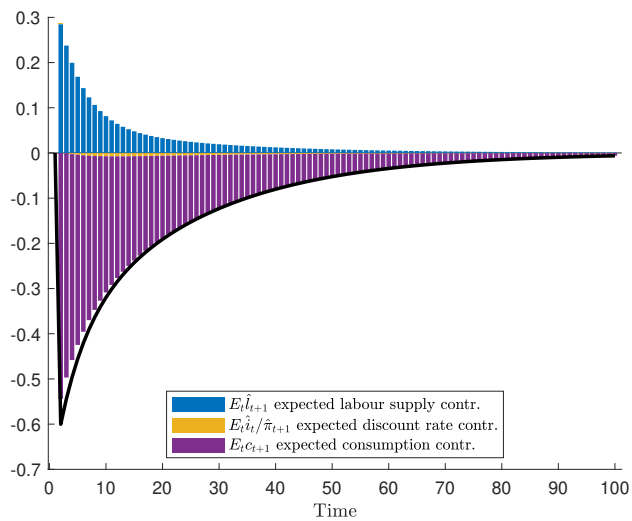
(b) Expectations contribution to $\hat{C}_t$ under RE in the RANK after a -1% productivity shock

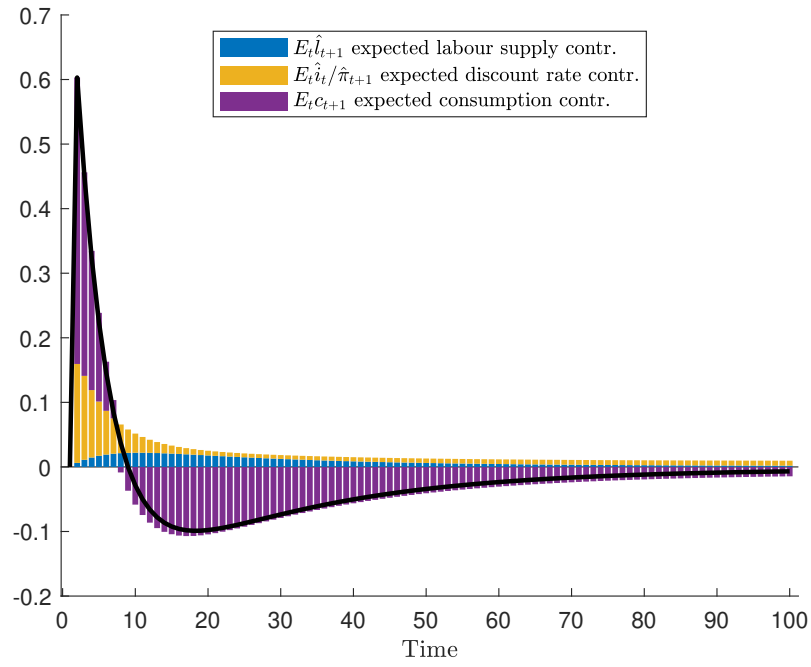Figure 10: Expectation contribution of the representative agent' consumption after a -1% productivity shock
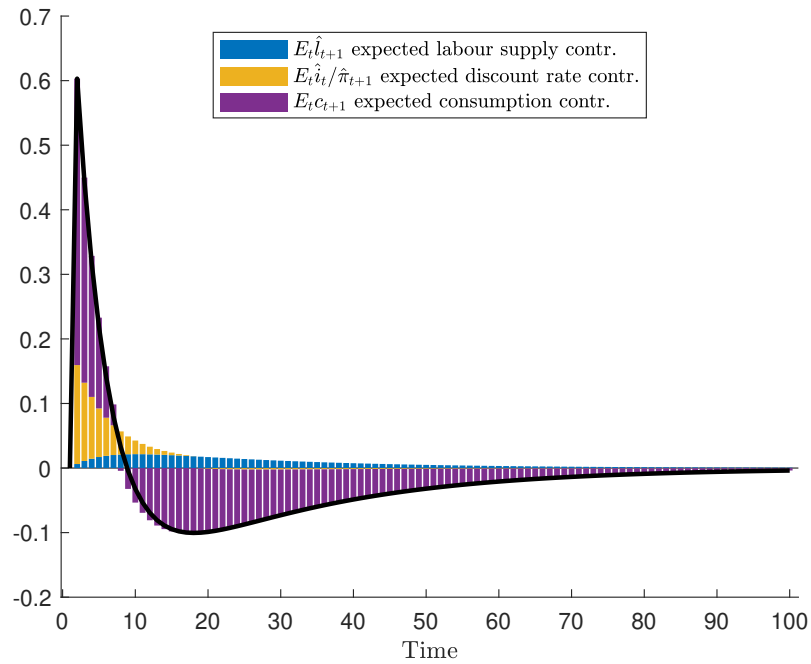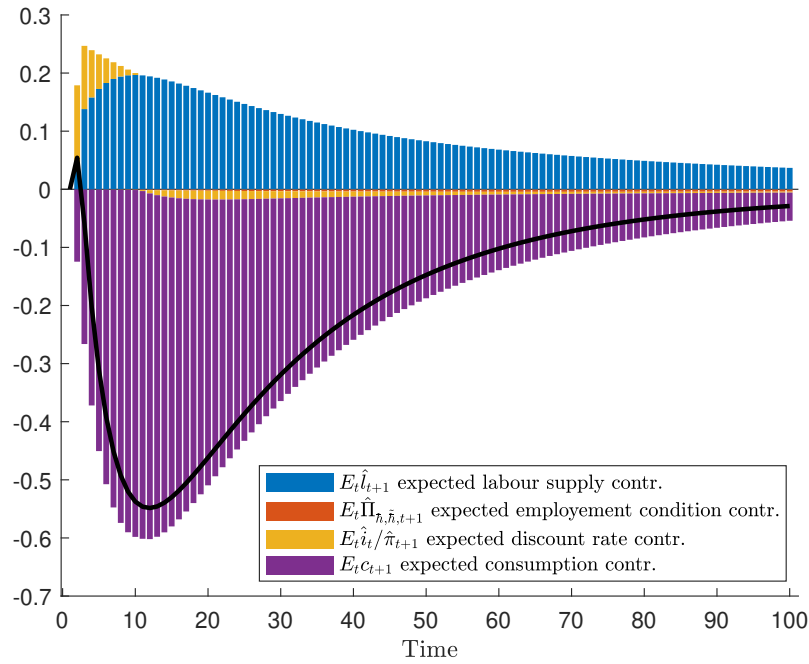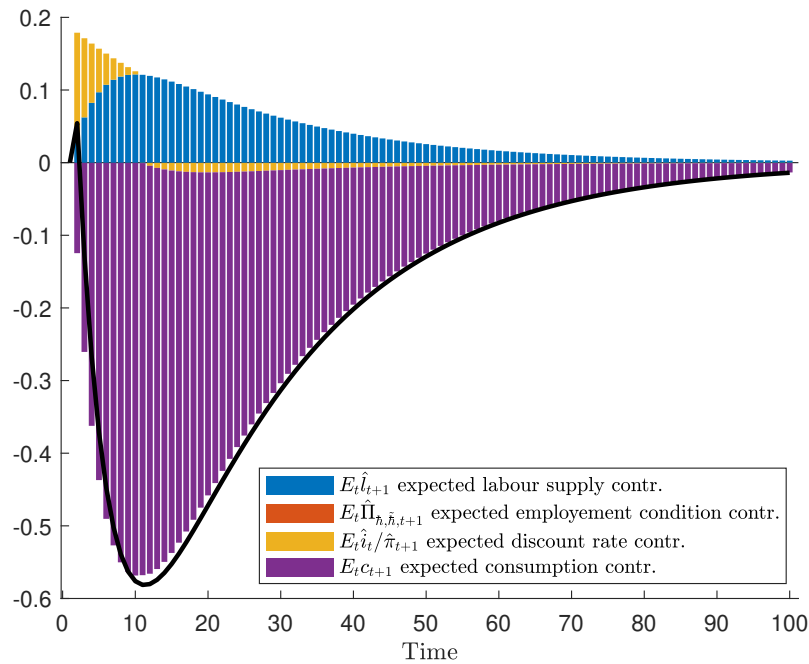
C Nominal rate shock IRFs
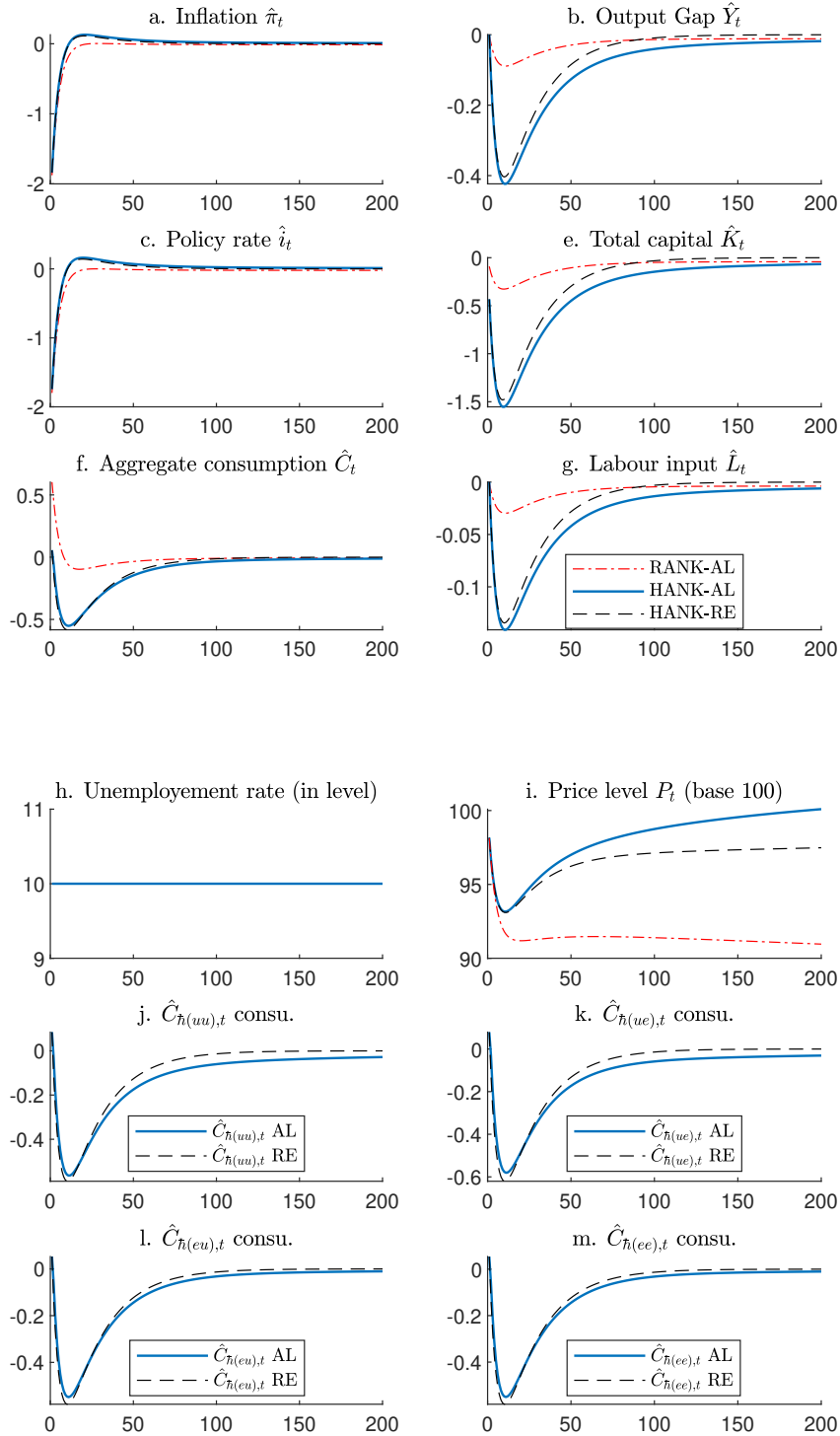
Notes: Results are in percentage point of the log deviation from the steady states. Price level and unemployment rate are in level.

Figure 11: Aggregate response to a +1% nominal rate shock

(a) Expectations contribution to $\hat{C}_t$ under AL in the RANK after +1% nominal rate shock

(b) Expectations contribution to $\hat{C}_t$ under RE in the RANK after +1% nominal rate shock

60

Figure 12: Expectation contribution of the representative agent' consumption after a +1% nominal rate shock

(a) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under AL in the HANK after a +1% nominal rate shock


(b) Expectations contribution to $\hat{c}_{\hbar(e,e),t}$ under RE in the HANK after +1% nominal rate shock

Notes: $\hat{C}_{\hbar(e,e),t}$ is the consumption of $\hbar(e,e)$, the history bin with agents employed in the last two periods. At the steady states 82% of the agents are in this bin.

Figure 13: Expectation contribution of the employed agents' consumption after a +1% nominal rate shock
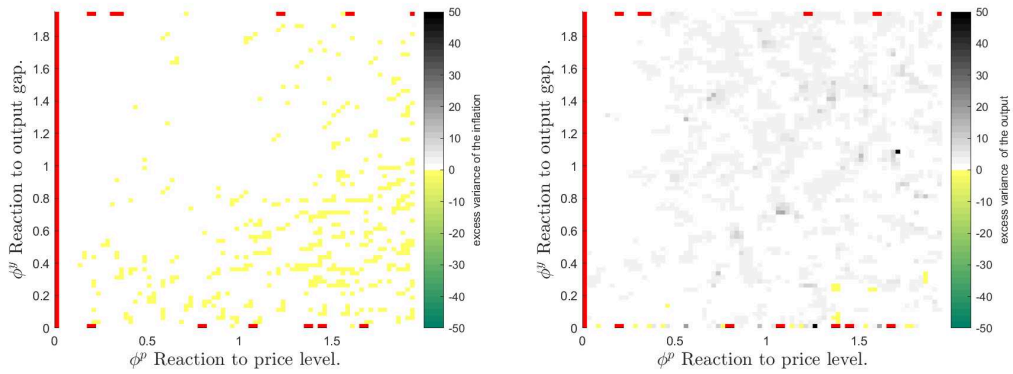
D Excess variances of the HANK AL
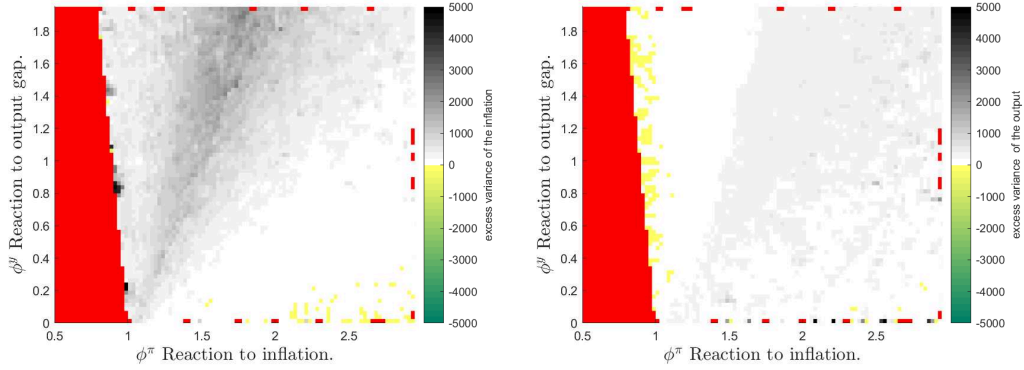
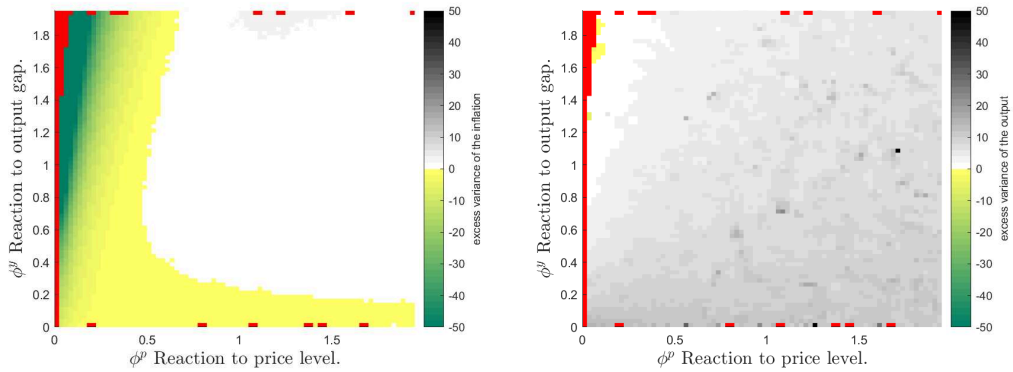(a) Excess variance in inflation w.r.t HANK(b) Excess variance in output w.r.t HANK RE
RE
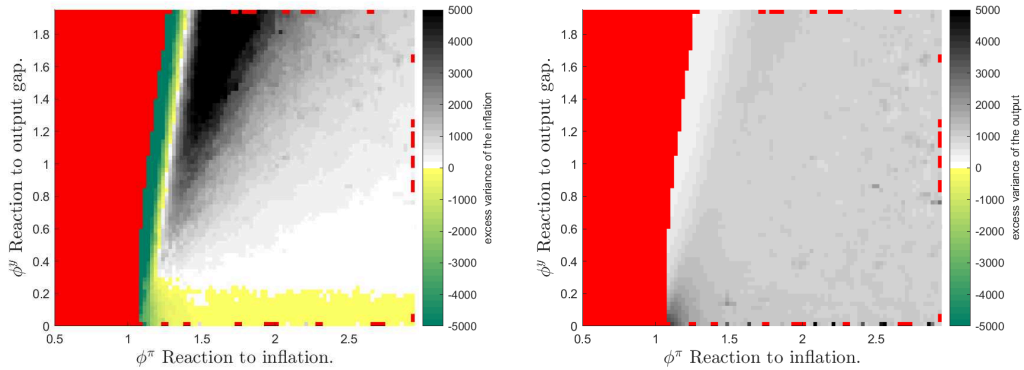
(c) Excess variance in inflation w.r.t RANK AL(d) Excess variance in output w.r.t RANK AL

Notes: The data is the absolute difference between the models and the HANK adaptive learning model. See
Figure 6 for more detail.

Figure 14: Excess output and inflation variances of the HANK-AL model w.r.t
the other models under IT

(a) Excess variance in inflation w.r.t HANK(b) Excess variance in output w.r.t HANK RE
RE

(c) Excess variance in inflation w.r.t RANK AL(d) Excess variance in output w.r.t RANK AL

Notes: The data is the absolute difference between the models and the HANK adaptive learning model. See
Figure 6 for more detail.

Figure 15: Excess output and inflation variances of the HANK-AL model w.r.t
the other models under PLT

63