PREDICTION OF MAXIMUM CRACK WIDTH FORMULA FOR RCC FLEXURAL MEMBER

Lakshmi T N, Jayasree S

Abstract—A formula for the maximum crack width has been developed by incorporating eight governing parameters such as steel stress, grade of concrete, area of steel reinforcement, diameter of bars, % of steel reinforcement, spacing of bars, yield stress of steel reinforcement and concrete cover based on statistical analysis of the author's test results reported in literatures using Statistical Package for Social Science (SPSS) software. An experimental investigation was also carried out on six RC beam models and compared with formula suggested in international codes such as BS 8110-1997/ IS 456-2000, ACI code 318, GBJ 10-89 1989, BS EN 1992-1-1: 2004 and ECP 203-2007. The performance of the proposed formula is checked with experimental results and it shows well correlation

Index Terms—Beam Model, Crack Width, International Codes, Predicted equation, Reinforced Concrete.

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1 INTRODUCTION

CRACKwidth calculation is one of the serviceability requirements in the structural members. The occurrence of cracks in RC structures is unavoidable because of the low tensile strength of concrete. Cracks form when the tensile stress in concrete exceeds its tensile strength [1]. Cracks in a RC member will always be a threat for the satisfactory performance and serviceability of the structure; it has significant influence on serviceability, durability, aesthetic and force transfer. Hence, an accurate estimation of the crack widths and its predictions are essential in structural design.

Limiting crack width is important, from the aesthetic point of view to ensure water tightness and to safe guard the reinforcement against corrosion [2].RC structures designed with low steel stresses under service loads undergo very limited cracking, expect for the cracks that occur due to shrinkage of concrete and temperature changes. In many cases no cracking is visible at all because many members are not subjected to their full service load and the concrete has some tensile strength. To minimize these adverse effects, serviceability limit states (SLS) for RC structures are usually applied to ensure their functionality and structural integrity under service loading condition.Prediction of crack width has been studied by many researchers such as Gergely and Lutz, 1968 [3]; R.J Frosch, 1999 [4]; B.B Broms, 1965 [5]; Kang et al., 1987 [6];N .Ganesan, 1998 [7]. As many variables influence the crack width and spacing of RC flexural members, due to the complexity of the problem, a number of methods have been developed in the past to determine the crack width. These methods are generally based partly on theoretical basis and partly on statistical analysis of test results. Therefore, the results predicted by this formula may vary by the type of specimens and the method of loading and so on.
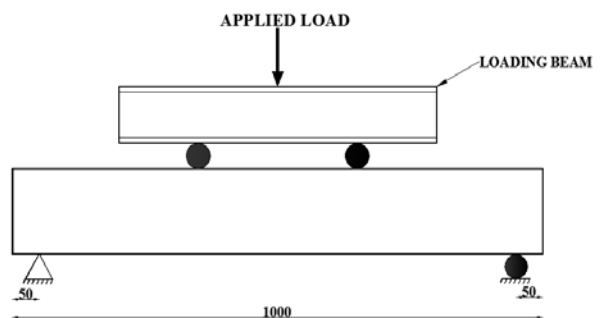
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2 EXPERIMENTAL INVESTIGATION

An experimental investigation was conducted on six RC beam specimens of size 100 x 150 x 1000 mm. The specimen details are shown in Table 1. All the beams were tested under two point loading in a Universal Testing Machine (UTM) of 1000kN capacity. A schematic representation of loading configuration is shown in Fig.1. The maximum crack width were measured at loads 10, 20, 30 and 40 kN using crack detection microscope of 50x magnification.

TABLE 1
SPECIMEN DETAILS

Beam Model	Parameters		
	f_{ck}	f_y	n & φ
1	M20	Fe415	2#8
2	M20	Fe415	3#8
3	M25	Fe415	2#8
4	M25	Fe415	3#8
5	M25	Fe500	2#8
6	M20	Fe500	2#8

Fig. 1. Loading configuration

2.1 Comparison of Crack Width with International Codes

The observed crack widths for all the beam models were compared with the formulas recommended in the British standard BS 8110 – 1997 [8], American standard ACI 318 – 1995, 05, 08 & 11 [9],[10],[11], Chinese code GBJ 10 – 1989 [12], European standard BS EN 1992 – 1 – 1: 2004 [13] and Egyptian standard ECP 203 – 2007 [14]. Its graphical representation is shown in Fig. 2 to 6. Where w_{theo} represents calculated crack width using International codes. It was observed that the values of crack width as predicted by the ACI code and Euro code correlated well with the experimental values.
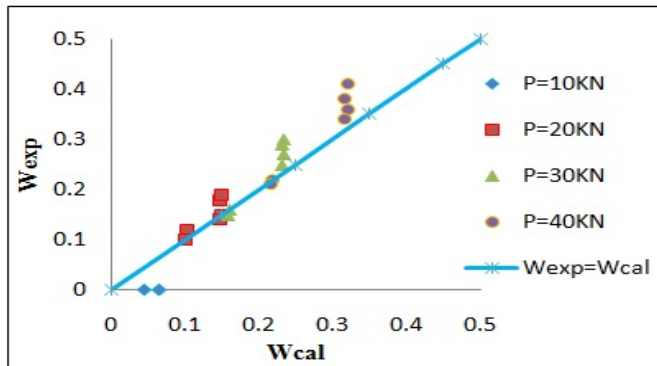

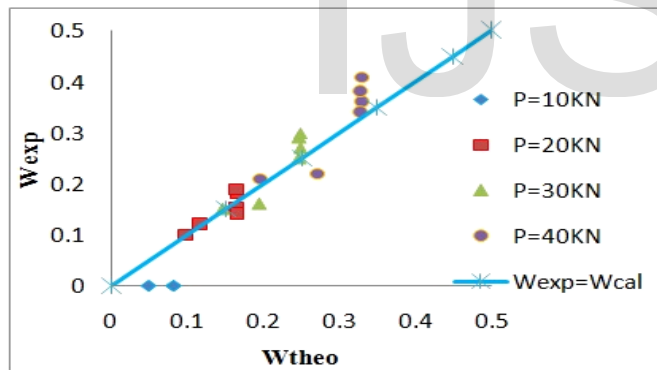Fig. 2. Comparison of Experimental crack width with BS 8110 code equation

Fig. 3. Comparison of Experimental crack width with ACI 318 code equation
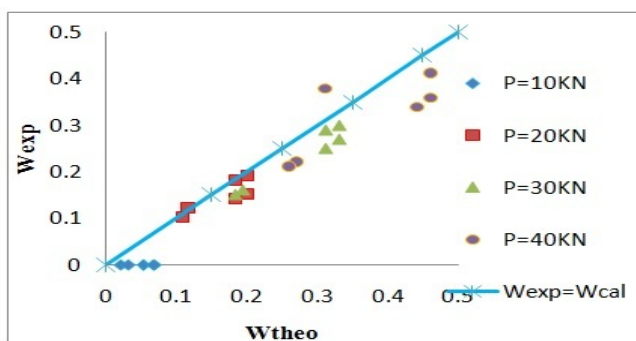

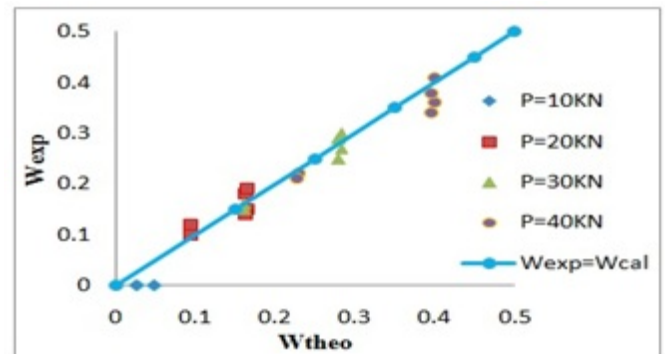Fig. 4. Comparison of Experimental crack width with Chinese code equation

Fig. 5. Comparison of Experimental crack width with Euro code equation
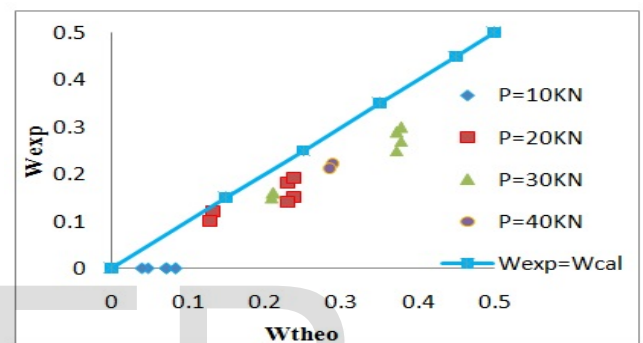

Fig. 6. Comparison of Experimental crack width with Egyptian code equation

3 PREDICTION OF MAXIMUM CRACK WIDTH EQUATION

A new formula for maximum crack width is predicted by incorporating the effect of steel stress, grade of concrete, area of steel reinforcement, diameter of bars, % of steel reinforcement, spacing of bars, yield stress of steel reinforcement and concrete cover based on statistical analysis of the author's test results reported in literatures using SPSS software. The relevant data used for regression analysis are 130 beam specimen results of various authors such as Hognestad, Kaar-Mattock, C&CA, Nawy and Syed [15].

Non linear regression is the method of finding a nonlinear model of arbitrary relationship between dependent variable and a set of independent variables; it is accomplished by using iterative estimation algorithms. Here dependent variable is crack width (wcal) and fourteen independent variables are steel stress (fs), modulus of elasticity of steel (Es), number and diameter of reinforcing bar (n & φ), yield stress of steel reinforcement (fy), grade of concrete (fck), spacing of bar (s), concrete cover (c), beam section (b, d & d') and percentage of steel reinforcement (μ). Parameter estimates and residual sum of squares are obtained for all iteration.

To predict the maximum crack width equation, the eight governing parameters given in heading 3 were incorporated in the statistical regression analysis along with three regression coefficients C1, C2 and C3 [16]. The regression equation takes the form,

$$w_{cal} = \frac{f_s}{E_s}\left[C1\,(\Phi/\mu) + C2\left(\frac{Sf_y}{nf_{ck}}\right) + C3\,c\right] \qquad (1)$$

The values of C1, C2 and C3 were determined from the statistical analysis and the solution for the regression coefficients led to the following equation.

$$w_{cal} = \frac{f_s}{E_s}\left[0.03\,(\Phi/\mu) + 0.021\left(\frac{Sf_y}{nf_{ck}}\right) + 1.4c\right] \qquad (2)$$

Where,

wcal	=	Maximum crack width in mm;
Φ	=	Diameter of bars in mm;
μ	=	Percentage steel reinforcement;
s	=	Spacing of bars in mm;
f_y	=	Yield stress in N/mm2;
n	=	Number of bars;
f_{ck}	=	Grade of concrete in N/mm2;
c	=	Concrete cover in mm.
f_s	=	Steel stress in N/mm2;
	=	$m\dfrac{M}{I_{cr}}(d-x)$ [7]
I_{cr}	=	Moment of inertia of cracked section;
	=	$\dfrac{bx^3}{3} + mA_{st}(d-x)^2$
m	=	modular ratio = $\dfrac{E_s}{E_c}$
Es	=	Modulus of elasticity for steel;
Ec	=	Modulus of elasticity for concrete;
M	=	Bending moment in Nmm;
d	=	Effective depth in mm;
x	=	Depth of neutral axis of cracked section;
Ast	=	Area of steel reinforcement in mm2;
b	=	Width of beam in mm.

SPSS provides a set of maximum crack width values corresponding to the equation (2) by checking 130 beam specimen parameters.

3.1 Modification of Predicted Equation

The observed crack widths were compared with equation (2) and its graphical representation is shown in Fig.7. It was observed that the maximum crack width value obtained from the predicted equation for all the beam models underestimates the experimental results.
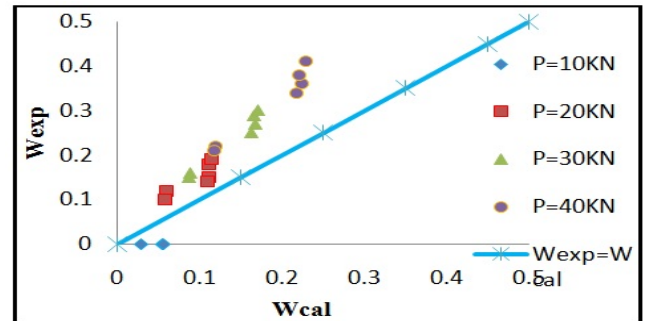
Fig. 7. Comparison of Experimental crack width with Predicted equation

Hence a correction factor is required for the equation (2) to correlate well with the experimental values. The correction factor is obtained by plotting a graph with the ratios of experimental and predicted crack width along y axis and ratios of steel stress (fs) and grade of concrete (fck) along x axis for all beam models. A best fit line was drawn in such a way that there is an equal distribution of points on either side. Slope of this line, gives the correction factor and is shown in Fig. 8.
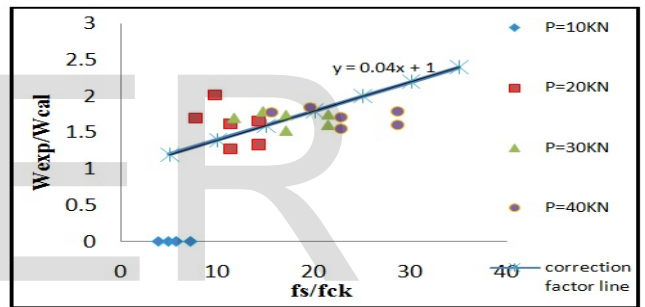
Fig. 8. Correction Factor Graph

The equation for the best fit line is,

$$y = 0.04x + 1 \qquad (3)$$

Hence the correction factor is taken as 0.04.

The predicted equation is modified to the proposed equation by substituting the values of x and y in equation (3).
Where,

$$x = \frac{f_s}{f_{ck}}; \qquad y = \frac{w_{exp}}{w_{cal}}$$

$$\frac{w_{exp}}{w_{cal}} = \left[0.04\left(\frac{f_s}{f_{ck}}\right) + 1\right] \qquad (4)$$

$$w_{exp} = \left[0.04\left(\frac{f_s}{f_{ck}}\right) + 1\right]w_{cal} \qquad (5)$$

Equation (5) can be written as,

$$w_{pro} = \psi w_{cal} \qquad (6)$$

This is the proposed Equation.

Where,

'ψ'	=	$\left[0.04\left(\dfrac{f_s}{f_{ck}}\right) + 1\right]$
w_{pro}	=	Proposed crack width in mm.

4 COMPARISION OF PROPOSED EQUATION WITH EXPERIMENTAL RESULTS

To check the relative performance of the proposed equation, crack widths obtained from equation (6) was again compared with experimental results and its graphical representation is shown in Fig. 9. It was observed that a good correlation is existing between the proposed and experimental values for all the beam models, since almost all data points lies within the limits and the scattering is acceptably small.

The mean values of the crack width ratios and the corresponding standard deviation, coefficient of variation for all beam models are shown in Table 2. It was observed that a good correlation exist between the proposed and experimental values for all the beam models. The proposed maximum crack width equation overestimate the crack width of beam models 1, 3 and 6 by 22, 20 and 4% respectively and shows a excellent correlation with the beam model 5.
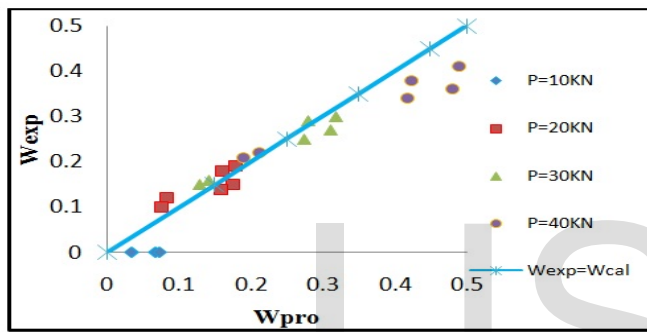

Fig. 9. Comparision of Proposed Equation with the Experimental crack width

TABLE 2
COMPARISON OF PROPOSED EQUATION WITH BEAM MODELS

Beam Model No.	Load (kN)	wexp	wpro	wpro / wexp
1	17.5	0.12	0.15	1.22
	20	0.15	0.18	1.17
	30	0.27	0.31	1.15
	40	0.36	0.48	1.33
	Average			1.22
	Standard Deviation			0.08
	Coefficient of Variation			0.0065
2	19	0.1	0.08	0.78
	20	0.12	0.08	0.69
	30	0.16	0.14	0.89
	40	0.22	0.21	0.96
	Average			0.83
	Standard Deviation			0.12
	Coefficient of Variation			0.0145
3	16	0.09	0.12	1.32
	20	0.14	0.16	1.14
	30	0.25	0.28	1.10
	40	0.34	0.42	1.23
	Average			1.20
	Standard Deviation			0.10
	Coefficient of Variation			0.0099

Beam Model No.	Load (kN)	wexp	wpro	wpro / wexp
4	16	0.08	0.06	0.70
	20	0.1	0.08	0.77
	30	0.15	0.13	0.87
	40	0.21	0.19	0.91
	Average			0.81
	Standard Deviation			0.09
	Coefficient of Variation			0.0088
5	13	0.09	0.09	1.04
	20	0.18	0.16	0.90
	30	0.29	0.27	0.96
	40	0.38	0.42	1.12
	Average			1
	Standard Deviation			0.09
	Coefficient of Variation			0.0088
6	17	0.15	0.14	0.96
	20	0.19	0.18	0.94
	30	0.3	0.32	1.06
	40	0.41	0.49	1.19
	Average			1.04
	Standard Deviation			0.12
	Coefficient of Variation			0.0152

4 CONCLUSION

A new formula was proposed for the determination of maximum crack width in RC flexural members incorporating the effect of steel stress, grade of concrete, area of steel reinforcement, diameter of bars, percentage of steel reinforcement, spacing of bars, yield stress of steel reinforcement and concrete cover, which is not available in literatures and international code formulas.

Compared with the experimental results, the proposed equation indicates well correlation. For better accuracy of the proposed equation, more and more number of test results and experimental results were required to obtain a statistically best fit equation for predicting the maximum crack width in RC flexural member, since cracking is highly random phenomena.

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