SAHETI SCHOOL

PRELIMINARY EXAMINATION 2017

MATHEMATICS GRADE 12

PAPER 1

Time: 3 hours Total: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 7 pages, graph paper, and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1:

a) Simplify $\left[\frac{(3x)^{-3}}{27x^{-3}}\right]^{\frac{-2}{3}}$ (3)

b) Solve for *x* and show all your working:

1) $2x^2 = x$ (2)

2) $3x^2 \geq 2 - 5x$ (4)

3) $\sqrt{20 - 2x} + x - 6 = 0$ (5)

4) $7^x - 8 + 7^{1-x} = 0$ (6)

[20]

QUESTION 2:

a) Study the pattern below:

Row 1: $4^2 - 3^2 + 2^2 - 1^2 = 10$

Row 2: $5^2 - 4^2 + 3^2 - 2^2 = 14$

Row 3: $6^2 - 5^2 + 4^2 - 3^2 = 18$

Row 23: (………………………...)

Row *n*: $a^2 - b^2 + c^2 - d^2 = T_n$

1) Complete pattern for Row 23 (2)

2) Determine *a, b, c, d* and T_n in terms of *n.* Simplify T_n as far as possible.
(6)

b) A certain quadratic pattern has the following characteristics:

$T_1 = p$; $T_2 = 18$; $T_4 = 4T_1$; $T_3 - T_2 = 10$
Determine the value of *p.* (6)

[14]

QUESTION 3:

a) Determine $\lim_{x \to 0} \frac{f(x+h)-f(x)}{h}$ (2)

b) State if the following is true or false:

1) The derivative of x^{π} is $\pi . x^{\pi}$ (2)

2) The derivative of $f(x) + g(x)$ is $f'(x) + g'(x)$ (2)

3) The derivative of $\frac{ax^n}{bx^n} = \frac{a}{b}$ (2)

4) If $\frac{df}{dx} = x^2$ and $\frac{dg}{dx} = x^2$ then $f(x) = g(x)$ (2)

c) Determine $f'(x)$ from first principles if $f(x) = -x^2 + \beta x$ (5)

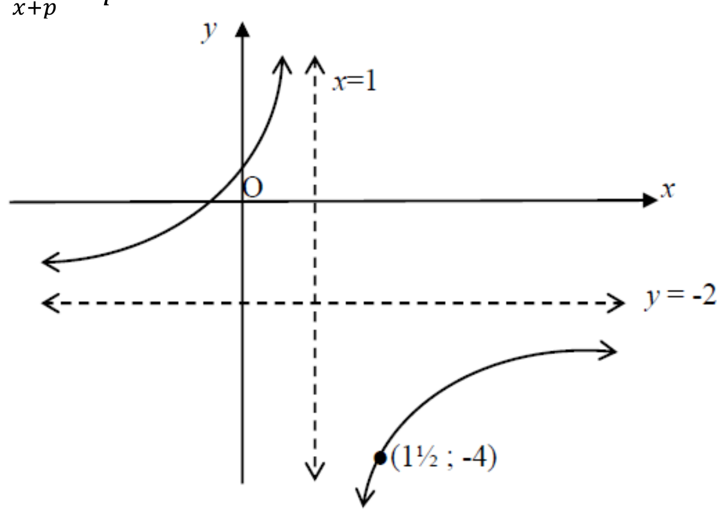
d) Determine $\frac{dy}{dx}$ of $y = \sqrt[n]{x^2} + \frac{3}{-2x}$ (4)

e) For which values of *a* is the graph of $y = x^3 + ax^2 + 9x$ concave up when $x = 3$? (4)

[23]

QUESTION 4:

Given $g(x) = \frac{a}{x+p} - q$.



a) Calculate the values of a, *p* and *q*. (4)

b) Write down the domain and range of $g(x)$. (2)

c) If $m(x) = g(-x)$, write down the equation $m(x)$. (3)

[9]

QUESTION 5:

a) Given $P(A) = 0{,}6$; $P(B) = 0{,}3$ and $P(A \text{ or } B) = 0{,}8$
Are the events A and B mutually exclusive? Justify your answer with appropriate calculations. (2)

b) Twenty–four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. All of the dogs in the kennel have at least one of the mentioned characteristics. Draw a Venn diagram **and** determine how many dogs are black with long hair but do not have short tails. (7)

[9]

SECTION B

QUESTION 6:

a) How many different arrangements of 20 lights can be made from a selection of 8 red, 6 yellow, 4 green and 2 blue lights? (4)

b) If each arrangement is equally likely, find the probability that an arrangement has the red colour bulb at each end. (4)

[8]

QUESTION 7:

a) Sunday took out a loan of R350 000 to purchase a plot. The financial institution offered him the full amount at 9% per annum-compounded semi-annually. Sunday will pay R21 000 every 6 months until he pays off the loan.

1) Calculate the balance outstanding of the loan after 3 years (4)

2) Calculate the total amount paid towards interest charges over 3-year period. (3)

b) Steph wins R3 200 000 on the lottery. She decides to invest the money, give up working and travel. She draws R35 000 per month from her winnings, starting in three months' time. The interest rate on the investment is 9½ % per annum, compounded monthly.
Calculate for how many years Steph's investment will support her. (5)

[12]

QUESTION 8:

a) In the arithmetic series, $k + 2k + 3k + \cdots .. + 200$; where *k* is a positive integer and *k* is a factor of 200

1) find, in terms of *k*, an expression for the number of terms in this series. (2)

2) find, in terms of *k*, an expression for the sum of this series. (3)

b) A geometric series has a first term of 5 and common ratio $\frac{4}{5}$. Calculate

1) the 20th term of the series, to 2 decimal places (2)

2) the sum to infinity of the series. (2)

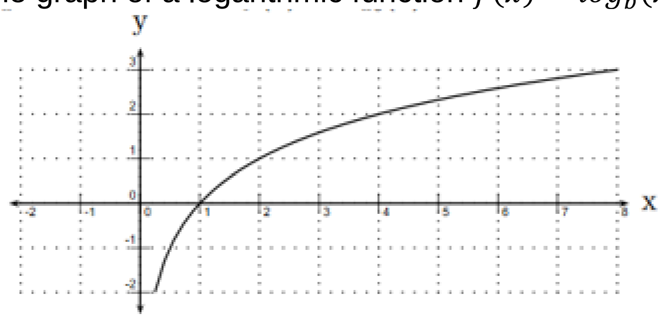
3) Given that the sum to *k* terms of the series is greater than 24.95, show that $k > \frac{log0.002}{log0.8}$ (4)

4) find the smallest possible value of *k* (1)

[14]

<u>QUESTION 9:</u>

a) The graph of a logarithmic function $f(x) = log_b(x)$ is shown below:



1) Calculate the value of *b.* (2)

2) Sketch the graph of $f^{-\prime}(x-2)$ (4)

3) Write down the values of *x* where $f'(x) \geq 0$ (2)

b) Given that $f(x) = (x+3)(2-x)$.

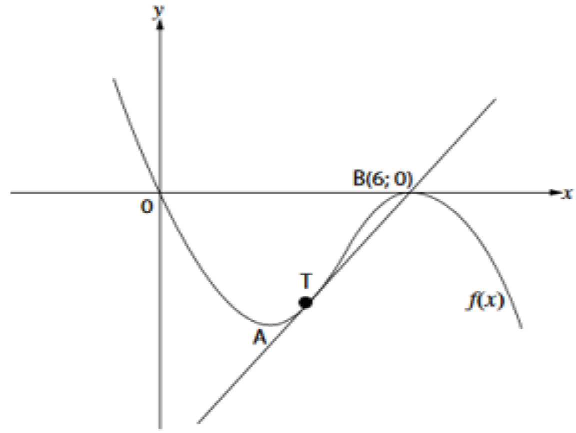
1) Sketch the graph of $f(x)$ showing all intercepts and turning point (4)

2) Write down the turning points of $g(x) = f\left(\frac{x}{2}\right) - 1$ (2)

[14]

QUESTION 10:

Given $f(x) = -x^3 + bx^2 - cx + d$. A and $B(6;0)$ are turning points of $f(x)$



Find:

a) the values of *b, c* and *d* (4)

b) the coordinates of A (3)

c) T has an *x*-coordinate of 3, find the equation of the tangent to the curve at T (5)

[12]

QUESTION 11:

a) The rabbit population on a small island is observed to be given by the function: $P(t) = 45t - 0.4t^3 + 500$, where *t* is the time in months since observations of the island began.

1) When is the maximum population attained, and what is that maximum population? (4)

2) When does the rabbit population disappear from the island? (3)

3) Sketch $P(t)$ (4)

b) When a person coughs, the trachea (windpipe) contracts causing air to flow faster through it. According to a mathematical model of coughing, the speed (*v*) of the airstream through the trachea is related to its radius (*r*) by the equation: $v = k(n-r)r^2$ provided that $\frac{n}{2} \leq r \leq n$. In the equation *k* is a constant and *n* is the normal radius. Determine to what fraction of its normal radius the trachea contracts when *v* is a maximum. (5)

[15]